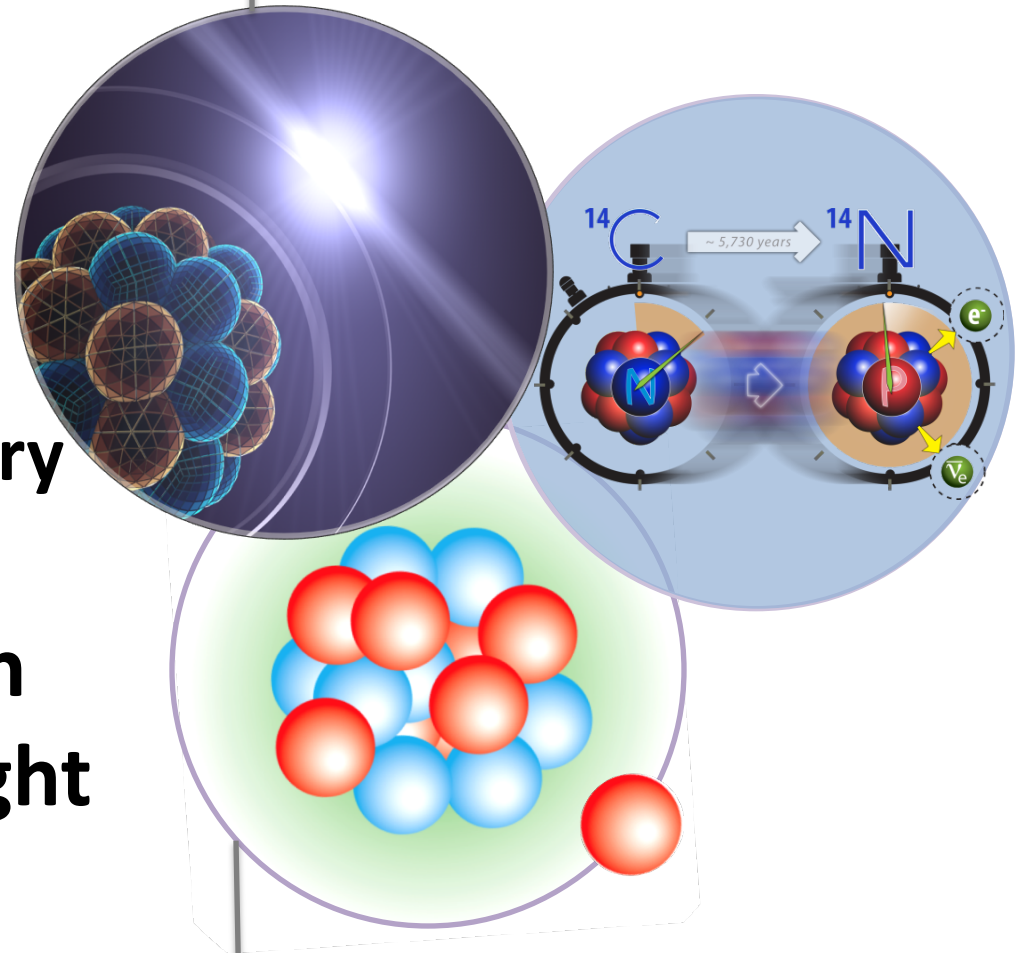


# Electroweak decays from coupled-cluster computations

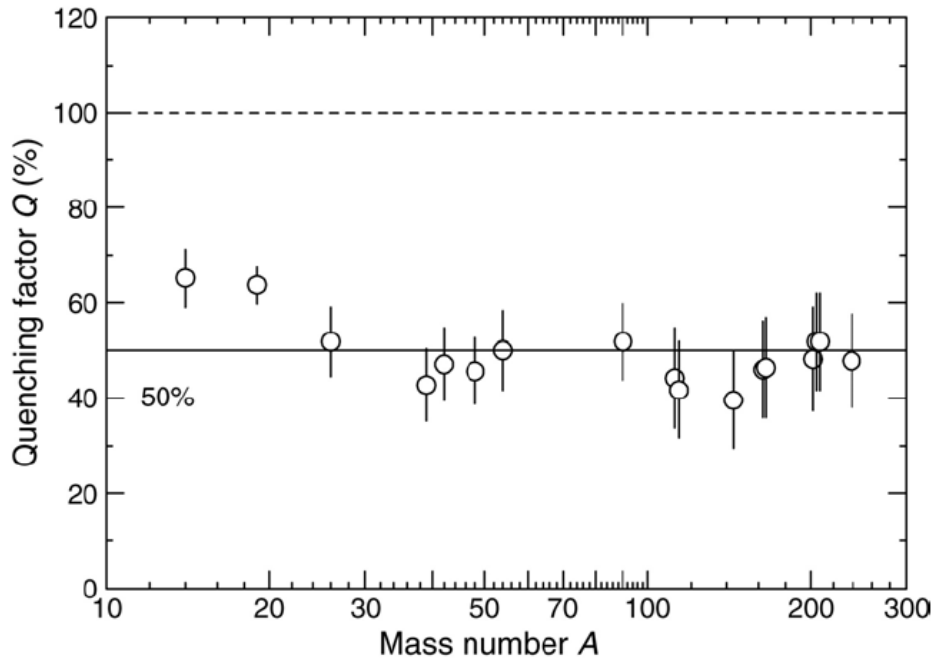
Gaute Hagen  
Oak Ridge National Laboratory

Fundamental Physics with  
Electroweak Probes of Light  
Nuclei

INT, Seattle June 13<sup>th</sup>, 2018



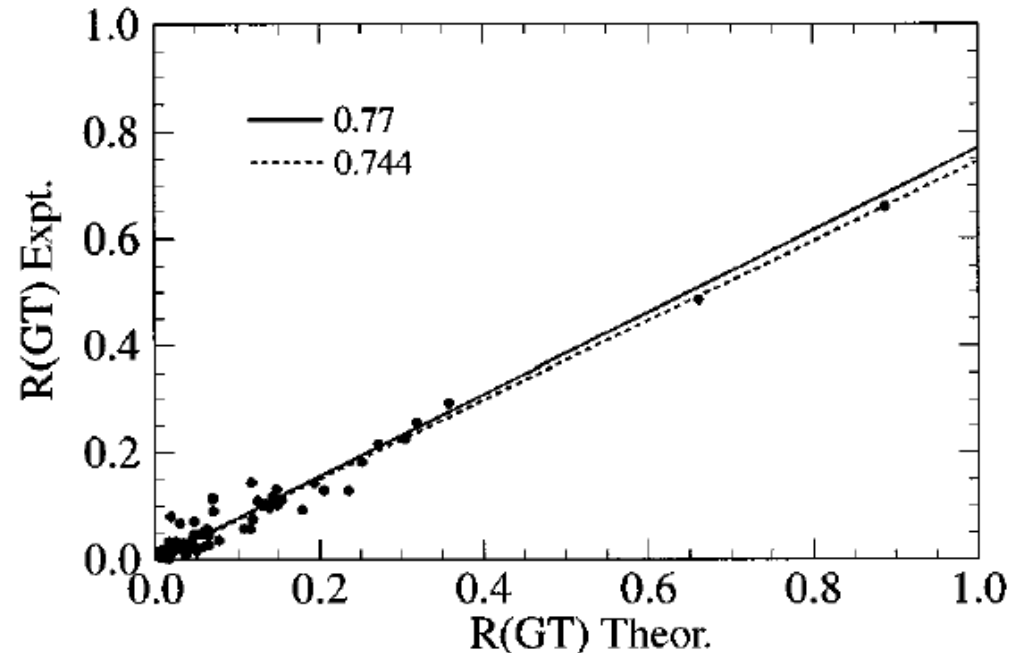
# A 50 year old problem: The puzzle of quenched of beta decays



Quenching obtained from charge-exchange ( $p,n$ ) experiments. (Gaarde 1983).

- Renormalizations of the Gamow-Teller operator?
- Missing correlations in nuclear wave functions?
- Model-space truncations?
- Two-body currents (2BCs)?

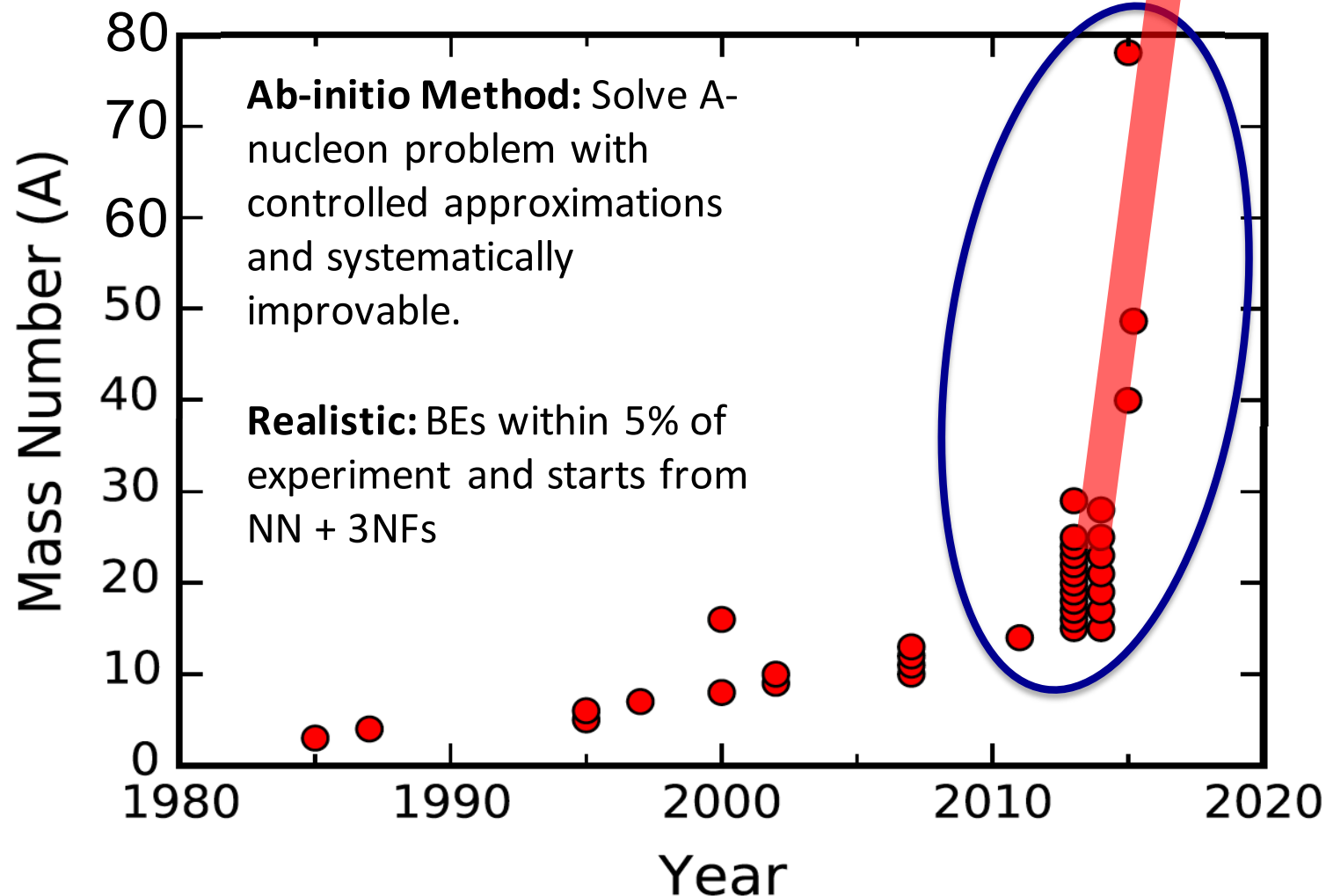
G. Martinez-Pinedo et al, PRC **53**, R2602 (1996)



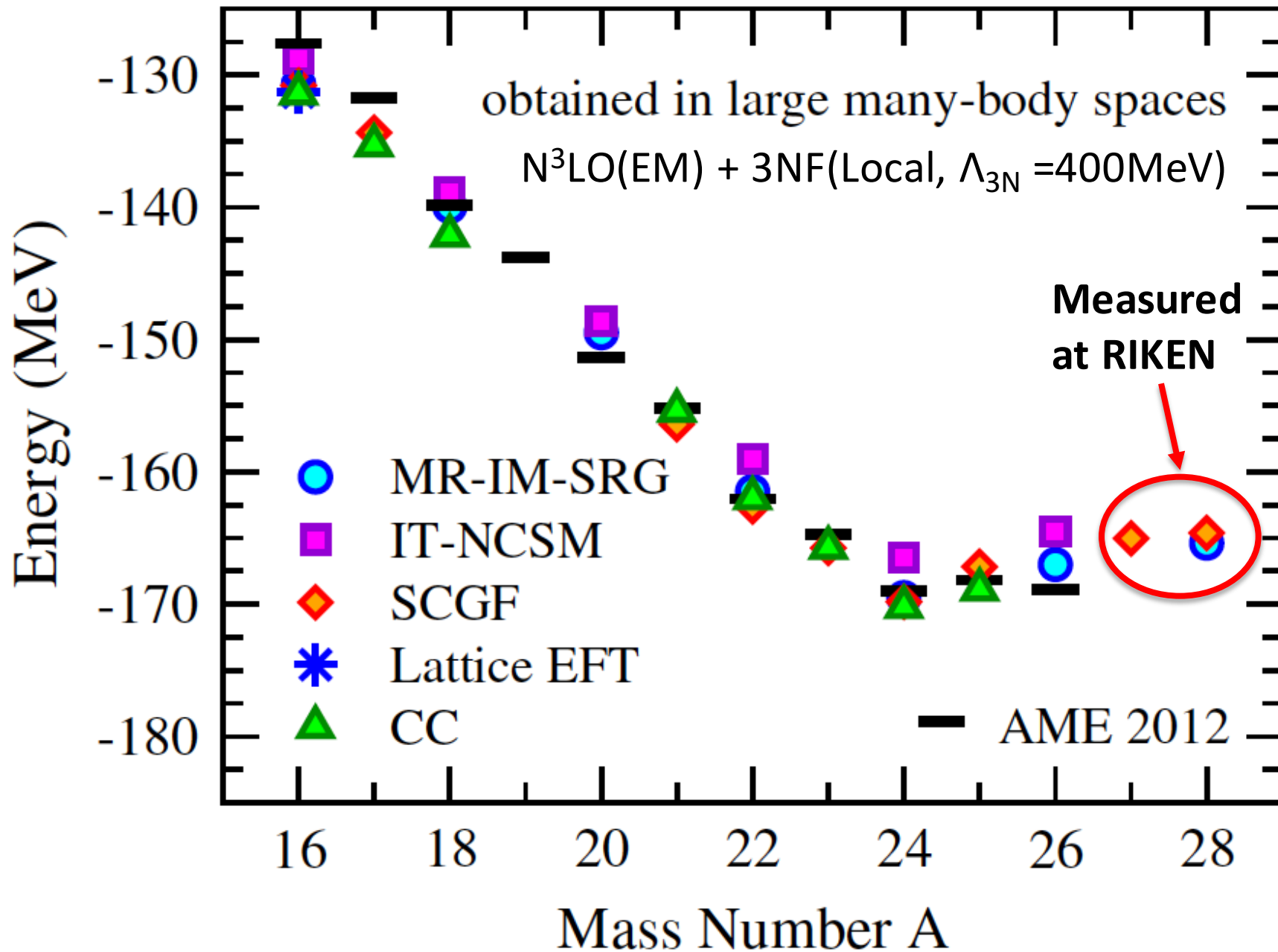
# Trend in realistic ab-initio calculations

**Explosion of many-body methods** (Coupled clusters, Green's function Monte Carlo, In-Medium SRG, Lattice EFT, MCSM, No-Core Shell Model, Self-Consistent Green's Function, UMOA, ...)

**Application of ideas from EFT and renormalization group** ( $V_{\text{low-k}}$ , Similarity Renormalization Group, ...)



