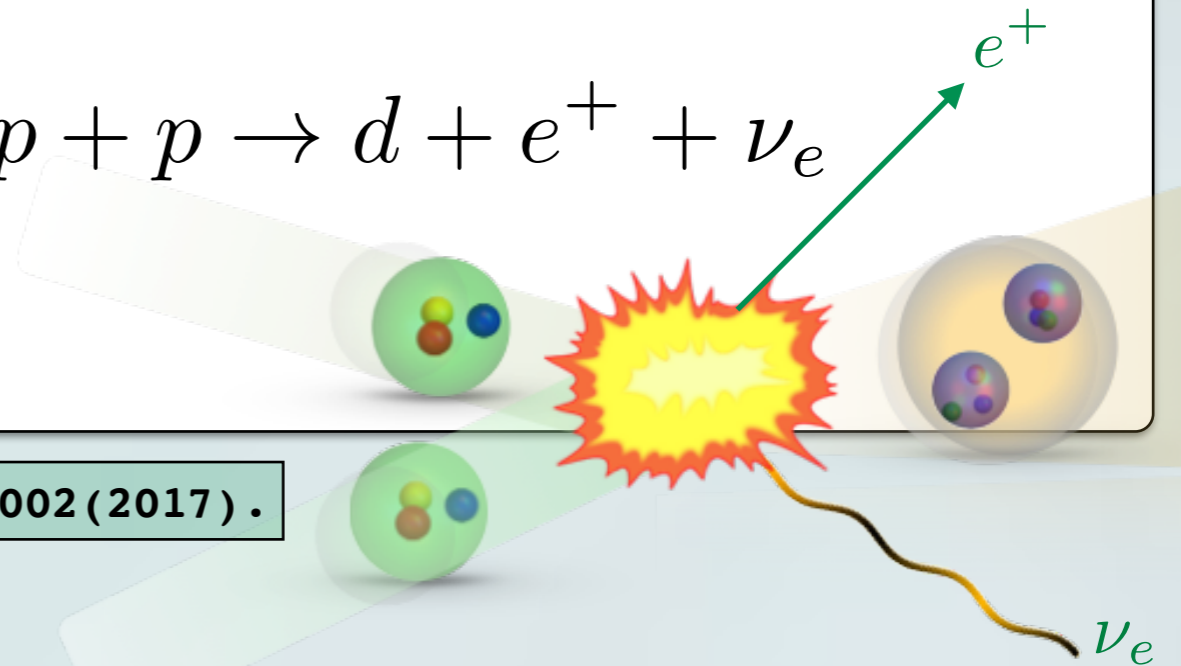
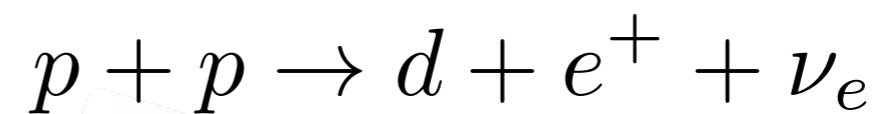
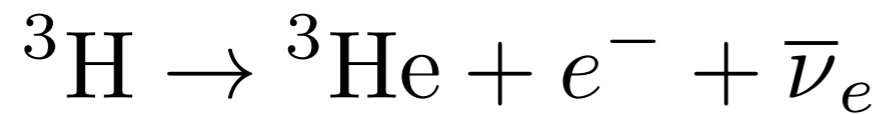


INT WORKSHOP ON FUNDAMENTAL PHYSICS WITH ELECTROWEAK PROBES OF LIGHT NUCLEI, JUNE 2018

# AXIAL PROPERTIES OF NUCLEI FROM LATTICE QCD AND EFT

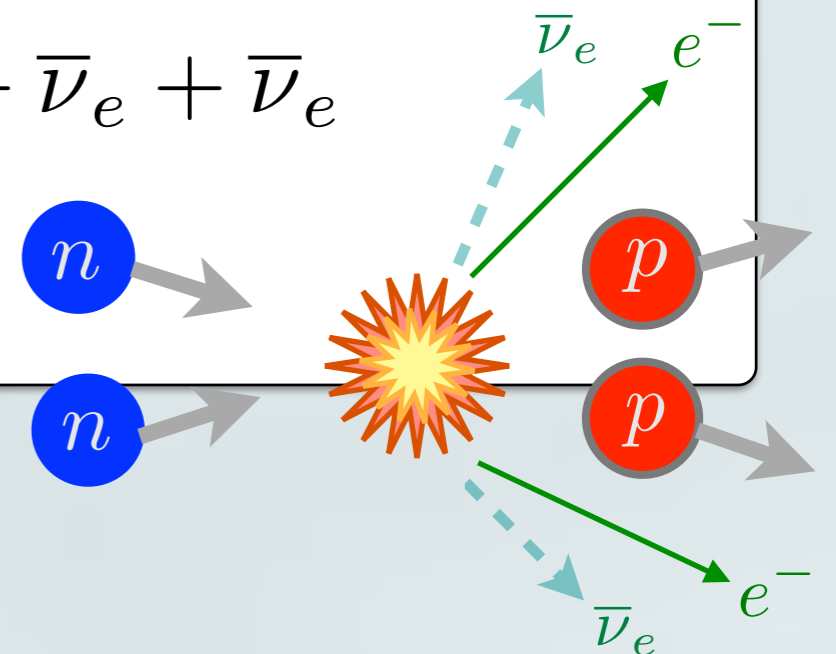
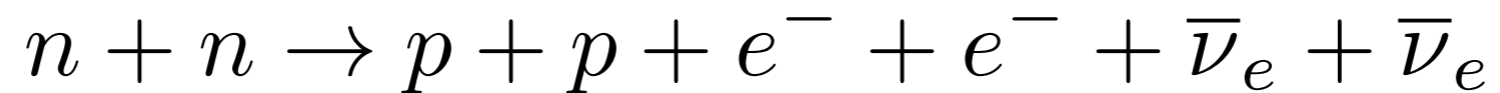
ZOHREH DAVOUDI, UNIVERSITY OF MARYLAND  
AND RIKEN FELLOW

## SINGLE-WEAK PROCESSES



Savage et al (NPLQCD), Phys.Rev.Lett.119,062002(2017).

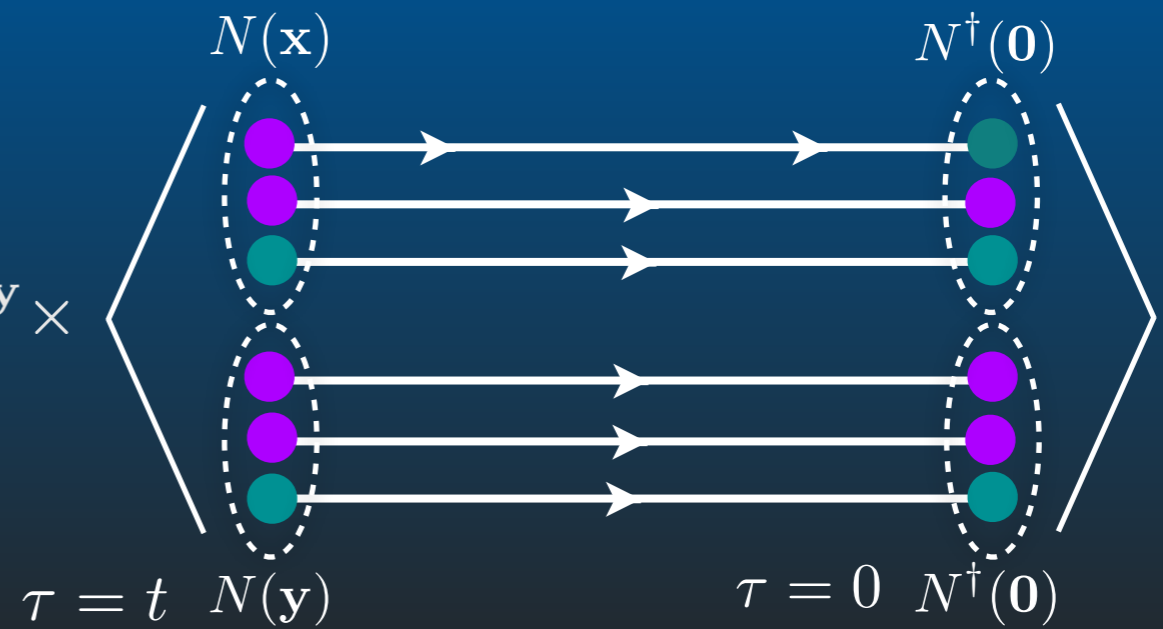
## A DOUBLY-WEAK PROCESS



Tiburzi et al (NPLQCD), Phys.Rev.D96,054505(2017),  
Shanahan et al (NPLQCD), Phys.Rev.Lett.119,062003(2017).

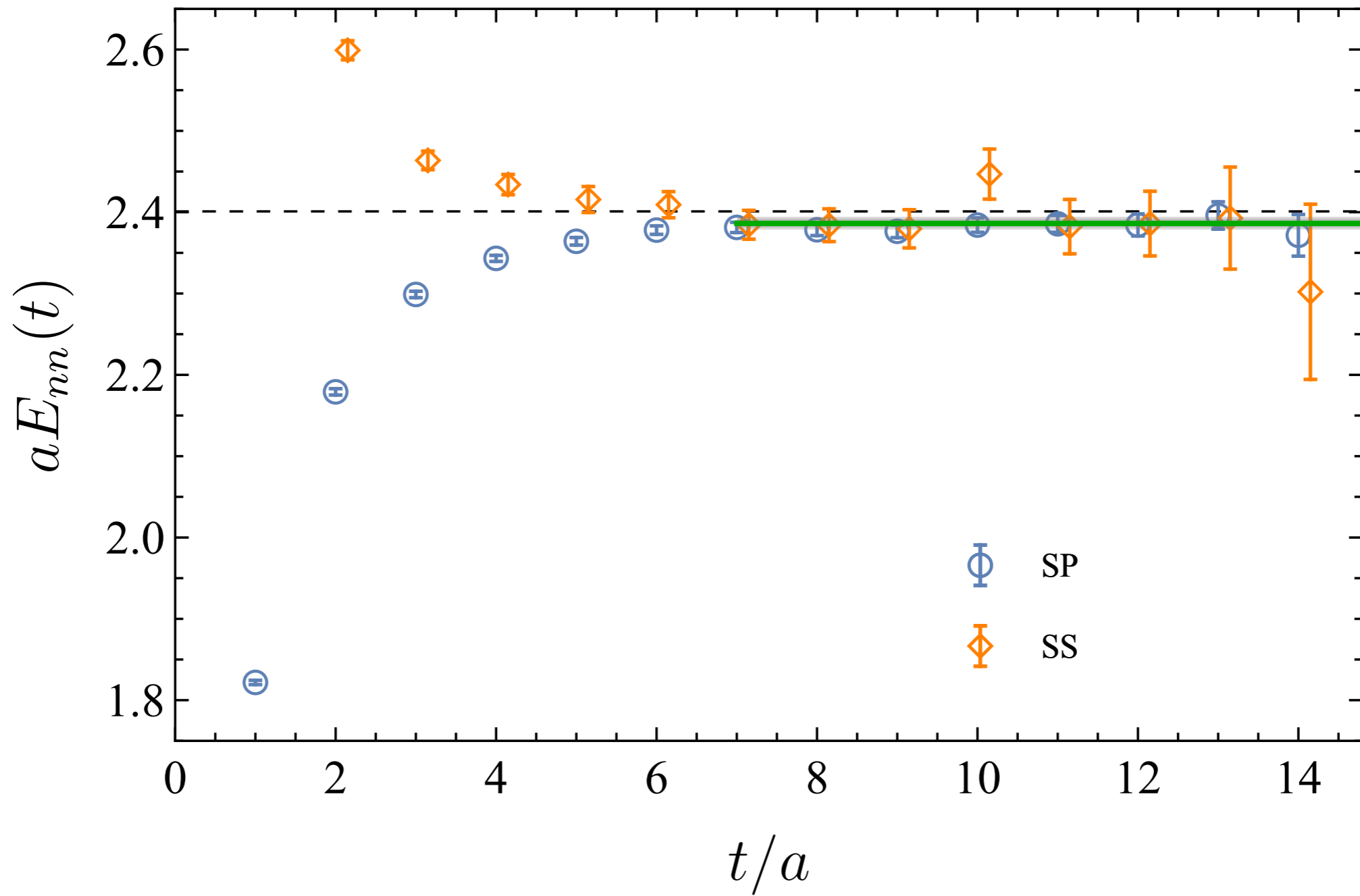
# ENERGIES FROM TWO-NUCLEON CORRELATION FUNCTIONS

$$C(\mathbf{P}; t, t_0) = \sum_{\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{P}} \sum_{\mathbf{x}, \mathbf{y}, \mathbf{z}} e^{i\mathbf{p}_1 \cdot \mathbf{x} + i\mathbf{p}_2 \cdot \mathbf{y}} \times$$



$$C_{\hat{O}, \hat{O}'}(\tau; \mathbf{d}) = \sum_{\mathbf{x}} e^{2\pi i \mathbf{d} \cdot \mathbf{x} / L} \langle 0 | \hat{O}'(\mathbf{x}, \tau) \hat{O}^\dagger(\mathbf{0}, 0) | 0 \rangle = \mathcal{Z}'_0 \mathcal{Z}_0^\dagger e^{-E^{(0)}\tau} + \mathcal{Z}'_1 \mathcal{Z}_1^\dagger e^{-E^{(1)}\tau} + \dots$$

