Nuclear Forces and Currents in Chiral Effective Field Theory

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Fundamental Physics with Electroweak Probes of Light Nuclei
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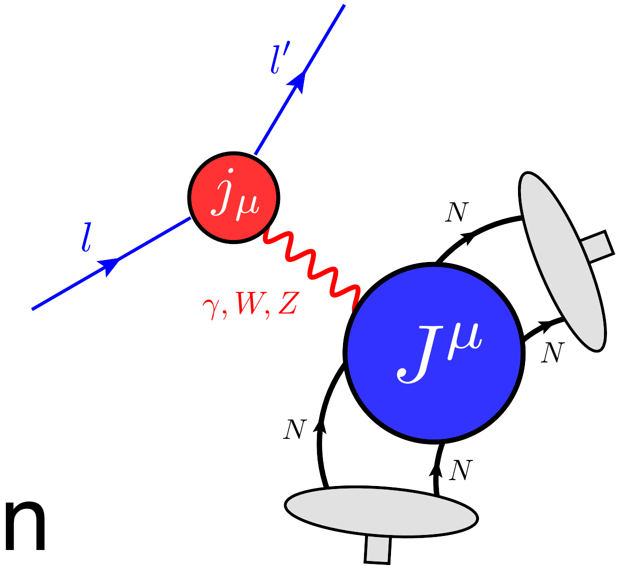
Outline

- Nuclear forces in chiral EFT
  - Introduction to chiral EFT
  - Nuclear forces up to N^{4}LO
    - Redundant short-range parameter
    - Uncertainty quantification

- Nuclear current in chiral EFT
  - Symmetries for currents
  - Nuclear currents up to N^{3}LO
  - Symmetry preserving regularization



ChPT and low energy QCD

Spontaneous + explicit (by small quark masses) breaking of chiral symmetry in QCD

$\Downarrow$

Existence of light weakly interacting Goldstone bosons

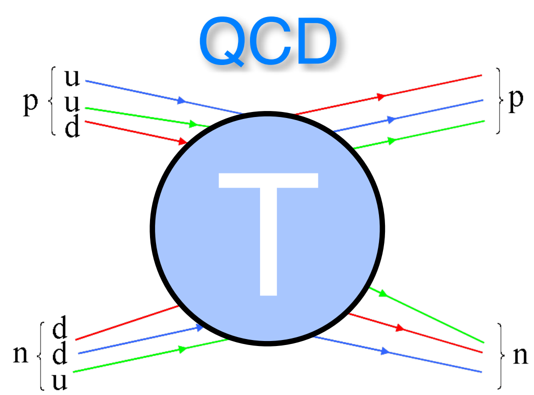
$\Downarrow$

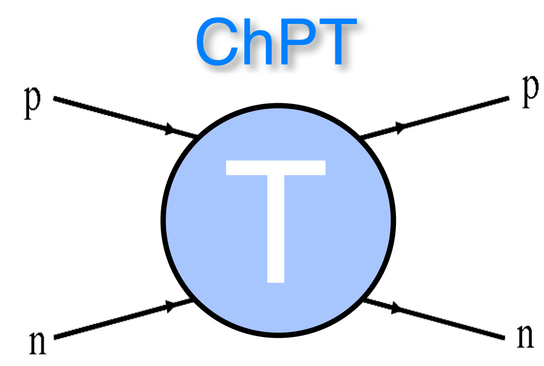
Chiral Perturbation theory (ChPT)
Expansion in small momenta and masses of Goldstone bosons

$\Downarrow$

Systematic description of QCD by ChPT in low energy sector
(low momenta and masses $q, M_\pi \ll \Lambda \simeq 1\,\mathrm{GeV}$)

From QCD to nuclear physics





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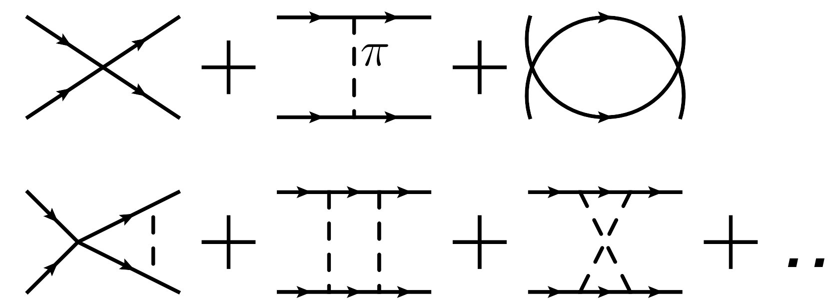
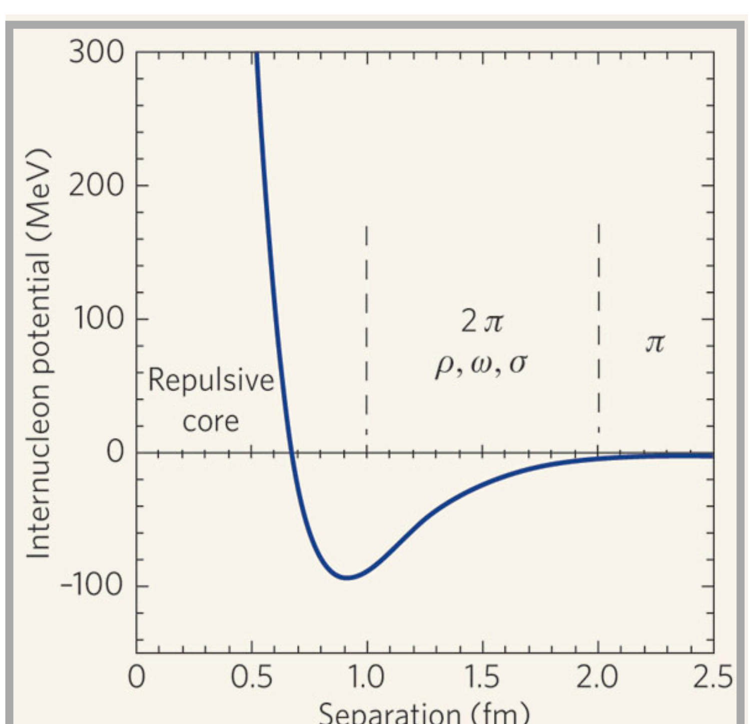
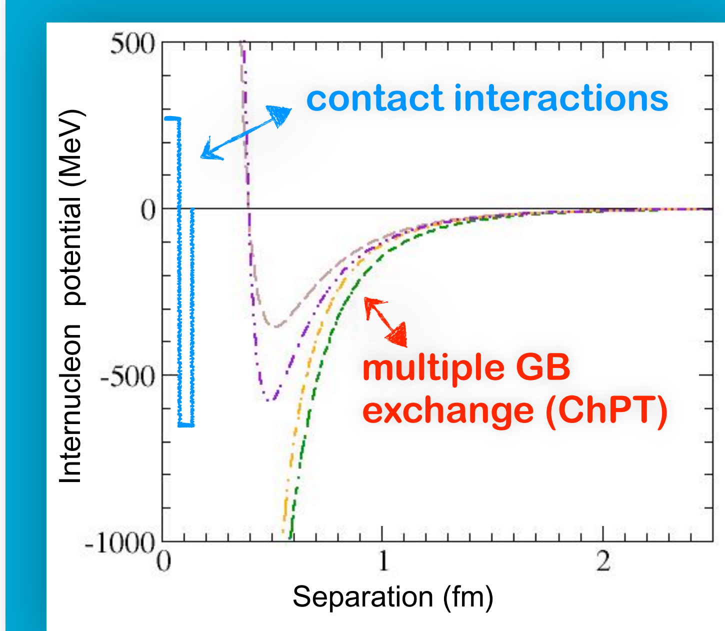


- **NN interaction is strong**: resummations/nonperturbative methods needed
- $1/m_N$ - expansion: nonrelativistic problem ($|\vec{p}_i| \sim M_\pi \ll m_N$) ⇨ the QM A-body problem

$$\left[\left(\sum_{i=1}^{A}\frac{-\vec{\nabla}_i^2}{2m_N}+\mathcal{O}(m_N^{-3})\right)+\underbrace{V_{2N}+V_{3N}+V_{4N}+\ldots}_{\textit{derived within ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$

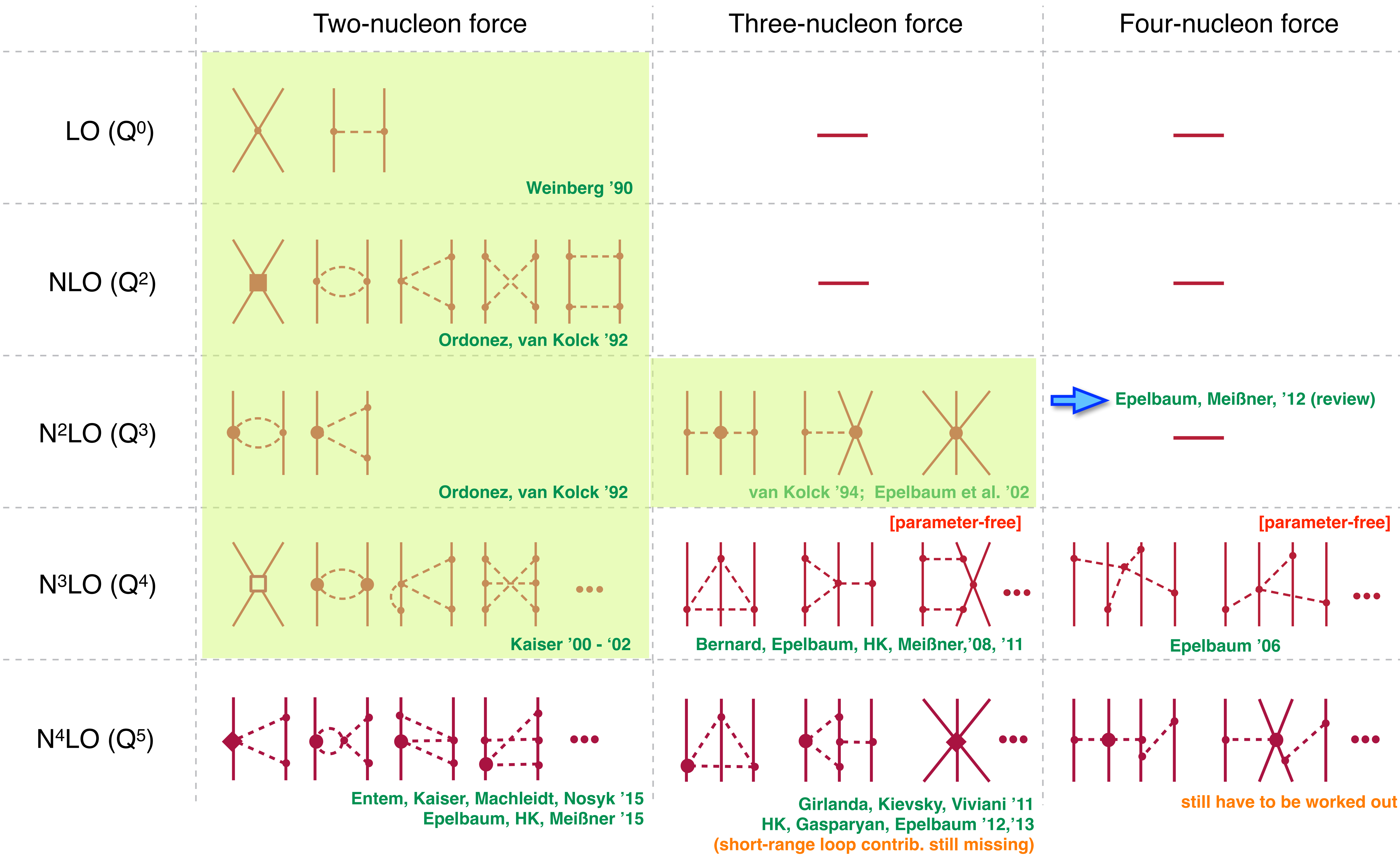
Weinberg '91





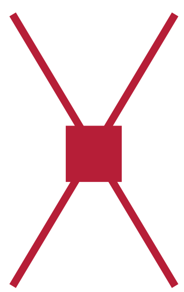
- unified description of $\pi\pi$, πN and NN
- consistent many-body forces and currents
- systematically improvable
- bridging different reactions (electroweak, π-prod., ...)
- precision physics with/from light nuclei