# Oxygen chain with interactions from chiral EFT



# Nuclear forces from chiral effective field theory

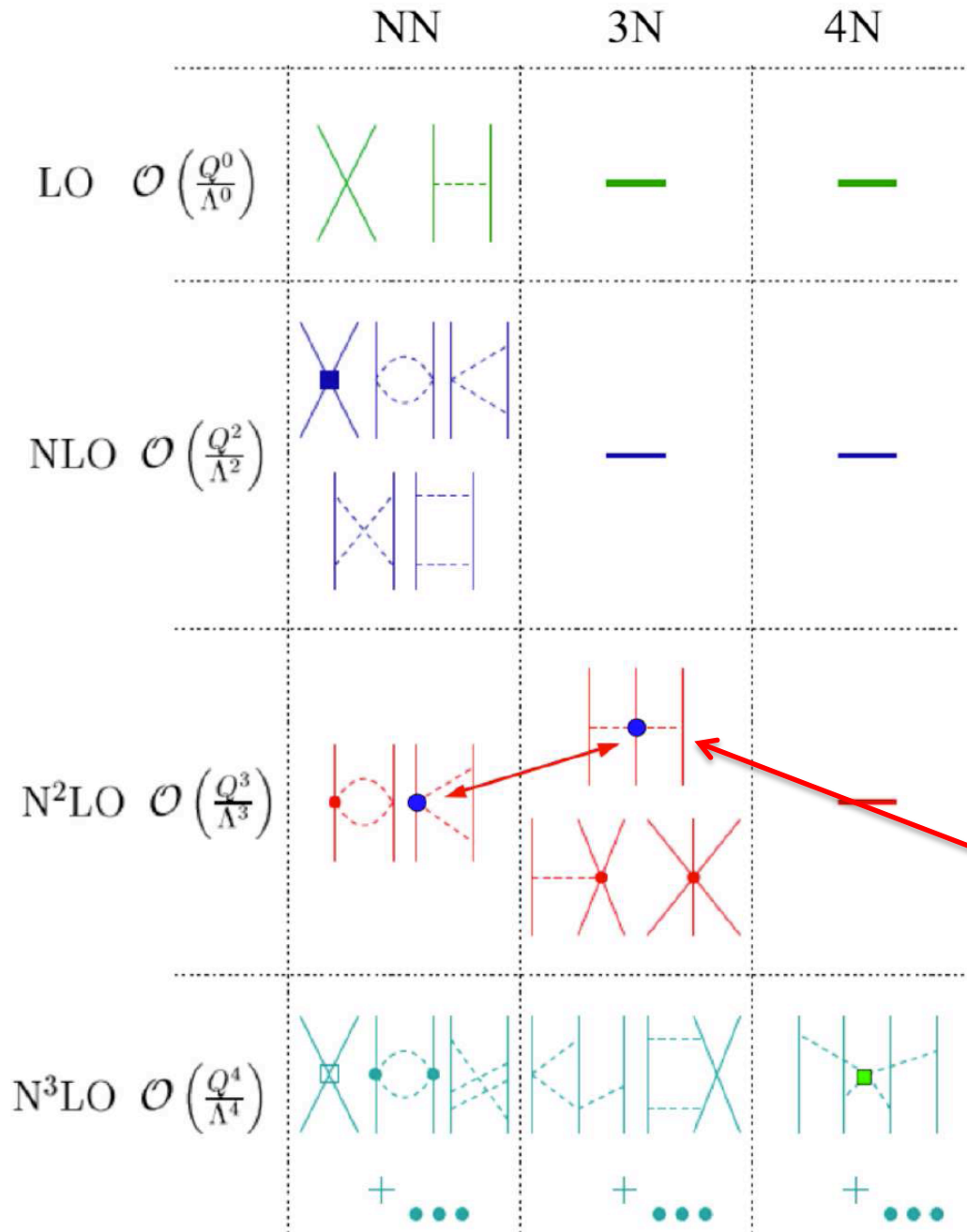
[Weinberg; van Kolck; Epelbaum *et al.*; Entem & Machleidt; ...]

		NN	3N	4N
LO	$\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$			
NLO	$\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$			
N <sup>2</sup> LO	$\mathcal{O}\left(\frac{Q^3}{\Lambda^3}\right)$			
N <sup>3</sup> LO	$\mathcal{O}\left(\frac{Q^4}{\Lambda^4}\right)$			
		+ ...	+ ...	+ ...

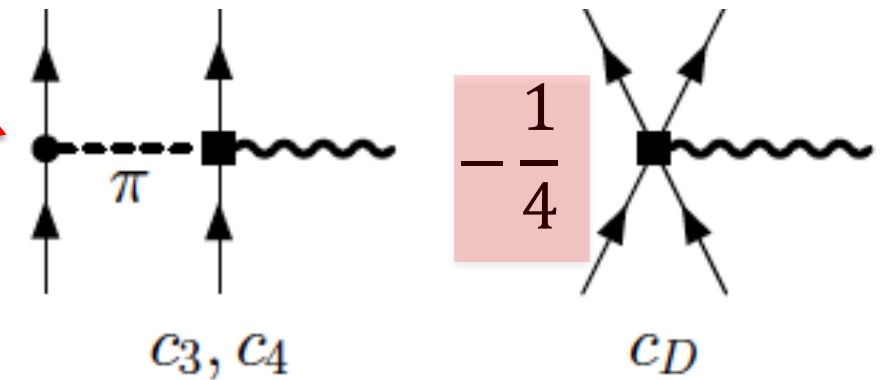
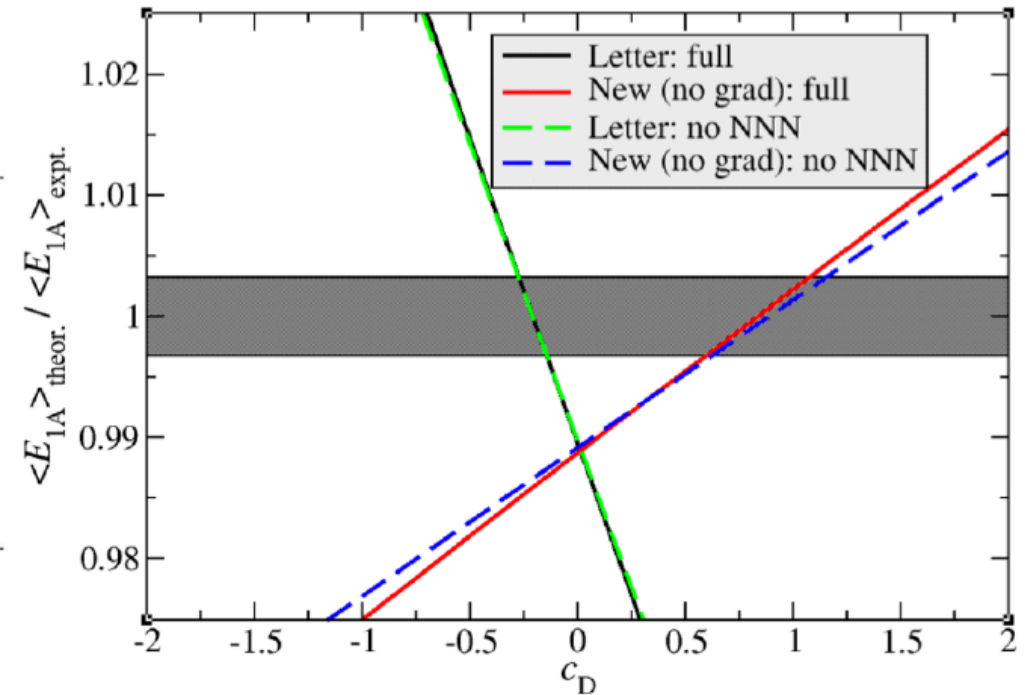
- Developing higher orders and higher rank (3NF, 4NF) [Epelbaum 2006; Bernard et al 2007; Krebs et al 2012; Hebeler et al 2015; Entem et al 2017, Reinert et al 2018...]
- Propagation of uncertainties on the horizon [Navarro Perez 2014, Carlsson et al 2015]
- Different optimization protocols [Ekström et al 2013, Carlsson et al 2016]
- Improved understanding/handling via SRG [Bogner et al 2003; Bogner et al 2007]
- local / semi-local / non-local formulations [Epelbaum et al 2015, Gezerlis et al 2013/2014, Reinert et al 2018]
- Chiral EFT's with explicit Delta isobars [Krebs et al 2018, Piarulli et al 2017, Ekstrom et al 2017]

# Nuclear forces from chiral effective field theory

[Weinberg; van Kolck; Epelbaum *et al.*; Entem & Machleidt; ...]

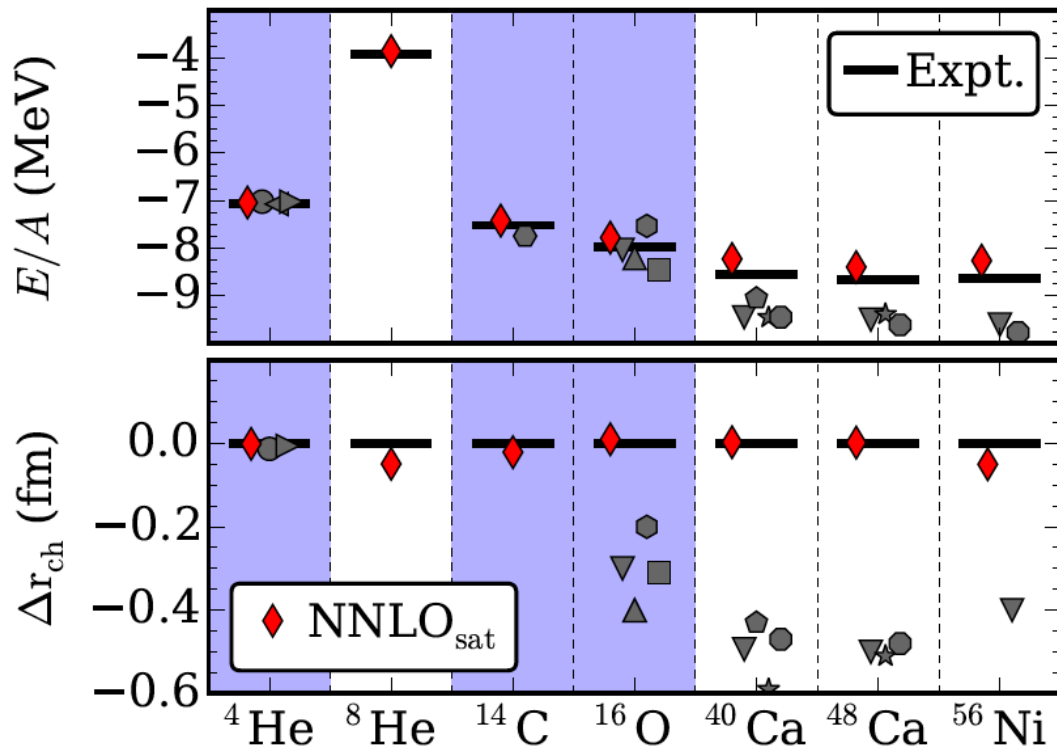


From Sofia Quaglioni and Kyle Wendt





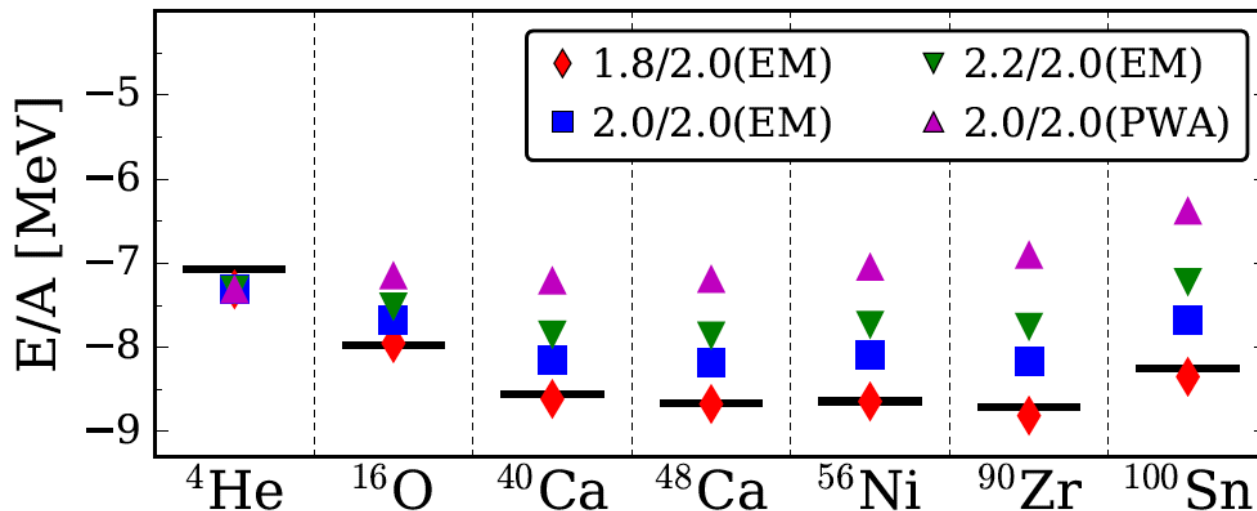
# A family of interactions from chiral EFT



$\text{NNLO}_{\text{sat}}$ : Accurate radii and BEs

- Simultaneous optimization of NN and 3NFs
- Include charge radii and binding energies of  ${}^3\text{H}$ ,  ${}^{3,4}\text{He}$ ,  ${}^{14}\text{C}$ ,  ${}^{16}\text{O}$  in the optimization
- Harder interaction: difficult to converge beyond  ${}^{56}\text{Ni}$

A. Ekström *et al*, Phys. Rev. C **91**, 051301(R) (2015).



1.8/2.0(EM): Accurate BEs

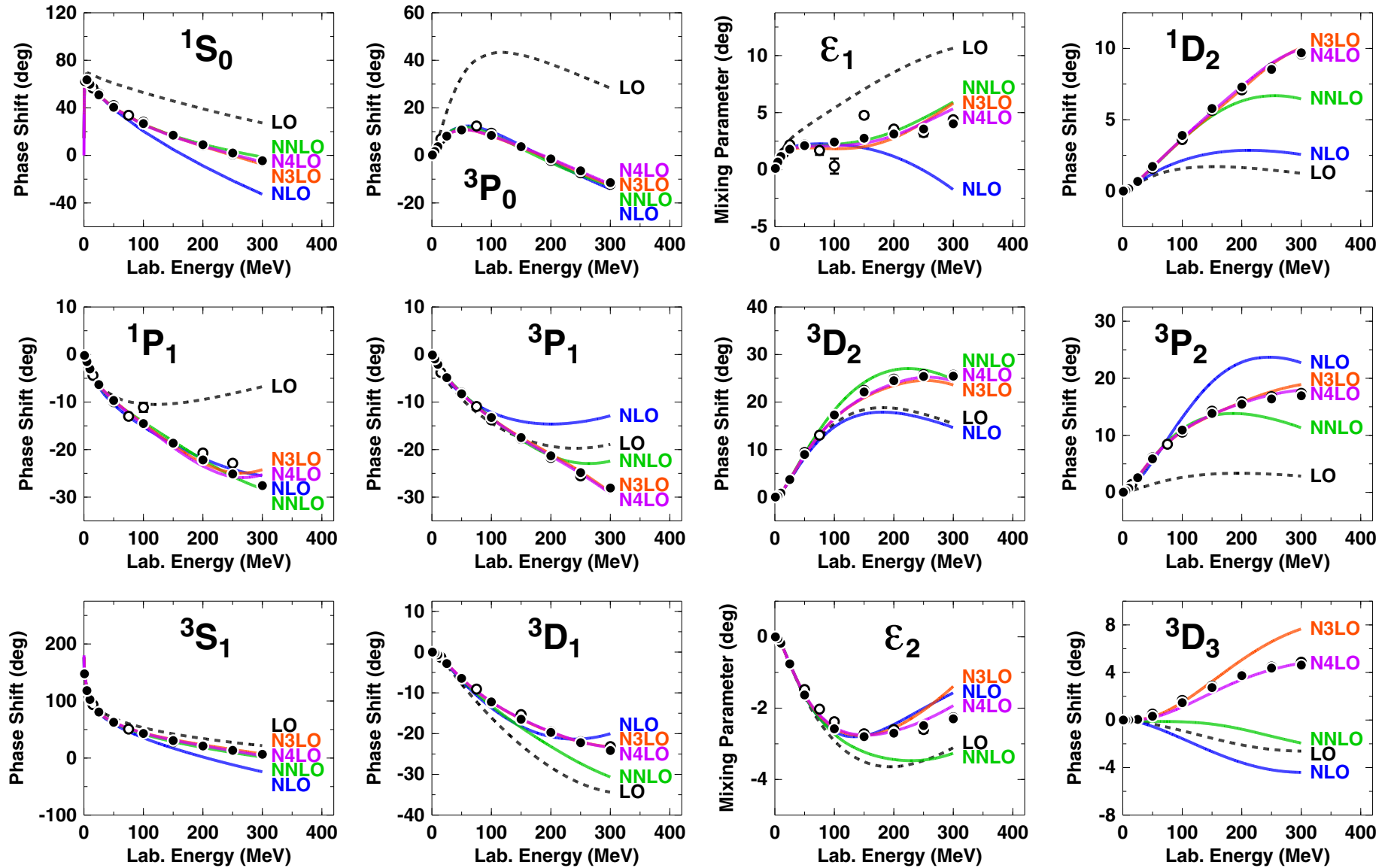
Soft interaction: SRG NN from Entem & Machleidt with 3NF from chiral EFT

