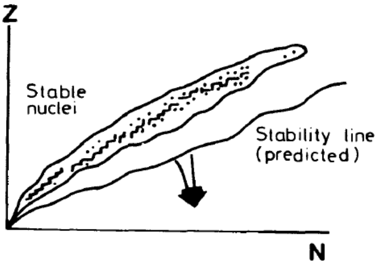
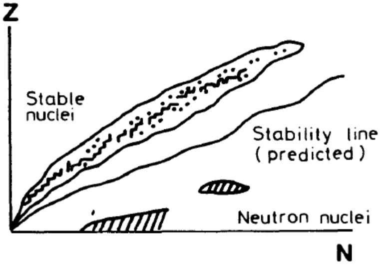
# NEUTRON CLUSTER.

Johannes Kirscher
יוהנס קירשר

The City College of New York



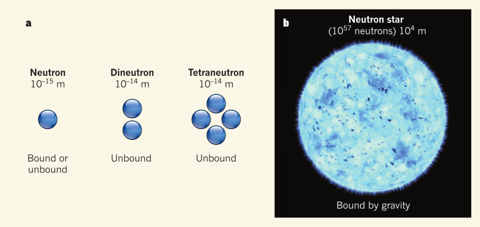




A. Ogloblin and Y. Penionzhkevich in Treatise on Heavy-Ion Science V. 8 (1989)

# NEUTRON CLUSTER.

Johannes Kirscher
יוהנס קירשר

The City College of New York

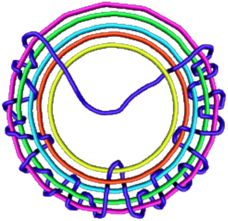



(C. BERTULANI, V. ZELEVINSKY, Nature 4/2016)

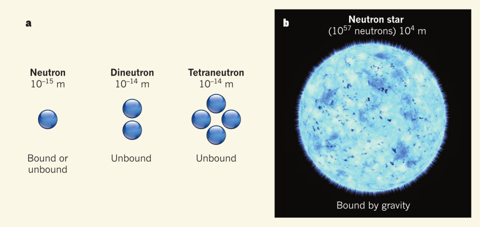
# Brunnian neutron cluster.

Johannes Kirscher
יוהנס קירשר

The City College of New York



(R. Scharein, KnotPlot)

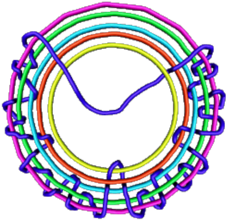



(C. BERTULANI, V. ZELEVINSKY, Nature 4/2016)

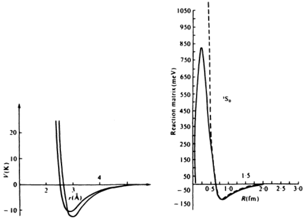
# Brunnian neutron cluster.

Johannes Kirscher
יוהנס קירשר

The City College of New York



(R. Scharein, KnotPlot)




A. Ogloblin and Y. Penionzhkevich in Treatise on Heavy-Ion Science V. 8 (1989)

# Theoretical search.

## Precision models of the nuclear force $\overset{?}{\Leftrightarrow}$ bound neutron cluster

I) Modify AV18/IL2[**]

- $\Delta V_{NN}(^1S_0)$ long range ($2\pi$ exchange)

  $\Rightarrow \quad \Delta\delta_{NN}(^1S_0) \approx 12^\circ$

  $\Rightarrow {}^4n \rightarrow {}^2n - {}^2n$

- $\Delta V_{NN}(^1S_0)$ short range

  $\Rightarrow {}^4n \rightarrow {}^2n - {}^2n$

- $V_{NNN}(T = 3/2)$

  $\Rightarrow B(^6\mathrm{Li}, {}^6\mathrm{He}) \gg B_{\mathrm{exp.}}$

- $V_{NNNN}(T = 2)$

  $\Rightarrow B(^5\mathrm{H}^{\|}) > 0$

II) Enhance[††] **only** $^3PF_2$ channel of AV14, Reid 93, Nijm II, AV18

Fine tuning is necessary to avoid devastating effects on other nuclear systems.

[**] S. C. Pieper, Phys. Rev. Lett. 90, 252501 (2003)

[††] R. Lazauskas and J. Carbonell, Phys. Rev. C 71, 044004 (2005)

# Theoretical search.

## Precision models of the nuclear force $\overset{?}{\Leftrightarrow}$ bound neutron cluster

I) Modify `AV18/IL2`[**]

•$\Delta V_{NN}(^1S_0)$ long range ($2\pi$ exchange)

$\Rightarrow \quad \Delta\delta_{NN}(^1S_0) \approx 12^\circ$

$\Rightarrow {}^4n \to {}^2n - {}^2n$

•$\Delta V_{NN}(^1S_0)$ short range

$\Rightarrow {}^4n \to {}^2n - {}^2n$

•$V_{NNN}(T = 3/2)$

$\Rightarrow B(^6\mathrm{Li}, ^6\mathrm{He}) \gg B_{\mathrm{exp.}}$

•$V_{NNNN}(T = 2)$

$\Rightarrow B(^5\mathrm{H}^{\|}) > 0$

II) Enhance[††] **only** $^3PF_2$ channel of `AV14, Reid 93, Nijm II, AV18`

Fine tuning is necessary to avoid devastating effects on other nuclear systems.

---

[**] S. C. Pieper, Phys. Rev. Lett. 90, 252501 (2003)

[††] R. Lazauskas and J. Carbonell, Phys. Rev. C 71, 044004 (2005)

# Theoretical search.

## Precision models of the nuclear force $\overset{?}{\Leftrightarrow}$ bound neutron cluster

I) Modify `AV18/IL2`**

• $\Delta V_{NN}(^1S_0)$ long range ($2\pi$ exchange)

$\Rightarrow \quad \Delta\delta_{NN}(^1S_0) \approx 12^\circ$

$\Rightarrow {}^4n \rightarrow {}^2n - {}^2n$

• $\Delta V_{NN}(^1S_0)$ short range

$\Rightarrow {}^4n \rightarrow {}^2n - {}^2n$

• $V_{NNN}(T = 3/2)$

$\Rightarrow B(^6\text{Li}, ^6\text{He}) \gg B_{\text{exp.}}$

• $V_{NNNN}(T = 2)$

$\Rightarrow B(^5\text{H}^{||}) > 0$

II) Enhance†† **only** $^3PF_2$ channel of `AV14, Reid 93, Nijm II, AV18`

Fine tuning is necessary to avoid devastating effects on other nuclear systems.

** S. C. Pieper, Phys. Rev. Lett. 90, 252501 (2003)

†† R. Lazauskas and J. Carbonell, Phys. Rev. C 71, 044004 (2005)

# Theoretical search.

## Precision models of the nuclear force $\overset{?}{\Leftrightarrow}$ bound neutron cluster

I) Modify `AV18/IL2`[**]

- $\Delta V_{NN}(^1S_0)$ long range ($2\pi$ exchange)

  $\Rightarrow \quad \Delta\delta_{NN}(^1S_0) \approx 12^\circ$

  $\Rightarrow {}^4n \to {}^2n - {}^2n$

- $\Delta V_{NN}(^1S_0)$ short range

  $\Rightarrow {}^4n \to {}^2n - {}^2n$

- $V_{NNN}(T = 3/2)$

  $\Rightarrow B(^6\mathrm{Li}, {}^6\mathrm{He}) \gg B_{\mathrm{exp.}}$

- $V_{NNNN}(T = 2)$

  $\Rightarrow B(^5\mathrm{H}^{\|}) > 0$

II) Enhance[††] **only** $^3PF_2$ channel of `AV14, Reid 93, Nijm II, AV18`

Fine tuning is necessary to avoid devastating effects on other nuclear systems.

