Nucleon Polarisabilities and χEFT: Bridging Between QCD and Data

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1. Two-Photon Response Explores System Dynamics
2. Polarisabilities, Compton Data, χEFT and Lattice-QCD
3. Spin Polarisabilities and Nucleon Spin Structure
4. Concluding Questions at the Intensity & Precision Frontier

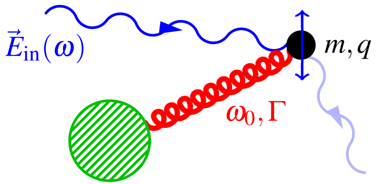
How do constituents of the nucleon react to external fields?
How to reliably extract proton, neutron, spin polarisabilities?
How to bridge between QCD and Nuclear Physics?

Comprehensive Theory Effort:
hg/JMcG/DRP/G. Feldman: **Prog. Part. Nucl. Phys. 67 (2012) 841**;
Polarisabilities & Bayes in χEFT for lattice-QCD: hg/JMcG/DRP *Eur. Phys. J.* **A52** (2016) 139

1. Two-Photon Response Explores System Dynamics

(a) Polarisabilities: Stiffness of Charged Constituents in El.- Mag. Fields

Example: induced electric dipole radiation from harmonically bound charge, damping Γ Lorentz/Drude 1900/1905



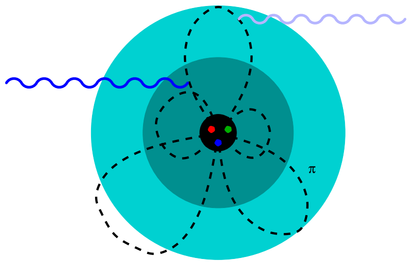
$$\vec{d}_{\text{ind}}(\omega) = \underbrace{\frac{q^2}{m} \frac{1}{\omega_0^2 - \omega^2 - \mathrm{i}\Gamma\omega}}_{=:\,4\pi\,\alpha_{E1}(\omega)} \vec{E}_{\text{in}}(\omega)$$

$=: 4\pi\,\alpha_{E1}(\omega)$ "displaced volume" $[10^{-4}\ \text{fm}^3]$

electric scalar dipole polarisability

Dis-entangle ***interaction scales, symmetries & mechanisms*** with & among constituents.

Fundamental hadron properties, like charge, mass, mag. moment, $\langle r_N^2 \rangle \ldots$ PDG

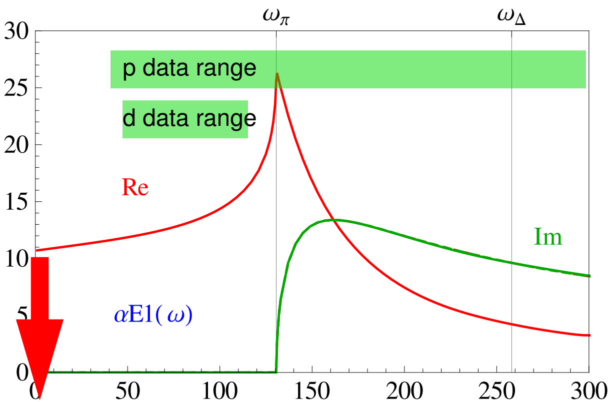


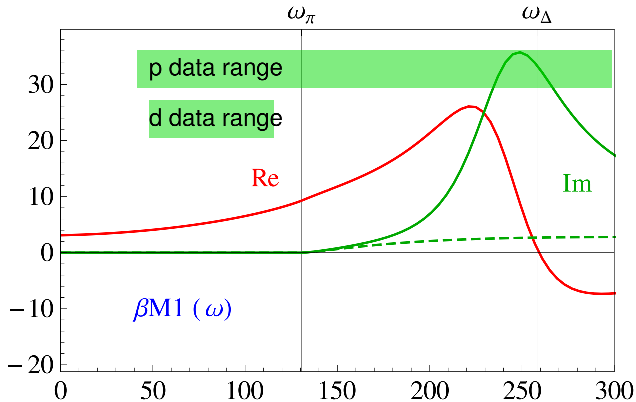
(b) Understanding Energy Dependence

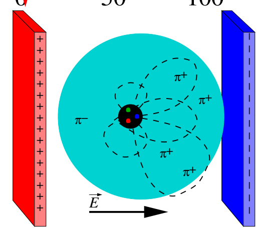
Polarisabilities: ***Energy-dependent Multipoles*** **of real Compton scattering.**

$$T = \begin{bmatrix} \textbf{Powell}\text{: point spin-}\frac{1}{2}\text{ with} \\ \text{anomalous mag. moment} \end{bmatrix} + 2\pi\,\omega^2 \Big[\underbrace{\alpha_{E1}(\omega)}_{\text{electric}} (\vec{\epsilon}'^\dagger \cdot \vec{\epsilon}) + \underbrace{\beta_{M1}(\omega)}_{\text{magnetic}} \Big((\hat{\vec{k}}' \times \vec{\epsilon}'^\dagger) \cdot (\hat{\vec{k}} \times \vec{\epsilon}) \Big) + \dots \Big]$$

Neither more nor less information about ***two-photon response*** of constituents, but **more readily accessible.**





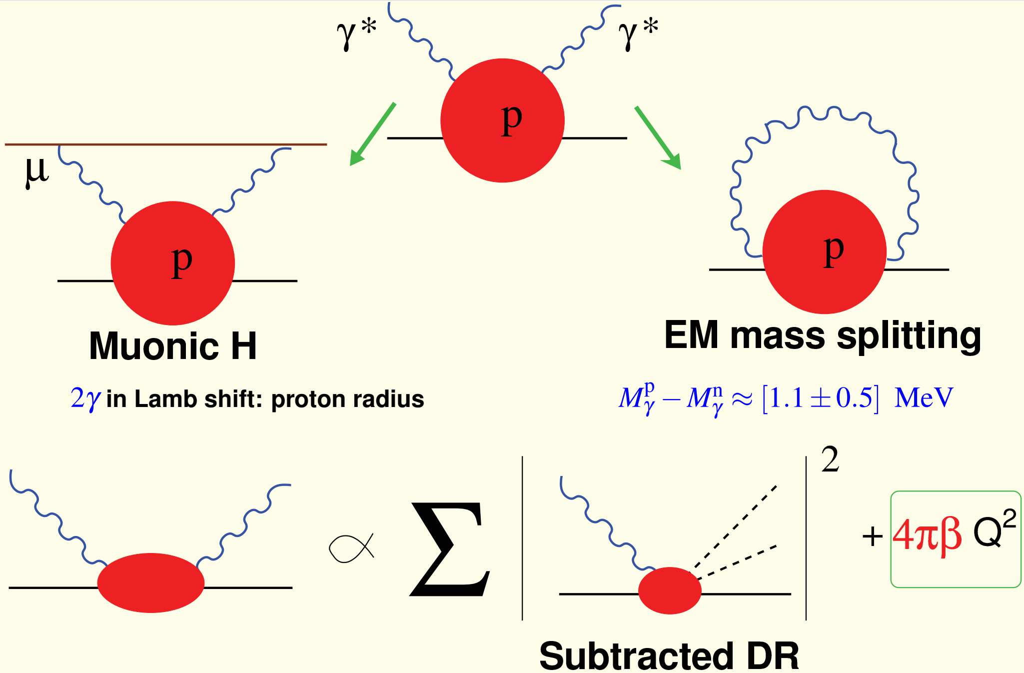


For $\omega \not\ll m_\pi$ more than "static+slope"! $\Longrightarrow$ Need to understand *dynamics*
to reliably extrapolate from data to *"the (static) polarisabilities"* $\alpha_{E1}(\omega = 0)$ etc.

$\Longrightarrow$ Compresses rich dynamics into few numbers.

(c) Example: Why the Magnetic Polarisability β_{M1} Matters

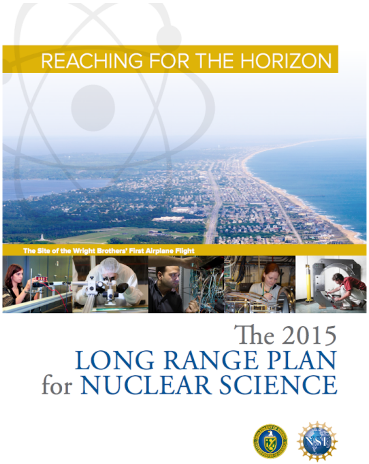
modified from McGovern's plenary at χDyn 2015



Dispersion Relations: Cottingham Sum Rule and VVCS

$$\overline{T}_1(\nu, Q^2) = -\nu^2 \int_{\nu_{th}^2}^{\infty} \frac{d\nu'^2}{\nu'^2} \frac{W_1(\nu', Q^2)}{\nu'^2 - \nu^2} + 4\pi\beta Q^2 + O(Q^4)$$

(d) A Word from Our Sponsors: The US Long Range Plan



The special status of pions and kaons in QCD and their marked impact on the long-distance structure of hadrons can be systematically encoded in an effective theory, applicable to processes at low energy. This effective theory, as well as emerging LQCD calculations, can provide benchmark predictions for so-called polarizabilities that parameterize the deformation of hadrons due to electromagnetic fields, spin fields, or even internal color fields. Great progress has been made in determining the electric and magnetic polarizabilities. Within the next few years, data are expected from the High Intensity Gamma-ray Source (HIγS) facility that will allow accurate extraction of proton-neutron differences and spin polarizabilities. JLab also explores aspects

[US NSAC LRP 2015 p. 14]

HIγS (DOE): a central goal; > 3000 hrs committed at $60 - 100$ MeV

proton doubly & beam pol. (E-06-09/10) deuteron beam pol. (E-18-09, running)

^{3}He unpol & doubly pol. (E-07-10, E-08-16) ^{4}He unpol ^{6}Li unpol. (E-15-11, first!)

A2 @ MAMI (DFG: 5-year SFB): running, data cooking and planned

proton $100 - 400$ MeV: beam & target pol. deuteron, ^{3}He, ^{4}He unpol., beam & target pol.

MAXlab: data cooking deuteron $100 - 160$ MeV: unpol.

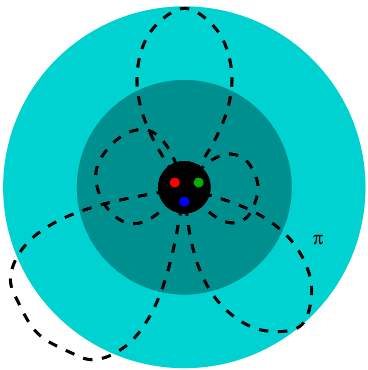
(a) The Low-Energy Method: Chiral Effective Field Theory

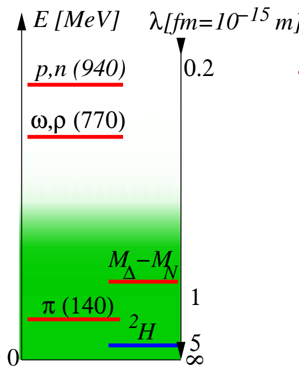
Degrees of freedom $\pi, N, \Delta(1232)$ + **all interactions allowed by symmetries:** Chiral SSB, gauge, iso-spin,...

$\Longrightarrow$ **Chiral Effective Field Theory χEFT $\equiv$ low-energy QCD**

$$\mathcal{L}_{\chi\text{EFT}} = (D_\mu \pi^a)(D^\mu \pi^a) - m_\pi^2\, \pi^a \pi^a + \dots + N^\dagger[\mathrm{i}\, D_0 + \frac{\vec{D}^2}{2M} + \frac{g_A}{2f_\pi}\, \vec{\sigma}\cdot\vec{D}\pi + \dots]N + C_0 \left(N^\dagger N\right)^2 + \dots$$

Controlled approximation $\Longrightarrow$ ***Model-independent, error-estimate*.**





Two Low-Energy Régimes:

Low régime: $\omega \lesssim m_\pi$: $\Delta(1232)$ suppressed

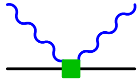
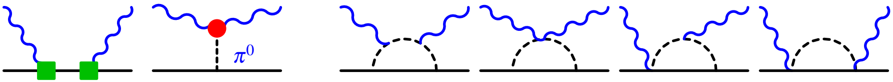
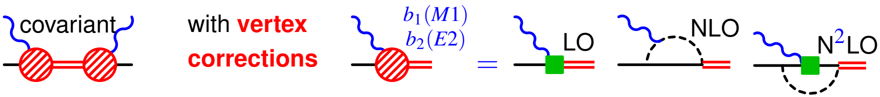
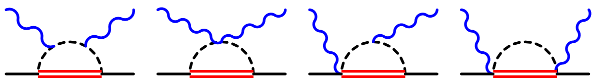
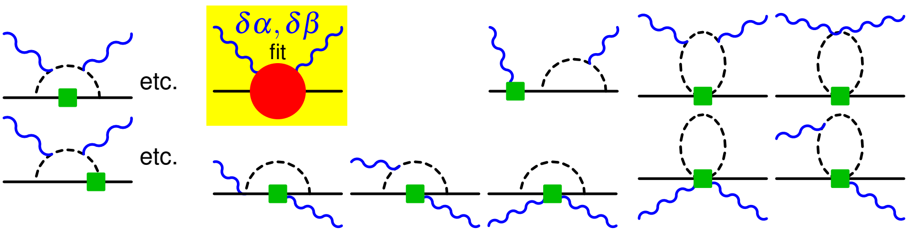
High régime: $\omega \approx M_\Delta - M_N \approx 300$ MeV:

$\Delta(1232)$ dominates $\Longrightarrow$ propagator:

$$\frac{1}{E - (M_\Delta - M_N) - \Sigma} + \text{relativity}$$

Expand in $\dfrac{\omega}{\Lambda_\chi}$ **and** $\delta = \dfrac{M_\Delta - M_N}{\Lambda_\chi \approx 1\,\mathrm{GeV}} \approx \sqrt{\dfrac{m_\pi}{\Lambda_\chi}} = \dfrac{p_{\text{typ}}}{\Lambda_\chi} \ll 1$ **(numerical fact)** Pascalutsa/Phillips 2002.