K. Hebeler *et al* PRC (2011).

T. Morris *et al*, PRL (2018).

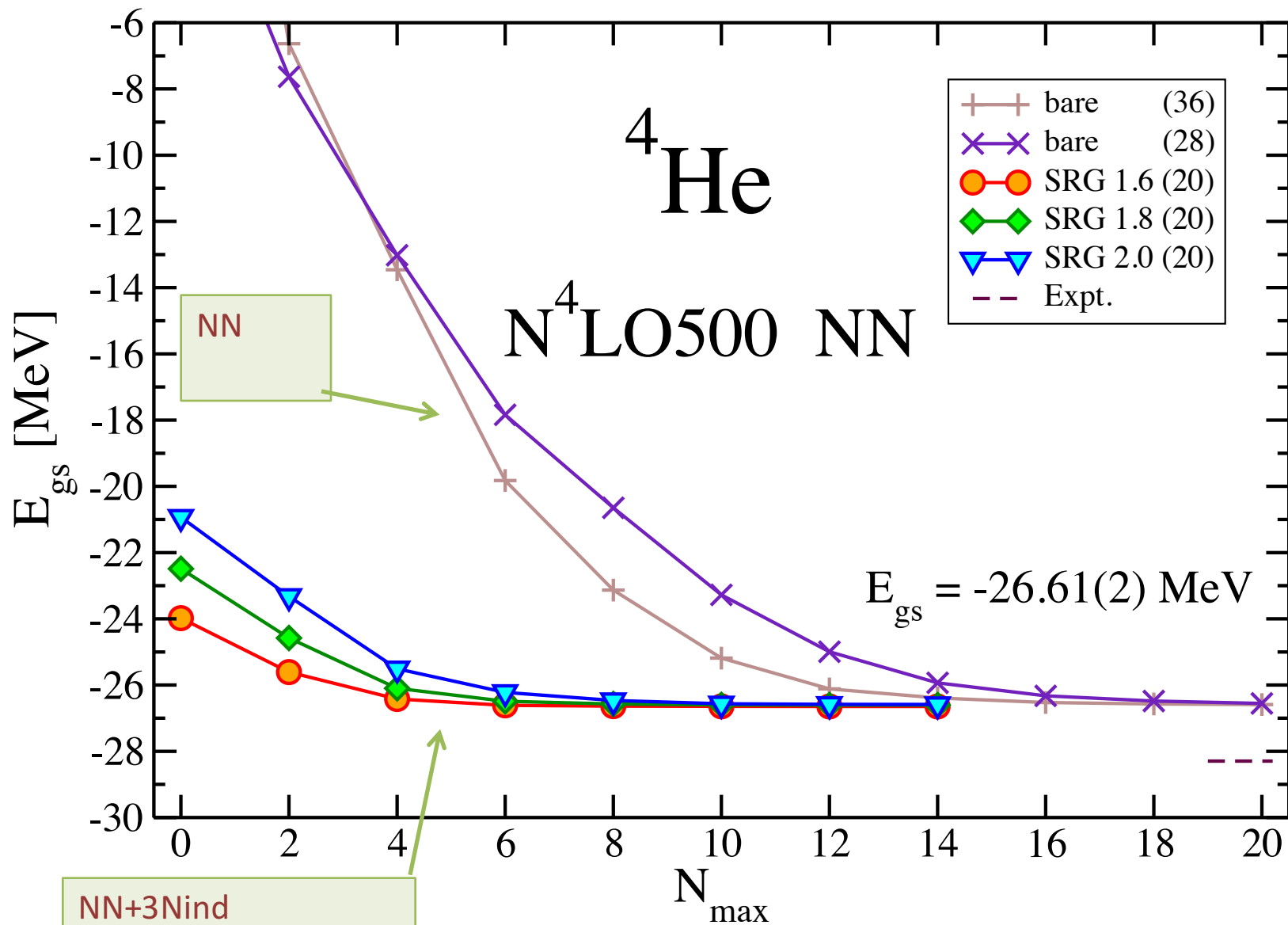
# High-quality two-nucleon potentials up to fifth order of the chiral expansion

D. R. Entem,<sup>1,\*</sup> R. Machleidt,<sup>2,†</sup> and Y. Nosyk<sup>2</sup>





# NCSM results for $^4\text{He}$ with N4LO (NN-only)



From: P. Navratil

# Fit 3N and 2BC contacts to triton half-life

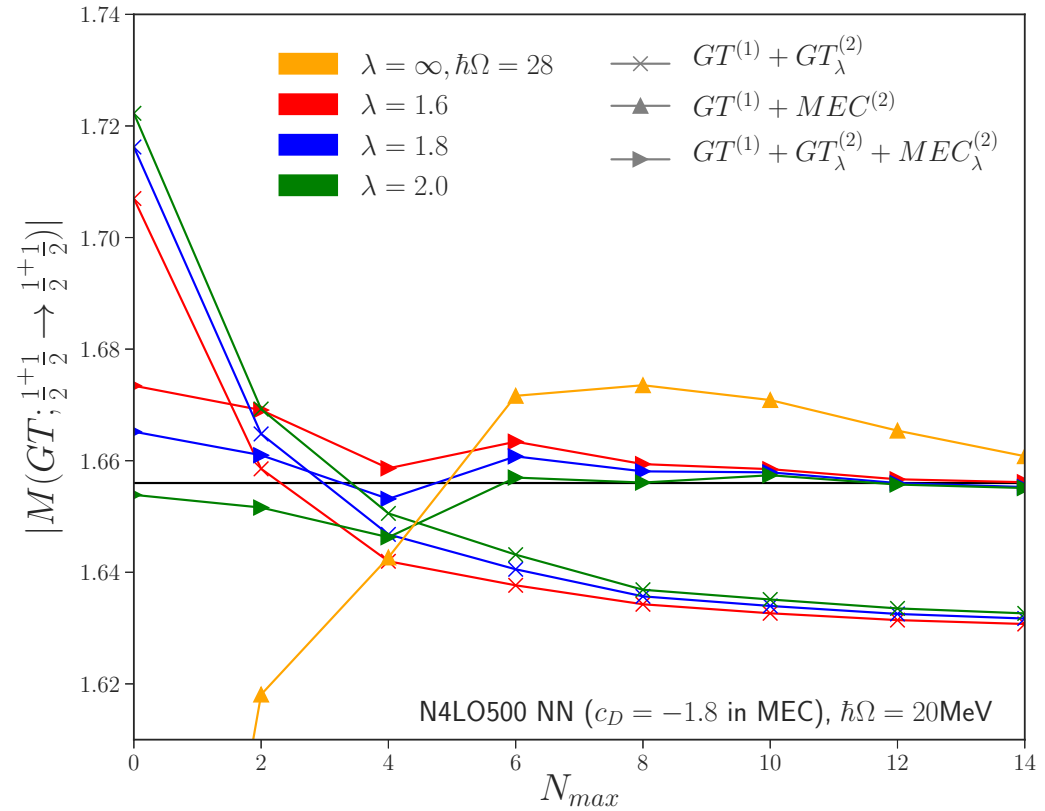
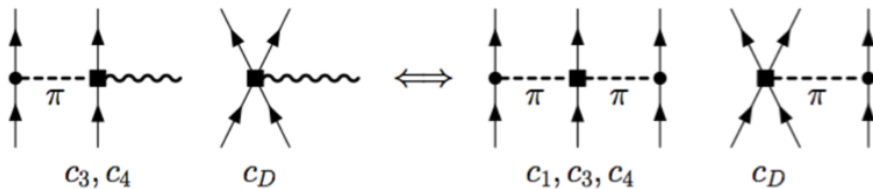
$$\hat{O} = GT^{(1)} + MEC^{(2)} \rightarrow \hat{O}_\alpha = GT^{(1)} + GT_\alpha^{(2)} + MEC_\alpha^{(2)} + \dots$$

## Operator:

Gamow-Teller (1-body) + chiral meson exchange current (2-body)  
Park (2003)

## Potential: "N<sup>4</sup>LO NN"

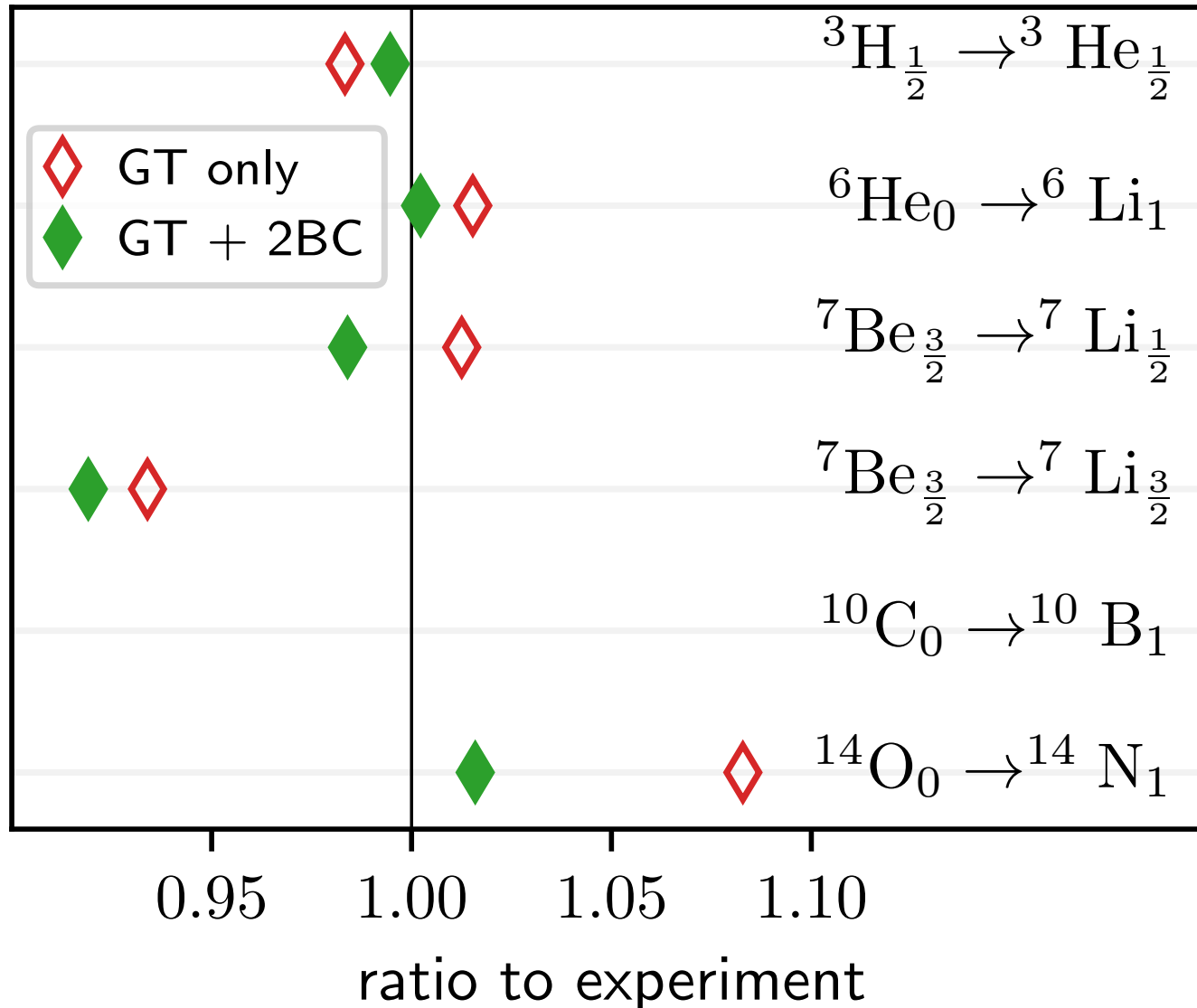
- chiral NN @ N<sup>4</sup>LO, Machleidt PRC96 (2017), 500MeV cutoff
- LEC  $c_D = -1.8$  determined



From: P. Gysbers, P. Navratil, S. Quaglioni

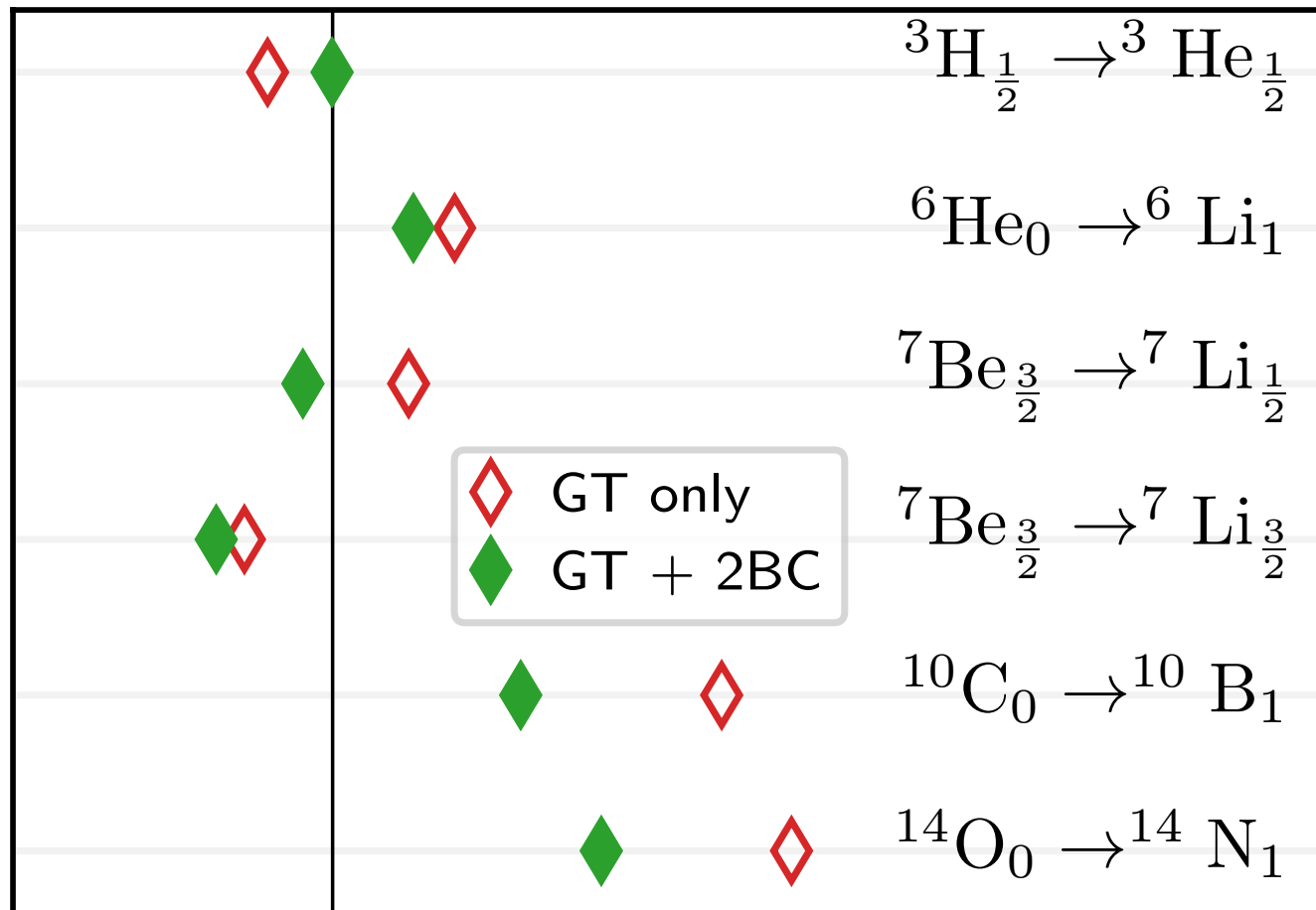
# Theory to experiment ratios for beta decays in light nuclei from NCSM