Chiral Expansion of the Nuclear Forces



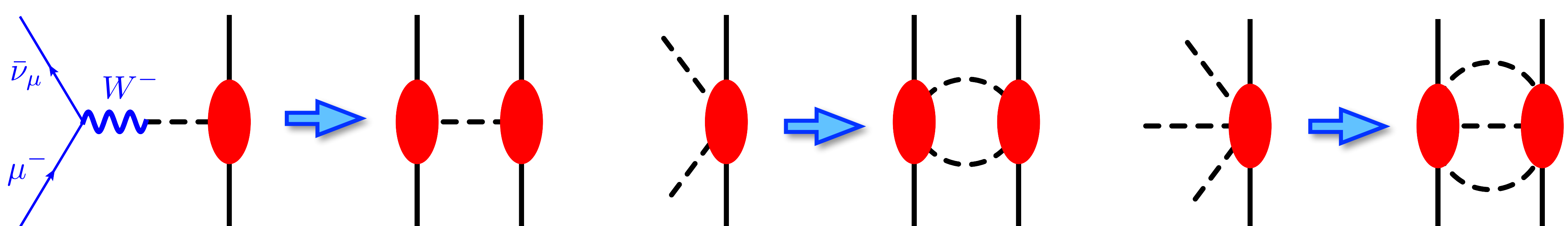
Long and Short Range Interactions

- Couplings of short-range interactions are fixed from NN - data.
  In the isospin limit we have:



LO [Q^0]: 2 operators (S-waves)
NLO [Q^2]: + 7 operators (S-, P-waves and ε_1)
N^{2}LO [Q^3]: no new terms
N^{3}LO [Q^4]: + 12 operators (S-, P-, D-waves and ε_1, ε_2)
N^{4}LO [Q^5]: no new terms

- Long range part of the nuclear forces are predictions (chiral symmetry of QCD) once couplings from single-nucleon subprocess are determined



NN parameter free predictions

Redundant Short-Range LECs

- Slow convergence of fits to data at N3LO & beyond → Redundancy of LECs

Hammer, Furnstahl '00, Beane, Savage '01, Wesolowski et al.'16

Short-range LECs at N³LO: $V_{\rm cont}^{(Q^4)} = D_1 q^4 + D_2 k^4 + D_3 q^2 k^2 + \cdots + D_{15} \vec\sigma_1 \cdot (\vec q \times \vec k) \vec\sigma_2 \cdot (\vec q \times \vec k)$

$U = e^{\gamma_1 T_1 + \gamma_2 T_2 + \gamma_3 T_3}$ with $T_1 = \frac{m_N}{2\Lambda_b^4} \vec k \cdot \vec q, \quad T_2 = \frac{m_N}{2\Lambda_b^4} \vec k \cdot \vec q \, \vec\sigma_1 \cdot \vec\sigma_2, \quad T_3 = \frac{m_N}{2\Lambda_b^4} \Big(\vec\sigma_1 \cdot \vec k \, \vec\sigma_2 \cdot \vec q + \vec\sigma_1 \cdot \vec q \, \vec\sigma_2 \cdot \vec k \Big)$

applied to kinetic energy operator generates short-range N³LO structures

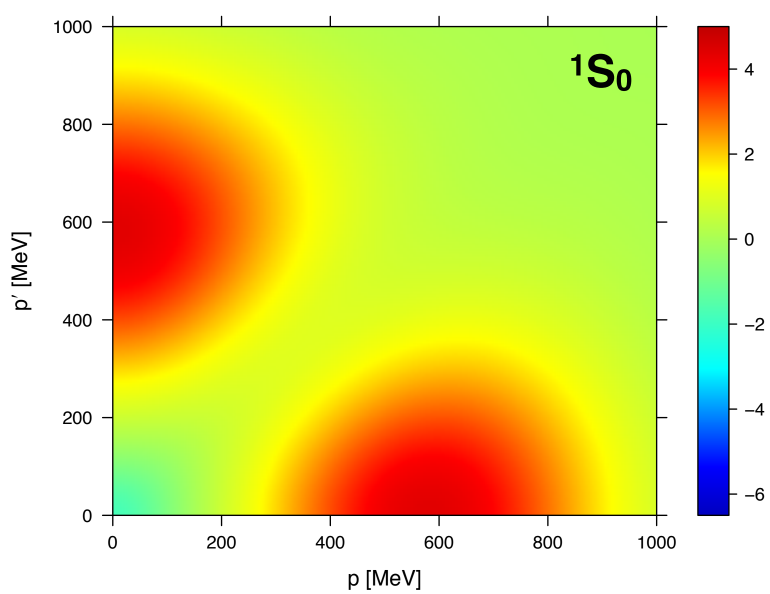
$$U^\dagger H_0 U = H_0 + \frac{\gamma_1}{\Lambda_b^4} (\vec k \cdot \vec q)^2 + \frac{\gamma_2}{\Lambda_b^4} (\vec k \cdot \vec q)^2 \vec\sigma_1 \cdot \vec\sigma_2 + \frac{\gamma_3}{\Lambda_b^4} \vec k \cdot \vec q \Big(\vec\sigma_1 \cdot \vec k \, \vec\sigma_2 \cdot \vec q + \vec\sigma_1 \cdot \vec q \, \vec\sigma_2 \cdot \vec k \Big) + \mathcal{O}(Q^5)$$

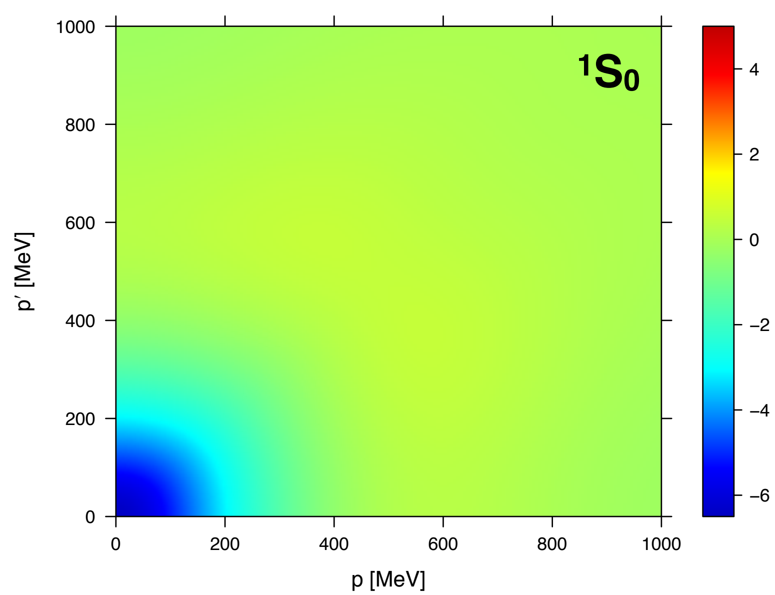
Conventional choice of $\gamma_{1,2,3}$

$D_{1S0}^{\rm off} = D_{3S1}^{\rm off} = D_{\epsilon 1}^{\rm off} = 0$

leads to softer NN interactions

$$\begin{aligned}
\langle {}^1S_0, p' | V_{\rm cont} | {}^1S_0, p \rangle &= \tilde C_{1S0} + C_{1S0}(p^2 + p'^2) + D_{1S0} \, p^2 p'^2 + D_{1S0}^{\rm off} (p^2 - p'^2)^2 \\
\langle {}^3S_1, p' | V_{\rm cont} | {}^3S_1, p \rangle &= \tilde C_{3S1} + C_{3S1}(p^2 + p'^2) + D_{3S1} \, p^2 p'^2 + D_{3S1}^{\rm off} (p^2 - p'^2)^2 \\
\langle {}^3S_1, p' | V_{\rm cont} | {}^3D_1, p \rangle &= C_{\epsilon 1} p^2 + D_{\epsilon 1} \, p^2 p'^2 + D_{\epsilon 1}^{\rm off} \, p^2 (p^2 - p'^2)
\end{aligned}$$





Softness of NN interaction for made choice of $\gamma_{1,2,3}$ confirmed by Weinberg eigenvalue analysis

Regularization of NN Force I

Regularization of LS-equation with finite cut-off range: $400\,\mathrm{MeV} < \Lambda < 550\,\mathrm{MeV}$

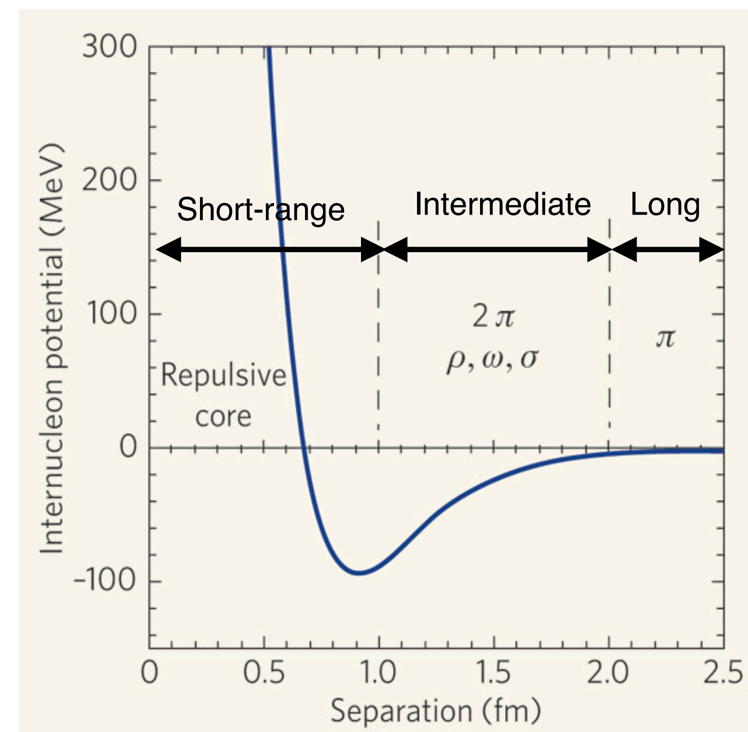
- Appearance of deeply-bound states for higher cut-offs make many-body calculations for A > 3 unfeasible

Regulator which minimizes finite-Λ artifacts is crucial!

Long-range behavior (prediction of chiral EFT) of the NN force should be unaffected by regulator

Short-range part strongly depends on regulator

- Short-range interactions are responsible for restoring regulator independence
- Increasing number of short-range interactions remove regulator dependence



Regularize nuclear force in coordinate space: *Epelbaum, HK, Meißner '15*

$$V_{\mathrm{long-range}}(\vec{r}) \to V_{\mathrm{long-range}}(\vec{r}) \left[1 - \exp\left(-\frac{r^2}{R^2} \right) \right]^n$$

✖ Inconvenient for currents (momentum space regulator preferable)

Regularization of NN Force II

- Regularize one-pion-exchange propagator: *Reinert, HK, Epelbaum '17 (inspired by Rijken '91)*

$$\frac{1}{q^2+M_\pi^2} \to \frac{\exp\left(-\frac{q^2+M_\pi^2}{\Lambda^2}\right)}{q^2+M_\pi^2} = \frac{1}{q^2+M_\pi^2} - \frac{1}{\Lambda^2} + \frac{q^2+M_\pi^2}{2\Lambda^4} + \dots$$