---

[**] S. C. Pieper, Phys. Rev. Lett. 90, 252501 (2003)

[††] R. Lazauskas and J. Carbonell, Phys. Rev. C 71, 044004 (2005)

# Theoretical search.

## Precision models of the nuclear force $\overset{?}{\Leftrightarrow}$ bound neutron cluster

I) Modify `AV18/IL2`**

•$\Delta V_{NN}(^1S_0)$ long range ($2\pi$ exchange)

$\Rightarrow \quad \Delta\delta_{NN}(^1S_0) \approx 12^\circ$

$\Rightarrow {}^4n \to {}^2n - {}^2n$

•$\Delta V_{NN}(^1S_0)$ short range

$\Rightarrow {}^4n \to {}^2n - {}^2n$

•$V_{NNN}(T = 3/2)$

$\Rightarrow B(^6\text{Li}, ^6\text{He}) \gg B_{\text{exp.}}$

•$V_{NNNN}(T = 2)$

$\Rightarrow B(^5\text{H}^{\|}) > 0$

II) Enhance†† **only** $^3PF_2$ channel of `AV14, Reid 93, Nijm II, AV18`

Fine tuning is necessary to avoid devastating effects on other nuclear systems.

---

** S. C. Pieper, Phys. Rev. Lett. 90, 252501 (2003)

†† R. Lazauskas and J. Carbonell, Phys. Rev. C 71, 044004 (2005)

‖ A. H. Wuosmaa *et al.*, Phys. Rev. C 95, 014310 (2017)

# Theoretical search.

## Precision models of the nuclear force $\overset{?}{\Leftrightarrow}$ bound neutron cluster

I) Modify `AV18/IL2`**

•$\Delta V_{NN}(^1S_0)$ long range ($2\pi$ exchange)

$\Rightarrow \quad \Delta\delta_{NN}(^1S_0) \approx 12^\circ$

$\Rightarrow {}^4n \to {}^2n - {}^2n$

•$\Delta V_{NN}(^1S_0)$ short range

$\Rightarrow {}^4n \to {}^2n - {}^2n$

•$V_{NNN}(T = 3/2)$

$\Rightarrow B(^6\text{Li}, ^6\text{He}) \gg B_{\text{exp.}}$

•$V_{NNNN}(T = 2)$

$\Rightarrow B(^5\text{H}^{\|}) > 0$

II) Enhance†† **only** $^3PF_2$ channel of `AV14, Reid 93, Nijm II, AV18`

$\Downarrow$

stable $^3n$

and

unbound $^2n$

Fine tuning is necessary to avoid devastating effects on other nuclear systems.

---

** S. C. Pieper, Phys. Rev. Lett. 90, 252501 (2003)

†† R. Lazauskas and J. Carbonell, Phys. Rev. C 71, 044004 (2005)

‖ A. H. Wuosmaa *et al.*, Phys. Rev. C 95, 014310 (2017)

# Theoretical search.

## Precision models of the nuclear force $\overset{?}{\Leftrightarrow}$ bound neutron cluster

I) Modify `AV18/IL2`**

- $\Delta V_{NN}(^1S_0)$ long range ($2\pi$ exchange)

  $\Rightarrow \quad \Delta\delta_{NN}(^1S_0) \approx 12^\circ$

  $\Rightarrow {}^4n \rightarrow {}^2n - {}^2n$

- $\Delta V_{NN}(^1S_0)$ short range

  $\Rightarrow {}^4n \rightarrow {}^2n - {}^2n$

- $V_{NNN}(T = 3/2)$

  $\Rightarrow B(^6\text{Li}, ^6\text{He}) \gg B_{\text{exp.}}$

- $V_{NNNN}(T = 2)$

  $\Rightarrow B(^5\text{H}^{\|}) > 0$

II) Enhance†† **only** $^3PF_2$ channel of `AV14, Reid 93, Nijm II, AV18`

$\Downarrow$

stable $^3n$

and

unbound $^2n$

Fine tuning is necessary to avoid devastating effects on other nuclear systems.

** S. C. Pieper, Phys. Rev. Lett. 90, 252501 (2003)

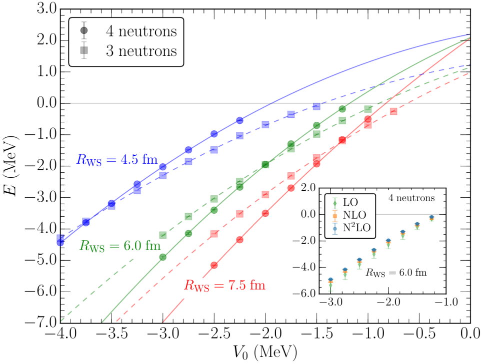
†† R. Lazauskas and J. Carbonell, Phys. Rev. C 71, 044004 (2005)

‖ A. H. Wuosmaa *et al.*, Phys. Rev. C 95, 014310 (2017)

# Theoretical search.

## Hierarchy‡‡ between 3- and 4-neutron resonances.

$$V_{\text{trap}}^{\text{Woods-Saxon}} + V_{nn}^{\chi} + V_{nnn}^{\chi}$$

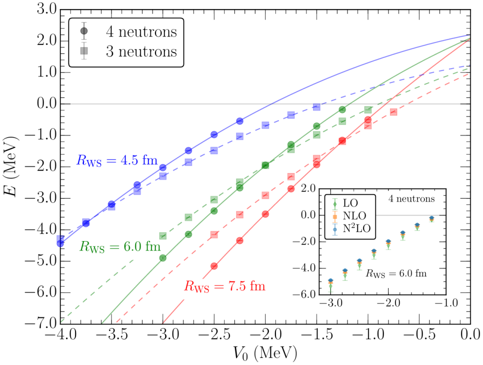



‡‡S. Gandolfi, H.-W. Hammer, P. Klos, J. E. Lynn, A. Schwenk, Phys. Rev. Lett. 118, 232501 (2017)

# Theoretical search.

## Hierarchy[‡‡] between 3- and 4-neutron resonances.

$$V_{\text{trap}}^{\text{Woods-Saxon}} + V_{nn}^{\chi} + V_{nnn}^{\chi}$$




$$E^*(^3n) < E^*(^4n)$$

‡‡ S. Gandolfi, H.-W. Hammer, P. Klos, J. E. Lynn, A. Schwenk, Phys. Rev. Lett. 118, 232501 (2017)

# Search for a stable neutron cluster.

How do significant $^{2}n$ uncertainties translate to the $^{N}n$ systems?

i) Relevant uncertainties.
ii) Appropriate parameterization of $\Delta(^{2}n)$.
iii) Most susceptible $^{N}n$ observables.

# Search for a stable neutron cluster.

How do significant $^2n$ uncertainties translate to the $^Nn$ systems?

i) Relevant uncertainties.
ii) Appropriate parameterization of $\Delta(^2n)$.
iii) Most susceptible $^Nn$ observables.

# Search for a stable neutron cluster.

How do significant $^2n$ uncertainties translate to the $^Nn$ systems?

i) Relevant uncertainties.

ii) Appropriate parameterization of $\Delta(^2n)$.

iii) Most susceptible $^Nn$ observables.

