Few-body systems with pionless effective field theory

Hilla De-Leon

Doron Gazit (HUJI), Lucas Platter (UTK)

האוניברסיטה
העברית
בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

INT program: "Fundamental Physics with Electroweak Probes of Light Nuclei", week #4

Outline

- Calculation of $A = 3$ bound state matrix element in pionless effective field theory
- Low energy magnetic reactions in $A \leq 3$ nuclear systems and uncertainty estimation.

Calculation of $A = 3$ bound state matrix element in pionless effective field theory

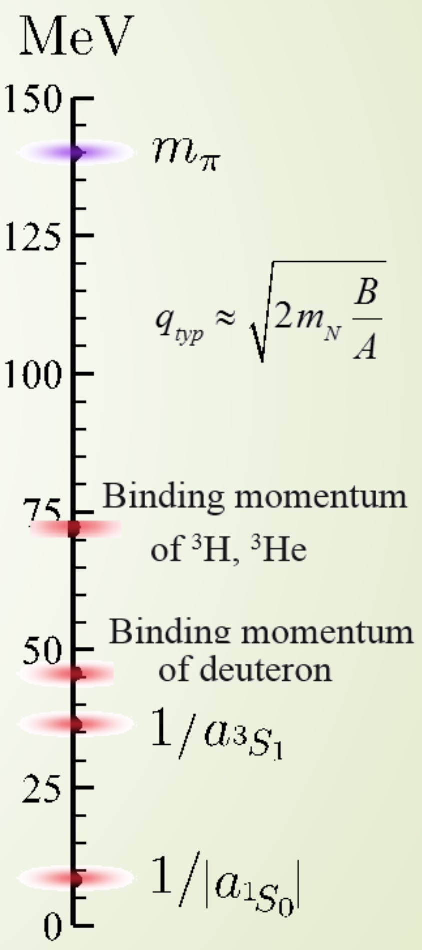
De-Leon, Platter, Gazit (2018), in prep.

Effective Field Theory

- The fundamental theory is Quantum Chromo-Dynamics (QCD), non-perturbative in the low energy regime.
- If the momentum scale, q, is small compared to the physical cutoff Λ_{cut}, a physical process can be described using Effective Field Theory.
- For low energies: $(q < \Lambda_{cut} = m_\pi)$, pion can be integrate out and only nucleons are left as effective degrees of freedom.

$$\text{QCD} \longrightarrow \not{\pi}\text{EFT}$$

- $\mathcal{L}_{\text{effective}} = \underbrace{\mathcal{O}(1)}_{LO} + \underbrace{\mathcal{O}\left(\frac{q}{\Lambda_{cut}}, \frac{r}{a}\right)}_{NLO} + \cdots +$

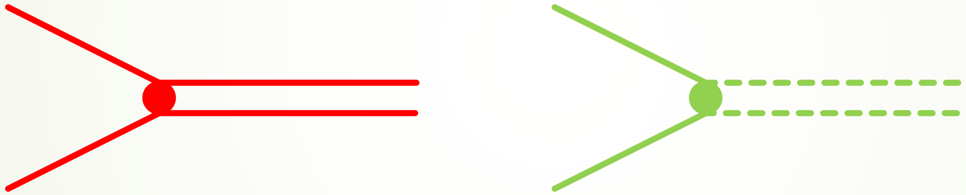


Kaplan, Savage ,Wise, (1998-1999), Bedaque, Hammer, van Kolck (1999)

Building $\not{\pi}$EFT Lagrangian

$$\mathcal{L}= N^T\left(iD_0+\frac{D^2}{2M_N}\right)N \quad -t^{i\dagger}\left(\sigma_t+iD_0+\frac{D^2}{4M_N}\right)t^i - s^{A\dagger}\left(\sigma_s+iD_0+\frac{D^2}{4M_N}\right)s^A$$

$$y_t[t^{i\dagger}(NP_t^iN)+h.c]+\ y_s[s^{A\dagger}(NP_s^AN)+h.c] \qquad +\mathcal{L}_3+\mathcal{L}_{\text{photon}}+\ \mathcal{L}_{\text{weak}}+\mathcal{L}_{\text{magnetic}}$$



Where:

$$t^i(^3S_1, I=0), \qquad s^A(^1S_0, I=1)$$

$$P_t^i=\frac{1}{\sqrt{8}}\sigma^2\sigma^i\tau^2 \quad ,P_s^A=\frac{1}{\sqrt{8}}\sigma^2\tau^2\tau^A$$

$$D_\mu=\partial_\mu+iA_\mu\hat{Q}$$

$$y_{t,s}^2=\frac{8\pi}{M_N^2\rho_{t,s}}$$

$$\sigma_{t,s}=\frac{2}{M_N\rho_{t,s}}\left(\frac{1}{a_{t,s}}-\mu\right)$$

Beane and Savage, (2001), Bedaque et al. (1999)

Building $\not\pi$EFT Lagrangian

$$y_{t,s}^2 = \frac{8\pi}{M^2 \rho_{t,s}}, \qquad \sigma_{t,s} = \frac{2}{M\rho_{t,s}}\left(\frac{1}{a_{t,s}} - \mu\right)$$

Scale separation:

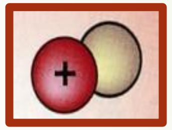
$$a \sim \frac{1}{q} \qquad \rho \sim \frac{1}{\Lambda_{\text{cut}}}$$

Parameter	Value	Parameter	Value
γ_t	45.701 MeV	ρ_t	1.765 fm
a_s	-23.714 fm	ρ_s	2.73 fm
a_p	-7.8063 fm	a_p	2.794 fm

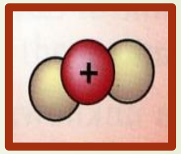
LO (left columns) **NLO** (right columns)

$\not\pi$EFT: $1 + 2 \neq 3$

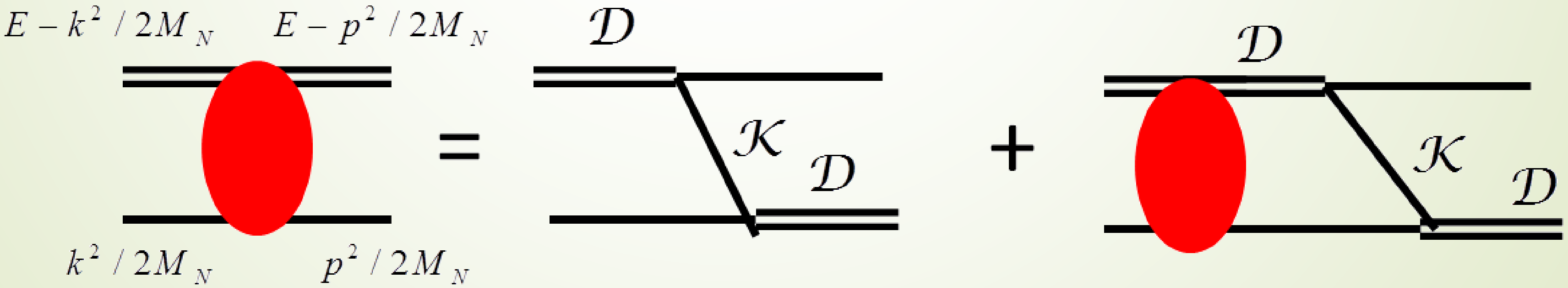
- For $\not\pi$EFT there is a **big difference** between a nuclear system with **2 particles and 3 particles**
- Deuteron: $\psi_d(k) = \frac{\sqrt{8\pi\gamma_t}}{k^2+\gamma_t^2}$



- Triton: $T(E,k,p) = \int_0^\Lambda d^3p' T(E,k,p')\mathcal{D}(E,p')\mathcal{K}(E,p',p)$



Summing over all possible amplitudes (Faddeev equation)



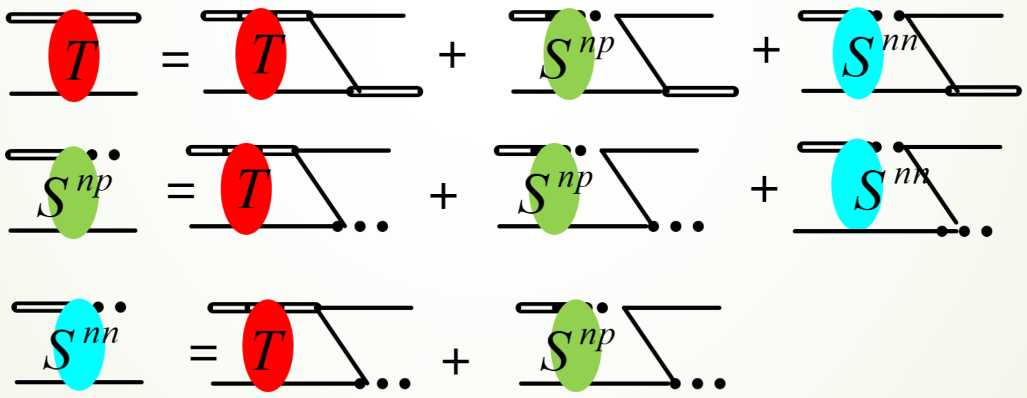
Bedaque, Hammer, van Kolck (1999), König and Hammer (2011), König et al (2014)

$\not{\pi}$EFT: $A = 3$ scattering amplitude

- For bound state: $T(E_B, k, p) = \frac{\mathcal{B}(k,E)^\dagger \mathcal{B}(p,E)}{E-E_B} + \mathcal{R}$

$$\mathcal{B}(p, E_B) = \int_0^\Lambda d^3p' \mathcal{B}(E_B, p') \mathcal{D}(E_B, p') \mathcal{K}(E_B, p', p)$$

- Triton, $J = \frac{1}{2}$, coupled channels Faddeev equation:

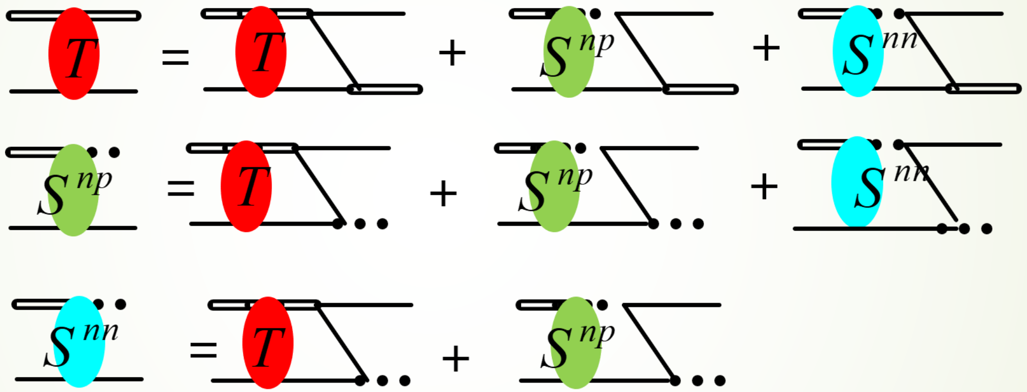


$$\mathcal{B} = \begin{pmatrix} \Gamma_t \\ \Gamma_{np} \\ \Gamma_{nn} \end{pmatrix}, \qquad \Gamma_\mu(E, p) = M_N \sum_{\nu=t,np,nn} y_\mu y_\nu a_{\mu\nu} K_0(E, p, p') \otimes D_\nu(E, p') \Gamma_\nu(E, p')$$

Bedaque, Hammer, van Kolck (1999), König and Hammer (2011), König et al (2014)

$\not{\pi}$EFT: $A = 3$ scattering amplitude

- Triton, $J = \frac{1}{2}$, coupled channels Faddeev equation:



$$\vec{\mathcal{B}} = \overleftrightarrow{\boldsymbol{M}} \times \vec{\mathcal{B}}$$

