High-momentum tails, short-range correlations, and low-momentum effective theories

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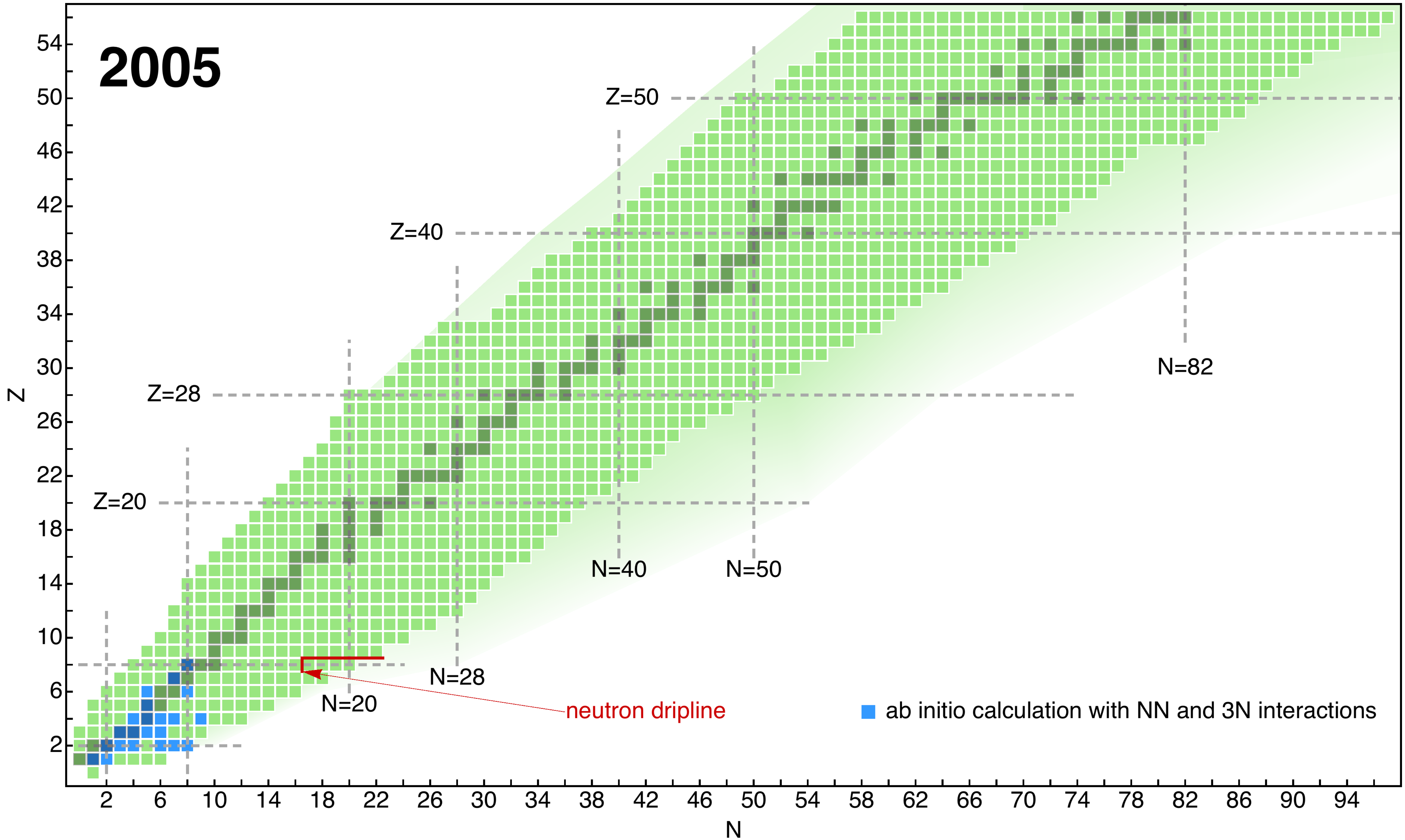
Outline

1. Effective operators and factorization
2. Scale dependence of Deuteron electrodisintegration
3. Scale dependence of short-range correlations in medium-mass nuclei

Progress in *Ab Initio* Calculations



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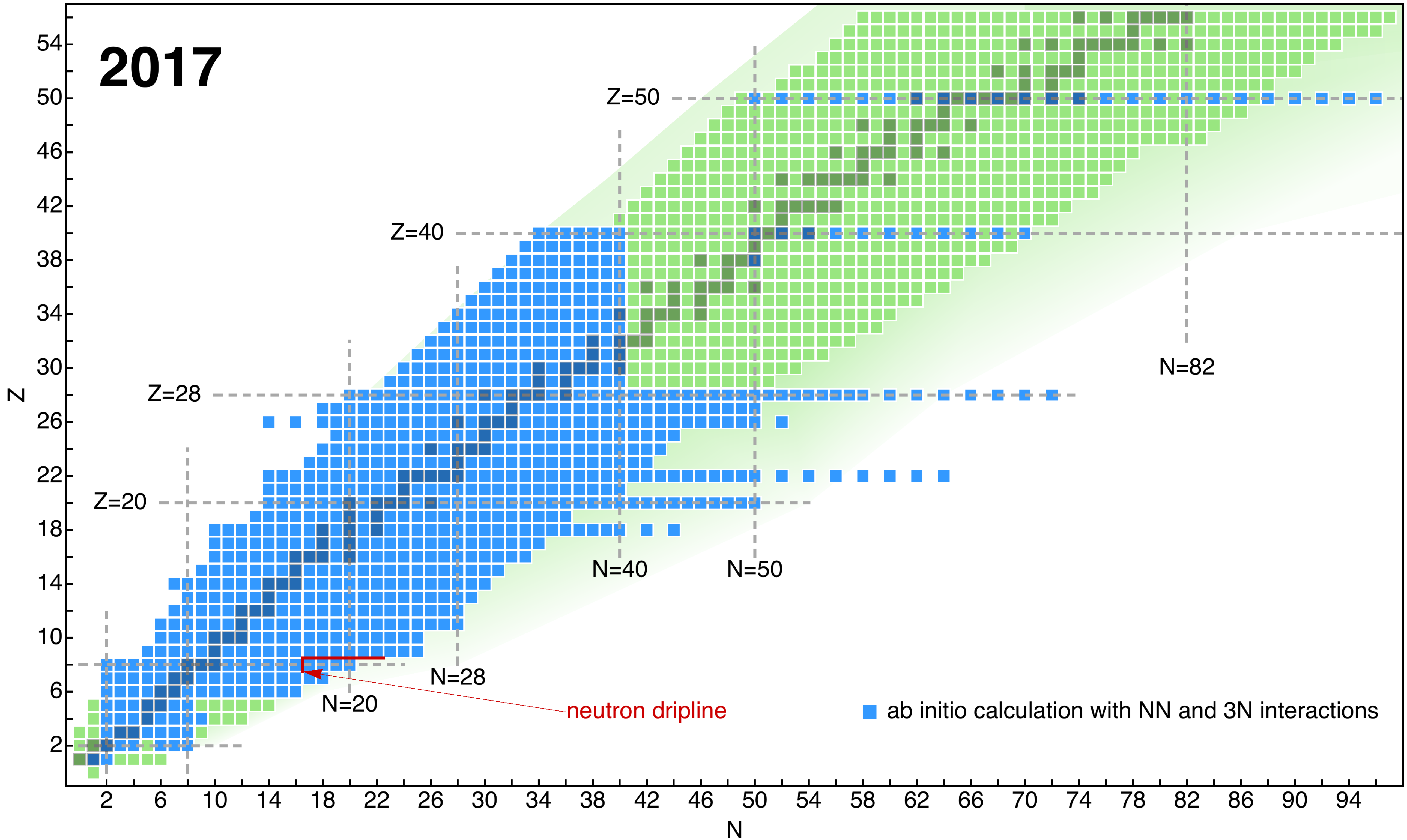

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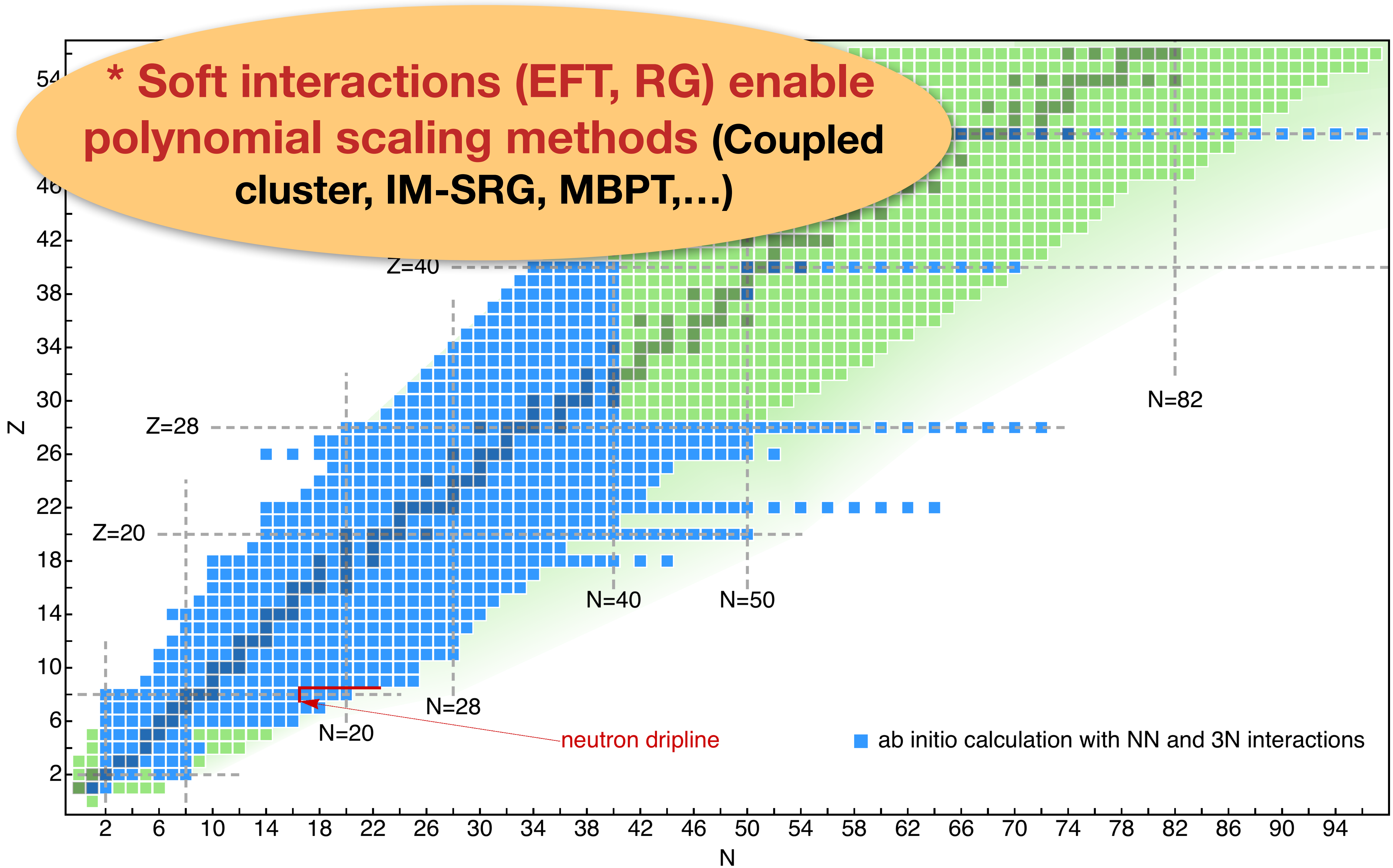
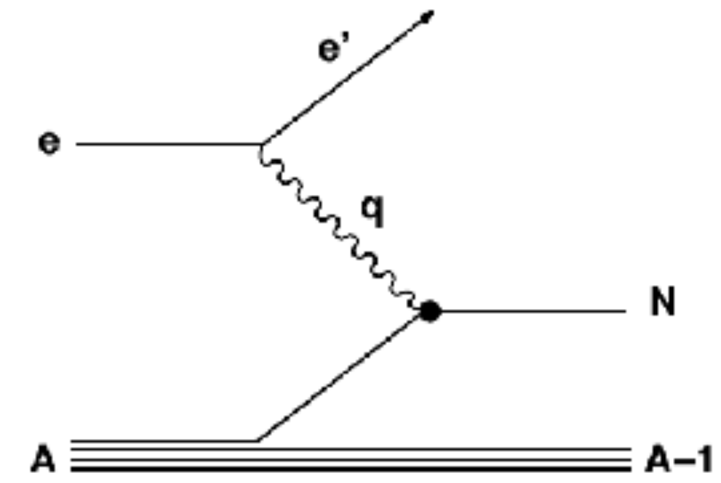


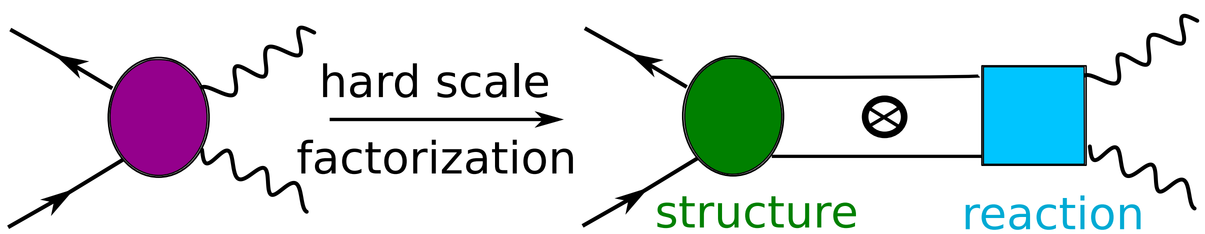
Bridging structure and reactions

- Goal: Extract nuclear properties from experiments and predict them from theory
- $$\frac{d\sigma}{d\Omega} \propto \left| \langle \psi_f | \widehat{O}(q) | \psi_i \rangle \right|^2$$

e.g., nucleon knockout reaction



- Factorization to isolate components and extract process-independent properties



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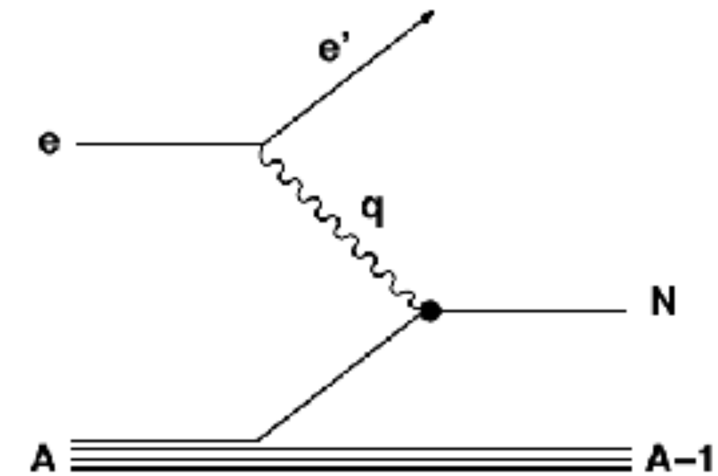

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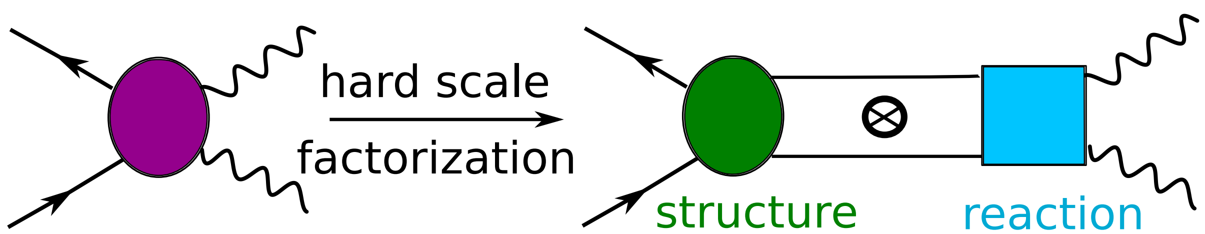
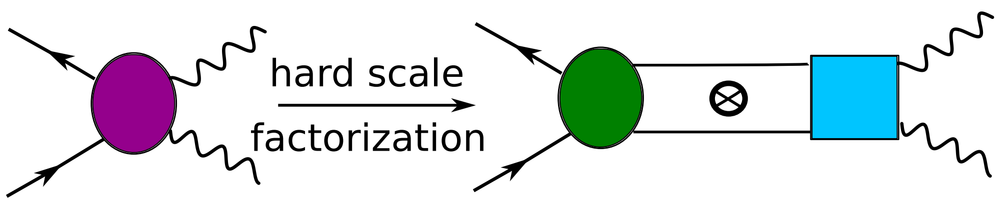


$$\underbrace{\langle\psi_f|}_{\text{structure}}\overbrace{\widehat{O}(q)}^{\text{reaction}}\underbrace{|\psi_i\rangle}_{\text{structure}} = \langle\psi_f|U_\lambda U_\lambda^\dagger \widehat{O}(q) U_\lambda U_\lambda^\dagger|\psi_i\rangle = \underbrace{\langle\psi_f^\lambda|}_{\text{structure}(\lambda)}\overbrace{\widehat{O}^\lambda(q)}^{\text{reaction}(\lambda)}\underbrace{|\psi_i^\lambda\rangle}_{\text{structure}(\lambda)}$$

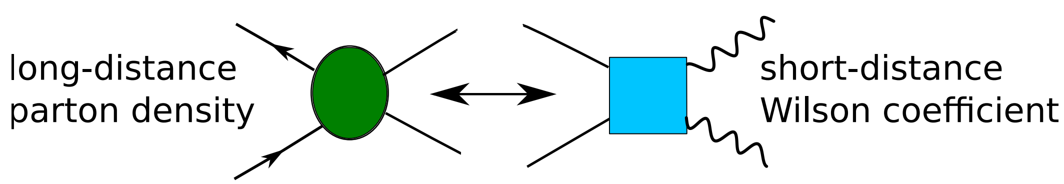
Factorization is scale-dependent (not unique)!!

Analogy with DIS in QCD

High-E QCD



$$F_2(x, Q^2) \sim \sum_a f_a(x, \mu_f) \otimes \widehat{F}_2^a(x, Q/\mu_f)$$



Low-E Nuclear

Observable: cross section

Structure model: spectroscopic factor

Reaction model: single-particle cross section

$$\sigma^{if} = \sum_{|J_i - J_f| \leq j \leq J_i + J_f} S_j^{if} \sigma_{sp}$$

- Separation not unique, depends on the scale μ_f
- Form factor F_2 independent of μ_f but pieces not
- $f_a(x, \mu_f)$ runs with $\mu_f^2 = Q^2$, but is process independent

Analogy with DIS in QCD

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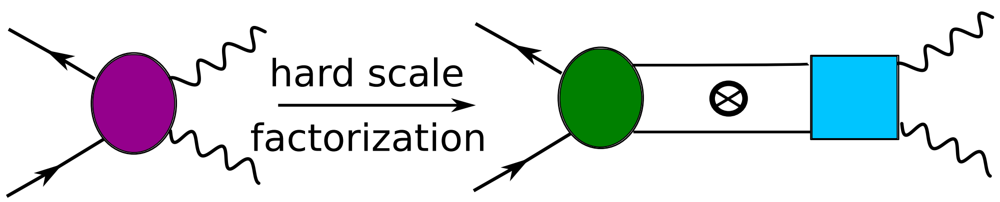

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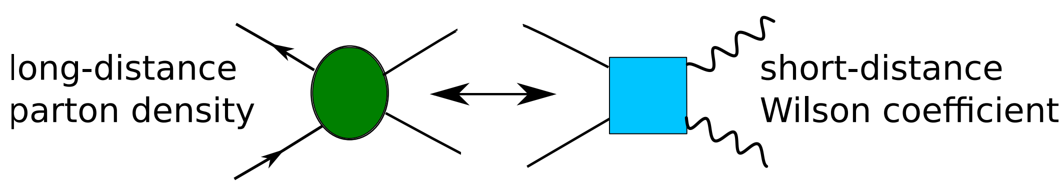

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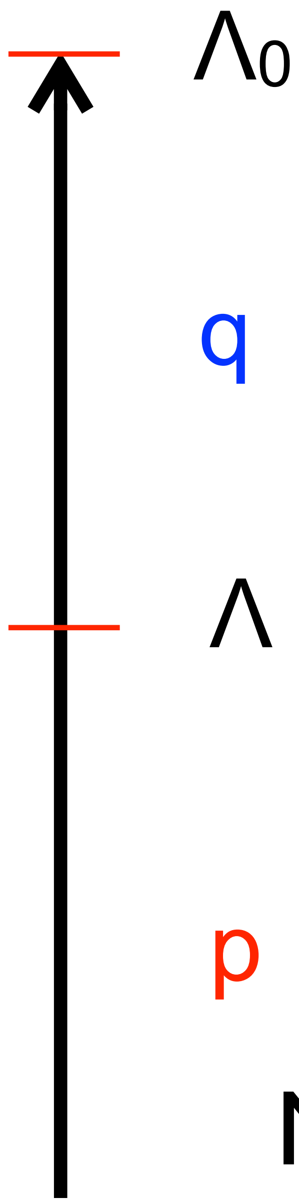
Open Questions

- When does factorization hold?
- What is the scale/scheme dependence of extracted props?
- Extract at one scale (e.g., to minimize FSI) and evolve to another?
- Scale/scheme dependence of interpretations? Are some better?
- Structure of evolved operators?

