Determining the subleading 3N contact interaction

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Motivation

Actual description of $A = 2, 3, 4$ systems using potential models:

- 2N system: χ^2 per datum ≈ 1
- 3N and 4N systems using a 2N interaction: χ^2 per datum $>> 1$
- 3N and 4N systems using a 2N+3N interaction: χ^2 per datum $>> 1$

Outline

Theoretical inputs:

- The 3N force models, leading and subleading terms
- $A = 3, 4$ systems using the Hyperspherical Harmonic method

Strategy:

- Matricial form of scattering states in $A = 3$
- Fitting the 3N force to 3N data
- χ^2 per datum and predictions

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Some comments

- During the 90's strong efforts have been done to describe the NN data base using a NN interaction
- Different strategies to construct the interaction have been used. We call these potentials realistic potentials. Examples are: AV18, CD bonn, Nijmegen I and II, Reid soft core (new version)
- At the same time a different framework appears: ChPT
- In this framework the different components of the NN and NNN interaction terms are organized in a perturbation series
- The convergence properties of the series have to be studied following the capability of the interaction to describe the data
- For example the LO term is: $V_{LO} = V_0\mathcal{P}_{01} + V_1\mathcal{P}_{10} + V_{OPEP}$ with V_0 and V_1 contact interactions
- Contact interaction are patological so a possibility is to extend its range: $V_S = C_S exp(-\Lambda^2 r^2)$

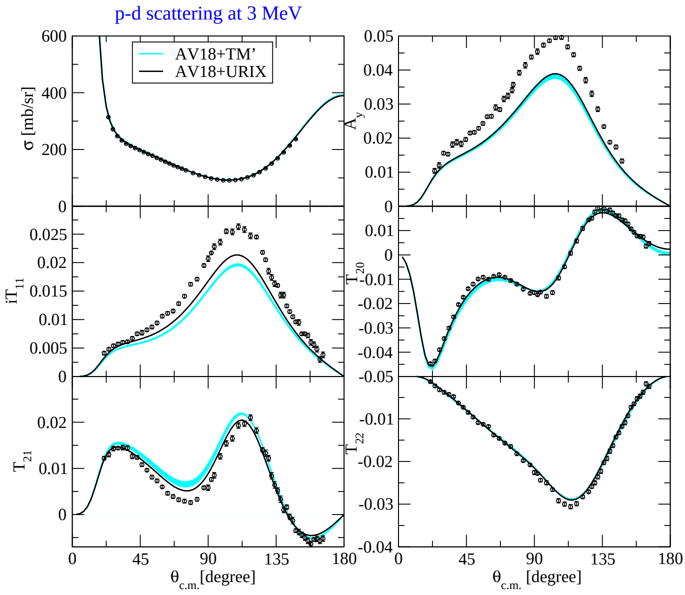
What about the 3N interaction

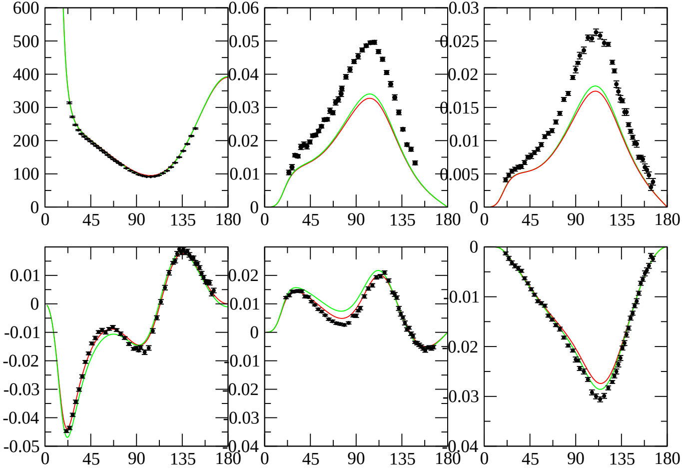
- Realistic potentials cannot reproduced the $B(3\mathrm{H})$ or/and $B(4\mathrm{He})$
- They were supplemented with NNN interactions. For example: URIX, TM, Brazil, IL7
- Widely used models (even today) are: AV18+UIX or CD Bonn+TM
- Using ChPT the first NNN interaction term appears at N2LO (we will comment on that) with two new LECs: c_E and c_D:
  $V^{3N}_{N2LO} = V_{sr} + V_{2\pi}$
  V_{sr} is the contact interaction containing the two LECS c_E, c_D
- At N3LO no new LECs appear, however the NNN interaction at that order has a very complicate structure. Its implementation did not help to improve the description of the 3N data
- New LECs appear in the short-range NNN interaction at N4LO
- Many calculations at present are done using the mixed order N3LO for the NN interaction and N2LO for the NNN interaction with c_E, c_D determined from $B(3\mathrm{H})$ and one more data that could be $B(4\mathrm{He})$, the doublet a_{nd} scattering length or tritium β decay.