# Search for a stable neutron cluster.

How do significant $^2n$ uncertainties translate to the $^Nn$ systems?

i) Relevant uncertainties.
ii) Appropriate parameterization of $\Delta(^2n)$.
iii) Most susceptible $^Nn$ observables.

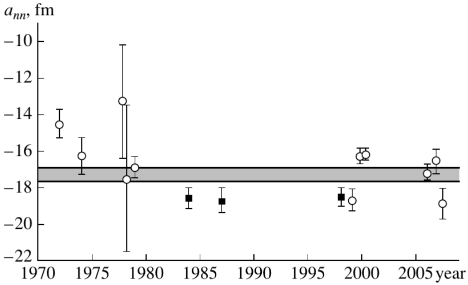
# Search for a stable neutron cluster.

How do significant $^2n$ uncertainties translate to the $^Nn$ systems?

i) Relevant uncertainties.

ii) Appropriate parameterization of $\Delta(^2n)$.

iii) Most susceptible $^Nn$ observables.




$$\Delta\left[a_{nn}(^1S_0)\right] \approx 3\%$$

* E. S. Konobeevski* , Yu. M. Burmistrov, S. V. Zuyev, M. V. Mordovskoy, and S. I. Potashev, Phys. At. Nuc., 73, 8, (2010)

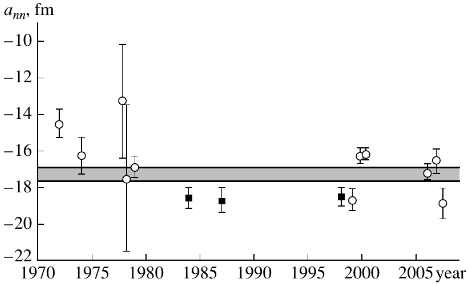
# SEARCH FOR A STABLE NEUTRON CLUSTER.

How do significant $^2n$ uncertainties translate to the $^Nn$ systems?

i) Relevant uncertainties.

ii) Appropriate parameterization of $\Delta(^2n)$.

iii) Most susceptible $^Nn$ observables.




$$\Delta\left[a_{nn}(^1S_0)\right] \approx 3\% \overset{\text{assume}}{\approx} \Delta\left[a_{nn}(^3P_J)\right]$$

* E. S. Konobeevski* , Yu. M. Burmistrov, S. V. Zuyev, M. V. Mordovskoy, and S. I. Potashev, Phys. At. Nuc., 73, 8, (2010)

# Search for a stable neutron cluster.

How do significant $^2n$ uncertainties translate to the $^Nn$ systems?

i) Relevant uncertainties.

ii) Appropriate parameterization of $\Delta(^2n)$.

iii) Most susceptible $^Nn$ observables.



$$S^I_\sigma = 2y_{ij}y_{jk}y_{ki} + \tfrac{2}{3}\sum_{cyc}(r^2_{ij}t_{ij}y_{jk}y_{ki} + C^2_j t_{ij}t_{jk}y_{ki}) - \tfrac{2}{3}C_iC_jC_k t_{ij}t_{jk}t_{ki}$$

$$+\left[\sum_{cyc}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\right]\left[\tfrac{2}{3}y_{ij}y_{jk}y_{ki} + \tfrac{1}{3}\sum_{cyc} r^2_{ij}t_{ij}y_{jk}y_{ki}\right] + \tfrac{1}{3}\sum_{cyc}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k C^2_j t_{ij}t_{jk}y_{ki}$$

$$-\tfrac{1}{3}\sum_{cyc}(\boldsymbol{\sigma}_i\cdot\mathbf{r}_{ij}\boldsymbol{\sigma}_j\cdot\mathbf{r}_{ij}t_{ij}y_{ki}y_{jk} + \boldsymbol{\sigma}_i\cdot\mathbf{r}_{ki}\boldsymbol{\sigma}_j\cdot\mathbf{r}_{ki}t_{ki}y_{jk}y_{ij} + \boldsymbol{\sigma}_i\cdot\mathbf{r}_{jk}\boldsymbol{\sigma}_j\cdot\mathbf{r}_{jk}t_{jk}y_{ij}y_{ki})$$

$$+\tfrac{1}{3}\sum_{cyc}C_k\boldsymbol{\sigma}_i\cdot\mathbf{r}_{jk}\boldsymbol{\sigma}_j\cdot\mathbf{r}_{ki}t_{ki}t_{jk}y_{ij} + \tfrac{1}{3}\sum_{cyc}\boldsymbol{\sigma}_i\cdot\mathbf{a}\boldsymbol{\sigma}_j\cdot\mathbf{a}(t_{ij}t_{jk}y_{ki} + t_{ij}y_{jk}t_{ki} + C_k t_{ij}t_{jk}t_{ki}), \qquad \text{(A4)}$$

$A^I_\sigma = \frac{i}{3}[$

$+$

and



### IV. PROJECTION INTO OPERATOR FORMAT

We can project the strong interaction potential given above from $S, T, T_z$ states into an operator format with 18 terms

$$v_{ij} = \sum_{p=1,18} v_p(r_{ij})O^p_{ij} . \qquad (25)$$

Here the first 14 operators are the same charge-independent ones used in the Argonne $v_{14}$ potential and are given by

$$O^{p=1,14}_{ij} = 1, \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j, \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j, (\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j), S_{ij}, S_{ij}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j), \mathbf{L}\cdot\mathbf{S}, \mathbf{L}\cdot\mathbf{S}(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j),$$
$$L^2, L^2(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j), L^2(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j), L^2(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j), (\mathbf{L}\cdot\mathbf{S})^2, (\mathbf{L}\cdot\mathbf{S})^2(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j) . \qquad (26)$$

# Search for a stable neutron cluster.

How do significant $^2n$ uncertainties translate to the $^Nn$ systems?

i) Relevant uncertainties.

ii) Appropriate parameterization of $\Delta(^2n)$.

iii) Most susceptible $^Nn$ observables.

For $\vec{k}_{nn} \to 0$, use EFT($\not{\pi}$):

$$\hat{H}_{\rm nucl} = -\sum_{i}^{A} \frac{\boldsymbol{\nabla}_i^2}{2m} + \sum_{i<j}^{A} \left[c_S^{\Lambda}\,\hat{P}_{S=0} + c_T^{\Lambda}\,\hat{P}_{S=1}\right] e^{-\frac{\Lambda^2}{4} r_{ij}^2} + \sum_{i<j<k}^{A} \sum_{\rm cyc} d_3^{\Lambda}\left(1/2 - 1/6\,\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j\right) e^{-\frac{\Lambda^2}{4}\left(r_{ij}^2 + r_{ik}^2\right)}$$

$$+ \sum_{i<j}^{A} \hat{P}_{T_z=-1}\;\left[\epsilon_1\,\boldsymbol{L}\cdot\boldsymbol{S} + \epsilon_2\,\hat{S}_{ij}\right] e^{-\frac{\Lambda^2}{4} r_{ij}^2}$$

(P. F. Bedaque, J.-W. Chen, H. W. Hammer, D. B. Kaplan, U. van Kolck, G. Rupak, M. J. Savage (199x-201y))