General form of RG-evolved operators

- Want to understand form of effective operators without getting bogged down in a particular scheme (Lee-Suzuki, SRG, etc.)



I only assume for **low-energy** states

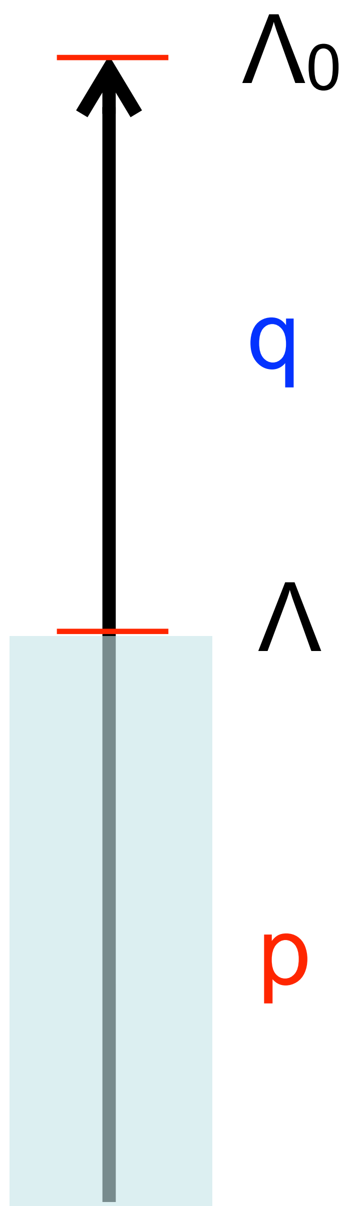
I) $P|\psi_n^{\Lambda_0}\rangle \approx Z(\Lambda)|\psi_n^{\Lambda}\rangle$

II) $Q|\psi_n^{\Lambda}\rangle \approx 0$

Met by all softening transformations I know of…

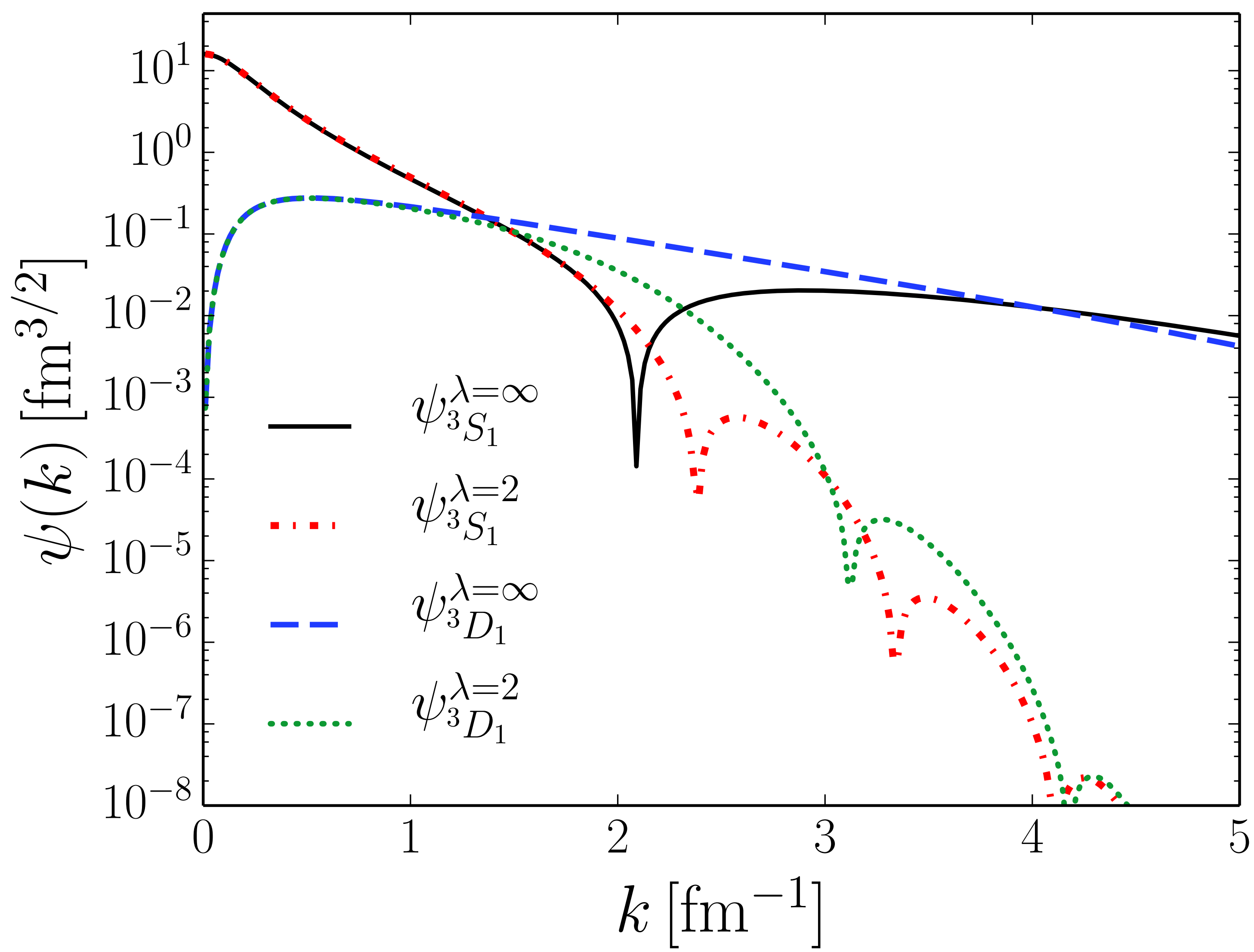
Wave function factorization

Consider low-k components of low-E wf's for A=2.



RG doesn't change long distance/IR structure

$$\psi_\alpha^{\Lambda_0}(\mathbf{p}) \approx Z_\Lambda \psi_\alpha^{\Lambda}(\mathbf{p})$$



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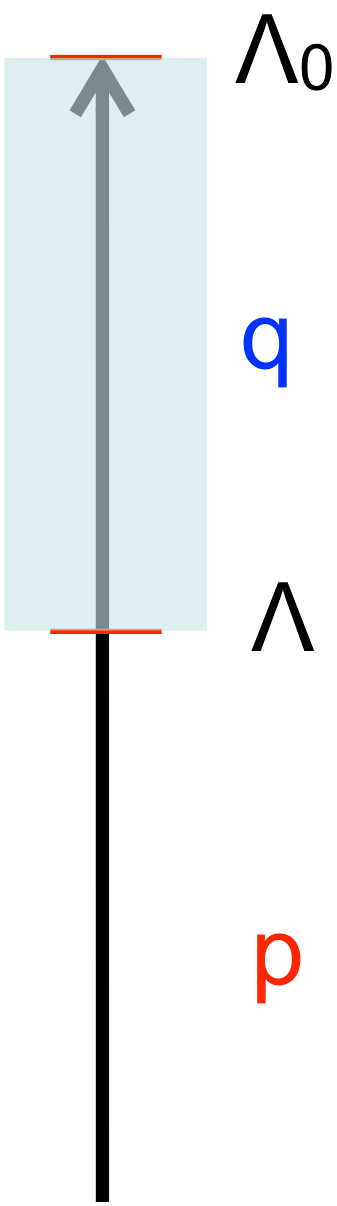

Scale separation ($E_\alpha << \Lambda^2 << q^2$)

$$\psi_\alpha^{\Lambda_0}(\mathbf{q}) \approx \gamma(\mathbf{q};\Lambda)\int_0^\Lambda d^3p\, Z_\Lambda \psi_\alpha^\Lambda(\mathbf{p}) + \eta(\mathbf{q};\Lambda)\int_0^\Lambda d^3p\, \mathbf{p}^2 Z_\Lambda \psi_\alpha^\Lambda(\mathbf{p}) \cdots$$

Anderson et al., PRC **82** (2010)
SKB and Roscher, PRC **86** (2012)

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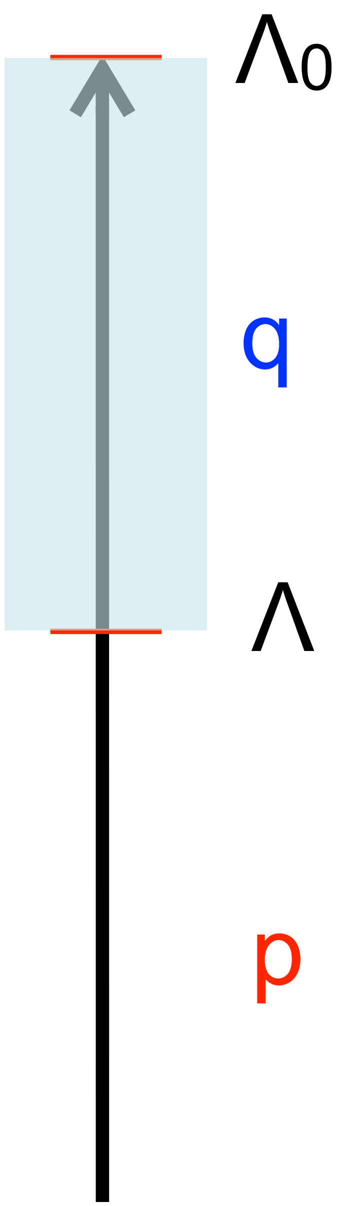

Scale separation ($E_\alpha \ll \Lambda^2 \ll q^2$)

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Operator **P**roduct **E**xpansion
of wave function a-la Lepage

$$\gamma(\mathbf{q};\Lambda) = -\int_\Lambda^{\Lambda_0} d\mathbf{q}'\,\langle\mathbf{q}|\frac{1}{QH^{\Lambda_0}Q}|\mathbf{q}'\rangle V^{\Lambda_0}(\mathbf{q}',0)$$

$$\beta(\mathbf{q};\Lambda) = -\int_\Lambda^{\Lambda_0} d\mathbf{q}'\,\langle\mathbf{q}|\frac{1}{QH^{\Lambda_0}Q}|\mathbf{q}'\rangle \frac{\partial^2}{\partial p^2} V^{\Lambda_0}(\mathbf{q}',\mathbf{p})\Big|_{\mathbf{p}=0}$$

State-independent
Wilson Coefficients

Anderson et al., PRC **82** (2010)
SKB and Roscher, PRC **86** (2012)

Wave function factorization

LO: $$\psi_\alpha^{\Lambda_0}(\mathbf{q}) \approx \gamma(\mathbf{q};\Lambda) \int_0^\Lambda d^3p\, Z_\Lambda \psi_\alpha^\Lambda(\mathbf{p})$$

state-independent ratio
for well-separated scales

$$\frac{\psi_\alpha^{\Lambda_0}(\mathbf{q})}{\psi_\alpha^\Lambda(\mathbf{r}=0)} \sim \gamma(\mathbf{q};\Lambda)$$

$$|E_\alpha| \lesssim \Lambda^2 \quad |\mathbf{q}| \gtrsim \Lambda$$

Wave function factorization

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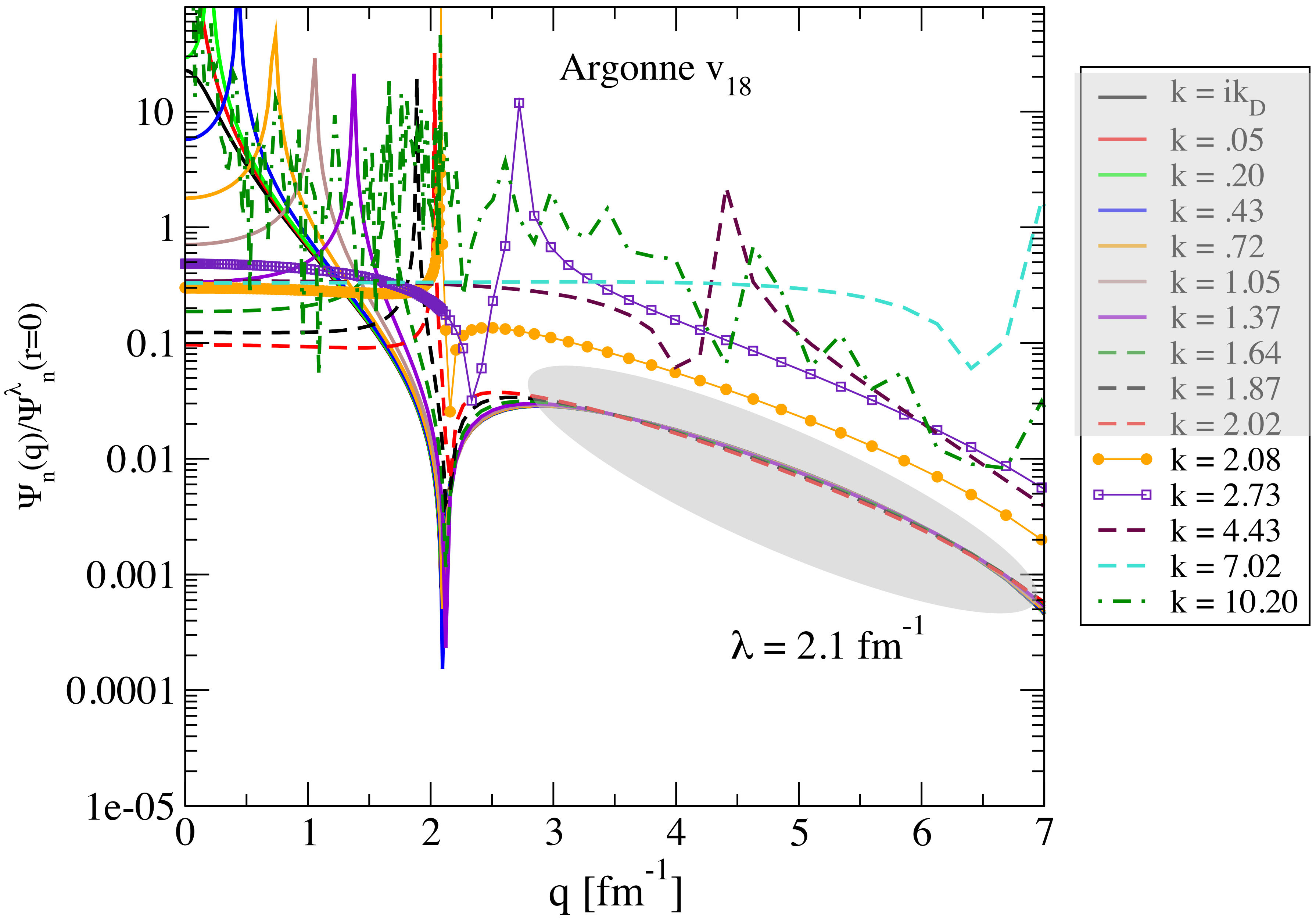

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Effective operators from w.f. factorization

$$\begin{aligned}\langle\psi_\alpha^{\Lambda_0}|\widehat{O}_{\Lambda_0}|\psi_\alpha^{\Lambda_0}\rangle &= \int_0^\Lambda dp\int_0^\Lambda dp'\,\psi_\alpha^{\Lambda_0 *}(p)O(p,p')\psi_\alpha^{\Lambda_0}(p') + \int_0^\Lambda dp\int_\Lambda^{\Lambda_0} dq\,\psi_\alpha^{\Lambda_0 *}(p)O(p,q)\psi_\alpha^{\Lambda_0}(q)\\ &+ \int_\Lambda^{\Lambda_0} dq\int_0^\Lambda dp\,\psi_\alpha^{\Lambda_0 *}(q)O(q,p)\psi_\alpha^{\Lambda_0}(p) + \int_\Lambda^{\Lambda_0} dq\int_\Lambda^{\Lambda_0} dq'\,\psi_\alpha^{\Lambda_0 *}(q)O(q,q')\psi_\alpha^{\Lambda_0}(q')\end{aligned}$$

Effective operators from w.f. factorization

$$\langle \psi_\alpha^{\Lambda_0} | \widehat{O}_{\Lambda_0} | \psi_\alpha^{\Lambda_0} \rangle = \int_0^\Lambda dp \int_0^\Lambda dp' \, \psi_\alpha^{\Lambda_0 *}(p) O(p,p') \psi_\alpha^{\Lambda_0}(p') + \int_0^\Lambda dp \int_\Lambda^{\Lambda_0} dq \, \psi_\alpha^{\Lambda_0 *}(p) O(p,q) \psi_\alpha^{\Lambda_0}(q)$$

$$+ \int_\Lambda^{\Lambda_0} dq \int_0^\Lambda dp \, \psi_\alpha^{\Lambda_0 *}(q) O(q,p) \psi_\alpha^{\Lambda_0}(p) + \int_\Lambda^{\Lambda_0} dq \int_\Lambda^{\Lambda_0} dq' \, \psi_\alpha^{\Lambda_0 *}(q) O(q,q') \psi_\alpha^{\Lambda_0}(q')$$

Now use:

$$\psi_\alpha^{\Lambda_0}(\mathbf{q}) \approx \gamma(\mathbf{q}; \Lambda) \int_0^\Lambda d^3p \, Z_\Lambda \psi_\alpha^\Lambda(\mathbf{p}) + \cdots$$ OPE for w.f.'s

$$\psi_\alpha^{\Lambda_0}(\mathbf{p}) \approx Z_\Lambda \psi_\alpha^\Lambda(\mathbf{p})$$ IR structure unaltered

$$O(q,p) \approx O(q,0) + \cdots$$ Scale separation

Effective operators from w.f. factorization

$$\langle \psi_\alpha^{\Lambda_0} | \widehat{O}_{\Lambda_0} | \psi_\alpha^{\Lambda_0} \rangle \approx Z_\Lambda^2 \langle \psi_\alpha^{\Lambda} | \widehat{O}_{\Lambda_0} | \psi_\alpha^{\Lambda} \rangle + g^{(0)}(\Lambda) \, \langle \psi_\alpha^{\Lambda} | \delta^{(3)}(\mathbf{r}) | \psi_\alpha^{\Lambda} \rangle + \cdots$$

state-independent
high-q physics
depends on operator

state dependent
soft m.e. (low-k)
same for all high-q operators

Effective operators from w.f. factorization

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E.g.,

$$g^{(0)}(\Lambda) \equiv 2Z_\Lambda^2 \int_\Lambda^{\Lambda_0} d\tilde{q}\, O(0,q)\gamma(q;\Lambda) + Z_\Lambda^2 \int_\Lambda^{\Lambda_0} d\tilde{q} \int_\Lambda^{\Lambda_0} d\tilde{q}'\, \gamma^*(q;\Lambda) O(q,q')\gamma(q';\Lambda)$$

Generically:

$$\widehat{O}_\Lambda = Z_\Lambda^2\, \widehat{O}_{\Lambda_0} + g^{(0)}(\Lambda)\, \delta(\mathbf{r}) + g^{(2)}(\Lambda)\, \nabla^2 \delta(\mathbf{r}) + \cdots$$

Scaling of high momentum operators

How does an operator that probes high-momentum w.f. components look in a low-momentum effective theory?

$$\langle \psi_\alpha^{\Lambda_0} | \widehat{O}_{\Lambda_0} | \psi_\alpha^{\Lambda_0} \rangle \approx Z_\Lambda^2 \langle \psi_\alpha^\Lambda | \widehat{O}_{\Lambda_0} | \psi_\alpha^\Lambda \rangle + g^{(0)}(\Lambda) \, \langle \psi_\alpha^\Lambda | \delta^{(3)}(\mathbf{r}) | \psi_\alpha^\Lambda \rangle + \cdots$$

= 0 since $P_\Lambda O_{\Lambda_0} P_\Lambda = 0$

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E.g., momentum distribution for q >> Λ

$$\langle \psi_\alpha^{\Lambda_0} | a_{\mathbf{q}}^\dagger a_{\mathbf{q}} | \psi_\alpha^{\Lambda_0} \rangle \approx \gamma^2(\mathbf{q};\Lambda) \, Z_\Lambda^2 |\langle \psi_\alpha^{\Lambda} | \delta(\mathbf{r}) | \psi_\alpha^{\Lambda} \rangle|^2$$

low-E states have the same large-q tails

Generalize to arbitrary **A-body** states?