all 1/Λ-corrections are short-range interactions

- Implement similar regularization for two-pion exchange

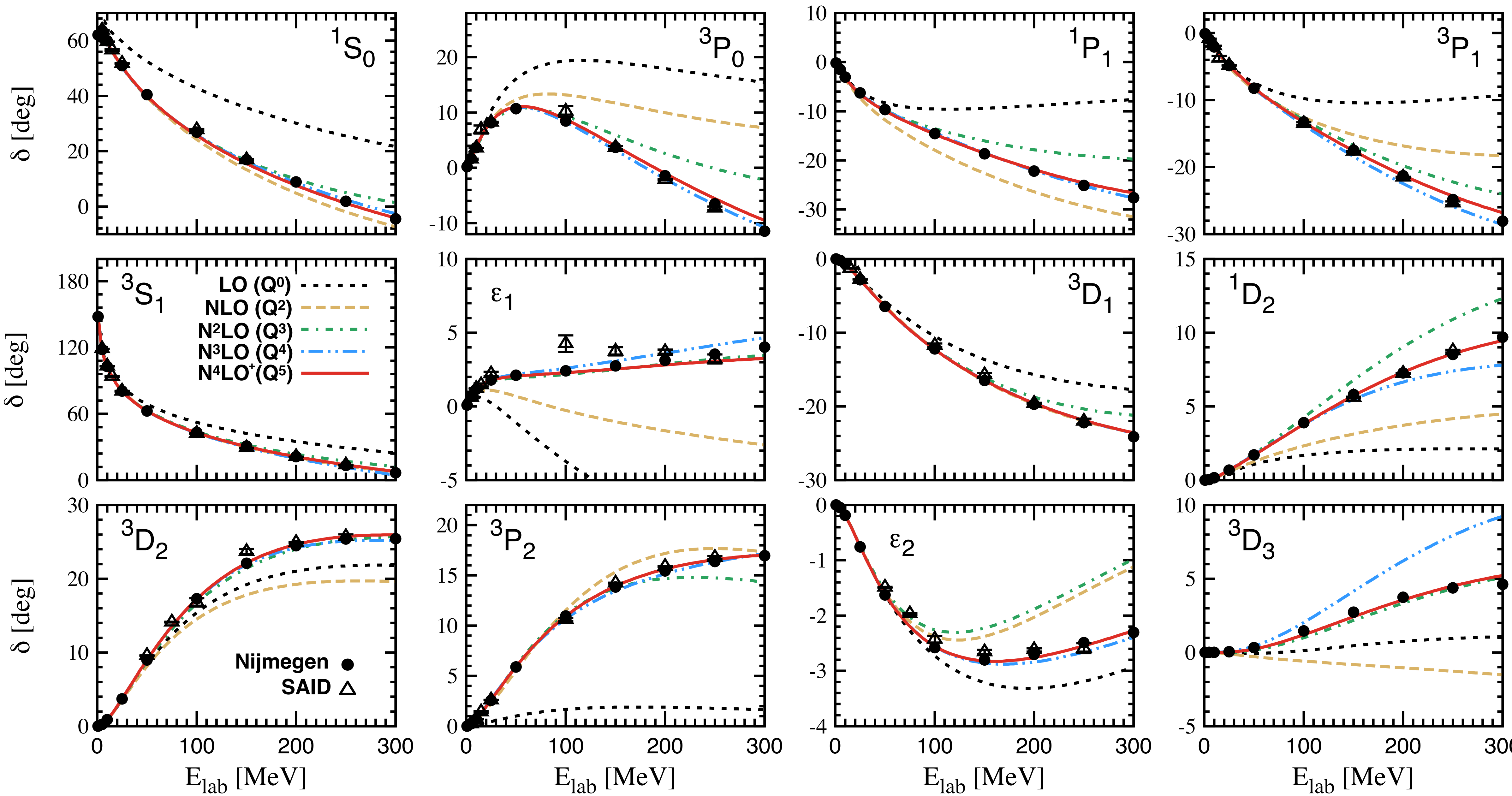
$$V(q) = \frac{2}{\pi}\int_{2M_\pi}^{\infty} d\mu\,\mu\frac{\rho(\mu)}{q^2+\mu^2} \to V_\Lambda(q) = \frac{2}{\pi}\int_{2M_\pi}^{\infty} d\mu\,\mu\frac{\rho(\mu)}{q^2+\mu^2}\exp\left(-\frac{q^2+\mu^2}{2\Lambda^2}\right)$$

- Compared to simple gaussian regulator $\exp\left(-\frac{q^2}{\Lambda^2}\right)$ πN-coupling gets quenched

$$g_A \to g_A \exp\left(-\frac{M_\pi^2}{2\Lambda^2}\right) < g_A$$

Chiral Expansion of np Phase Shifts

Reinert, HK, Epelbaum '17



- Good convergence of chiral expansion & excellent agreement with NPWA data
- Chiral potential match in precision phenomenological potentials (CD Bonn, Av18,…) with around 40% less parameter

Uncertainty Quantification

Reinert, HK, Epelbaum '17

Effective range, deuteron properties and phase-shift with quantified uncertainty

Example: deuteron asymptotic normalization

	$\Lambda = 400$ MeV	$\Lambda = 450$ MeV	$\Lambda = 500$ MeV	$\Lambda = 550$ MeV
A_S (fm$^{-1/2}$)	$0.8847^{(+3)}_{(-3)}(6)(4)(4)$	$0.8847^{(+3)}_{(-3)}(3)(5)(1)$	$0.8849^{(+3)}_{(-3)}(1)(7)(0)$	$0.8851^{(+3)}_{(-3)}(3)(8)(1)$
η	$0.0255^{(+1)}_{(-1)}(1)(3)(2)$	$0.0255^{(+1)}_{(-1)}(1)(4)(1)$	$0.0257^{(+1)}_{(-1)}(1)(5)(1)$	$0.0258^{(+1)}_{(-1)}(1)(5)(1)$

statistical error, truncation error, πN LECs, variation of E_{max}

$$A_S = 0.8847^{(+3)}_{(-3)}(3)(5)(1)\ \mathrm{fm}^{-1/2}$$

$$\eta \equiv \frac{A_D}{A_S} = 0.0255^{(+1)}_{(-1)}(1)(4)(1)$$

Exp: $A_S = 0.8781(44)\ \mathrm{fm}^{-1/2}$, $\eta = 0.0256(4)$

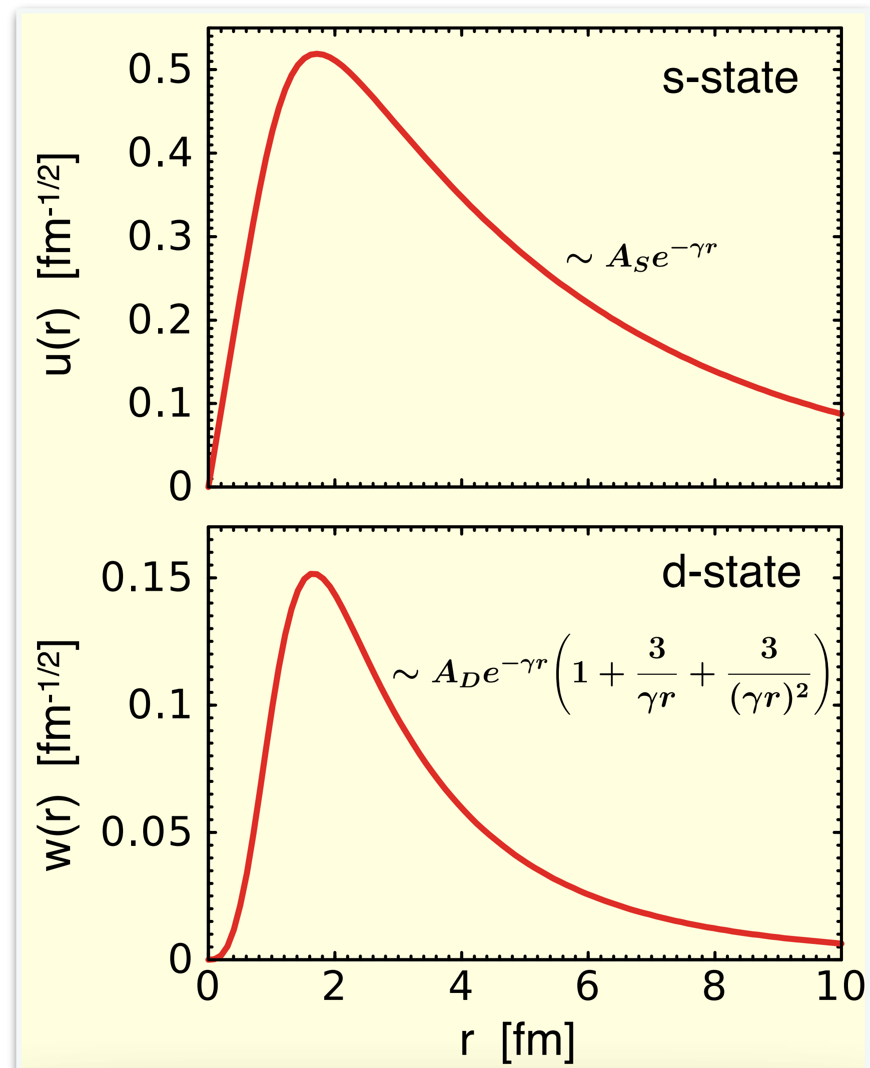
Borbely et al. '85 *Rodning, Knutson '90*

Nijmegen PWA [errors are „educated guesses"] *Stoks et al. '95*

$$A_S = 0.8845(8)\ \mathrm{fm}^{-1/2}, \quad \eta = 0.0256(4)$$

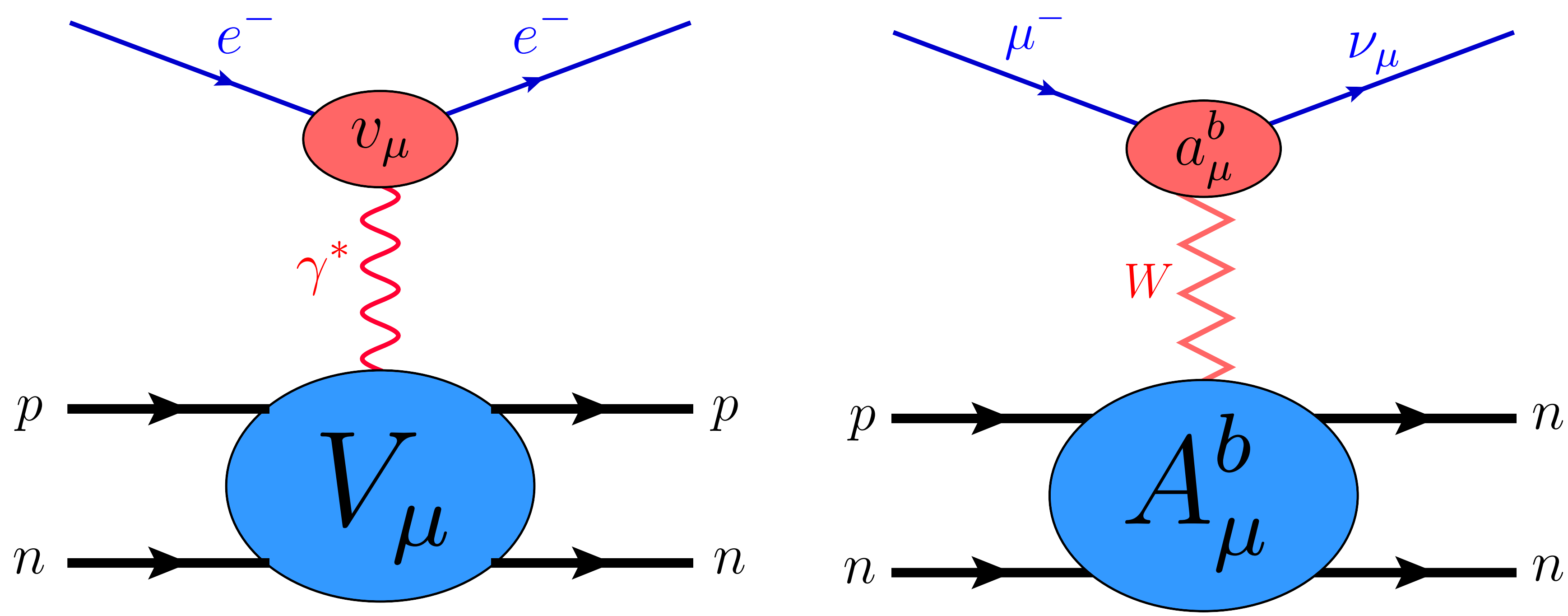
Granada PWA [errors purely statistical] *Navarro Perez et al. '13*