# Search for a stable neutron cluster.

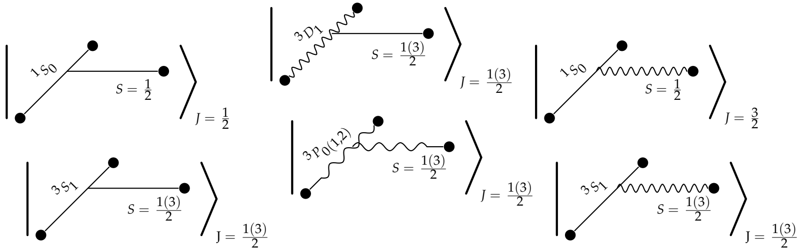
How do significant $^2n$ uncertainties translate to the $^Nn$ systems?

i) Relevant uncertainties.

ii) Appropriate parameterization of $\Delta(^2n)$.

iii) Most susceptible $^Nn$ observables.

$$J^\pi = \frac{1}{2}^+ : \qquad {}^3\mathrm{H}$$



# Search for a stable neutron cluster.

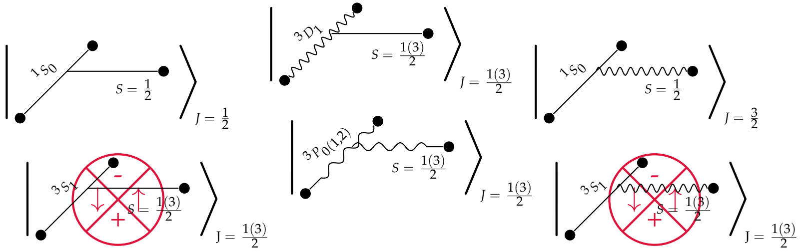
How do significant $^2n$ uncertainties translate to the $^Nn$ systems?

i) Relevant uncertainties.

ii) Appropriate parameterization of $\Delta(^2n)$.

iii) Most susceptible $^Nn$ observables.

$$J^\pi = \frac{1}{2}^+: \qquad {}^3n$$



# Search for a stable neutron cluster.

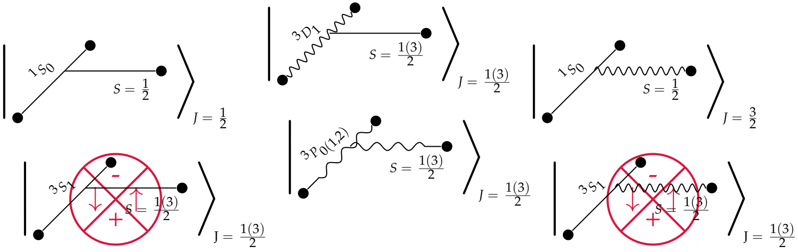
How do significant $^{2}n$ uncertainties translate to the $^{N}n$ systems?

i) Relevant uncertainties.

ii) Appropriate parameterization of $\Delta(^{2}n)$.

iii) Most susceptible $^{N}n$ observables.

$$J^{\pi} = \frac{1}{2}^{+}: \qquad {}^{3}n$$




2 consequences of anti-symmetrization { Exclusion of components. / Sign inversion of effective interactions.

# Search for a stable neutron cluster.

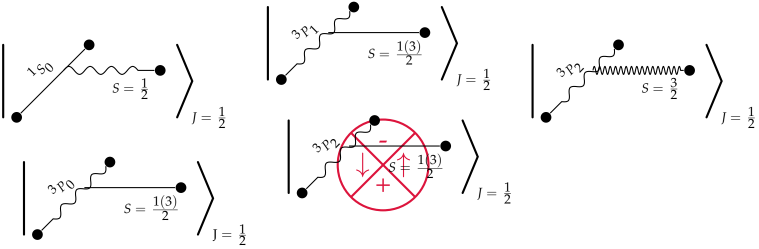
How do significant $^{2}n$ uncertainties translate to the $^{N}n$ systems?

i) Relevant uncertainties.

ii) Appropriate parameterization of $\Delta(^{2}n)$.

iii) Most susceptible $^{N}n$ observables.

$$J^{\pi} = \frac{1}{2}^{-}: \qquad {}^{3}n$$



# SEARCH FOR A STABLE NEUTRON CLUSTER.

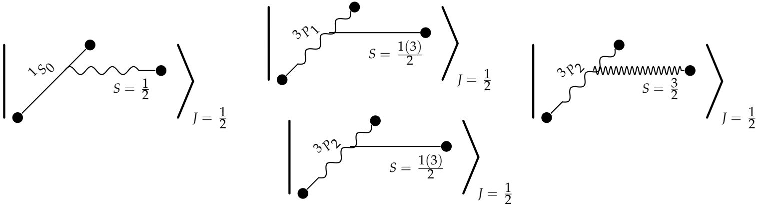
How do significant $^2n$ uncertainties translate to the $^Nn$ systems?

i) Relevant uncertainties.

ii) Appropriate parameterization of $\Delta(^2n)$.

iii) Most susceptible $^Nn$ observables.

$$J^\pi = \frac{3}{2}^- : \qquad ^3n$$



# Search for a stable neutron cluster.

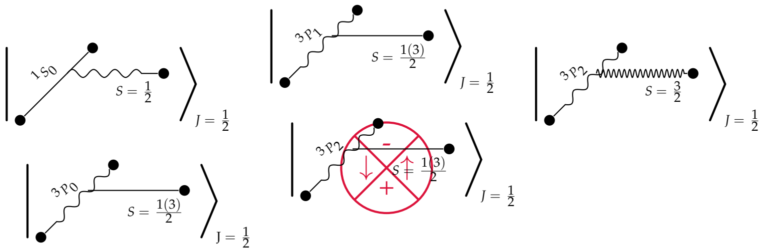
How do significant $^2n$ uncertainties translate to the $^Nn$ systems?

i) Relevant uncertainties.

ii) Appropriate parameterization of $\Delta(^2n)$.

iii) Most susceptible $^Nn$ observables.

$$J^\pi = \frac{1}{2}^- : \qquad ^3n$$

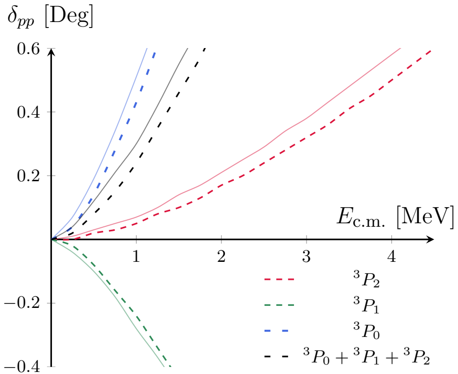



**Lore –** $J^\pi$ of ground state maximizes the number of configurations in minimal angular-momentum states with attractive pair and effective interactions.

"A bound state has no overlap with a basis state which contains repellent – assuming E-independence – substructures."

# Isospin symmetry.

## Effect of a (non-perturbative) tensor interaction: **neutron-neutron** vs. **proton-proton**



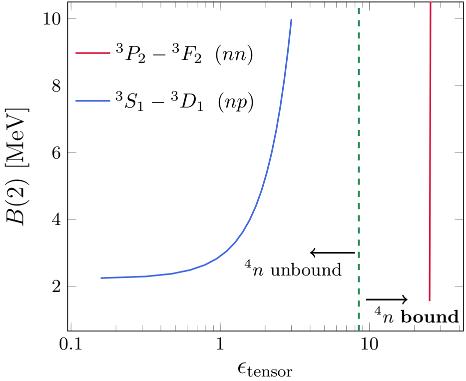

i) Comparison between PWA (- - -) and EFT($\not\pi$) (**tetra-neutron** critical, —) :

Seemingly small difference in low-energy $P$-wave phase shifts.