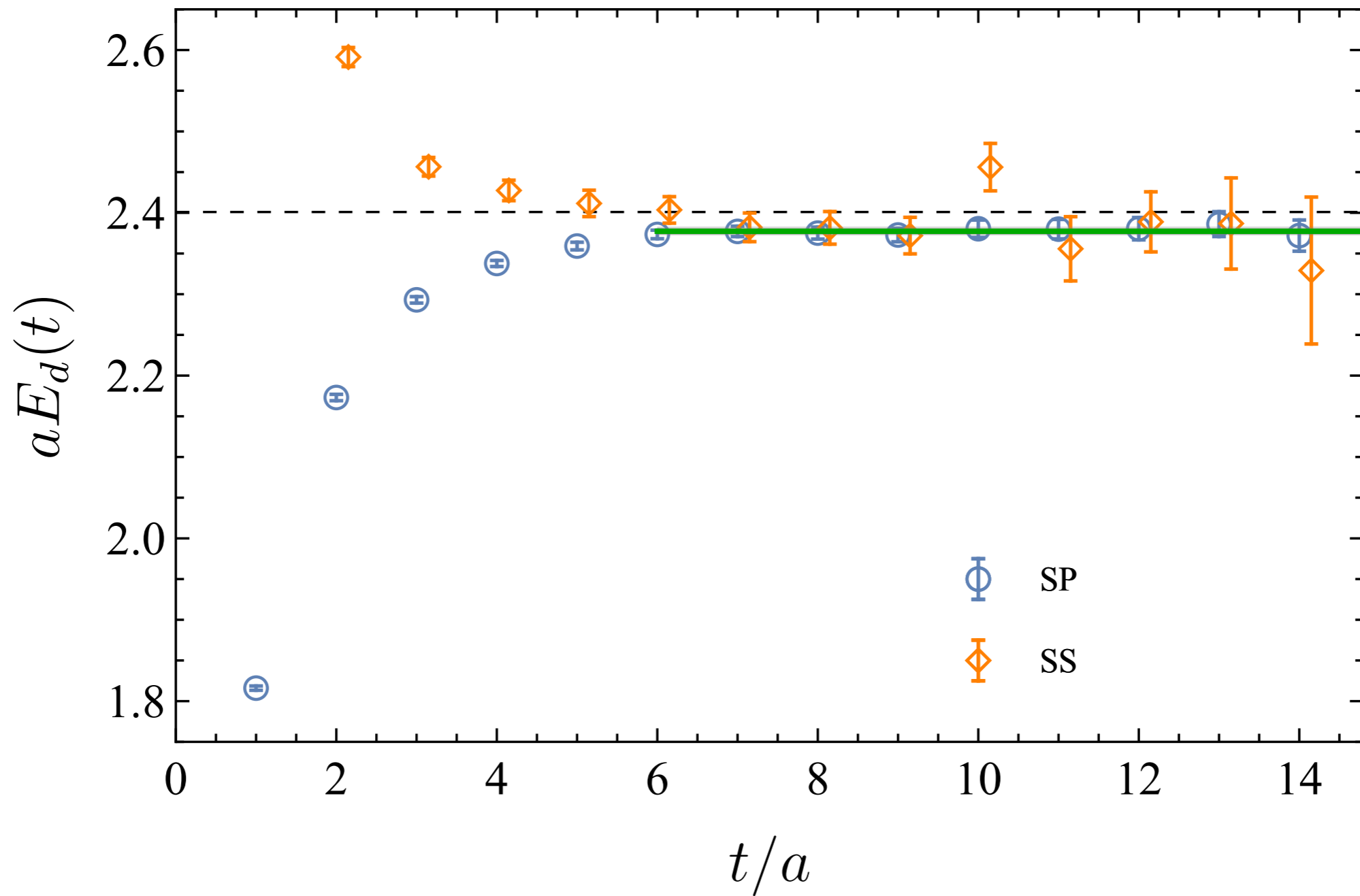
# ENERGIES FROM TWO-BARYON CORRELATION FUNCTIONS



$N_f = 3, m_\pi = 0.806 \text{ GeV}, a = 0.145(2) \text{ fm}$

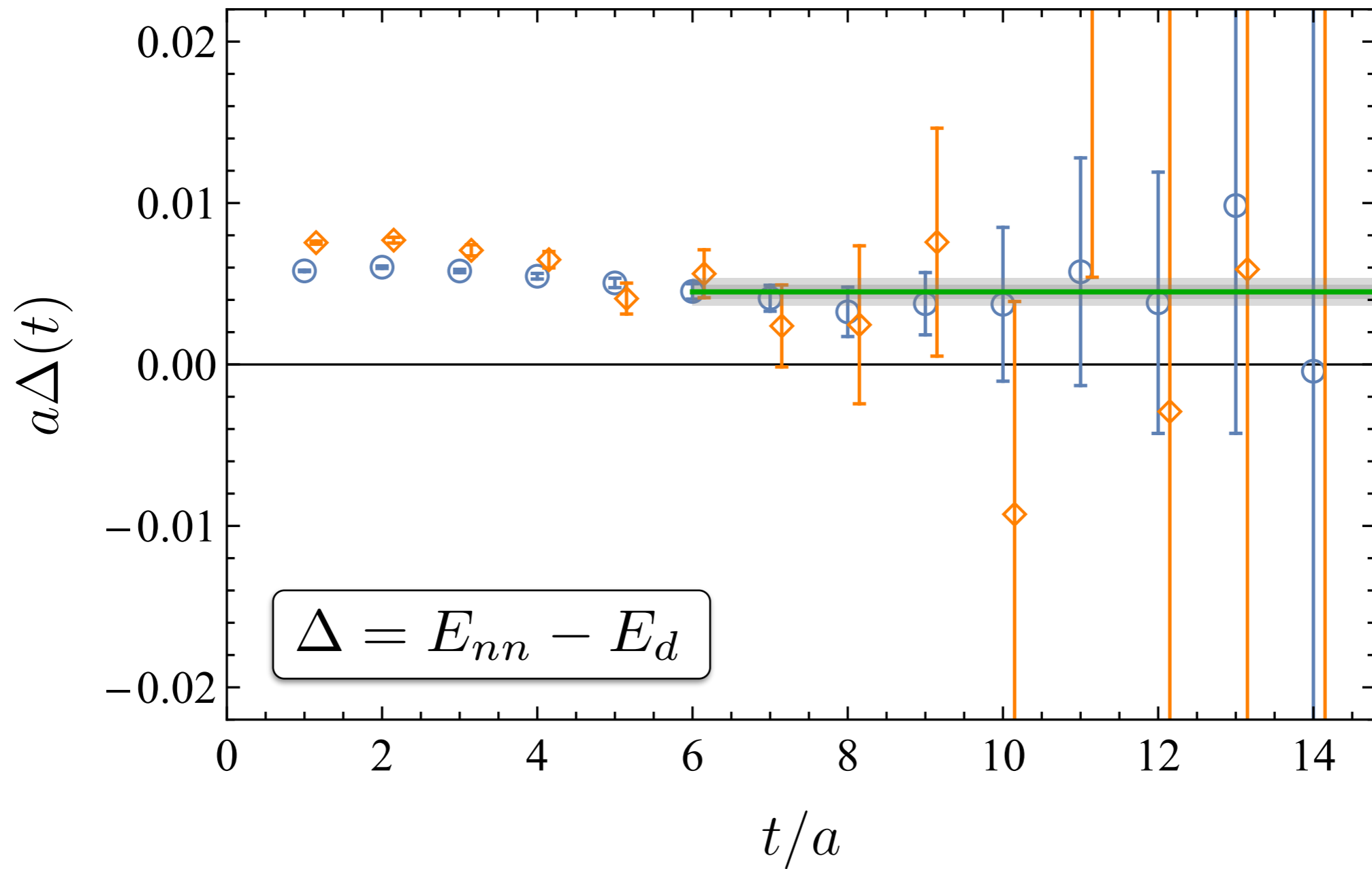


# ENERGIES FROM TWO-BARYON CORRELATION FUNCTIONS



$N_f = 3, m_\pi = 0.806 \text{ GeV}, a = 0.145(2) \text{ fm}$

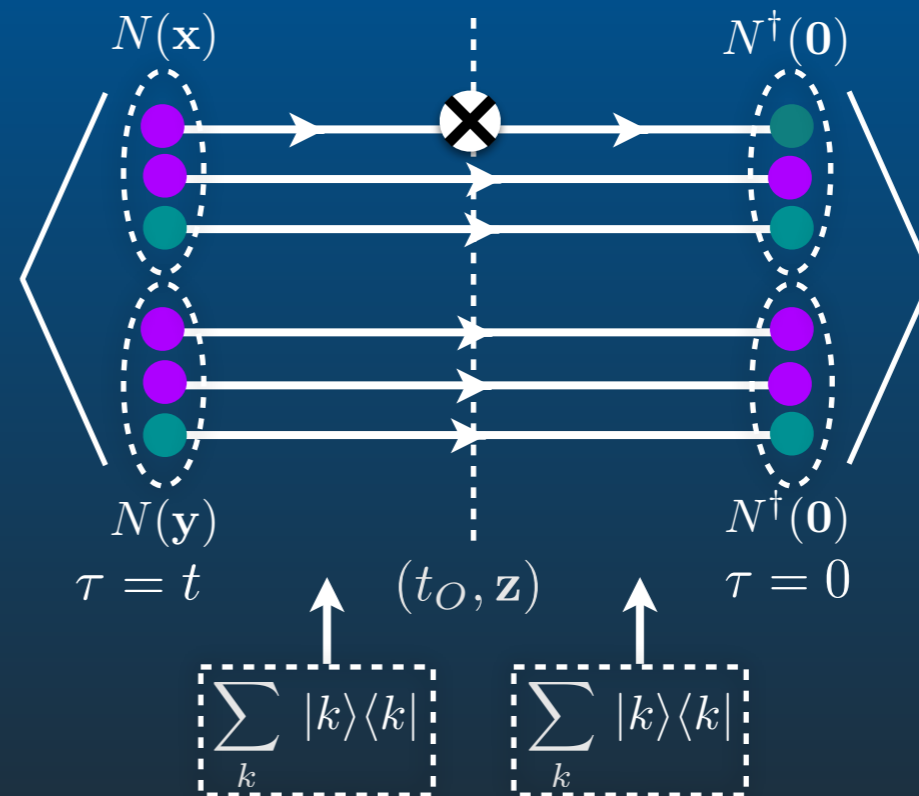
# ENERGIES FROM TWO-BARYON CORRELATION FUNCTIONS



$N_f = 3, m_\pi = 0.806 \text{ GeV}, a = 0.145(2) \text{ fm}$

# TRADITIONAL MATRIX ELEMENT CALCULATIONS: 3-POINT FUNCTIONS

$$C(\mathbf{P}; t, t_0) = \sum_{\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{P}} \sum_{\mathbf{x}, \mathbf{y}, \mathbf{z}} e^{i\mathbf{p}_1 \cdot \mathbf{x} + i\mathbf{p}_2 \cdot \mathbf{y}} \times$$



$$= Z_{0,pp}^{\text{src}} Z_{0,d}^{\text{snk}\dagger} e^{-E_{0,pp}t_0} e^{-E_{0,d}(t-t_0)} \langle pn | A | pp \rangle_L + \dots$$

# MATRIX ELEMENTS FROM A COMPOUND PROPAGATOR/BACKGROUND FIELD

$$S_{\lambda_q; \Gamma}^{(q)}(x, y) = S^{(q)}(x, y) + \lambda_q \int dz S^{(q)}(x, z) \Gamma S^{(q)}(z, y)$$

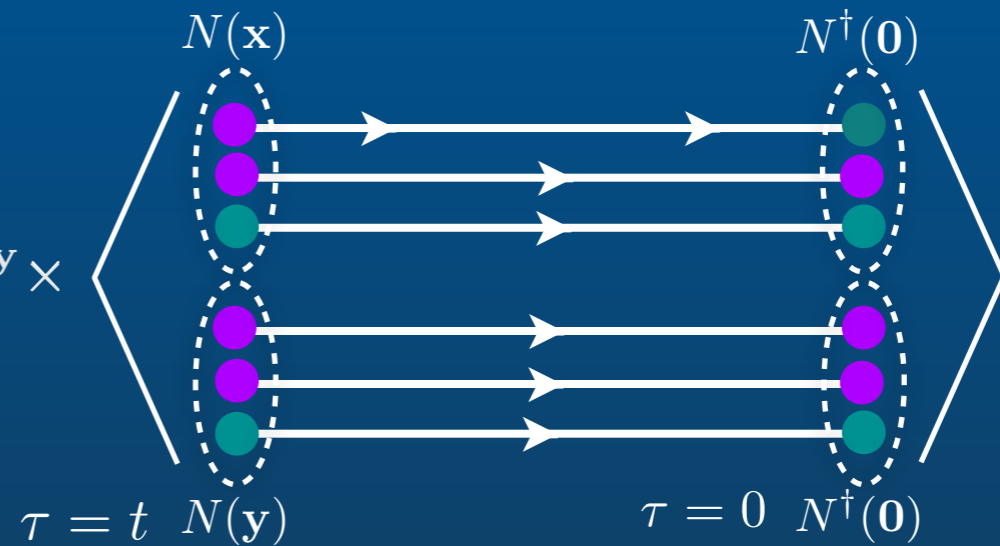


Savage et al (NPLQCD), Phys.Rev.Lett.119,062002(2017).

Buochard et al (CALLATT), Phys.Rev.D96,014504(2017).