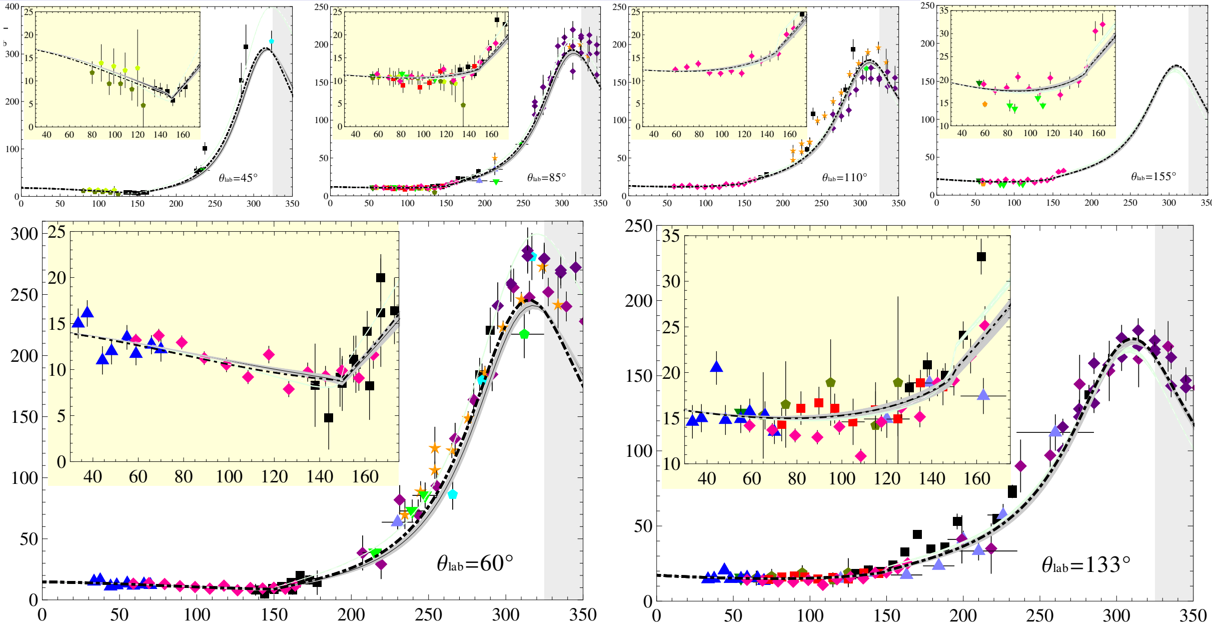
Unified Amplitude: gauge & RG invariant set of all contributions which are
in low régime $\omega \lesssim m_\pi$ at least N^{4}LO $(e^2\delta^4)$: accuracy $\delta^5 \lesssim 2\%$;
or in high régime $\omega \sim M_\Delta - M_N$ at least NLO $(e^2\delta^0)$: accuracy $\delta^2 \lesssim 20\%$.

	$\omega \lesssim m_\pi$	$\sim M_\Delta - M_N \approx 300$ MeV
	$e^2\delta^0$ ***LO***	$e^2\delta^0$ ↘NLO
	$e^2\delta^2$ N^{2}LO	$e^2\delta^1$ N^{2}LO
	$e^2\delta^3$ N^{3}LO	$e^2\delta^{-1}$ ↗***LO***
	$e^2\delta^3$ N^{3}LO	$e^2\delta^1$ N^{2}LO
	$e^2\delta^4$ N^{4}LO	$e^2\delta^2$ N^{3}LO

Unknowns: short-distance $\delta\alpha, \delta\beta \Longleftrightarrow$ Fit static α_{E1}, β_{M1} (offset). $\Longrightarrow$ *Predict* ω-dependence.

(c) Nucleon Polarisabilities from a Consistent Database

McGovern/Phillips/hg 2013
database: +Feldman PPNP 2012



Noisy database, partially conflicting $\Longrightarrow$ reproducible trimming necessary.

Fit focuses on different Physics in different regions:

> 200 MeV: $\Delta(1232)$ **fit** $b_1 = 3.61 \pm 0.02 \Longleftrightarrow < 170$ MeV: **polarisabilities**

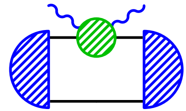
(d) Neutron Polarisabilities & Nuclear Binding

hg/…/+Phillips/+McGovern 2004-2014
MECs: Beane/…1999-2005

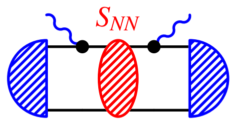
χEFT: consistency between wave functions, potentials, currents, meson-exchange, 1-N and few-N.

- **Nucleon Structure: average of neutron & proton polarisabilities:**

  χEFT, Disp. Rel.: p-n difference is small hg/Pasquini/…2005



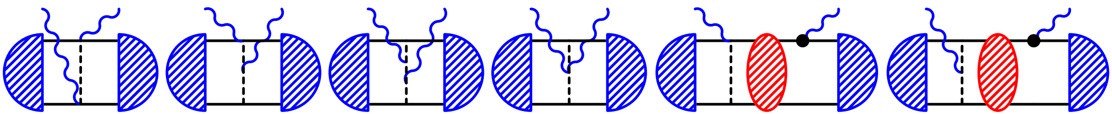
- **Parameter-free coherent Rescattering Contributions:**

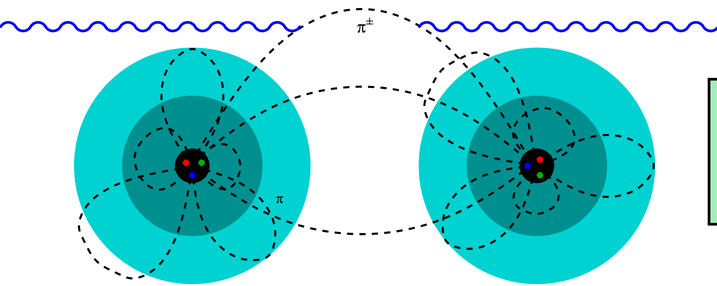


$$\frac{\mathrm{i}}{B_d \pm \omega - \frac{q^2}{M}} : \text{2N} \begin{cases} \text{coherent for } \omega \sim 20\ \text{MeV} \\ \text{incoherent for } \omega \sim m_\pi \end{cases}$$

$\Longrightarrow$ less relevant for $\omega \gtrsim 40\ \text{MeV}$

- **Parameter-free charged Meson-Exchange Currents are large,** dictated by gauge & chiral symmetry:





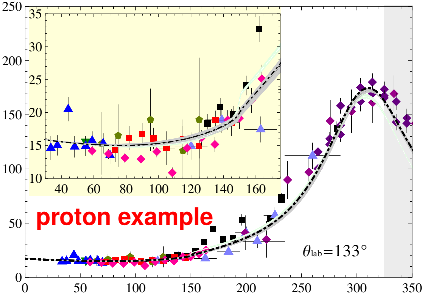
Model-independently subtract binding

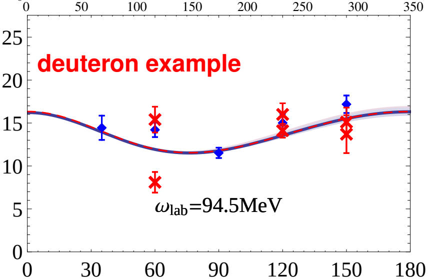
$\Longrightarrow$ χEFT: quantify reliable uncertainties.

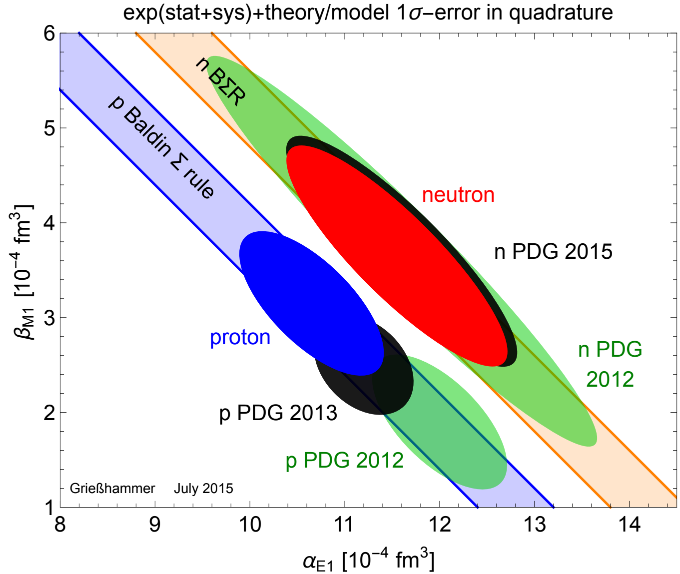
Test charged-pion component of NN force.

(e) Scalar Polarisabilities from Consistent p & d Databases

database: JMcG/DRP/hg/ Feldman PPNP 2012





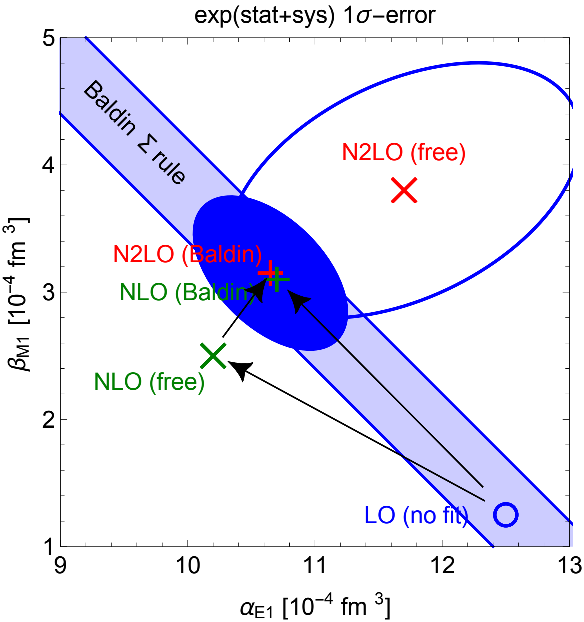


	α_{E1} [10^{-4} fm^3]	β_{M1} [10^{-4} fm^3]	χ^2/d.o.f.
proton (Baldin, N^{2}LO) McGovern/Phillips/hg EPJA 2013	$10.65 \pm 0.35_{stat} \pm 0.2_{\Sigma} \pm 0.3_{theory}$	$3.15 \mp 0.35_{stat} \pm 0.2_{\Sigma} \mp 0.3_{theory}$	$113.2/135$
neutron, with data from **Compton@MAXlab** COMPTON@MAX-lab PRL 2014	$11.55 \pm 1.25_{stat} \pm 0.2_{\Sigma} \pm 0.8_{theory}$	$3.65 \mp 1.25_{stat} \pm 0.2_{\Sigma} \mp 0.8_{theory}$	$45.2/44$

$\Longrightarrow$ **neutron ≈ proton polarisabilities:** $\alpha_{E1}^{p-n} = -0.9 \pm 1.6_{tot}$; **exp. error dominates.**

CottinghamΣR explains $M_\gamma^p - M_\gamma^n$ **with** $\alpha_{E1}^{p-n} = -1.7 \pm 0.4_{tot}$. **Gasser/Hoferichter/Leutwyler/ Rusetzky 1506.06747**

(f) Fit Discussion: Parameters and Uncertainties



Consistency of Fit Error:

Example 1σ-contours for proton

Consistent with Baldin Σ Rule

$$\alpha_{E1}+\beta_{M1}=\frac{1}{2\pi^2}\int\limits_{\nu_0}^{\infty}\mathrm{d}\nu\;\frac{\sigma(\gamma p\to X)}{\nu^2}$$

$$=13.8\pm0.4$$ Olmos de Leon 2001

need more forward data to constrain.

Fit Stability: floating norms within exp. sys. errors; vary dataset, b_1, vertex dressing,. . .

(g) What Does "Conservative" Theory Uncertainty Mean?