† Workman, Briscoe, Strakovsky, NN PWA, PRC (2016)

# Isospin symmetry.

## Effect of a (non-perturbative) tensor interaction: **deuteron** & **neuteron**



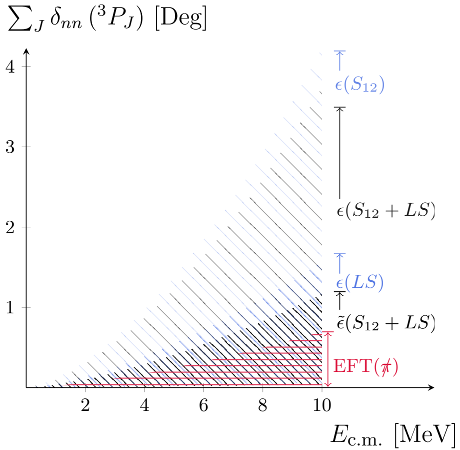

i) Coupled "low-$L$" neutron-proton system most $\epsilon$-sensitive;

ii) Many "higher-$L$" pairs more sensitive than one;

iii) Hence, without $\hat{P}_{T_z=-1}$, the iterated tensor is inconsistent with $B(np) = 2.22$ MeV.

# Isospin symmetry.

Comparison of renormalization schemes: $\epsilon_{\text{spin-orbit}}$ , $\epsilon_{\text{tensor}}$ , $\epsilon_{LS+S_{12}}$ & $\Lambda$.

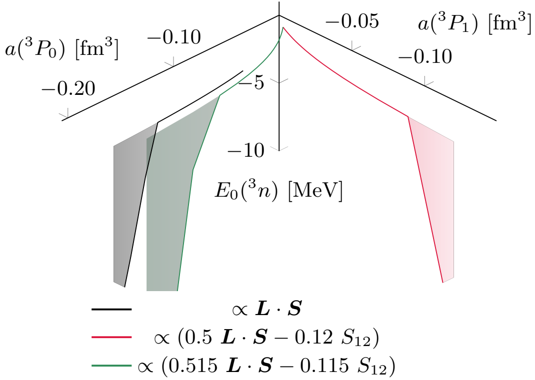



i) $\lim_{\Lambda\to\infty} \delta_{nn}(P,\text{contact}) = 0$ **but** $\lim_{\Lambda\to\infty} \delta_{nn}(P,\epsilon_c) = \text{finite}$

ii) The tetraneutron can be stabilized with an interaction with smaller impact on the same 2-neutron partial wave.

# Stability through increasing neutron number.

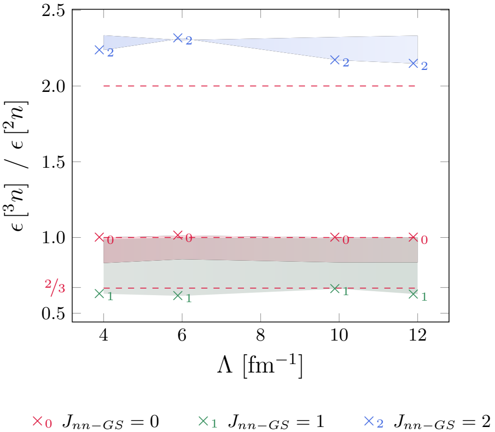
## Comparison of critical interaction strengths: **dineutron & trineutron**




i) An increasing attraction in the relative *nn* *P*-wave binds the trineutron ($J^\pi = \frac{1}{2}^-$) before **any** $^3P_J$ dineutron.

# Stability through increasing neutron number.

## Comparison of critical interaction strengths: **dineutron & trineutron**



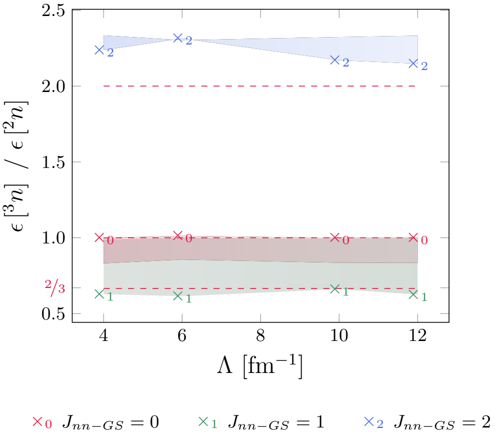

i) Effective $^3P_2 \overset{D}{-} n$ interaction too weak to stabilize $^3n$ before $^2n$ ...

ii) The $\epsilon$ disturbance **cannot** overcome the limit set by the multiplicity of a $^3P_0$ or $^3P_1$ **dineutron** in the $\frac{1}{2}^-$ **trineutron** state.

iii) $\Lambda$ (RG) invariance.

# Stability through increasing neutron number.

## Comparison of critical interaction strengths: **dineutron & trineutron**



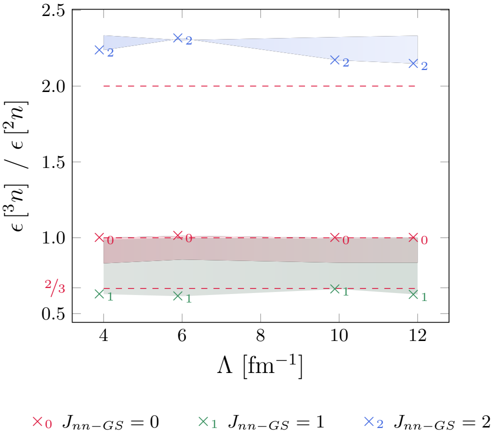

i) and for $J^\pi = \frac{1}{2}^-$, there is **no** contribution from a $^3P_2 \overset{S}{-} n$ state.

ii) The $\epsilon$ disturbance **cannot** overcome the limit set by the multiplicity of a $^3P_0$ or $^3P_1$ **dineutron** in the $\frac{1}{2}^-$ **trineutron** state.

iii) $\Lambda$ (RG) invariance.

# Stability through increasing neutron number.

Comparison of critical interaction strengths: **dineutron & trineutron**



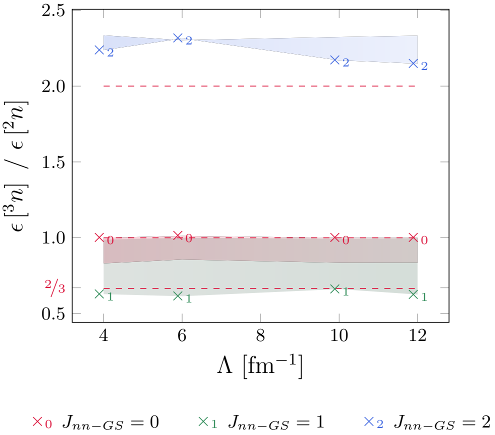

i) and for $J^\pi = \frac{1}{2}^-$, there is **no** contribution from a $^3P_2 \overset{S}{-} n$ state.

ii) The $\epsilon$ disturbance **cannot** overcome the limit set by the multiplicity of a $^3P_0$ or $^3P_1$ **dineutron** in the $\frac{1}{2}^-$ **trineutron** state.

iii) $\Lambda$ (RG) invariance.