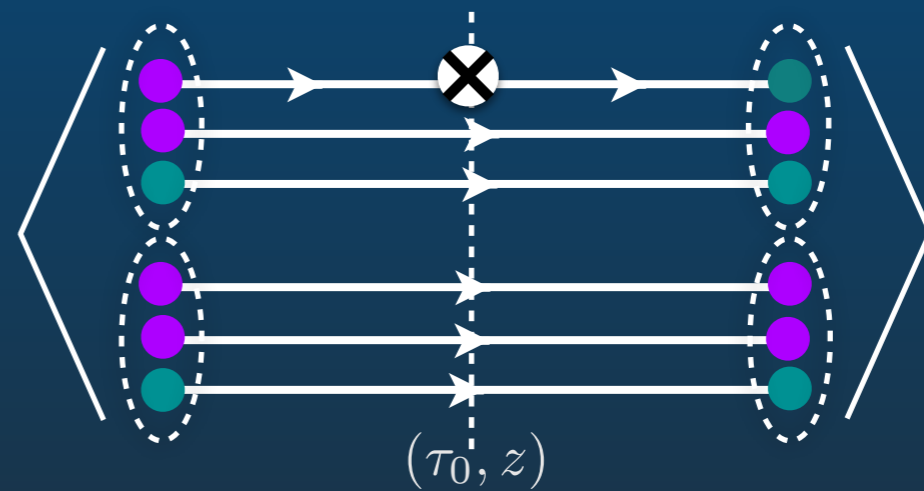
# MATRIX ELEMENTS FROM A COMPOUND PROPAGATOR/BACKGROUND FIELD

$$C_\lambda(\mathbf{P}; t, t_0) = \sum_{\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{P}} \sum_{\mathbf{x}, \mathbf{y}, \mathbf{z}} e^{i\mathbf{p}_1 \cdot \mathbf{x} + i\mathbf{p}_2 \cdot \mathbf{y}} \times$$



ALL  
POSSIBILITIES

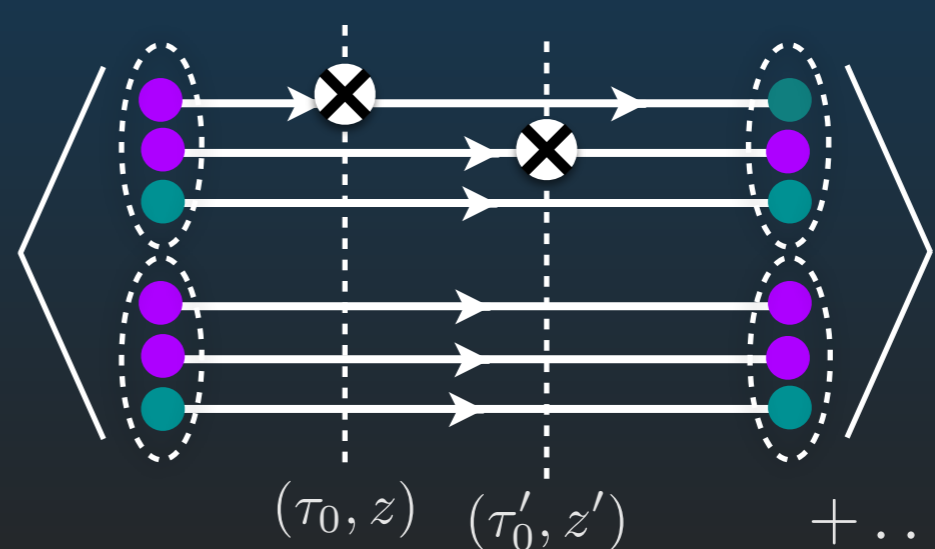
$$\longrightarrow + \lambda \sum_{\tau_0=0}^T \sum_z$$



TIME-ORDERED  
PRODUCT

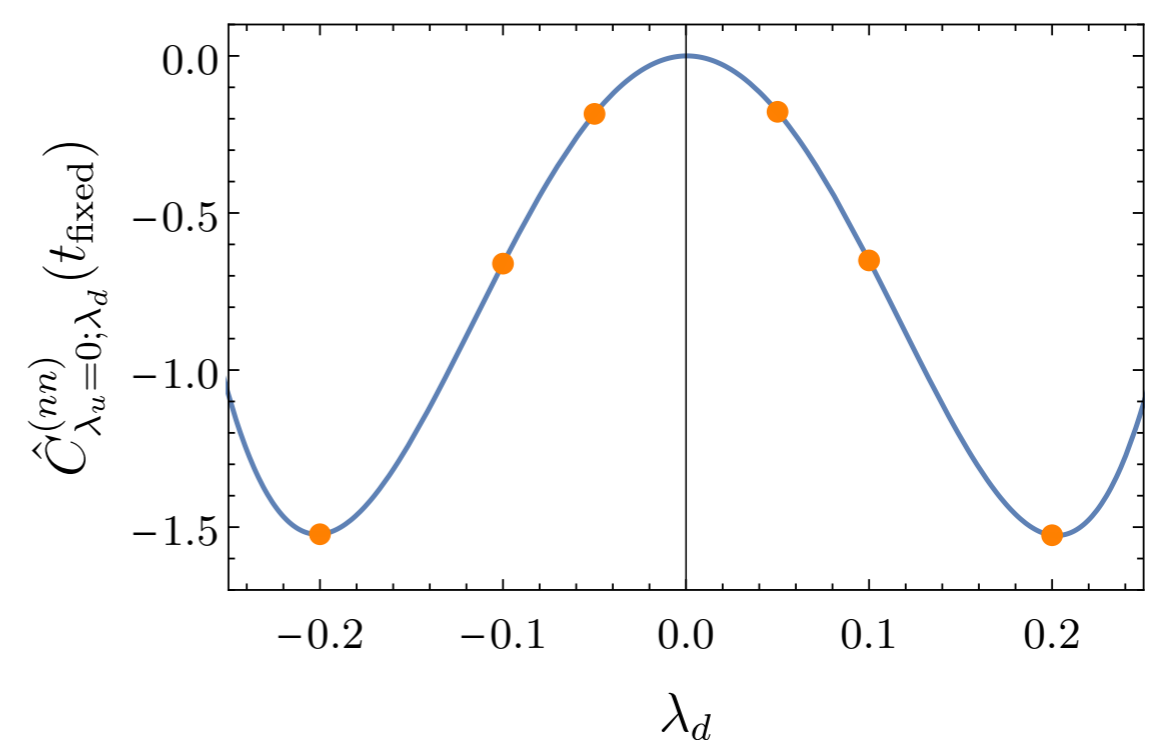
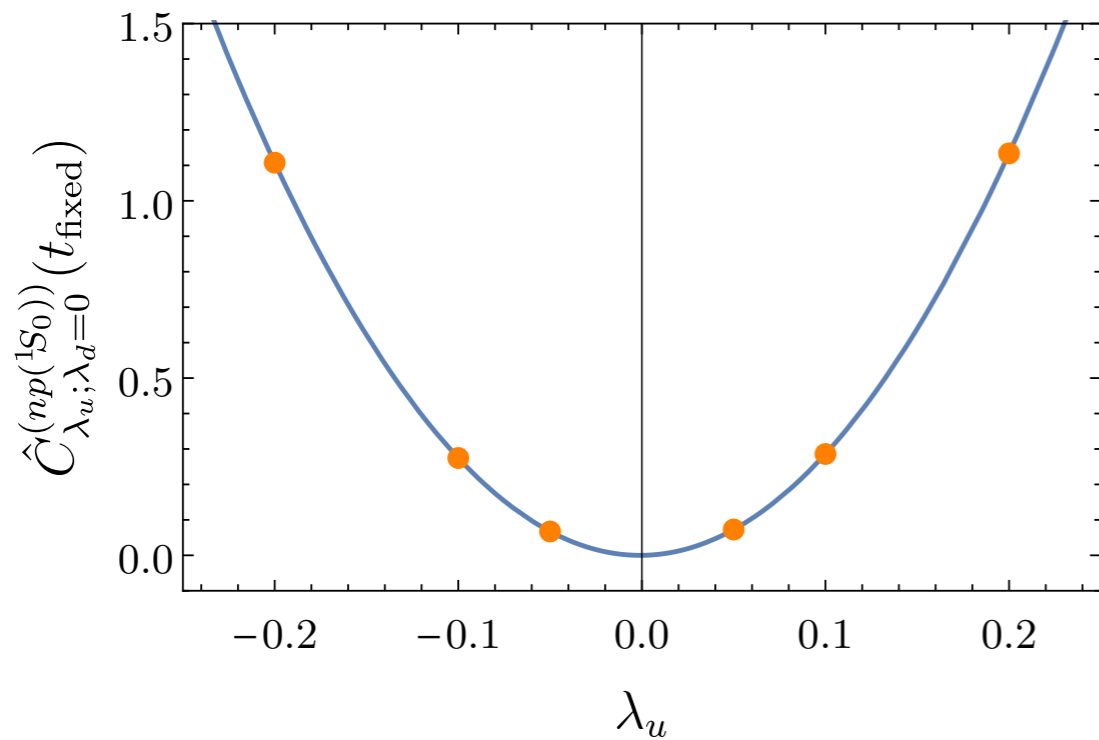
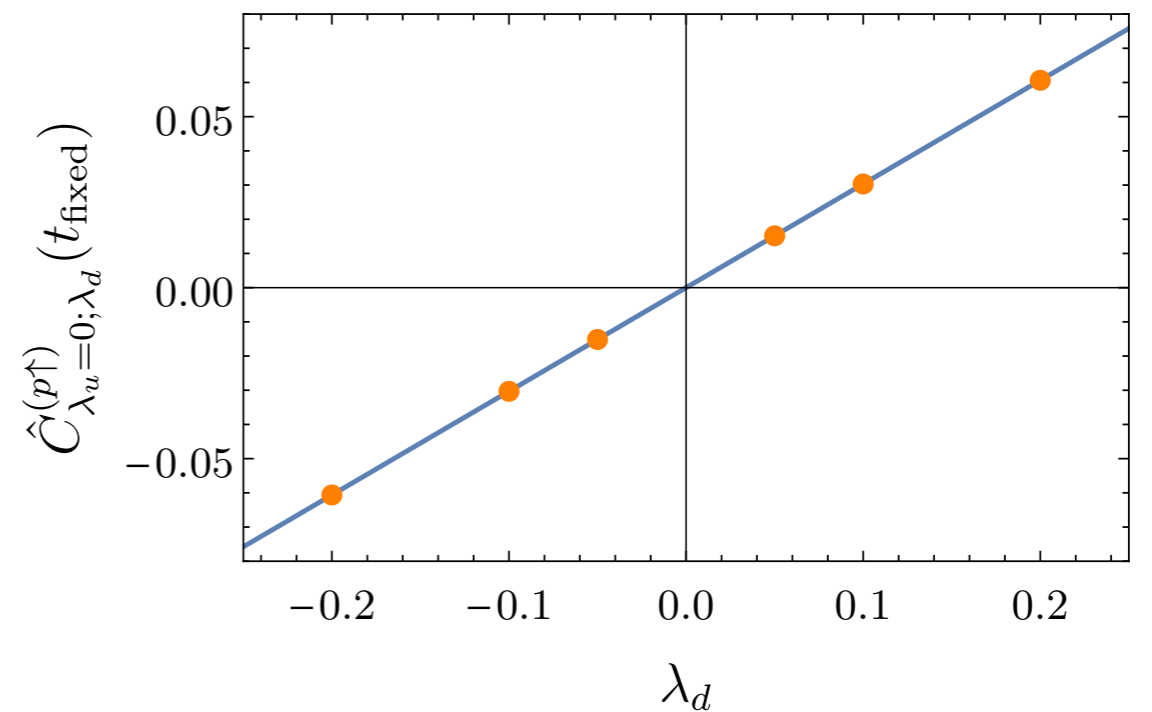
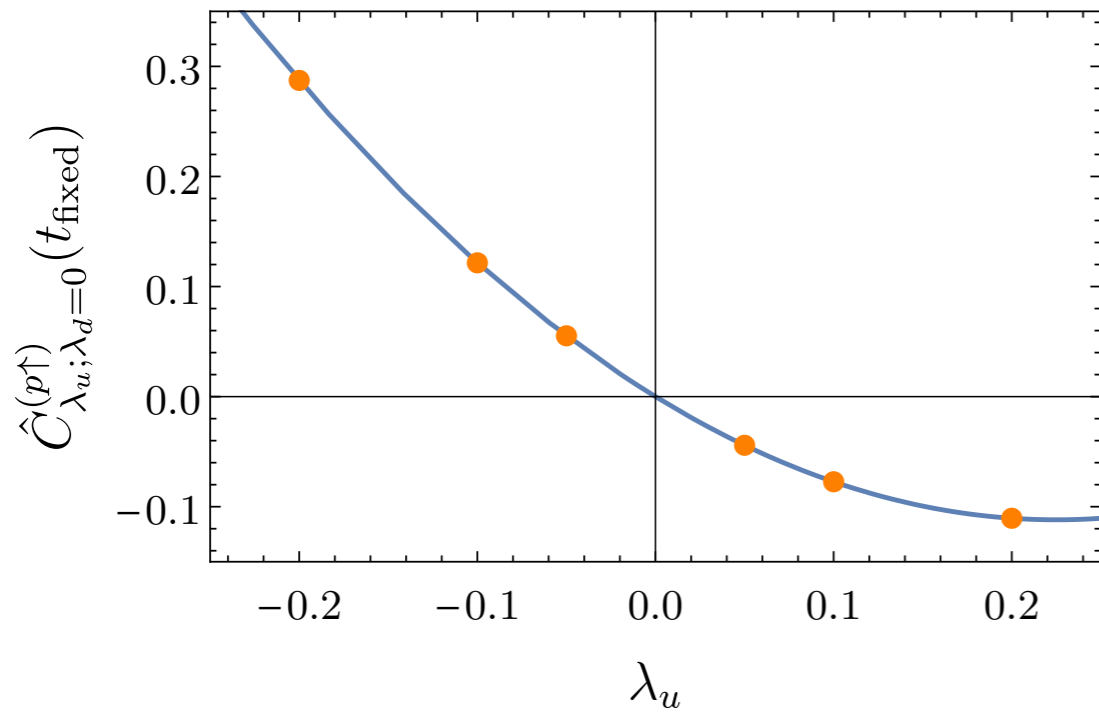
ALL  
POSSIBILITIES

$$\longrightarrow + \lambda^2 \sum_{\tau_0=0}^T \sum_{\tau'_0=0}^T \sum_z \sum_{z'}$$

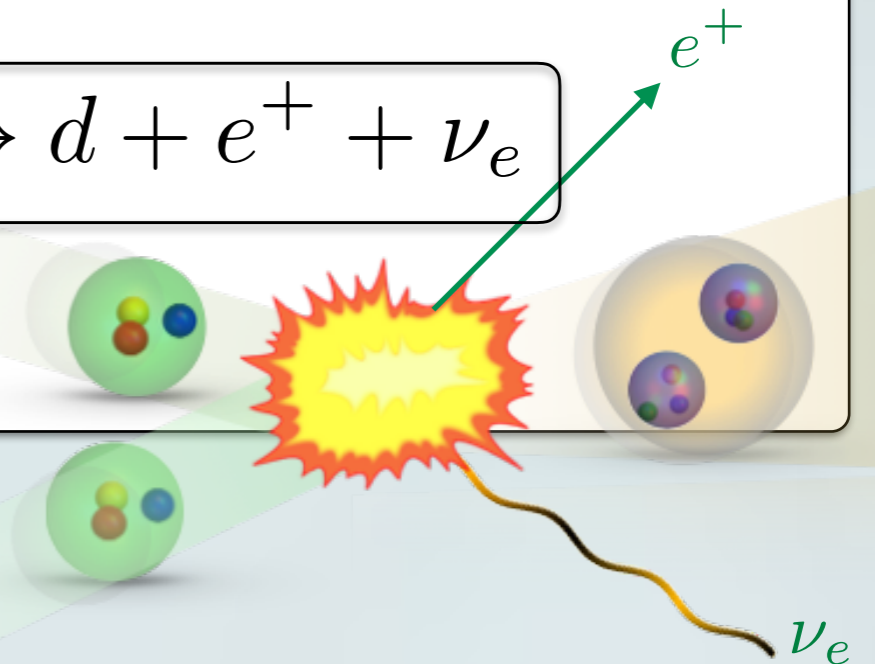
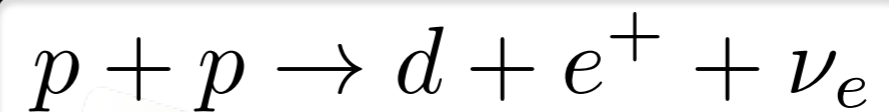
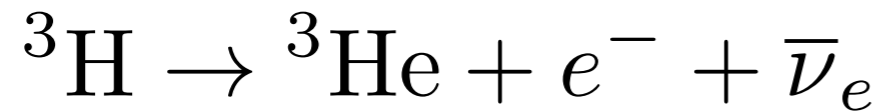


DOUBLE-CURRENT MES ARE EXACT  
FOR ISOTENSOR QUANTITIES.