Scaling of high momentum tails

SKB and Roscher, PRC **86** (2012)

Creation/annihilation operators under RG evolution:

$$a_{\mathbf{q}}^{(\Lambda)\dagger} = a_{\mathbf{q}}^{\dagger} + \sum_{\mathbf{k_1},\mathbf{k_2}} C_{\mathbf{q}}^{\Lambda}(\mathbf{k_1},\mathbf{k_2}) a_{\mathbf{k_1}}^{\dagger} a_{\mathbf{k_2}}^{\dagger} a_{\mathbf{k_1}+\mathbf{k_2}-\mathbf{q}} + \cdots \equiv a_{\mathbf{q}}^{\dagger} + \delta a_{\mathbf{q}}^{(\Lambda)\dagger}$$

fixed from RGE in A=2 system

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Scale separation (Λ << q < Λ₀):

$$\begin{aligned}\langle \psi_{\alpha,A}^{\Lambda_0} | a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} | \psi_{\alpha,A}^{\Lambda_0} \rangle &= \langle \psi_{\alpha,A}^{\Lambda} | a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} + \delta a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} + a_{\mathbf{q}}^{\dagger} \delta a_{\mathbf{q}} + \delta a_{\mathbf{q}}^{\dagger} \delta a_{\mathbf{q}} | \psi_{\alpha,A}^{\Lambda} \rangle \\ &\approx \langle \psi_{\alpha,A}^{\Lambda} | \delta a_{\mathbf{q}}^{\dagger} \delta a_{\mathbf{q}} | \psi_{\alpha,A}^{\Lambda} \rangle\end{aligned}$$

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Scale separation ($\Lambda << q < \Lambda_0$):

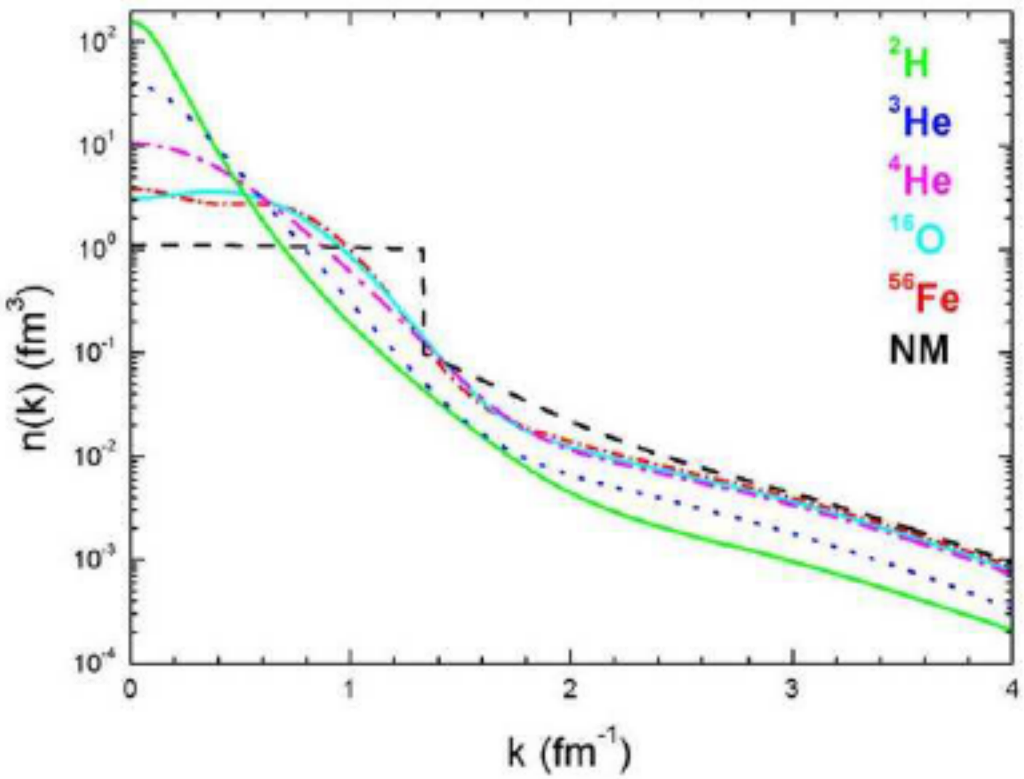
$$\begin{aligned}
\langle \psi^{\Lambda_0}_{\alpha,A} | a^\dagger_{\mathbf{q}} a_{\mathbf{q}} | \psi^{\Lambda_0}_{\alpha,A} \rangle &= \langle \psi^{\Lambda}_{\alpha,A} | a^\dagger_{\mathbf{q}} a_{\mathbf{q}} + \delta a^\dagger_{\mathbf{q}} a_{\mathbf{q}} + a^\dagger_{\mathbf{q}} \delta a_{\mathbf{q}} + \delta a^\dagger_{\mathbf{q}} \delta a_{\mathbf{q}} | \psi^{\Lambda}_{\alpha,A} \rangle \\
&\approx \langle \psi^{\Lambda}_{\alpha,A} | \delta a^\dagger_{\mathbf{q}} \delta a_{\mathbf{q}} | \psi^{\Lambda}_{\alpha,A} \rangle \\
&\approx \gamma^2(\mathbf{q};\Lambda) \times \sum^{\Lambda}_{\mathbf{k},\mathbf{k}',\mathbf{K}} Z^2_\Lambda \langle \psi^{\Lambda}_{\alpha,A} | a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'} | \psi^{\Lambda}_{\alpha,A} \rangle
\end{aligned}$$

- hard (high q) physics
- Universal (state-indep)
- fixed from A=2

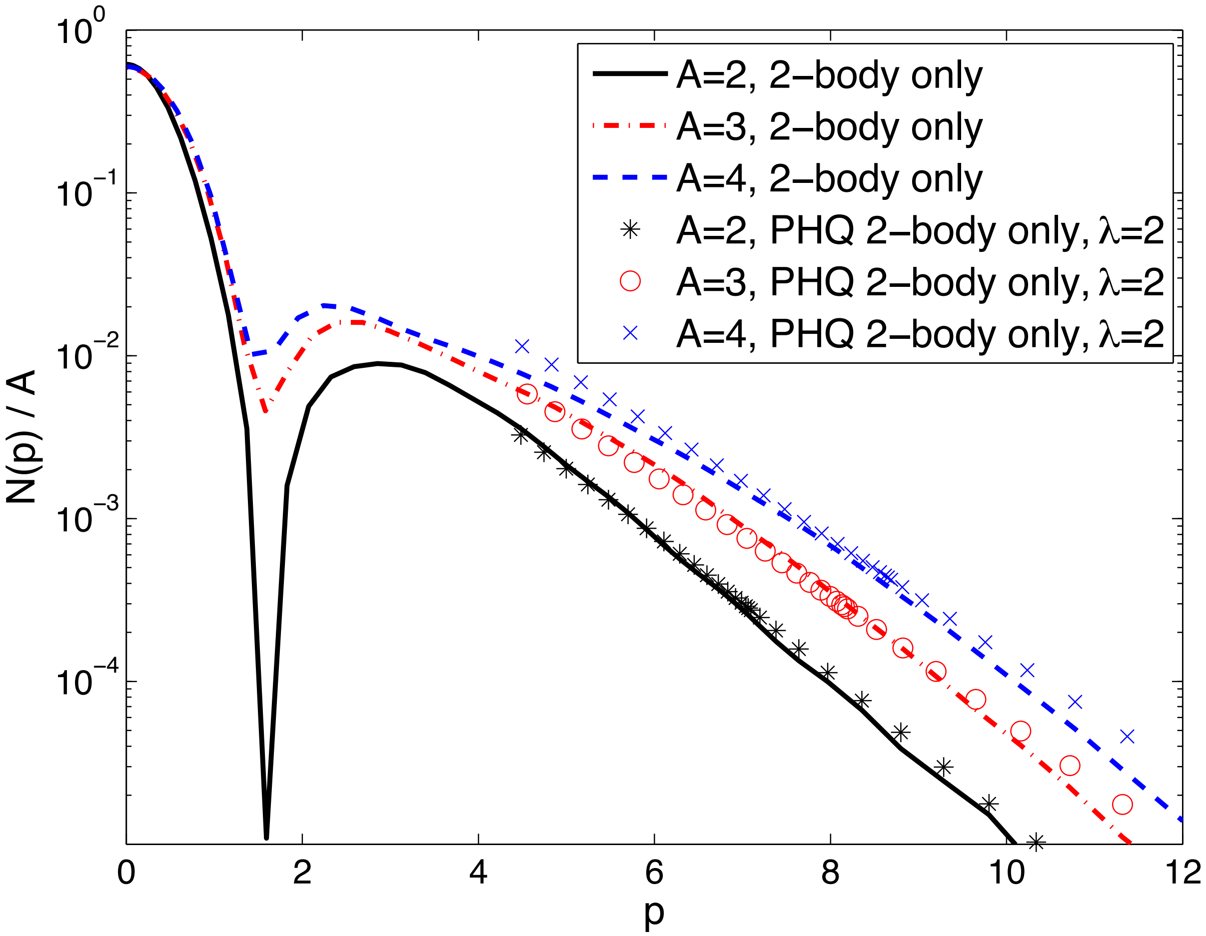
X

- soft (low-k) m.e.
- same for all high-q probes
- A-dependent scale factor

Scaling of high momentum tails



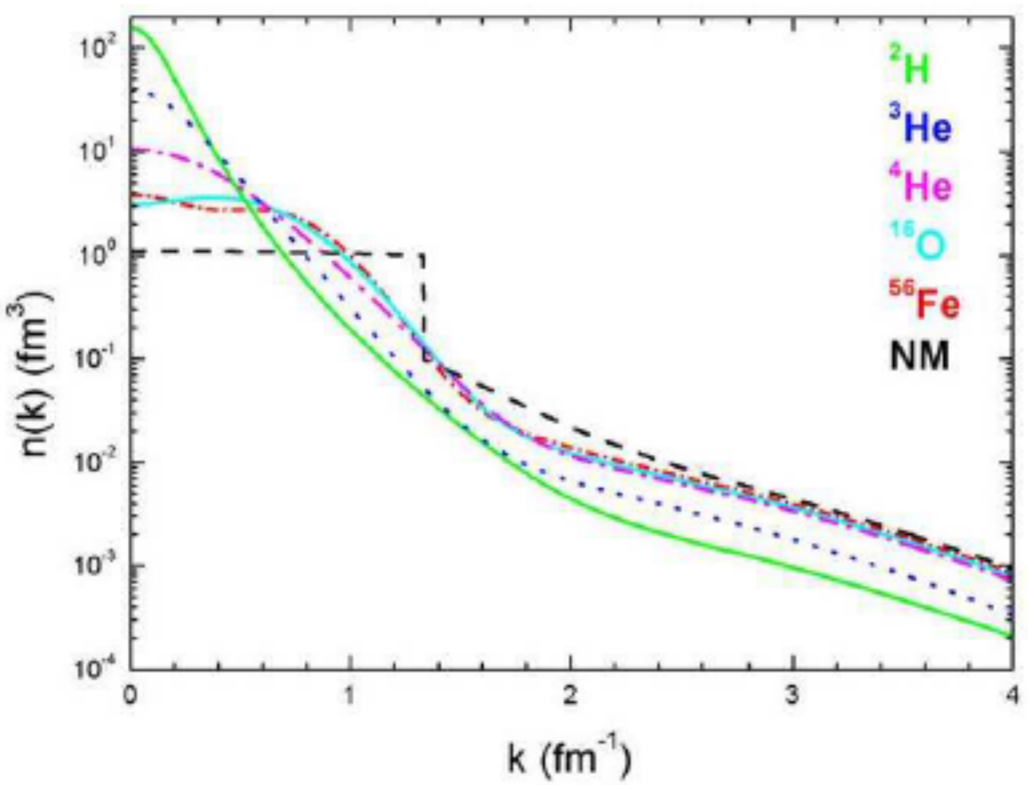
[From C. Ciofi degli Atti and S. Simula]



natural explanation why high-q tails scale

$$C(A,2) \equiv \frac{n_A(\mathbf{q})}{n_D(\mathbf{q})} \sim \frac{\sum_{\mathbf{k},\mathbf{k}',\mathbf{K}} \langle \psi^{\Lambda}_{\alpha,A} | a^{\dagger}_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^{\dagger}_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'} | \psi^{\Lambda}_{\alpha,A} \rangle}{\sum_{\mathbf{k},\mathbf{k}',\mathbf{K}} \langle \psi^{\Lambda}_{\alpha,D} | a^{\dagger}_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^{\dagger}_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'} | \psi^{\Lambda}_{\alpha,D} \rangle}$$

Scaling of high momentum tails



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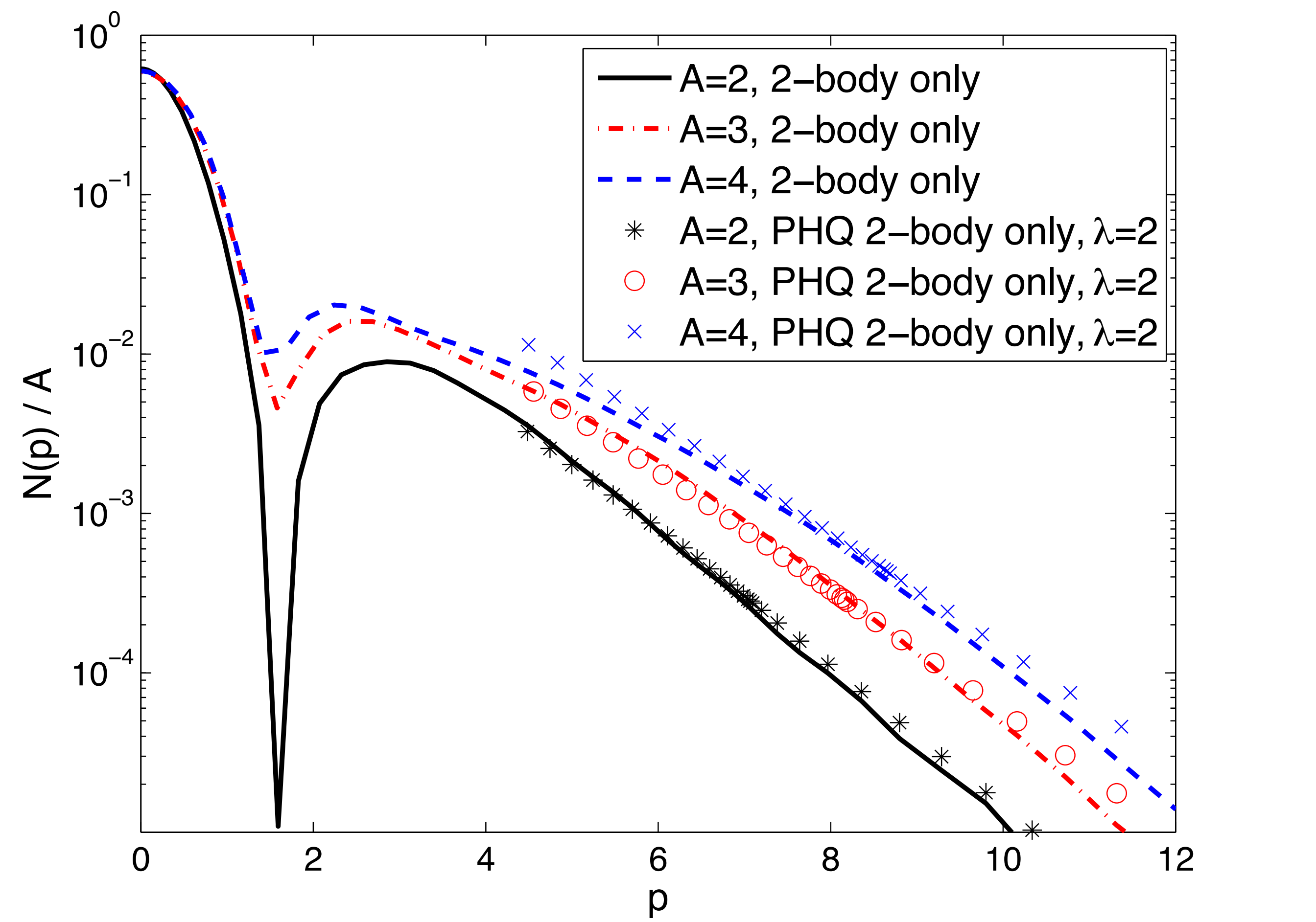

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cf. "contact formalism" of Barnea et al (see Or Hen's talk)

Scaling of high momentum tails

E.g., static structure functions $\widehat{S}(\mathbf{q}) = \widehat{\rho}^\dagger(\mathbf{q})\widehat{\rho}(\mathbf{q})$

$$\langle \psi^{\Lambda_0}_{\alpha,A}|\widehat{S}(\mathbf{q})|\psi^{\Lambda_0}_{\alpha,A}\rangle \approx \left\{2\gamma(\mathbf{q};\Lambda) + \sum_{\mathbf{P}} \gamma(\mathbf{P}+\mathbf{q};\Lambda)\gamma(\mathbf{P};\Lambda)\right\}$$

$$\times \sum_{\mathbf{K},\mathbf{k},\mathbf{k}'}^{\Lambda} Z^2_\Lambda \langle \psi^{\Lambda}_{\alpha,A}|a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}} a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}'}|\psi^{\Lambda}_{\alpha,A}\rangle$$