$$A_S = 0.8829(4)\ \mathrm{fm}^{-1/2}, \quad \eta = 0.0249(1)$$



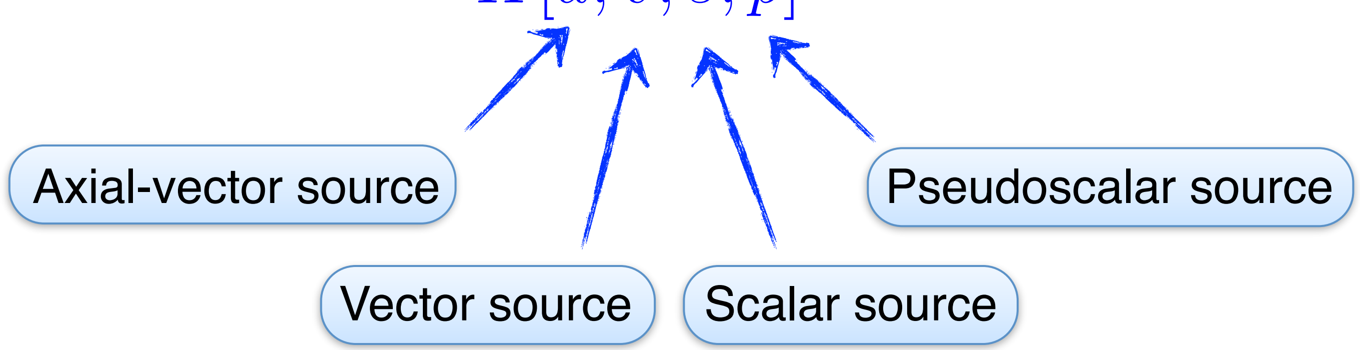
Nuclear currents in chiral EFT

Electroweak probes on nucleons and nuclei can be described by current formalism



Chiral EFT Hamiltonian depends on external sources

$$H[a, v, s, p]$$

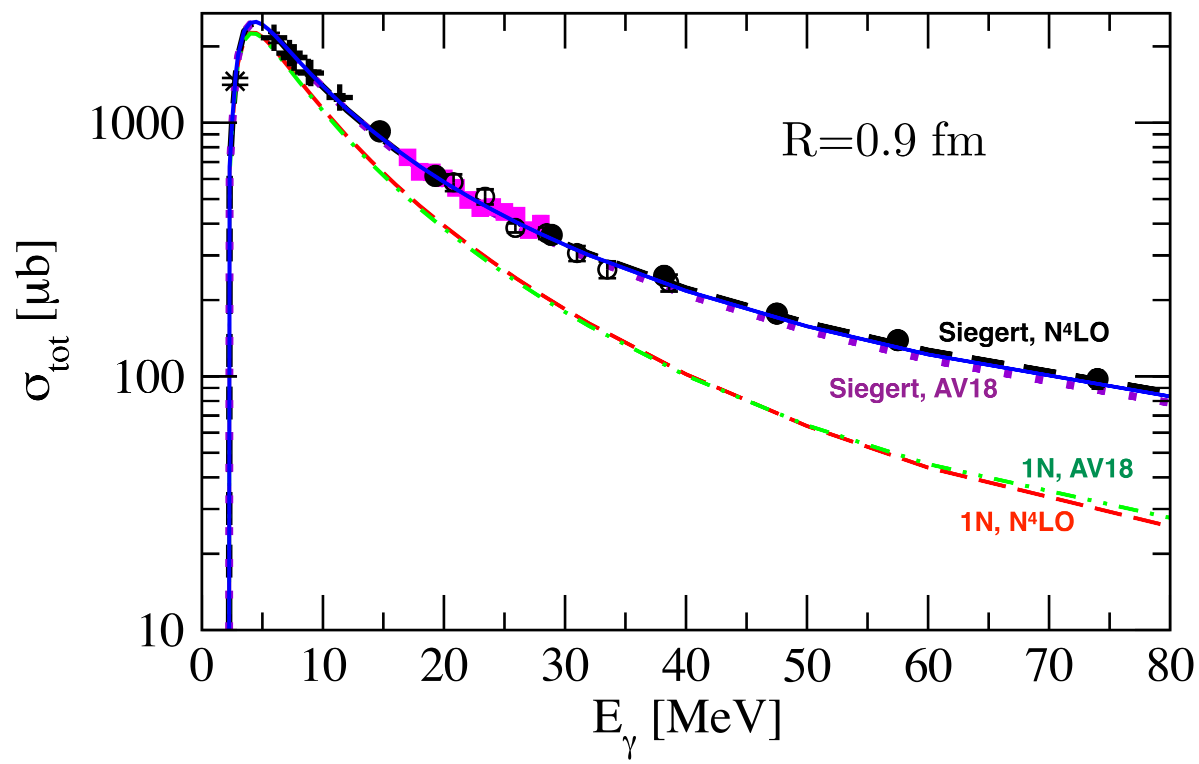


Siegert theorem + N4LO

Skibinski et al. PRC93 (2016) no. 6, 064002

Generate longitudinal component of NN current by continuity equation

$\left[H_{\text{strong}}, \boldsymbol{\rho}\right] = \vec{k} \cdot \vec{\boldsymbol{J}} \leftarrow$ regularized longitudinal current (Siegert theorem)

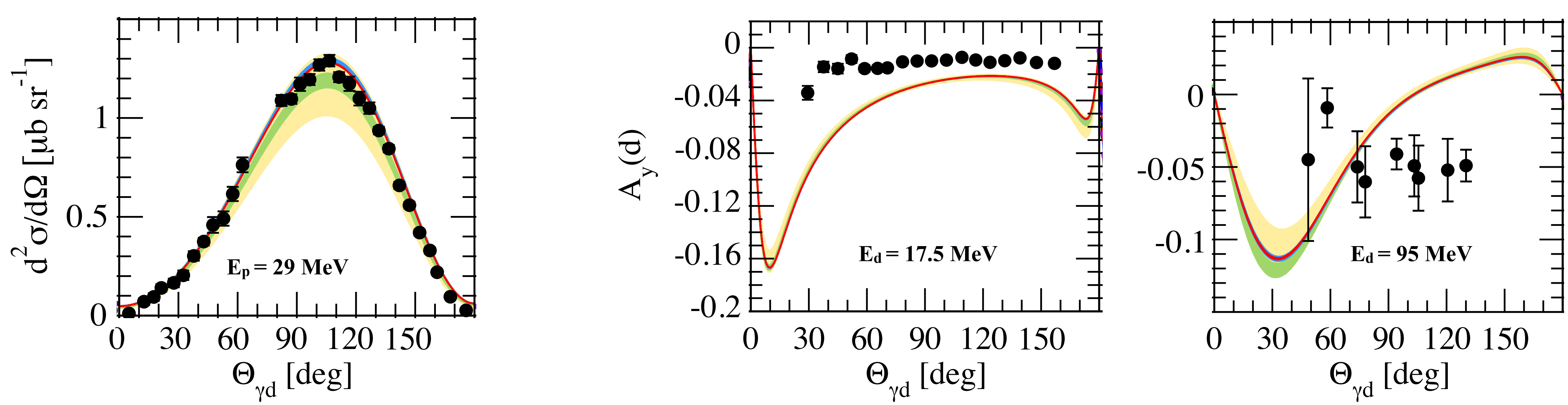


Deuteron photo-disintegration

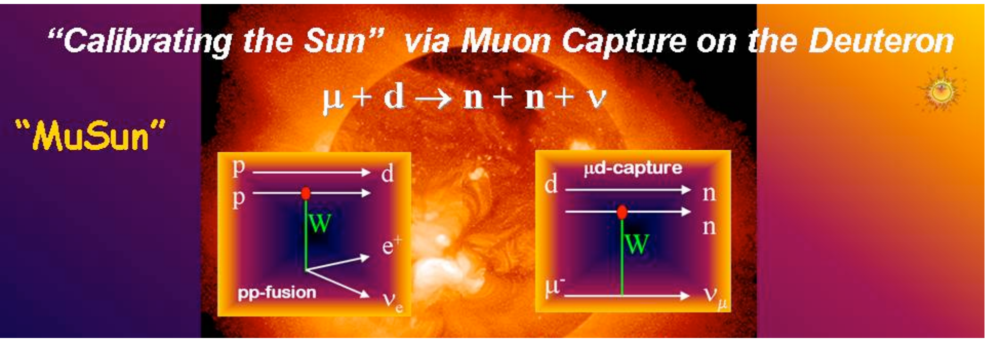
$\gamma + d \rightarrow p + n$

- consistent regularization via cont. eq.
- improvement by 1N+Siegert
- implementation of transverse part & exchange currents work in progress

Nucleon-deuteron radiative capture: $p(n) + d \rightarrow^3 \mathrm{H}(^3\mathrm{He}) + \gamma$

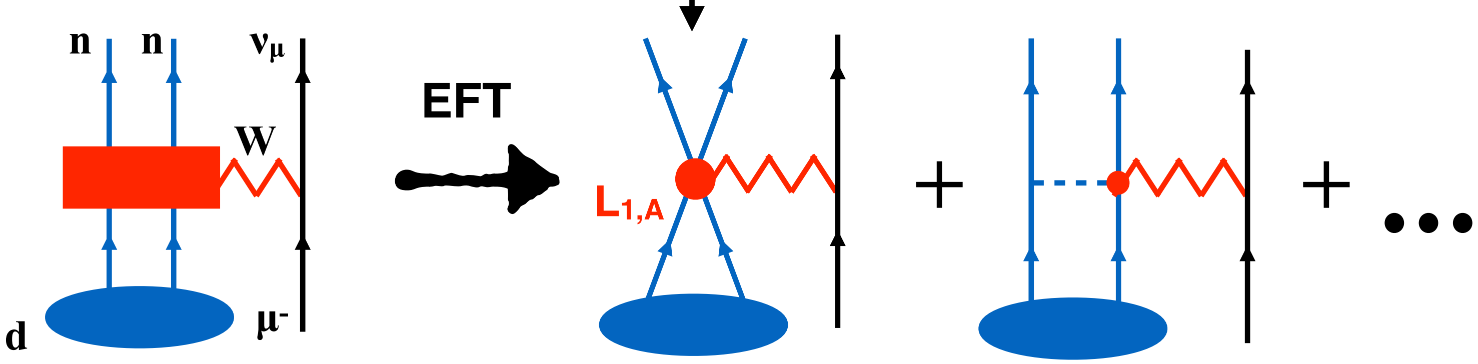


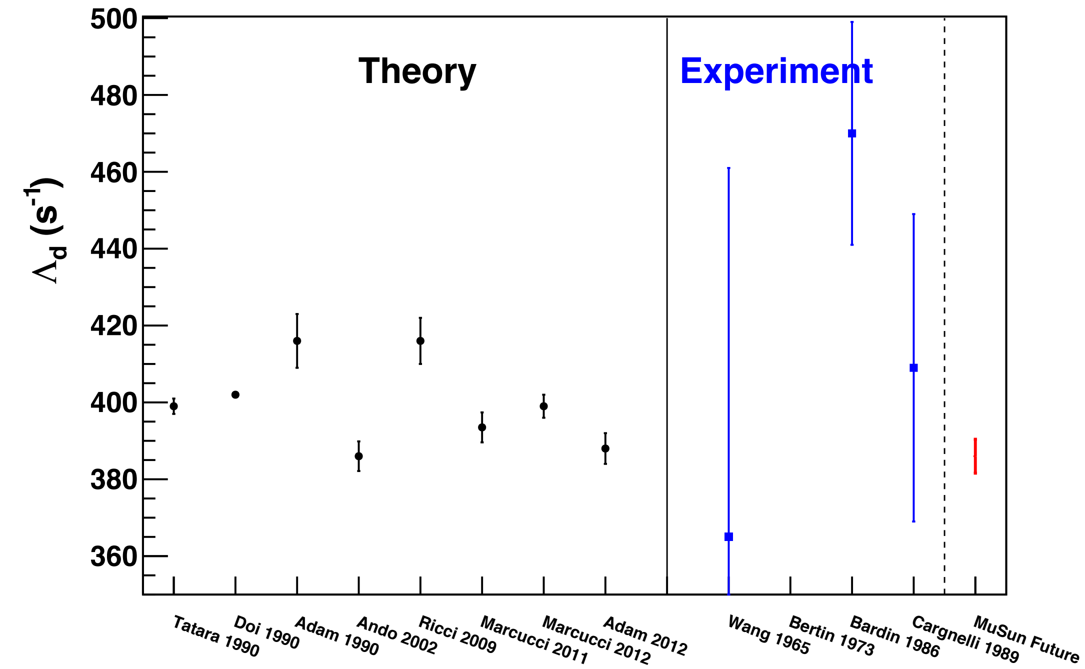
MuSun experiment at PSI



Main goal: measure the doublet capture rate Λ_d in
$\mu^- + d \rightarrow \nu_\mu + n + n$ with the accuracy of ~ 1.5%

This will strongly constrain the short-range axial current





The resulting axial exchange current can be used to make precision calculations for

- triton half life, $fT_{1/2} = 1129.6 \pm 3.0$ s, and the muon capture rate on ^{3}He, $\Lambda_0 = 1496 \pm 4\ s^{-1}$ → precision tests of the theory
- weak reactions of astrophysical interest such as e.g. the pp chain of the solar burning:

$$p + p \rightarrow d + e^+ + \nu_e$$
$$p + p + e^- \rightarrow d + \nu_e$$
$$p + {}^3He \rightarrow {}^4He + e^+ + \nu_e$$
$${}^7Be + e^- \rightarrow {}^7Li + \nu_e$$
$${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$$

- $L_{1,A}$ governs the leading 3NF

Historical remarks

- Meson-exchange theory, Skyrme model, phenomenology, …
  Brown, Adam, Mosconi, Ricci, Truhlik, Nakamura, Sato, Ando, Kubodera, Riska, Sauer, Friar, Gari …

- First derivation within chiral EFT to leading 1-loop order using TOPT
  Park, Min, Rho Phys. Rept. 233 (1993) 341; NPA 596 (1996) 515;
  Park et al., Phys. Rev. C67 (2003) 055206
  – only for the threshold kinematics
  – pion-pole diagrams ignored
  – box-type diagrams neglected
  – renormalization incomplete