NNLO<sub>sat</sub> ( $c_D = 0.82$ )



# Theory to experiment ratios for beta decays in light nuclei from NCSM

N4LO(EM) + 3N<sub>int</sub> SRG-evolved to 2.0fm<sup>-1</sup> (c<sub>D</sub> = -1.8)

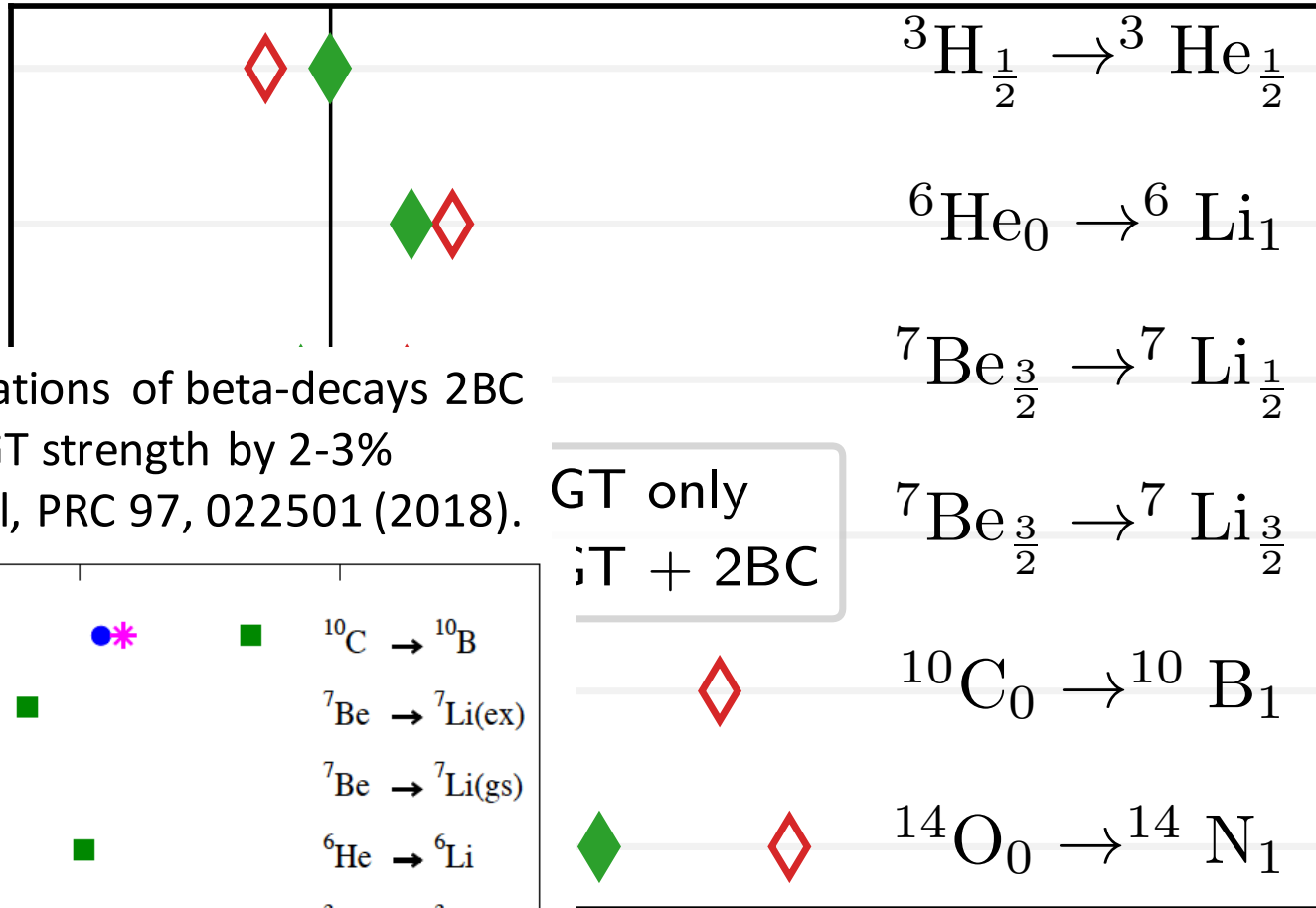


0.95 1.00 1.05 1.10  
ratio to experiment

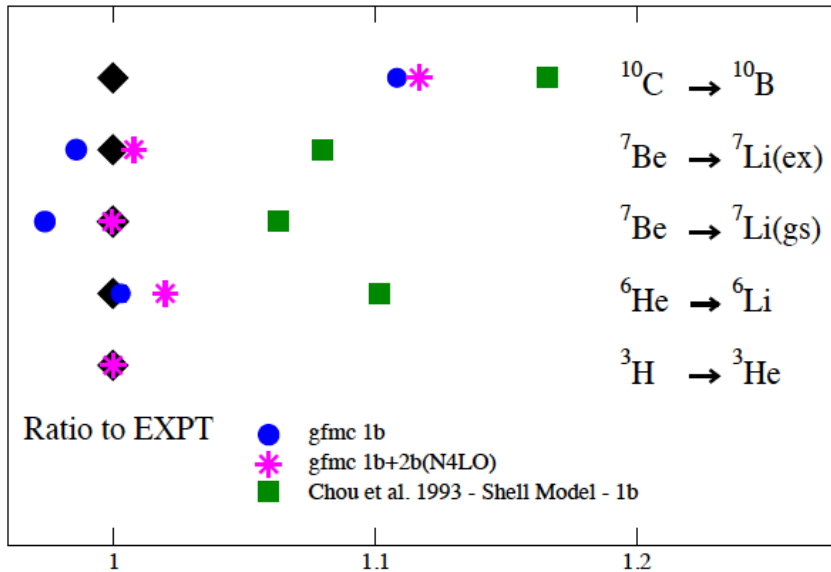
Entem, Machleidt & Nosyk,  
PRC 96, 024004 (2017)

# Theory to experiment ratios for beta decays in light nuclei from NCSM

N4LO(EM) +  $3N_{\text{Inl}}$  SRG-evolved to  $2.0\text{fm}^{-1}$  ( $c_D = -1.8$ )



In QMC calculations of beta-decays 2BC increase the GT strength by 2-3%  
S. Pastore et al, PRC 97, 022501 (2018).

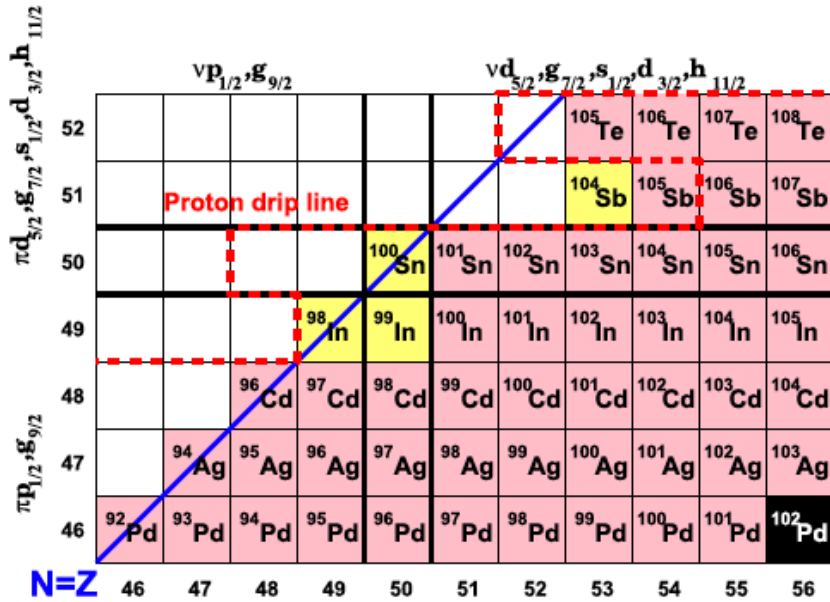


GT only  
;T + 2BC

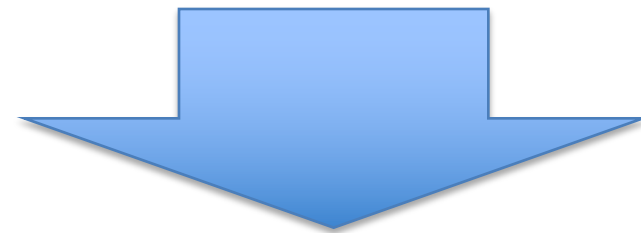
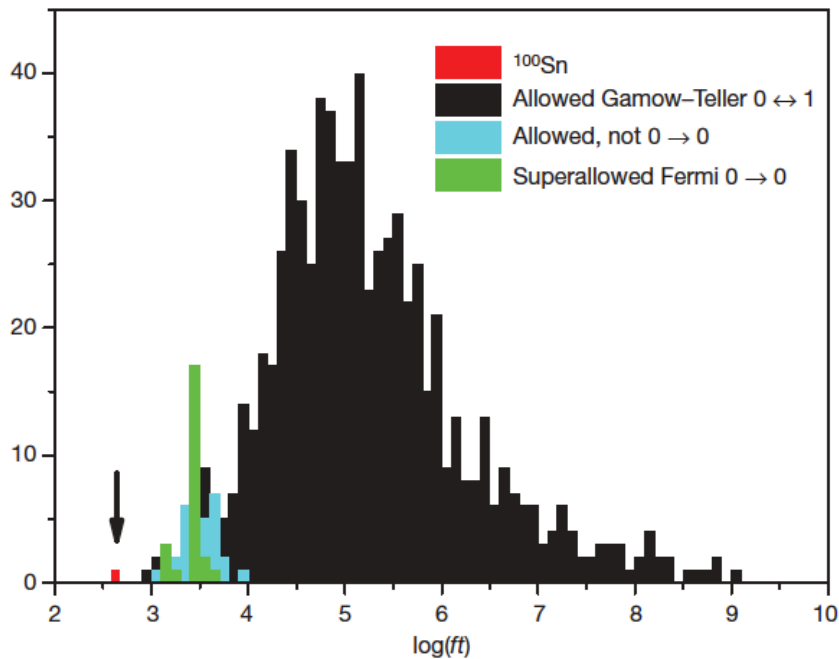
1.10  
o experiment

Entem, Machleidt & Nosyk,  
PRC 96, 024004 (2017)

# $^{100}\text{Sn}$ – a nucleus of superlatives



- Heaviest self-conjugate doubly magic nucleus
- Largest known strength in allowed nuclear  $\beta$ -decay
- Ideal nucleus for high-order CC approaches

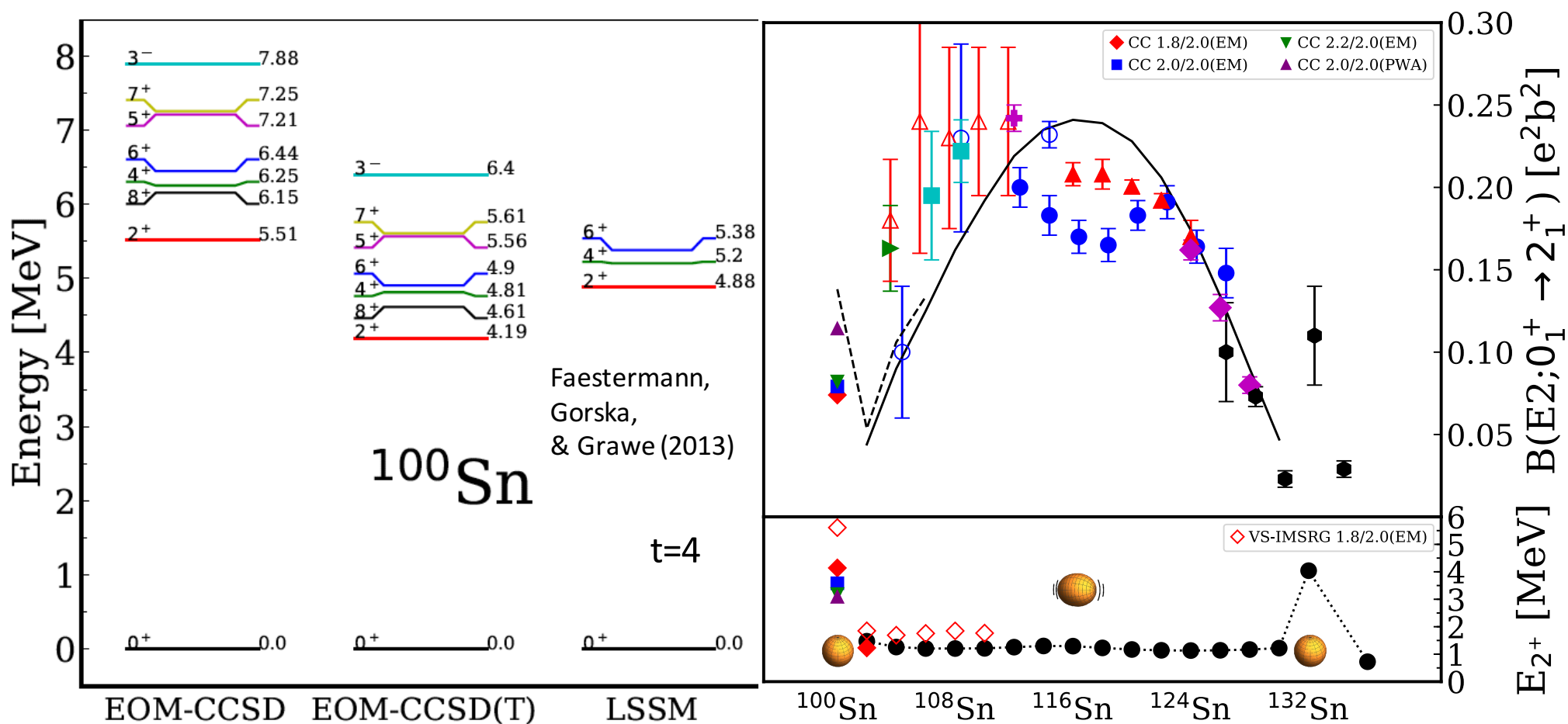


Quantify the effect of quenching from correlations and 2BCs

Hinke et al, Nature (2012)

### Structure of the Lightest Tin Isotopes

T. D. Morris,<sup>1,2</sup> J. Simonis,<sup>3,4</sup> S. R. Stroberg,<sup>5,6</sup> C. Stumpf,<sup>3</sup> G. Hagen,<sup>2,1</sup> J. D. Holt,<sup>5</sup> G. R. Jansen,<sup>7,2</sup>  
 T. Papenbrock,<sup>1,2</sup> R. Roth,<sup>3</sup> and A. Schwenk<sup>3,4,8</sup>