^{3}H and ^{4}He Bound States and $n-d$ scattering length

Potential(NN)	Method	^{3}H[MeV]	^{4}He[MeV]	$^2a_{nd}$[fm]
AV18	HH	7.624	24.22	1.258
	FE/FY Bochum	7.621	24.23	1.248
	FE/FY Lisbon	7.621	24.24	
CDBonn	HH	7.998	26.13	
	FE/FY Bochum	8.005	26.16	0.925
	FE/FY Lisbon	7.998	26.11	
	NCSM	7.99(1)		
N3LO-Idaho	HH	7.854	25.38	1.100
	FE/FY Bochum	7.854	25.37	
	FE/FY Lisbon	7.854	25.38	
	NCSM	7.852(5)	25.39(1)	
NV2-Ia	HH	7.818	24.15	1.215

Potential(NN+NNN)				
AV18/UIX	HH	8.479	28.47	0.590
	FE/FY Bochum	8.476	28.53	0.578
CDBonn/TM	HH	8.474	29.00	
	FE/FY Bochum	8.482	29.09	0.570
N3LO-Idaho/N2LO	HH	8.474	28.37	0.675
	NCSM	8.473(5)	28.34(2)	
NV2+3-Ia	HH	8.476	28.32	0.645
Exp.		8.48	28.30	0.645 ± 0.010


p-d scattering at 3 MeV
AV18+TM'
AV18+URIX
σ [mb/sr]
A_y
iT_{11}
T_{20}
T_{21}
T_{22}
$\theta_{c.m.}$ [degree]
$\theta_{c.m.}$ [degree]