- Leading one-loop expressions using TOPT for general kinematics (still incomplete, e.g. no 1/m corrections)
  Pastore, Girlanda, Schiavilla, Goity, Viviani, Wiringa;
  PRC78 (2008) 064002; PRC80 (2009) 034004; PRC84 (2011) 024001 ← Vector current

  Baroni, Girlanda, Pastore, Schiavilla, Viviani;
  PRC93 (2016) 015501, Erratum: PRC 93 (2016) 049902 ← Axial vector current

Complete derivation to leading one-loop order using the method of UT

Kölling, Epelbaum, HK, Meißner;
PRC80 (2009) 045502; PRC84 (2011) 054008 ← Vector current

HK, Epelbaum, Meißner, Ann. Phys. 378 (2017) 317 ← Axial vector current

Diagonalization via Okubo

- Decomposition of the Fock space $\mathcal{H}$

Projector operators: $\eta + \lambda = 1$

$$\mathcal{H}_M \xleftarrow{\eta} \mathcal{H} = \mathcal{H}_M \oplus \mathcal{H}_R \xrightarrow{\lambda} \mathcal{H}_R$$

Model-space including only pure nucleon states

Remainder-space including states with at least one pion

$$H|\Psi\rangle = (H_0 + H_I)|\Psi\rangle = E|\Psi\rangle \quad \Longleftrightarrow \quad \begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} \begin{pmatrix} \eta|\Psi\rangle \\ \lambda|\Psi\rangle \end{pmatrix} = E \begin{pmatrix} \eta|\Psi\rangle \\ \lambda|\Psi\rangle \end{pmatrix}$$

- Block-diagonalization by applying unitary transformation

$$\tilde{H} = U^\dagger H U = \begin{pmatrix} \eta \tilde{H} \eta & 0 \\ 0 & \lambda H \lambda \end{pmatrix}$$

$$V_{\rm eff} = \eta(\tilde{H} - H_0)\eta$$

$V_{\rm eff}$ is E - indep. $\Rightarrow$ important for few-nucleon simulations

Possible parametrization by Okubo '54

$$U = \begin{pmatrix} \eta(1 + A^\dagger A)^{-1/2} & -A^\dagger(1 + AA^\dagger)^{-1/2} \\ A(1 + A^\dagger A)^{-1/2} & \lambda(1 + AA^\dagger)^{-1/2} \end{pmatrix}$$

With decoupling eq. $\lambda(H - [A, H] - AHA)\eta = 0$

Can be solved perturbatively within ChPT

Epelbaum, Glöckle, Meißner, ´98

Unitary transformations for currents

- Step 1: $\tilde{H} \to \tilde{H}[a,v,s,p] = U^\dagger H[a,v,s,p]U$

Okubo transf. or further strong unitary transf. are not enough to renormalize the currents

- Step 2: additional (time-dependent) unitary transformations

$$i\frac{\partial}{\partial t}\Psi = H\Psi \longrightarrow i\frac{\partial}{\partial t}U(t)U^\dagger(t)\Psi = U(t)i\frac{\partial}{\partial t}U^\dagger(t)\Psi + \left(i\frac{\partial}{\partial t}U(t)\right)U^\dagger(t)\Psi = HU(t)U^\dagger(t)\Psi$$

$$\Psi' = U^\dagger(t)\Psi \longrightarrow i\frac{\partial}{\partial t}\Psi' = \left[U^\dagger(t)HU(t) - U^\dagger(t)\left(i\frac{\partial}{\partial t}U(t)\right)\right]\Psi'$$

Explicit time-dependence through source terms

$$\tilde{H}[a,v,s,p] \to \underbrace{U^\dagger[a,v]\tilde{H}[a,v,s,p]U[a,v] + \left(i\frac{\partial}{\partial t}U^\dagger[a,v]\right)U[a,v]}_{=:\, H_{\text{eff}}[a,\dot{a},v,\dot{v}]}$$

$$A_\mu^b(\vec{x},t) := \frac{\delta}{\delta a^{\mu,b}(\vec{x},t)} H_{\text{eff}}[a,\dot{a},v,\dot{v}]\Big|_{a=v=0}$$

Due to time-derivatives ($\dot{a}$,$\dot{v}$) the currents depend on energy transfer if transformed into momentum space

Chiral symmetry constraints

Chiral symmetry transformations on the path integral level

Gasser, Leutwyler Ann. Phys. (1984) 142: $v_\mu = \frac{1}{2}(r_\mu + l_\mu)$ and $a_\mu = \frac{1}{2}(r_\mu - l_\mu)$

$$\langle 0_{\rm out}|0_{\rm in}\rangle_{a,v,s,p} = \exp\left(i\, Z[a,v,s,p]\right) = \exp\left(i\, Z[a',v',s',p']\right) = \langle 0_{\rm out}|0_{\rm in}\rangle_{a',v',s',p'}$$

$$\begin{aligned} r_\mu &\to r'_\mu = R\, r_\mu R^\dagger + iR\,\partial_\mu R^\dagger\ , \\ l_\mu &\to l'_\mu = L\, l_\mu L^\dagger + iL\,\partial_\mu L^\dagger\ , \\ s + i\,p &\to s' + i\,p' = R(s + i\,p)L^\dagger\ , \\ s - i\,p &\to s' - i\,p' = L(s - i\,p)R^\dagger\ . \end{aligned}$$

Chiral $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ rotation does not change the generating functional $\longrightarrow$ Ward identities

Chiral symmetry transformations on the Hamiltonian level

- There exists a unitary transformation $U(R,L)$ such that from Schrödinger eq.

$$i\frac{\partial}{\partial t}\Psi = H_{\rm eff}[a,v,s,p]\Psi \quad \text{takes the form} \quad i\frac{\partial}{\partial t}U^\dagger(R,L)\Psi = H_{\rm eff}[a',v',s',p']U^\dagger(R,L)\Psi$$

Transformed Hamiltonian is unitary equivalent to the untransformed one

$$H_{\rm eff}[a',\dot a',v',\dot v',s',p'] = U^\dagger(R,L)H_{\rm eff}[a,\dot a,v,\dot v,s,p]U(R,L) + \left(i\frac{\partial}{\partial t}U^\dagger(R,L)\right)U(R,L)$$

Continuity equation

Infinitesimally we have $R = 1 + \frac{i}{2}\boldsymbol{\tau}\cdot\boldsymbol{\epsilon}_R(x)$ and $L = 1 + \frac{i}{2}\boldsymbol{\tau}\cdot\boldsymbol{\epsilon}_L(x)$

Expressed in $\boldsymbol{\epsilon}_V = \frac{1}{2}(\boldsymbol{\epsilon}_R + \boldsymbol{\epsilon}_L)$ and $\boldsymbol{\epsilon}_A = \frac{1}{2}(\boldsymbol{\epsilon}_R - \boldsymbol{\epsilon}_L)$ we have

$$\begin{aligned} \boldsymbol{v}_\mu &\to \boldsymbol{v}'_\mu = \boldsymbol{v}_\mu + \boldsymbol{v}_\mu \times \boldsymbol{\epsilon}_V + \boldsymbol{a}_\mu \times \boldsymbol{\epsilon}_A + \partial_\mu \boldsymbol{\epsilon}_V \\ \boldsymbol{a}_\mu &\to \boldsymbol{a}'_\mu = \boldsymbol{a}_\mu + \boldsymbol{a}_\mu \times \boldsymbol{\epsilon}_V + \boldsymbol{v}_\mu \times \boldsymbol{\epsilon}_A + \partial_\mu \boldsymbol{\epsilon}_A \end{aligned} \longrightarrow \begin{aligned} \dot{v}_\mu &\to \dot{v}'_\mu = \partial_\mu \dot{\epsilon}_V + \ldots \\ \dot{a}_\mu &\to \dot{a}'_\mu = \partial_\mu \dot{\epsilon}_A + \ldots \end{aligned}$$

$$H_{\text{eff}}[a', \dot{a}', v', \dot{v}', s', p'] = U^\dagger(R,L) H_{\text{eff}}[a, \dot{a}, v, \dot{v}, s, p] U(R,L) + \left(i\frac{\partial}{\partial t} U^\dagger(R,L)\right) U(R,L)$$

- $H_{\text{eff}}[a', \dot{a}', v', \dot{v}', s', p']$ is a function of $\boldsymbol{\epsilon}_V, \dot{\boldsymbol{\epsilon}}_V, \ddot{\boldsymbol{\epsilon}}_V, \boldsymbol{\epsilon}_A, \dot{\boldsymbol{\epsilon}}_A, \ddot{\boldsymbol{\epsilon}}_A$

$$\longrightarrow U = \exp\left(i \int d^3x \left[\boldsymbol{R}_0^v(\vec{x}) \cdot \boldsymbol{\epsilon}_V(\vec{x}, t) + \boldsymbol{R}_1^v(\vec{x}) \cdot \dot{\boldsymbol{\epsilon}}_V(\vec{x}, t) + \boldsymbol{R}_0^a(\vec{x}) \cdot \boldsymbol{\epsilon}_A(\vec{x}, t) + \boldsymbol{R}_1^a(\vec{x}) \cdot \dot{\boldsymbol{\epsilon}}_A(\vec{x}, t) \right] \right)$$

Expanding both sides in $\vec{\epsilon}_V, \vec{\epsilon}_A$, comparing the coefficients and transforming to momentum space we get the continuity equation

$$\mathcal{C}(\vec{k}, k_0) = \left[H_{\text{strong}}, \boldsymbol{A}_0(\vec{k}, k_0) \right] - \vec{k} \cdot \vec{\boldsymbol{A}}(\vec{k}, k_0) + i\, m_q \boldsymbol{P}(\vec{k}, k_0)$$

$$\mathcal{C}(\vec{k}, 0) + \underbrace{\left[H_{\text{strong}}, \frac{\partial}{\partial k_0} \mathcal{C}(\vec{k}, k_0) \right]}_{\text{new term}} = 0$$

Unitary ambiguities

34 different unitary transformations are possible at the order Q

$$U_i(a) = \exp\left(S_i^{\text{ax}} - h.c.\right)$$

$$S_1^{ax} = \alpha_1^{ax}\eta A_{2,0}^{(0)}\eta H_{2,1}^{(1)}\lambda^1 \frac{1}{E_\pi^3} H_{2,1}^{(1)}\eta,$$

$$S_2^{ax} = \alpha_2^{ax}\eta H_{2,1}^{(1)}\lambda^1 \frac{1}{E_\pi^2} A_{2,0}^{(0)}\lambda^1 \frac{1}{E_\pi} H_{2,1}^{(1)}\eta$$

$$\ldots$$

Vertices without axial source are denoted by $H_{n,p}^{(\kappa)}$

Vertices with one axial source are denoted by $A_{n,p}^{(\kappa)}$

n — number of nucleons

p — number of pions

a — number of axial sources

$\kappa = d + \frac{3}{2}n + p + a - 4 \longleftarrow$ inverse mass dimension

Large unitary ambiguity is related to appearance of the axial-vector-one-pion interaction $A_{0,1}^{(-1)}$ (30 out of 34 transformations depend on it)

Reasonable constraints come from

- Perturbative renormalizability of the current

$$l_i = l_i^r(\mu) + \gamma_i\lambda =: \frac{1}{16\pi^2}\bar{l}_i + \gamma_i\lambda + \frac{\gamma_i}{16\pi^2}\ln\left(\frac{M_\pi}{\mu}\right),$$

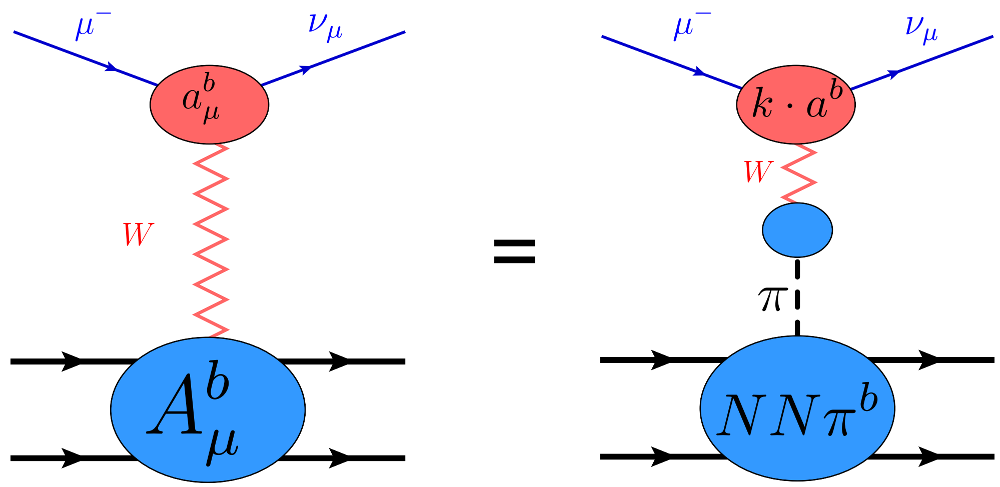
$$d_i = d_i^r(\mu) + \frac{\beta_i}{F^2}\lambda =: \bar{d}_i + \frac{\beta_i}{F^2}\lambda + \frac{\beta_i}{16\pi^2F^2}\ln\left(\frac{M_\pi}{\mu}\right)$$

$$\begin{aligned}
\gamma_3 &= -\frac{1}{2}\,,\\
\gamma_4 &= 2\,,\\
\beta_2 = -2\beta_5 = \frac{1}{2}\beta_6 &= -\frac{1}{12}(1+5g_A^2)\,,\\
\beta_{15} = \beta_{18} = \beta_{22} = \beta_{23} &= 0\,,\\
\beta_{16} &= \frac{1}{2}g_A + g_A^3\,.
\end{aligned}$$