# MATRIX ELEMENTS FROM A COMPOUND PROPAGATOR/BACKGROUND FIELD



## SINGLE-WEAK PROCESSES

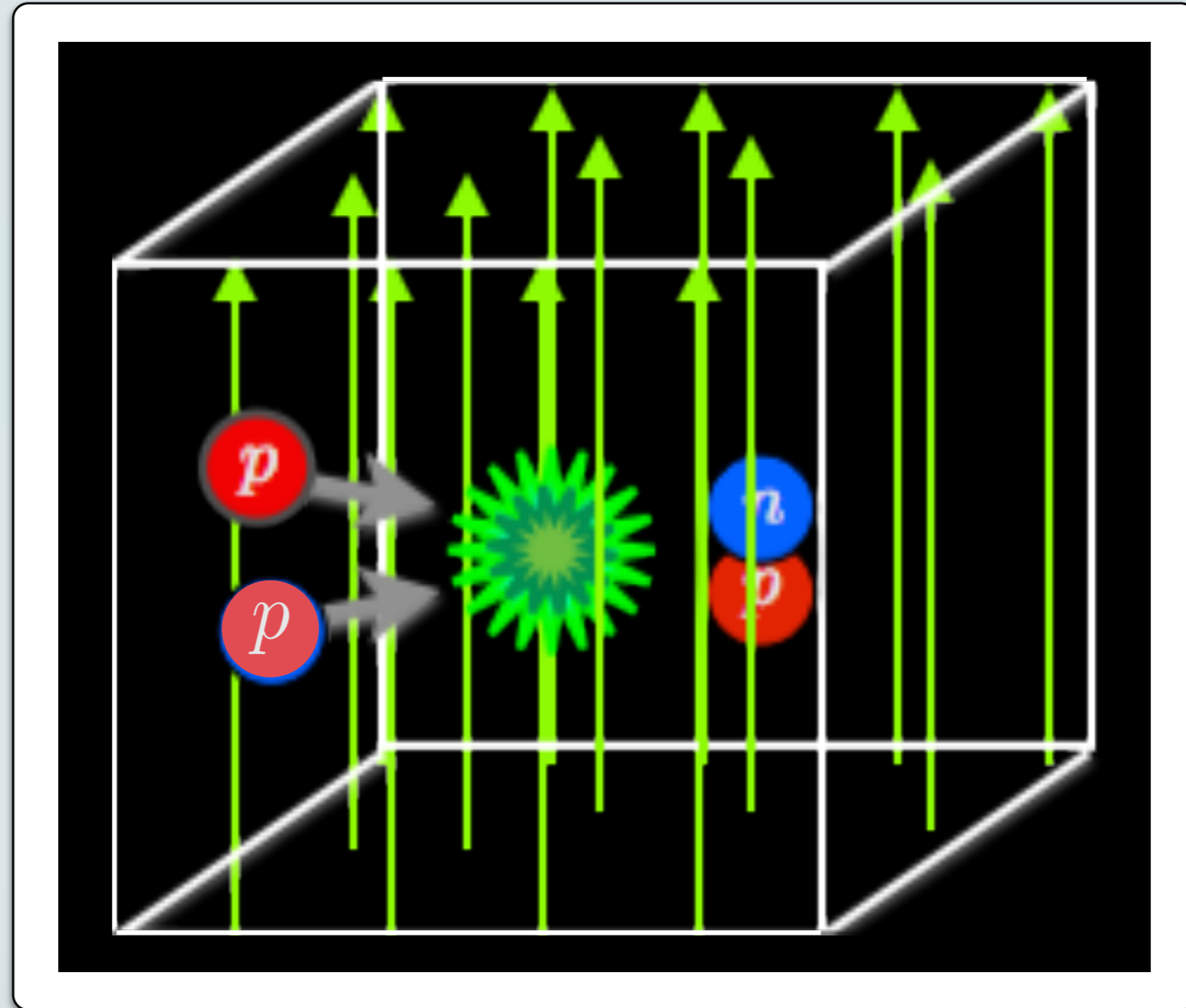


Savage et al (NPLQCD), Phys.Rev.Lett.119,062002(2017).

PRIMARY REACTION IN THE  $pp$  CHAIN THAT POWERS SUN.  
UNCERTAINTIES LARGE AT LOW INCIDENT VELOCITIES  
RELEVANT TO ENERGY PRODUCTION IN SUN.

Savage et al (NPLQCD), Phys.Rev.Lett.119,062002(2017).

# FIRST-ORDER RESPONSE TO AN AXIAL BACKGROUND FIELD

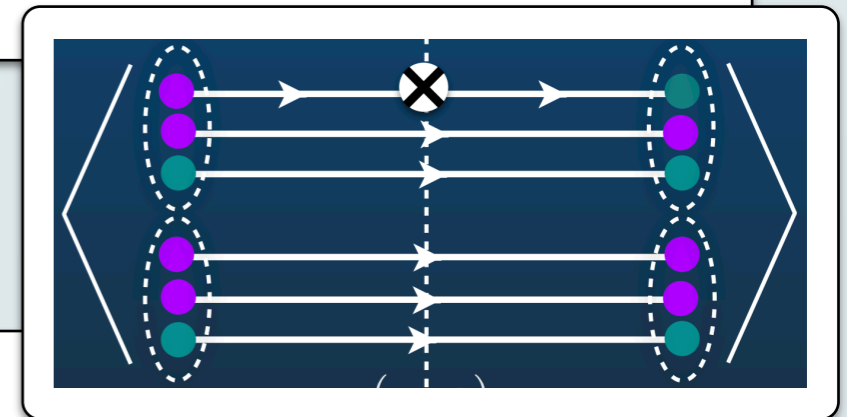




# FIRST-ORDER RESPONSE TO AN AXIAL BACKGROUND FIELD

$$C_{\lambda_u; \lambda_d=0}^{(3S_1, 1S_0)}(t) = \lambda_u \sum_{t_1=0}^t \sum_{\mathbf{x}, \mathbf{y}} \langle 0 | \chi_{3S_1}(\mathbf{x}, t) J_3^{(u)}(\mathbf{y}, t_1) \chi_{1S_0}^\dagger(0) | 0 \rangle + c_2 \lambda_u^2 + c_3 \lambda_u^3$$

$$C_{\lambda_u=0; \lambda_d}^{(3S_1, 1S_0)}(t) = \lambda_d \sum_{t_1=0}^t \sum_{\mathbf{x}, \mathbf{y}} \langle 0 | \chi_{3S_1}(\mathbf{x}, t) J_3^{(d)}(\mathbf{y}, t_1) \chi_{1S_0}^\dagger(0) | 0 \rangle + b_2 \lambda_d^2 + b_3 \lambda_d^3,$$



$$C_{\lambda_u; \lambda_d=0}^{(3S_1, 1S_0)}(t) \Big|_{\mathcal{O}(\lambda_u)} = \underbrace{Z_d Z_{np(1S_0)}^\dagger}_{\text{OVERLAP FACTORS}} e^{-\bar{E}t} \left[ \sinh\left(\frac{\Delta t}{2}\right) \left\{ \frac{\langle d | \tilde{J}_3^{(u)} | np(1S_0) \rangle}{a\Delta/2} + c_- \right\} + \cosh\left(\frac{\Delta t}{2}\right) c_+ + \mathcal{O}(e^{-\tilde{\delta}t}) \right]$$

DINEUTRON-DEUTERON MASS DIFFERENCE
GROUND-STATE MATRIX ELEMENT

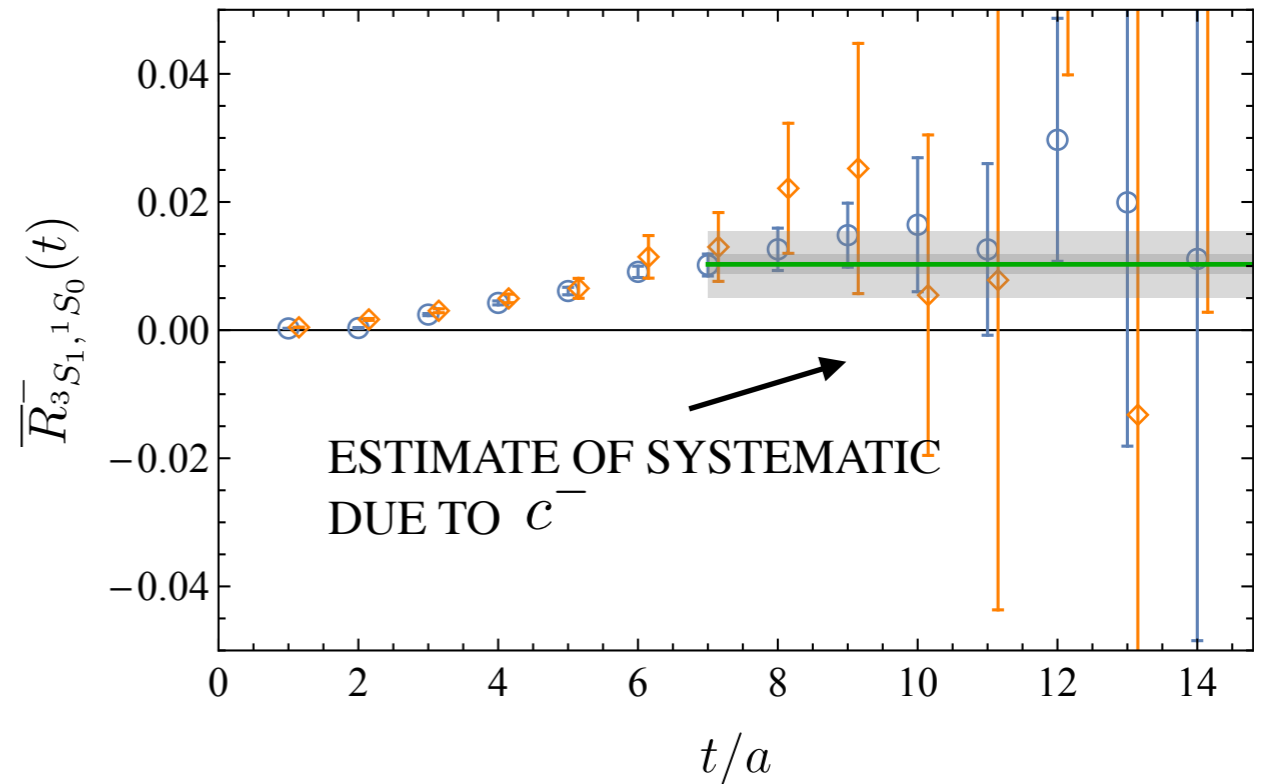
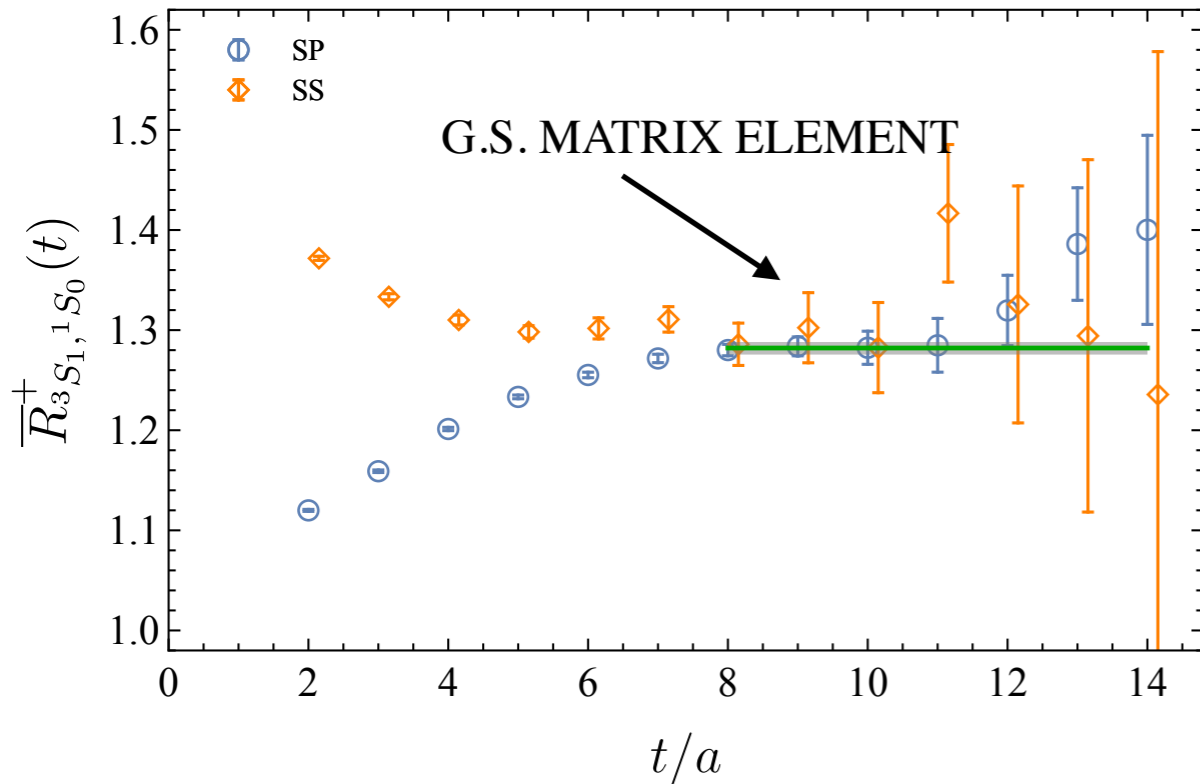
DEPEND ON EXCITED-STATES

SINCE  $a\Delta < 0.01$ , WE ARE ABLE TO FIT THE GROUND-STATE ME UP TO A SMALL SYSTEMATICS.