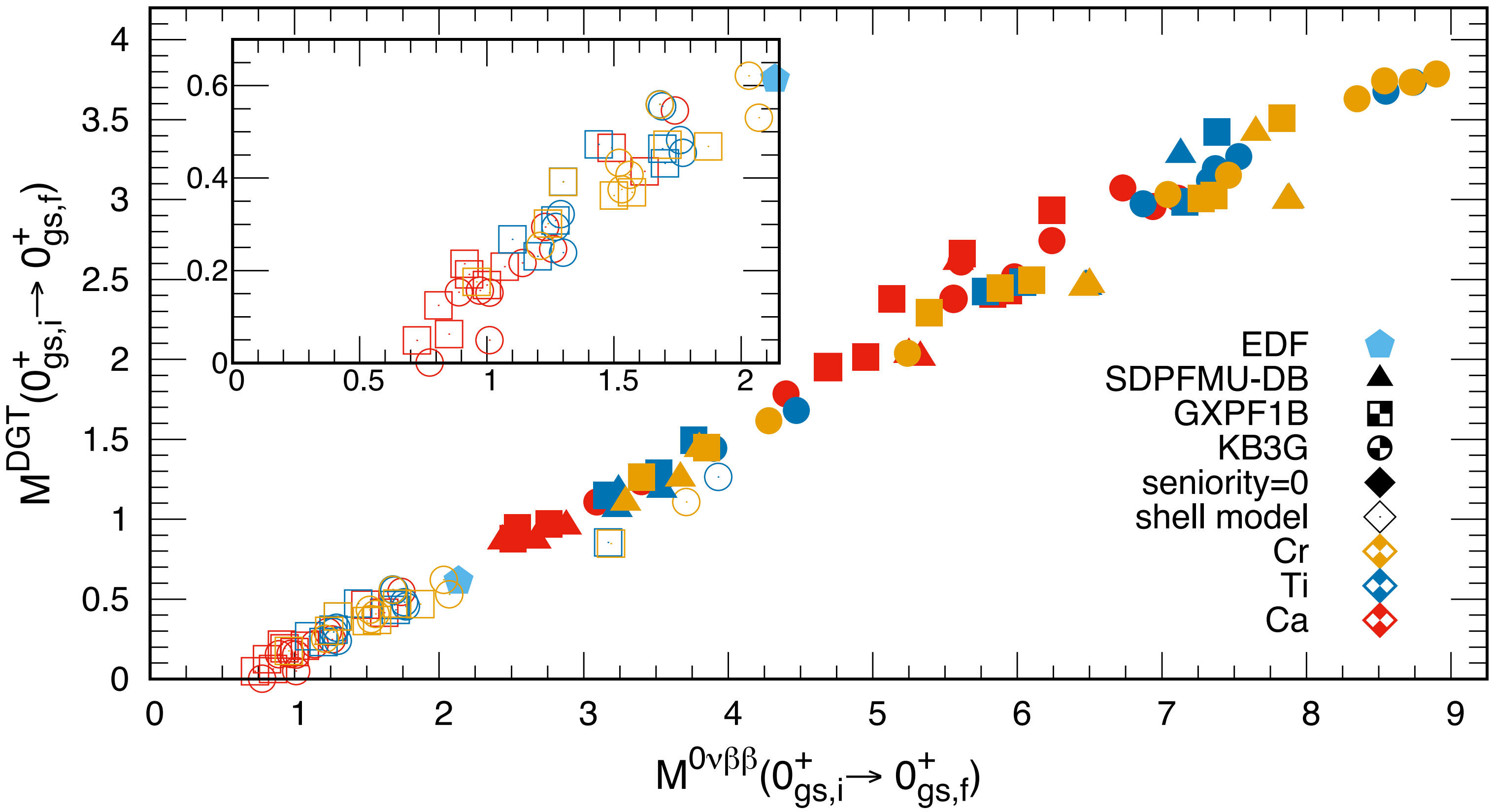
Universal (state-indep) q-dependence => connects few-body and A-body

State dependence encoded in low-k m.e. =>

linear correlations between observables with same leading OPE

Ex: 0νββ and Double GT correlation

Shimizu, Menendez, Yako PRL 120 (2018)



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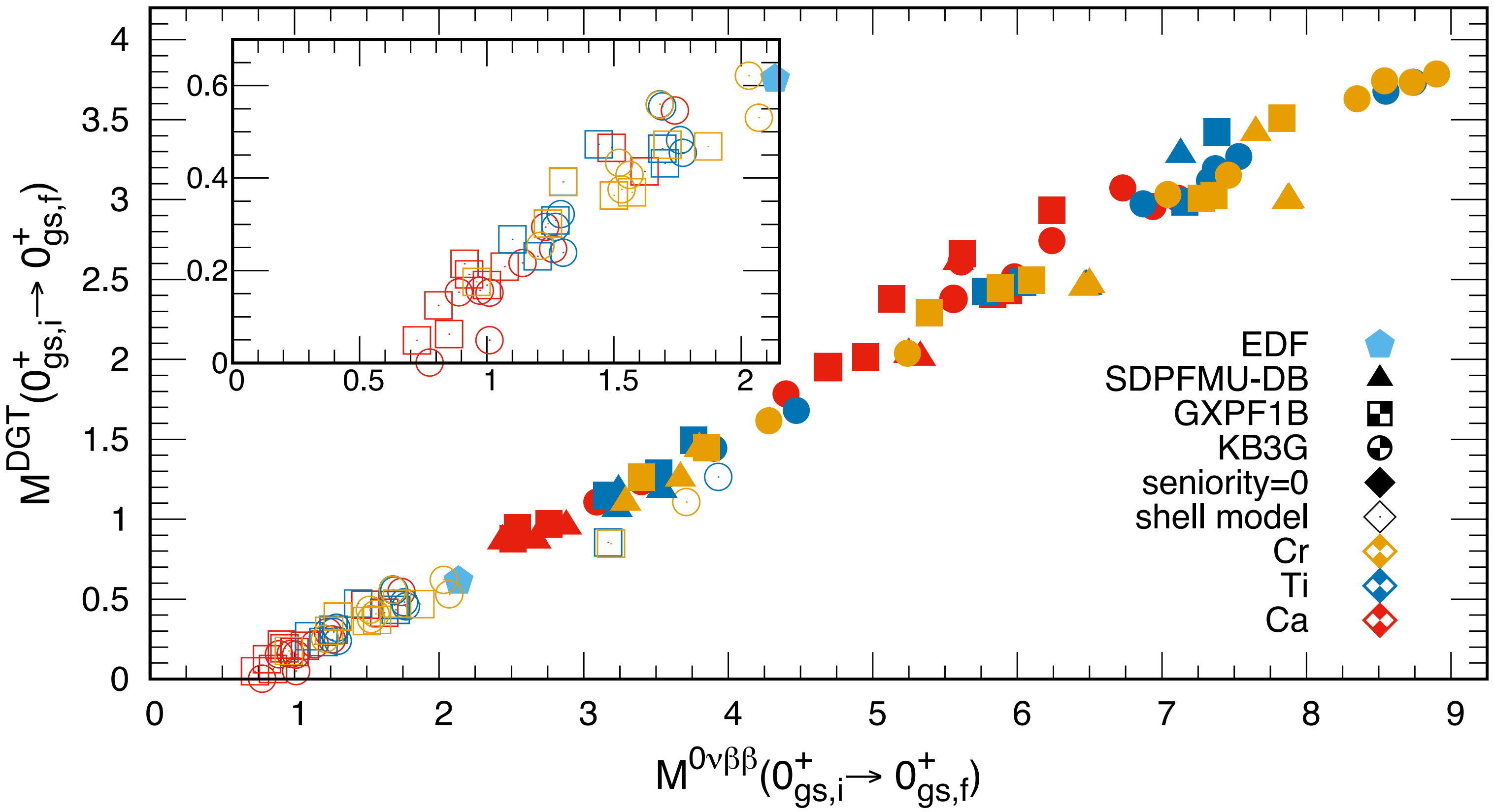

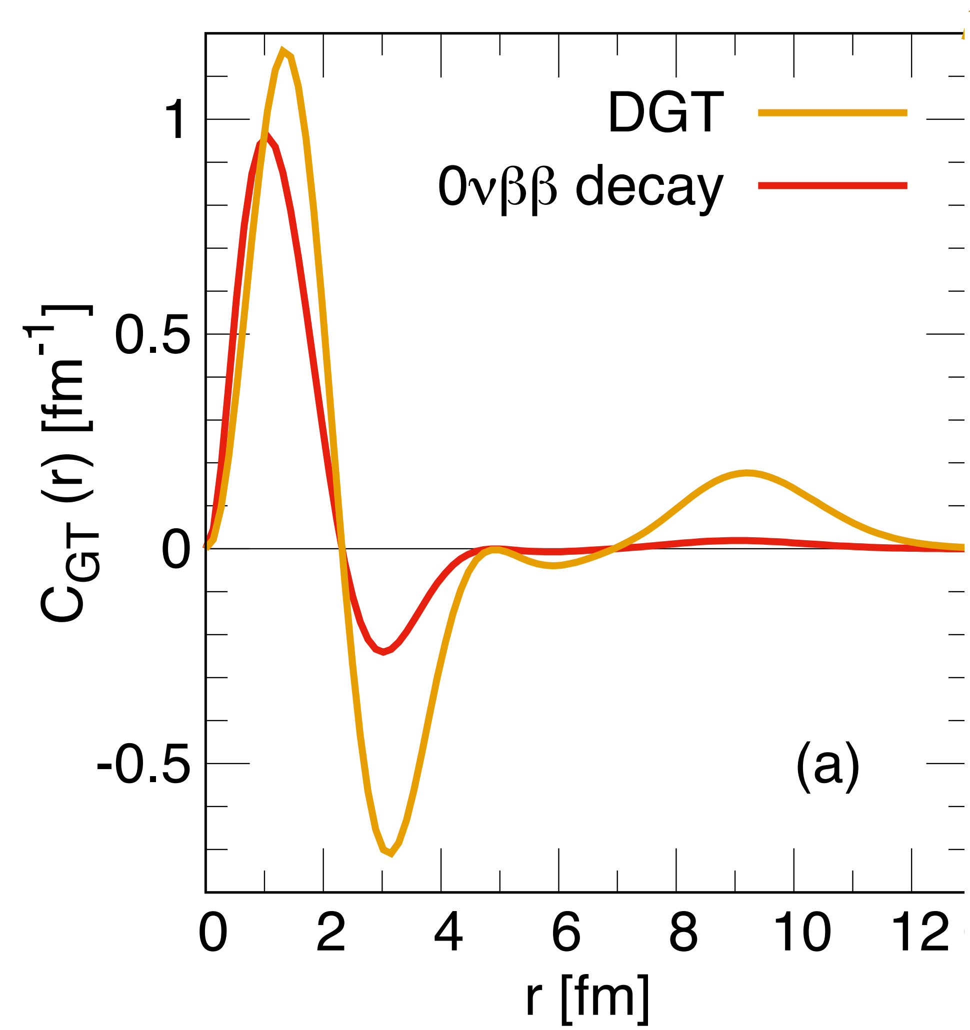

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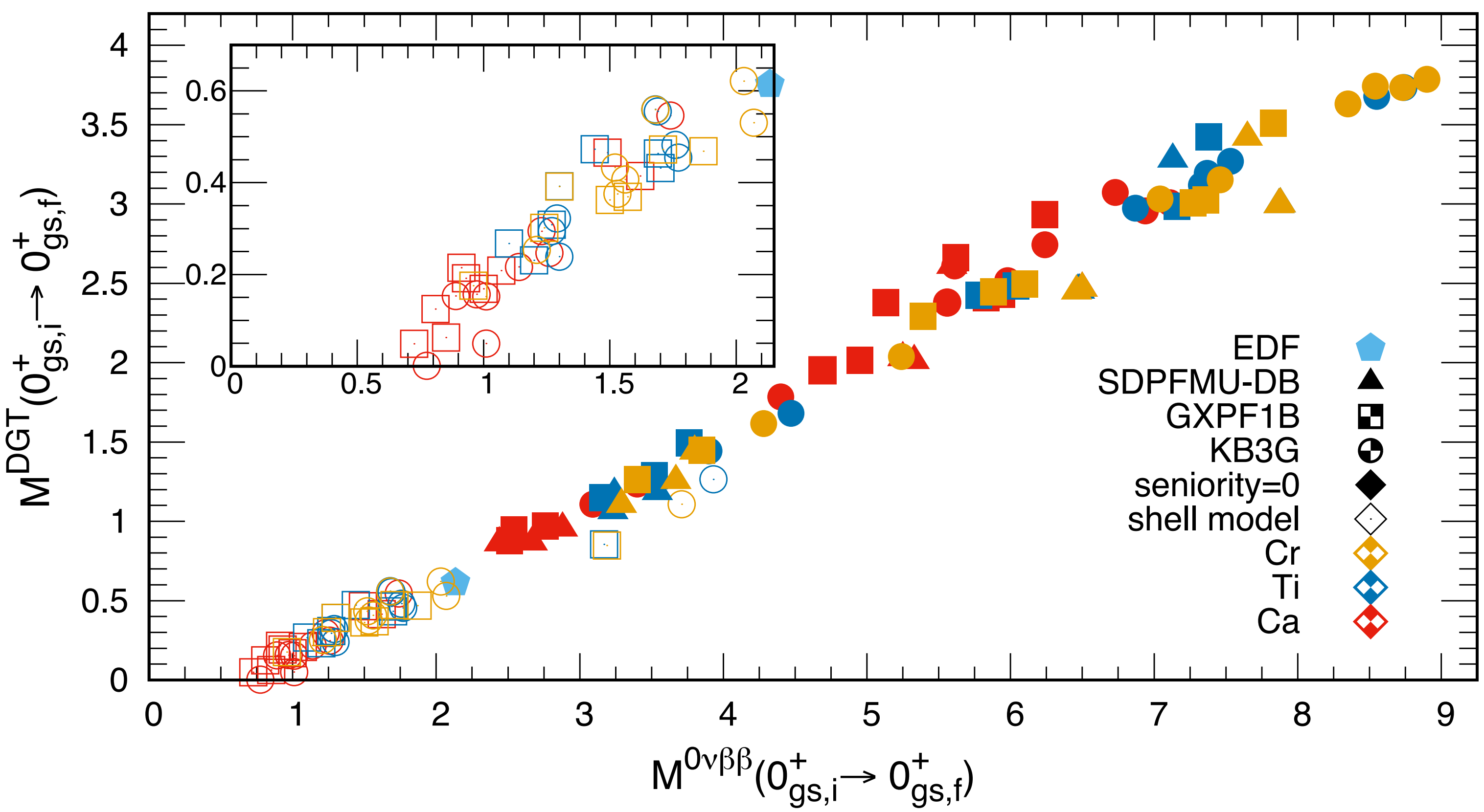
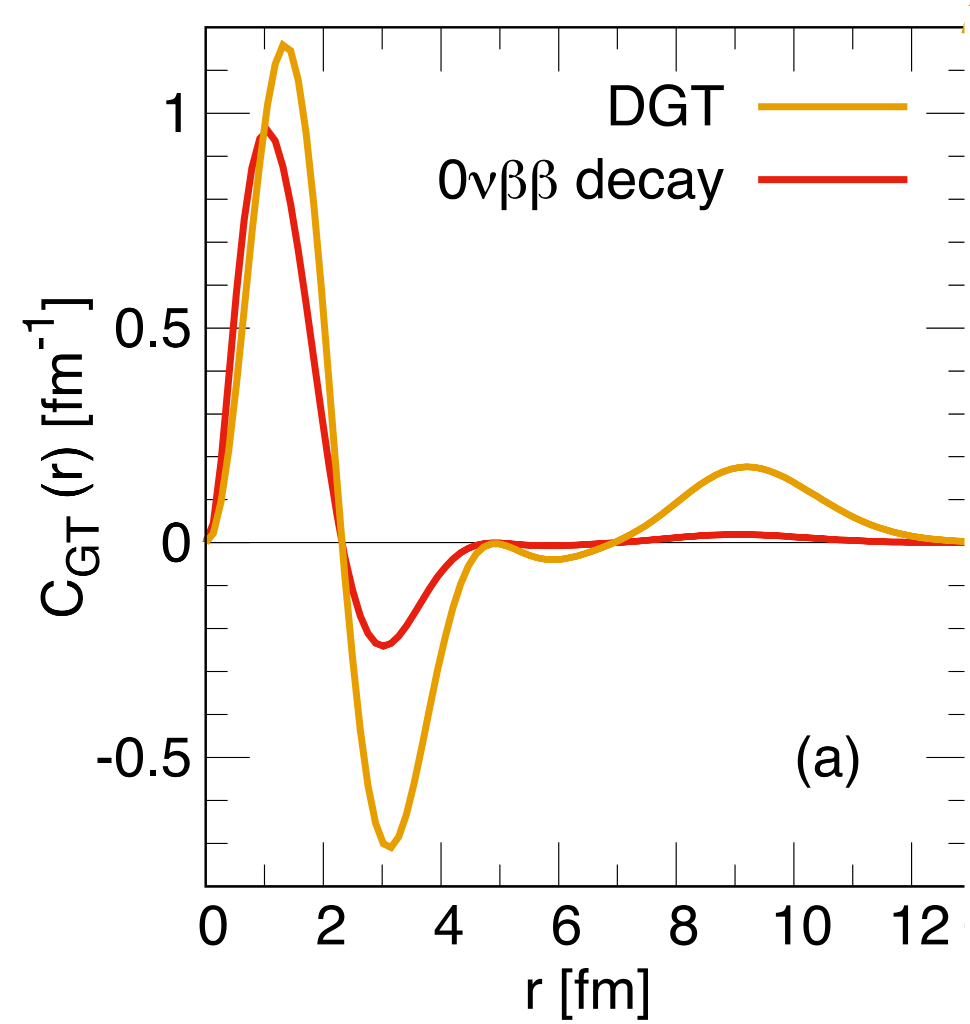
Shimizu, Menendez, Yako PRL 120 (2018)