Eigen value problem

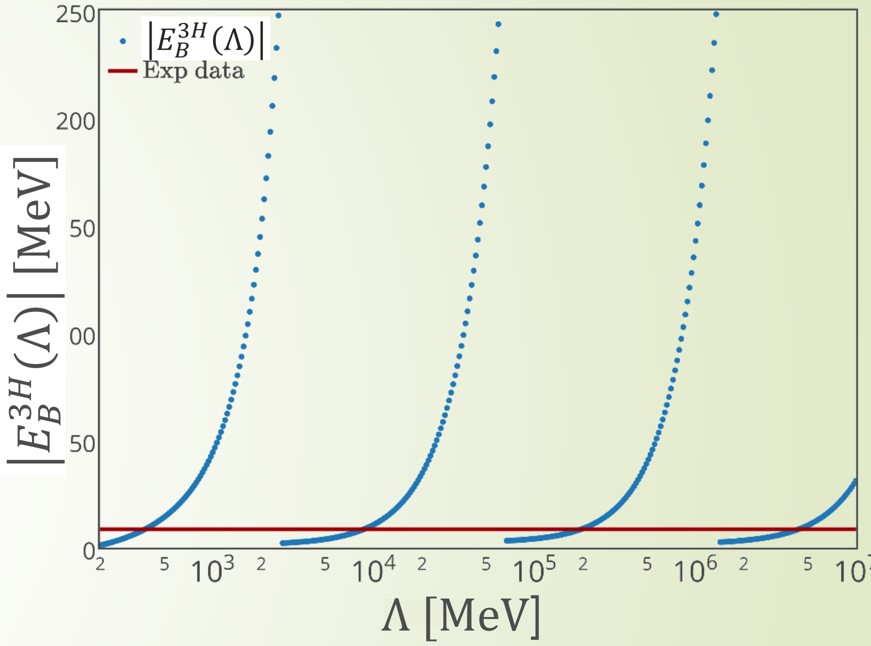
$$\vec{\mathcal{B}} = \begin{pmatrix} \Gamma_t \\ \Gamma_{np} \\ \Gamma_{nn} \end{pmatrix}, \ \boldsymbol{M}_{\mu,\nu} = M_N y_\mu y_\nu a_{\mu\nu} K_0(E,p,p') D_\nu(E,p')$$

Bedaque, Hammer, van Kolck (1999), König and Hammer (2011), König et al (2014)

Binding energy:

- Deuteron: $E_B = -\frac{\gamma_t^2}{M}$
- Triton:

$$\Gamma_\mu(E,p) = M \sum_{\nu=t,np,nn} y_\mu y_\nu a_{\mu\nu} K_0(E,p,p') \otimes D_\nu(E,p')\Gamma_\nu(E,p')$$



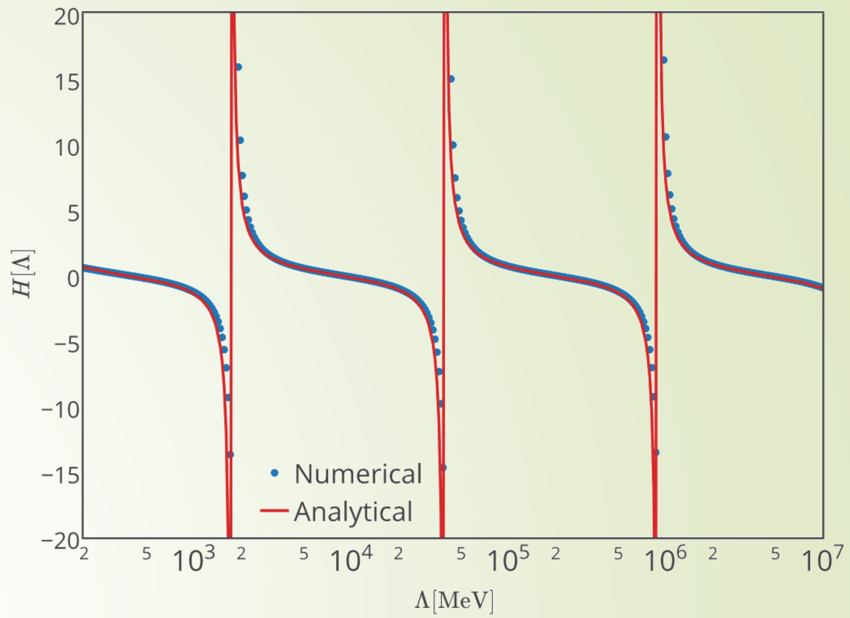
- $E_B = E_B(\Lambda)$, Efimov effect:
- 3-body system has strong

cutoff dependence → add 3-body force at LO.

$$\Gamma_\mu(E,p) = M \sum_{\nu=t,np,nn} y_\mu y_\nu \left[a_{\mu\nu} K_0(E,p,p') + b_{\mu\nu}\frac{H(\Lambda)}{\Lambda^2}\right] \otimes D_\nu(E,p')\Gamma_\nu(E,p')$$

Bedaque, Hammer, van Kolck (1999)

Binding energy:



- Deuteron: $E_B = -\frac{\gamma_d^2}{M}$
- Triton:

$$\Gamma_\mu(E,p) = M \sum_{\nu=t,np,nn} y_\mu y_\nu a_{\mu\nu} K_0(E,p,p') \otimes D_\nu(E,p')\Gamma_\nu(E,p')$$

- $E_B = E_B(\Lambda)$, Efimov effect:
- 3-body system has strong cutoff dependence → add 3-body force at LO.

$$\Gamma_\mu(E,p) = M \sum_{\nu=t,np,nn} y_\mu y_\nu \left[a_{\mu\nu} K_0(E,p,p') + b_{\mu\nu}\frac{H(\Lambda)}{\Lambda^2}\right] \otimes D_\nu(E,p')\Gamma_\nu(E,p')$$

Bedaque, Hammer, van Kolck (1999)

Normalization of the wave-function:

- Deuteron: $Z_d^{-1} = \frac{i\partial}{\partial E}\frac{1}{iD_t(E,p)}|_{E=E_d,p=0}$, $\quad Z_d = \frac{1}{1-\gamma_t\rho_t}$
- Triton: $\Gamma_\mu(E,p) = M\sum_{\nu=t,np,nn} y_\mu y_\nu \left[a_{\mu\nu}K_0(E,p,p') + b_{\mu\nu}\frac{H(\Lambda)}{\Lambda^2}\right] \otimes D_\nu(E,p')\Gamma_\nu(E,p')$
- Bethe-Salpeter (B.S.) normalization condition:

$$\langle\hat{1}\rangle = \sum_{\mu\nu}\left\langle \underbrace{\Gamma_\mu(E,p)D_\mu(E,p)}_{\psi_\mu(E,p)} \left| \frac{\partial}{\partial E}\left[\hat{I}_{\mu\nu}(E,p,p') - a_{\mu\nu}K_0(E,p,p')\right] \right| \underbrace{D_\nu(E,p')\Gamma_\nu(E,p')}_{\psi_\nu(E,p')} \right\rangle$$

$$\hat{I}_{\mu\nu} = D_\mu(E,p)^{-1}\frac{2\pi^2}{p'^2}\delta(p-p')\delta_{\mu,\nu}$$

$$1 = \frac{1}{Z^{^3\mathrm{H}}}\sum_{\mu\nu}\left\langle \psi_\mu(E,p) \left| \frac{\partial}{\partial E}\left[\hat{I}_{\mu\nu}(E,p,p') - a_{\mu\nu}K_0(E,p,p')\right] \right| \psi_\nu(E,p') \right\rangle$$

König and Hammer (2011), De-Leon, Platter and Gazit (2018)

Normalization of the wave-function:

- Deuteron: $Z_d^{-1} = \frac{i\partial}{\partial E}\frac{1}{iD_t(E,p)}|_{E=E_d,p=0}$, $\quad Z_d = \frac{1}{1-\gamma_t\rho_t}$
- Triton: $\Gamma_\mu(E,p) = M\sum_{\nu=t,np,nn} y_\mu y_\nu \left[a_{\mu\nu}K_0(E,p,p') + b_{\mu\nu}\frac{H(\Lambda)}{\Lambda^2}\right] \otimes D_\nu(E,p')\Gamma_\nu(E,p')$
- Bethe-Salpeter (B.S.) normalization condition:

$$\langle\hat{1}\rangle = \sum_{\mu\nu}\left\langle \underbrace{\Gamma_\mu(E,p)D_\mu(E,p)}_{\psi_\mu(E,p)}\left|\frac{\partial}{\partial E}\left[\hat{I}_{\mu\nu}(E,p,p') - a_{\mu\nu}K_0(E,p,p')\right]\right|\underbrace{D_\nu(E,p')\Gamma_\nu(E,p')}_{\psi_\nu(E,p')}\right\rangle$$

$$\hat{I}_{\mu\nu} = D_\mu(E,p)^{-1}\frac{2\pi^2}{p'^2}\delta(p-p')\delta_{\mu,\nu}$$

$$\mathbf{1} = \frac{\mathbf{1}}{Z^{^3\mathrm{H}}}\sum_{\mu\nu}\left\langle \psi_\mu(E,p)\left|\frac{\partial}{\partial E}\left[\hat{I}_{\mu\nu}(E,p,p') - a_{\mu\nu}K_0(E,p,p')\right]\right|\psi_\nu(E,p')\right\rangle$$

Matrix Element!

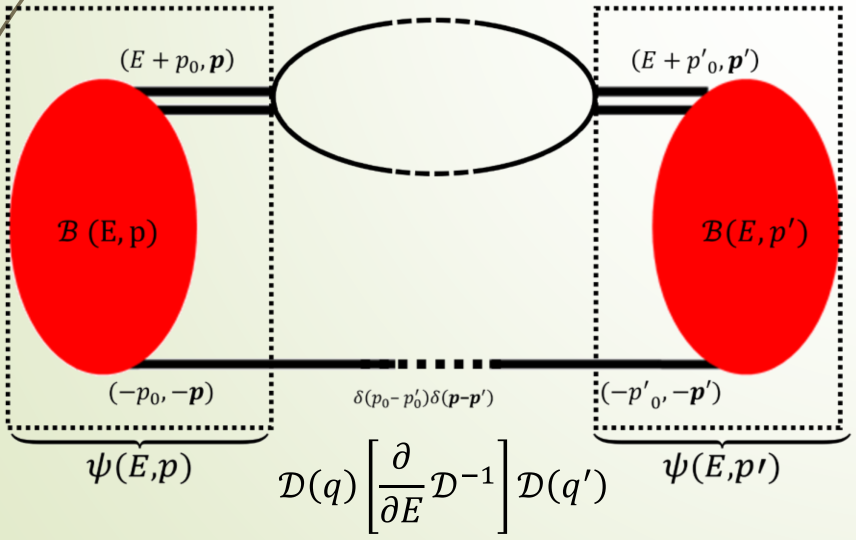
König and Hammer (2011), De-Leon, Platter and Gazit (2018)

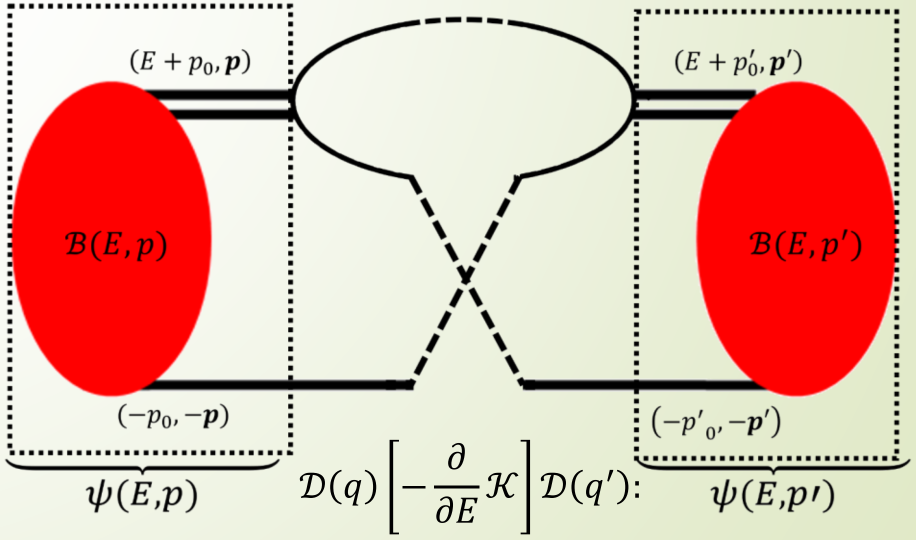
Normalization of the wave-function:

$$1 = \frac{1}{Z^{^3\mathrm{H}}} \sum_{\mu\nu} \left\langle \psi_\mu(E,p) \left| \frac{\partial}{\partial E} \left[\hat{I}_{\mu\nu}(E,p,p') - a_{\mu\nu} K_0(E,p,p') \right] \right| \psi_\nu(E,p') \right\rangle$$

$$\frac{\partial S(E,p)}{\partial E} = \mathrm{S(E,p)S(E,p')}\delta(p-p')$$

We show that the normalization is equivalent to all possible connections between two identical bubbles:



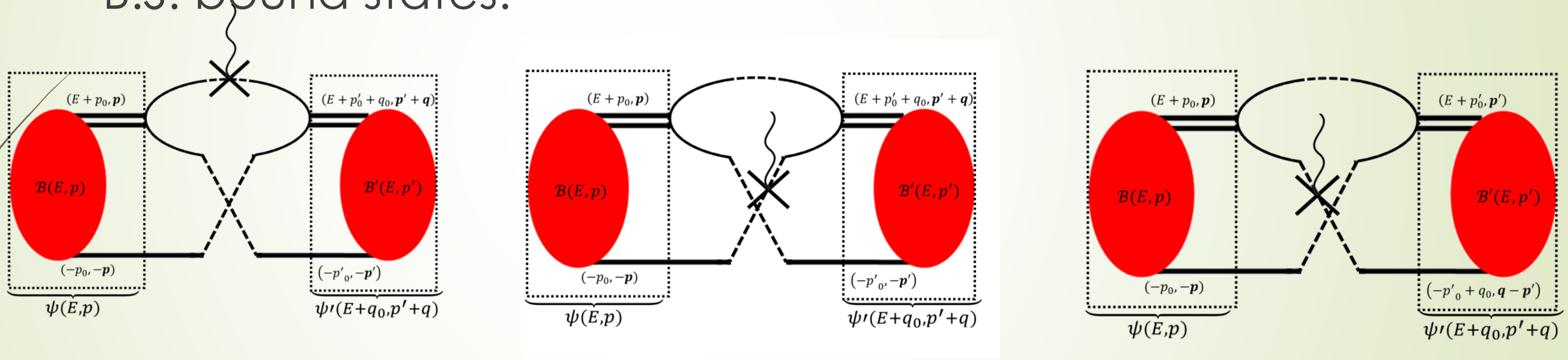


De-Leon, Platter and Gazit (2018)

General matrix element

$$\mathcal{O}|S,S_z,I,I_z,E\rangle \propto |S,S'_z,I,I'_z,E',q\rangle\ ,\ \langle\mathcal{O}\rangle = a^J\langle S,S'_z,I,I'_z,E',q|\mathcal{O}^J\mathcal{O}^I\mathcal{O}^q|S,S_z,I,I_z\ ,E\rangle$$

- A matrix element is equivalent to all possible connections, with B.S. bound states:

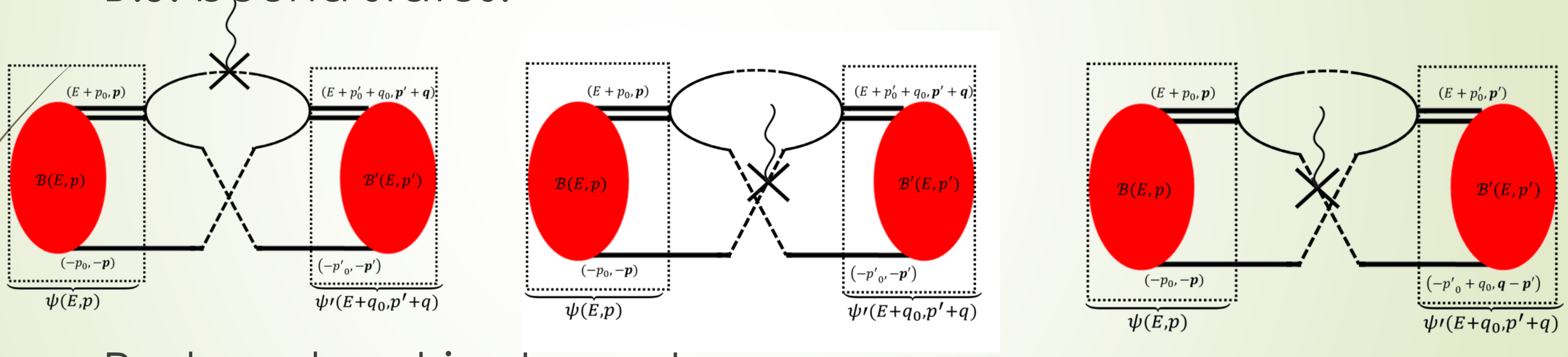


De-Leon, Platter and Gazit (2018)

General matrix element

$$\mathcal{O}|S,S_z,I,I_z,E\rangle \propto |S,S'_z,I,I'_z,E',q\rangle\ ,\ \langle\mathcal{O}\rangle = a^J\left\langle S,S'_z,I,I'_z,E',q\middle|\mathcal{O}^J\mathcal{O}^I\mathcal{O}^q\middle|S,S_z,I,I_z\ ,E\right\rangle$$

- A matrix element is equivalent to all possible connections, with B.S. bound states:

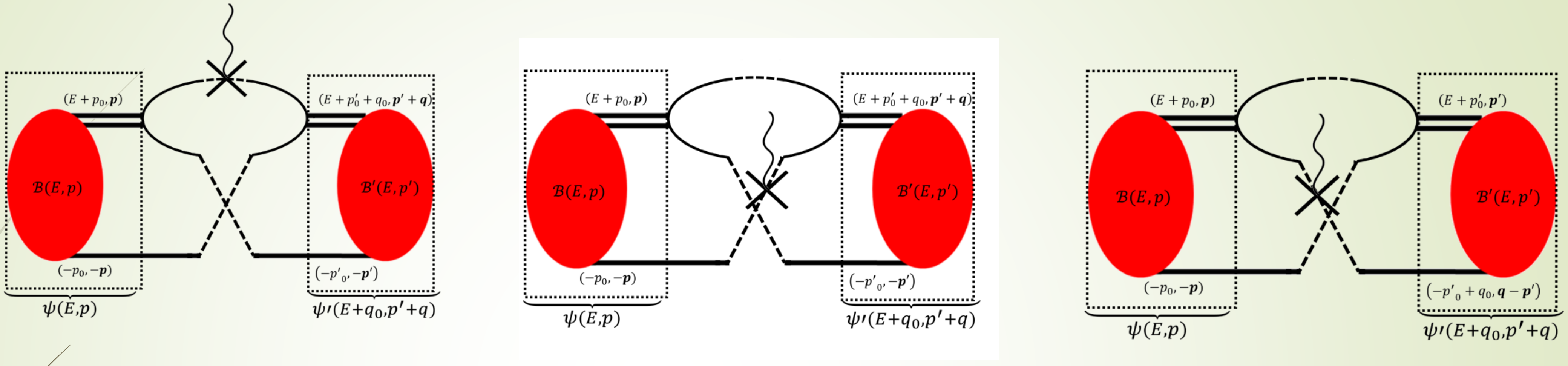


- Reduced matrix element:

$$\langle\mathcal{O}\rangle = a^J\left\langle\frac{1}{2}\middle\|\mathcal{O}^J\middle\|\frac{1}{2}\right\rangle\left\langle\frac{1}{2}\middle|\mathcal{O}^I\middle|\frac{1}{2}\right\rangle\sum_{\mu,\nu}\left\langle\psi^j_\nu(E',p+q)\middle|a^{i,j}_{\mu\nu}\mathcal{K}^q(E,p,p')+d^{i,j}_{\mu\nu}\mathcal{J}^{\,q}(E,p,p')\middle|\psi^i_\mu(E,p)\right\rangle$$

De-Leon, Platter and Gazit (2018)

Reduced matrix element



- Reduced matrix element

$$\langle \mathcal{O} \rangle = a^J \left\langle \frac{1}{2} \| \mathcal{O}^J \| \frac{1}{2} \right\rangle \left\langle \frac{1}{2} \middle| \mathcal{O}^I \middle| \frac{1}{2} \right\rangle \sum_{\mu,\nu} \left\langle \psi_\nu^j(E', p+q) \middle| a_{\mu\nu}^{i,j} \mathcal{K}^q(E,p,p') + d_{\mu\nu}^{i,j} \mathcal{J}^q(E,p,p') \middle| \psi_\mu^i(E,p) \right\rangle$$

- For the case that $i = j, q = 0, E' = E$:

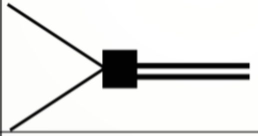
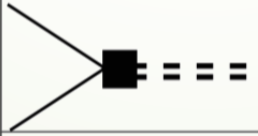
$$a_{\mu\nu}^{i,j} = a_{\mu\nu}$$
$$d_{\mu\nu}^{i,j} = \delta_{\mu,\nu}$$

De-Leon, Platter and Gazit (2018)

General matrix element

- A typical $\not{\pi}$EFT interaction contains also the following two-body interactions up to NLO:

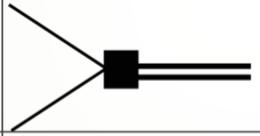
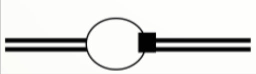
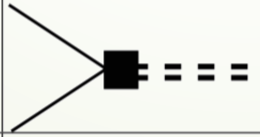
$$t^\dagger\, t\delta[q_0-(E-E')],\, s^\dagger\, s\delta[q_0-(E-E')],\, (t^\dagger\, s+h.c)\delta[q_0-(E-E')]$$

(1a)	$t^\dagger(NP^sN)+h.c$		$\frac{1}{\sqrt{2\pi\rho_t}}\left(\mu-\frac{1}{a_t}\right)\left[\frac{1}{\sqrt{8}}\sigma^2\tau^2\tau^A+h.c\right]$
(1b)	$t^\dagger(NP^sN)+h.c$		$\frac{1}{2\pi\sqrt{\rho_t\rho_s}}\left(\mu-\frac{1}{a_t}\right)\left(\mu-\frac{1}{a_s}\right)\left[\frac{1}{\sqrt{8}}\sigma^2\tau^2\tau^A+h.c\right]$
(2a)	$s^\dagger(NP^tN)+h.c$		$\frac{1}{\sqrt{2\pi\rho_s}}\left(\mu-\frac{1}{a_s}\right)\left[\frac{1}{\sqrt{8}}\tau^2\sigma^2\sigma^i+h.c\right]$
(2b)	$s^\dagger(NP^tN)+h.c$		$\frac{1}{2\pi\sqrt{\rho_s\rho_t}}\left(\mu-\frac{1}{a_s}\right)\left(\mu-\frac{1}{a_t}\right)\left[\frac{1}{\sqrt{8}}\tau^2\sigma^2\sigma^i+h.c\right]$
(3)	$s^\dagger t+h.c$		$\frac{1}{2\pi\sqrt{\rho_t\rho_s}}\left(\mu-\frac{1}{a_t}\right)\left(\mu-\frac{1}{a_s}\right)$

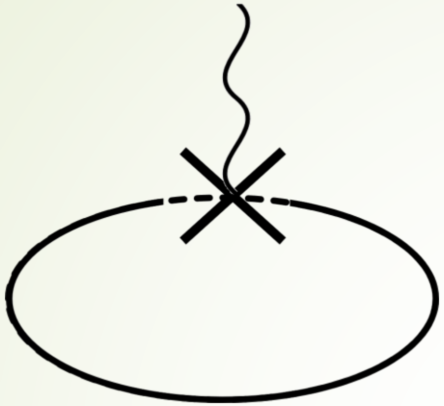
General matrix element

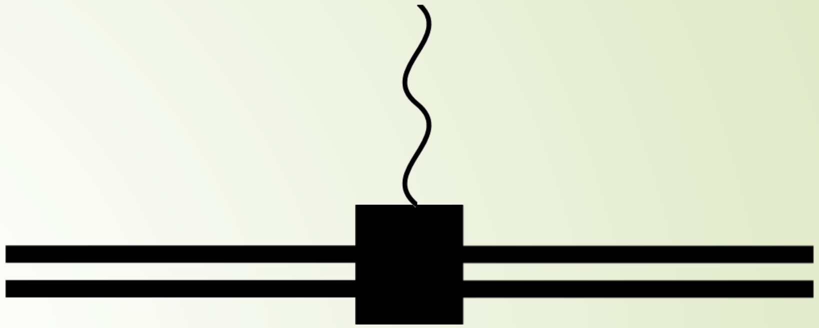
- A typical $\not{\pi}$EFT interaction contains also the following two-body interactions up to NLO:

$$t^\dagger\, t\delta[q_0-(E-E')], s^\dagger\, s\delta[q_0-(E-E')], (t^\dagger\, s+h.c)\delta[q_0-(E-E')]$$