# MATRIX ELEMENT FROM QCD

$N_f = 3, m_\pi = 0.806 \text{ GeV}, a = 0.145(2) \text{ fm}$

Savage et al (NPLQCD), arXiv:1610.04545.



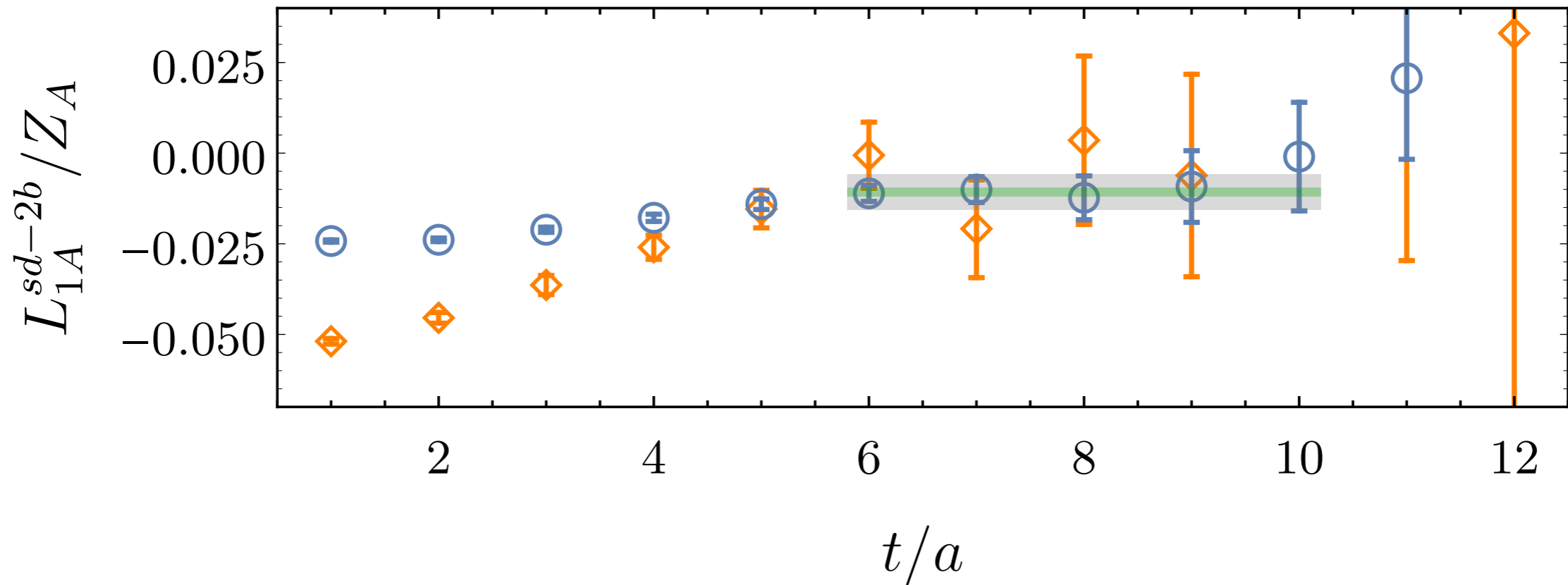
AVERAGE AND DIFFERENCE OF CORRELATOR  
AND TIME-REVERSED CORREALTOR

$$R_{3S_1, 1S_0}^\pm(t) = \frac{1}{2} \frac{C_{\lambda_u; \lambda_d=0}^\pm(t) \Big|_{\mathcal{O}(\lambda_u)} - C_{\lambda_u=0; \lambda_d}^\pm(t) \Big|_{\mathcal{O}(\lambda_d)}}{\sqrt{C_{0;0}^{(3S_1)}(t) C_{0;0}^{(1S_0)}(t)}}$$

# MATRIX ELEMENT FROM QCD

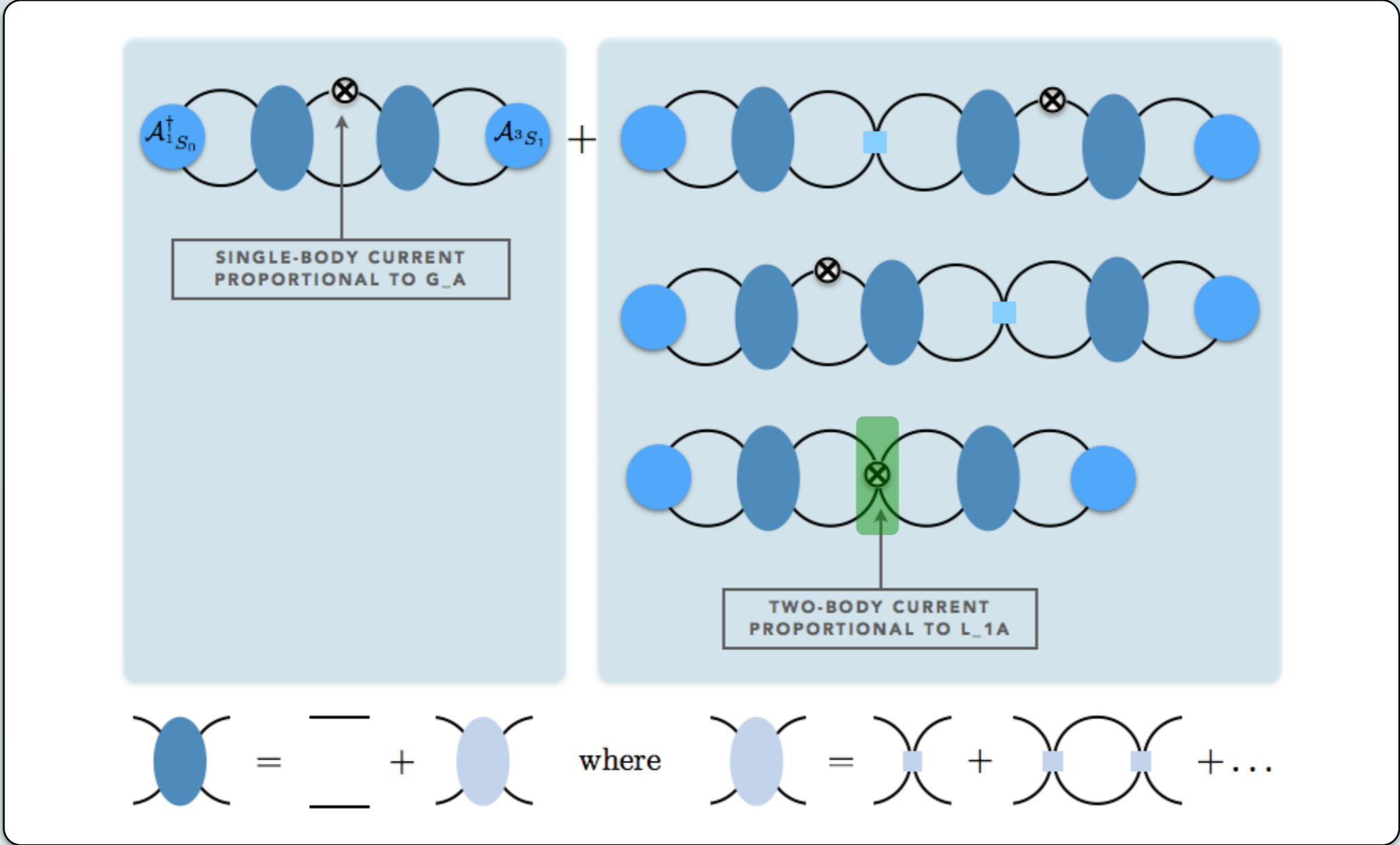
$N_f = 3, m_\pi = 0.806 \text{ GeV}, a = 0.145(2) \text{ fm}$

Savage et al (NPLQCD), arXiv:1610.04545.



$$L_{1,A}^{sd-2b} \equiv \frac{|\langle pp | A_3^+ | d \rangle| - g_A}{Z_A} = -0.011(01)(15)$$

# MATRIX ELEMENT FROM EFT



Detmold and Savage, Nucl.Phys. A743 (2004) 170.  
 Briceno and ZD, Phys. Rev. D 88, 094507 (2013).

Chen et al, Phys.Rev. C67 (2003) 025801.

$$|\langle d; j | A_k^- | pp \rangle| \equiv g_A C_\eta \sqrt{\frac{32\pi}{\gamma^3}} \Lambda(p) \delta_{jk}$$

$$\Lambda(0) = \frac{1}{\sqrt{1-\gamma\rho}} \{ e^\chi - \gamma a_{pp} [1 - \chi e^\chi \Gamma(0, \chi)] + \frac{1}{2} \gamma^2 a_{pp} \sqrt{r_1 \rho} \} - \frac{1}{2g_A} \gamma a_{pp} \sqrt{1-\gamma\rho} L_{1,A}^{sd-2b}$$

## TWO-NUCLEON SHORT-DISTANCE COUPLING

FROM TRITON LIFETIME:

$$L_{1,A} \approx 2.0(2.4) \text{ fm}^3 @ \mu = m_{\pi}^{\text{phys.}} = 140 \text{ MeV}$$

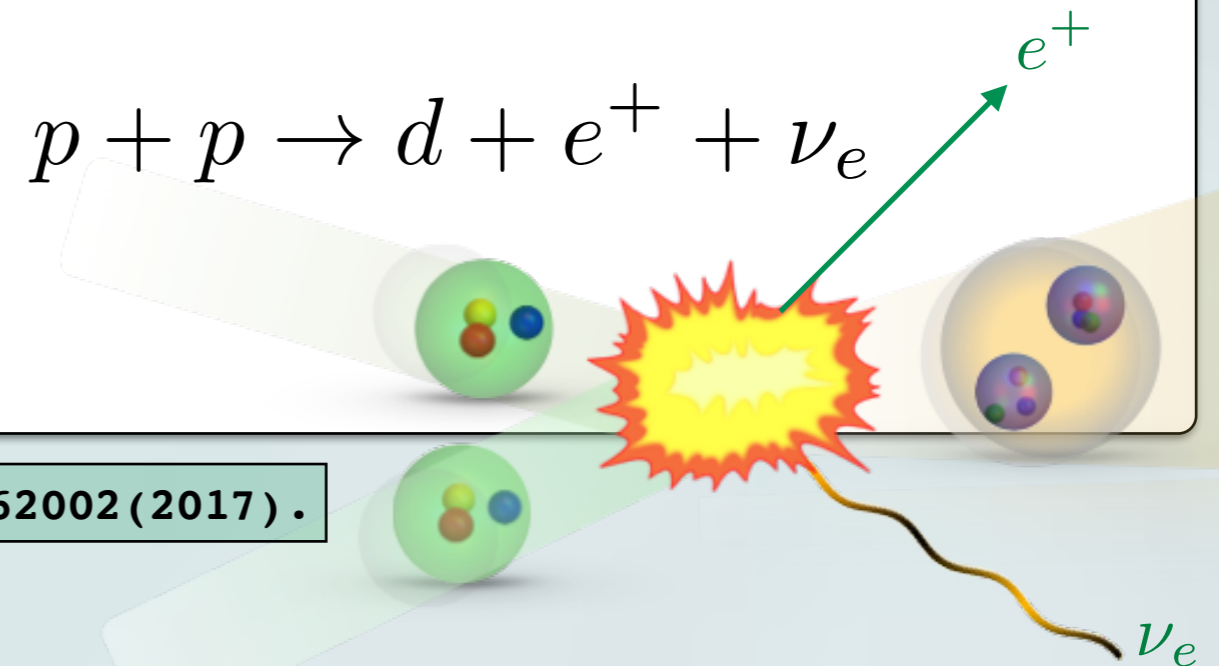
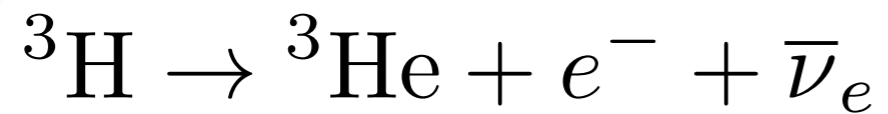
De-Leon, Platter and Gazit, arXiv:1611.10004 (2016).

THIS WORK:

$$L_{1,A} \approx 3.9(0.2)(1.0)(0.4)(0.9) \text{ fm}^3 @ \mu = m_{\pi}^{\text{phys.}} = 140 \text{ MeV}$$

Savage et al (NPLQCD), Phys.Rev.Lett.119,062002(2017).

## SINGLE-WEAK PROCESSES



Savage et al (NPLQCD), Phys.Rev.Lett.119,062002(2017).