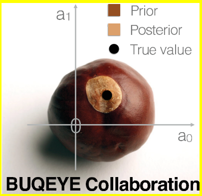
hg/JMcG/DRP 1511.01952
follows BUQEYE 1506.01343+1511.03618

χEFT $\alpha_{E1}^{(p)} - \beta_{M1}^{(p)}\ [10^{-4}\ \mathrm{fm}^3]: 7.5 \pm 0.7_{\mathrm{stat}} \pm 0.6_{\mathrm{th}} = 11.2_{\mathrm{LO}} - 3.6_{\mathrm{NLO}} - 0.1_{\mathrm{N^2LO}} \pm 0.6_{\mathrm{th}}$

Observable as series: $\mathcal{O} = c_0 + c_1\,\delta^1 + c_2\,\delta^2 + \text{unknown} \times \delta^3$

Assuming $\delta \simeq 0.4$: $11.2 \quad -9.1\,\delta^1 \quad -0.6\,\delta^2 \quad \pm(11.2 \times \delta^3 \approx 0.7??)$

$\Longrightarrow$ **Estimate next term *"most conservatively"* as** $|$unknown $c_3| \lesssim \max\{|c_0|;|c_1|;|c_2|\}$.



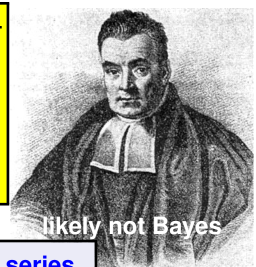
No infinite sampling pool; data fixed; more data changes confidence.

$\Longrightarrow$ **Call upon the Reverend Bayes!**

New information (new order) increases level of confidence.

$\Longrightarrow$ **Smaller corrections, more reliable uncertainties.**

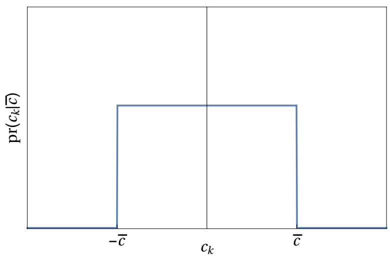
see e.g. **BUQEYE collaboration Furnstahl/Phillips/…1506.01343**



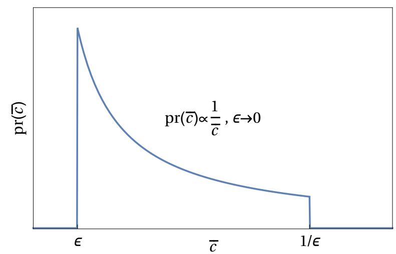
Bayes makes you specify your premises/assumptions about series.

Priors: leading-omitted term dominates ($\delta \ll 1$); putative distributions of *all* c_k's and of largest value $\bar{c}$ in series.

"Least informed/informative": All values c_k equally likely, given upper bound $\bar{c}$ of series.



"Any upper bound": $\ln$-uniform prior sets no bias on scale of $\bar{c}$.

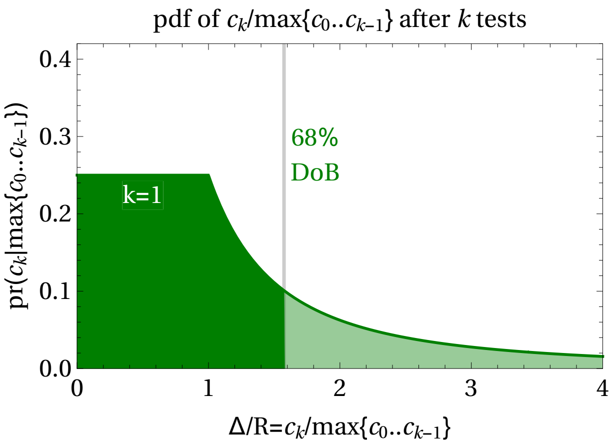


Input: Expansion parameter $\delta \simeq 0.4$, number of orders $k = 1$ (LO) and
probable "largest number" $R = \delta^{k=1} \times \max\{|c_0 = 11.2| \quad\} = 4.5$.

Result: Posterior $\equiv$ *Degree of Belief (DoB)* that next term $c_k\delta^k$ differs from order-k central value by Δ.

BUQEYE 1506.01343 eq. (22)

$$\mathrm{pr}(\Delta|\max.\,R, \text{order } k) \propto \int_0^\infty \mathrm{d}\bar{c}\;\mathrm{pr}(\bar{c})\;\mathrm{pr}(c_k = \frac{\Delta}{\delta^k}|\bar{c}) \prod_n^{k-1} \mathrm{pr}(c_n|\bar{c}) \to \frac{k}{k+1}\,\frac{1}{2R} \begin{cases} 1 & |\Delta| \le R \\ \left(\frac{R}{|\Delta|}\right)^{k+1} & |\Delta| > R \end{cases}$$



order	DOB in $\pm R$	σ: 68%	$\Delta(95\%)$
LO	$\frac{1}{2} = 50\%$	$1.6\,R$	$11R = 7\sigma$
Gauß	68.27%	$1.0\,R$	2.0σ

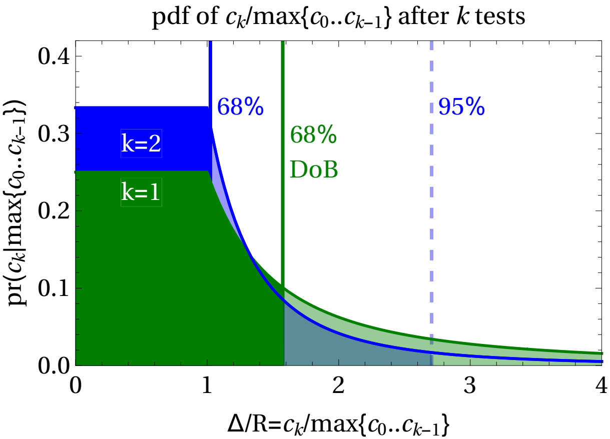
Quantifying One's Beliefs in $\mathcal{O} = \delta^n(c_0 + c_1\,\delta^1 + c_2\,\delta^2 + \ldots) = 11.2 - 9.1\,\delta^1 - 0.6\,\delta^2 \pm 0.6_{\text{th}}$

Input: Expansion parameter $\delta \simeq 0.4$, number of orders $k = 2$ (NLO) and probable "largest number" $R = \delta^{k=2} \times \max\{|c_0 = 11.2|; |c_1 = -9.1| \quad\} = 1.7$.

Result: Posterior $\equiv$ ***Degree of Belief (DoB)*** that next term $c_k\delta^k$ differs from order-k central value by Δ.

BUQEYE 1506.01343 eq. (22)

$$\text{pr}(\Delta|\text{max. } R, \text{order } k) \propto \int_0^\infty \text{d}\bar{c}\; \text{pr}(\bar{c})\; \text{pr}(c_k = \frac{\Delta}{\delta^k}|\bar{c}) \prod_n^{k-1} \text{pr}(c_n|\bar{c}) \to \frac{k}{k+1}\,\frac{1}{2R} \begin{cases} 1 & |\Delta| \le R \\ \left(\frac{R}{|\Delta|}\right)^{k+1} & |\Delta| > R \end{cases}$$



order	DOB in $\pm R$	σ: 68%	$\Delta(95\%)$
LO	$\frac{1}{2} = 50\%$	$1.6\,R$	$11R = 7\sigma$
NLO	$\frac{2}{3} = 66.7\%$	$1.0\,R$	$2.7R = 2.6\sigma$
Gauß	68.27%	$1.0\,R$	2.0σ

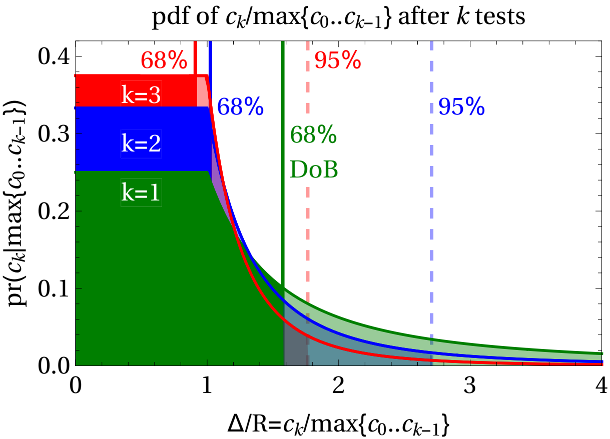
Quantifying One's Beliefs in $\mathcal{O} = \delta^n(c_0 + c_1\,\delta^1 + c_2\,\delta^2 + \dots) = 11.2 - 9.1\,\delta^1 - 0.6\,\delta^2 \pm 0.6_{\text{th}}$

Input: Expansion parameter $\delta \simeq 0.4$, number of orders $k = 3$ (N^{k-1}LO) and probable "largest number" $R = \delta^k \times \max\{|c_0 = 11.2|; |c_1 = -9.1|; |c_2 = -0.6|; \dots; |c_{k-1}|\} = 0.7$.

Result: Posterior $\equiv$ ***Degree of Belief (DoB)*** that next term $c_k\delta^k$ differs from order-k central value by Δ.

BUQEYE 1506.01343 eq. (22)

$$\text{pr}(\Delta|\text{max. } R, \text{order } k) \propto \int_0^\infty \text{d}\bar{c}\; \text{pr}(\bar{c})\; \text{pr}(c_k = \frac{\Delta}{\delta^k}|\bar{c}) \prod_n^{k-1} \text{pr}(c_n|\bar{c}) \to \frac{k}{k+1}\,\frac{1}{2R} \begin{cases} 1 & |\Delta| \leq R \\ \left(\frac{R}{|\Delta|}\right)^{k+1} & |\Delta| > R \end{cases}$$



order	DOB in $\pm R$	σ: 68%	$\Delta(95\%)$
LO	$\frac{1}{2} = 50\%$	$1.6\,R$	$11R = 7\sigma$
NLO	$\frac{2}{3} = 66.7\%$	$1.0\,R$	$2.7R = 2.6\sigma$
N^{2}LO	$\frac{3}{4} = 75\%$	$0.9\,R$	$1.8R = 1.9\sigma$
N^{k-1}LO k terms	$\frac{k}{k+1}$	$0.68\frac{k+1}{k}R\,(k \geq 2)$	
Gauß	68.27%	$1.0\,R$	2.0σ

For "high enough" order, largest number R limits $\gtrsim 68\%$ **degree-of-belief interval.**

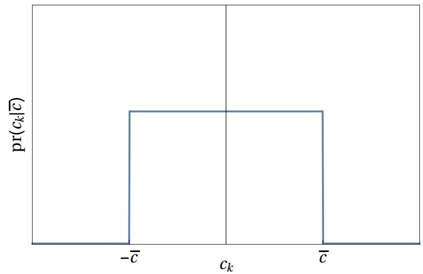
Varying priors: When $k \geq 2$ orders known, DoBs with different assumptions about $\bar{c}$, c_n vary by $\lesssim \pm 20\%$.

Posterior pdf *not Gauß'ian:* Plateau & power-law tail.– Do not add in quadrature in convolution!

$\Longrightarrow$ ***Interpretation of all theory uncertainties, with these priors; "$A \pm \sigma$": 68% DoB interval $[A-\sigma; A+\sigma]$.***

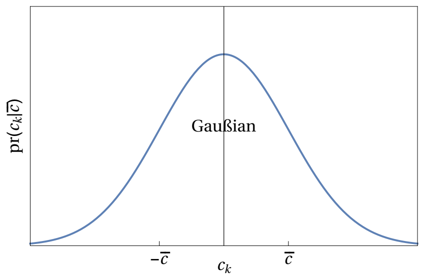
(h) Prior Choice: What is "Natural Size"? (SCOTUS: I Know It When I see It.)

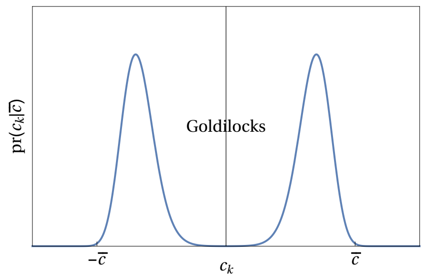
Observable/Series $\mathcal{O} = c_0 + c_1\delta^1 + c_2\delta^2 +$ unknown $\times\, \delta^3$ with *"naturally-sized coefficients"* c_i.



"Least informative/informed":

characterised by 1 number: $\bar{c}$.





More informed choices: more complicated structures, more thought, more parameters: $\bar{c}$, typ. size, spread,. . .

BUQEYE (Wesolowski/Klco/. . .): When $k \geq 2$ orders known, DoBs with different assumptions about $\bar{c}$, c_n vary by $\lesssim \pm 20\%$ for some "reasonable priors".

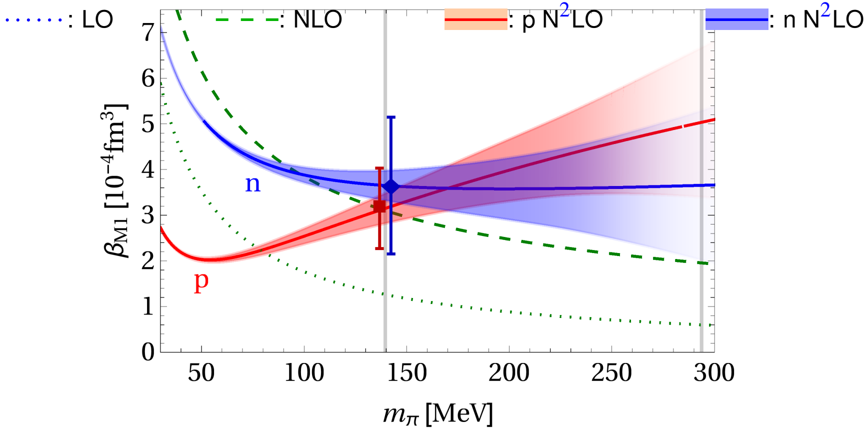
(i) Chiral Corridors of Uncertainties: m_π-Dependence Reveals Fine-Tuning

Observable $\mathcal{O} = c_0(m_\pi) + c_1(m_\pi)\delta^1 + c_2(m_\pi)\delta^2 +$ unknown $\times \delta^3$.