# Stability through increasing neutron number.

## Comparison of critical interaction strengths: **dineutron & trineutron**



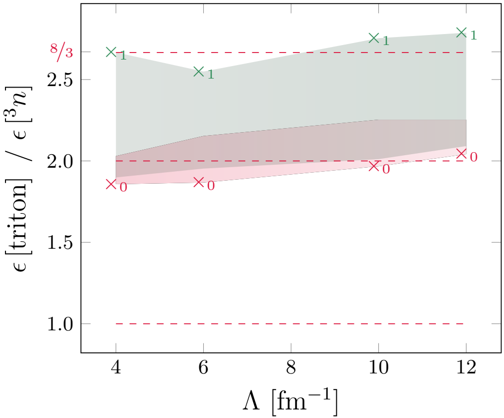

i) and for $J^\pi = \frac{1}{2}^-$, there is **no** contribution from a $^3P_2 \overset{S}{-} n$ state.

ii) The $\epsilon$ disturbance **cannot** overcome the limit set by the multiplicity of a $^3P_0$ or $^3P_1$ **dineutron** in the $\frac{1}{2}^-$ **trineutron** state.

iii) $\Lambda$ (RG) invariance.

# Stability through increasing neutron number.

### Comparison of critical interaction strengths: **triton** & **trineutron**

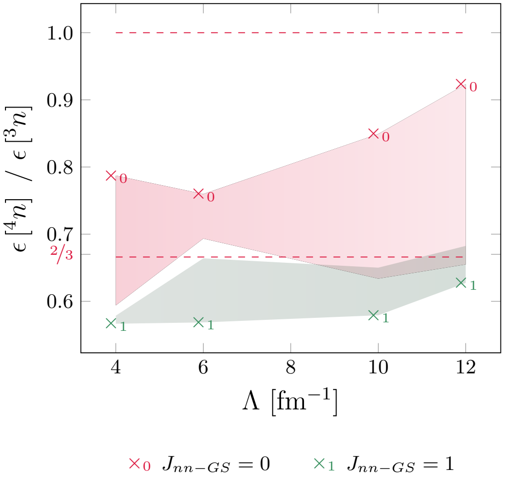



$\times_0$ $J_{nn-GS} = 0$ —— $J_{nn-GS} = 1$

The stability of $^3$H reflects the insignificance of relative *P*-waves in its ground state.

# Stability through increasing neutron number.

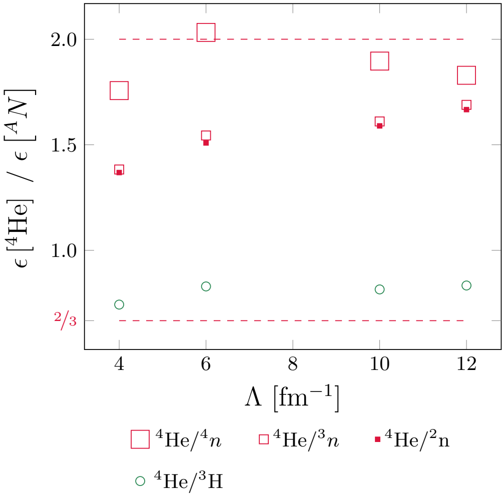
## Comparison of critical interaction strengths: **trineutron** & **tetraneutron**




The tetraneutron is significantly more **sensitive** *wrt.* *P*-wave attraction compared with the trineutron.

# Stability through increasing neutron number.

## Comparison of critical interaction strengths: ${}^{A<4}n$ & **tetraneutron**




Numerical refinements $\Rightarrow$ $\Delta\epsilon(A) < \Delta\epsilon(A') < 0$ for $A' < A$.

# EFFECTIVE $^2n - {}^2n$ AND $^3n - n$ INTERACTIONS

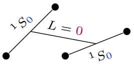
$\epsilon$ = "UNPHYSICAL" TO BIND THE FRAGMENTS.

Attractive $nn$ $\begin{smallmatrix} {}^1S_0 \\ {}^3P_\Sigma \end{smallmatrix}$ imply effective trimer-fermion and dimer-dimer interactions $\Rightarrow$
Ground-state quantum numbers of $^4n$.

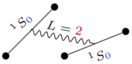
# Effective $^2n-{}^2n$ and $^3n-n$ interactions

$\epsilon$ = "unphysical" to bind the fragments.

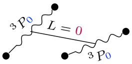
Attractive $nn$ $\begin{matrix}{}^1S_0\\ {}^3P_\Sigma\end{matrix}$ imply effective trimer-fermion and dimer-dimer interactions ⇒ Ground-state quantum numbers of $^4n$.



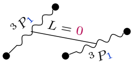

$[0 \otimes 0]^0 \otimes 0]^0$
$\left[ \left[\frac{1}{2} \otimes \frac{1}{2}\right]^0 \otimes \left[\frac{1}{2} \otimes \frac{1}{2}\right]^0 \right]^0$




$[[0 \otimes 0]^0 \otimes 2]^2$
$\Rightarrow J^\pi = 2^+ \neq 0^+$




$[1 \otimes 0]^1 \otimes 1]^{0,1,2}$
$\left[ \left[\frac{1}{2} \otimes \frac{1}{2}\right]^1 \otimes \left[\frac{1}{2} \otimes \frac{1}{2}\right]^1 \right]^{0,1,2}$




$[1 \otimes 0]^1 \otimes 1]^{0,1,2}$
$\left[ \left[\frac{1}{2} \otimes \frac{1}{2}\right]^1 \otimes \left[\frac{1}{2} \otimes \frac{1}{2}\right]^1 \right]^{0,1,2}$

# Effective $^2n - {}^2n$ and $^3n - n$ interactions

### $\epsilon$ = "unphysical" to bind the fragments.

Attractive $nn$ $\begin{matrix} ^1S_0 \\ ^3P_\Sigma \end{matrix}$ imply effective trimer-fermion and dimer-dimer interactions $\Rightarrow$ Ground-state quantum numbers of $^4n$.


$[0 \otimes 1]^1 \otimes 1]^1$ $\left[\left[\left[\frac{1}{2} \otimes \frac{1}{2}\right]^0 \otimes \frac{1}{2}\right]^{\frac{1}{2}} \otimes \frac{1}{2}\right]^1$

$[1 \otimes 0]^1 \otimes 1]^{0,1,2}$ $\left[\left[\left[\frac{1}{2} \otimes \frac{1}{2}\right]^1 \otimes \frac{1}{2}\right]^{\frac{1}{2},\frac{3}{2}} \otimes \frac{1}{2}\right]^{0,1(1,2)}$

$[1 \otimes 0]^1 \otimes 1]^{0,1,2}$ $\left[\left[\left[\frac{1}{2} \otimes \frac{1}{2}\right]^1 \otimes \frac{1}{2}\right]^{\frac{1}{2},\frac{3}{2}} \otimes \frac{1}{2}\right]^{0,1(1,2)}$

$[1 \otimes 0]^1 \otimes 1]^{0,1,2}$ $\left[\left[\left[\frac{1}{2} \otimes \frac{1}{2}\right]^1 \otimes \frac{1}{2}\right]^{\frac{1}{2},\frac{3}{2}} \otimes \frac{1}{2}\right]^{0,1(1,2)}$

# EFFECTIVE $^2n - {}^2n$ AND $^3n - n$ INTERACTIONS

$\epsilon =$ “UNPHYSICAL” TO BIND THE FRAGMENTS.

Attractive $nn$ $\begin{matrix} {}^1S_0 \\ {}^3P_\Sigma \end{matrix}$ imply effective trimer-fermion and dimer-dimer interactions $\Rightarrow$ Ground-state quantum numbers of $^4n$.