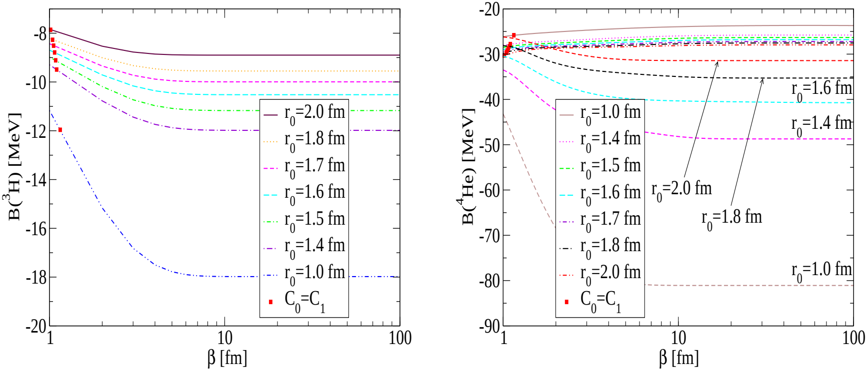


p-d sacttering at 3 MeV with INV2a and INV2a+TNI

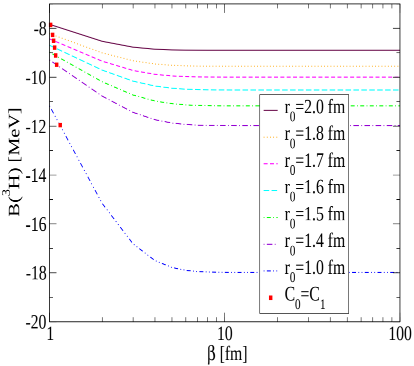
pionless EFT

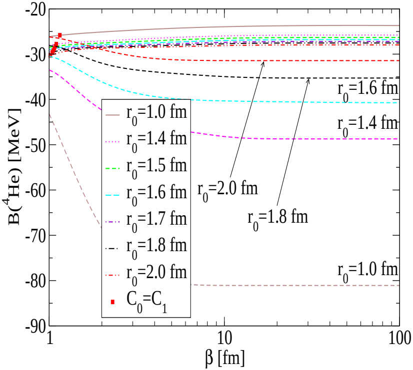
There is another way of organizing the nuclear interaction based on an EFT in which the pion degrees of freedom are integrated out. The 2N LO potential is
$V_{LO}^{(2N)} = V_0\mathcal{P}_{01} + V_1\mathcal{P}_{10}$
This potential produces the Thomas collapse in the 3N sector. A contact 3N term is added for regularization and the complete LO potential is
$V_{LO}^{\not\pi} = V_{LO}^{(2N)} + V_{LO}^{(3N)} = V_0\mathcal{P}_{01} + V_1\mathcal{P}_{10} + W_0(3N)$
The LO potential in pionless EFT and ChPT is different. This might produces differences in the LO description of nuclei as the number of particles increases. As alternative a LO potential including a 3N contact term can be consider:

$$V_{LO} = V_{LO}^{(2N)} + V_{LO}^{(3N)} = V_0\mathcal{P}_{01} + V_1\mathcal{P}_{10} + V_{OPEP} + W_0(3N)$$



The motivation of this discussion is to consider the possibility of promoting the contact term to LO and the subleading terms to N2LO





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3N Subleading contact interaction

$$V_{3N}^{CT} = V^{(0)} + V^{(2)}$$

$$V^{(0)} = \sum_{i \neq j \neq k} E_0 Z_0(r_{ij}) Z_0(r_{ik})$$

$$V^{(2)} = \sum_{i \neq j \neq k} (E_1 + E_2 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + E_3 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + E_4 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)$$

$$\left[Z_0''(r_{ij}) + 2 \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik})$$

$$+(E_5 + E_6 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) S_{ij} \left[Z_0''(r_{ij}) - \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik})$$

$$+(E_7 + E_8 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k)(\mathbf{L} \cdot \boldsymbol{S})_{ij} \frac{Z_0'(r_{ij})}{r_{ij}} Z_0(r_{ik})$$

$$+(E_9 + E_{10} \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k) \boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{ij} \boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{ik} Z_0'(r_{ij}) Z_0'(r_{ik})$$

with the profile function

$$Z_0(r;\Lambda) = \int \frac{d^3p}{(2\pi)^3} e^{i\mathbf{p}\cdot\mathbf{r}} F(p^2;\Lambda)$$

with the cutoff function

$$F(p^2;\Lambda) = exp\left[-\left(\frac{p^2}{\Lambda^2}\right)^2 \right]$$

The Hamiltonian of the system is:

$$H = T + V_{2N} + V_{3N} + V_{3N}^{CT} = T + V_{2N} + V_{3N} + V^{(0)} + V^{(2)}$$

Here I will show a hybrid scheme to analyse the capability of the subleading term $V^{(2)}$ to improve the description of the 3N data in which $V_{2N} + V_{3N}$= AV18+URIX with the profile function

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Determination of the subleading LECs

The N-d scattering wave function is

$$\Psi_{LSJJ_z} = \Psi_{\rm C} + \Psi_{\rm A}$$

the internal part, $\Psi_{\rm C}$ is expanded in The HH basis

$$\Psi_C = \sum_\mu c_\mu \Phi_\mu$$

the asymptotic part, $\Psi_{\rm A}$ is

$$\Psi_A^{LSJJ_z} = \Omega^R_{LSJJ_z} + \sum_{L'S'} \mathcal{R}^J_{LS,L'S'}(q)\Omega^I_{L'S'JJ_z}$$

The Kohn variational principle requires the functional

$$\left[\mathcal{R}^J_{LS,L'S'}(q)\right] = \mathcal{R}^J_{LS,L'S'}(q) - \langle \Psi_{L'S'JJ_z} | H - E | \Psi_{LSJJ_z} \rangle$$

to be stationary

The linear equations

The stationary condition implies: $\sum_{\tilde{L}\tilde{S}} \mathcal{R}^J_{LS,\tilde{L}\tilde{S}} X_{L'S',\tilde{L}\tilde{S}} = Y_{LS,L'S'}$

where

$X_{LS,L'S'} = \langle \Omega^I_{LSJJ_z} + \Psi^I_C | H - E | \Omega^I_{L'S'JJ_z} \rangle$
$Y_{LS,L'S'} = -\langle \Omega^I_{LSJJ_z} + \Psi^R_C | H - E | \Omega^I_{L'S'JJ_z} \rangle$
and the internal functions Ψ^λ_C are solutions of