(4a)	$t^\dagger(NP^tN)+h.c$		$\frac{1}{\sqrt{2\pi\rho_t}}\left(\mu-\frac{1}{a_t}\right)\left[\frac{1}{\sqrt{8}}\sigma^2\tau^2\sigma^i+h.c\right].$
(4b)	$t^\dagger(NP^tN)+h.c$		$\frac{1}{2\pi\rho_t}\left(\mu-\frac{1}{a_t}\right)^2\left[\frac{1}{\sqrt{8}}\sigma^2\tau^2\sigma^i+h.c\right]$
(5)	$t^\dagger t$		$\frac{1}{2\pi\rho_t}\left(\mu-\frac{1}{a_t}\right)^2$
(6a)	$s^\dagger(NP^sN)+h.c$		$\frac{1}{\sqrt{2\pi\rho_s}}\left(\mu-\frac{1}{a_s}\right)\left[\frac{1}{\sqrt{8}}\sigma^2\tau^2\tau^A+h.c\right]$
(6b)	$s^\dagger(NP^sN)+h.c$		$\frac{1}{2\pi\rho_s}\left(\mu-\frac{1}{a_s}\right)^2\left[\frac{1}{\sqrt{8}}\sigma^2\tau^2\tau^A+h.c\right]$
(7)	$s^\dagger s$		$\frac{1}{2\pi\rho_s}\left(\mu-\frac{1}{a_s}\right)^2$

Deuteron matrix element





- Reduced matrix element:

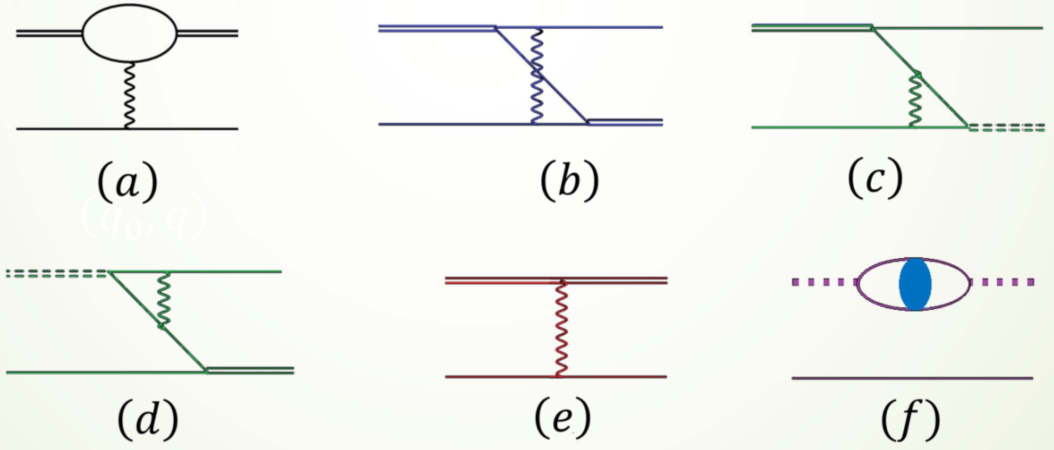
$$\langle \mathcal{O} \rangle = a^J \left[\left\langle \frac{1}{2} \| \mathcal{O}^J \| \frac{1}{2} \right\rangle \langle \psi_t \ (E', p+q) | d_{t,t} \mathcal{J}^{\,q} (E, p, p') | \psi_t^i (E, p) \rangle + L_2 \langle \psi_t \ (E', p+q) | \psi_t^i (E, p) \rangle \right]$$

This implies that $\not{\pi}$EFT is consistent for $A = 2 \leftrightarrow 3$ transitions for bound states.

De-Leon, Platter and Gazit (2018)

Adding photons – perturbative and non perturbative approaches:

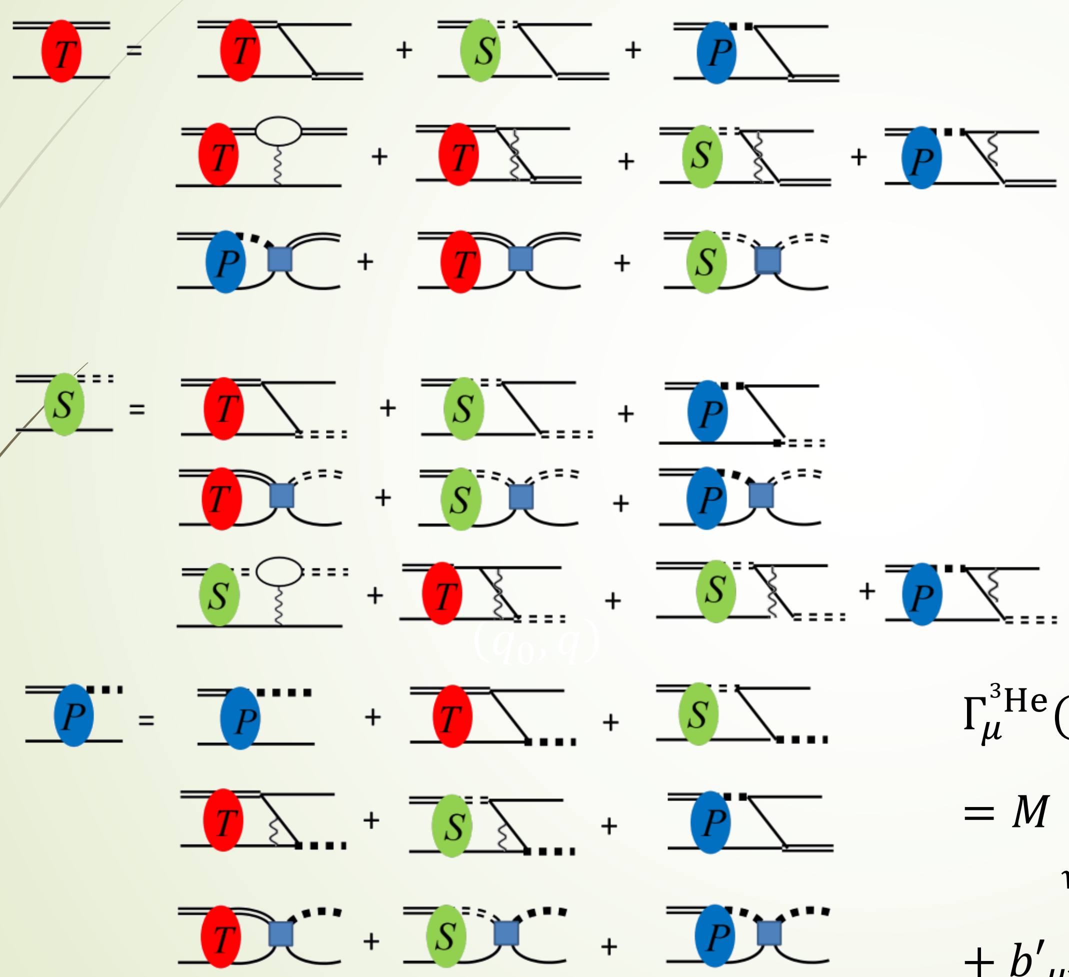
- For the bound state the typical momentum $Q \geq \sqrt{M_N E_{^3\mathrm{He}}} \sim 85\mathrm{MeV}]$, one photon exchange - $\frac{\alpha \mathrm{M_N}}{Q} \ll 1.$
- The Columbic correction :



- Diagrams a-d: $\sim \mathcal{O}(\mathcal{K})\frac{\alpha \mathrm{M_N}}{Q}$, diagram e$\sim \mathcal{O}(a)\frac{Q}{\Lambda}$ which is NLO, diagram f: result of the pp propagator.

König, Hammer (2011), Vannase et al (2015), König et al (2014-2016)

^{3}He – non perturbative photons:



$$\Gamma_\mu^{^3\mathrm{He}}(E_{^3\mathrm{He}}, p)$$
$$= M \sum_{\nu=t,np,pp} y_\mu y_\nu \Big[a'_{\mu\nu} K_0(E_{^3\mathrm{He}}, p, p') + c'_{\mu\nu}\, K^C_{\mu\nu}(E_{^3\mathrm{He}}, p, p'$$
$$+ b'_{\mu\nu} \frac{H(\Lambda)}{\Lambda^2} \Big] \otimes D_\nu(E, p')\Gamma_\nu(E_{^3\mathrm{He}}, p')$$

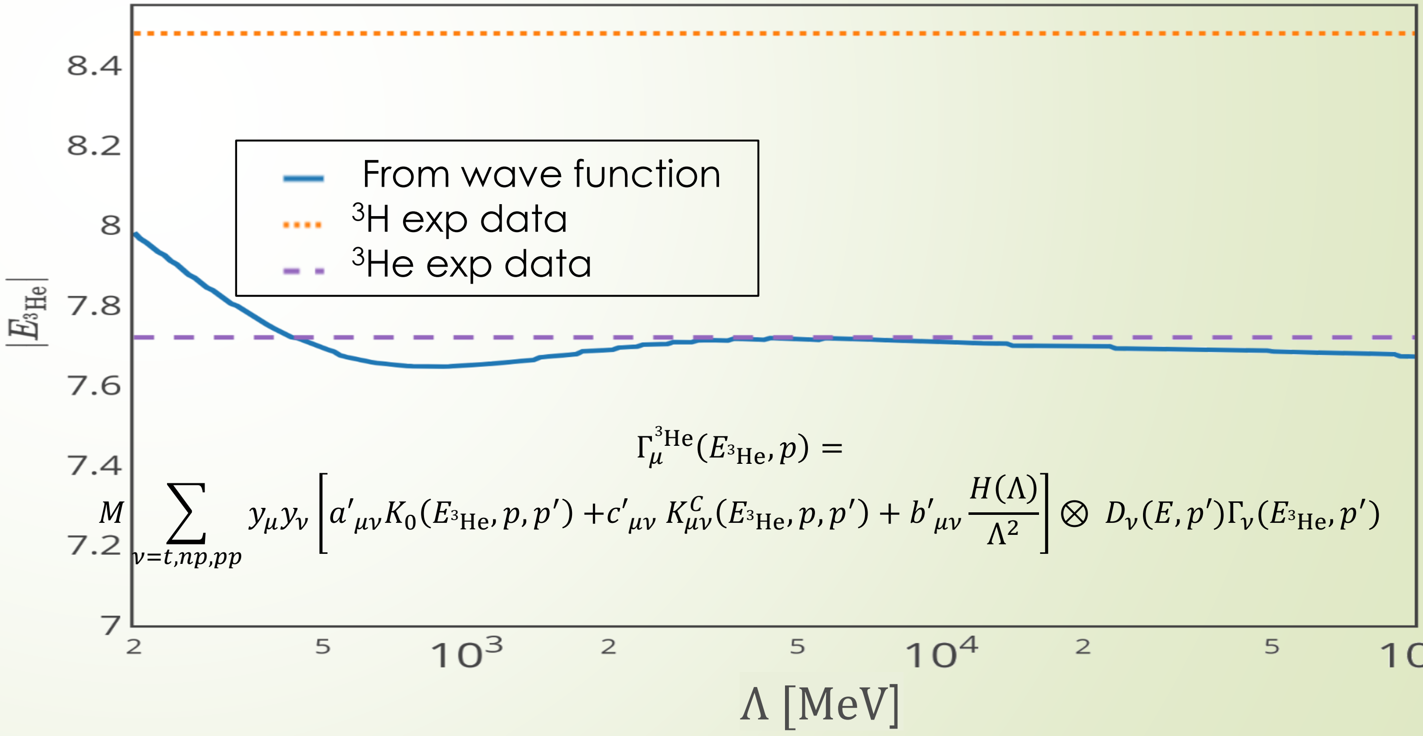
König, Hammer (2011), Vannase et al (2015), König et al (2014-2016)

^{3}He – non perturbative photons:

- Binding energy:

Two ways to find ^{3}He binding energy difference:

- Find the pole of the non-perturbative solution of the homogenous Faddeev equation with Coulomb interaction.