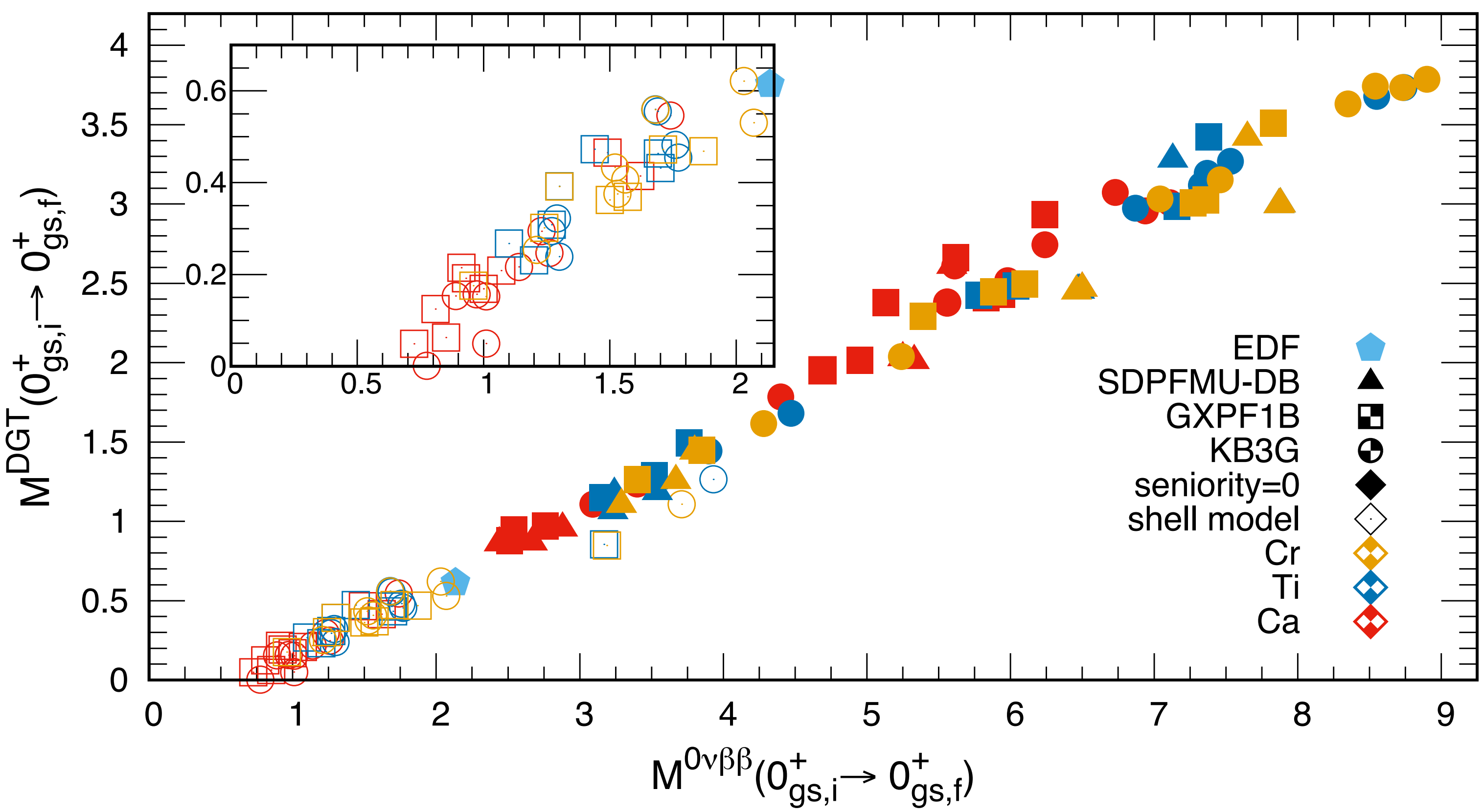


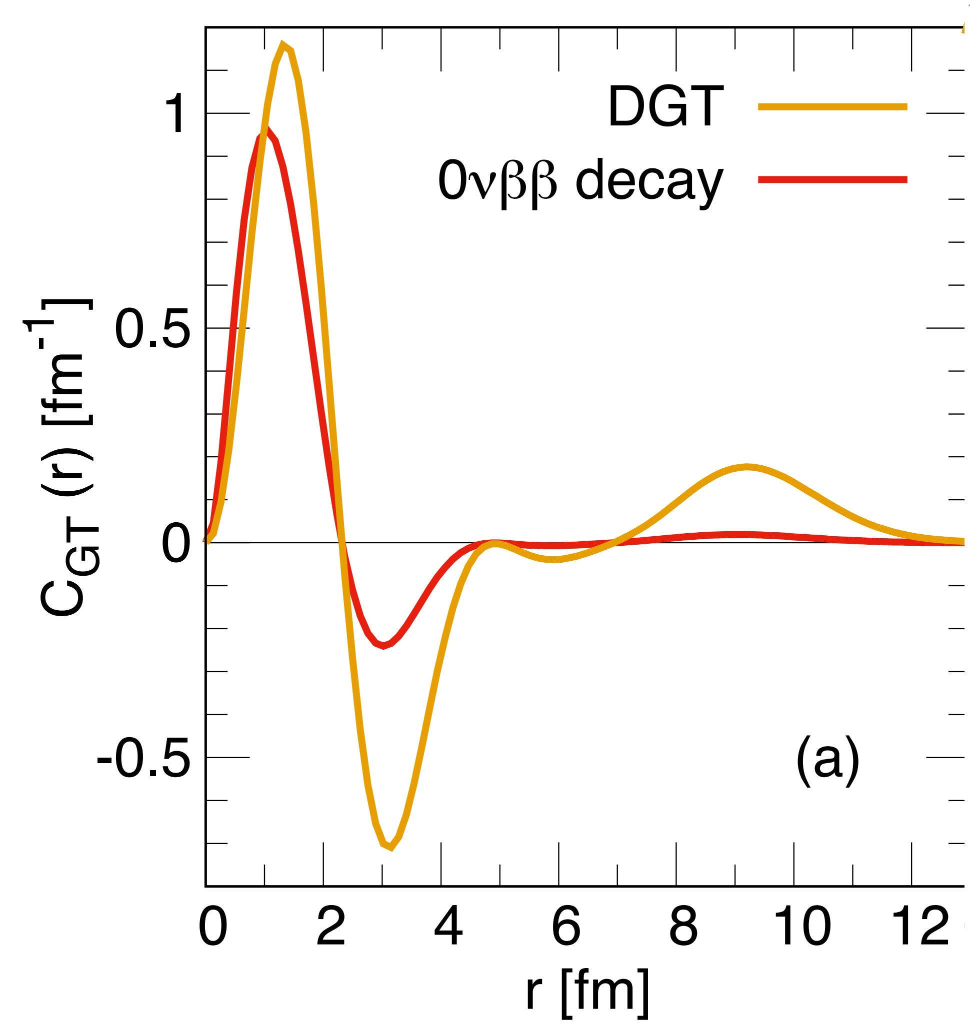
DGT and 0νββ operators main contribution for r < 2 fm

Same leading operator in OPE => linear relation

Ex: 0νββ and Double GT correlation

Shimizu, Menendez, Yako PRL 120 (2018)





DGT and 0νββ operators main contribution for r < 2 fm

Wilson Coeff's from A=2

Same leading operator in OPE => linear relation

slope at ~10-15% level

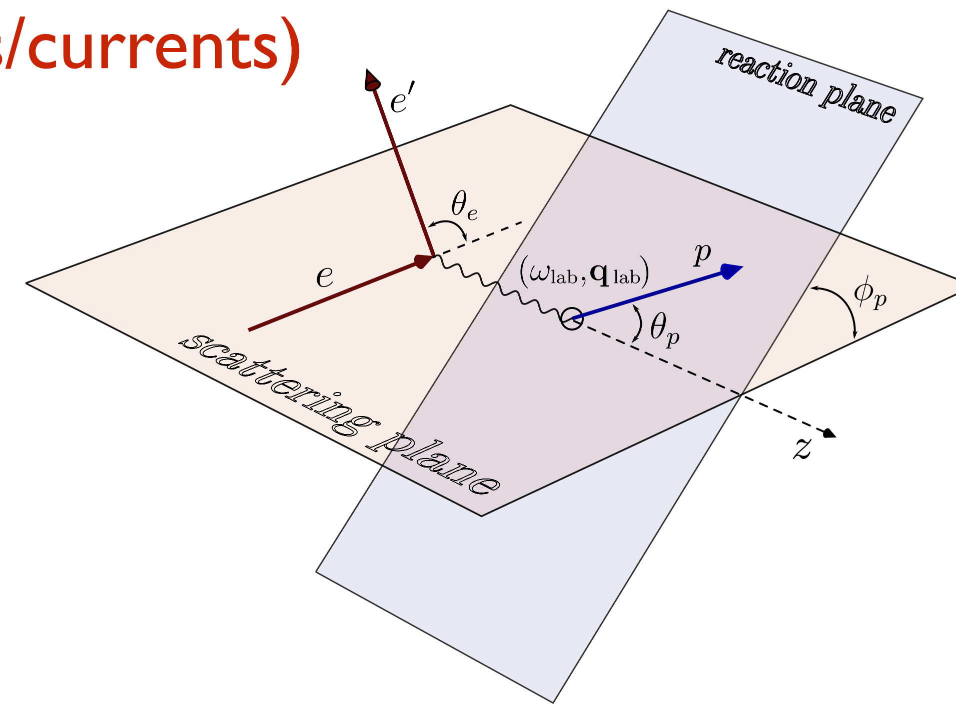
Running summary part 1

- Simple decoupling + scale separation arguments generically give the form of effective operators softened by OLS, SRG, …
- Can we use scaling of A-body tails w.r.t. few-body systems to constrain the form of short-distance contributions to NMEs and other quantities?
- Can we use factorization/OPE-like arguments to identify quantities that correlate w/0vBB NME?
- How do interpretations change as Λ varied by RG transformations? (See part 2)

Scale dependence of deuteron electrodisintegration

Test ground: ^{2}H(e,e′p)n

- Simplest knockout process (no induced 3N forces/currents)
- Focus on longitudinal structure function f_L

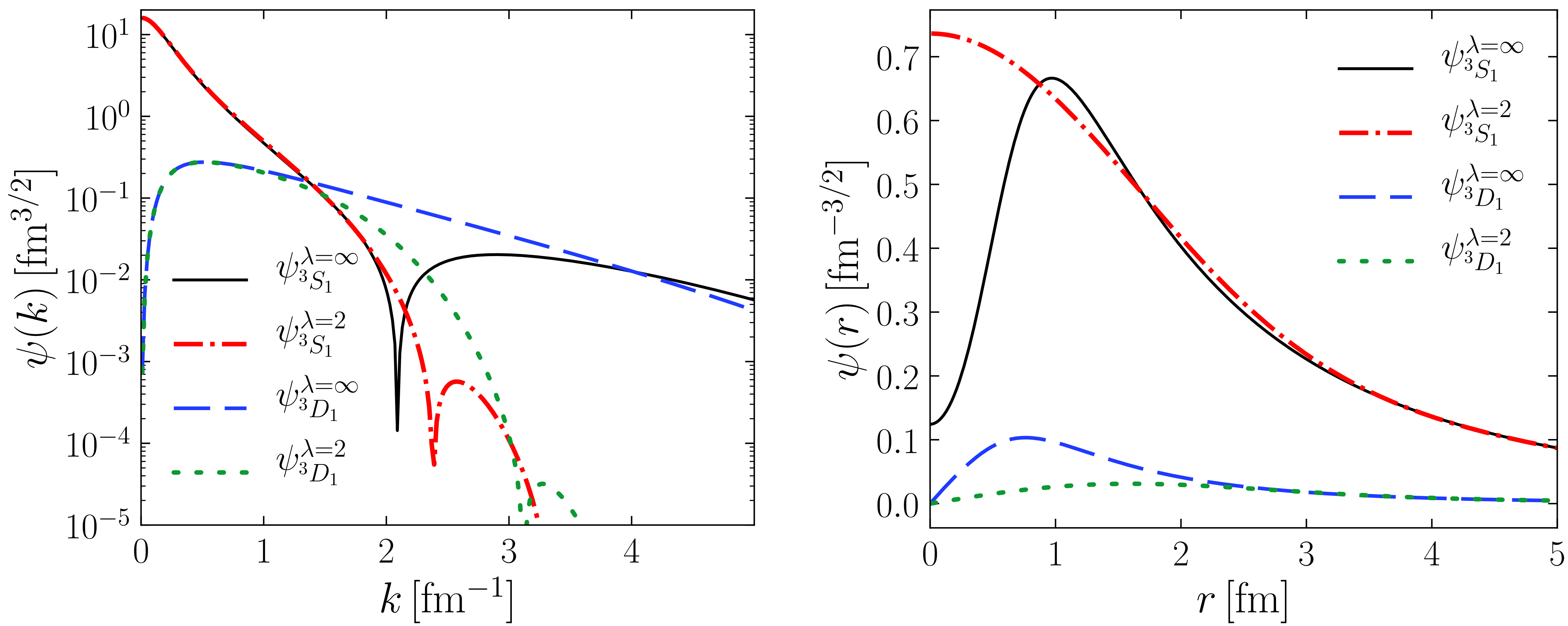


$$f_L \sim \sum_{m_s, m_J} \left|\langle \psi_f | J_0 | \psi_i \rangle\right|^2$$

- $$f_L^\lambda \sim \left|\underbrace{\langle \psi_f | U_\lambda^\dagger}_{\psi_f^\lambda} \underbrace{U_\lambda J_0 U_\lambda^\dagger}_{J_0^\lambda} \underbrace{U_\lambda | \psi_i \rangle}_{\psi_i^\lambda}\right|^2; \quad U_\lambda^\dagger U_\lambda = I; \quad f_L^\lambda = f_L$$

- Components (deuteron wf, transition operator, FSI) scale-dependent, total is not.
- Are some resolutions “better” than others? E.g., in a given kinematics, can FSI be minimized with different choices of λ??

Deuteron wave function evolution



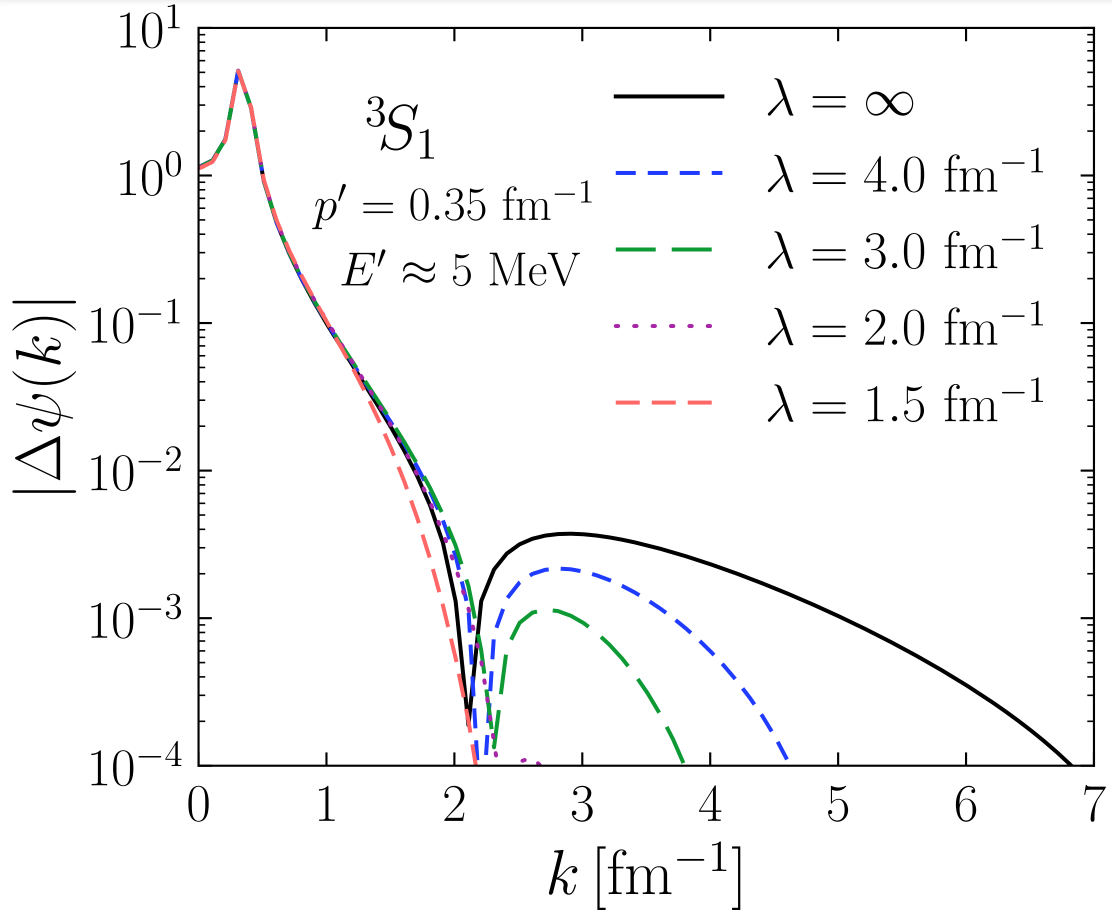
k < λ components invariant <==> RG preserves long-distance physics

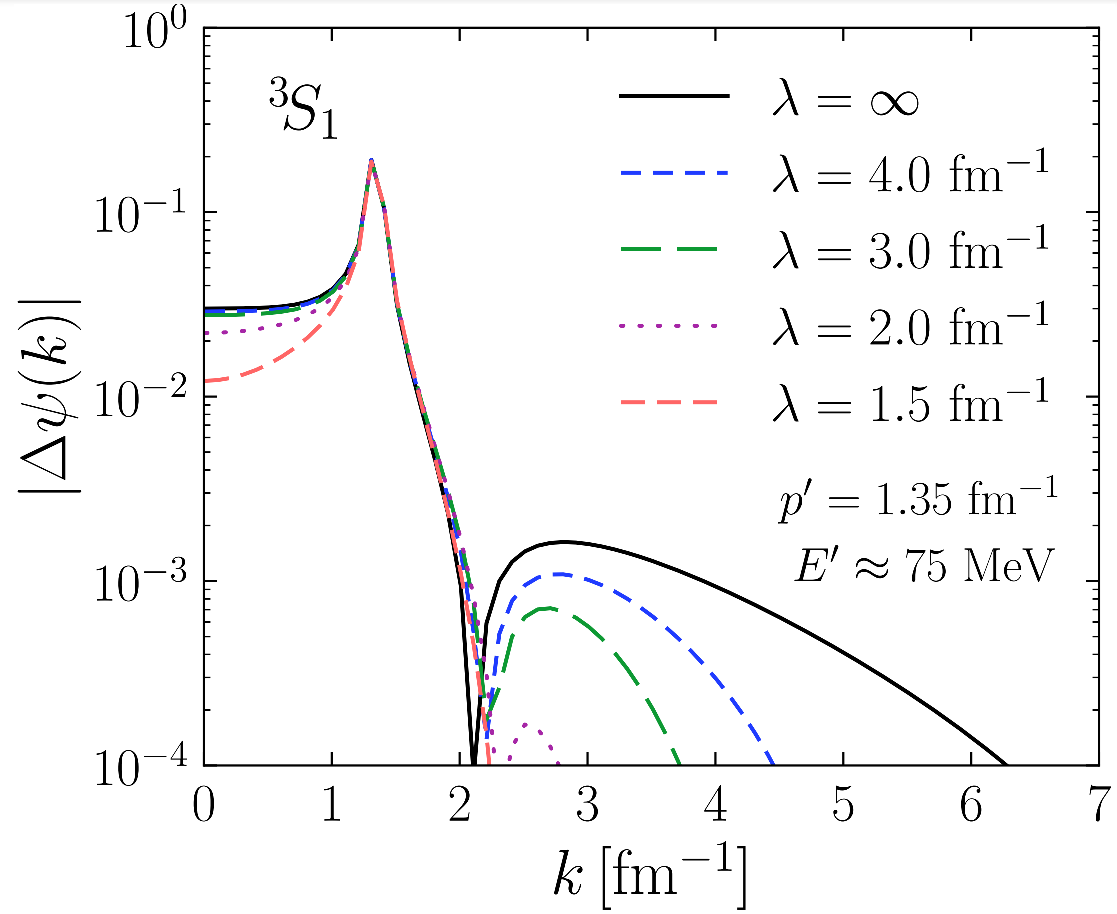
k > λ components suppressed <==> short-range correlations blurred out

Folklore: Simple wave functions at low λ <==> more complicated operators?

especially for high-q processes?

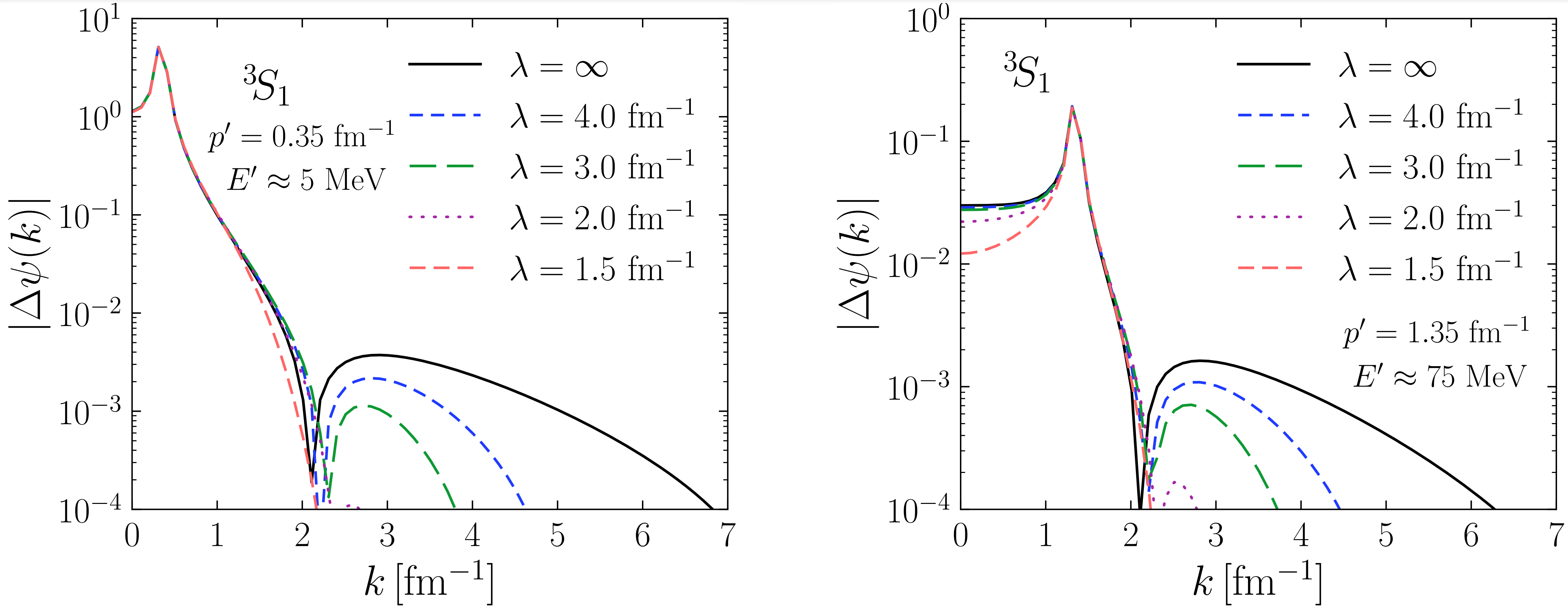
Final-state wave function evolution





$$\psi_f^\lambda(p';k) = \underbrace{\phi_{p'}}_{\text{IA}} + \underbrace{\Delta\psi_\lambda(p';k)}_{\text{FSI}}$$