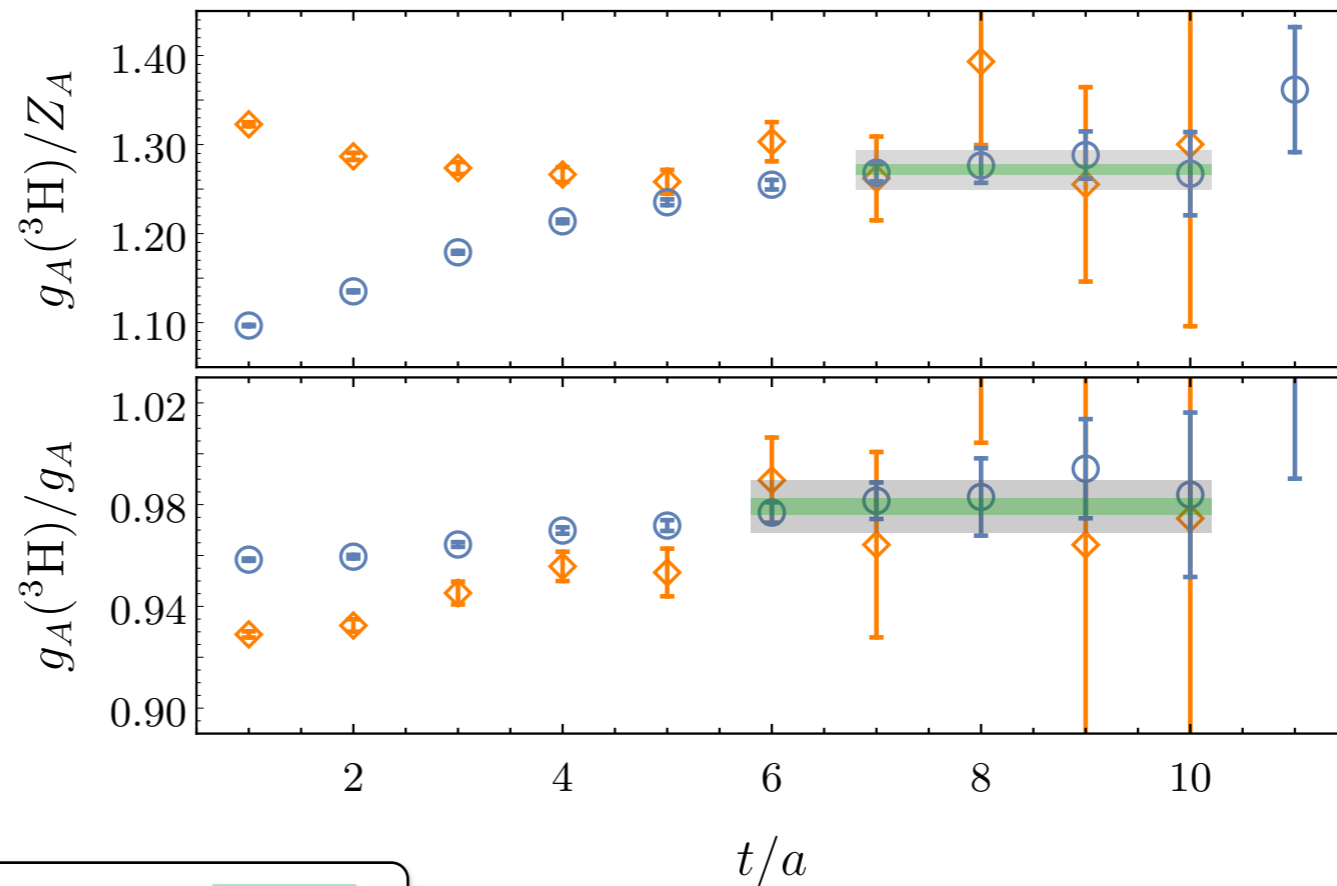
A SUPER-ALLOWED PROCESS, PROVIDES CONSTRAINTS ON THE ANTI-NEUTRINO MASS. THEORETICAL UNCERTAINTIES ARISE FROM GAMOV-TELLER MATRIX ELEMENTS (POOR CONSTRAINTS ON L<sub>1A</sub> OF PIONLESS EFT).

Savage et al (NPLQCD), Phys.Rev.Lett.119,062002(2017).

# MATRIX ELEMENT FROM QCD

$$N_f = 3, m_\pi = 0.806 \text{ GeV}, a = 0.145(2) \text{ fm}$$

AXIAL CHARGE  
OF THE TRITON



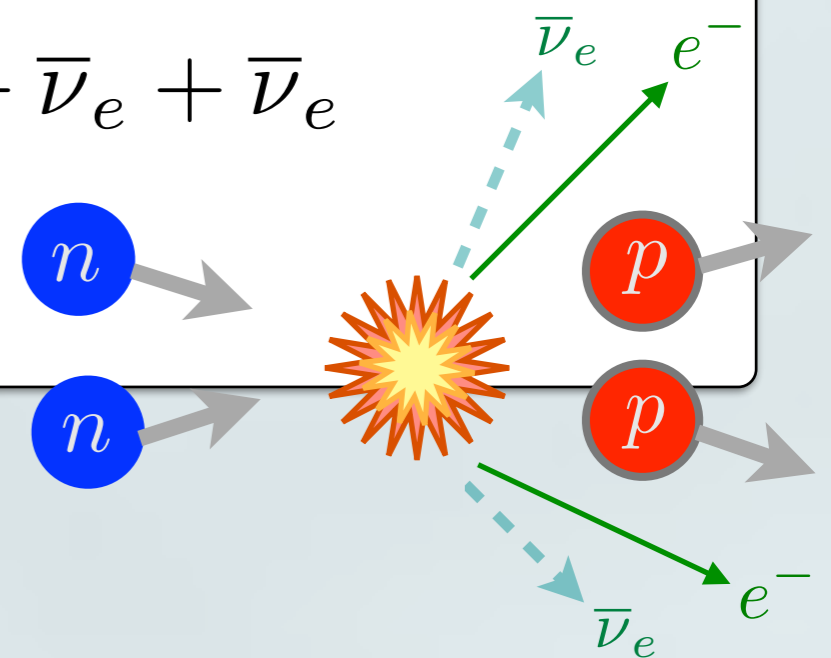
$$\langle ^3\text{He} | \bar{q} \gamma_k \gamma_5 \tau^+ q | ^3\text{H} \rangle = \bar{u} \gamma_k \gamma_5 \tau^+ u g_A \langle \mathbf{GT} \rangle$$

$$\frac{g_A(^3\text{H})}{g_A} = 0.979(3)(10)$$

$$\frac{(1 + \delta_R) f_V}{K/G_V^2} t_{1/2} = \frac{1}{\langle \mathbf{F} \rangle^2 + f_A/f_V g_A^2 \langle \mathbf{GT} \rangle^2}$$

## A DOUBLY-WEAK PROCESS

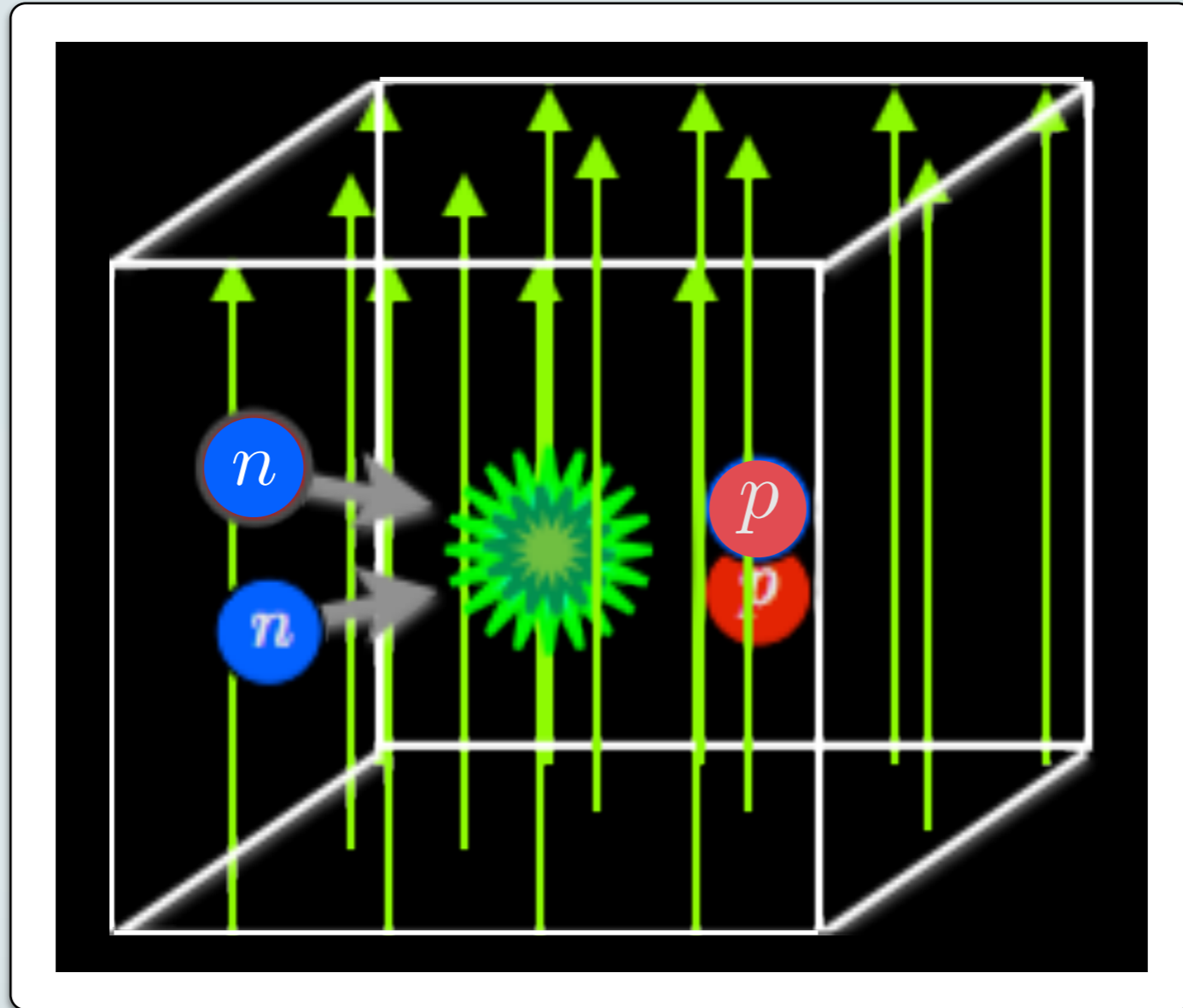
$$n + n \rightarrow p + p + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$



Tiburzi et al (NPLQCD), Phys.Rev.D96,054505(2017),  
Shanahan et al (NPLQCD), Phys.Rev.Lett.119,062003(2017).



## SECOND-ORDER RESPONSE TO AN AXIAL BACKGROUND FIELD



Tiburzi et al (NPLQCD), *Phys.Rev.D*96,054505(2017),  
Shanahan et al (NPLQCD), *Phys.Rev.Lett.*119,062003(2017).

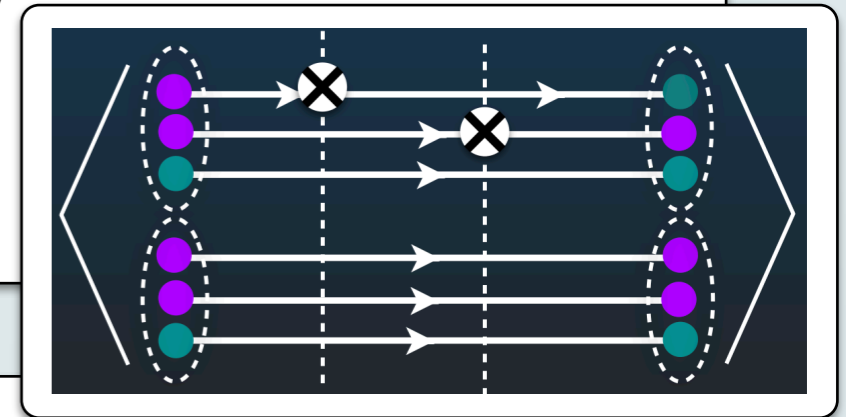
# SECOND-ORDER RESPONSE TO AN AXIAL BACKGROUND FIELD

$$C_{nn \rightarrow pp}(t) = 2 C_{\lambda_u; \lambda_d=0}^{(np(1S_0))}(t) \Big|_{\mathcal{O}(\lambda_u^2)} - C_{\lambda_u; \lambda_d=0}^{(nn)}(t) \Big|_{\mathcal{O}(\lambda_u^2)} - C_{\lambda_u=0; \lambda_d}(t) \Big|_{\mathcal{O}(\lambda_d^2)}$$



$$C_{\lambda_u; \lambda_d=0}^{(np(1S_0))}(t) = \sum_{\mathbf{x}} \langle 0 | \chi_{np}(\mathbf{x}, t) \chi_{np}^\dagger(0) | 0 \rangle + \lambda_u \sum_{\mathbf{x}, \mathbf{y}} \sum_{t_1=0}^t \langle 0 | \chi_{np}(\mathbf{x}, t) J_3^{(u)}(\mathbf{y}, t_1) \chi_{np}^\dagger(0) | 0 \rangle$$

$$+ \frac{\lambda_u^2}{2} \sum_{\mathbf{x}, \mathbf{y}, \mathbf{z}} \sum_{t_1=0}^t \sum_{t_2=0}^t \langle 0 | \chi_{np}(\mathbf{x}, t) J_3^{(u)}(\mathbf{y}, t_1) J_3^{(u)}(\mathbf{z}, t_2) \chi_{np}^\dagger(0) | 0 \rangle + g_3 \lambda_u^3,$$



ALREADY CONSTRAINED FROM  
ZEROth AND FIRST ORDERS!