$$\sum_{\mu'} \langle \Phi_\mu | H - E | \Phi_{\mu'} \rangle c^\lambda_{\mu'} = -\langle \Phi_\mu | H - E | \Omega^\lambda_{LSJJ_z} \rangle,$$

with $\lambda = R, I$. Decomposing the hamiltonian as

$$H = T + V = T + V_{2N} + V_{3N} = H_L + V^{(0)} + V^{(2)}$$

the linear equations can be put in the matricial form

$$\sum_{\mu'} (H^L_{\mu\mu'} + \sum_{i=0,10} E_i V^i_{\mu\mu'} - EN_{\mu\mu'}) c^\lambda_{\mu'} = -H^L_{\mu\lambda} + \sum_{i=0,10} E_i V^i_{\mu\lambda} - EN_{\mu\lambda}.$$

Constructing the observables

The transition matrix is

$$M^{SS'}_{\nu\nu'}(\theta) = f_c(\theta)\delta_{SS'}\delta_{\nu\nu'} + \frac{\sqrt{4\pi}}{k}\sum_{L,L',J}\sqrt{2L+1}(L0S\nu|J\nu)(L'M'S'\nu'|J\nu)$$
$$\times \exp[i(\sigma_L + \sigma_{L'} - 2\sigma_0)]\ {}^J T^{SS'}_{LL'}\ Y_{L'M'}(\theta,0)$$

the matrix M is 6×6, due to the couplings of the spin 1 (deuteron) and spin $1/2$ (nucleon) to $S,S' = 1/2, 3/2$ with projections ν and ν'.

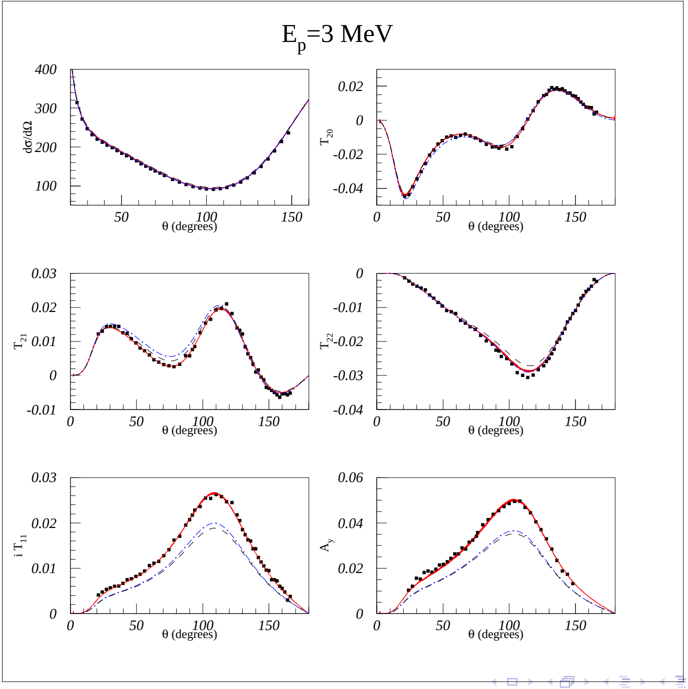
Solving the linear system for different J^π states, the observables are then calculated: $\sigma = tr(M^\dagger M)$, $A_y = tr(M^\dagger \sigma_y M)$, ...

The LECs $E_0,\ldots,E_{10}$ are obtained minimizing the χ^2 function

$$\chi^2 = \sum_i \frac{(d_i^{\rm exp} - d_i^{\rm th})^2}{(\sigma_i^{\rm exp})^2},$$