Final-state wave function evolution

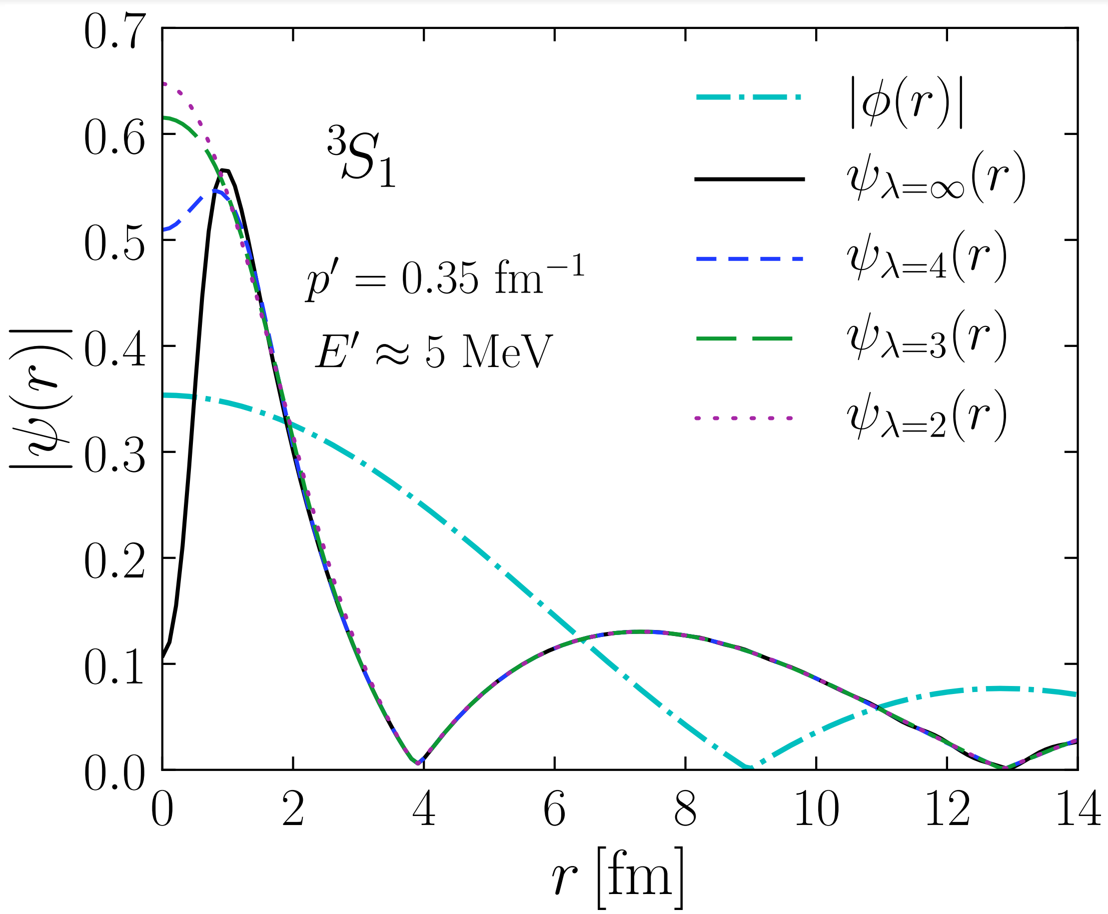


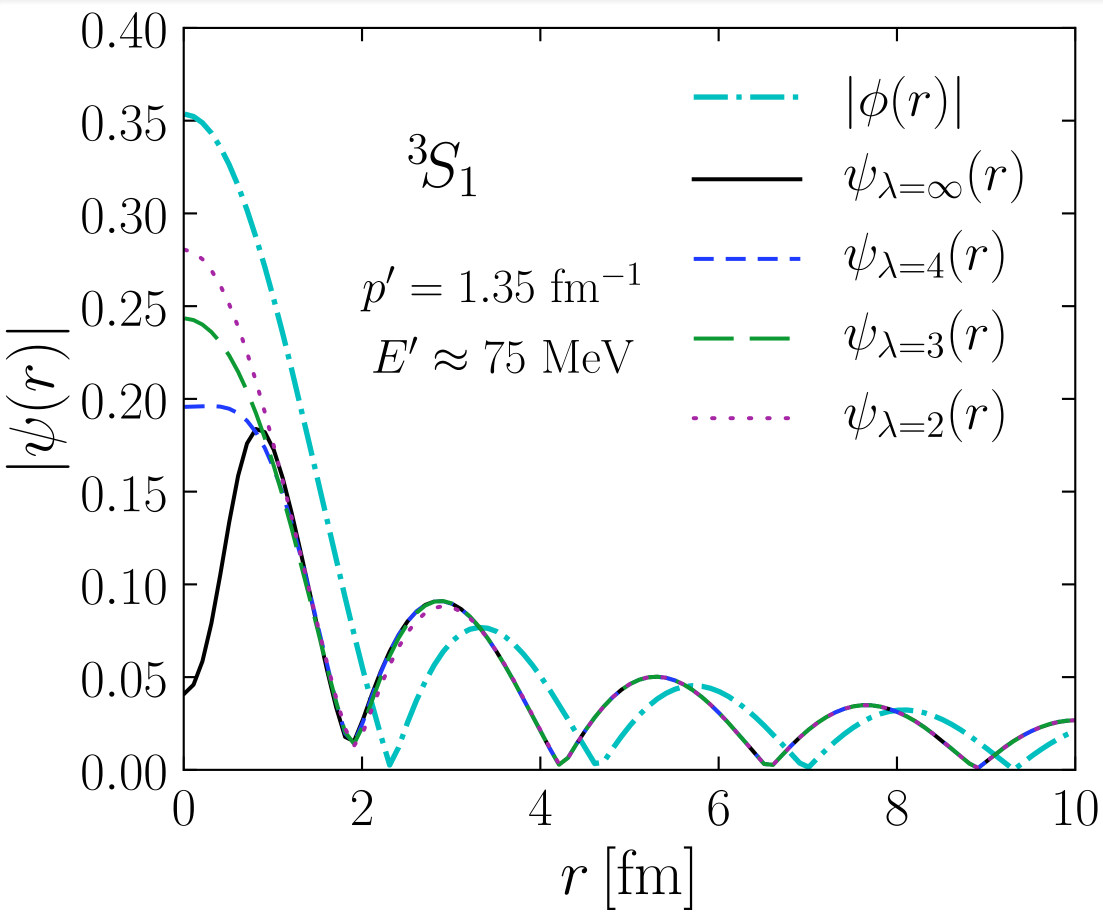
$$\psi_f^\lambda(p';k) = \underbrace{\phi_{p'}}_{\text{IA}} + \underbrace{\Delta\psi_\lambda(p';k)}_{\text{FSI}}$$

- High-k tail suppressed with evolution
- For $p' \gtrsim \lambda$, $\Delta\psi_f^\lambda(p';k)$ localized around outgoing p'

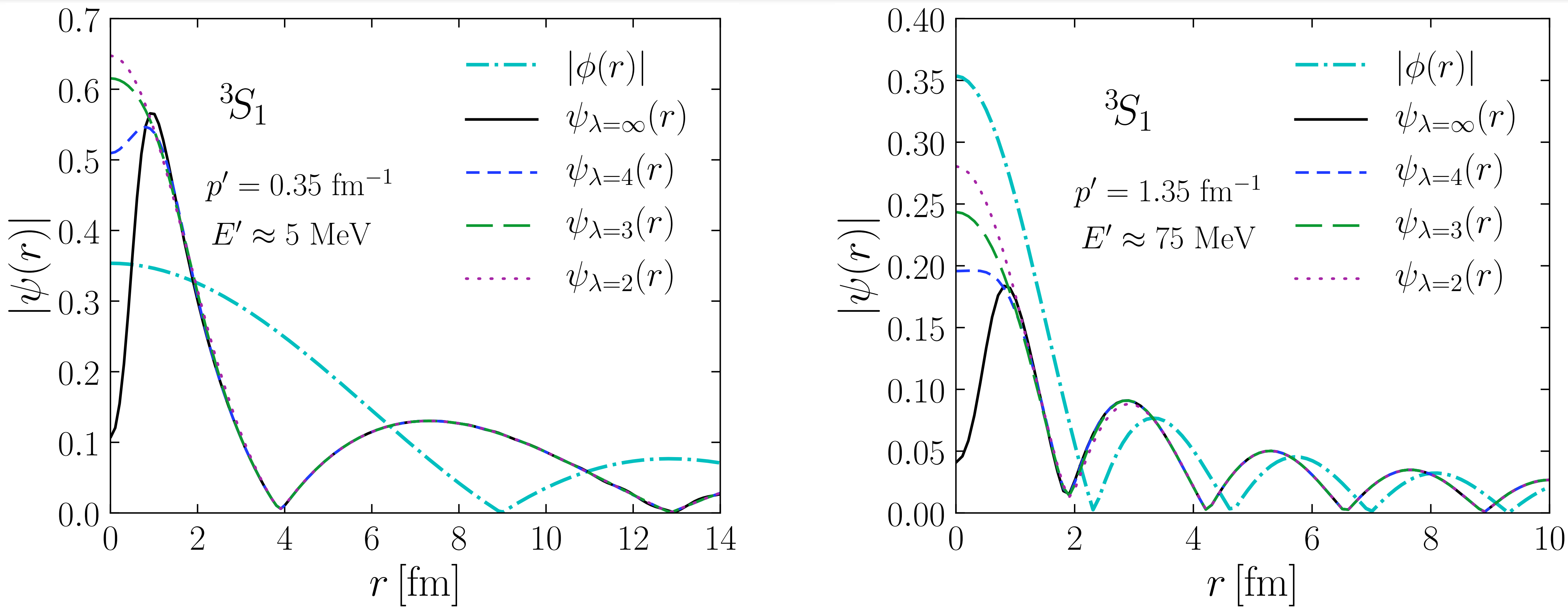
"local decoupling" Dainton et al. PRC 89 (2014)

Final-state wave function evolution





Final-state wave function evolution

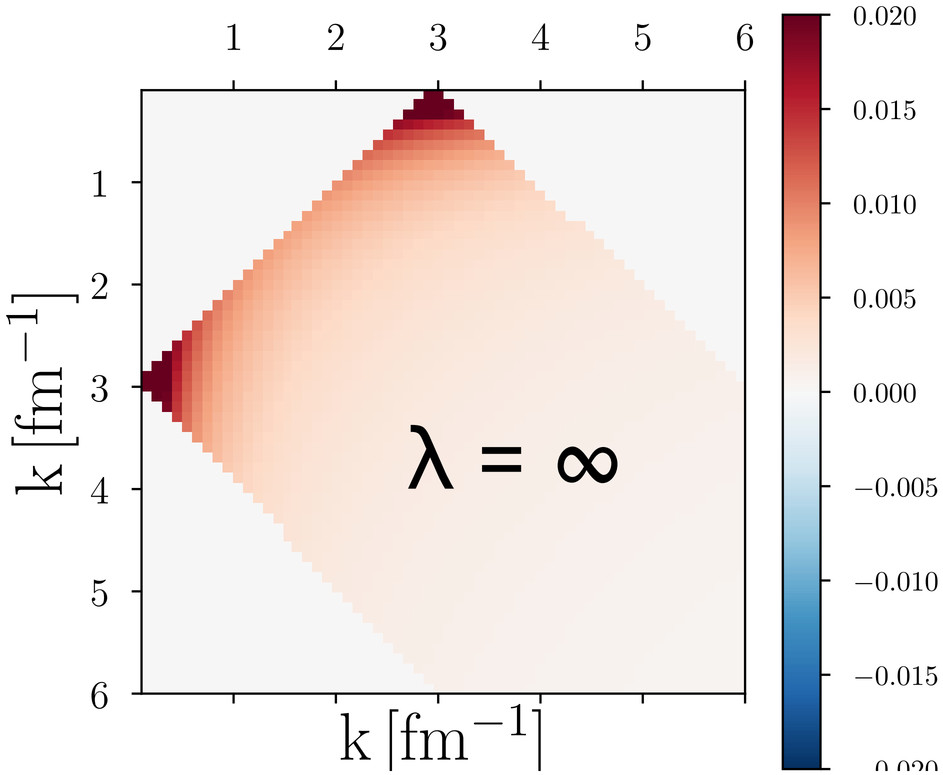


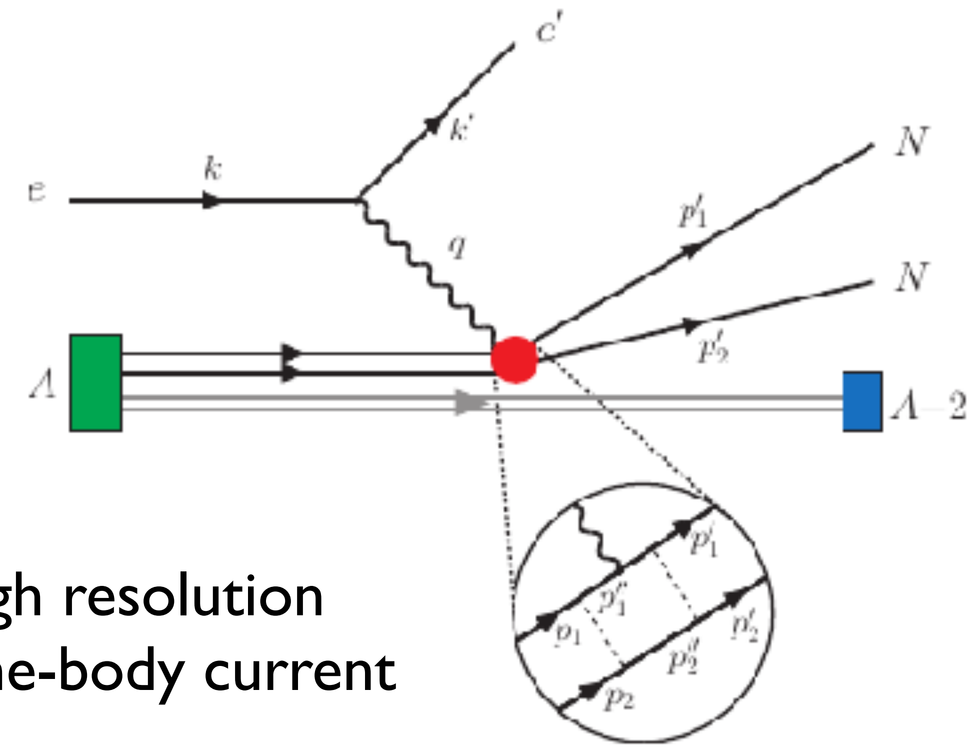
- Correlation "wound" at small r smoothed out under evolution
- Long-distance tail invariant (phase shifts preserved)