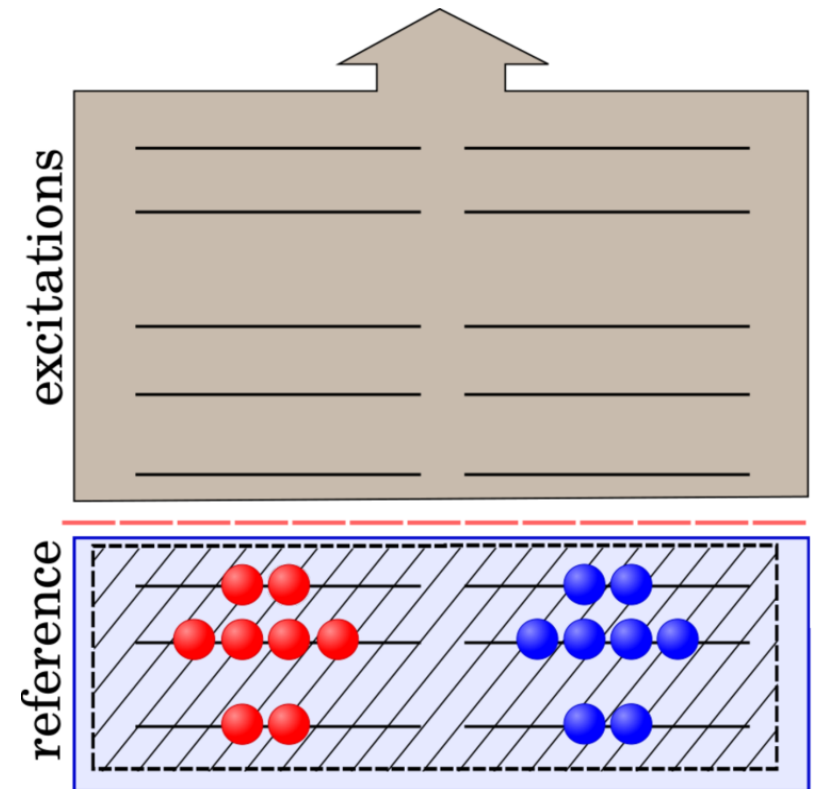
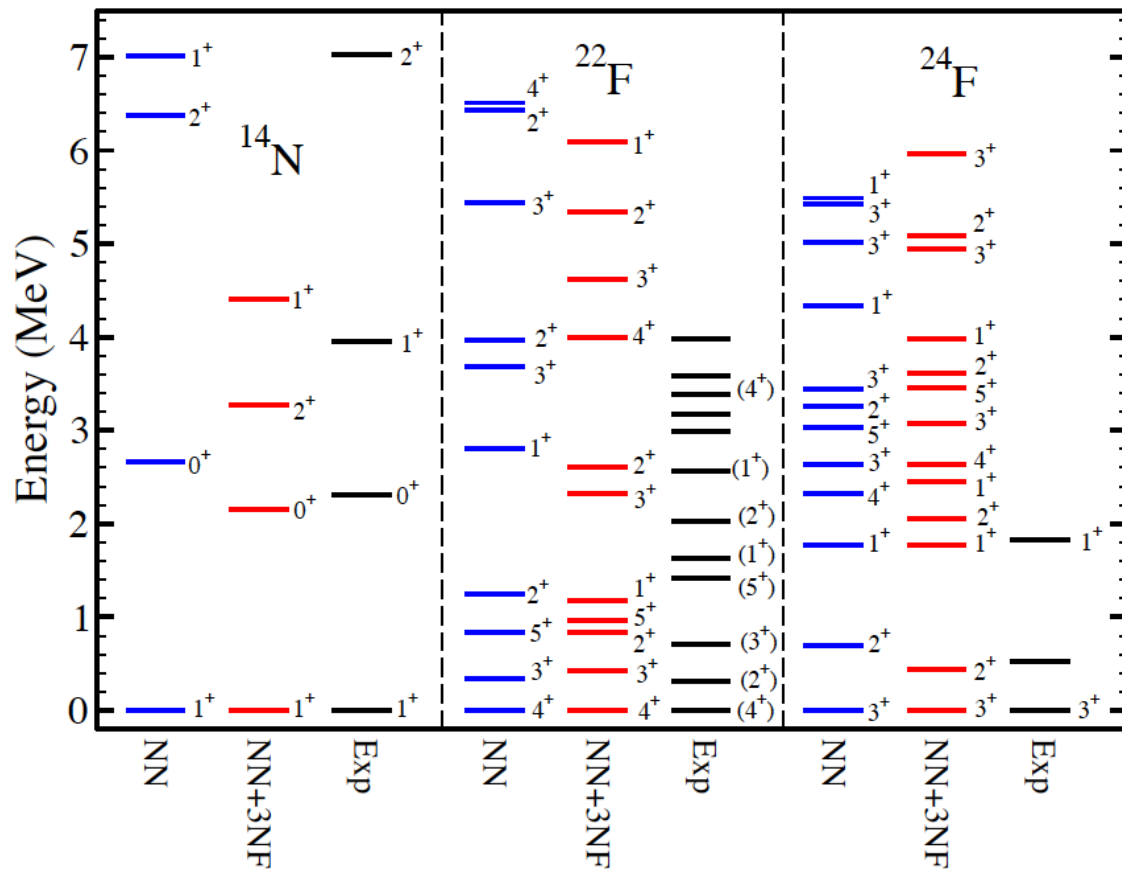




# Coupled cluster calculations of beta-decay partners

Diagonalize  $\overline{H} = e^{-T} H_N e^T$  via a novel equation-of-motion technique:

$$R_v = \sum r_i^a p_a^\dagger n_i + \frac{1}{4} \sum r_{ij}^{ab} p_a^\dagger N_b^\dagger N_j n_i + \frac{1}{36} \sum r_{ijk}^{abc} p_a^\dagger N_b^\dagger N_c^\dagger N_k N_j n_i$$

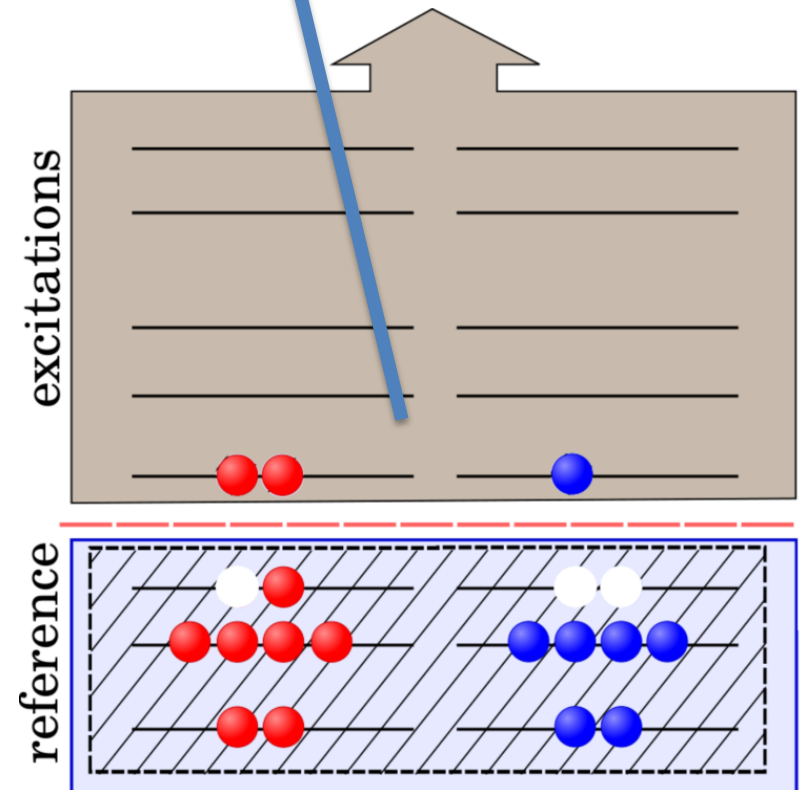
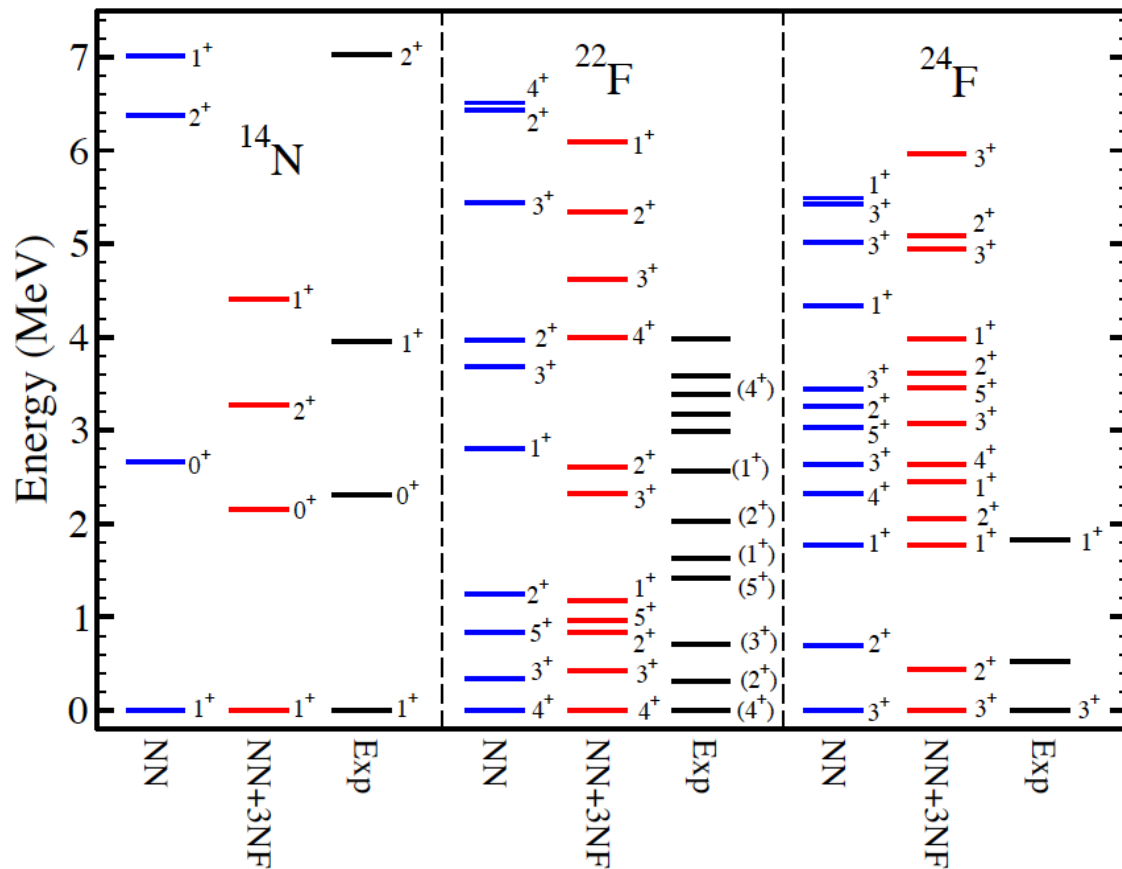


A. Ekström, G. Jansen, K. Wendt et al, PRL 113 262504 (2014)

# Coupled cluster calculations of beta-decay partners

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A. Ekström, G. Jansen, K. Wendt et al, PRL 113 262504 (2014)

# Charge exchange EOM-CCSDT-1

$$\bar{H}_{CCSDT-1} = \begin{array}{c} \text{P-space} \\ \left[ \begin{array}{cc|c} \langle S|\bar{H}|S\rangle & \langle D|\bar{H}|S\rangle & \langle T|V|S\rangle \\ \langle S|\bar{H}|D\rangle & \langle D|\bar{H}|D\rangle & \langle T|V|D\rangle \\ \hline \langle S|V|T\rangle & \langle D|V|T\rangle & \langle T|F|T\rangle \end{array} \right] \\ \text{Q-space} \end{array}$$

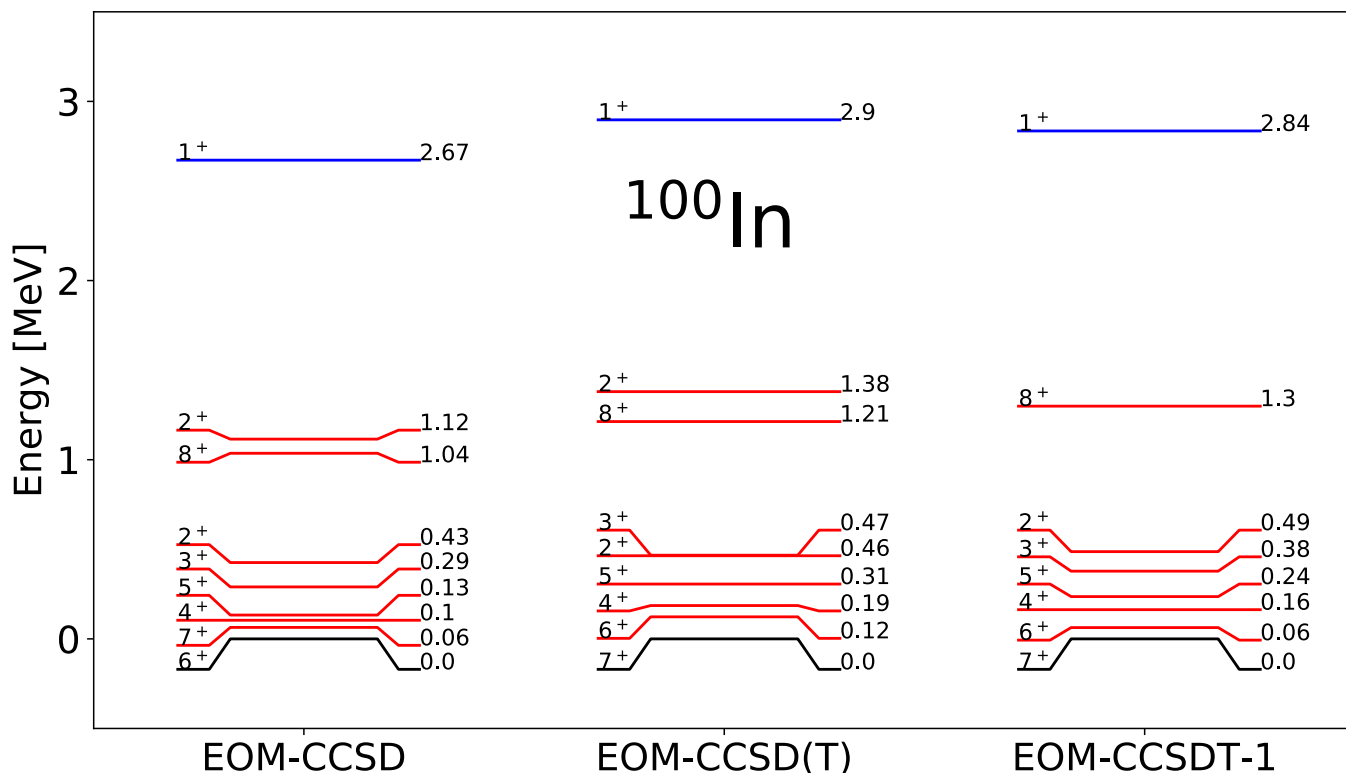
- Bloch-Horowitz is exact; iterative solution poss.

$$\bar{H}_{PP}R_P + \bar{H}_{PQ}(\omega - \bar{H}_{QQ})^{-1}\bar{H}_{QP}R_P = \omega R_P$$