SHORT-DISTANCE G.S. TO G.S. ME

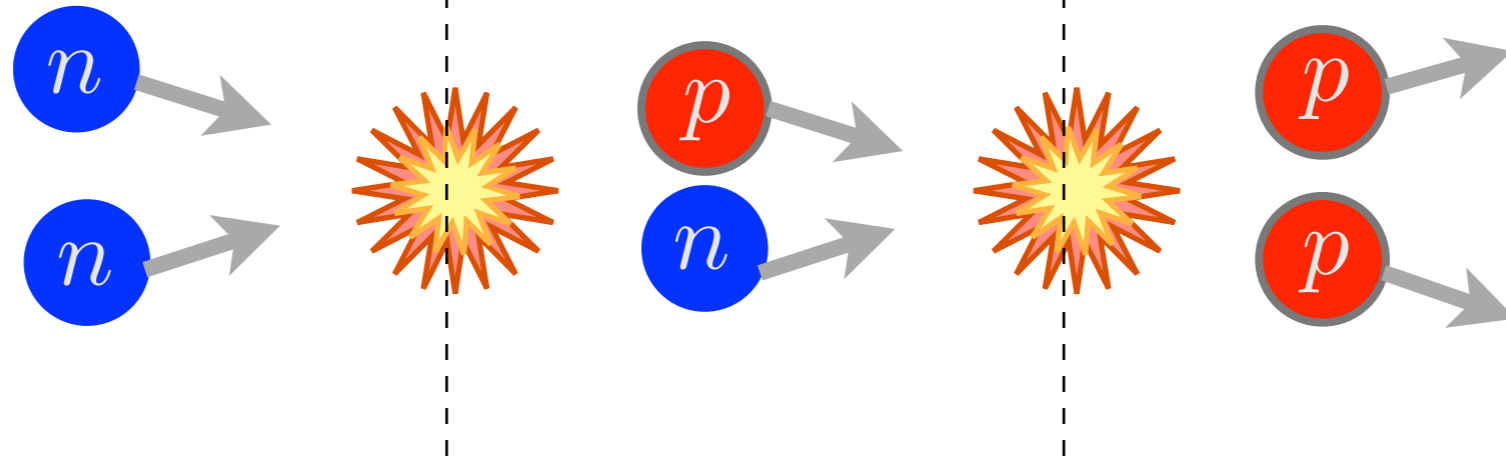
$$a^2 \mathcal{R}_{nn \rightarrow pp}(t) = \left[ -t + \frac{e^{\Delta t} - 1}{\Delta} \right] \frac{\langle pp | \tilde{J}_3^+ | d \rangle \langle d | \tilde{J}_3^+ | nn \rangle}{\Delta} + t \sum_{l' \neq d} \frac{\langle pp | \tilde{J}_3^+ | l' \rangle \langle l' | \tilde{J}_3^+ | nn \rangle}{\delta_{l'}}$$

STUFF THAT DEPEND ON EXCITED STATES

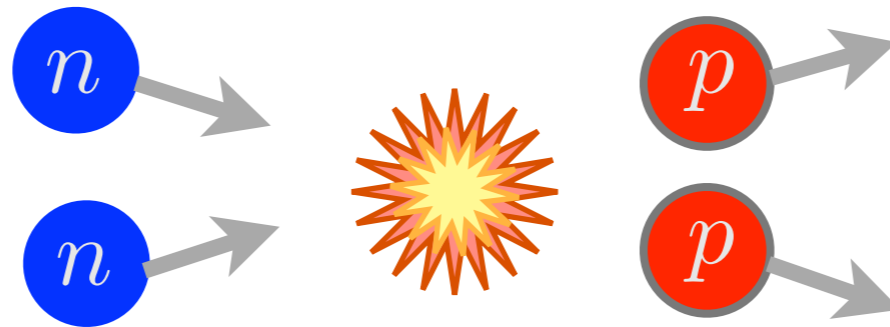
$$+ c + d e^{\Delta t} + \mathcal{O}(e^{-\delta t}, e^{-\delta' t}),$$

HERE THE FACT THAT  $\Delta \neq 0$  RESCUES US!

PROPAGATING  
DEUTERON



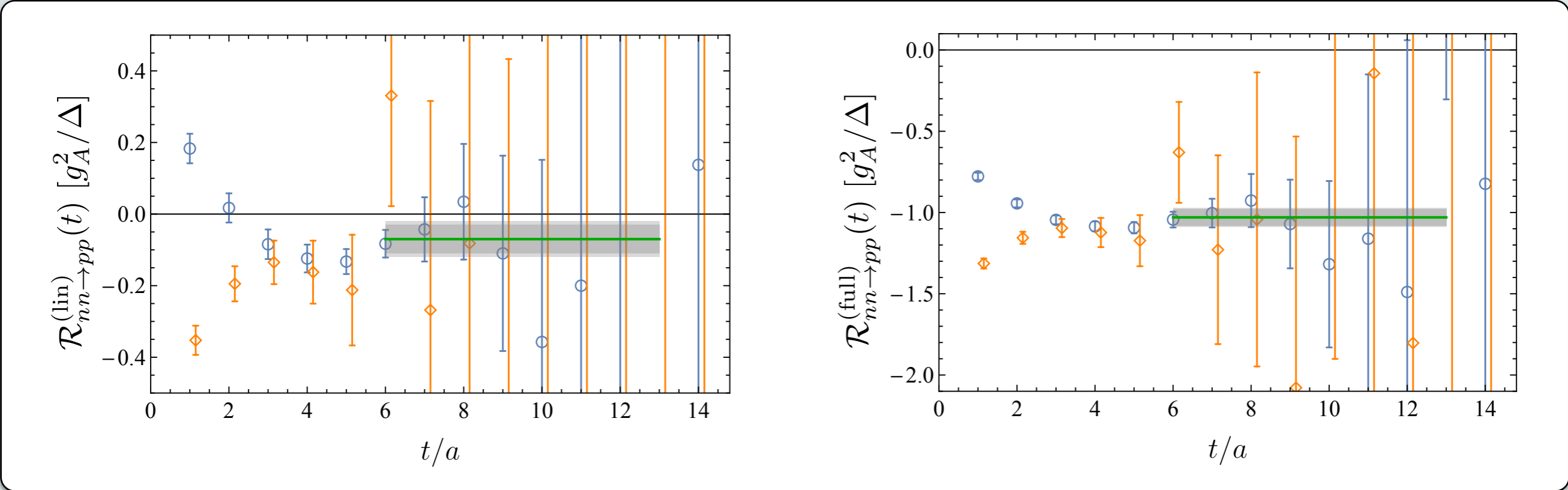
LONG-DISTANCE PIECE



SHORT-DISTANCE PIECE

# MATRIX ELEMENT FROM QCD

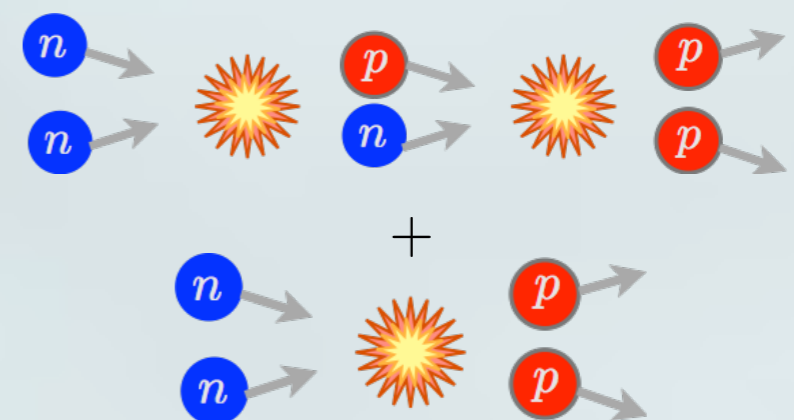
$N_f = 3, m_\pi = 0.806 \text{ GeV}, a = 0.145(2) \text{ fm}$



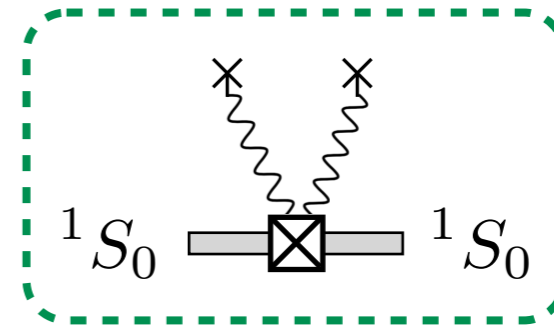
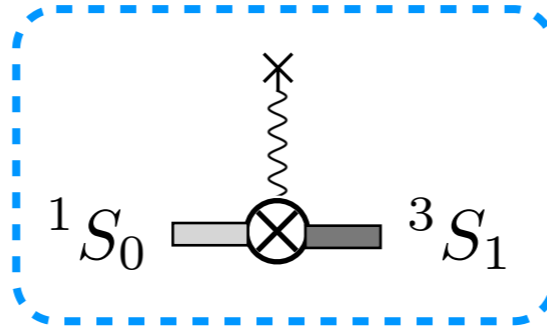
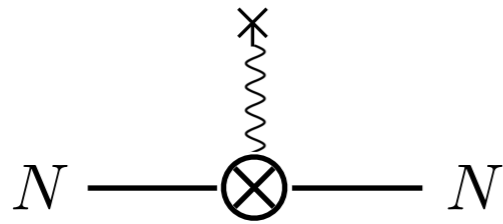
SHORT-DISTANCE CONTRIBUTION



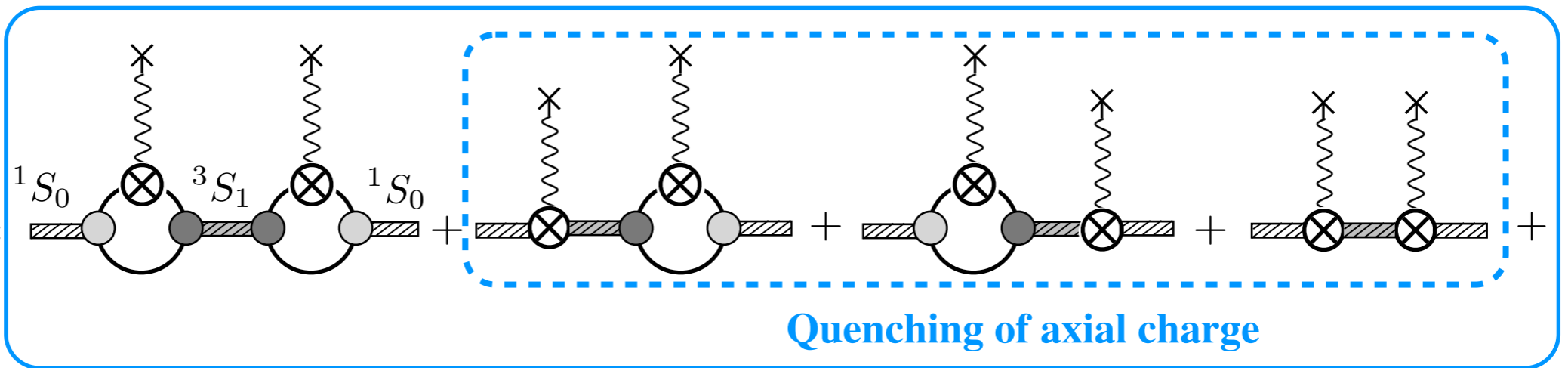
FULL CONTRIBUTION



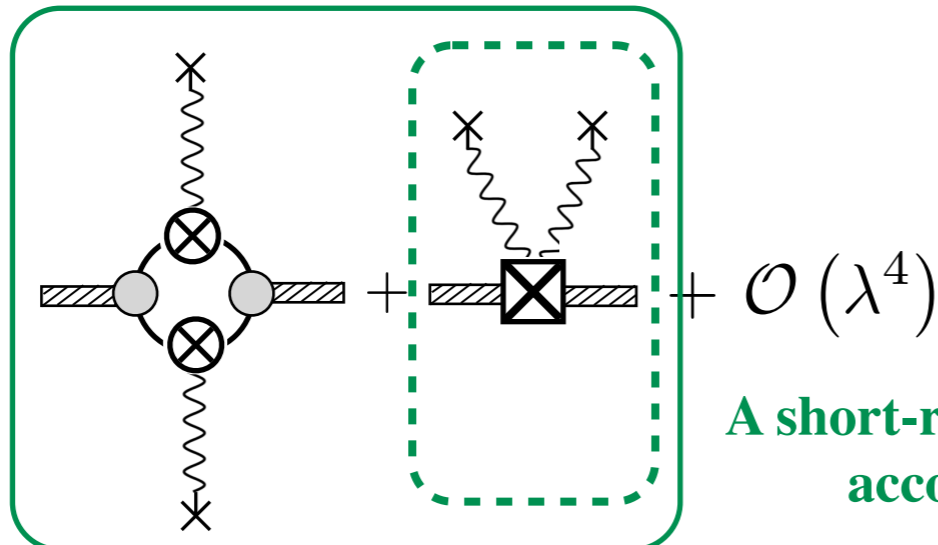
# EFT VERTICES AND CORRELATION FUNCTIONS USING DIBARYONS



$$i\mathcal{C}_{nn \rightarrow pp} =$$



**Give partly the dominant long-range contribution**

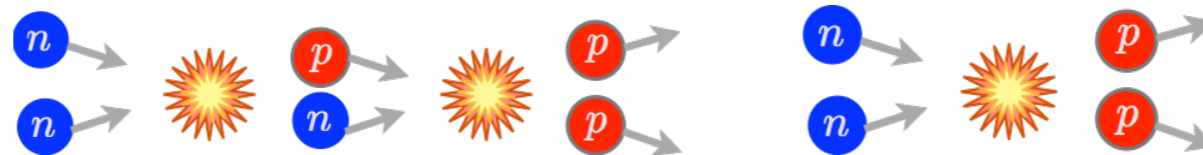


# EFT CONSTRAINED WITH LQCD.

$$M_{pp \rightarrow d} = g_A(1 + S) + \mathbb{L}_{1,A}$$

AND

$$M_{nn \rightarrow pp} = \frac{|M_{pp \rightarrow d}|^2}{\Delta} + \frac{M g_A^2}{4\gamma_s^2} - \mathbb{H}_{2,S}$$



$$\mathbb{H}_{2,S} = 4.7(1.3)(1.8) \text{ fm}$$

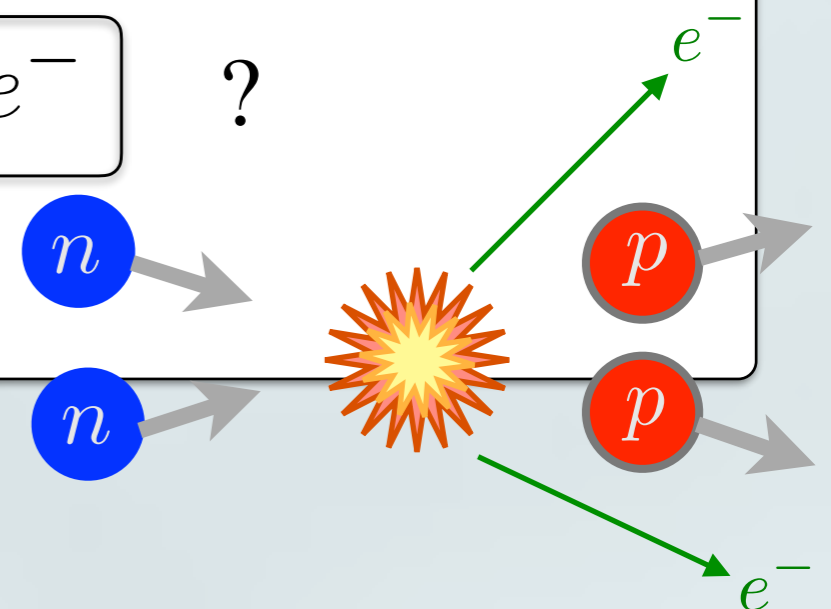
$$@ m_\pi \approx 800 \text{ MeV}$$

LESSON FROM 800 MEV WORLD:

AXIAL POLARIZABILITY COULD BE IMPORTANT. CANNOT BE CONSTRAINED BY SINGLE-BETA DECAY PROCESSES.