$$\Gamma_\mu^{^3\mathrm{He}}(E_{^3\mathrm{He}},p) = M\sum_{\nu=t,np,pp} y_\mu y_\nu \left[a'_{\mu\nu}K_0(E_{^3\mathrm{He}},p,p') + c'_{\mu\nu}\,K^C_{\mu\nu}(E_{^3\mathrm{He}},p,p') + b'_{\mu\nu}\frac{H(\Lambda)}{\Lambda^2}\right]\otimes D_\nu(E,p')\Gamma_\nu(E_{^3\mathrm{He}},p')$$

König et al (2014-2016)

^{3}He perturbative photons:

- Binding energy:

  Two ways to find ^{3}He binding energy difference:

  - Find the pole of the non-perturbative solution of the homogenous Faddeev equation with Coulomb interaction.
  - Since Coulomb interaction is perturbative in ^{3}He, one can calculate the energy shift in the one-photon approximations as a matrix element.

$$\Delta E = \frac{1}{Z^{^3\mathrm{H}}} \sum_{\mu,\nu=t,np,nn} y_\mu y_\nu \left[\Gamma_\mu^{^3\mathrm{H}}(E_{^3\mathrm{H}},p) D_\mu(E_{^3\mathrm{H}},p)\right] \otimes c_{\mu\nu} K_{\mu\nu}^C(p,p',E) \otimes \left[D_\nu(E_{^3\mathrm{H}},p')\Gamma_\nu^{^3\mathrm{H}}(E_{^3\mathrm{H}},p')\right] +$$

$$\frac{1}{Z^{^3\mathrm{H}}} \sum_{\mu=t,np,nn} \left[\Gamma_\mu^{^3\mathrm{H}}(E_{^3\mathrm{H}},p) D_\mu(E_{^3\mathrm{H}},p)\right] \otimes \left[a_{\mu nn} K_0(p,p',E) + b_{\mu nn}\frac{H(\Lambda)}{\Lambda^2}\right] \otimes$$

$$\left\{\left[D_{pp}(E_{^3\mathrm{H}},p') - D_{nn}(E_{^3\mathrm{H}},p')\right]\Gamma_{nn}^{^3\mathrm{H}}(E_{^3\mathrm{H}},p')\right\} =$$

$$\underbrace{\frac{1}{Z^{^3\mathrm{H}}} \sum_{\mu=t,np,nn} \left[\Gamma_\mu^{^3\mathrm{H}}(E_{^3\mathrm{H}},p) D_\mu(E_{^3\mathrm{H}},p)\right] \otimes c_{\mu\nu} K_{\mu\nu}^C(p,p',E) \otimes \left[D_\nu(E_{^3\mathrm{H}},p')\Gamma_\nu^{^3\mathrm{H}}(E_{^3\mathrm{H}},p')\right]}_{\text{one body}} +$$

$$\underbrace{\frac{1}{Z^{^3\mathrm{H}}} \Gamma_{nn}^{^3\mathrm{H}}(E_{^3\mathrm{H}},p) \otimes \frac{2\pi^2}{p'^2}\delta(p-p') \otimes \left\{\left[D_{pp}(E_{^3\mathrm{H}},p') - D_{nn}(E_{^3\mathrm{H}},p')\right]\Gamma_{nn}^{^3\mathrm{H}}(E_{^3\mathrm{H}},p')\right\}}_{\text{two body}} =$$

$$\sum_{\mu=t,np,nn} \left\langle \psi_\mu^{^3\mathrm{H}}(E_{^3\mathrm{H}},p) \middle| \mathcal{O}_{\mu\nu}^{q(1)}(E_{^3\mathrm{H}},p,p') + \mathcal{O}_{\mu\nu}^{q(2)}(E_{^3\mathrm{H}},p,p') \middle| \psi_\mu^{^3\mathrm{H}}(E_{^3\mathrm{H}},p')\right\rangle.$$

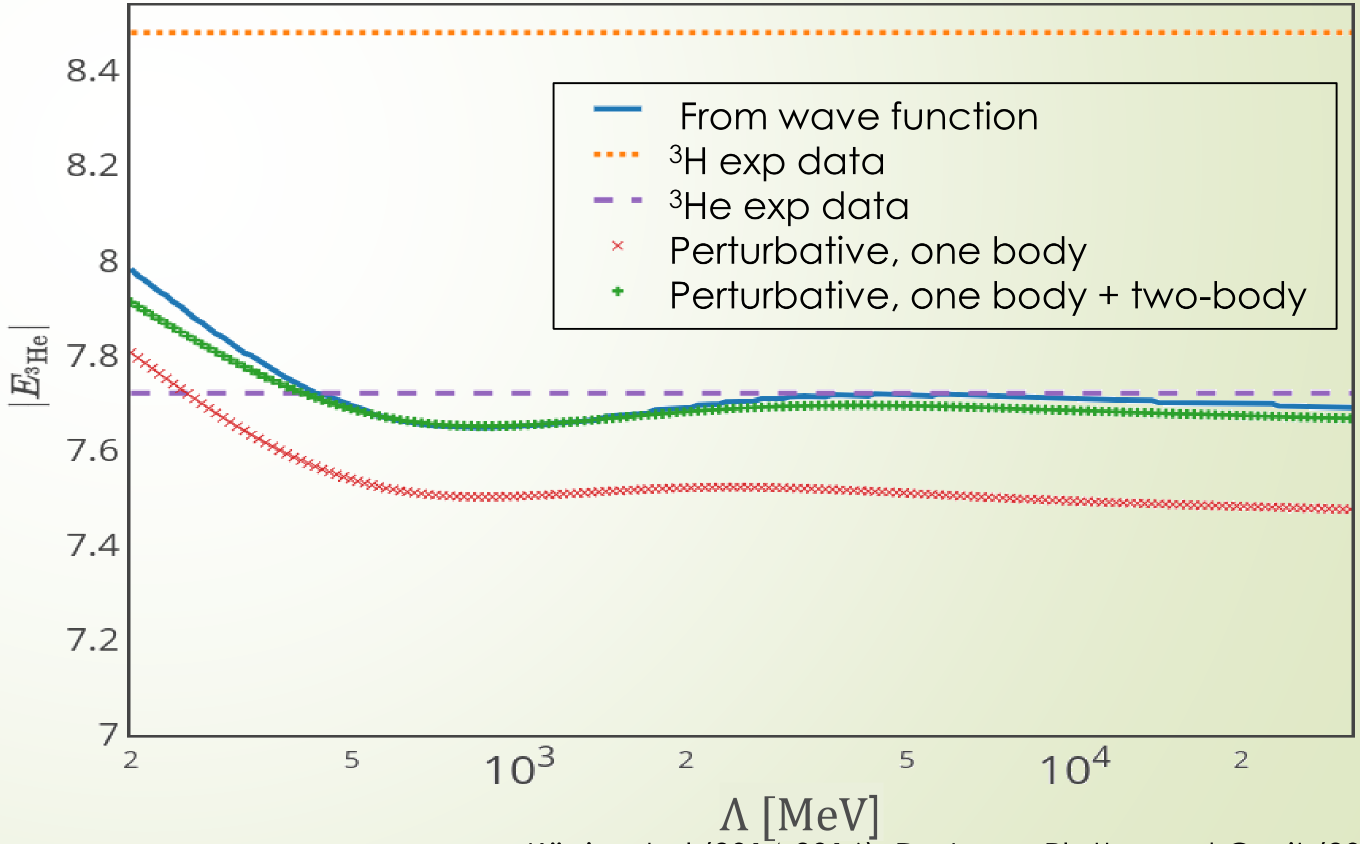
König et al (2014-2016), De-Leon, Platter and Gazit (2018)

^{3}He perturbative photons:

- Binding energy:

Two ways to find ^{3}He binding energy difference:

- Find the pole of the non-perturbative solution of the homogenous Faddeev equation with Coulomb interaction.
- Since Coulomb interaction is perturbative in ^{3}He, one can calculate the energy shift in the one-photon approximations as a matrix element.



König et al (2014-2016), De-Leon, Platter and Gazit (2018)

NLO A=3, Binding Energy

For triton:

$$\Delta E_B = \lim_{E \to E_B} \frac{(E - E_B)^2 T^{\mathrm{NLO}}(E,k,p)}{Z^{LO}(k,p)} =$$

$$\langle \psi^{\mathrm{LO}}(E,p) | \mathcal{O}^{\mathrm{NLO}}(E,p,p') | \psi^{\mathrm{LO}}(E,p') \rangle = f(\Lambda)$$

$$\mathrm{E_B^{LO}} = \mathrm{E_B^{NLO}} \rightarrow \Delta E_B = f(\Lambda) = 0$$

Determines the three –body force up to NLO:

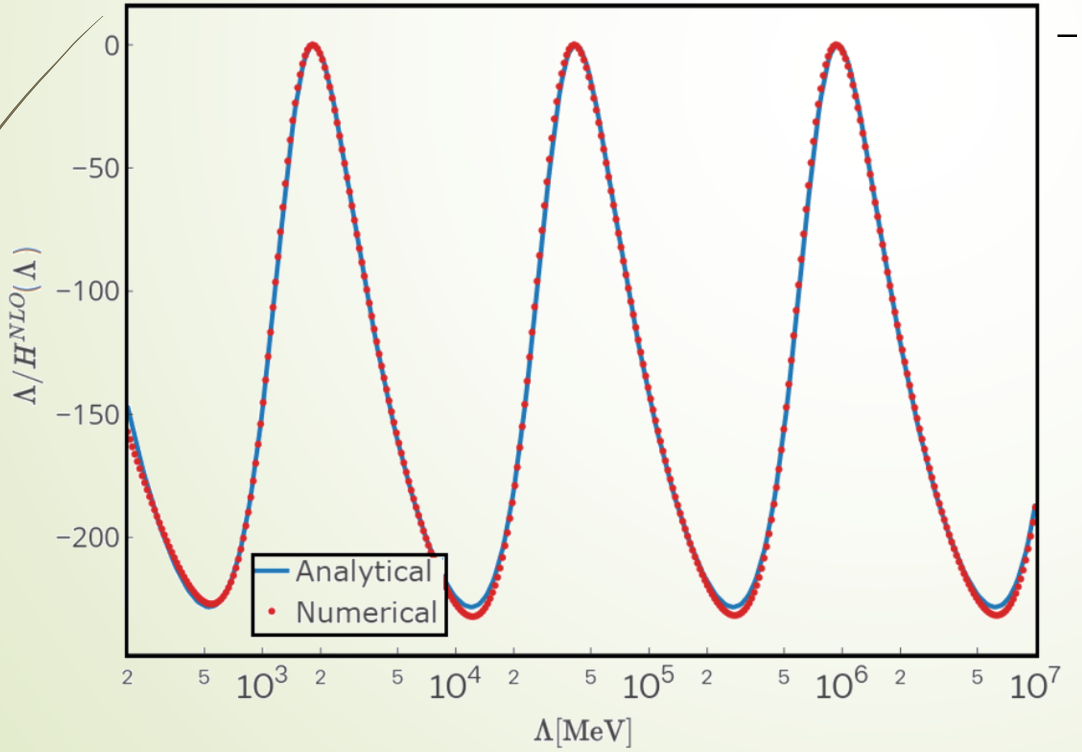
Ji et al (2012), Vanasse et al (2014).

NLO A=3, Binding Energy

For triton:

$$E_B^{LO} = E_B^{NLO} \rightarrow \Delta E_B = f(\Lambda) = 0$$

Determines the three –body force up to NLO:



$$-\frac{H^{NLO}(\Lambda)}{\pi^2\Lambda^2} = \frac{1}{4\pi M}\left[\frac{\rho_t}{y_t^2}\int_0^\Lambda dp'\left(\Gamma_t^{3H}(E_{3H},p')\right)2\frac{\sqrt{\frac{3p'^2}{4}-ME_{3H}}}{\left(\sqrt{\frac{3p'^2}{4}-ME_{3H}}-\gamma_t\right)^2}\right.$$

$$+\frac{\rho_s}{y_s^2}\int_0^\Lambda dp'\left(\Gamma_{np}^{3H}(E_{3H},p')\right)2\frac{\sqrt{\frac{3p'^2}{4}-ME_{3H}}}{\left(\sqrt{\frac{3p'^2}{4}-ME_{3H}}-\frac{1}{a_s}\right)^2}$$

$$\left.+\frac{\rho_s}{y_s^2}\int_0^\Lambda dp'\left(\Gamma_{nn}^{3H}(E_{3H},p')\right)2\frac{\sqrt{\frac{3p'^2}{4}-ME_{3H}}}{\left(\sqrt{\frac{3p'^2}{4}-ME_{3H}}-\frac{1}{a_s}\right)^2}\right]$$

$$\times\left[\frac{My_t^2}{4\pi^4}\int_0^\Lambda \psi_t\,(E_{3H},p')p'^2dp' + \frac{My_s^2}{4\pi^4}\int_0^\Lambda \psi_t\,(E_{3H},p')p'^2dp'\right.$$

$$\left.+\frac{My_s^2}{4\pi^4}\int_0^\Lambda \psi_{nn}\,(E_{3H},p')p'^2dp'\right]^{-2}$$

Ji et al (2012), Vanasse et al (2014).