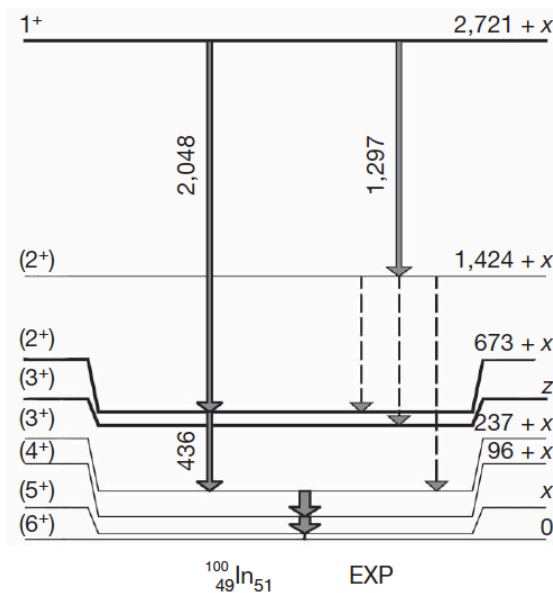
- Q-space is restricted to:  $\tilde{E}_{pqr} = \tilde{e}_p + \tilde{e}_q + \tilde{e}_r \leq \tilde{E}_{3\max}$
- No large memory required for lanczos vectors
- Can only solve for one state at a time
- Reduces matrix dimension from  $\sim 10^9$  to  $\sim 10^6$

# $^{100}\text{In}$ from charge exchange coupled-cluster equation-of-motion method

1.8/2.0 (EM)



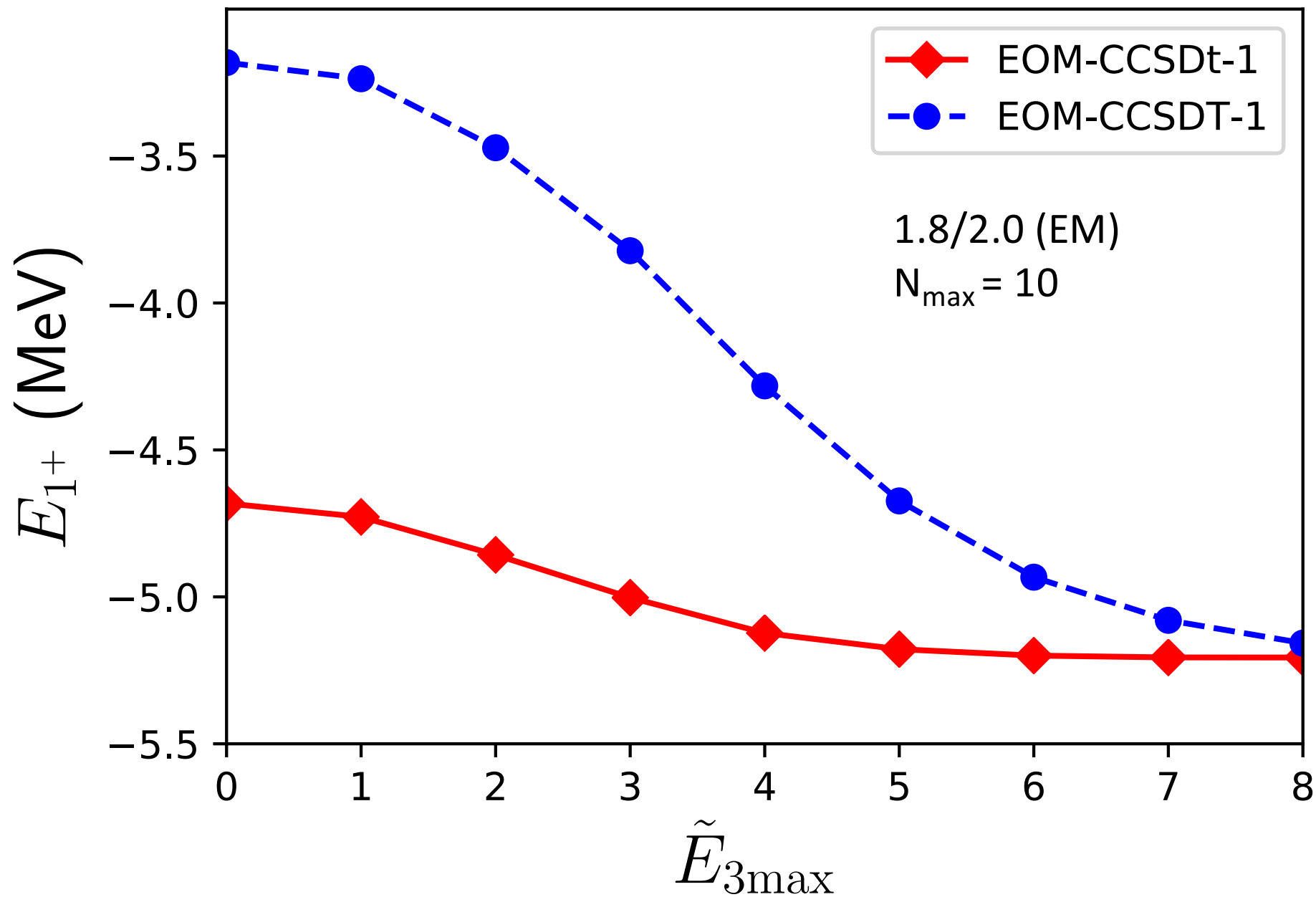
Hinke et al, Nature (2012)



Charge-exchange EOM-CC with perturbative corrections accounting for excluded 3p3h states:

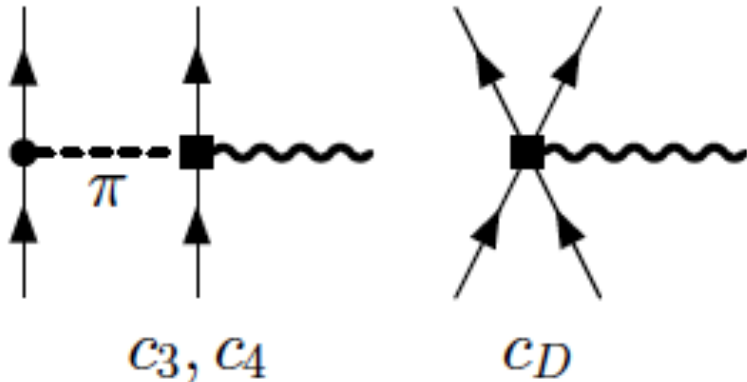
$$\Delta\omega_{\mu} = \langle \Phi_0 | L_{\mu} \bar{H}_{PQ'} (\omega_{\mu} - \bar{H}_{Q'Q'})^{-1} \bar{H}_{Q'P} R_{\mu} | \Phi_0 \rangle$$

# Convergence of excited states in $^{100}\text{In}$



# Normal ordered one- and two-body current

Gamow-Teller matrix element:  $\hat{O}_{GT} \equiv \hat{O}_{GT}^{(1)} + \hat{O}_{GT}^{(2)} \equiv g_A^{-1} \sqrt{3\pi} E_1^A$



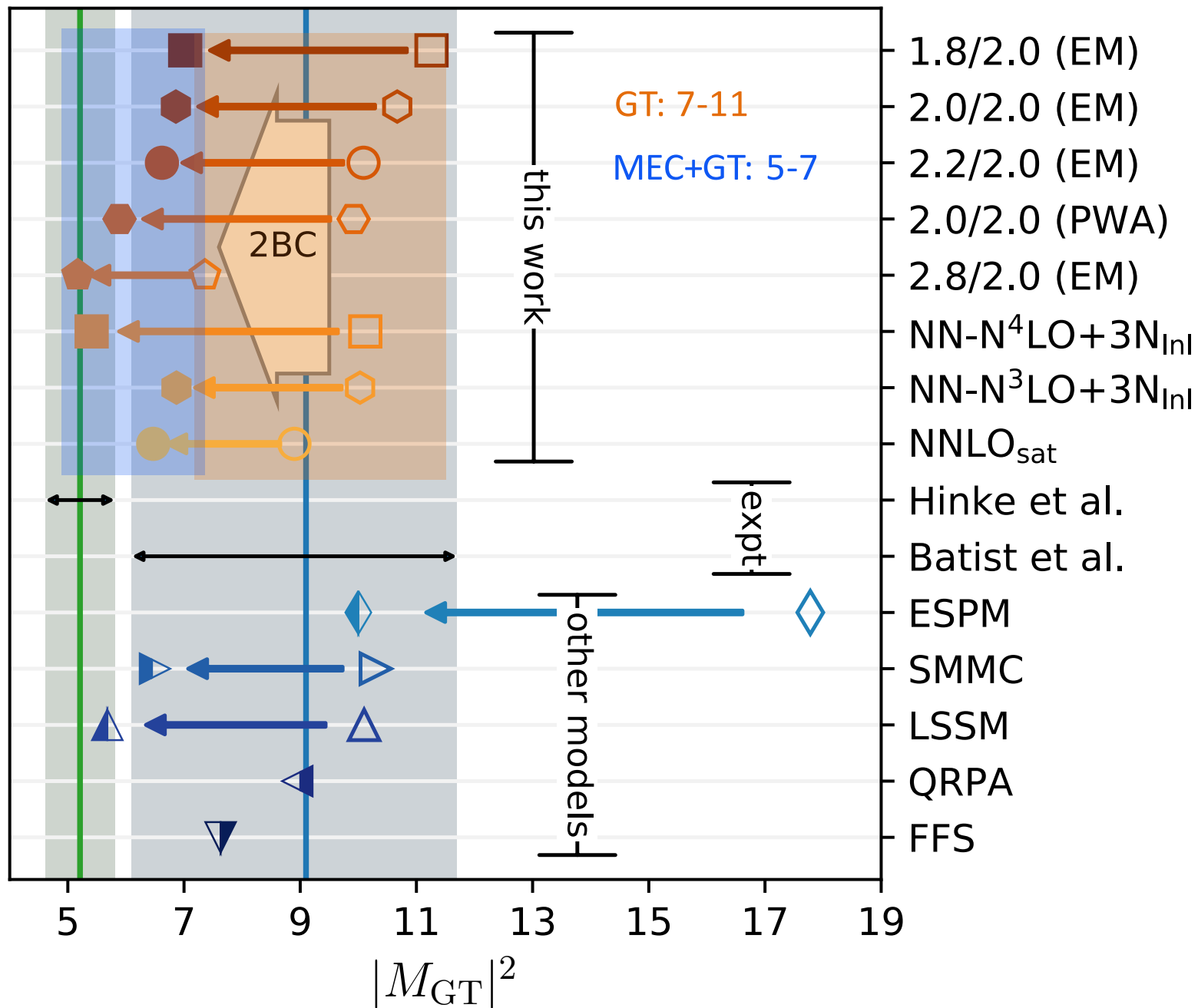
Normal ordered operator:

$$\hat{O}_{GT} = O_N^1 + \cancel{O_N^2}$$

Benchmark between NCSM and CC for the large transition in  $^{14}\text{O}$  using  $\text{NNLO}_{\text{sat}}$

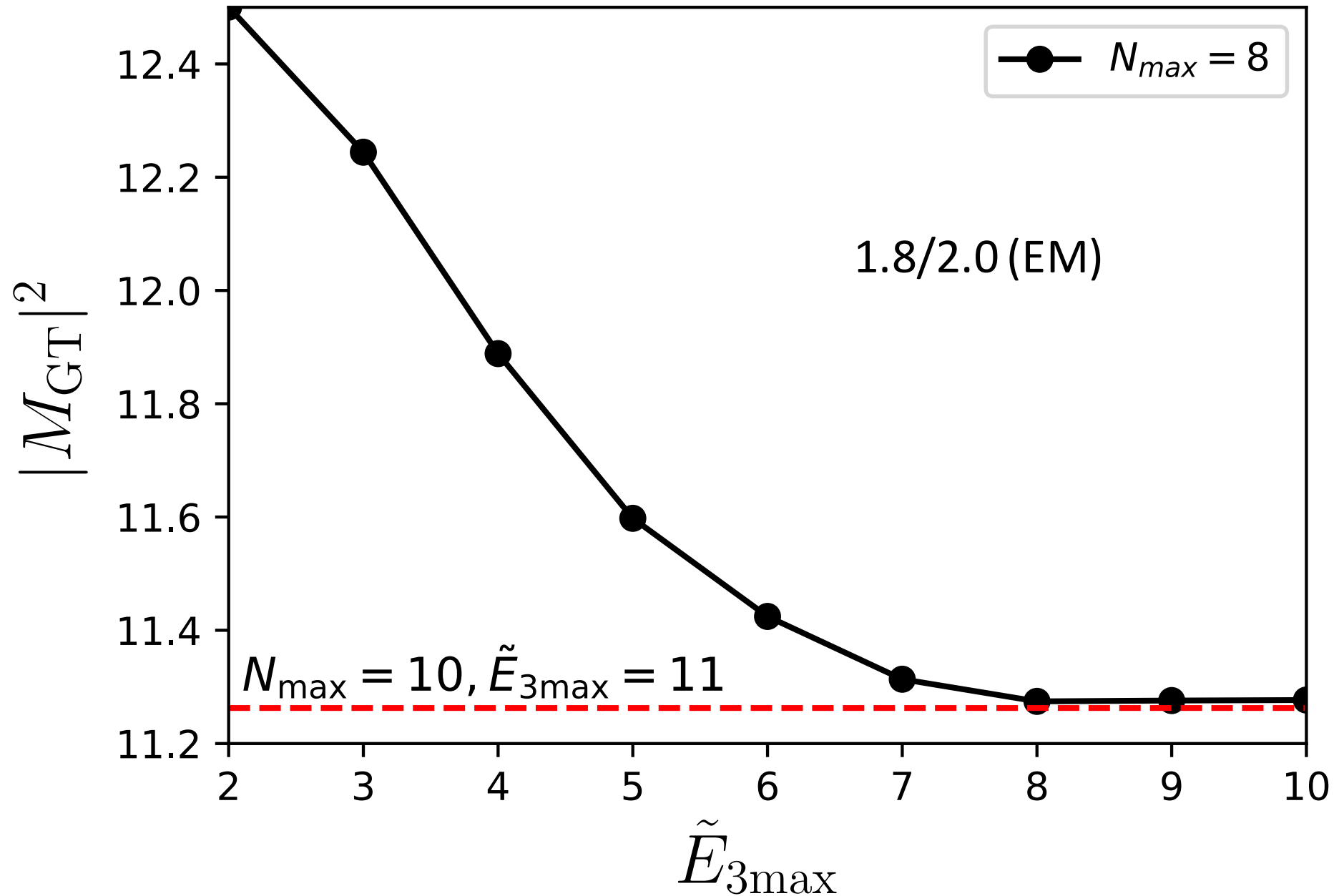
Method	$ M_{GT}(\sigma\tau) $	$ M_{GT} $
EOM-CCSD	2.15	2.08
EOM-CCSDT-1	1.77	1.69
NCSM	1.80(3)	1.69(3)