$[0 \otimes 1]^1 \otimes 1]^1$ $\quad$ $[1 \otimes 0]^1 \otimes 1]^{0,1,2}$ $\quad$ $[1 \otimes 0]^1 \otimes 1]^{0,1,2}$ $\quad$ $[1 \otimes 0]^1 \otimes 1]^{0,1,2}$

$\left[\left[\left[\frac{1}{2} \otimes \frac{1}{2}\right]^0 \otimes \frac{1}{2}\right]^{\frac{1}{2}} \otimes \frac{1}{2}\right]^1$ $\quad$ $\left[\left[\left[\frac{1}{2} \otimes \frac{1}{2}\right]^1 \otimes \frac{1}{2}\right]^{\frac{1}{2},\frac{3}{2}} \otimes \frac{1}{2}\right]^{0,1(1,2)}$ $\quad$ $\left[\left[\left[\frac{1}{2} \otimes \frac{1}{2}\right]^1 \otimes \frac{1}{2}\right]^{\frac{1}{2},\frac{3}{2}} \otimes \frac{1}{2}\right]^{0,1(1,2)}$ $\quad$ $\left[\left[\left[\frac{1}{2} \otimes \frac{1}{2}\right]^1 \otimes \frac{1}{2}\right]^{\frac{1}{2},\frac{3}{2}} \otimes \frac{1}{2}\right]^{0,1(1,2)}$

$L_{\text{rel}} =$ odd inaccessible for bosons $\curvearrowright$ $J^\pi(^4n) = 0^+$

* D. S. Petrov, C. Salomon, and G. V. Shlyapnikov, PRL 93 (2004), P. Pieri and G. C. Strinati, PRB 61 (2000)

# Effective $^2n - ^2n$ and $^3n - n$ interactions

$\epsilon =$ "unphysical" to bind the fragments.

fermion-fermion attractive ⇒ dimer-dimer attractive:

$$\lim_{a_{nn}/r \to \infty} \frac{a_{DD}}{a_{nn}} \approx ^{*} 0.6$$

* D. S. Petrov, C. Salomon, and G. V. Shlyapnikov, PRL 93 (2004), P. Pieri and G. C. Strinati, PRB 61 (2000)

# Effective $^2n - {}^2n$ and $^3n - n$ interactions

### $\epsilon$ = "unphysical" to bind the fragments.

2-neutron attraction (assumed) insufficient for bound dimers.
$\Rightarrow$ Detune to analyze $\hat{V}_{\text{eff}}$.

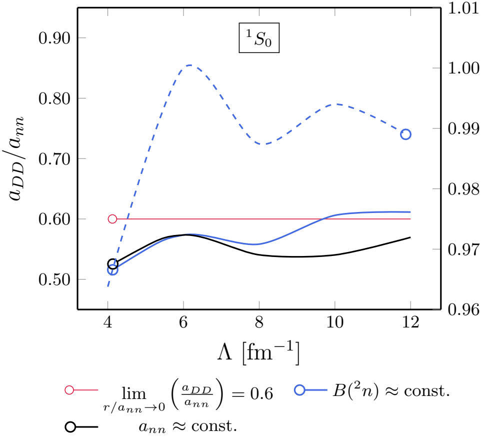
* D. S. Petrov, C. Salomon, and G. V. Shlyapnikov, PRL 93 (2004), P. Pieri and G. C. Strinati, PRB 61 (2000)

# EFFECTIVE $^2n - {}^2n$ AND $^3n - n$ INTERACTIONS

$\epsilon =$ "UNPHYSICAL" TO BIND THE FRAGMENTS.

Bosonic *S*-wave dimers:

$0 < a_{DD} < a_{nn}$ and $-a^{-1} = k \cot \delta$ implies $\delta_{DD} > \delta_{nn}$




---

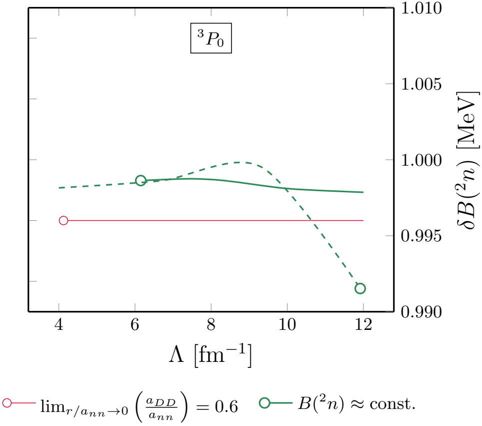
* D. S. Petrov, C. Salomon, and G. V. Shlyapnikov, PRL 93 (2004), P. Pieri and G. C. Strinati, PRB 61 (2000)

# EFFECTIVE $^2n - {}^2n$ AND $^3n - n$ INTERACTIONS

$\epsilon =$ "UNPHYSICAL" TO BIND THE FRAGMENTS.

Bosonic *P*-wave dimers:

$$0 < a_{DD} < a_{nn} \;\text{ and }\; -a^{-1} = k^3 \cot\delta \;\text{ implies }\; \delta_{DD} > \delta_{nn}$$

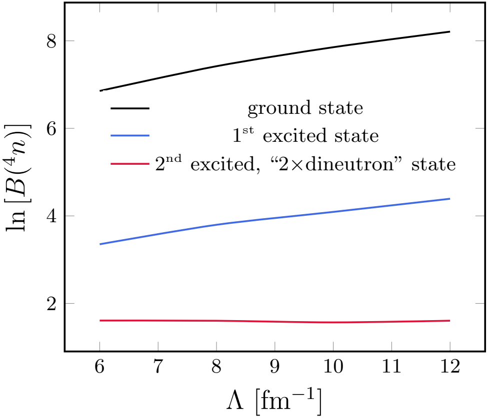


# Effective $^2n - {}^2n$ and $^3n - n$ interactions

$\epsilon =$ "unphysical" to bind the fragments.

$\Rightarrow {}^4n$ sustains **2** bound states.

$$\lim_{\Lambda\to\infty} B^{(0,1)}({}^4n) = \lim_{r/a\to 0} B^{(0,1)}({}^4n) = \infty$$



# Effective $^2n - {}^2n$ and $^3n - n$ interactions

$\epsilon$ = "unphysical" to bind the fragments.

Why is $^4n$ not bound although the 2-2 interaction is more attractive than the 1-1, which sustains a stable $^2n$?

## Effective $^2n - ^2n$ and $^3n - n$ interactions

### $\epsilon$ = "unphysical" to bind the fragments.

Why is $^4n$ not bound although the 2-2 interaction is more attractive than the 1-1, which sustains a stable $^2n$?

For energies $E(^4n) > 2 \times B(^2n)$ other states could be accessible, *e.g.* , the $\frac{1}{2}^-$ trineutron.

# EFFECTIVE $^2n - {}^2n$ AND $^3n - n$ INTERACTIONS

$\epsilon$ = "UNPHYSICAL" TO BIND THE FRAGMENTS.

Why is $^4n$ not bound although the 2-2 interaction is more attractive than the 1-1, which sustains a stable $^2n$?

For energies $E(^4n) > 2 \times B(^2n)$ other states could be accessible, *e.g.* , the $\frac{1}{2}^-$ trineutron.

Such a channel increases the likelihood of a bound $^4n$ if the trineutron-neutron effective interaction is attractive in the $L_{\mathrm{rel}}$ which are relevant for $J^\pi(^4n) = 0^+$ .