χEFT: explicit m_π-dependence, parameters fixed at m_π^{phys}.

Propagating Uncertainties: Bayesian order-by-order as before, now at each m_π.

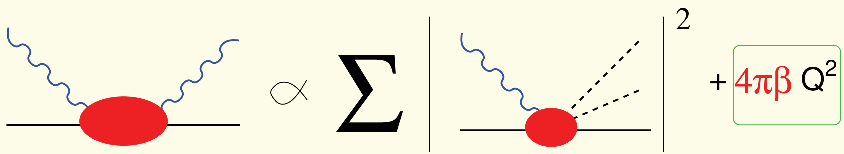
Some new terms linear in m_π. $\Longrightarrow$ Conservatively expand in $\delta(m_\pi) = 0.4 \times \frac{m_\pi}{m_\pi^{\text{phys}}}$, fade as $m_\pi \nearrow \frac{m_\pi^{\text{phys}}}{0.4}$.



At physical $m_\pi = 140$ MeV: paramagnetic $\Delta(1232)$ fine-tuned against diamagnetic NLO πN loops.

Only physical point has no substantial isospin splitting: stat. significant only for $m_\pi \lesssim 120$ MeV.

$\Longrightarrow$ SPECULATION – NO ERROR BARS



Subtracted DR

$$\overline{T}_1(\nu,Q^2) = -\nu^2 \int_{\nu_{th}^2}^{\infty} \frac{d\nu'^2}{\nu'^2} \frac{W_1(\nu',Q^2)}{\nu'^2-\nu^2} + 4\pi\beta Q^2 + O(Q^4)$$

Cottingham Σ Rule: $\beta_{M1}^{p-n} \Longleftrightarrow$ proton-neutron self-energy difference: $M_{p-n} = M_{p-n}^{\text{strong}} + M_{p-n}^{\text{em,elastic}} - A\ \beta_{M1}^{p-n}$

If $-A\beta_{M1}^{p-n} \approx 0.5\ \text{MeV}$ ***and If*** dispersive $A \propto \int_0^\Lambda dQ^2 \left(\frac{m_\rho^2 Q}{m_\rho^2+Q^2} \right)^2$ weakly m_π-dependent Walker-Loud/Carlson/Miller 2012

Then $\left.\frac{dM_{p-n}^{\beta}(m_\pi)}{d\ln m_q}\right|_{m_\pi^{\text{phys}}} = -0.65\ \text{MeV}$: ***Might not be negligible*** vs. $\left.\frac{dM_{p-n}^{\text{strong}}}{d\ln m_q}\right|_{m_\pi^{\text{phys}}} \approx -2.1\ \text{MeV}$ Bedaque/Luu/Platter 2011

Impact on p-n mass difference?: $-A\beta_{M1}^{\text{p-n}} \approx 0.5\ \text{MeV}$ wants more stable n as $m_q \searrow$, competes with $M_{\text{p-n}}^{\text{strong}}$.

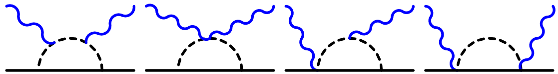
$\to$ Neutron lifetime $\to$ Big Bang Nucleosynthesis $\to$ Anthropic Principle?

(k) It's A Bit More Complicated...

Bernard/Kaiser/Meißner 1992-4, Butler/Savage/Springer 1992-3, Hemmert/... 1998
Kumar/McGovern/Birse 2000, McGovern 2001, JMcG/DRP/hg 2013 + 1511.01952

Both *magnitude* and *relative* importance of contributions change with m_π: | $\sim m_\pi^{\text{phys}}$ | $\sim M_\Delta - M_N \approx 300$ MeV

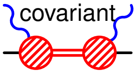
charged pion cloud
infinite in chiral limit

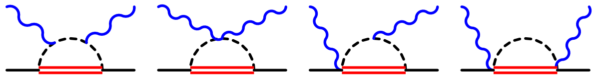


$e^2\delta^2$ LO

$e^2\epsilon^1$ **LO**
isoscalar only

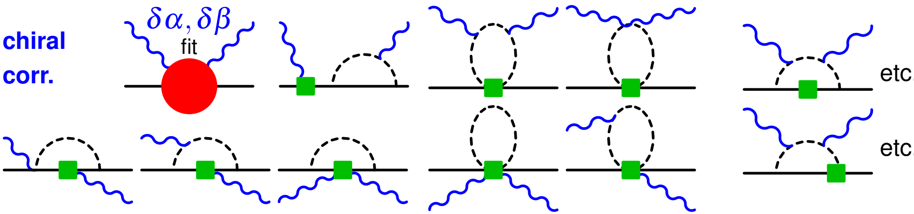
$\Delta(1232)$
+ **its π cloud**





$e^2\delta^3$ NLO

chiral corr.



$e^2\delta^4$ N^{2}LO

$e^2\epsilon^2$ NLO
incomplete:
no χcorrection to Δ & $\Delta\pi$;
isovector incomplete

(i) Close to m_π^{phys} $\Longrightarrow$ $\sqrt{\dfrac{m_\pi}{\Lambda_\chi \approx 800 \text{ MeV}}} \approx \dfrac{M_\Delta - M_N}{\Lambda_\chi}$ $=:$ **δ-counting** Pascalutsa/Phillips 2002

(ii) Close to 300 MeV $\Longrightarrow$ $\dfrac{m_\pi \sim (M_\Delta - M_N)}{\Lambda_\chi}$ $=:$ **ϵ-counting** Manohar/Jenkins 1994,...

(iii) Beyond $\Lambda_\chi \approx 800$ MeV $\Longrightarrow$ no small parameter, no convergence $\Longrightarrow$ ***at best qualitatively useful!***

Use unified amplitude: $\Longrightarrow$ **Accuracy N^{2}LO ($\sim 6\%$) for $m_\pi \sim 140$ MeV, LO ($\sim 40\%$) for $m_\pi \sim 300$ MeV.**

Gradual loss of accuracy, isovector incomplete, more sensitive to Bayesian prior when only LO.

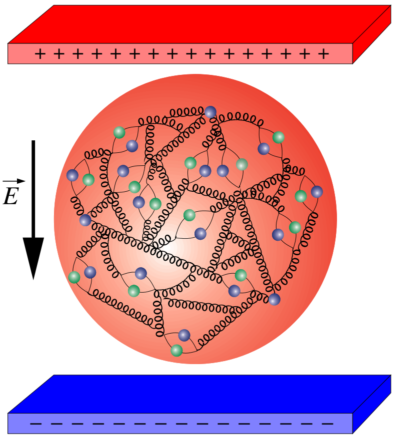
$\Longrightarrow$ ***Fade corridors out beyond*** ~ 250 **MeV.**

At this order, $g_A, f_\pi, M_N, (M_\Delta - M_N), \ldots$ independent of m_π.

(I) Chiral Extrapolations for Polarisabilities in Lattice QCD

Towards comparable uncertainties in experiment, χEFT and lattice QCD.

χEFT: reliable error estimate for $\frac{m_\pi}{\Lambda_\chi}$ extrapolation. $\Longrightarrow$ ***Fading corridors beyond*** ~ 250 MeV.

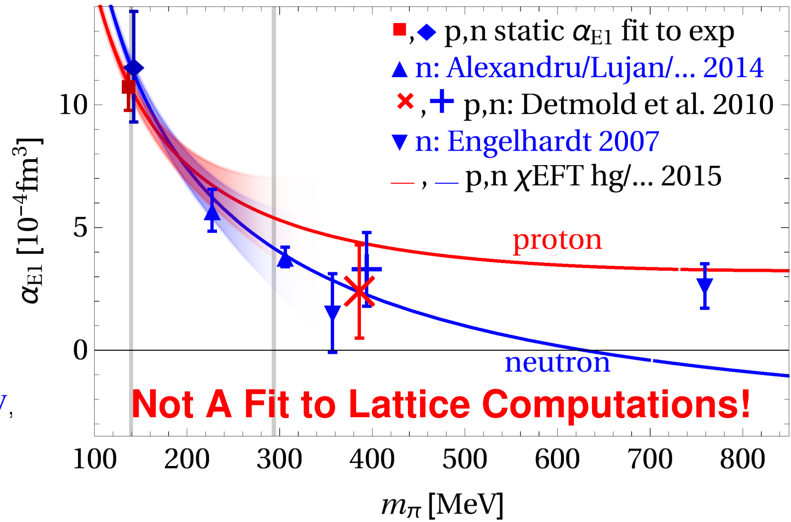


Ongoing: charged sea, $m_\pi \searrow 200$ MeV,
larger volumes, more statistics,. . .

Active lattice groups:

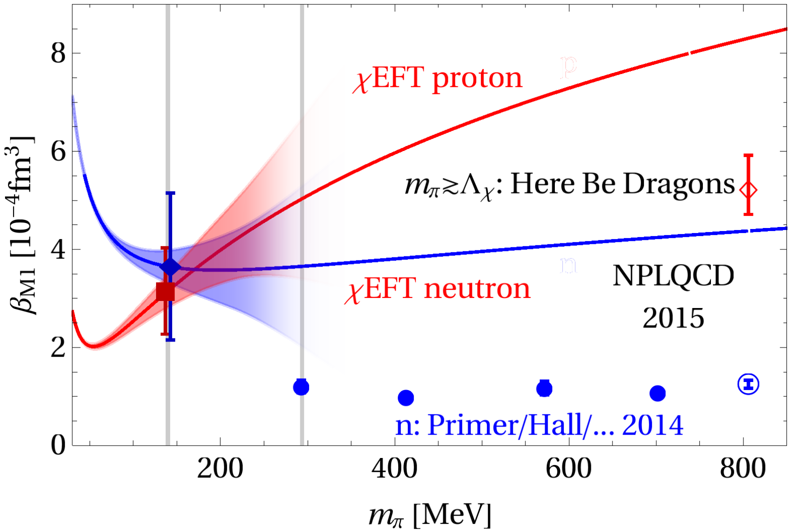
Alexandru/Lee/. . . 2005-;
Engelhardt/LHPC 2006-;
EMC/NPLQCD 2006-, 2015-;
Leinweber/Primer/Hall/. . . 2013-

Example: static electric polarisability α_{E1}

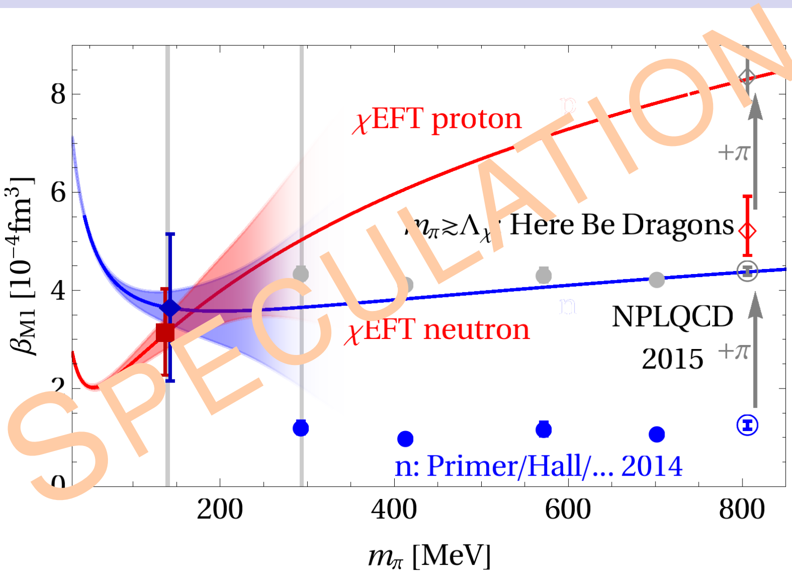


(m) Magnetic Polarisabilities: Surprises and Numerology

hg/JMcG/DRP
1511.01952



(m) Magnetic Polarisabilities: Surprises and Numerology

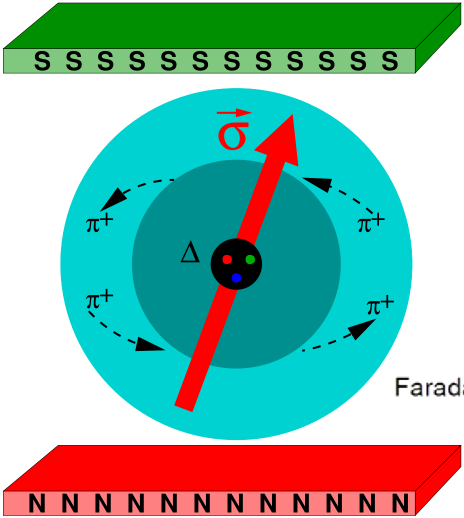


Why m_π-independent offset? **Why isoscalar off by "exactly" $\pi \times 10^{-4}$ fm^3?**

Why isovector "exactly" matched? ***Principle of Chiral Persistence*?**

(a) Spin Polarisabilities: Nucleonic Bi-Refringence and Faraday Effect

Optical Activity: Response of spin-degrees of freedom, complements JLab spin programme.



$$\mathcal{L}_{\text{pol}} = 4\pi\, N^\dagger \times \Bigg\{ \frac{1}{2} \Big[\alpha_{E1}\, \vec{E}^2 + \beta_{M1}\, \vec{B}^2 \Big]$$ **scalar dipole**

$$+ \frac{1}{2} \Big[\gamma_{E1E1}\, \vec{\sigma}\cdot(\vec{E}\times\dot{\vec{E}}) + \gamma_{M1M1}\, \vec{\sigma}\cdot(\vec{B}\times\dot{\vec{B}})$$

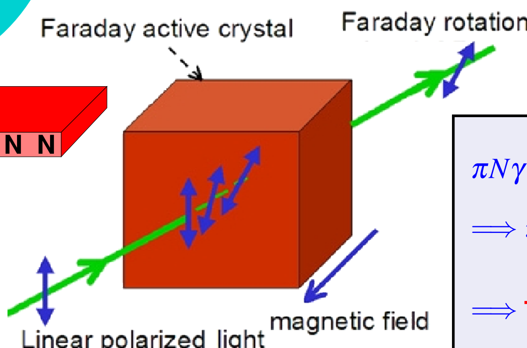
"pure" spin-dependent dipole

$$-2\,\gamma_{M1E2}\, \sigma_i\, B_j\, E_{ij} + 2\,\gamma_{E1M2}\, \sigma_i\, E_j\, B_{ij} \Big] + \dots \Bigg\}\, N$$

"mixed" spin-dependent dipole

+ quadrupole etc.