After renormalizing LECs l_i from $\mathcal{L}_\pi^{(4)}$ and d_i from $\mathcal{L}_{\pi N}^{(\bar{3})}$ and using well known β- and γ-functions (*Gasser et al. Eur. Phys. J. C26 (2002), 13*) we require the current to be finite

Matching to nuclear forces

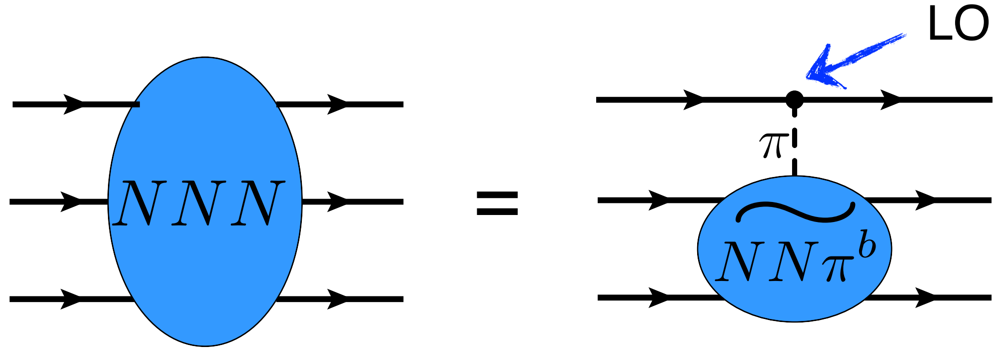
Dominance of the pion production operator at the pion-pole (axial-vector current)



+ Non-pole contributions

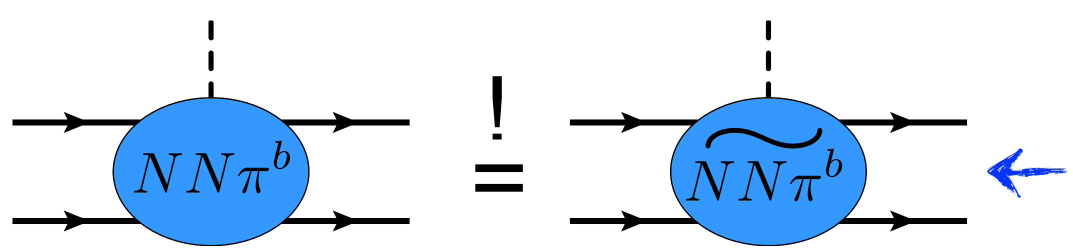
Dominance of the pion production operator at the pion-pole (three-nucleon force)

LO pion-nucleon vertex



+ Non-pole contributions

Consistent regularization of nuclear forces and currents calls for matching requirement between pion-production operators in different processes

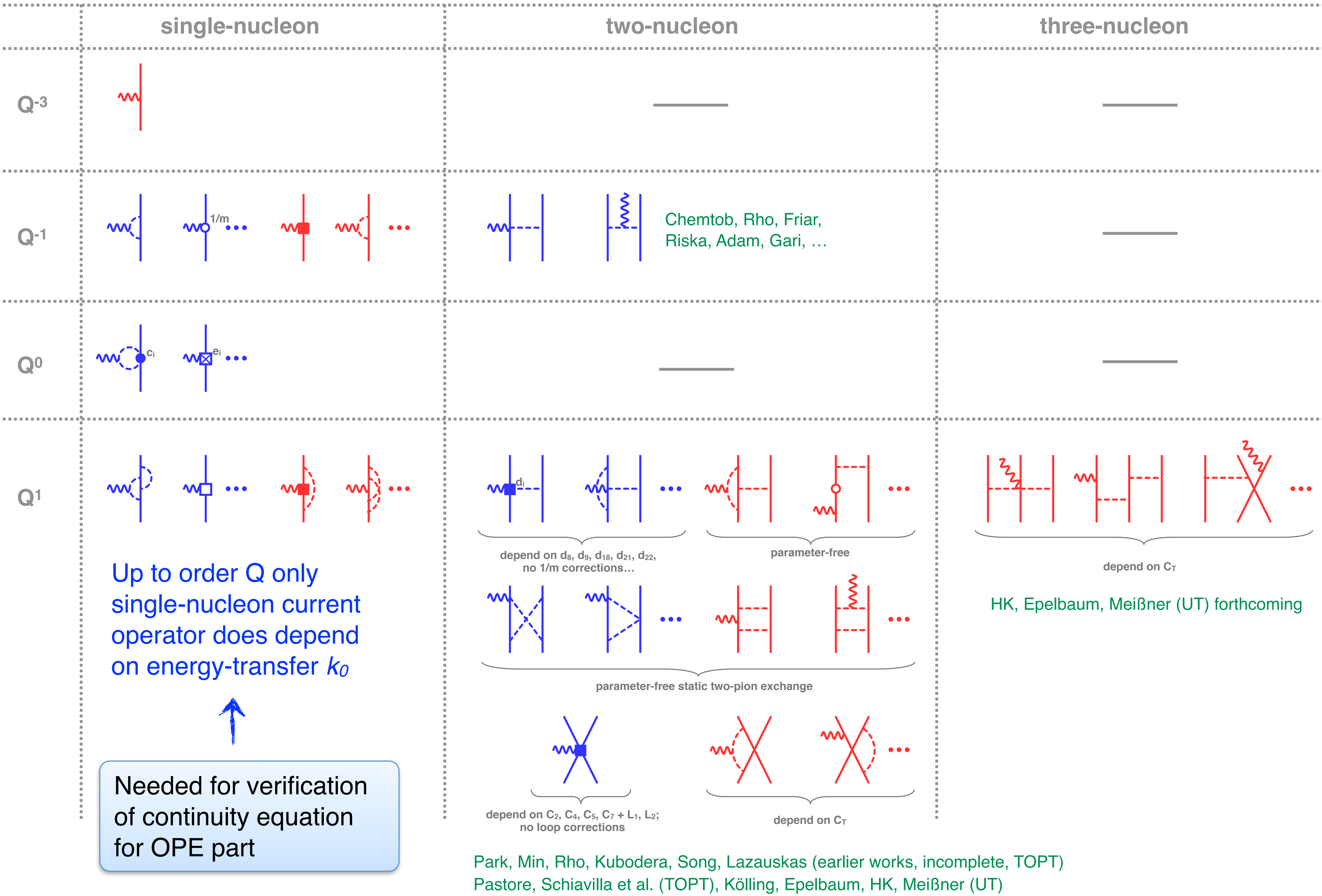


Matching requirement is fulfilled only for particular choice of unitary phases

After renormalizability and matching requirement there are no further unitary ambiguities!

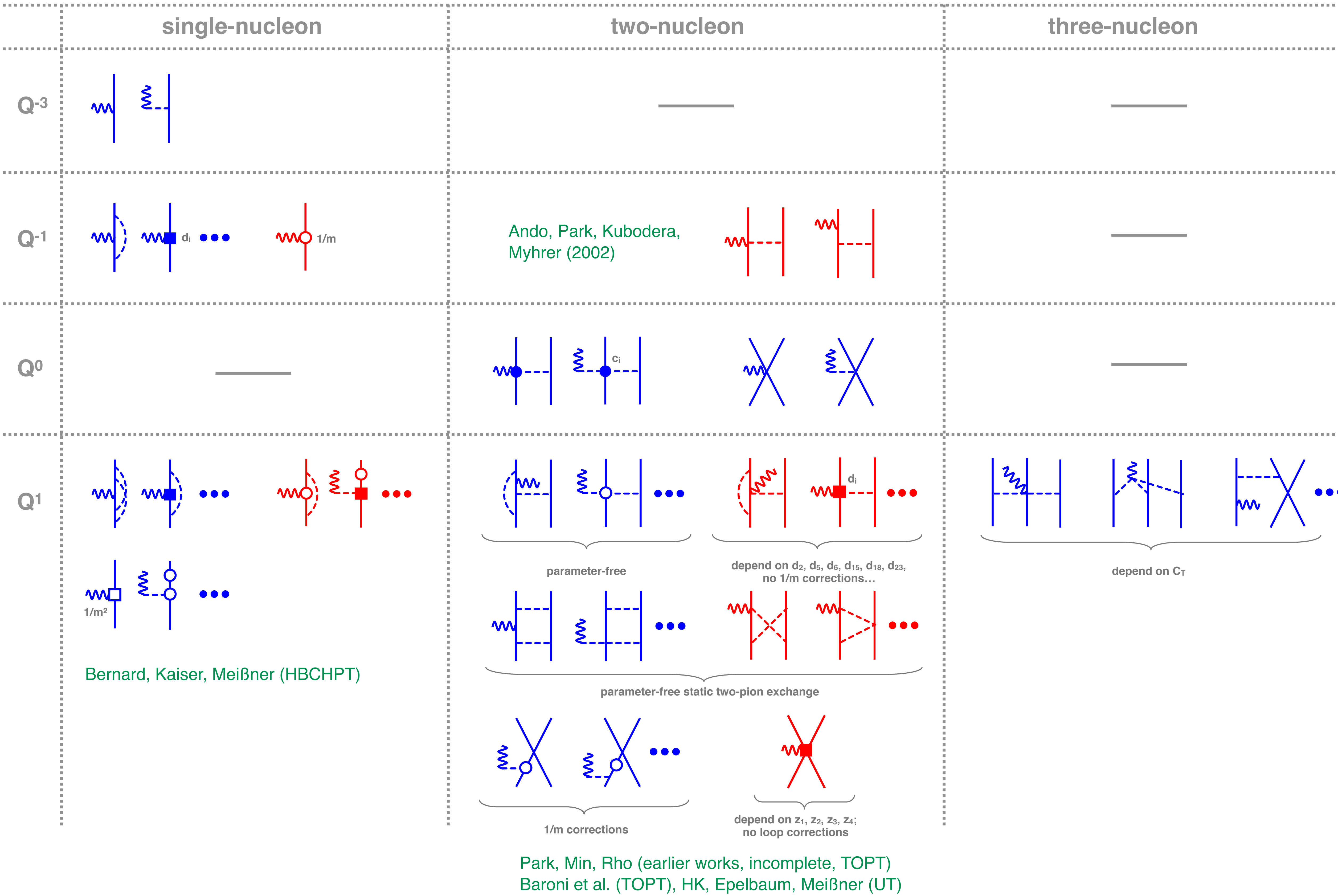
Vector currents in chiral EFT

Chiral expansion of the electromagnetic **current** and **charge** operators



Axial vector operators in chiral EFT

Chiral expansion of the axial vector **current** and **charge** operators



Compare with Baroni et al.