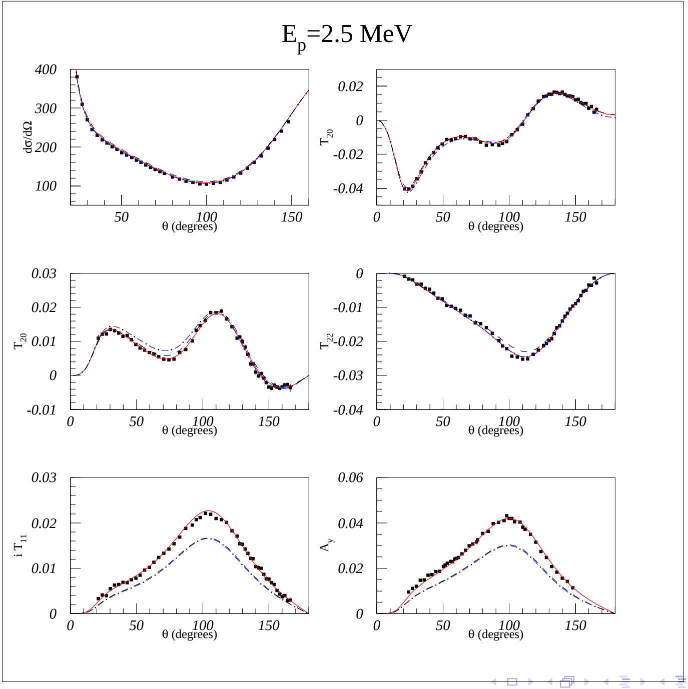
Results for $p - d$ scattering at 3 MeV

Λ (MeV)	200	300	400	500
χ^2/d.o.f.	1.6	1.6	1.7	1.8
e_0	6.174	1.963	-0.336	2.460
e_1	3.031	-1.083	-0.068	-0.973
e_2	-4.230	-2.453	-2.402	-1.264
e_3	1.311	3.226	0.098	1.202
e_4	0.579	-1.302	1.004	-0.524
e_5	-0.710	-0.899	-1.048	-2.112
e_6	0.502	-0.401	-0.023	-0.248
e_7	1.938	7.446	7.934	4.209
e_8	-0.152	2.196	1.987	-0.805
e_9	-1.239	-5.620	-3.96	-1.787
e_{10}	1.159	-1.451	-0.973	-2.687
a_2 (fm)	0.631	0.643	0.644	0.642
a_4 (fm)	6.33	6.33	6.32	6.32

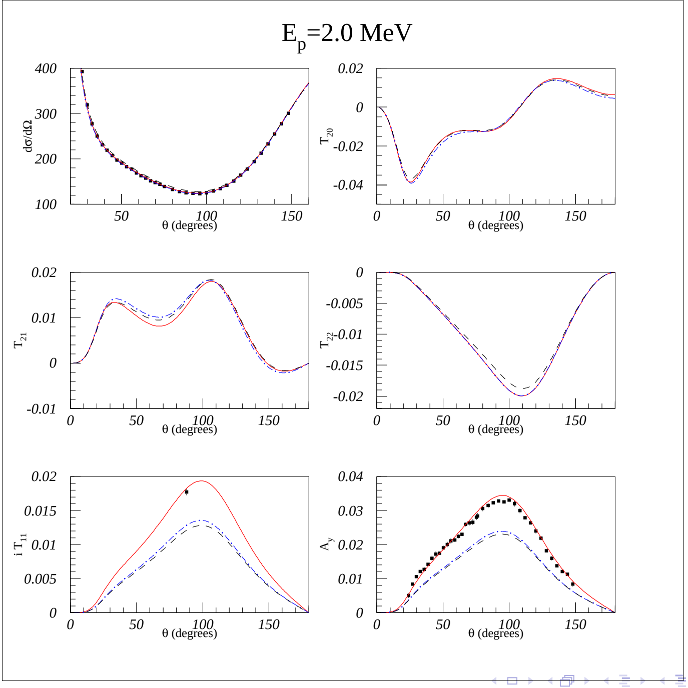


E_p=3 MeV
dσ/dΩ
T_{20}
T_{21}
T_{22}
i T_{11}
A_y
θ (degrees)