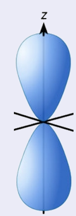
$E_{ij} := \frac{1}{2}(\partial_i E_j + \partial_j E_i)$ etc.



$\pi N\gamma: -\frac{g_A}{2f_\pi}\vec{\sigma}\cdot(\vec{q}+e\vec{\epsilon})+\dots$

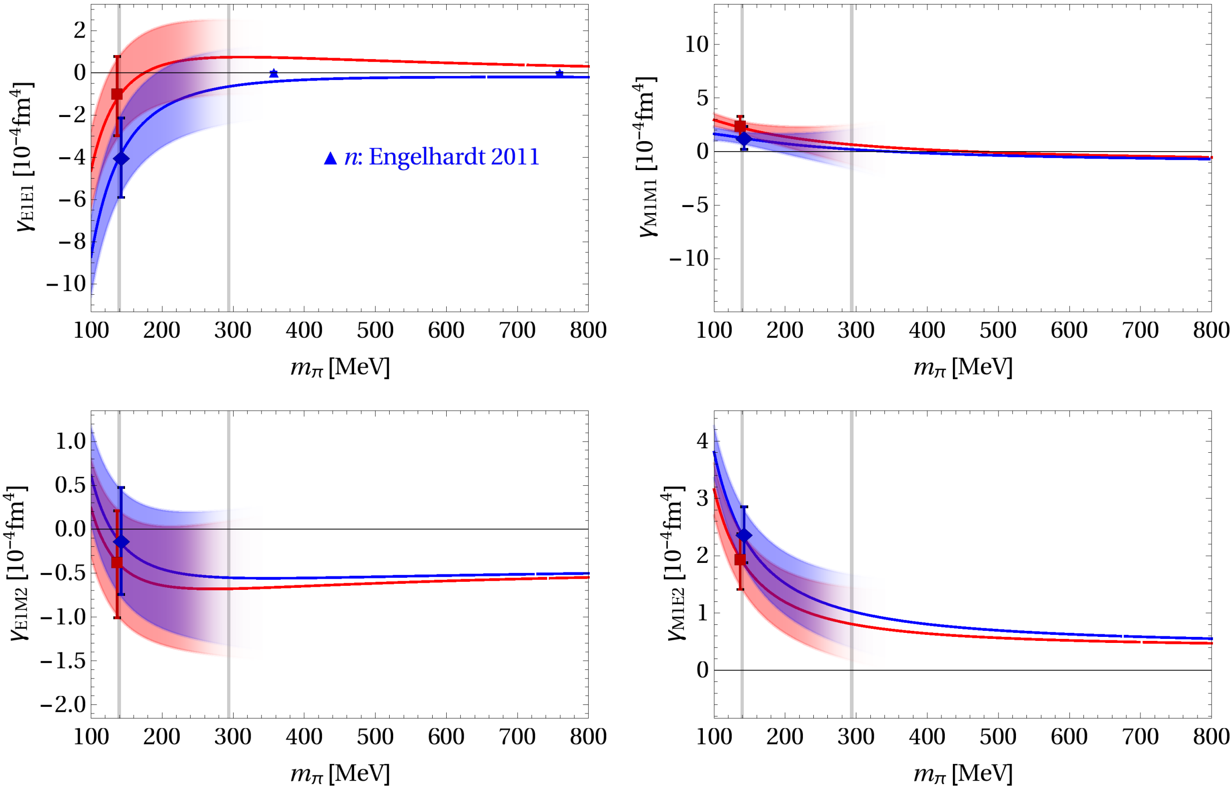
$\Longrightarrow$ π emission/absorption depends on N spin.

$\Longrightarrow$ **Test χiral Symmetry!**



(b) Spin Polarisabilities: Theory Speaks

χEFT: Parameter-free predictions; lattice-QCD: Ramping up.



(c) Plethora of Observables for Polarised Beams on Polarised Targets/Recoils

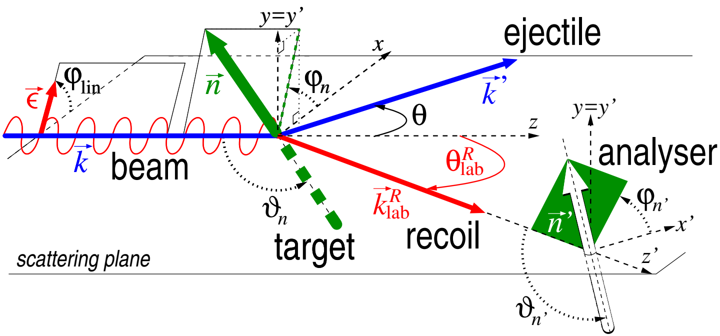
Unpolarised/linear/circular beam on scalar/vector/tensor target/recoil:

Proton/^{3}He (spin-$\frac{1}{2}$): 7 **Asymmetries:** 1 **beam,** 1 **target,** 2 **circpol. double,** 3 **linpol. double**

5 **Polarisation Transfers:** 2 **circpol. beam on pol. recoil,** 3 **linpol. beam on pol. recoil**

Deuteron (spin-1): 17 **Asymmetries:** 1 **beam,** 4 **target,** 4 **circpol. double,** 8 **linpol. double**

12 **Polarisation Transfers:** 4 **circpol. beam on pol. recoil,** 8 **linpol. beam on pol. recoil**



$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right|_{\text{unpol}} \times \Big[1+\Sigma^{\text{lin}}(\omega,\theta)P^{(\gamma)}_{\text{lin}}\cos 2\phi_{\text{lin}}$$

$$+\sum_{\substack{J=1,2\\0\le M\le J}} T_{JM}(\omega,\theta)P^{(\mathrm{d})}_J d^J_{M0}(\theta)\cos[M\phi-\frac{\pi}{2}\delta_{J1}]$$

$$+\sum_{\substack{J=1,2\\0\le M\le J}} T^{\text{circ}}_{JM}(\omega,\theta)P^{(\gamma)}_{\text{circ}}P^{(\mathrm{d})}_J d^J_{M0}(\theta)\sin[M\phi+\frac{\pi}{2}\delta_{J1}]$$

$$+\sum_{\substack{J=1,2\\-J\le M\le J}} T^{\text{lin}}_{JM}(\omega,\theta)P^{(\gamma)}_{\text{lin}}P^{(\mathrm{d})}_J d^J_{M0}(\theta)\cos[M\phi-2\phi_{\text{lin}}-\frac{\pi}{2}\delta_{J1}]\Big]$$

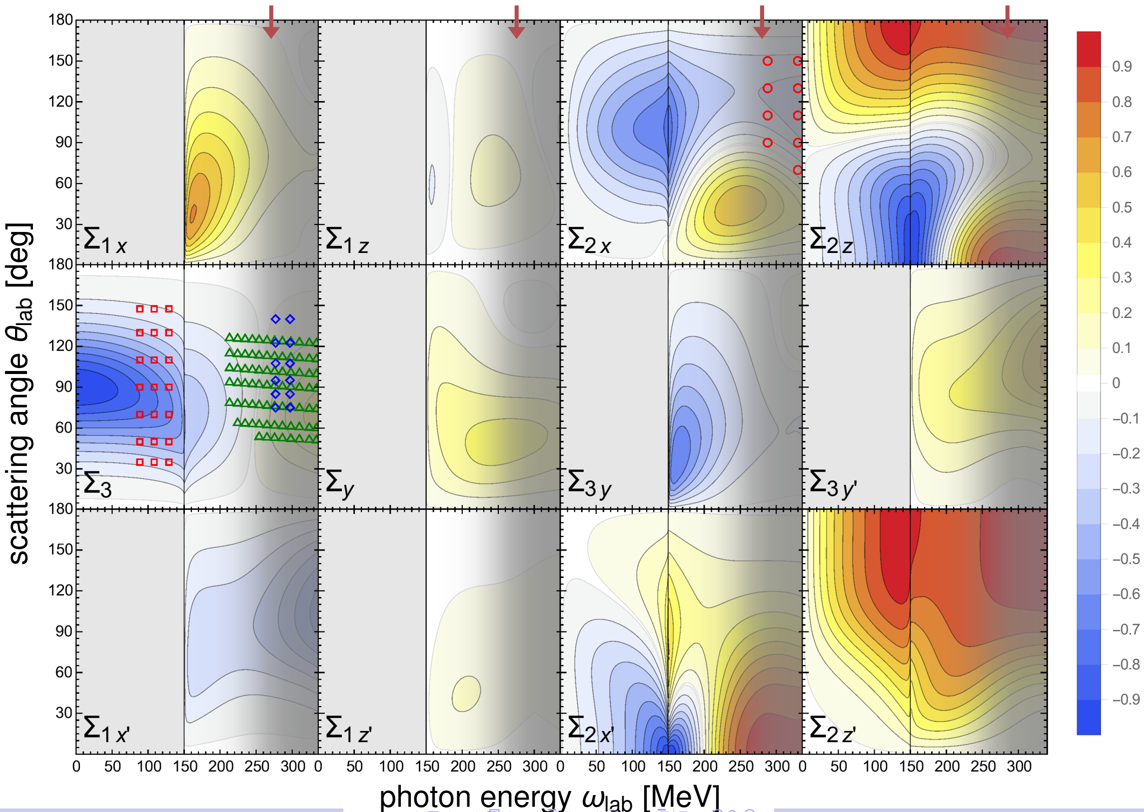
6 p & n polarisabilities $+$ **constraints on** $\alpha_{E1}+\beta_{M1}$, γ_0,...; **experiment: detector settings, feasibilities,...**

No single measurement to provide definitive answers: multi-parameter extractions, systematics, validation.

$\Longrightarrow$ **Experiment & Theory collaborate to identify** ***observables with biggest impact.***

(d) The 12 Proton Observables: Not Lots Of Data, and Wrong Region

Fading Colours for $\omega \gtrsim 250$ MeV indicate breakdown of χEFT expansion.

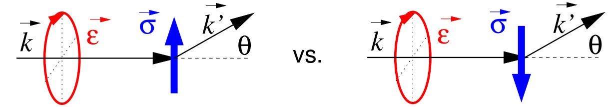


(e) Spin Polarisabilities from Polarised Photons

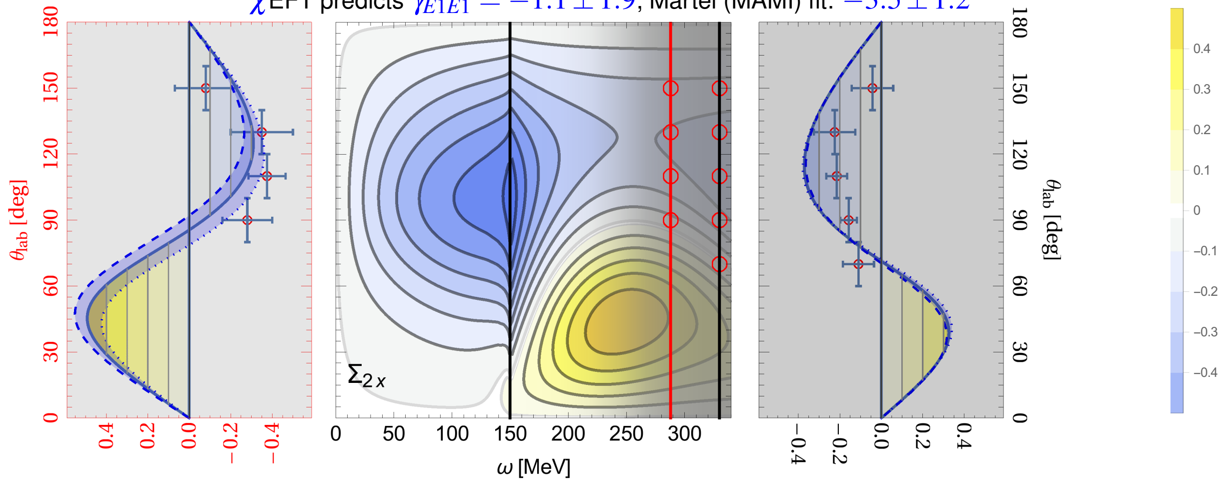
$\mathcal{O}(e^2\delta^3)$: hg/Hildebrandt/...2003
$\mathcal{O}(e^2\delta^4)$: hg/McGovern/Phillips 1511.0952&1711.11546
exp MAMI: Martel/... PRL 2014; Collicott/... t.b.a.