NLO A=3, wave function

For triton:

- Photon perturbation:

$$\Delta E_B^C = f(\Lambda) = \sum_\mu \langle \psi_\mu^{\mathrm{LO}}(E,p) | \mathcal{O}_{\mu,\nu}^C(E,p,p') | \psi_\nu^{\mathrm{LO}}(E,p') \rangle$$

$$\Gamma_\mu^{{}^3\mathrm{He}}(E_{3He},p) = \sum_\nu \Bigg[\underbrace{\mathcal{O}_{\mu,\nu}^S}_{\text{identical to }{}^3\mathrm{H}} + \mathcal{O}_{\mu,\nu}^C \Bigg] \otimes D_\nu(E_{{}^3\mathrm{He}},p')\Gamma_\nu^{{}^3\mathrm{He}}(E_{{}^3\mathrm{He}},p'),$$

- NLO perturbation:

$$\Delta E_B^{\mathrm{NLO}} = f(\Lambda) = \sum_\mu \langle \psi_\mu^{\mathrm{LO}}(E,p) | \mathcal{O}_{\mu\nu}^{\mathrm{NLO}}(E,p,p') | \psi_\nu^{\mathrm{LO}}(E,p') \rangle$$

$$\Gamma_\mu^{\mathrm{LO}}(E_{{}^3\mathrm{H}},p) + \Gamma_\mu^{NLO}(E_{{}^3\mathrm{H}},p) = \sum_\nu \Bigg[\underbrace{\mathcal{O}_{\mu,\nu}^S}_{\mathrm{LO}} + \underbrace{\mathcal{O}_{\mu,\nu}^{\mathrm{NLO}}}_{\mathrm{NLO}} \Bigg] \otimes D_\nu(E_{{}^3\mathrm{H}},p')\, \Gamma_\nu^{\mathrm{LO}}(E_{{}^3\mathrm{H}},p'),$$

Vanasse et al (2014), De-Leon, Platter and Gazit (2018)

NLO A=3, general EW matrix element

$$\langle \mathcal{O}_{EW}^{\mathrm{LO}} \rangle + \langle \mathcal{O}_{EW}^{\mathrm{NLO}} \rangle =$$

$$\underbrace{\langle \psi^{\mathrm{LO}} | \mathcal{O}_{EW}^{\mathrm{LO}} | \psi^{LO} \rangle}_{\mathcal{O}_{EW}^{LO}} + \underbrace{\langle \psi^{\mathrm{NLO}} | \mathcal{O}_{EW}^{\mathrm{LO}} | \psi^{LO} \rangle + \langle \psi^{\mathrm{LO}} | \mathcal{O}_{EW}^{\mathrm{NLO}} | \psi^{\mathrm{LO}} \rangle + \langle \psi^{\mathrm{LO}} | \mathcal{O}_{EW}^{\mathrm{LO}} | \psi^{\mathrm{NLO}} \rangle}_{\mathcal{O}_{EW}^{NLO}}$$

$$\psi_\mu(E,p) = D_\mu^{\mathrm{NLO}}(E,p)\Gamma_\mu^{\mathrm{LO}}(E,p) + D_\mu^{\mathrm{LO}}(E,p)\Gamma_\mu^{\mathrm{NLO}}(E,p) +$$

Vanasse et al (2014) , De-Leon, Platter and Gazit (2018)

Low energy magnetic reactions in $A \leq 3$ nuclear systems and uncertainty estimation.

De-Leon and Gazit (2018), in prep.

Pionless: Kirscher et al. (2017), Vanasse (2017)
chiral: Pastore et al (2013), Bacca and Pastore (2014)

Electroweak analogues: low energy observables $A < 4$

- To examine the consistency of $\not{\pi}\mathrm{EFT}$ we need to find a set of $\mathrm{A} < 4$ reactions all well measured.

Electroweak analogues: low energy observables $A < 4$

- To examine the consistency of $\not\pi$EFT we need to find a set of $\mathrm{A} < 4$ reactions all well measured.

	Weak	Electromagnetic	
A=2	Proton-proton fusion: $p + p \to d + \nu_e + e^+$	Radiative capture: $n + p \to d + \gamma$	Deuteron magnetic moment: $\langle \mu_d \rangle$
A=3	$^3\mathrm{H}$ β decay: $^3\mathrm{H} \to \overline{\nu_e} + e^- + {}^3\mathrm{He}$	$^3\mathrm{H}$ magnetic moment: $\langle \mu_{^3\mathrm{H}} \rangle$	$^3\mathrm{He}$ magnetic moment: $\langle \mu_{^3\mathrm{He}} \rangle$

- All the Electromagnetic interactions for $A < 4$ are well measured.

Electroweak analogues: low energy observables $A < 4$

- To examine the consistency of $\not{\pi}\mathrm{EFT}$ we need to find a set of $\mathrm{A} < 4$ reactions all well measured.

	EM	Weak
1-body LEC	κ_0, κ_1	g_A
1-body operator	$\sigma, \sigma\tau^0$	$\tau^{+,-}, \sigma\tau^{+,-}$
2-body operator	$L_1 s^\dagger d, L_2 d^\dagger d$	$L_{1A} s^\dagger d$
$A = 2, q \approx 0$ obs.	$\sigma_{np}, \langle\mu_d\rangle$	Λ_{pp}
$A = 3, q \approx 0$ obs.	$\langle\mu_{^3\mathrm{H}}\rangle, \langle\mu_{^3\mathrm{He}}\rangle$	$^3\mathrm{H}\ \beta$ decay

- All the Electromagnetic interactions for $A < 4$ are well measured.

Magnetic interaction in $\not{\pi}$EFT :

- The one body Lagrangian of the magnetic interaction is given by:

$$\mathcal{L}^1_{\text{magnetic}} = \frac{e}{2M_N} N^\dagger \, (\kappa_0 + \kappa_1 \tau_3)\sigma \cdot BN$$

- The two body Lagrangian of the magnetic interaction is given by:

$$\mathcal{L}^2_{\text{magnetic}} = -[\kappa_1 L'_1 (t^\dagger s + s^\dagger t) \cdot \vec{B} - \kappa_0 L'_2 (t^\dagger t) \cdot \vec{B}]$$

$$L'_1 = -\frac{\rho_t + \rho_s}{\sqrt{\rho_t \rho_s}} + l_1(\mu)$$

$$L'_2 = -2 + l_2(\mu)$$

$$l_1(\mu) = \frac{M}{\pi \sqrt{\rho_t \rho_s} \kappa_1} L_1 \left(\mu - \frac{1}{a_t}\right)\left(\mu - \frac{1}{a_s}\right)$$

$$l_2(\mu) = 2 \frac{M}{\pi \rho_t \kappa_0} L_2 \left(\mu - \frac{1}{a_t}\right)^2$$

De-Leon and Gazit (2018)

Magnetic interaction in $\not\pi$EFT :

A=2

- $n + p \rightarrow d + \gamma$

$$\sigma_{np} \propto (Y')^2$$

$$Y' = \sqrt{Z_d^{NLO}} \underbrace{\left(1 - \frac{1}{a_s \gamma_t}\right)}_{\mathcal{O}(0)} + \sqrt{Z_d} \left[\underbrace{-\frac{\gamma_t(\rho_s + \rho_t)}{4} + l'_1(\mu)}_{\mathcal{O}(1)} \right]$$

- $\langle \mu_d \rangle$

$$\langle \mu_d \rangle = (2\kappa_0) \left\{ Z_d^{NLO} - Z_d \left[\underbrace{\gamma_t \rho_t - l'_2(\mu)}_{\mathcal{O}(1)} \right] \right\}$$

$$Y', \langle \mu_d \rangle \approx 1$$

All calculations were done up to NLO, and we keep consistency in Z_d.

Deuteron normalization

- Deuteron normalization: $Z_d^{-1} = \frac{i\partial}{\partial p_0}\frac{1}{iD_t(p_0,p)}\Big|_{p_0=-\frac{\gamma_t}{M_N^2},p=0}$, $Z_d = \frac{1}{1-\gamma_t\rho_t}$

Up to NLO there are two alternatives to arrange the EFT expansion

- Effective range expansion (ERE) , $\gamma_t\rho_t$ is the small parameter:

$$Z_d = \frac{1}{1-\gamma_t\rho_t} = \underbrace{1}_{LO} + \underbrace{\gamma_t\rho_t}_{NLO} + \underbrace{(\gamma_t\rho_t)^2}_{N^2LO} + \underbrace{(\gamma_t\rho_t)^3}_{N^3LO}$$

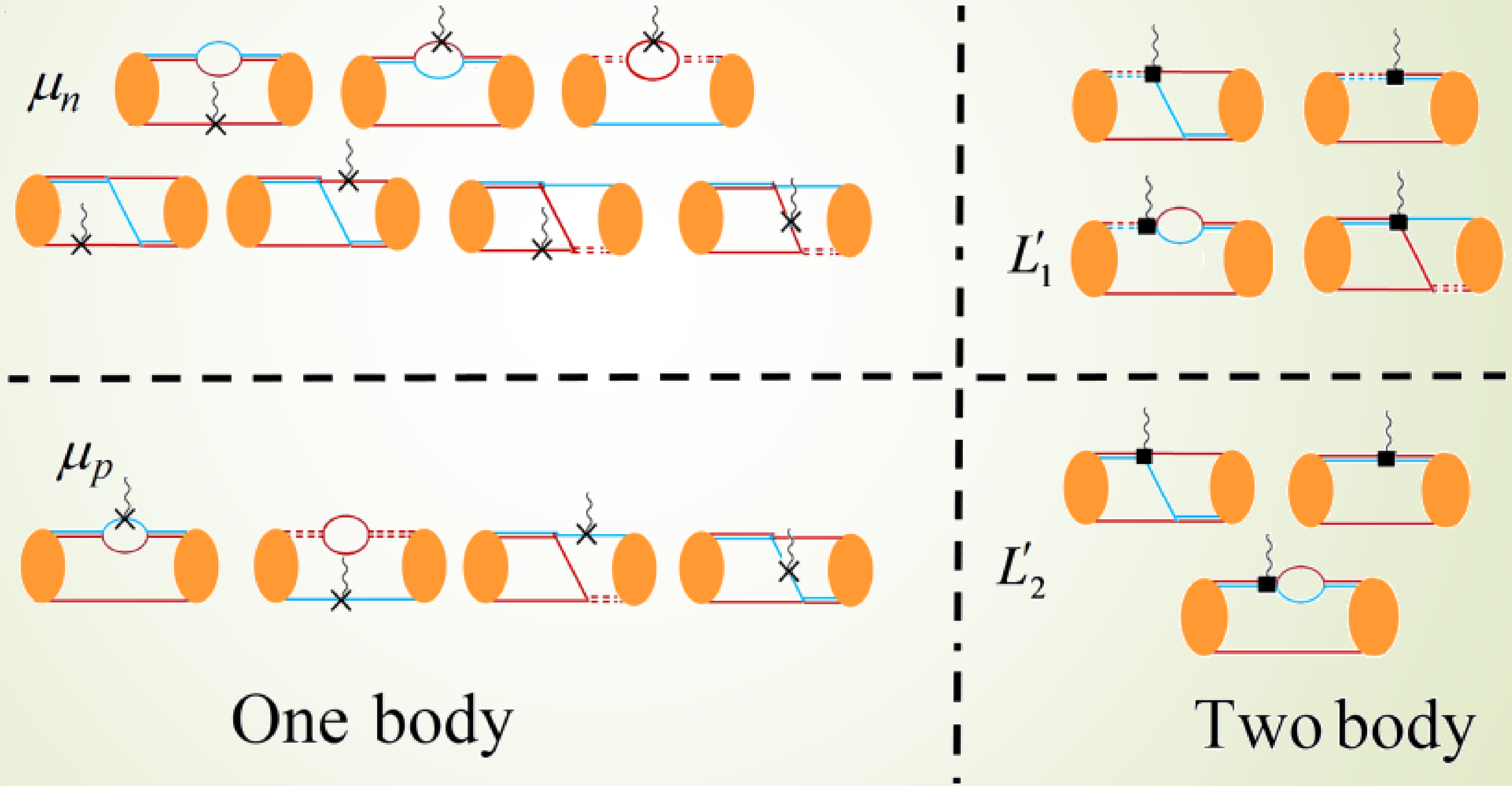
- Z-parameterization, $Z_d - 1$ is the small parameter:

$$Z_d = \frac{1}{1-\gamma_t\rho_t} = \underbrace{1}_{LO} + \underbrace{Z_d - 1}_{NLO}\ \underbrace{0}_{N^2LO} + \underbrace{0}_{N^3LO}$$