Baroni et al. PRC94 (2016) no. 2, 024003; Erratum PRC95 (2017) no. 5, 059902;
PRC93 (2016) no. 1, 015501; Erratum PRC93 (2016) no. 4, 049902

At zero momentum transfer the result of Baroni et al. is

$$\mathbf{j}_{\pm}^{\rm N4LO}(\mathrm{OPE};\mathbf{k}) = \frac{g_A^5\, m_\pi}{256\,\pi\, f_\pi^4}\left[18\,\tau_{2,\pm}\,\mathbf{k} - (\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_\pm\,\boldsymbol{\sigma}_1\times\mathbf{k}\right]\boldsymbol{\sigma}_2\cdot\mathbf{k}\,\frac{1}{\omega_k^2} + (1 \rightleftharpoons 2)\ , \tag{5}$$

$$\begin{aligned}\mathbf{j}_{\pm}^{\rm N4LO}(\mathrm{MPE};\mathbf{k}) = {} & \frac{g_A^3}{32\,\pi f_\pi^4}\,\tau_{2,\pm}\left[W_1(k)\,\boldsymbol{\sigma}_1 + W_2(k)\,\mathbf{k}\,\boldsymbol{\sigma}_1\cdot\mathbf{k} + Z_1(k)\left(2\,\mathbf{k}\,\boldsymbol{\sigma}_2\cdot\mathbf{k}\frac{1}{\omega_k^2} - \boldsymbol{\sigma}_2\right)\right] \\ & + \frac{g_A^5}{32\,\pi f_\pi^4}\,\tau_{1,\pm}\,W_3(k)\,(\boldsymbol{\sigma}_2\times\mathbf{k})\times\mathbf{k} - \frac{g_A^3}{32\,\pi f_\pi^4}\,(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_\pm\,Z_3(k)\,\boldsymbol{\sigma}_1\times\mathbf{k} \\ & \times\boldsymbol{\sigma}_2\cdot\mathbf{k}\,\frac{1}{\omega_k^2} + (1 \rightleftharpoons 2)\ ,\end{aligned} \tag{6}$$

$$\begin{aligned}
W_1(k) &= \frac{M_\pi}{2}\left(1+g_A^2\left(-9+\frac{4M_\pi^2}{k^2+4M_\pi^2}\right)\right)+\frac{1}{2}\left((1-5g_A^2)k^2+4(1-2g_A^2)M_\pi^2\right)A(k), \\
W_2(k) &= \frac{M_\pi}{2k^2(k^2+4M_\pi^2)}\left(\left(1+3g_A^2\right)k^2+4\left(1+2g_A^2\right)M_\pi^2\right)-\frac{1}{2k^2}\left((-1+g_A^2)k^2+4(1+2g_A^2)M_\pi^2\right)A(k) \\
W_3(k) &= \cancel{-\frac{1}{6M_\pi}-\frac{4}{3}A(k)} = -2A(k), \\
Z_1(k) &= 2M_\pi+2(k^2+2M_\pi^2)A(k), \\
Z_3(k) &= \frac{M_\pi}{2}+\frac{1}{2}(k^2+4M_\pi^2)A(k).
\end{aligned}$$

Baroni et al. PRC94 (2016) no. 2, 024003;
Erratum PRC95 (2017) no. 5, 059902

The current of Baroni et al. does ~~not~~ exist in the chiral limit!

$$\begin{aligned}\vec{j}_a^{\,\rm N4LO}(\mathrm{MPE},\vec{q}_1\,) - \vec{A}^{a\,(Q)}_{\rm 2N:2\pi} - \vec{A}^{a\,(Q)}_{\rm 2N:1\pi} &= -\vec{q}_1\,\frac{g_A^5 A(q_1)(4M_\pi^2+q_1^2)\vec{q}_1\cdot\vec{\sigma}_2\tau_1^a}{32\pi F_\pi^4 q_1^2} \\ &+ \ \text{rational function in }\vec{q}_1 + 1\leftrightarrow 2\end{aligned}$$

where $A(q) = \frac{1}{2q}\arctan\frac{q}{2M_\pi}$

Two currents have different long range parts!

Higher Derivative Regularization

Based on ideas: Slavnov, NPB31 (1971) 301;
Djukanovic et al. PRD72 (2005) 045002; Long and Mei PRC93 (2016) 044003

- Change leading order pion - Lagrangian (modify free part)

$$S_\pi^{(2)} = \int d^4x \frac{1}{2}\vec{\pi}(x)\left(-\partial^2 - M_\pi^2\right)\vec{\pi}(x) \to S_{\pi,\Lambda}^{(2)} = \int d^4x \frac{1}{2}\vec{\pi}(x)\left(-\partial^2 - M_\pi^2\right)\exp\left(\frac{\partial^2 + M_\pi^2}{\Lambda^2}\right)\vec{\pi}(x)$$

$$\frac{1}{q^2 + M_\pi^2} \to \frac{\exp\left(-\frac{q^2+M_\pi^2}{\Lambda^2}\right)}{q^2 + M_\pi^2}$$

$\mathcal{L}_{\pi,\Lambda}^{(2)}$ has to be invariant under $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \times \mathrm{U}(1)_\mathrm{V}$

- Every derivative should be covariant one
- Lagrangian $\mathcal{L}_{\pi,\Lambda}^{(2)}$ should be formulated in terms of $U(\vec{\pi}(x)) \in \mathrm{SU}(2)$

Gasser, Leutwyler ´84, ´85; Bernard, Kaiser, Meißner ´95

Building blocks $\chi = 2B(s + ip)$

$$\nabla_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu)U$$

Higher Derivative Lagrangian

- To construct a parity-conserving regulator it is convenient to work with building-blocks

$$u_\mu = i\, u^\dagger \nabla_\mu U u^\dagger, \quad D_\mu = \partial_\mu + \Gamma_\mu, \quad \Gamma_\mu = \frac{1}{2}\left[u^\dagger, \partial_\mu u\right] - \frac{i}{2} u^\dagger r_\mu u - \frac{i}{2} u\, l_\mu u^\dagger$$

$$\chi_\pm = u^\dagger \chi\, u^\dagger \pm u \chi^\dagger u, \quad \chi = 2B(s + i\, p), \quad u = \sqrt{U}, \quad \mathrm{ad}_A B = [A, B]$$

Possible ansatz for higher derivative pion Lagrangian

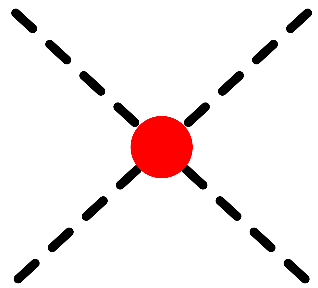
$$\mathcal{L}^{(2)}_{\pi,\Lambda} = \mathcal{L}^{(2)}_\pi + \frac{F^2}{4} \mathrm{Tr}\left[\mathrm{EOM} \frac{1 - \exp\left(\frac{\mathrm{ad}_{D_\mu} \mathrm{ad}_{D^\mu} + \frac{1}{2}\chi_+}{\Lambda^2}\right)}{\mathrm{ad}_{D_\mu} \mathrm{ad}_{D^\mu} + \frac{1}{2}\chi_+} \mathrm{EOM}\right]$$

$$\mathcal{L}^{(2)}_\pi = \frac{F^2}{4} \mathrm{Tr}\left[u_\mu u^\mu + \chi_+\right] \qquad \mathrm{EOM} = -\left[D_\mu, u^\mu\right] + \frac{i}{2}\chi_- - \frac{i}{4}\mathrm{Tr}\left(\chi_-\right)$$

Expand $\mathcal{L}^{(2)}_{\pi,\Lambda}$ in D_0 $\longrightarrow$ Lorentz-invariance only perturbatively

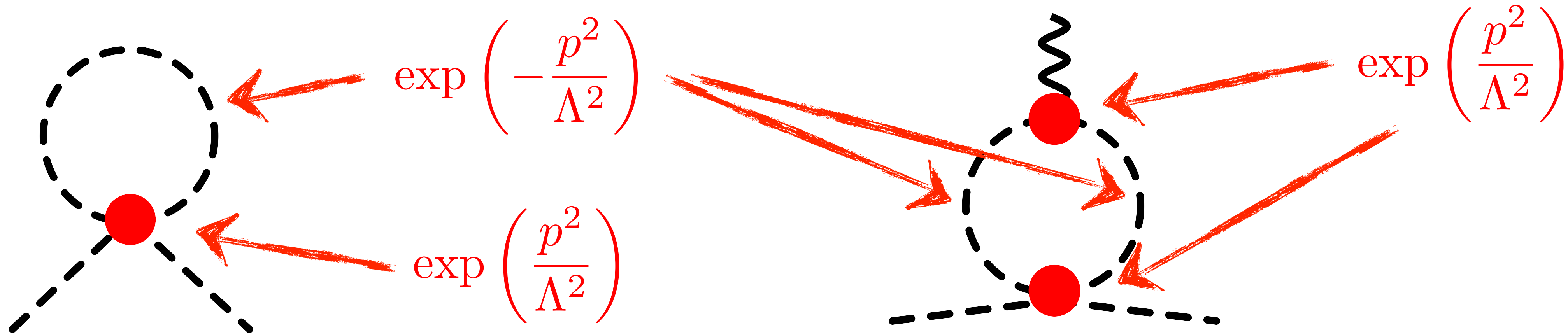
Use dimensional regularization on top of higher derivative one for regularization of zeroth component of momenta

Modified Vertices



- Enhanced by $\exp\left(\frac{p^2}{\Lambda^2}\right)$
- Every propagator is suppressed by $\exp\left(-\frac{p^2}{\Lambda^2}\right)$

Pionic sector becomes unregularized



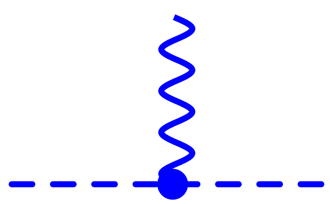
- Use dimensional on top of high derivative regularization
- Dimensional regularization will not affect effective potential and Schrödinger or LS equations but will regularize pionic sector

Regularization of Vector Current

- Modify pion-propagators in a vector current

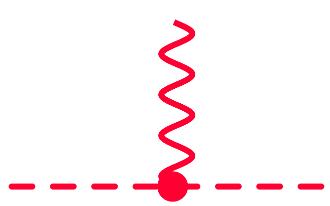
$$= \frac{1}{q^2+M^2} \to \frac{\exp\left(-\frac{q^2+M^2}{\Lambda^2}\right)}{q^2+M^2} =$$

- Modify two-pion-photon vertex



$$= e\,\epsilon_\mu (q_2^\mu - q_1^\mu)\epsilon_{3,a_1,a_2}$$

$\downarrow$

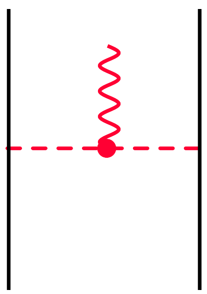


$$= e\,\epsilon_\mu (q_2^\mu - q_1^\mu)\epsilon_{3,a_1,a_2} \times \frac{1}{q_1^2 - q_2^2}\left[(q_1^2+M^2)\exp\left(\frac{q_1^2+M^2}{\Lambda^2}\right) - (q_2^2+M^2)\exp\left(\frac{q_2^2+M^2}{\Lambda^2}\right)\right]$$

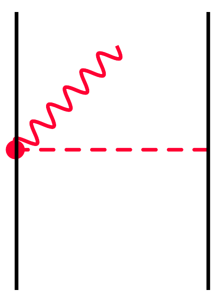
Modified two-pion-photon vertex leads to exponential increase in momenta

Regularization of Vector Current

Regularization of pion-exchange vector current



$$= \frac{i\,e\,g_A^2}{4F^2}\vec{q}_1\cdot\vec{\sigma}_1\vec{q}_2\cdot\vec{\sigma}_2[\tau_1\times\tau_2]_3\frac{\vec{\epsilon}\cdot(\vec{q}_2-\vec{q}_1)}{q_1^2-q_2^2}\left[\frac{\exp\left(-\frac{q_2^2+M^2}{\Lambda^2}\right)}{q_2^2+M^2}-\frac{\exp\left(-\frac{q_1^2+M^2}{\Lambda^2}\right)}{q_1^2+M^2}\right]$$



$$= -\frac{i\,e\,g_A^2}{4F^2}\vec{\epsilon}\cdot\vec{\sigma}_1\vec{q}_2\cdot\vec{\sigma}_2[\tau_1\times\tau_2]_3\frac{\exp\left(-\frac{q_1^2+M^2}{\Lambda^2}\right)}{q_1^2+M^2}+(1\leftrightarrow 2)$$

Riska prescription: longitudinal part of the current can be derived from continuity equation

Riska, Prog. Part. Nucl. Phys. 11 (1984) 199

$$\left[H_{\text{strong}},\boldsymbol{\rho}\right] = \vec{k}\cdot\vec{\boldsymbol{J}}$$

Higher orders $\longrightarrow$ work in progress

Summary on Nuclear Forces

- Chiral nuclear NN forces are calculated up to N^4LO
- Phase-shifts, deuteron properties, … with uncertainty quantification
- Chiral NN force match in precision phenomenological potentials (CD Bonn, Av18,…) with around 40% less parameter

Outlook

- Implementation of isospin breaking corrections
- Deltafull analysis up to N^3LO

Summary on Currents

- Electroweak currents are analyzed up to order Q
- There is a high degree of unitary ambiguity
- Modified continuity equation
- Renormalizability and matching to nuclear forces conditions lead to unique axial-vector current
- Differences in long range part between our results and Baroni et al.