Proton best: Incoming γ circularly polarised, sum over final states. N-spin in $(\vec{k},\vec{k}')$-plane, perpendicular to $\vec{k}$:

Σ_{2x} :



χEFT predicts $\gamma_{E1E1} = -1.1 \pm 1.9$; Martel (MAMI) fit: -3.5 ± 1.2



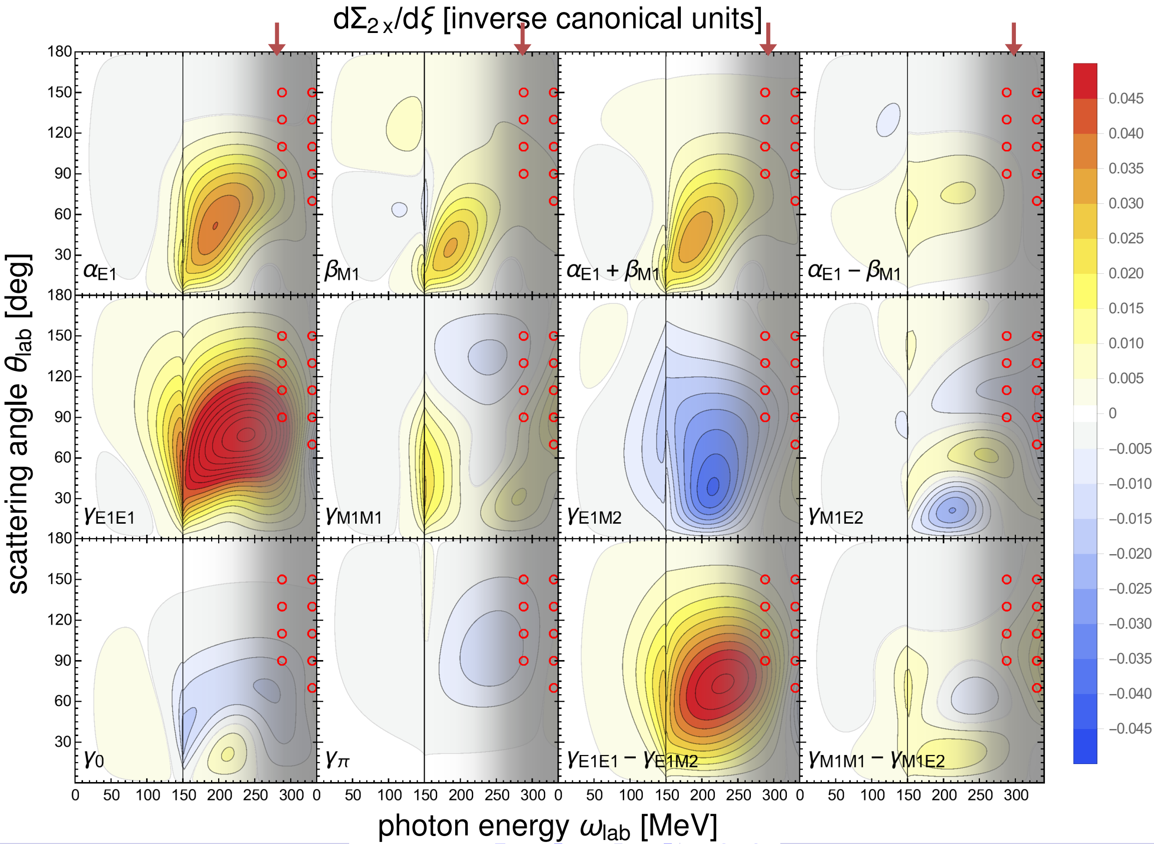
$\mathcal{O}(e^2\delta^4)$ χ**EFT prediction** hg/McGovern/Phillips 2014 **vs. MAMI extraction** Martel/...2014

static $[10^{-4}\ \text{fm}^4]$	γ_{E1E1}	γ_{M1M1}	γ_{E1M2}	γ_{M1E2}
MAMI 2014 proton	-3.5 ± 1.2	3.2 ± 0.9	-0.7 ± 1.2	2.0 ± 0.3
χEFT proton **predicted**	$-1.1 \pm 1.9_{\text{th}}$	$2.2 \pm 0.5_{\text{stat}} \pm 0.6_{\text{th}}$ **fit to unpol.**	$-0.4 \pm 0.6_{\text{th}}$	$1.9 \pm 0.5_{\text{th}}$
χEFT neutron **predicted**	$-4.0 \pm 1.9_{\text{th}}$	$1.3 \pm 0.5_{\text{stat}} \pm 0.6_{\text{th}}$	$-0.1 \pm 0.6_{\text{th}}$	$2.4 \pm 0.5_{\text{th}}$

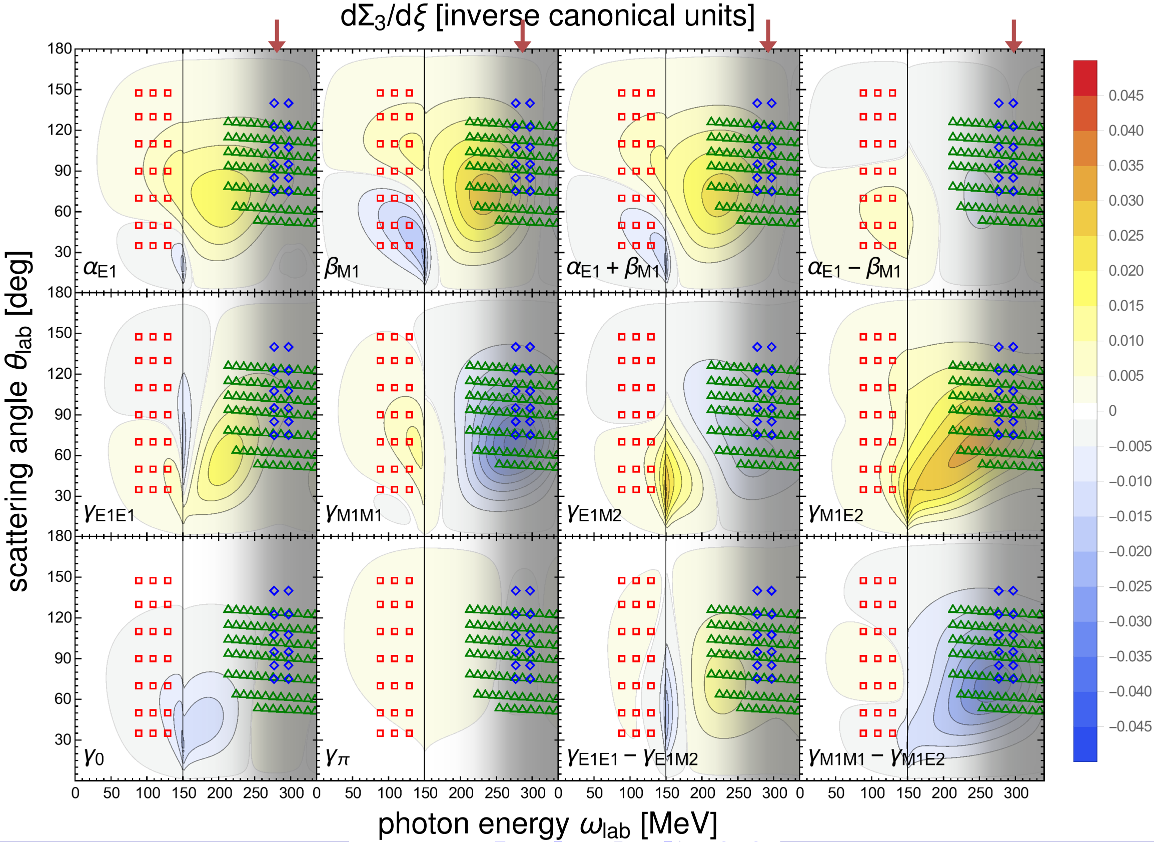
(f) Zooming In On Sensitivity of Σ_{2x}: Circpol Beam on Linpol Proton

JMcG/hg/DRP
1711.11546

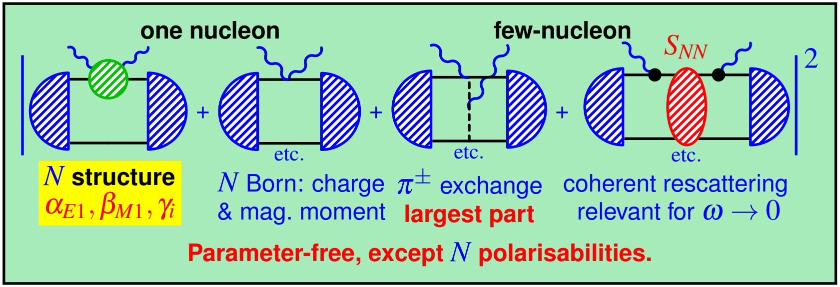
Fading Colours for $\omega \gtrsim 250$ MeV indicate breakdown of χEFT expansion.

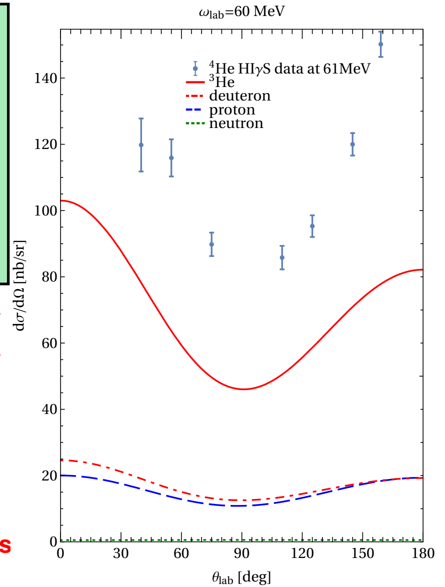


Fading Colours for $\omega \gtrsim 250\ \mathrm{MeV}$ indicate breakdown of χEFT expansion.



deuteron: hg/.../+Phillips/+McGovern 2004-
MECs: Beane/...1999-2005
^{3}**He:** Shukla/...2009 +Strandberg/Margaryan/hg/...1804.01206





Experiment: More charge & MECs $\Longrightarrow$ more counts $\Longrightarrow$ *heavier nuclei*

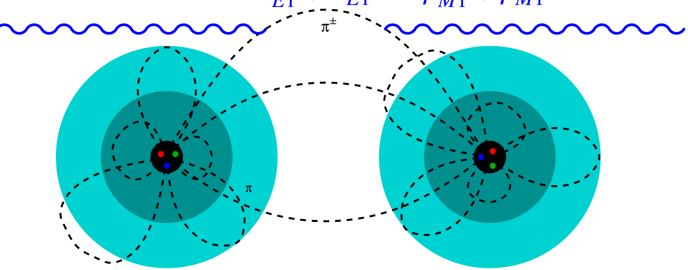
Theory: Reliable only if nuclear binding & levels accurate $\Longrightarrow$ *lighter nuclei*

Find *sweet-spot* between competing forces: deuteron, ^{3}He, ^{4}He.

Use Complementing Targets of Opportunity.

Deuteron, ^{4}He: sensitive to average p+n polarisabilities $\Longrightarrow$ **neutron pols**

^{3}**He**: sensitive to $2\alpha_{E1}^{p} + \alpha_{E1}^{n}$ & $2\beta_{M1}^{p} + \beta_{M1}^{n}$ $\Longrightarrow$ **neutron pols.**



Model-independently subtract binding effects.

$\Longrightarrow$ **χEFT: quantify reliable uncertainties.**

Chirally consistent 1N & few-N: potentials, wave functions, currents, π-exchange.

Test charged-pion component of NN force.