	Z_d^{LO}	Z_d^{NLO}	ρ_t^{LO}	$\rho_t^{NLO} = \frac{Z_d^{NLO}-1}{\gamma_t}$
ERE	1	1.408	0	Physical
Z	1	Physical	0	$0.69/\gamma_t$

Griesshammer (2004), Philips (2000)

A=3 magnetic moments calculations:



De-Leon and Gazit (2018)

A=3 magnetic moments calculations:

$$\langle \mu^{^3\mathrm{H}} \rangle = \frac{\langle \frac{1}{2} \| \boldsymbol{\sigma} \| \frac{1}{2} \rangle}{\sqrt{3}} \sum_{\mu,\nu} \left\langle \psi_\mu(E_{^3\mathrm{H}}, P) \middle| a^{i,j}_{\mu,\nu} \mathcal{K}^{q=0}(E,p,p') + d^{i,j}_{\mu,\nu} \mathcal{J}^{\,q=0}(E,p,p') \middle| \psi_\nu(E_{^3\mathrm{H}}, P) \right\rangle$$

-

$$-L_1' \left(\langle \psi_t(E_{^3\mathrm{H}}, P) | \psi_{np}(E_{^3\mathrm{H}}, P) \rangle + \langle \psi_{np}(E_{^3\mathrm{H}}, P) | \psi_t(E_{^3\mathrm{H}}, P) \rangle \right) + \frac{3}{2} L'_2 \langle \psi_t(E_{^3\mathrm{H}}, P) | \psi_s(E_{^3\mathrm{H}}, P) \rangle$$

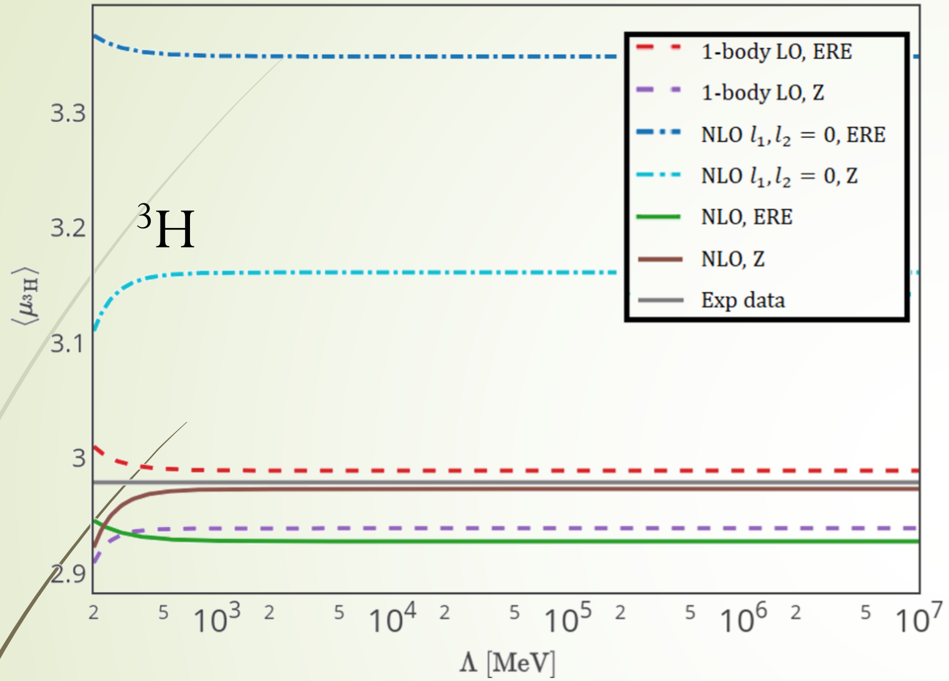
$$\langle \mu^{^3\mathrm{He}} \rangle = \frac{\langle \frac{1}{2} \| \boldsymbol{\sigma} \| \frac{1}{2} \rangle}{\sqrt{3}} \sum_{\mu,\nu} \left\langle \psi_\mu(E_{^3\mathrm{He}}, P) \middle| a'^{i,j}_{\mu,\nu} \mathcal{K}^{q=0}(E,p,p') + d'^{i,j}_{\mu,\nu} \mathcal{J}^{\,q=0}(E,p,p') \middle| \psi_\nu(E_{^3\mathrm{He}}, P) \right\rangle$$

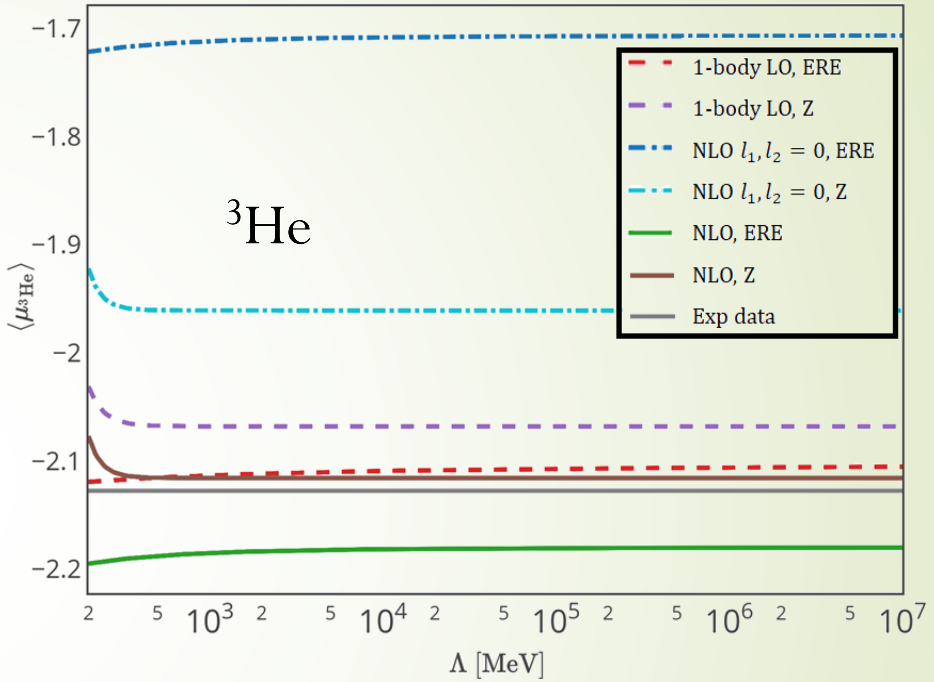
-

$$-L_1' \left(\langle \psi_t(E_{^3\mathrm{He}}, P) | \psi_{np}(E_{^3\mathrm{He}}, P) \rangle + \langle \psi_{np}(E_{^3\mathrm{He}}, P) | \psi_t(E_{^3\mathrm{He}}, P) \rangle \right) + \frac{3}{2} L_1' \ \langle \psi_t(E_{^3\mathrm{He}}, P) | \psi_s(E_{^3\mathrm{He}}, P) \rangle$$

De-Leon and Gazit (2018)

A=3 magnetic moments calculations:



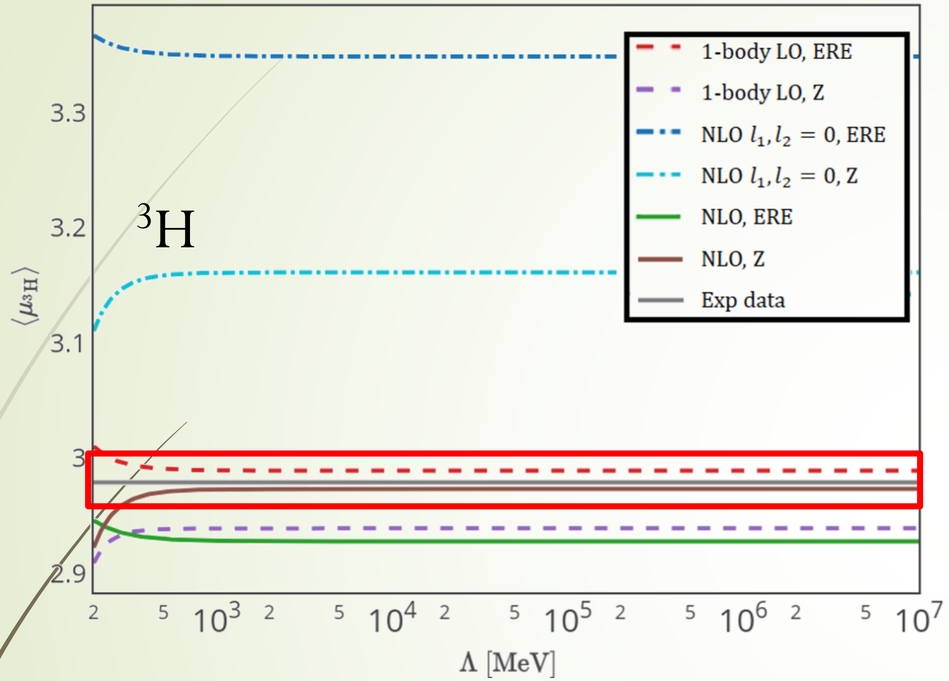


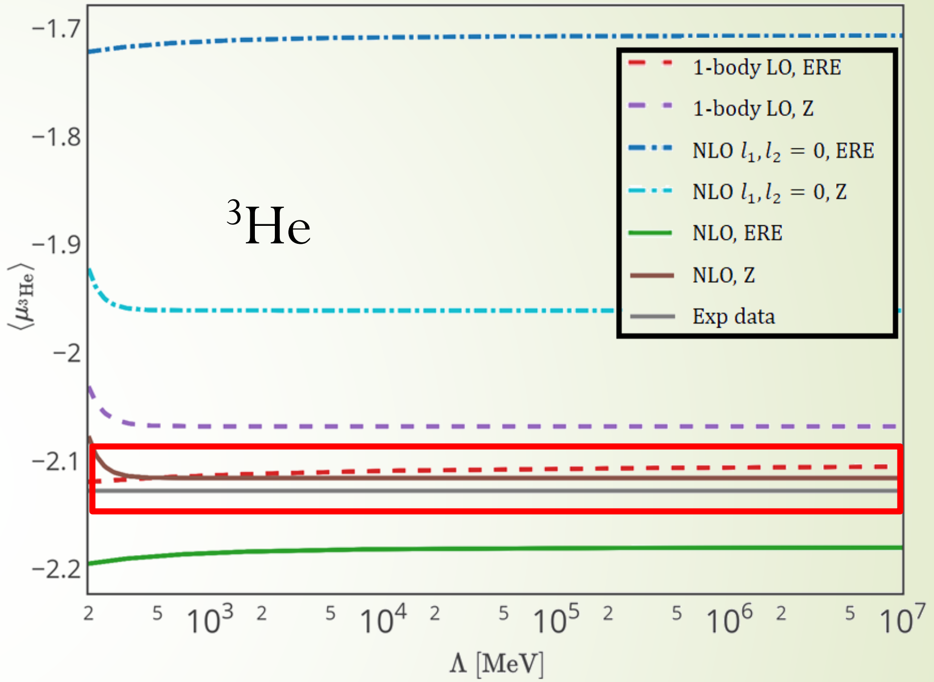
- **Cutoff independence.**
- When l'_1 and $l'2$ are fixed from **A=2 observables**:

	$\mu^{NLO}_{^3\mathrm{H}}$	$\mu^{NLO}_{^3\mathrm{He}}$
NLO	2.97 (2.92)	-2.11 (−2.18)
EXP	2.9789	−2.1276

De-Leon and Gazit (2018)