Current operator evolution

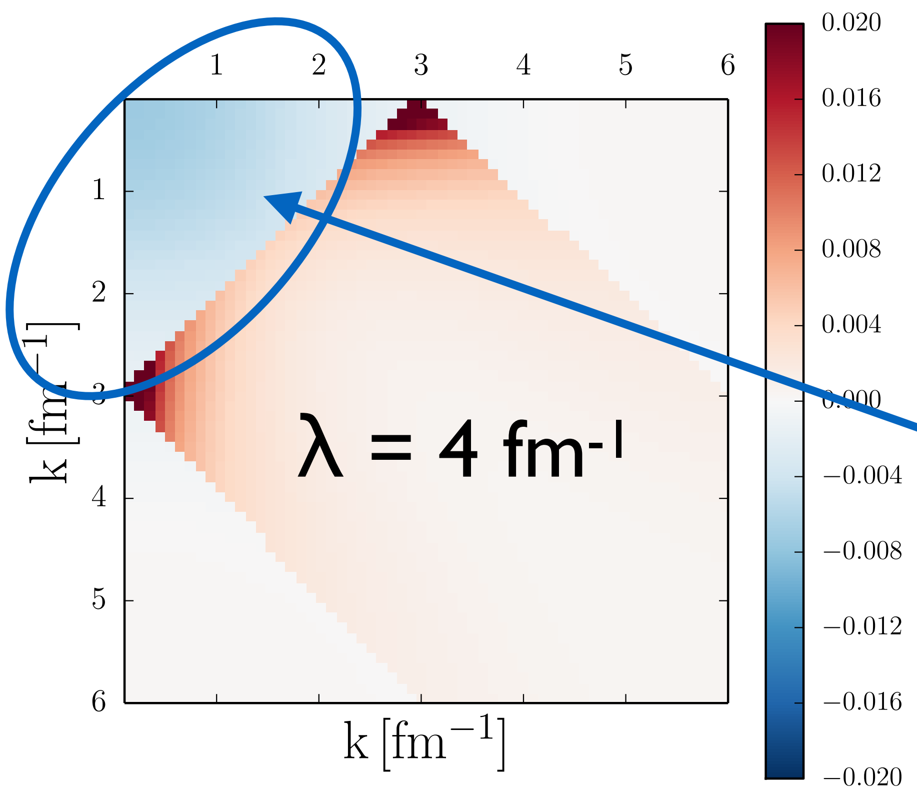
$^3S_1$ channel

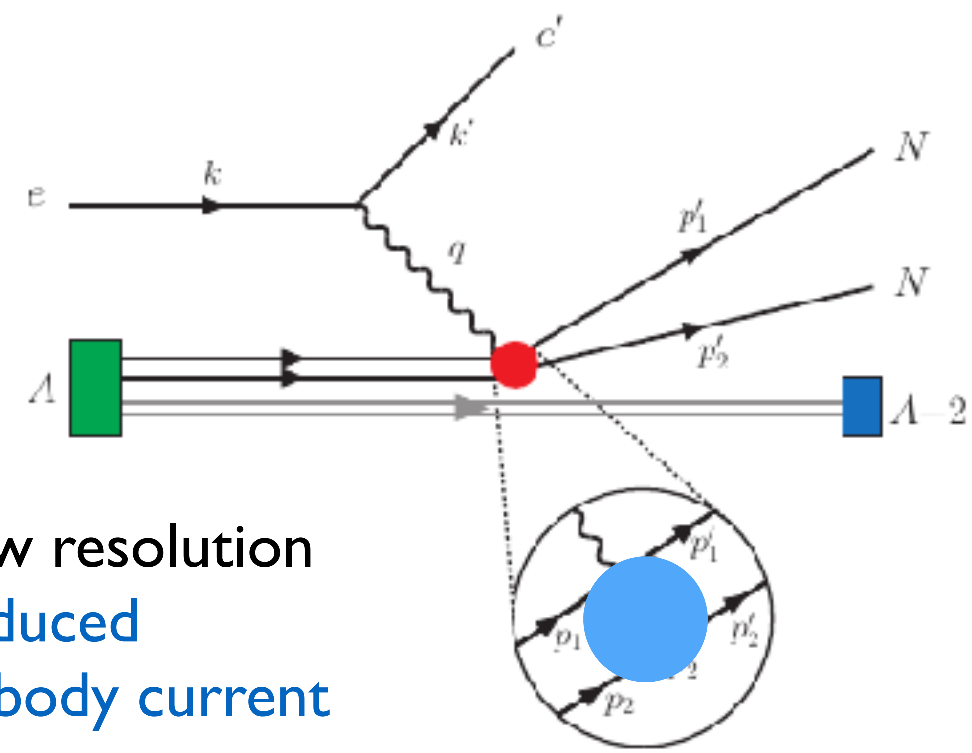
$q^2 = 36\ \mathrm{fm}^{-2}$





high resolution
one-body current



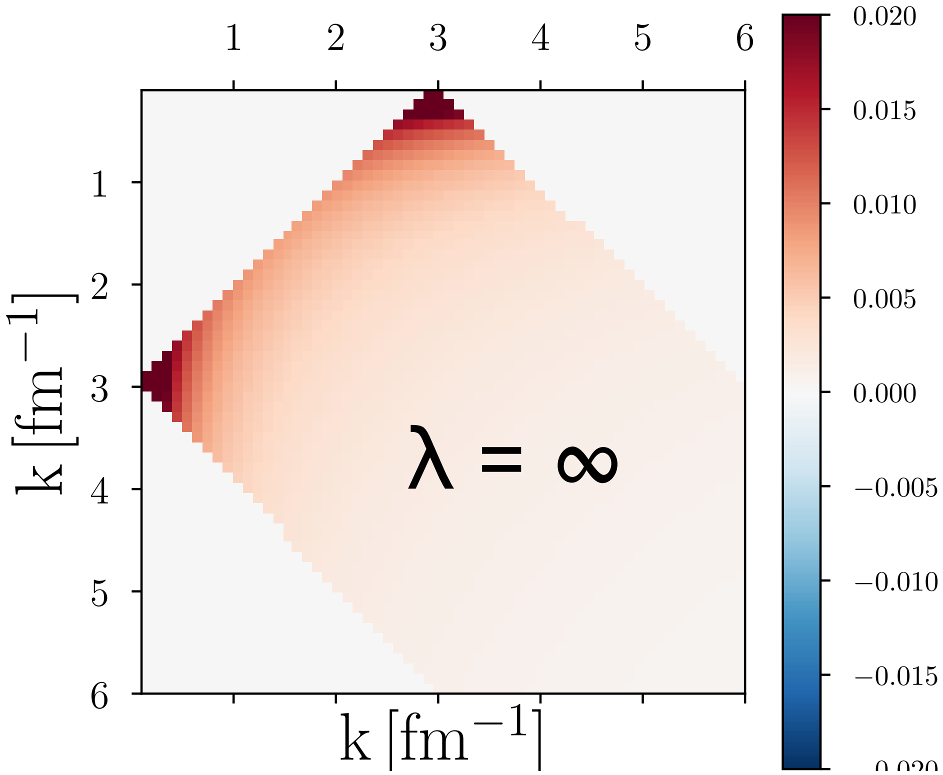


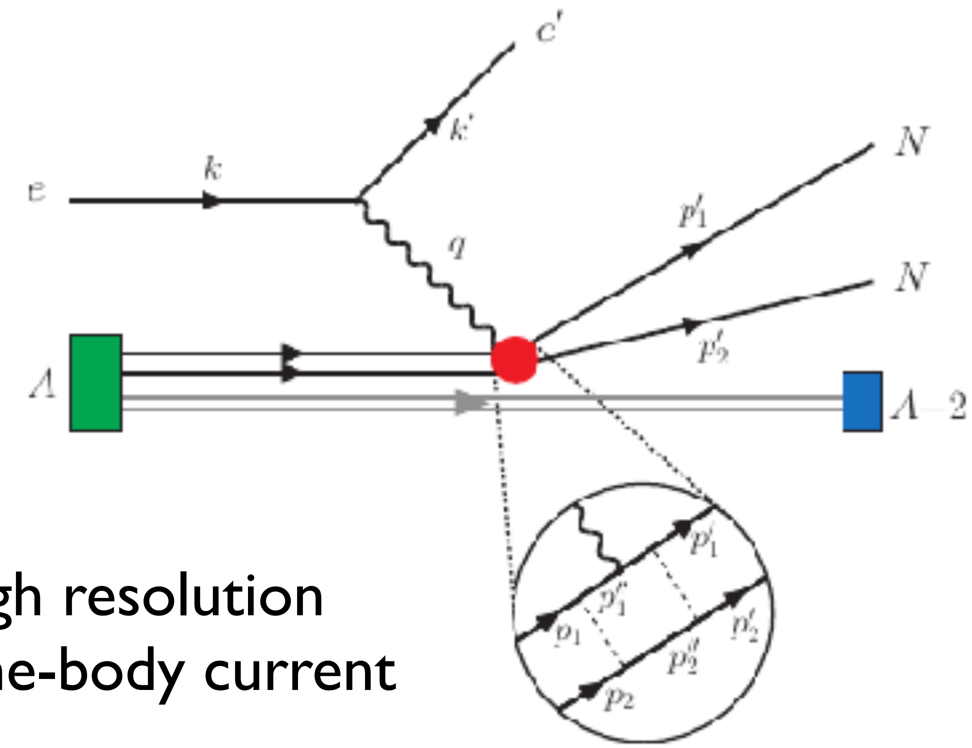
low resolution
induced
2-body current

Current operator evolution

$^3S_1$ channel

$q^2 = 36\ fm^{-2}$





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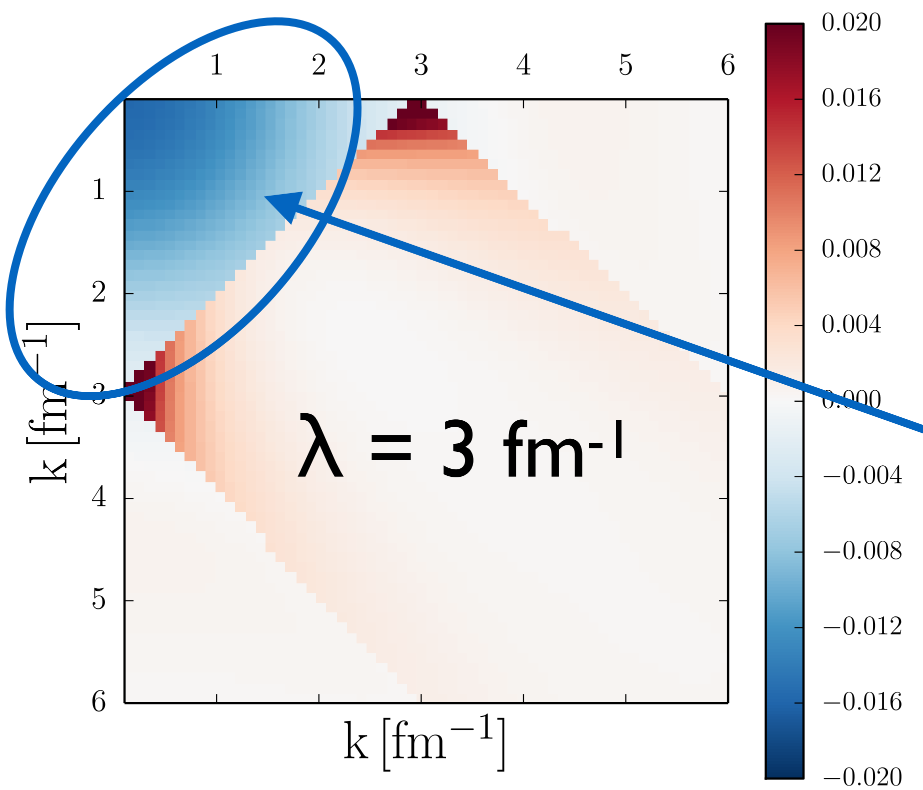

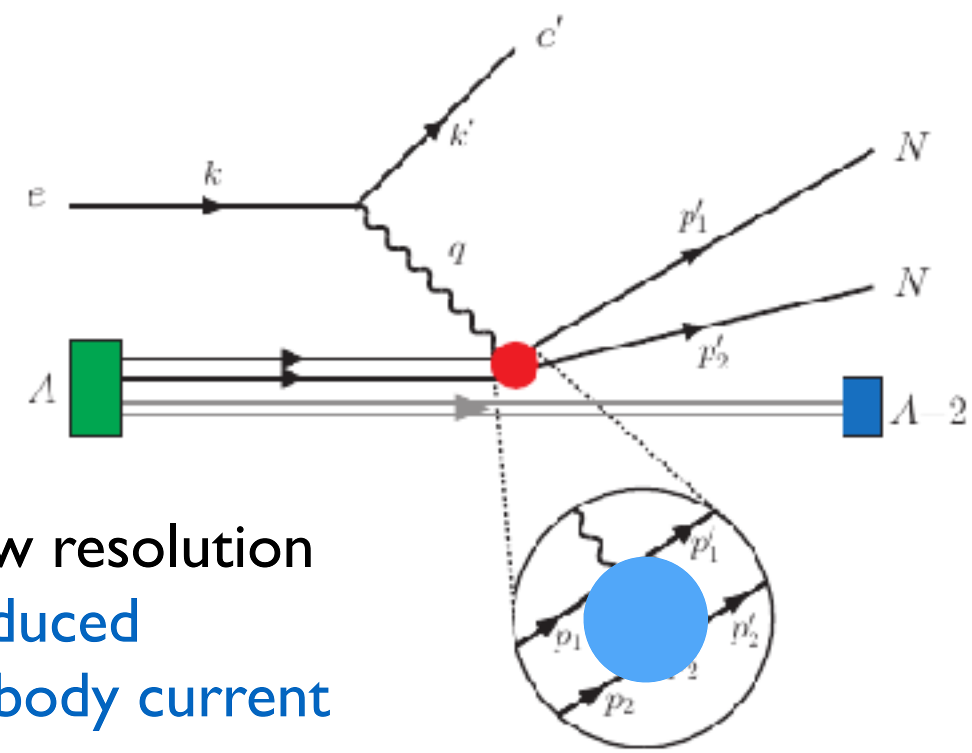

low resolution
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Current operator evolution

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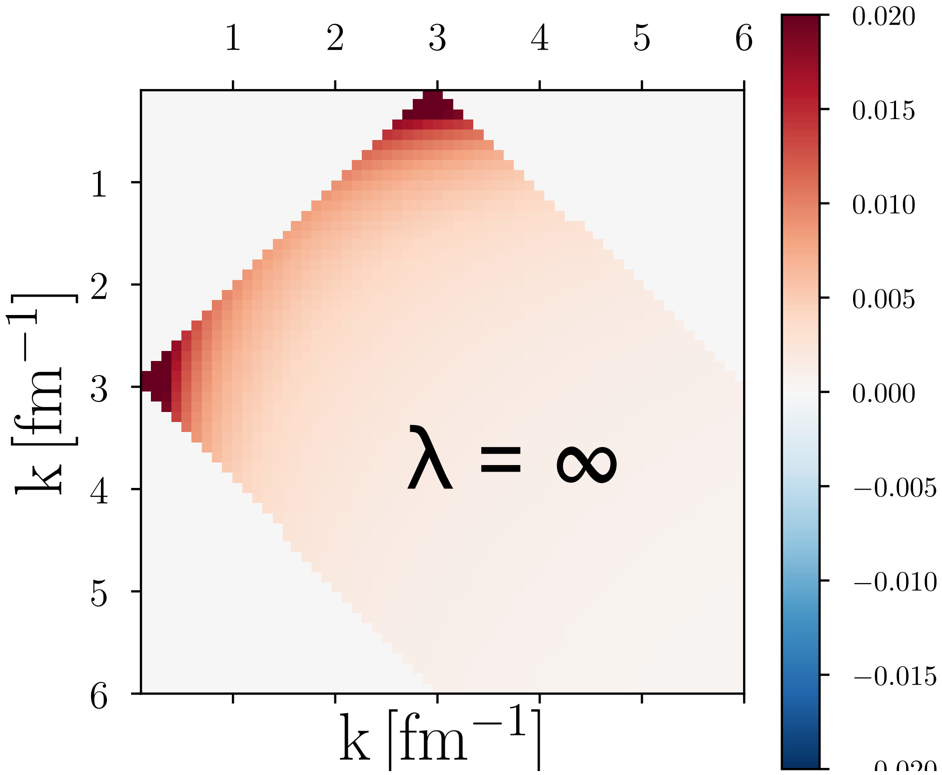
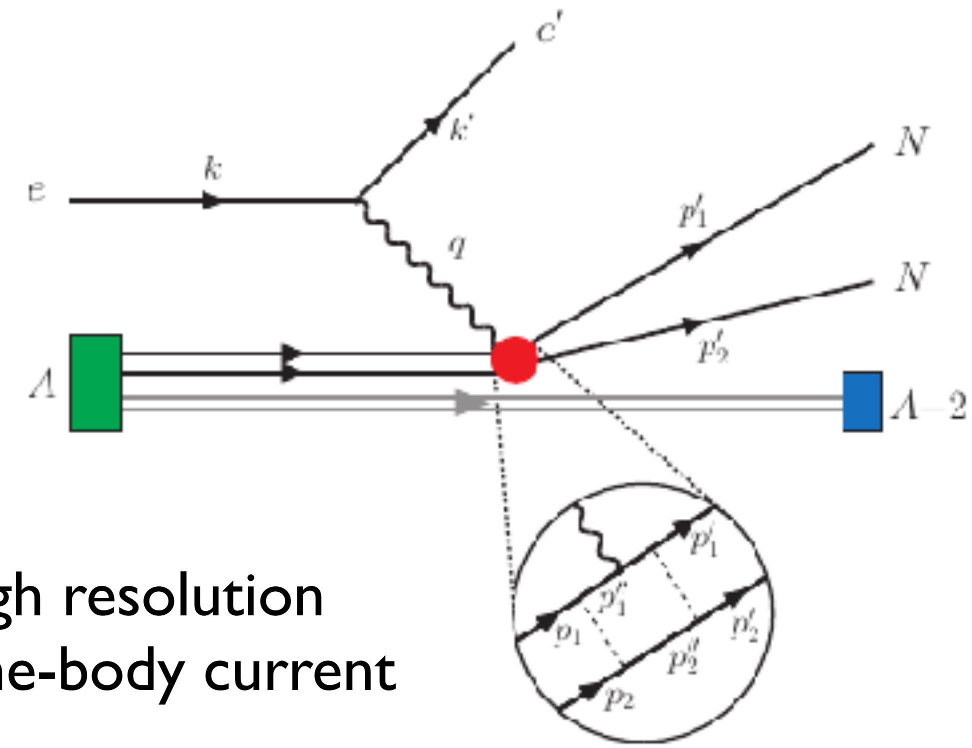
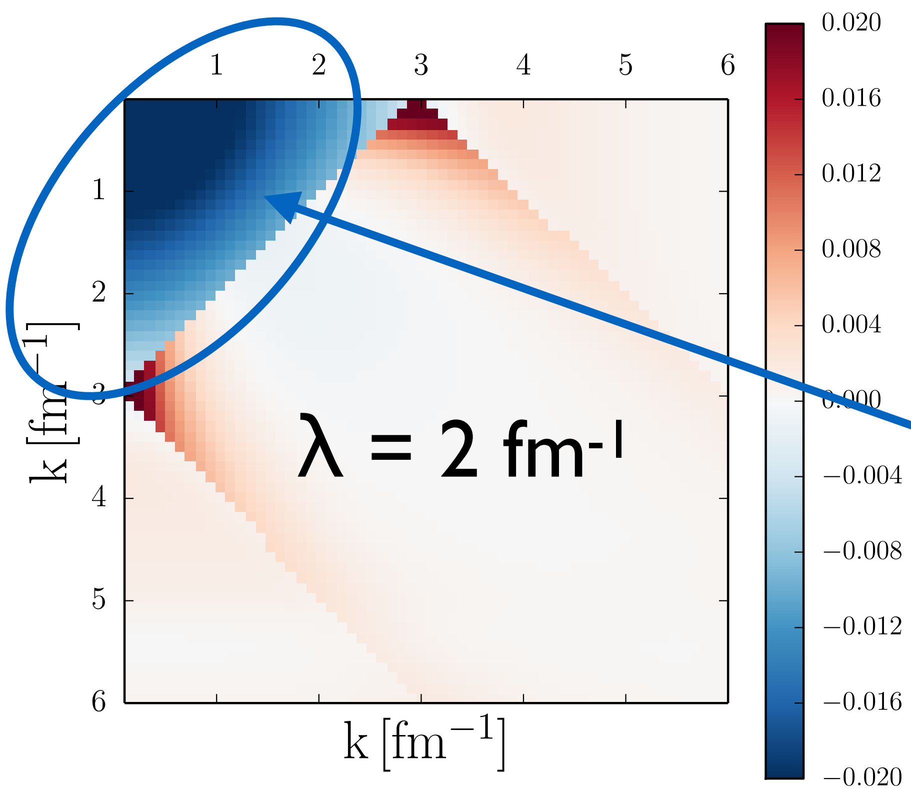
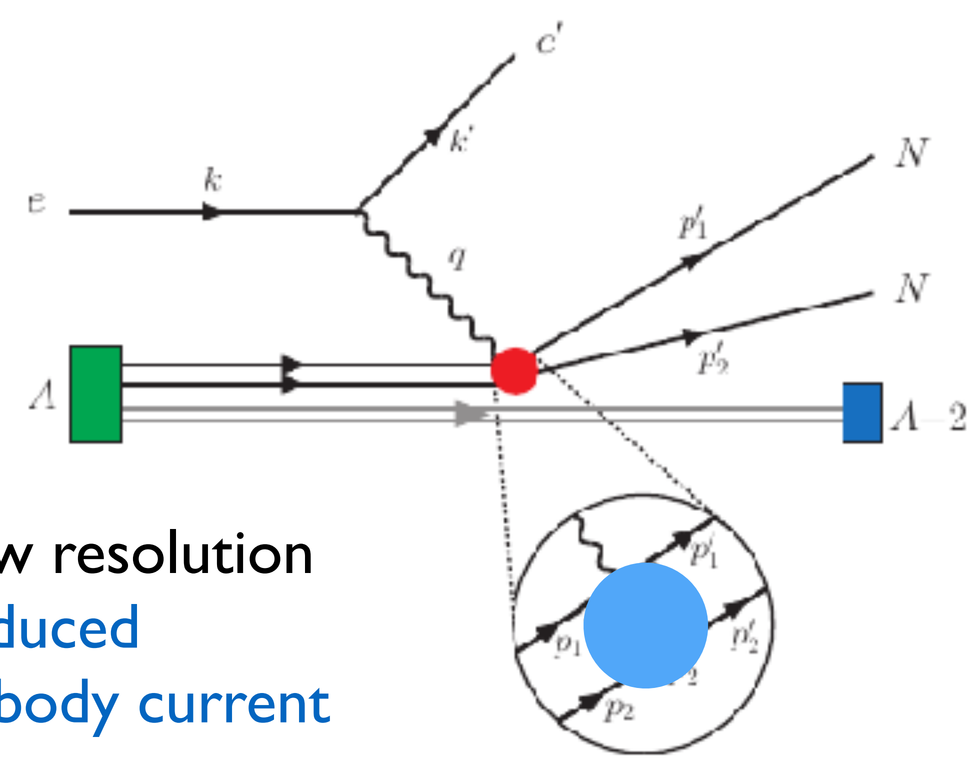




high resolution
one-body current

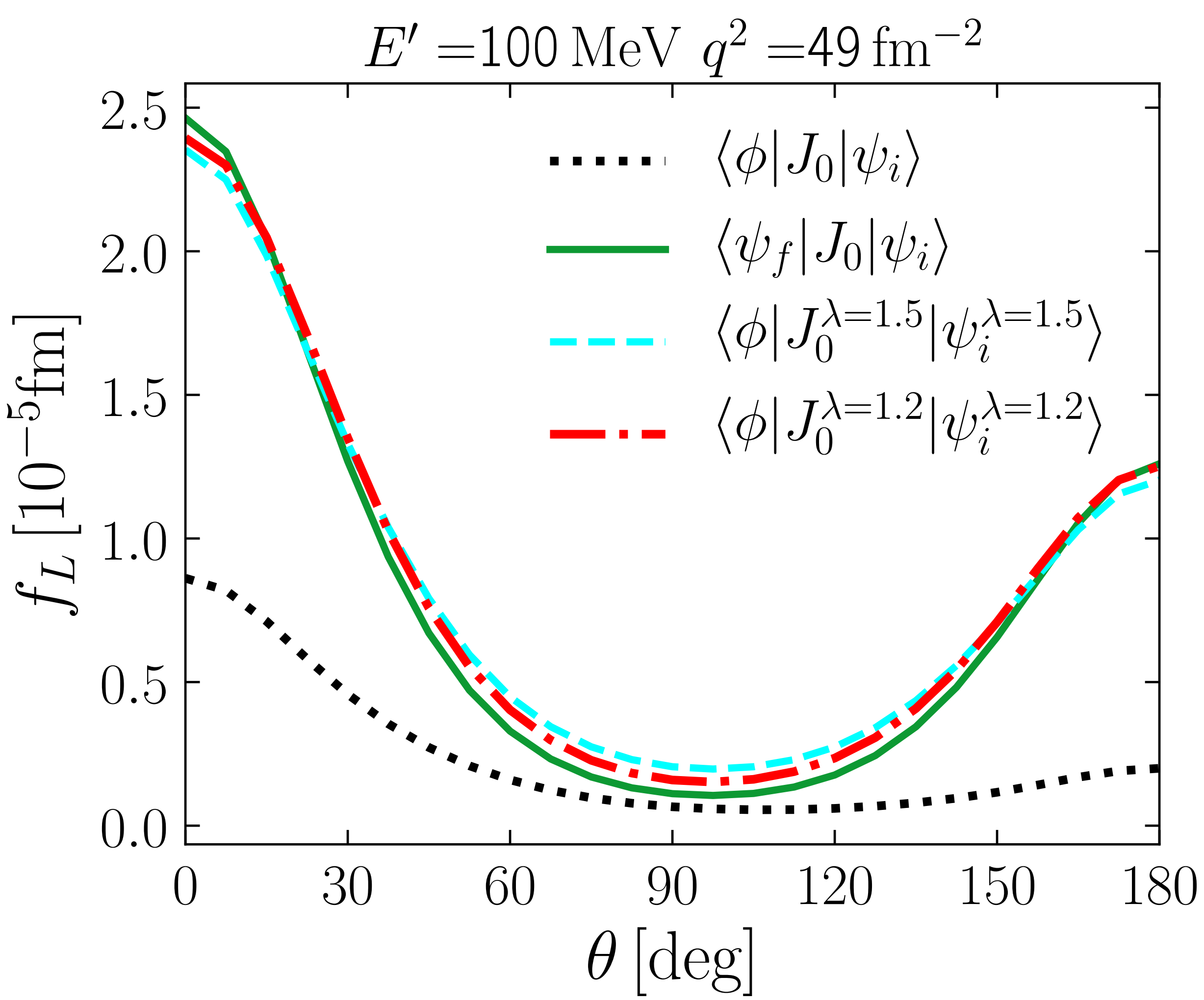




low resolution
induced
2-body current

λ dependence of Final State Interactions

Look at kinematics relevant to SRC studies



x_B=1.64, Q^2= 1.78 GeV^2

FSI sizable at large λ
but negligible at low-resolution!