## A DOUBLY-WEAK PROCESS

$$n + n \rightarrow p + p + e^{-} + e^{-} \quad ?$$

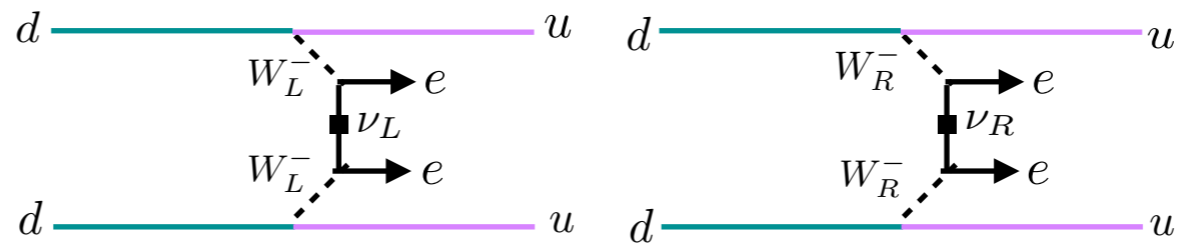


Tiburzi et al (NPLQCD), Phys.Rev.D96,054505(2017),  
Shanahan et al (NPLQCD), Phys.Rev.Lett.119,062003(2017).

# MATCHING HIGH SCALE TO LOW SCALE

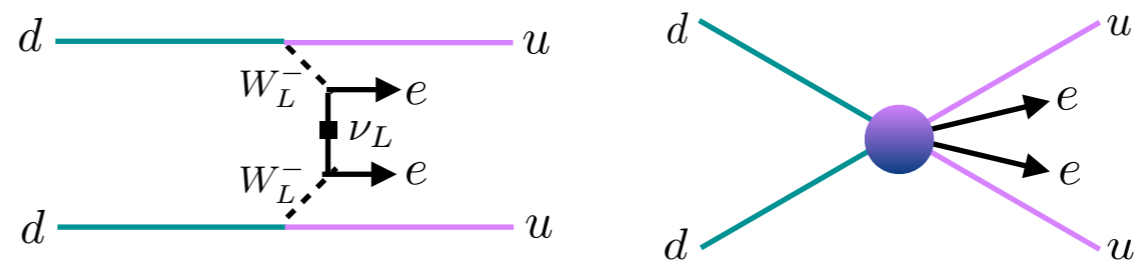
$\Lambda > \text{TeV}$

START WITH YOUR FAVORITE HIGH-SCALE MODEL, E.G.:



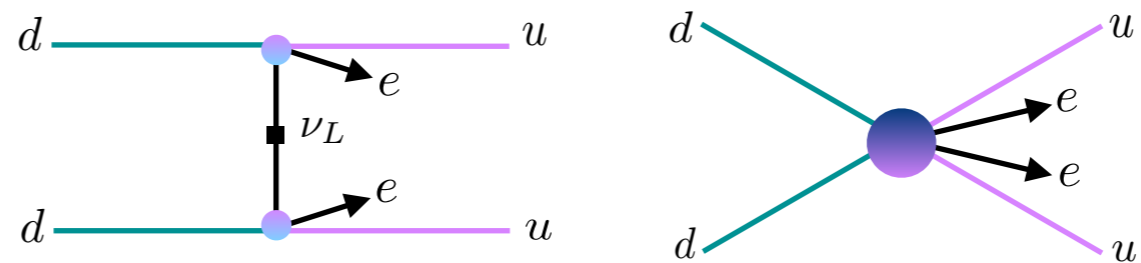
$\Lambda \sim 10^2 \text{ GeV}$

RUN IT DOWN TO THE SCALE WHERE THE HIGH-SCALE PHYSICS CAN BE INTEGRATED OUT:



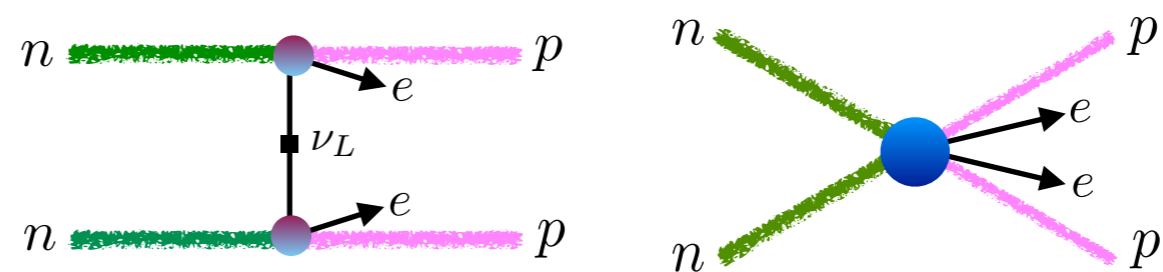
$\Lambda \sim 2 \text{ GeV}$

RUN IT DOWN TO PERTURBATIVE QUARK-LEVEL MATRIX ELEMENTS:



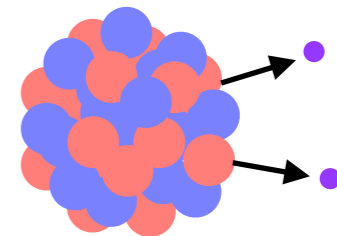
$\Lambda < \text{GeV}$

RUN IT DOWN TO THE HADRONIC SCALE:



$\Lambda < \text{MeV}$

USE NUCLEAR MANY-BODY CALCULATION TO MATCH IT TO NUCLEAR MATRIX ELEMENTS:



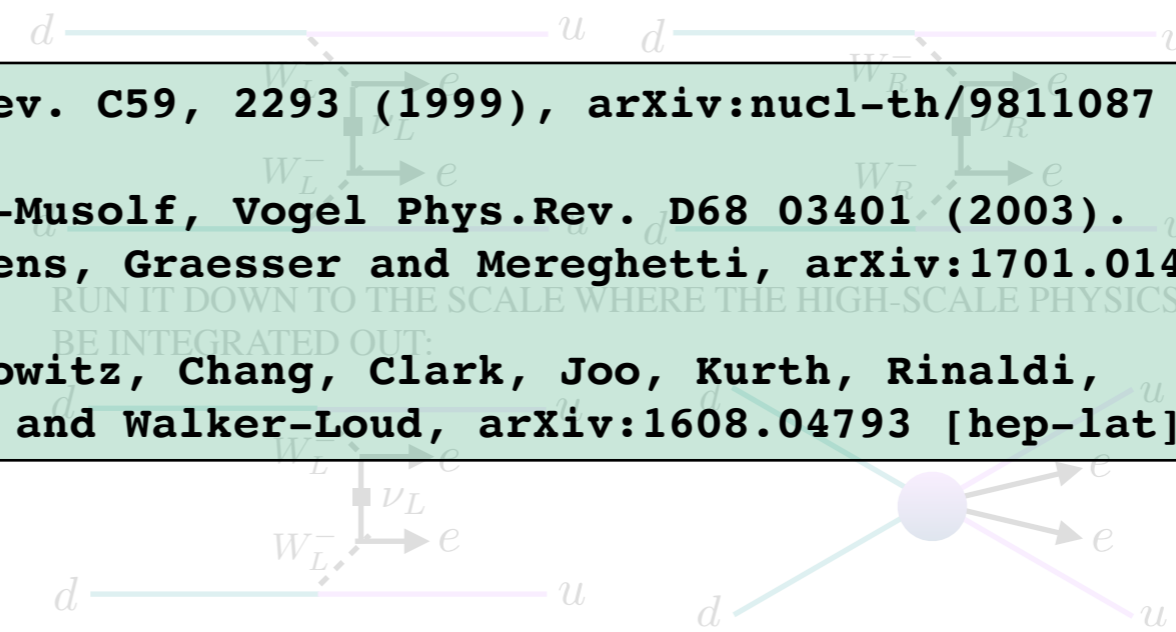


# MATCHING HIGH SCALE TO LOW SCALE

$\Lambda > \text{TeV}$

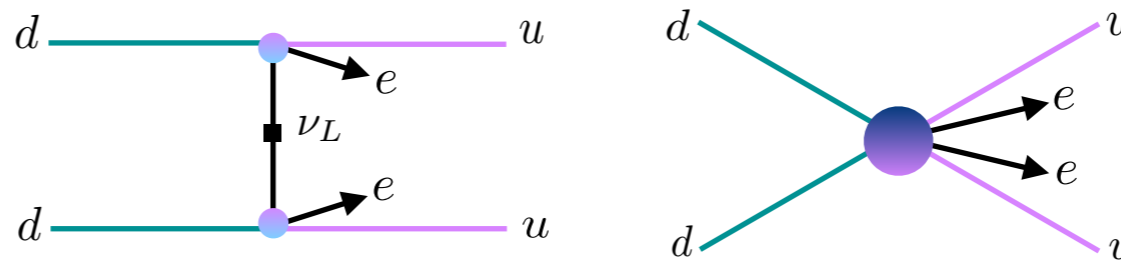
START WITH YOUR FAVORITE HIGH-SCALE MODEL, E.G.:

Savage, Phys. Rev. C59, 2293 (1999), arXiv:nucl-th/9811087 [nucl-th].  
 Prezeau, Ramsey-Musolf, Vogel Phys.Rev. D68 03401 (2003).  
 Cirigliano, Dekens, Graesser and Mereghetti, arXiv:1701.01443 [hep-ph].  
 Nicholson, Berkowitz, Chang, Clark, Joo, Kurth, Rinaldi, Tiburzi, Vranas and Walker-Loud, arXiv:1608.04793 [hep-lat]



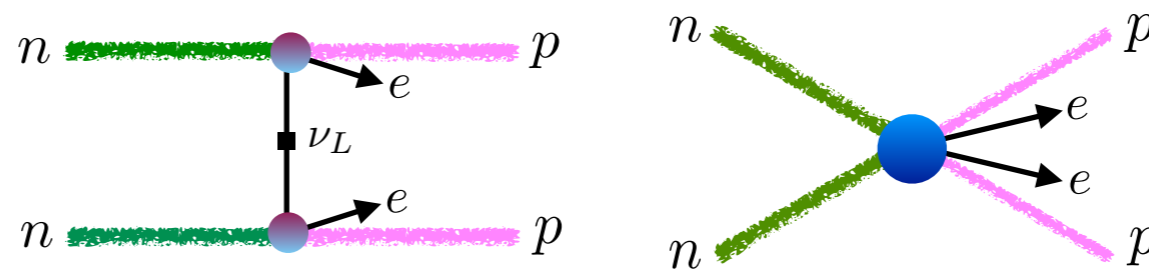
$\Lambda \sim 2 \text{ GeV}$

RUN IT DOWN TO PERTURBATIVE QUARK-LEVEL MATRIX ELEMENTS:



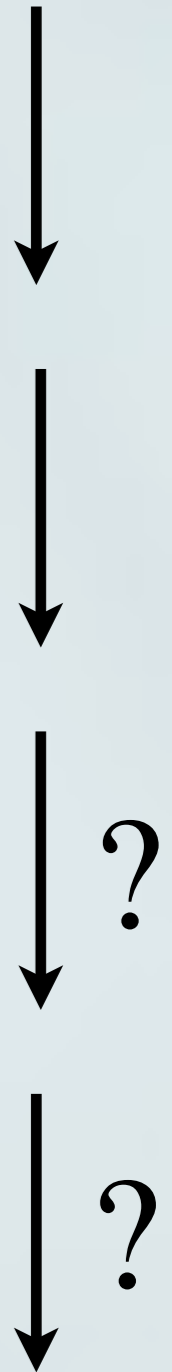
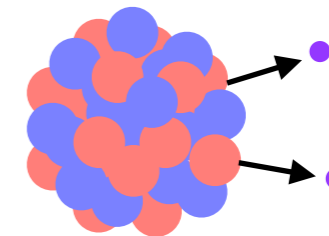
$\Lambda < \text{GeV}$

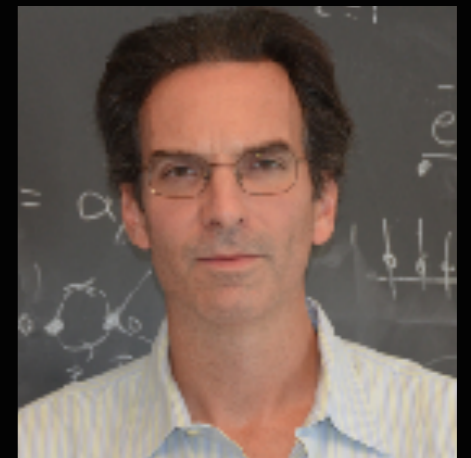
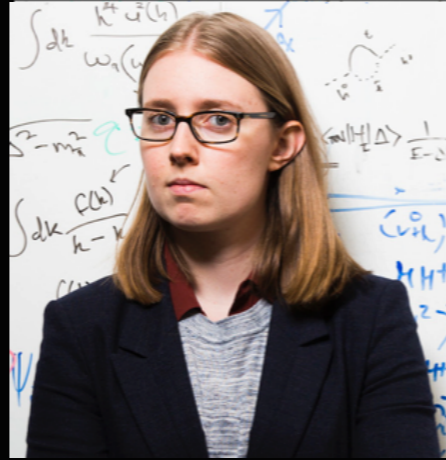
RUN IT DOWN TO THE HADRONS SCALE:



$\Lambda < \text{MeV}$

USE NUCLEAR MANY-BODY CALCULATION TO MATCH IT TO NUCLEAR MATRIX ELEMENTS:





THANK YOU