Outlook

- Regularization and PWD of the currents
- Electroweak currents up to order Q^2

Uncertainty Estimate

Epelbaum, HK, Meißner '15

- Uncertainties in the experimental data
- Uncertainties in the estimation of πN LECs
- Uncertainties in the determination of contact interaction LECs
- Uncertainties of the fits due to the choice of E_{max}
- Systematic uncertainty due to truncation of the chiral expansion at a given order

Estimate the uncertainty via expected size of higher-order corrections

For a N^{4}LO prediction of an observable $X^{\mathrm{N^4LO}}$ we get an uncertainty

$$\begin{aligned}\Delta X^{\mathrm{N^4LO}}(p) \;=\; \max\Big(& Q \times |X^{\mathrm{N^3LO}}(p) - X^{\mathrm{N^4LO}}(p)|, Q^2 \times |X^{\mathrm{N^2LO}}(p) - X^{\mathrm{N^3LO}}(p)|, \\ & Q^3 \times |X^{\mathrm{NLO}}(p) - X^{\mathrm{N^2LO}}(p)|, Q^4 \times |X^{\mathrm{LO}}(p) - X^{\mathrm{NLO}}(p)|, Q^6 \times |X^{\mathrm{LO}}(p)|\Big)\end{aligned}$$

with chiral expansion parameter $Q = \max\left(\frac{p}{\Lambda_b}, \frac{M_\pi}{\Lambda_b}\right)$

For σ_{tot} errors $\rightarrow$ 68% degree-of-belief intervals(Bayesian analysis): *Furnstahl et al. '15*

Pion-Nucleon Scattering

- Effective chiral Lagrangian:

low-energy constants

$$\mathcal{L}_\pi = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \dots$$

$$\mathcal{L}_{\pi N} = \underbrace{\bar{N}\left(i\gamma^\mu D_\mu[\pi] - m + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu[\pi]\right)N}_{\mathcal{L}_{\pi N}^{(1)}} + \underbrace{\sum_i \boldsymbol{c}_i \bar{N}\hat{O}_i^{(2)}[\pi]N}_{\mathcal{L}_{\pi N}^{(2)}} + \underbrace{\sum_i \boldsymbol{d}_i \bar{N}\hat{O}_i^{(3)}[\pi]N}_{\mathcal{L}_{\pi N}^{(3)}} + \underbrace{\sum_i \boldsymbol{e}_i \bar{N}\hat{O}_i^{(4)}[\pi]N}_{\mathcal{L}_{\pi N}^{(4)}} + \dots$$

- Pion-nucleon scattering is calculated up to Q^4 in heavy-baryon ChPT

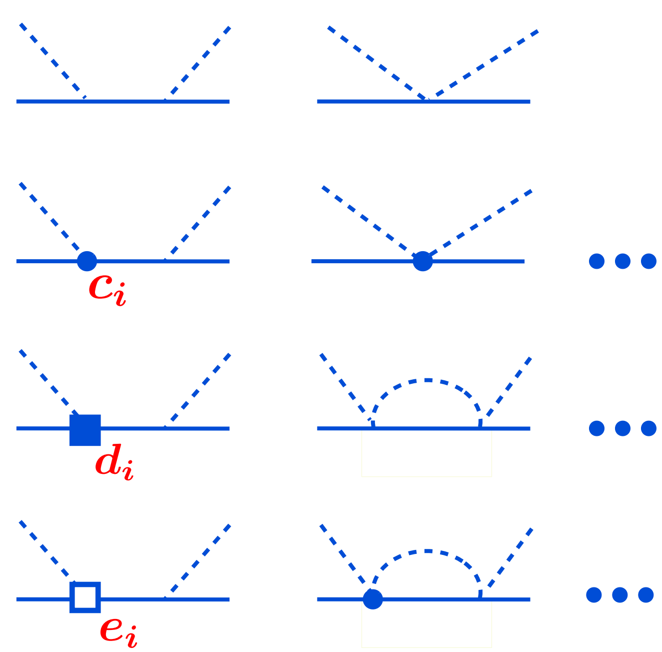
Fettes, Meißner '00; HK, Gasparyan, Epelbaum '12

Order Q:

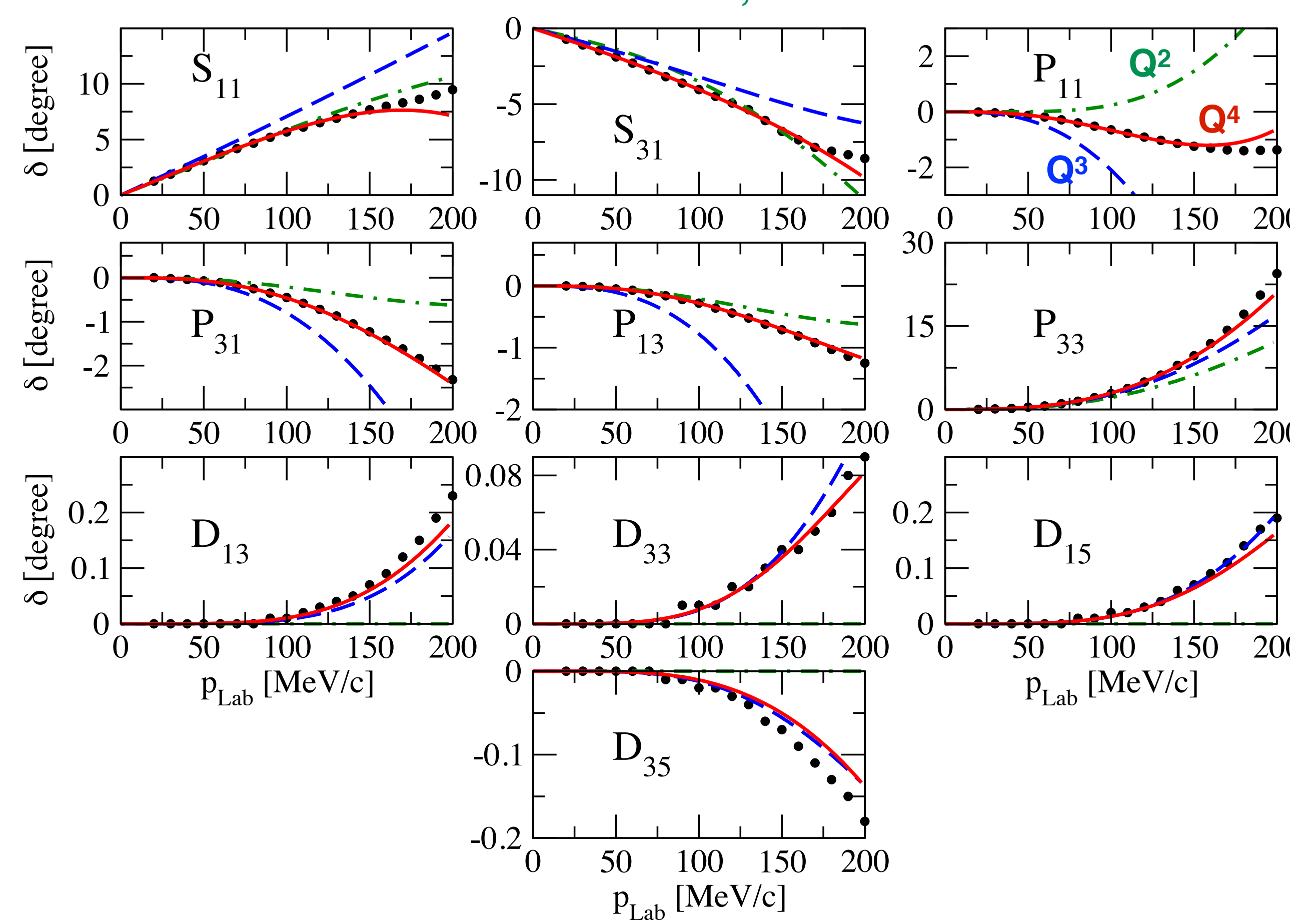
Order Q^2:

Order Q^3:

Order Q^4:



Karlsruhe-Helsinki PWA, Koch et al. '86

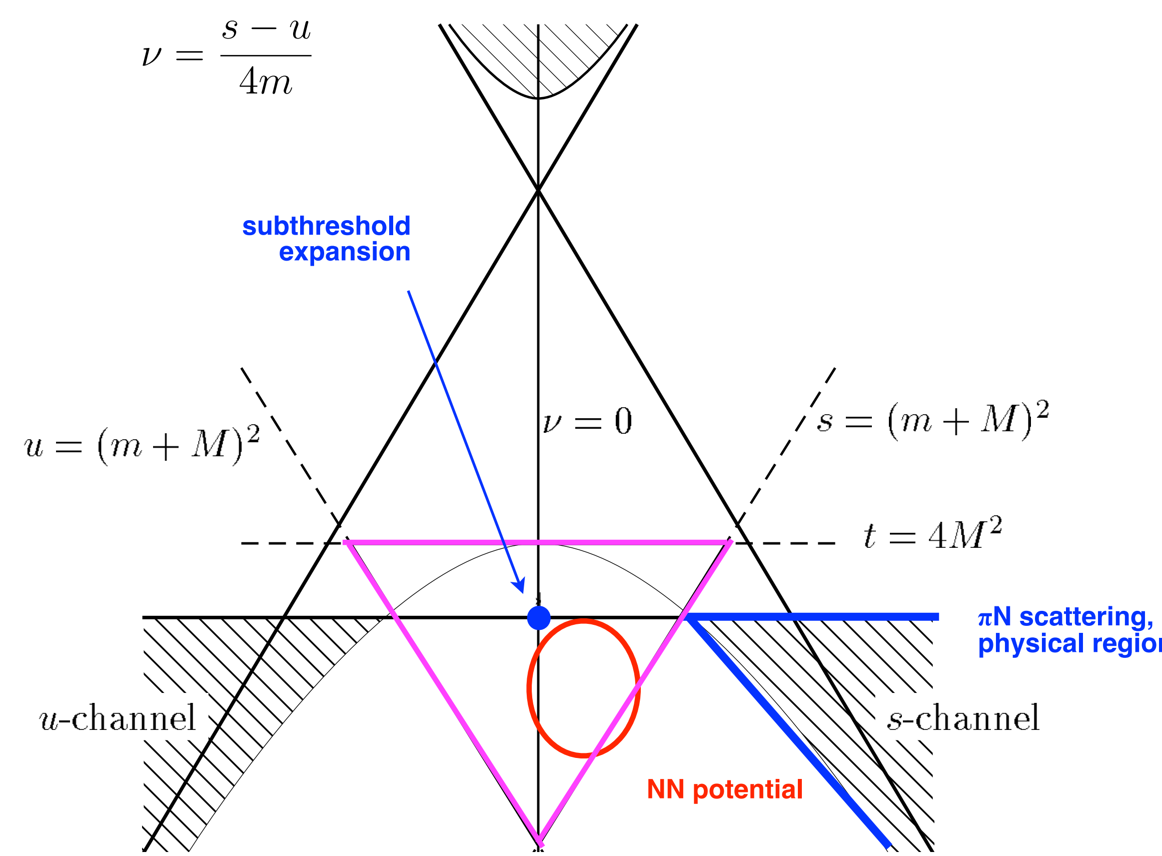


Dispersive analysis of πN scattering

- Roy-Steiner equations for πN scattering
  Hoferichter et al., Phys. Rept. 625 (16) 1

  Partial Wave Decomposition of
  Hyperbolic dispersion relations
  πN → πN & ππ → $\bar{\mathrm{N}}$N channels

  Input:
  S- and P-waves above $s_{\mathrm{m}} = (1.38\,\mathrm{GeV})^2$
  Higher partial waves for all *s*
  Inelasticities for $s < s_m$ and scattering lengths



  Output:
  S- and P-waves with error bands, σ-term,
  Subthreshold coefficients $\bar{X} = \sum_{m,n} x_{mn}\, \nu^{2m+k} t^n, \qquad X = \{A^{\pm},\, B^{\pm}\}$

- c_i, d_i, e_i are fixed from subthreshold coefficients (within Mandelstam triangle where one expects best convergence of chiral expansion)

- Subthreshold point is closer to kinematical region of NN force than the physical region of πN scattering

NN Data Used in the Fits

Reinert, HK, Epelbaum '17

- From 1950 on around 3000 proton-proton + 5000 neutron-proton scattering data below 350 MeV have been measured

- Not all of these data are compatible. Rejections are required to get a reasonable fit

- Granada 2013 base used: *Navarro Perez et al. '13* rejection by 3σ-criterion

  ⇨ 31% of np + 11% of pp data have been rejected

Resulting data base consists of 2697 np + 2158 pp data for E_{lab}=0-300 MeV