(i) Improve on the Neutron: Target ^{3}He

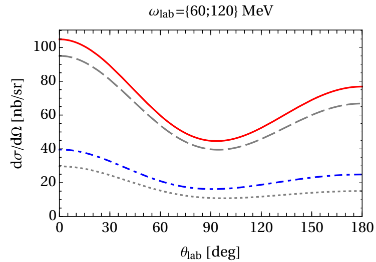
Shukla/Phillips/Nogga 2009
+Strandberg/Margaryan/hg/McG/Ph 1804.01206

Correction of Code's Isospinology

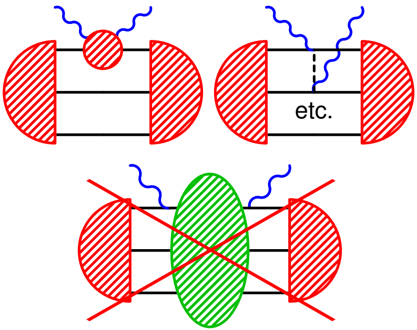
of $\mathcal{O}(e^2\delta^2)$ (no $\Delta(1232)$)

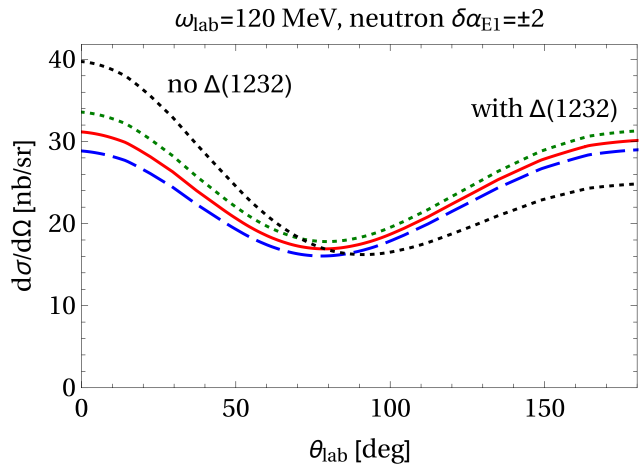
increases rates found by Margaryan

erratum to Shukla/…2009 in press



Example unpolarised ^{3}He: Sensitivity on $\Delta(1232)$ and α_{E1}^n at $\omega_{\rm lab} = 120$ MeV



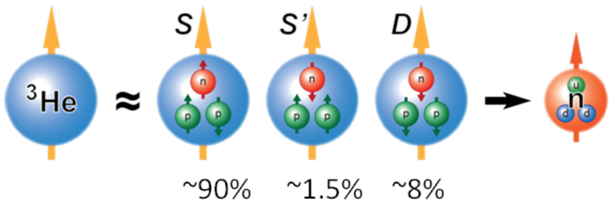


– Beyond $\omega \in [80;120]$ MeV: rescattering (Thomson, T_{NN}); explicit $\Delta(1232)$ also in MECs

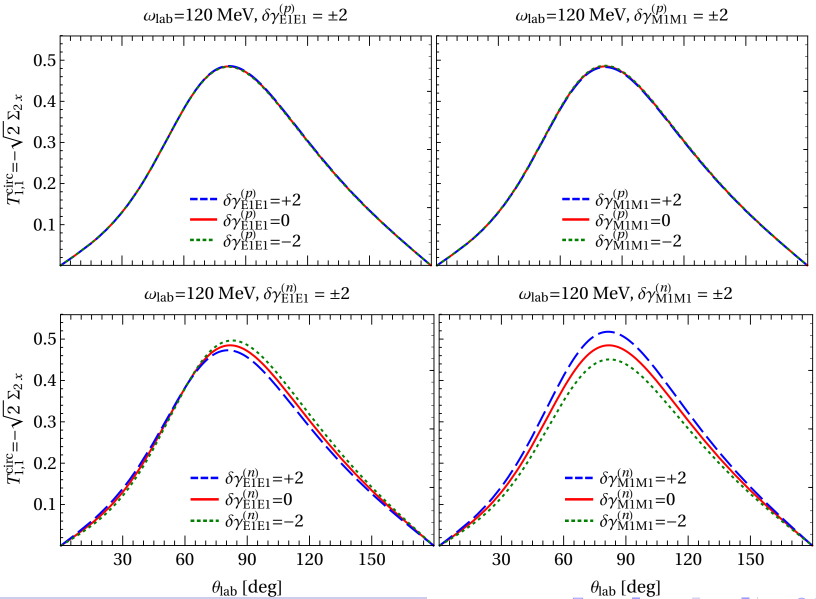
(j) Double-Polarised on ^{3}He: Effective n Spin Target

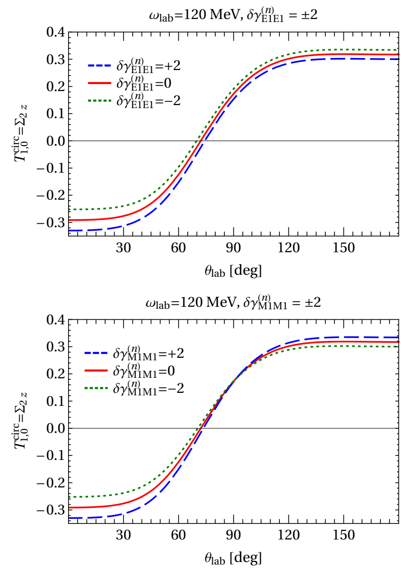
Choudhury Shukla/Phillips/Nogga 2006-09
hg/Phillips/Strandberg/Margaryan 1804.01206

^{3}He as "effective" spin target: *sensitivity* to neutron spin, not to proton spin.



Sensitivity to γ_i's
at $\omega_{\mathrm{lab}} = 120$ MeV enhanced by
interference with charged Born+MEC.





(k) Experiment and Theory in Sync at the Precision and Intensity Frontier

"At present, single and double polarised data is sorely missing." **Theory letter [arXiv:1409.1512]**

No single measurement will provide definitive answer: multi-parameter extraction, systematics, validation.

$\Longrightarrow$ **Experiment & Theory collaborate to identify *observables with biggest impact.***

Photon energy ω=120MeV — Reference frame: cm | lab

Deuteron vector polarisation $P_1^{(d)}$=1.\`

Deuteron tensor polarisation $P_2^{(d)}$=0.53\`

Photon right–circular polarisation $P_{circ}^{(\gamma)}$=−0.5

−0.5

Photon linear polarisation $P_{lin}^{(\gamma)}$=1.\`

Configuration 1

Deuteron polarisation quantisation axis θ_{d1}=0°

ϕ_{d1}=0°

Photon linear polarisation angle ϕ_{lin1}=90°

Configuration 2

Deuteron polarisation quantisation axis θ_{d2}=90°

ϕ_{d2}=270°

Photon linear polarisation angle ϕ_{lin2}=90°

Variation by ±2 of: $\delta\beta_{M1}$

χEFT order: $e^2\delta^3=\epsilon^3$: with $\Delta(1232)$ | $e^2\delta^2=Q^3$: no $\Delta(1232)$

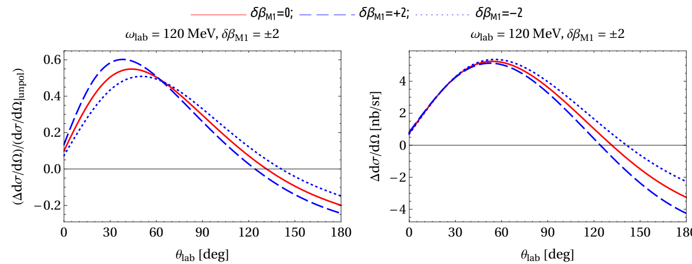
Deuteron wave function: NNLO Epelbaum 650MeV | AV18 — NN potential: AV18

Range on y–axis: All

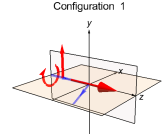
Normalise left plot to: $\frac{d\sigma}{d\Omega}\big|_{unpol}$ | $\Sigma\frac{d\sigma}{d\Omega}$: sum of configurations

Export $\Delta\frac{d\sigma}{d\Omega}$ and $\Sigma\frac{d\sigma}{d\Omega}$ of this configuration? — File name: "out.dat"

Example double-polarised on deuteron

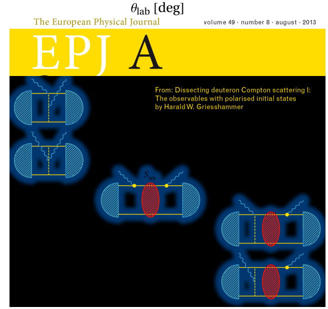


Configuration 1



Probability of spin projection $M_{\vec d}$:

Cartesian polarisation along $\vec d$:



hg EPJA**49** (2013) 100, Europhysics News HIGHLIGHT May 2013

Cross section difference of configurations: $\Delta\frac{d\sigma}{d\Omega}=\frac{d\sigma}{d\Omega}\big|_{unpol}\times[0+0.78\,T_{1,-1}^{lin}-0.55\,T_{1,0}^{circ}-0.78\,T_{1,1}+0.78\,T_{1,1}^{lin}-0.32\,T_{2,-2}^{lin}+0.8\,T_{2,0}-0.8\,T_{2,0}^{lin}+0.32\,T_{2,2}-0.32\,T_{2,2}^{lin}]$

Cross section sum of configurations: $\Sigma\frac{d\sigma}{d\Omega}=\frac{d\sigma}{d\Omega}\big|_{unpol}\times[2+-2.\,\Sigma^{lin}+-0.78\,T_{1,-1}^{lin}-0.55\,T_{1,0}^{circ}+0.78\,T_{1,1}-0.78\,T_{1,1}^{lin}+0.32\,T_{2,-2}^{lin}+0.26\,T_{2,0}-0.26\,T_{2,0}^{lin}-0.32\,T_{2,2}+0.32\,T_{2,2}^{lin}]$

4. Concluding Questions at the Intensity & Precision Frontier

Polarisabilities: **ω-dependence maps out scales, symmetries & mechanisms of interactions:**
χiral symmetry of pion-cloud, $\Delta(1232)$ properties, nucleon spin-constituents.

Spin Polarisabilities: **Stiffness of Spin Constituents; Nuclear Faraday Effect.**
χEFT: parameter-free predictions, lattice QCD catching up.

Target	Opportunities	Theory Status
proton	p spin pols.	"done" well ahead of exp.
deuteron	sensitive to $p+n$ average polarised, d-wave interference: mixed spin pols $\gamma_{E1M2}, \gamma_{M1E2}$	$\omega \lesssim 120\ \mathrm{MeV}$ done
^{3}He: increased rates	unpolarised: sensitive to $2p+n$ polarised: "n-spin" $\Longrightarrow$ sensitive to γ_i^n	$\omega \in [50; 120]\ \mathrm{MeV}$ done
^{4}He: increased rates	sensitive to $p+n$ average	starting
$\gamma X \to NY\gamma$ quasifree	tag n or p directly – both in one go?	$\gamma d \to np\gamma$ done

We Need Data: **elastic & inelastic cross-sections & asymmetries –** ***reliable systematics!***

Only *combination of dedicated experiments* meaningful! (Not "one datum for one answer".)

$\Longrightarrow$ ***Synergy of Experiment, Low-Energy Theory & Lattice QCD, competitive uncertainties!***

$\Longrightarrow$ **Compton Community programme outlined in White Paper for a**
***Next Generation Laser Compton Gamma-ray Beam Facility*, sent to DoE.**

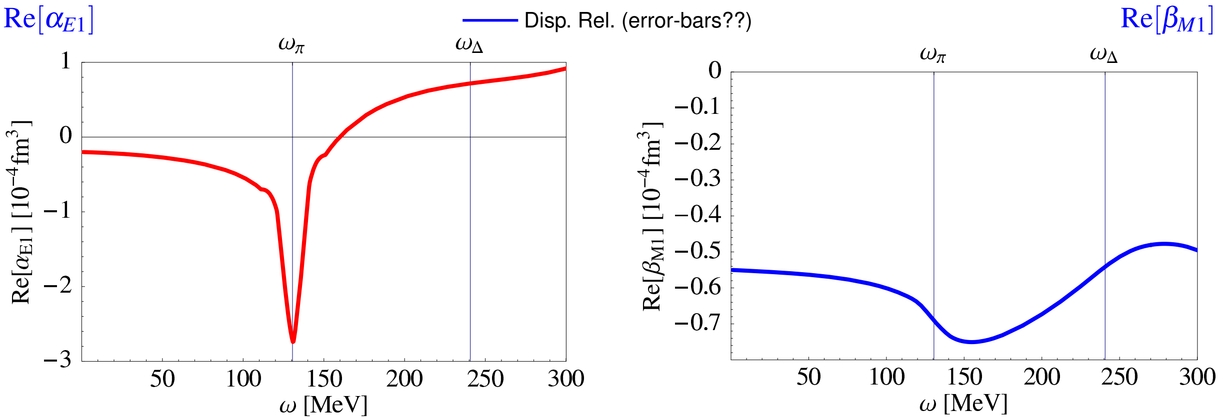
The efficient person gets the job done right. The effective person gets the right job done.

(a) Iso-Vector Polarisabilities

Proton-neutron difference $\alpha_{E1}^{v} := \alpha_{E1}^{p} - \alpha_{E1}^{n}$ etc. probes details:

Explicit χiral-symmetry-breaking in pion-cloud,. . . , elmag. p-n self-energy difference $[0 \pm 1]$ MeV $\propto \beta_{M1}^{p} - \beta_{M1}^{n}$

$\mathcal{O}(e^2\delta^4)$ **(N^{2}LO) in χEFT; compatible with $\approx \frac{\text{iso-scalar}}{10}$ in Dispersion Relations.** hg/Pasquini/. . . 2005



No free neutron targets $\Longrightarrow$ χEFT for model-independent subtraction of nuclear binding.

5. The Promise of Reliable Error Bars

(a) (Dis)Agreement Significant Only When All Error Sources Explored

Editorial PRA 83 (2011) 040001

PHYSICAL REVIEW A **83**, 040001 (2011)

Editorial: Uncertainty Estimates

The purpose of this Editorial is to discuss the importance of including uncertainty estimates in papers involving theoretical calculations of physical quantities.