# Super allowed Gamow-Teller decay of $^{100}\text{Sn}$

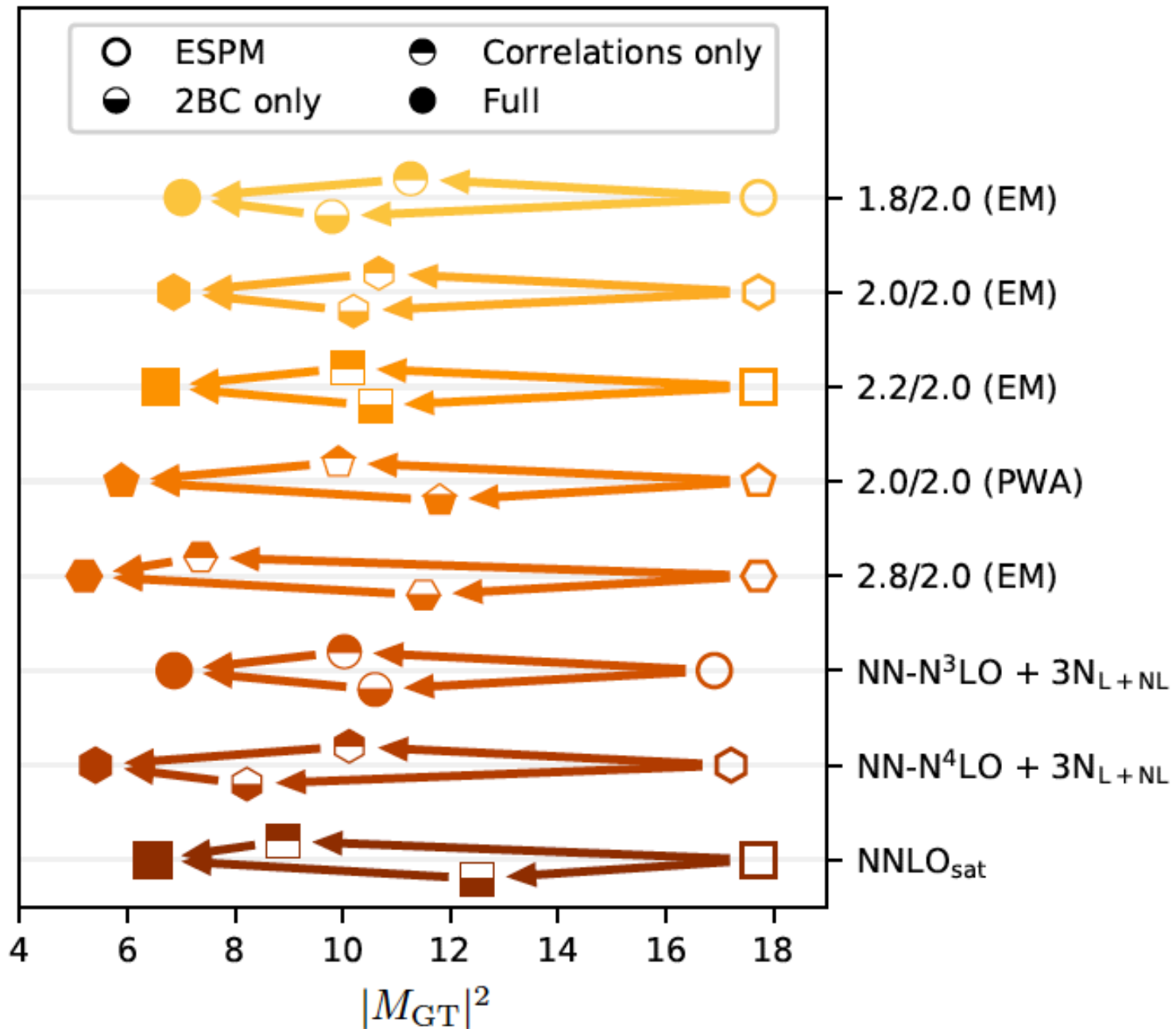




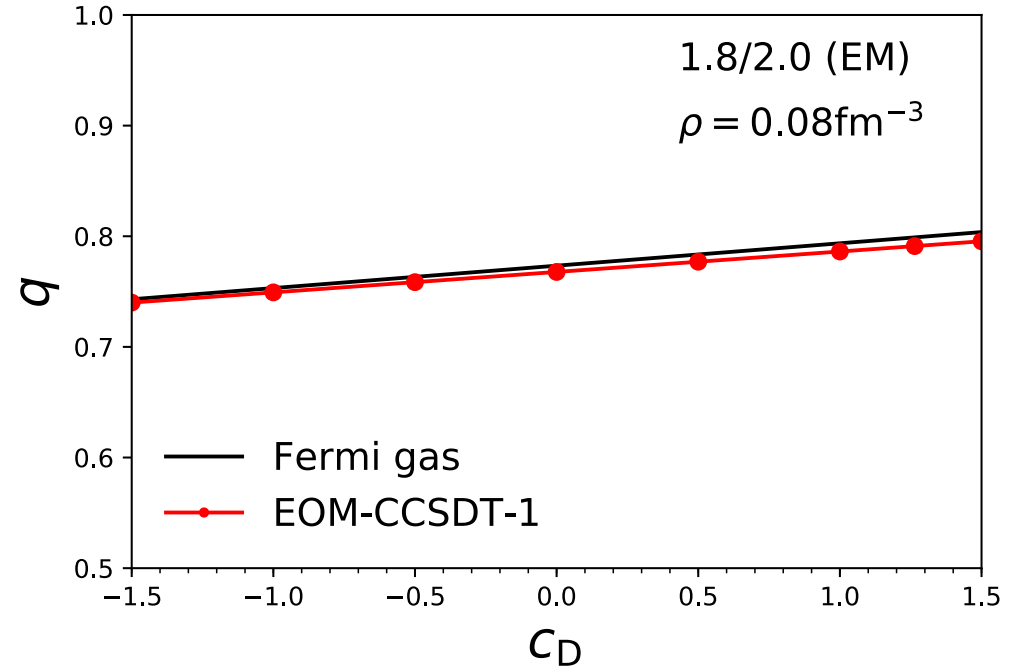
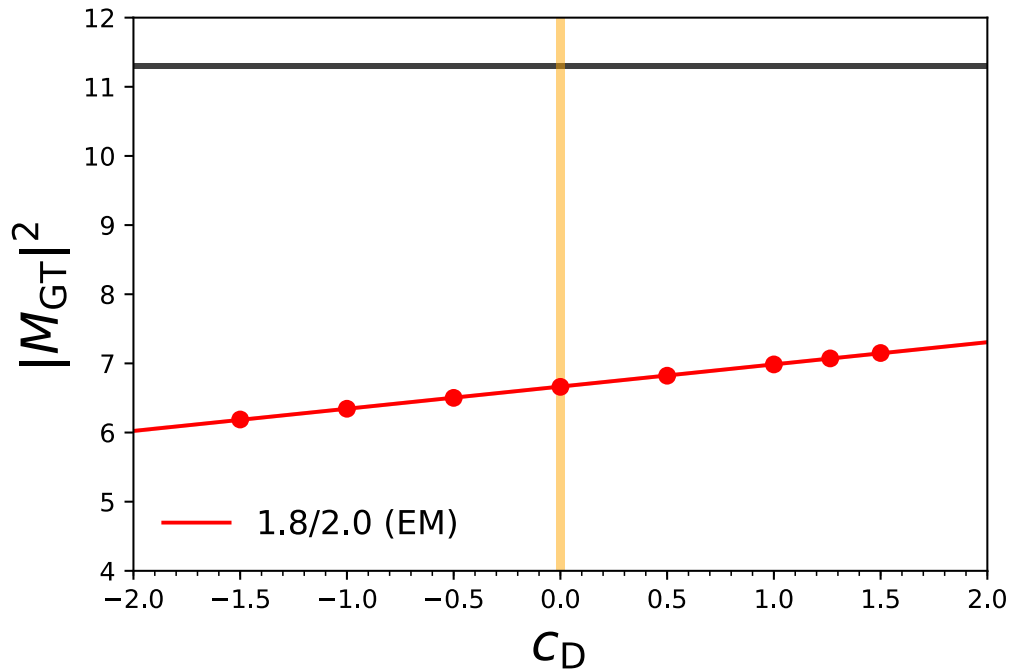
# Convergence of GT transition in $^{100}\text{Sn}$



# Role of 2BC and correlations in $^{100}\text{Sn}$



# The small role of short-ranged 2BC on GT decay

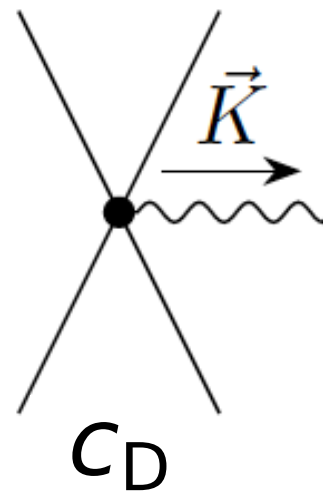


J. Menéndez, D. Gazit, A. Schwenk

PRL 107, 062501 (2011)

One-body normal ordering of 2BC in free Fermi gas

$$q \approx 1 - \frac{\rho \hbar^3 c^3}{F_\pi^2} \left( -\frac{c_D}{4g_A \Lambda} + \frac{I}{3}(2c_4 - c_3) + \frac{I}{6m} \right)$$

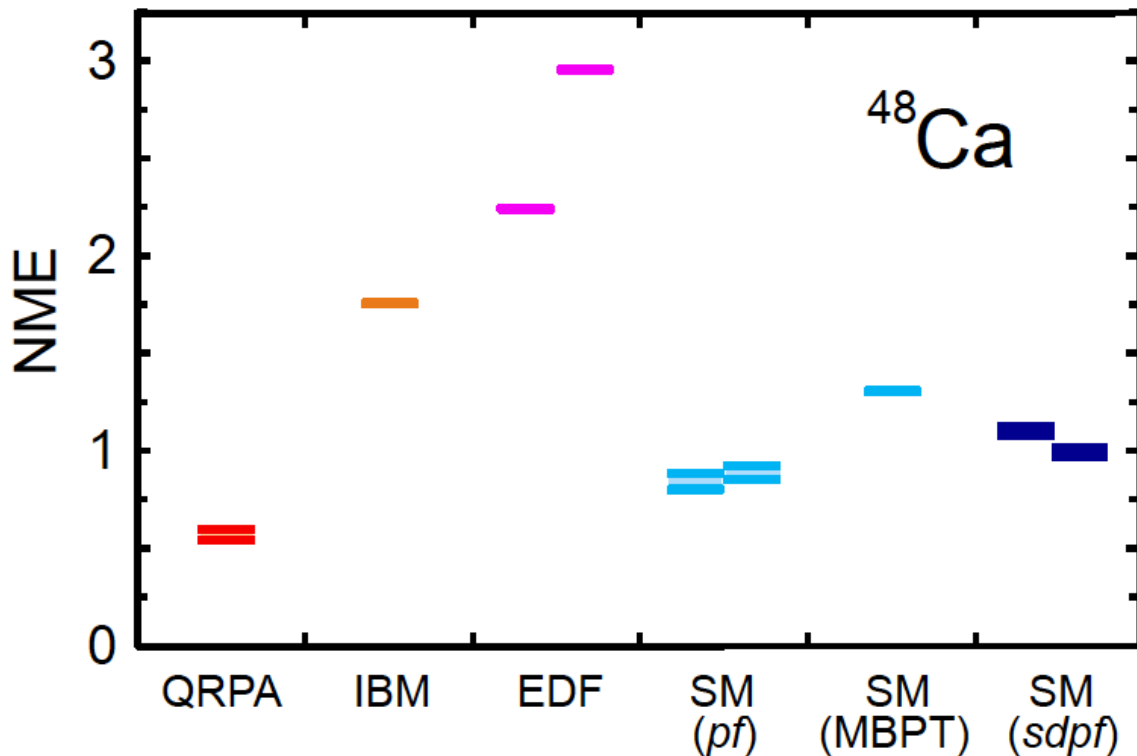


Short-ranged contact term of 2BC (heavy meson exchange)

# Neutrinoless $\beta\beta$ -decay of $^{48}\text{Ca}$

$$\left[ T_{1/2}^{0\nu} \left( 0_i^+ \rightarrow 0_f^+ \right) \right]^{-1} = G^{0\nu} |M^{0\nu}|^2 \left( \frac{\langle m_{\beta\beta} \rangle}{m_e} \right)^2$$

$0\nu\beta\beta$



- The NME for  $0\nu\beta\beta$  differ by a factor two to six depending on the method
- Need to determine the NME more precisely with quantified uncertainties
- What does ab-initio calculations add to this picture?

Nuclear matrix element for neutrinoless double beta decay in  $^{48}\text{Ca}$  using different methods. From Y. Iwata et al, PRL (2016).

## Neutrinoless $\beta\beta$ -decay of $^{48}\text{Ca}$

$$|\langle ^{48}\text{Ti} | O | ^{48}\text{Ca} \rangle|^2 = \langle ^{48}\text{Ti} | O | ^{48}\text{Ca} \rangle \langle ^{48}\text{Ca} | O^\dagger | ^{48}\text{Ti} \rangle$$

$$= \langle \Phi_0 | L_0 \bar{O}_N | \Phi_0 \rangle \langle \Phi_0 | (1 + \Lambda) \bar{O}_N^\dagger R_0 | \Phi_0 \rangle$$

Closure approximation with  
Gamow-Teller, Fermi and Tensor  
contributions:

$$M_{GT}^{0\nu} + \left( \frac{g_V}{g_A} \right)^2 M_F^{0\nu} + M_T^{0\nu}$$

Compute  $^{48}\text{Ti}$  using a double charge exchange equation of motion  
method:  $\bar{H}_N R_\mu | \Phi_0 \rangle = E_\mu R_\mu | \Phi_0 \rangle$

$$R_\mu = \frac{1}{4} \sum_{ijab} r_{ij}^{ab} p_a^\dagger p_b^\dagger n_i n_j + \frac{1}{36} \sum_{ijkabc} r_{ijk}^{abc} p_a^\dagger p_b^\dagger N_c^\dagger N_k n_i n_j$$

$$L_\mu = \frac{1}{4} \sum_{ijab} l_{ab}^{ij} p_b p_a n_i^\dagger n_j^\dagger + \frac{1}{36} \sum_{ijkabc} l_{abc}^{ijj} p_a p_b N_c N_k^\dagger n_i^\dagger n_j^\dagger$$

## $\beta\beta$ -decay of $^{48}\text{Ca}$

$$\begin{aligned} M^{2\nu} &= \sum_{\mu} \frac{\langle 0_f^+ | O_{\text{GT}} | 1_{\mu}^+ \rangle \langle 1_{\mu}^+ | O_{\text{GT}} | 0_i^+ \rangle}{E_{\mu} - E_i + Q_{\beta\beta}/2} \\ &= \langle 0_f^+ | O_{\text{GT}} \frac{1}{H - E_i + Q_{\beta\beta}/2} O_{\text{GT}} | 0_i^+ \rangle \\ &= \langle \Phi_0 | L_0 \bar{O}_{\text{GT}} \frac{1}{\bar{H} - E_i + Q_{\beta\beta}/2} \bar{O}_{\text{GT}} | \Phi_0 \rangle \end{aligned}$$

# Lanczos continued fraction method

$$M^{2\nu} = \langle \Phi_0 | L_0 \bar{O}_{\text{GT}} \frac{1}{\bar{H} - E_i + Q_{\beta\beta}/2} \bar{O}_{\text{GT}} | \Phi_0 \rangle$$