A=3 magnetic moments calculations:





Z – parameterization gives better predictions

	$\mu^{NLO}_{^3\mathrm{H}}$	$\mu^{NLO}_{^3\mathrm{He}}$
NLO	2.97 (2.92)	-2.11 (−2.18)
EXP	2.9789	−2.1276

De-Leon and Gazit (2018)

A=3 magnetic moments calculations:

- l'_1 and l'_2 are fixed from **A=3** observables **simultaneously**:

	l'_1	l'_2
ERE	$9.81 \cdot 10^{-2}$	$15.2 \cdot 10^{-2}$
Z	$3.91 \cdot 10^{-2}$	$-2.12 \cdot 10^{-2}$

- l'_1 and l'_2 are fixed from **A=2** observables:

	l'_1	l'_2
ERE	$8.18 \cdot 10^{-2}$	$-2.25 \cdot 10^{-2}$
Z	$3.86 \cdot 10^{-2}$	-2.25 $\cdot 10^{-2}$

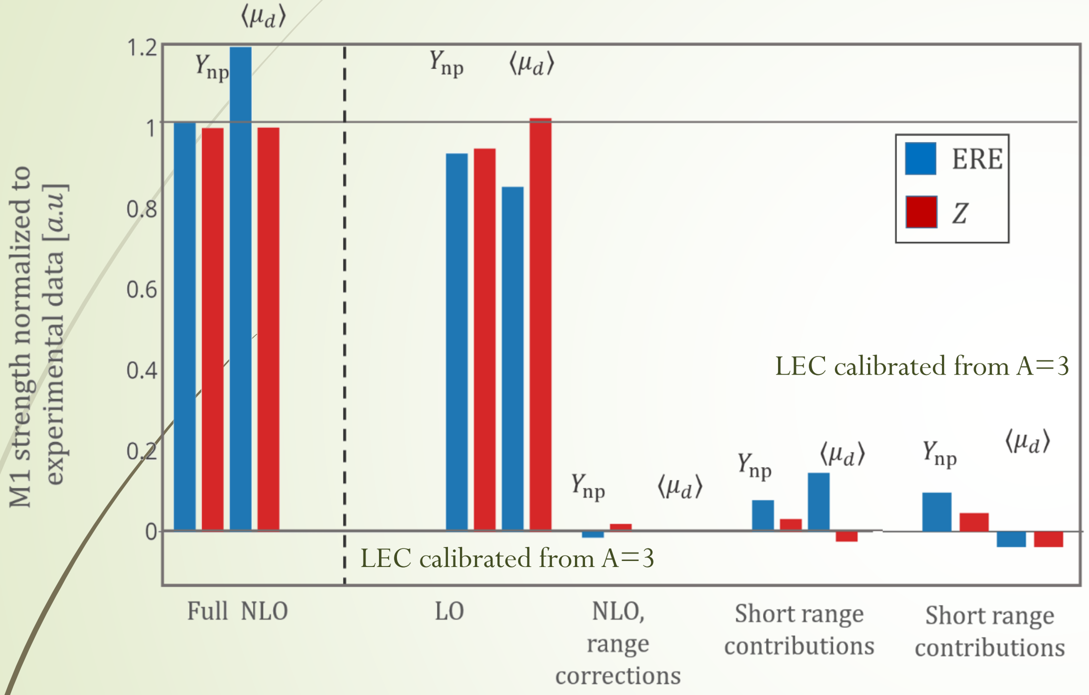
- When l'_1 and l'_2 are fixed from **A=3** observables **simultaneously**:

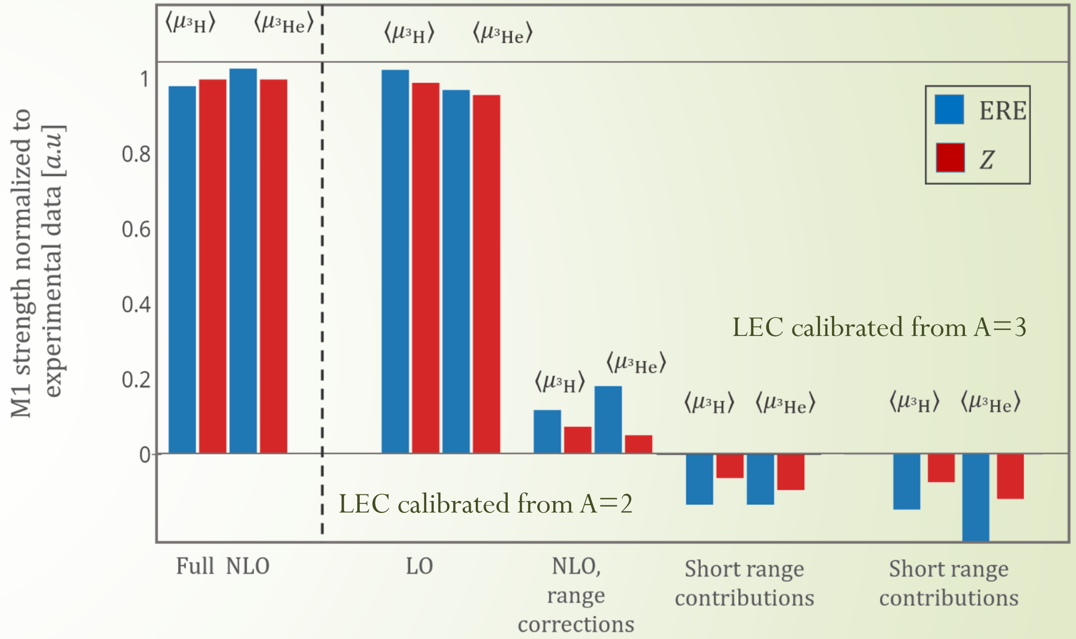
	Y'_{np}	$\langle \mu_d \rangle$
ERE	1.2613	1.02
Z	1.2455	0.8587
Exp	1.2450 ± 0.0019	0.8574

***Z* – parameterization gives better predictions.**

De-Leon and Gazit (2018)

Electromagnetic as study case: theoretical uncertainty





Fixing LECs from A=2 or from A=3 leads to the same result – consistent "measurements".

- Small NLO contributions for Z-parameterization

De-Leon and Gazit (2018)

Electromagnetic as study case: theoretical uncertainty

- For Z-parameterization, similar small NLO contributions.
- Post-dictions accurate to <1% for Z-parameterizations.
- All observables are consistent with each other in the Z-parameterization.
- ERE parameterization postdictions of A=2 and A=3 inconsistent at NLO.
- We can estimate the theoretical uncertainty $< 1\%$

De-Leon and Gazit (2018)

Summary and outlook

- B.S. normalization is equivalent to all possible connections between two amplitudes with identity insertion.
- Summing over all the one-and-two body photon exchange diagrams perturbatively yields the energy difference between ^3He and ^3H. This implies that the Coulomb interaction can be treated perturbatively
- At NLO, we proved that a consistent diagrammatic expansion is just the sum of all possible diagrams with a single NLO perturbation insertion.

Summary and outlook

- The small NLO contribution leads to a minor breaking of the SU(4) symmetry (for the Z−parameterization $\rho_t \sim \rho_s$).
- $\not{\pi}$EFT is consistent for the A=2↔3 transitions, for the Z−parameterization.
- The strong qualitive analogue between the weak and electrometric operators, implies that we can assume the same consistency for the weak interactions.

A=3 magnetic moments calculations:

$$\langle \mu^{^3\mathrm{H}} \rangle = \frac{\langle \frac{1}{2} \| \boldsymbol{\sigma} \| \frac{1}{2} \rangle}{\sqrt{3}} \sum_{\mu,\nu} \left\langle \psi_\mu(E_{^3\mathrm{H}}, P) \middle| a^{i,j}_{\mu,\nu} \mathcal{K}^{q=0}(E,p,p') + d^{i,j}_{\mu,\nu} \mathcal{J}^{q=0}(E,p,p') \middle| \psi_\nu(E_{^3\mathrm{H}}, P) \right\rangle -$$

$$-L_1' \left(\langle \psi_t(E_{^3\mathrm{H}}, P) | \psi_{np}(E_{^3\mathrm{H}}, P) \rangle + \langle \psi_{np}(E_{^3\mathrm{H}}, P) | \psi_t(E_{^3\mathrm{H}}, P) \rangle \right) + \frac{3}{2} L'_2 \langle \psi_t(E_{^3\mathrm{H}}, P) | \psi_s(E_{^3\mathrm{H}}, P) \rangle$$

Magnetic Coefficients:

$$d^{i,j}_{\mu,\nu} = \begin{bmatrix} & d & np & nn \\ d & \frac{(2\mu_p + \mu_n)}{3} & (\mu_n - \mu_p) & 0 \\ np & (\mu_n - \mu_p) & \mu_n & 0 \\ nn & 0 & 0 & \mu_p \end{bmatrix}$$

$$a^{i,j}_{\mu,\nu} = \begin{bmatrix} & d & np & nn \\ d & -\left(\frac{5}{3}\mu_p - \frac{2}{3}\mu_n\right) & (\mu_\mathrm{p} + 2\mu_\mathrm{n}) & 3\mu_\mathrm{p} \\ np & \frac{2}{3}\mu_n + \frac{1}{3}\mu_p & 2\mu_n - \mu_p & -\mu_p \\ nn & 2\mu_p & -2\mu_p & 0 \end{bmatrix}$$

Normalization Coefficients:

$$d_{\mu,\nu} = \begin{bmatrix} & d & np & nn \\ d & 1 & 0 & 0 \\ np & 0 & 1 & 0 \\ nn & 0 & 0 & 1 \end{bmatrix}$$

$$a_{\mu,\nu} = \begin{bmatrix} & d & np & nn \\ d & 1 & 3 & 3 \\ np & 1 & 1 & -1 \\ nn & 2 & -2 & 0 \end{bmatrix}$$

A=3 magnetic moments calculations:

$$\langle\mu^{^3\mathrm{He}}\rangle = \frac{\left\langle\frac{1}{2}\|\boldsymbol{\sigma}\|\frac{1}{2}\right\rangle}{\sqrt{3}}\sum_{\mu,\nu}\left\langle\psi_{\mu}(E_{^3\mathrm{He}},P)\middle|a'^{i,j}_{\mu,\nu}\mathcal{K}^{q=0}(E,p,p') + d'^{i,j}_{\mu,\nu}\mathcal{J}^{\,q=0}(E,p,p')\middle|\psi_{\nu}(E_{^3\mathrm{He}},P)\right\rangle\text{-}$$

$$-L\Big(\big\langle\psi_t(E_{^3\mathrm{He}},P)\big|\psi_{np}(E_{^3\mathrm{He}},P)\big\rangle + \big\langle\psi_{np}(E_{^3\mathrm{He}},P)\big|\psi_t(E_{^3\mathrm{He}},P)\big\rangle\Big) + \frac{3}{2}l_2\langle\psi_t(E_{^3\mathrm{He}},P)|\psi_s(E_{^3\mathrm{He}},P)\rangle$$

Magnetic Coefficients:

$$d'^{i,j}_{\mu,\nu} = \begin{bmatrix} & d & np & nn \\ d & \frac{(2\mu_n+\mu_p)}{3} & (\mu_p-\mu_n) & 0 \\ np & (\mu_p-\mu_p) & \mu_p & 0 \\ nn & 0 & 0 & \mu_n \end{bmatrix}$$

$$a'^{i,j}_{\mu,\nu} = \begin{bmatrix} & d & np & nn \\ d & -\left(\frac{5}{3}\mu_n - \frac{2}{3}\mu_p\right) & (\mu_\mathrm{n}+2\mu_\mathrm{p}) & 3\mu_\mathrm{n} \\ np & \frac{2}{3}\mu_p + \frac{1}{3}\mu_n & 2\mu_p-\mu_n & -\mu_n \\ nn & 2\mu_n & -2\mu_n & 0 \end{bmatrix}$$

Normalization Coefficients:

$$d'_{\mu,\nu} = \begin{bmatrix} & d & np & nn \\ d & 1 & 0 & 0 \\ np & 0 & 1 & 0 \\ nn & 0 & 0 & 1 \end{bmatrix}$$

$$a'_{\mu,\nu} = \begin{bmatrix} & d & np & nn \\ d & 1 & 3 & 3 \\ np & 1 & 1 & -1 \\ nn & 2 & -2 & 0 \end{bmatrix}$$