It is not unusual for manuscripts on theoretical work to be submitted without uncertainty estimates for numerical results. In contrast, papers presenting the results of laboratory measurements would usually not be considered acceptable for publication

The question is to what extent can the same high standards be applied to papers reporting the results of theoretical calculations. It is all too often the case that the numerical results are presented without uncertainty estimates. Authors sometimes say that it is difficult to arrive at error estimates. Should this be considered an adequate reason for omitting them? In order to answer this question, we need to consider the goals and objectives of the theoretical (or computational) work being done. Theoretical papers

physical effects not included in the calculation from the beginning, such as electron correlation and relativistic corrections. It is of course never possible to state precisely what the error is without in fact doing a larger calculation and obtaining the higher accuracy. However, the same is true for the uncertainties in experimental data. The aim is to estimate the uncertainty, not to state the exact amount of the error or provide a rigorous bound.

Theoretical uncertainty: Truncation of Physics

EFT claim: systematic in $Q = \frac{\textit{typ. low scale } p_{typ}}{\textit{typ. high scale } \overline{\Lambda}_{EFT}}$

Does Nuclear Structure emerge from QCD?

Beyond-Standard-Model Physics from Supernovae?

Religion	Science: Degree of Belief
Thou Shalt Believe!	***Conjecture*** ⟲ ***Evidence***

Scientific Method: Quantitative results with corridor of theoretical uncertainties for *falsifiable predictions*.

Need procedure which is established, economical, reproducible: room to argue about “error on the error”.

“Double-Blind” Theory Errors: Assess with pretense of no/very limited data.

(b) Statistical Interpretation of the Max-Criterion: A Simple Example

I take this table of πN scattering parameters in χEFT with effective $\Delta(1232)$ degrees of freedom from the talk by Jacobo Ruiz de Elvira. Here, I am not interested in the Physics, but use it as series $c_i = c_{i0} + c_{i1}\epsilon^1 + c_{i2}\epsilon^2$ in a small expansion parameter.

parameter $[\mathrm{GeV}^{-1}]$	LO total	NLO total	N^{2}LO total	expansion $= c_{i0} + c_{i1}\epsilon^1 + c_{i2}\epsilon^2$	perturbative expansion $\epsilon \approx 0.4$ (guess)
c_1	-0.69	-1.24	-1.11	$= -0.69 + 0.55 - 0.13$	$= -0.69 + 1.38\epsilon^1 - 0.81\epsilon^2$
c_2	$+0.81$	$+1.13$	$+1.28$	$= +0.81 - 0.32 - 0.15$	$= +0.81 - 0.80\epsilon^1 - 0.94\epsilon^2$
c_3	-0.45	-2.75	-2.04	$= -0.45 + 2.30 - 0.71$	$= -0.45 + 5.75\epsilon^1 - 4.44\epsilon^2$
c_4	$+0.64$	$+1.58$	$+2.07$	$= +0.64 - 0.94 - 0.49$	$= +0.64 - 2.35\epsilon^1 - 3.06\epsilon^2$

Now pick the largest absolute coefficient to estimate typical size of next-order correction $c_{i(n+1)} = c_{i3}$ in our case:

Max-Criterion: $c_{i(n+1)} \lesssim \max\limits_{n\in\{0;1;2\}} \{|c_{in}|\} =: R$ **is labelled as red in the table.**

This criterion has been applied since "Time Immemorial"
See example on the next slide which predates EKM by 4 years.

Multiply that number with ϵ^3 to finally get a corridor of uncertainty/typical size of the ϵ^3 contribution.

For c_1: $\max\limits_{n\in\{0;1;2\}} \{|-0.69|;|1.38|;|-0.81|\} = 1.38 \Longrightarrow$ error $\pm 1.38 \times (\epsilon = 0.4)^3 \approx 0.09 \Longrightarrow c_1 = -0.69 \pm 0.09$.

Similar: $c_2 = 1.28 \pm 0.06$, $c_3 = -2.04 \pm 0.37$, $c_4 = 2.07 \pm 0.20$ (round significant figures conservatively).

But what's the statistical interpretation? $\Longrightarrow$ Next slide!

Notes: **(1)** Provide a theoretical error *estimate* that is *reproducible*. You can then discuss with others who have different opinions. No estimate, no discussion possible. – **(2)** Sometimes, one discards the LO$\rightarrow$NLO correction if it's anomalously large. That is a "prior information" you need to disclose as "bias" of your estimate. – **(3)** Coefficients c_{in} appear "more natural" for c_1 and c_2 than for c_4 – c_4 not that well-converging? – **(4)** The uncertainty estimate is agnostic about the Physics details. Somebody just handed me a table. – **(5)** If you are not happy with the input "$\epsilon \approx 0.4$", pick another number. BUQEYE 1511.03618 developed the Bayesian technology to extract degrees of belief on what value of the expansion parameter the series suggests. – **(6)** The c_i are not observables, but they are renormalised couplings which – according to Renormalisation – should follow a perturbative expansion.

(b) Statistical Interpretation of the Max-Criterion: A Simple Example

The Bayesian interpretation of the max-criterion on the next slide will provide probability distribution (pdf)/degree-of-belief functions using a "reasonable" set of assumptions ("priors") which give nice, analytic expressions. That's one choice of assumptions, but other reasonable assumptions provide very similar pdf's see BUQEYE: 1506.01343, 1511.03618,. . . .

But before that, let's do something intuitive which gives the same statistical likeliness interpretation of the max-criterion as the Bayesian one. The Bayesian analysis formalises the example and provides actual pdf's.

Estimating a Largest Number: Given a finite set of (finite, positive) numbers in an urn. You get to draw one number at a time.

Your mission, should you choose to accept it: Guess the largest number in the urn from a limited number of drawings.

For c_1, we first draw $c_{10} = 0.69$. I would say it's "natural" to guess that there is a 1-in-$2 = 50\%$ chance that the next number is lower. But there is also a pretty good chance that if it is higher, then its distribution up there is not Gauß'ian but with a stronger tail.

Next, we draw $c_{11} = 1.38$ which is larger. So I revise my largest-number projection to $R = 1.38$, but I also get more confident that this may be pretty high (if not he highest already). After all, I already found one number which is lower, namely $c_{10} = 0.69$. With 2 pieces of information (0.69 and 1.38), it's "natural" that the 3rd drawing has a 2-in-3 or $2/3$ chance to be lower.

Next, we draw $c_{12} = 0.81 < R$. Looking at my set of 3 numbers, I am even more confident that $R = c_{11} = 1.38$ is the largest number, with 3-in-4 or 75% confidence. For c_1, evil forces interfere and we have no more drawings to draw information from.

But if we could reach into the urn k times and look at the collected k results, every time revising our max-estimate, it's "natural" to assign a $100\% \times k/(k+1)$ confidence that I have actually gotten the largest number R.

The Bayesian procedure on the next slide provides the same result. Read the BUQEYE papers for details and formulae!

In our example, we had $k = 3$ terms (drawings) for c_1. So the confidence that $R = 1.38$ is indeed the highest number is $3/4 = 75\%$, which is larger than $p(1\sigma) \approx 68\%$. For a 1σ corridor, I reasonably assume that the numbers are equi-distributed between 0 and the maximum R. Then, the 68%-error corridor is set by $\pm 68\% \times (k+1)/k \times R$ amongst the known numbers.

Now, I multiply that number with 3 powers of the expansion parameter $\epsilon \approx 0.4$ (estimate N^{3}LO terms!) (but see **Note (5)** on the previous slide): $\pm 1.38 \times (68\%/75\%) \times 0.4^3 = \pm 0.08$ is a good uncertainty estimate for a traditional 68% confidence region. I also get a feeling that the probabilities outside the interval $[0;R]$ may not be Gauß'ian-distributed. Bayes will confirm that.

(c) Isovector Contributions At The Physical Point

Isovector polarisabilities $\xi^v := \frac{1}{2}(\xi^p - \xi^n)$ at $\mathrm{N^2LO}$; parameter-free. $\implies \sim 20\%$ of LO?

Fits:

$$\alpha_{E1}^{p-n} = -0.9 \pm 1.3_{\text{tot}}$$
$$\beta_{M1}^{p-n} = -0.5 \pm 1.3_{\text{tot}}$$

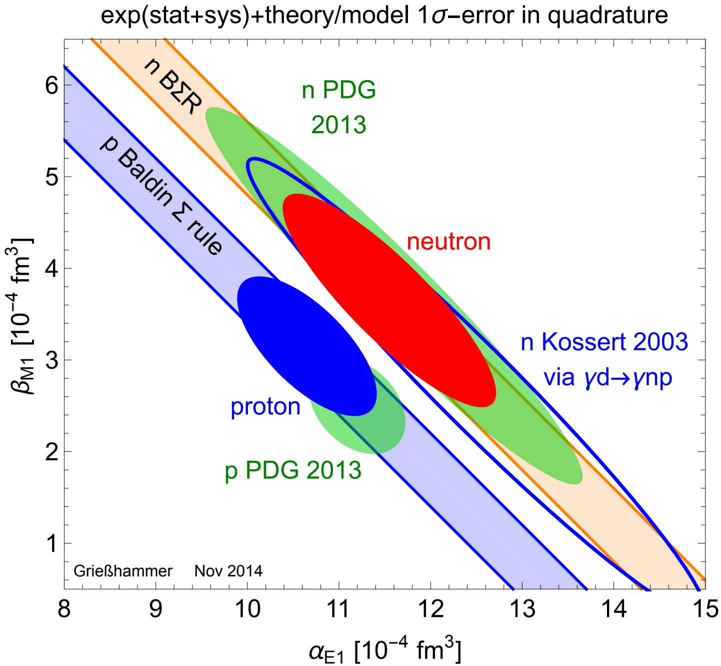
$\implies$ Consider m_q-dependence!

$$\left.\frac{\mathrm{d}\beta_{M1}^v}{\mathrm{d}\ln m_{\mathrm{q}}}\right|_{m_\pi^{\text{phys}}} = 0.65 \pm 0.4_{\text{th}}$$

$$\left.\frac{\mathrm{d}\alpha_{E1}^v}{\mathrm{d}\ln m_{\mathrm{q}}}\right|_{m_\pi^{\text{phys}}} = 0.7 \pm 0.4_{\text{th}}$$

HW: Get σ! Know isovector only at LO: $k = 1$

solution: $\sigma = 1.6R = 1.6 \times \mathrm{LO} \times (\delta = 0.4)$

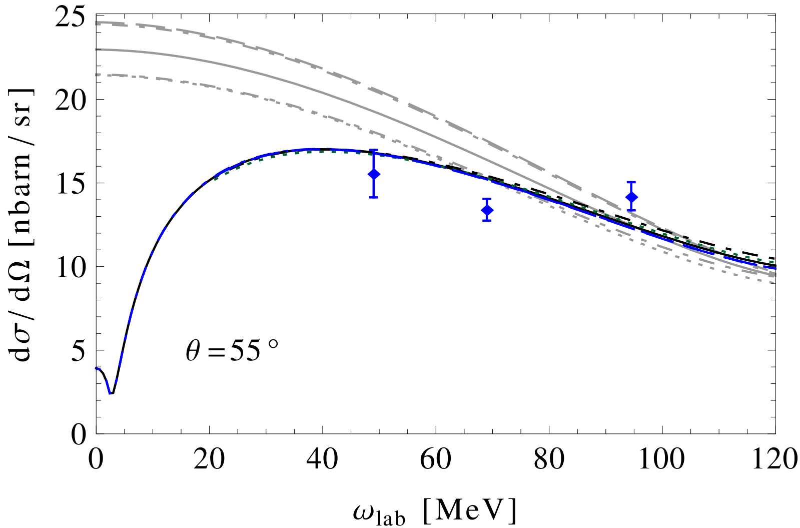


Possible fine-tuning at m_π^{phys} (statistically weak signal).

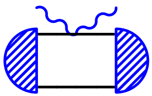
Low-Energy Theorem: Thomson limit $\mathcal{A}(\omega=0) = -\frac{e^2}{M_d}\vec{\epsilon}\cdot\vec{\epsilon}'$.

Thirring 1950, Friar 1975, Arenhövel 1980: Thomson limit $\Longleftrightarrow$ current conservation $\Longleftrightarrow$ gauge invariance.

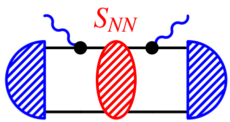
Exact Theorem $\Longrightarrow$ At each χEFT order $\Longrightarrow$ Checks numerics.



Amplitudes at $\omega = 0$:



$$\langle\Psi_d| - \frac{e^2}{M_N}\vec{\epsilon}\cdot\vec{\epsilon}'|\Psi_d\rangle = 2\mathcal{A}_{\text{Thomson}}$$



$= -A_{\text{Thomson}}$

Significantly reduces cross section for $\omega \lesssim 50$ MeV, but **less important at $\omega \gtrsim 50$ MeV**. Urbana, Lund data

Wave function & potential dependence significantly reduced even as $\omega \to 150$ MeV $\Longrightarrow$ **gauge invariance**.

(e) Myers et al. 2014: MAX-lab Doubles & Improves Database

Illinois 1994 ●, Lund 2003 ▲, Saskatoon 2000 ♦, **Lund 2014** × — $N\pi + \Delta +$ stat. error, Baldin constrained