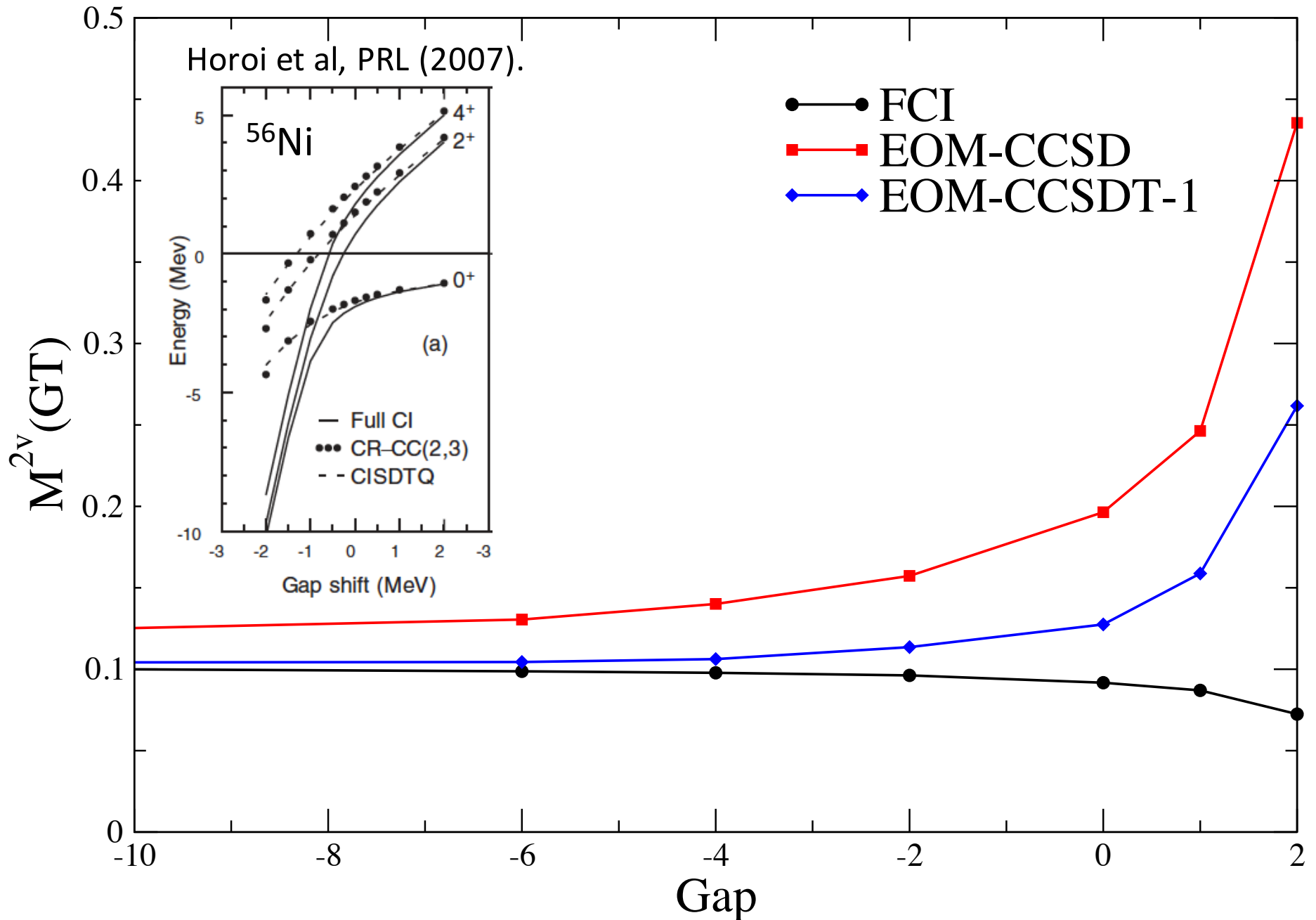
Define left/right Lanczos pivots:  $\langle \tilde{\nu}_0 | = \langle \Phi_0 | L_0 \bar{O}_{\text{GT}}$   $|\nu_0\rangle = \bar{O}_{\text{GT}} | \Phi_0 \rangle$

$$M^{2\nu} = \langle \tilde{\nu}_0 | \nu_0 \rangle \left\{ \frac{1}{(a_0 - Q_{\beta\beta}/2) - \frac{b_0^2}{(a_1 - Q_{\beta\beta}/2) - \frac{b_1^2}{(a_2 - Q_{\beta\beta}/2) - \dots}}} \right\}$$

- Lanczos continued fraction method, see e.g. Engel, Haxton, Vogel PRC (1992), Haxton, Nollett, Zurek PRC (2005), Miorelli et al PRC (2016).
- Matrix element is converged to machine precision after  $\sim 10$  iterations.
- Need more than 50  $1^+$  states converged in  $^{48}\text{Sc}$  (300-400 Lanczos iterations) if we sum explicitly over intermediate states

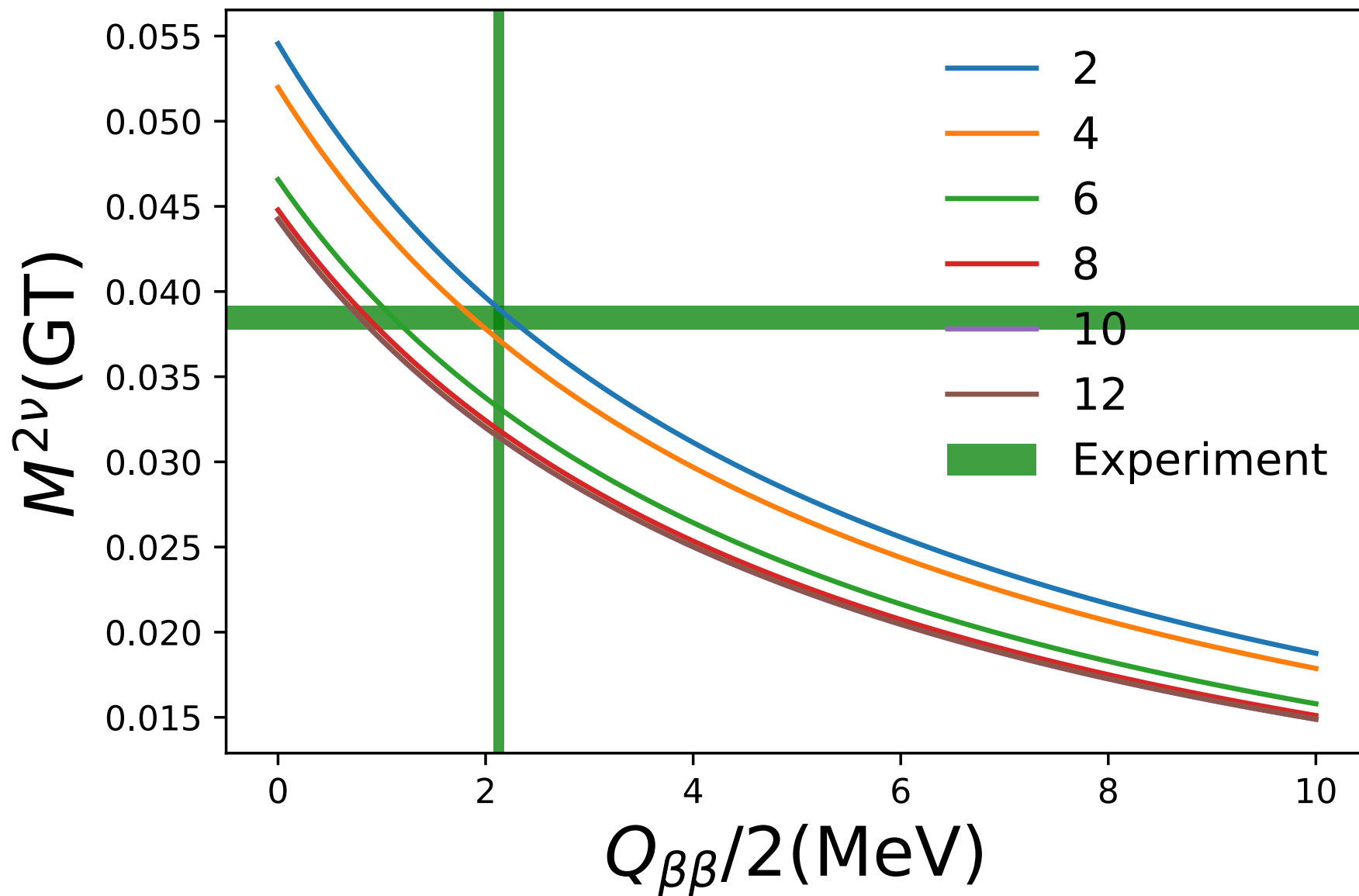


# $\beta\beta$ -decay of $^{48}\text{Ca}$ with KB3G shell-model interaction



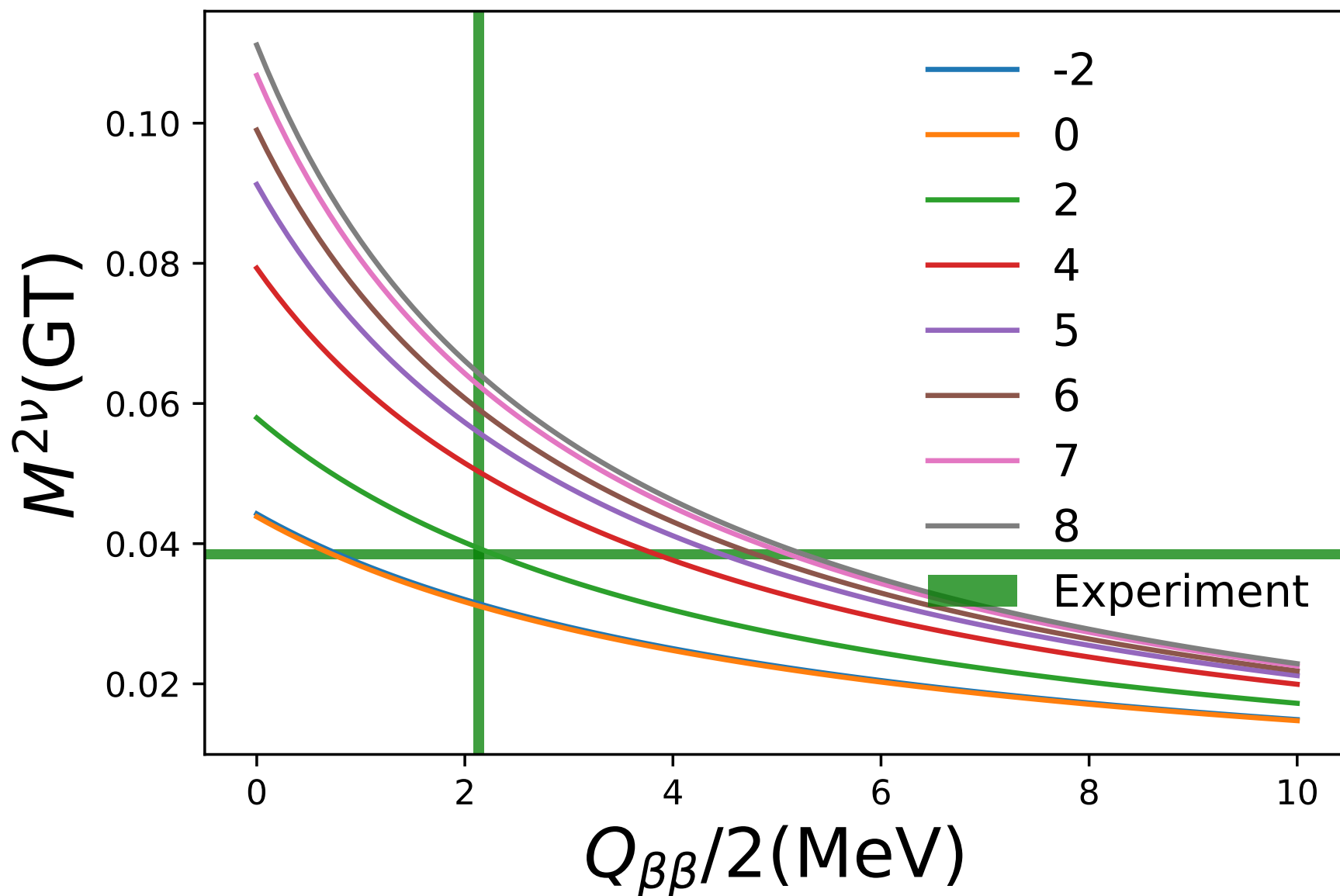
# $\beta\beta$ -decay of $^{48}\text{Ca}$

The role of 3p3h excitations in the ground-state of  $^{48}\text{Ti}$



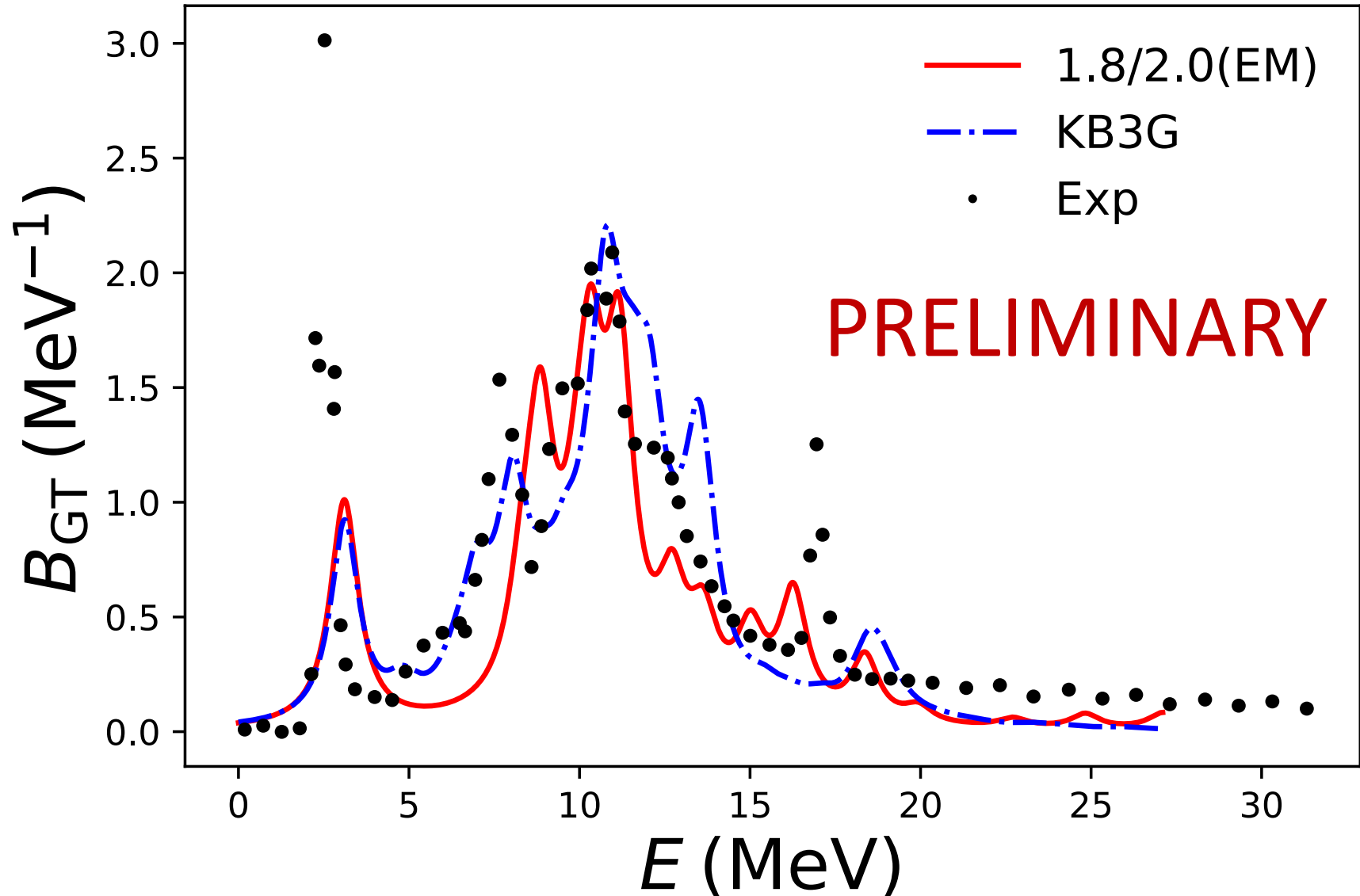
# $\beta\beta$ -decay of $^{48}\text{Ca}$

The role of 3p3h excitations in the intermediate  $1^+$  states of  $^{48}\text{Sc}$



# Gamow-Teller strengths in $^{48}\text{Ca}$

GT-strength computed using the Lanczos method and EOM-CCSDT-1  
No 2BCs, strength function folded with a Lorentzian of width 0.5MeV.



# Collaborators

@ ORNL / UTK: G. R. Jansen, **Sam Novario**, T. **Morris**, T. Papenbrock, **Z. H. Sun**

@ TU Darmstadt: **C. Drischler**, **C. Stumpf**, K. Hebeler, R. Roth, A. Schwenk, **J. Simonis**

@ LLNL: **K. Wendt**

@ Mainz: S. Bacca

@ TRIUMF: **P. Gysbers**, J. Holt, P. Navratil

@ Reed College: **S. R. Stroberg**