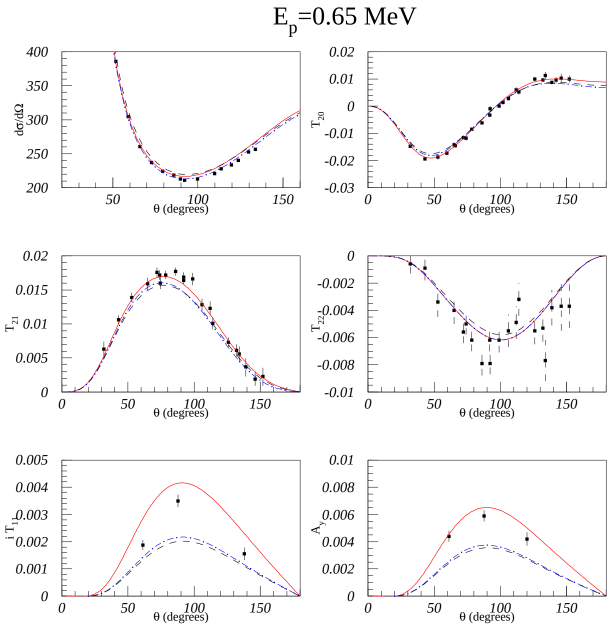
predictions at lower energies



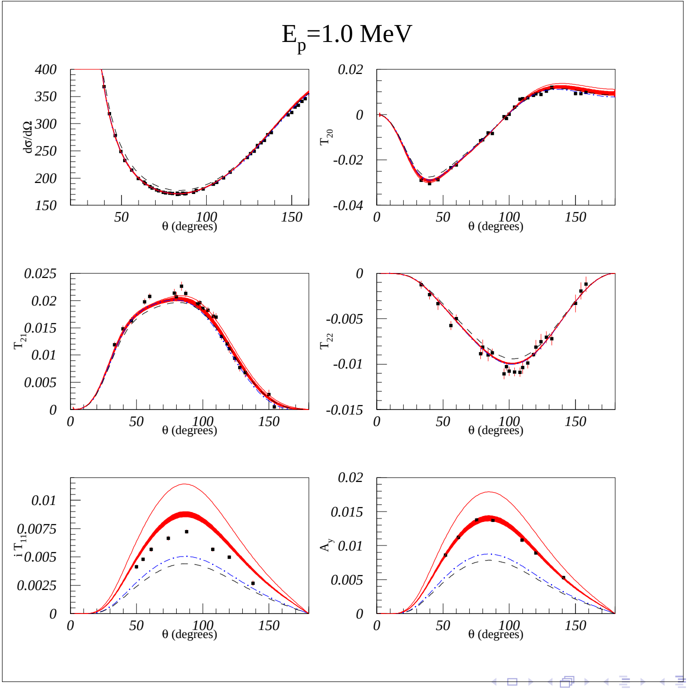
E_p=2.5 MeV
dσ/dΩ
400
300
200
100
50
100
150
θ (degrees)
T20
0.02
0
-0.02
-0.04
0
50
100
150
θ (degrees)
T20
0.03
0.02
0.01
0
-0.01
0
50
100
150
θ (degrees)
T22
0
-0.01
-0.02
-0.03
-0.04
0
50
100
150
θ (degrees)
i T11
0.03
0.02
0.01
0
0
50
100
150
θ (degrees)
Ay
0.06
0.04
0.02
0
0
50
100
150
θ (degrees)



E_p=2.0 MeV
dσ/dΩ
400
300
200
100
50
100
150
θ (degrees)
T_20
0.02
0
-0.02
-0.04
0
50
100
150
θ (degrees)
T_21
0.02
0.01
0
-0.01
0
50
100
150
θ (degrees)
T_22
0
-0.005
-0.01
-0.015
-0.02
0
50
100
150
θ (degrees)
i T_11
0.02
0.015
0.01
0.005
0
0
50
100
150
θ (degrees)
A_y
0.04
0.03
0.02
0.01
0
0
50
100
150
θ (degrees)



E_p=0.65 MeV
dσ/dΩ
T20
T21
T22
i T11
Ay
θ (degrees)



E_p=1.0 MeV
dσ/dΩ
400
350
300
250
200
150
50
100
150
θ (degrees)
T_{20}
0.02
0
-0.02
-0.04
0
50
100
150
θ (degrees)
T_{21}
0.025
0.02
0.015
0.01
0.005
0
0
50
100
150
θ (degrees)
T_{22}
0
-0.005
-0.01
-0.015
0
50
100
150
θ (degrees)
i T_{11}
0.01
0.0075
0.005
0.0025
0
0
50
100
150
θ (degrees)
A_y
0.02
0.015
0.01
0.005
0
0
50
100
150
θ (degrees)

Conclusions

- The subleading 3N contact interaction has been used to improve the description of the low energy 3N data
- Widely used realistic potentials consisting in 2N+ 3N interactions describe low energy p-d data with $\chi^2 \approx 100$
- Using the subleading terms we were able to fit p-d data with a $\chi^2 < 2$, comparable to the 2N case
- The fit was done at a single energy
- Predictions at lower energies were acceptable

- Work in progress:
  - Multi energy fit
  - Energies above the deuteron breakup threshold
  - Predictions in the four-nucleon system
  - Predictions in $A > 4$

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