# EFFECTIVE $^2n - ^2n$ AND $^3n - n$ INTERACTIONS

$\epsilon =$ "UNPHYSICAL" TO BIND THE FRAGMENTS.

Why is $^4n$ not bound although the 2-2 interaction is more attractive than the 1-1, which sustains a stable $^2n$?

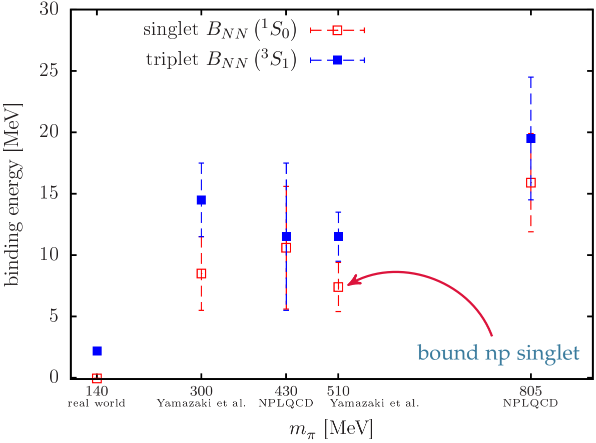
For energies $E(^4n) > 2 \times B(^2n)$ other states could be accessible, *e.g.* , the $\frac{1}{2}^-$ trineutron.

Such a channel increases the likelihood of a bound $^4n$ if the trineutron-neutron effective interaction is attractive in the $L_{\mathrm{rel}}$ which are relevant for $J^{\pi}(^4n) = 0^+$ .

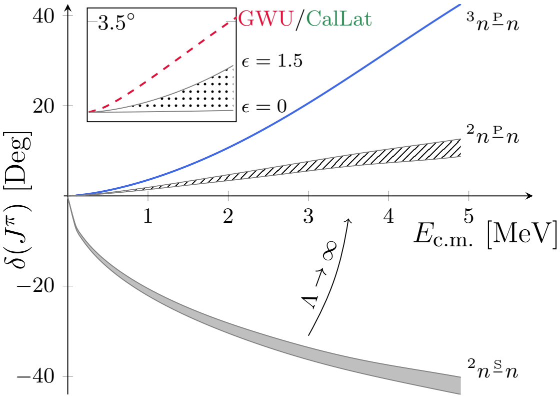
Nucleons in a $m_\pi = 806$ MeV universe provide a concrete example:

Nuclei from QCD — Nuclear data at $m_\pi = 806$ MeV

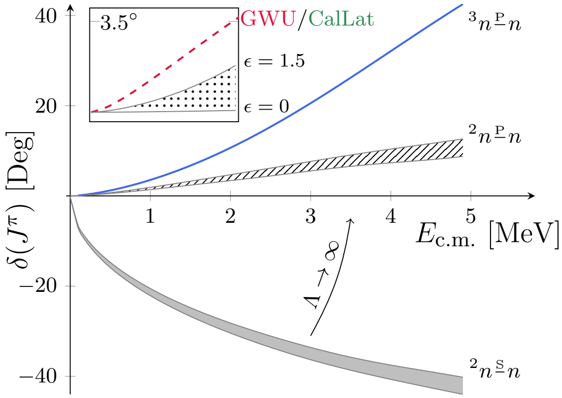
# 2 NUCLEONS AT $m_\pi = 806$ MEV. ⇝



# Elastic $^N n - n$ scattering @ $m_\pi = 806$ MeV.



# Elastic $^N n - n$ scattering @ $m_\pi = 806$ MeV.

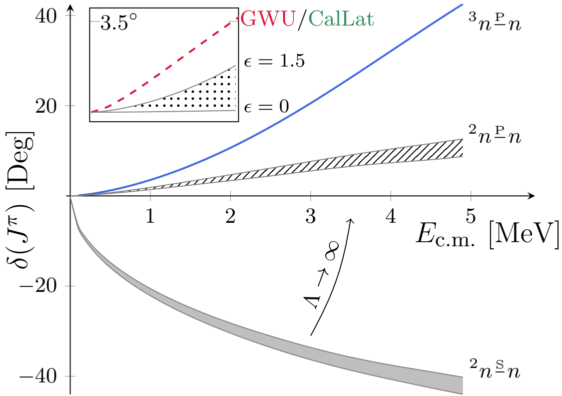



א)

$$\epsilon(^4n) < \epsilon(^3n) < \epsilon(^2n)$$

ב) With **our** $\Lambda$-$\epsilon$ parametrization, shallow $^N n$ states are fine-tuned.

# Elastic $^{N}n - n$ scattering @ $m_\pi = 806$ MeV.

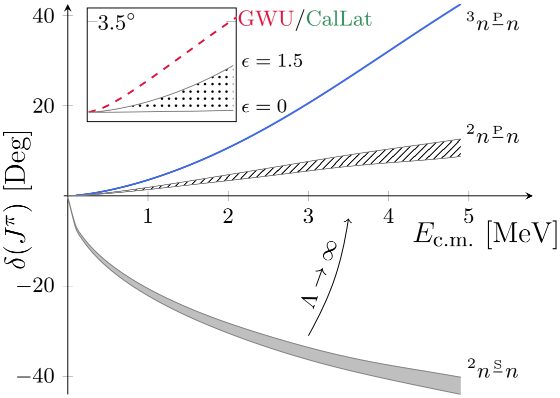



א)

$$\epsilon(^{4}n) < \epsilon(^{3}n) < \epsilon(^{2}n)$$

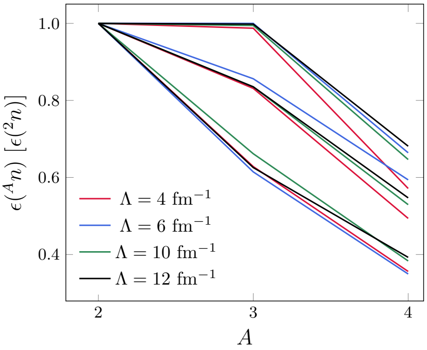
ב) With **our** $\Lambda$-$\epsilon$ parametrization, shallow $^{N}n$ states are fine-tuned.

# Elastic $^{N}n - n$ scattering @ $m_\pi = 806$ MeV.



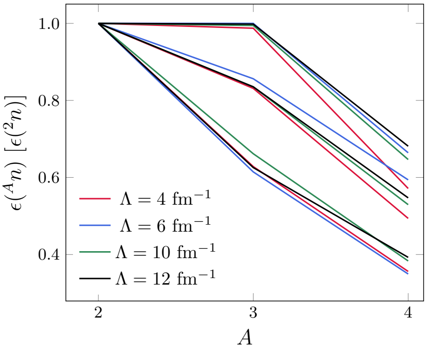

$$\epsilon(^4n) < \epsilon(^3n) < \epsilon(^2n)$$

[...] I scorn the man who is not trying
On his own work to meditate. [...]†




† Friedrich von Schiller, *The Song of the Bell* (1798)

# [...] I scorn the man who is not trying On his own work to meditate. [...]†




$\exists A_{\mathrm{crit}}$ :

$A_{\mathrm{crit}}$-neutron nucleus is bound at nuclear scales

$\forall M < A_{\mathrm{crit}}$ :

the $M$-neutron is either unstable or decoupled.

† Friedrich von Schiller, *The Song of the Bell* (1798)