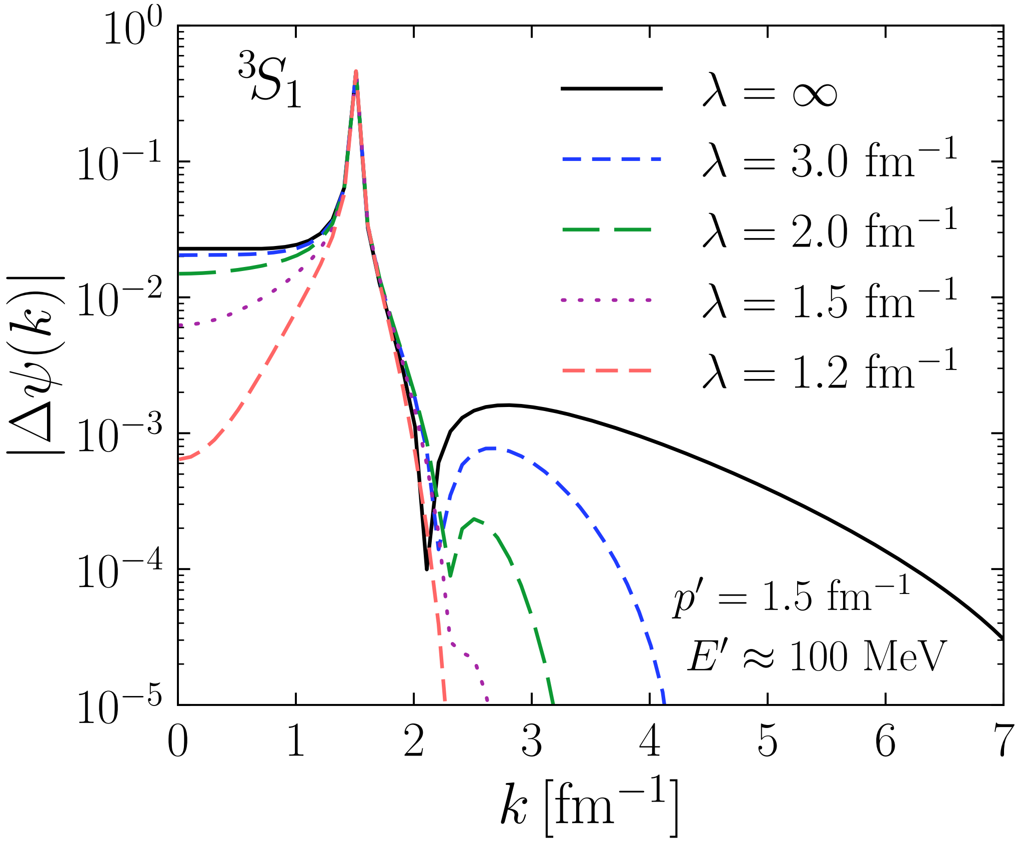
Folklore:

shouldn't hard processes
be complicated in low resolution
(λ << q) pictures?

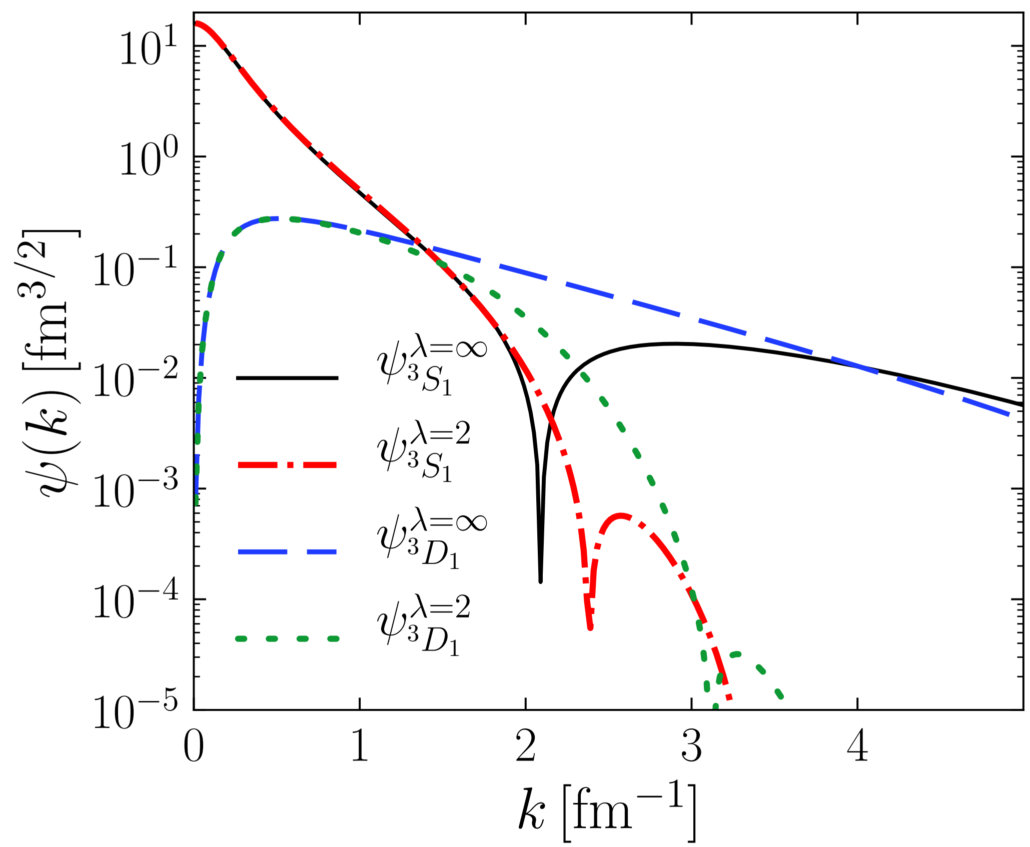
Why are FSI so small at low λ ?

λ dependence of Final State Interactions

final state wf (interacting piece)



initial state (deuteron) wf

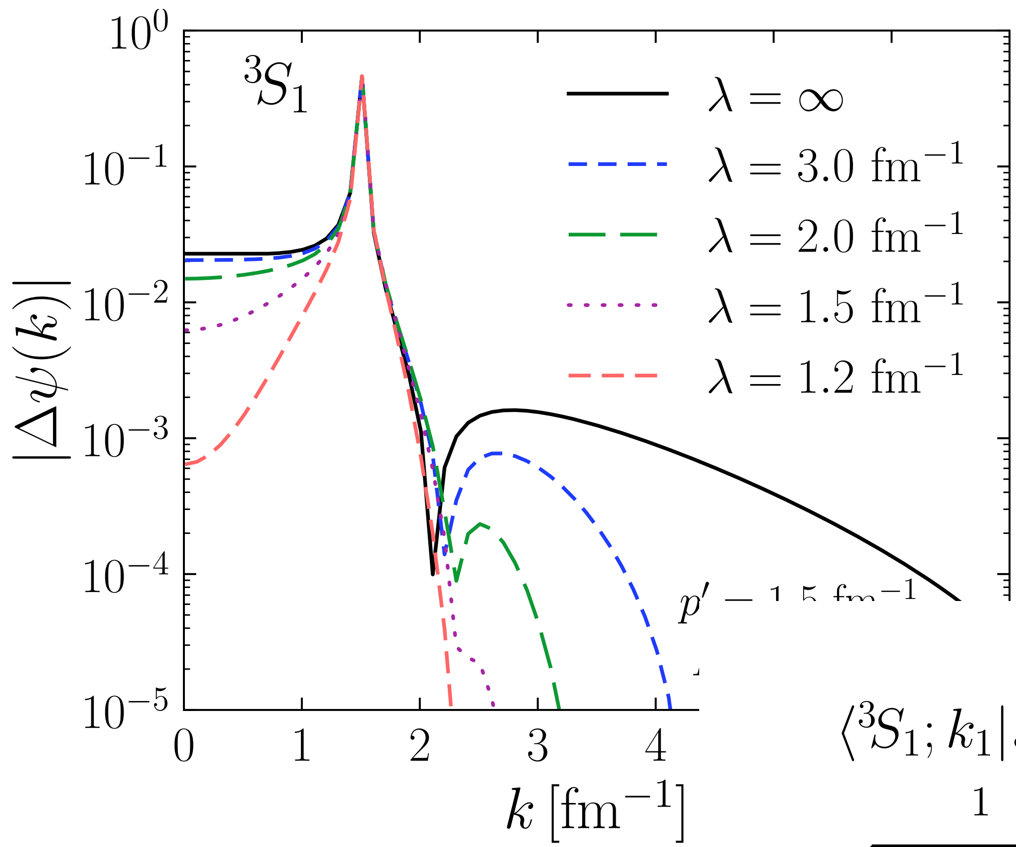


For p′ ≳ λ, interacting part of final state wf localized at k ≈ p′

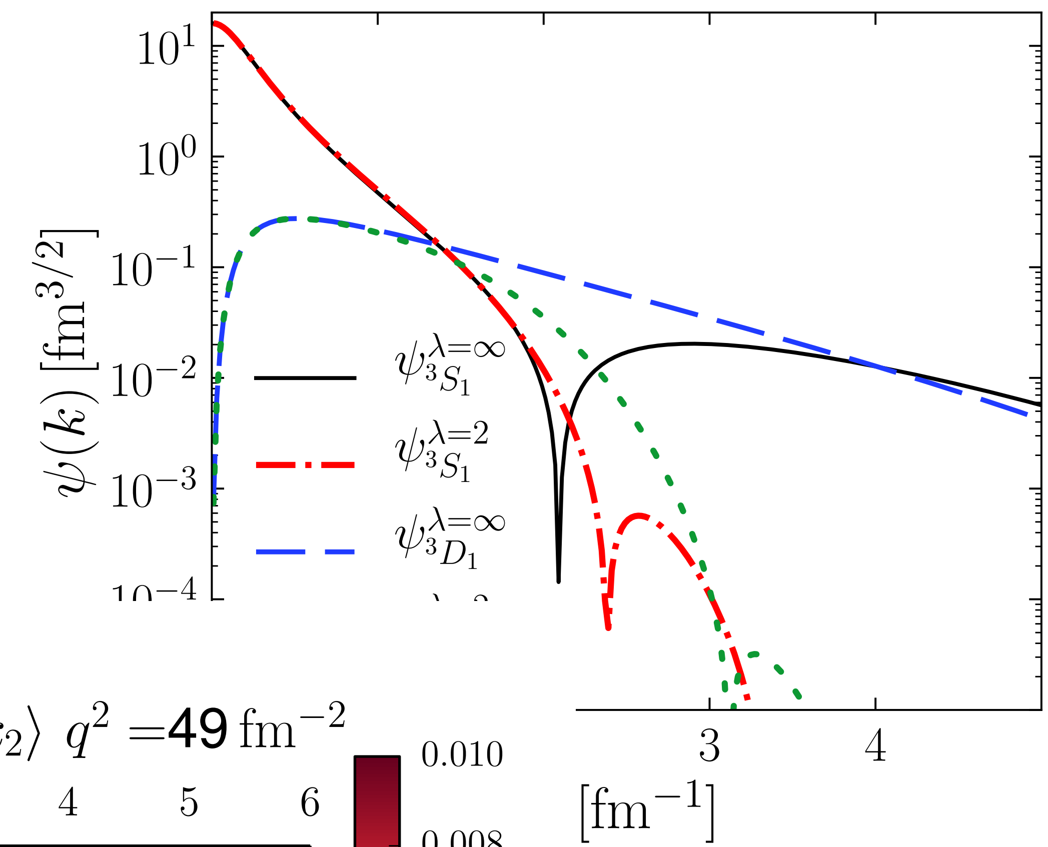
Dominant support of deuteron wf at k ≲ λ

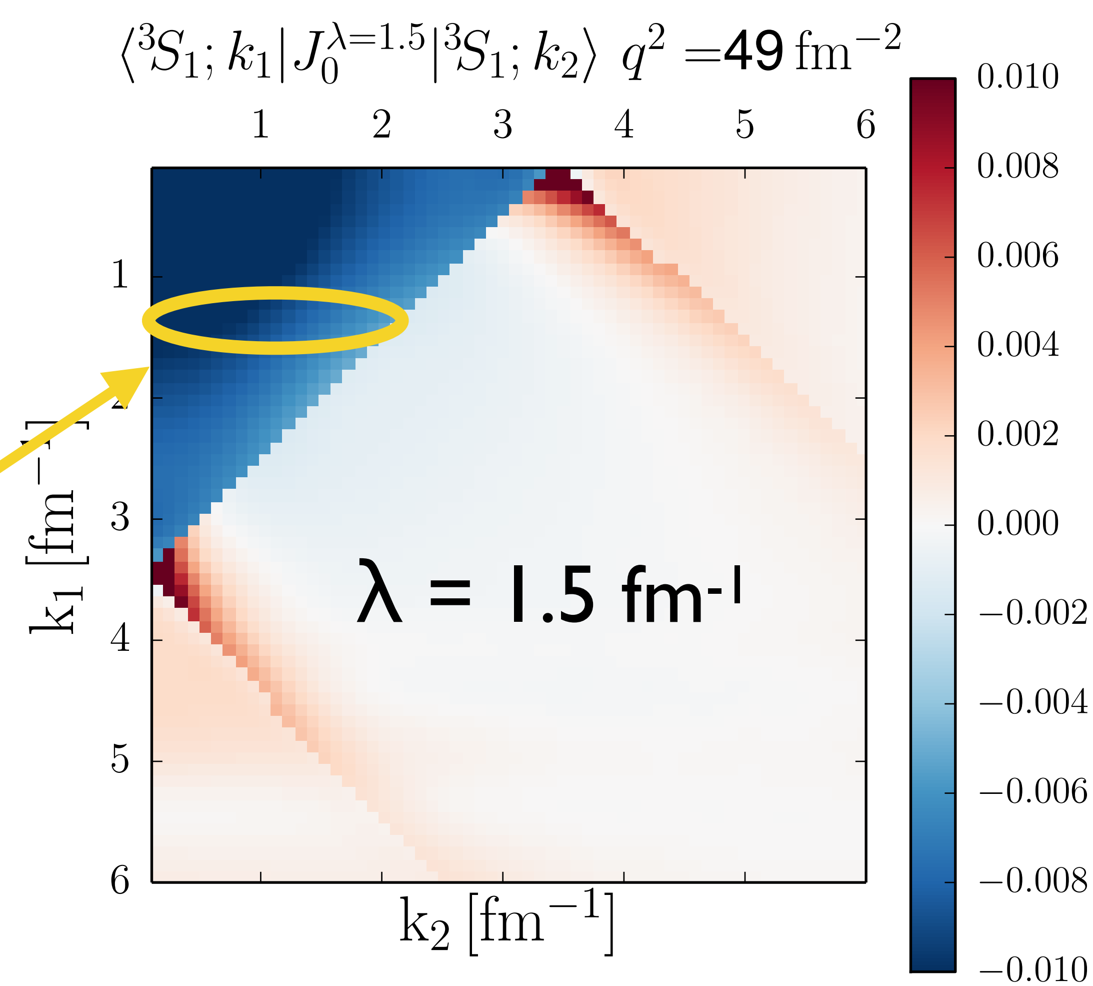
λ dependence of Final State Interactions

final state wf (interacting piece)



initial state (deuteron) wf



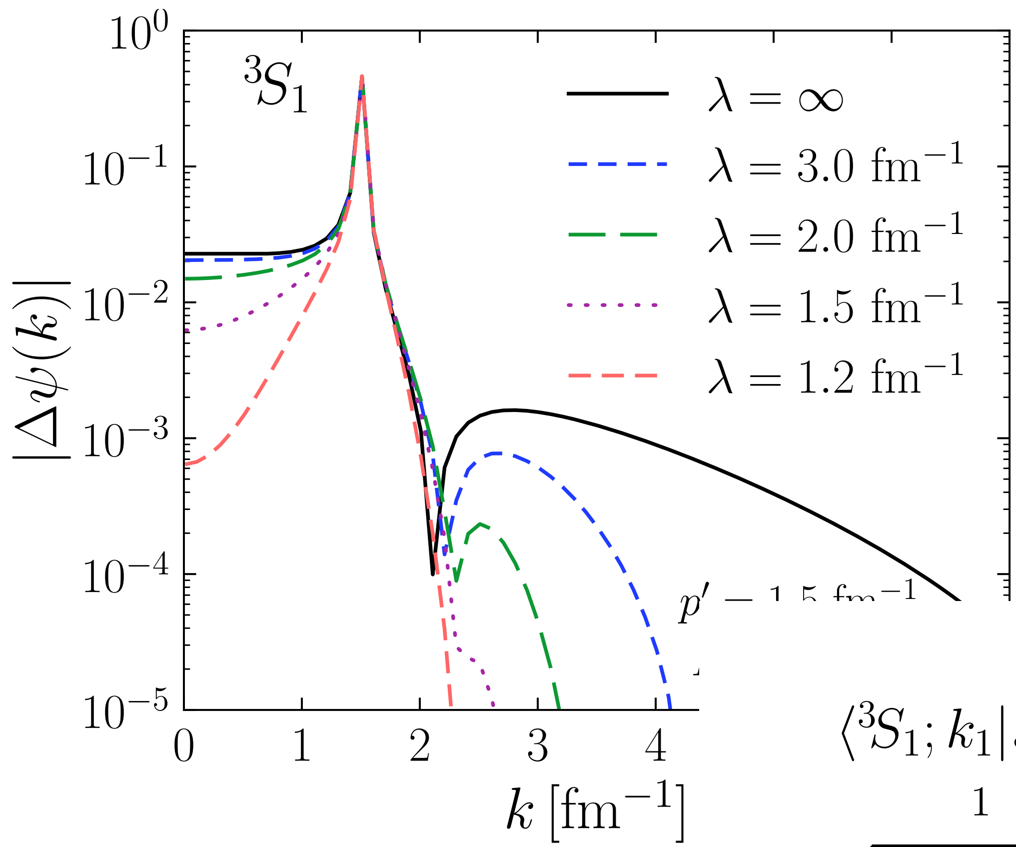


$J^\lambda_{\mathbf{q}}(k', k)$
probed by
transition

(smooth and non-singular)

λ dependence of Final State Interactions

final state wf (interacting piece)



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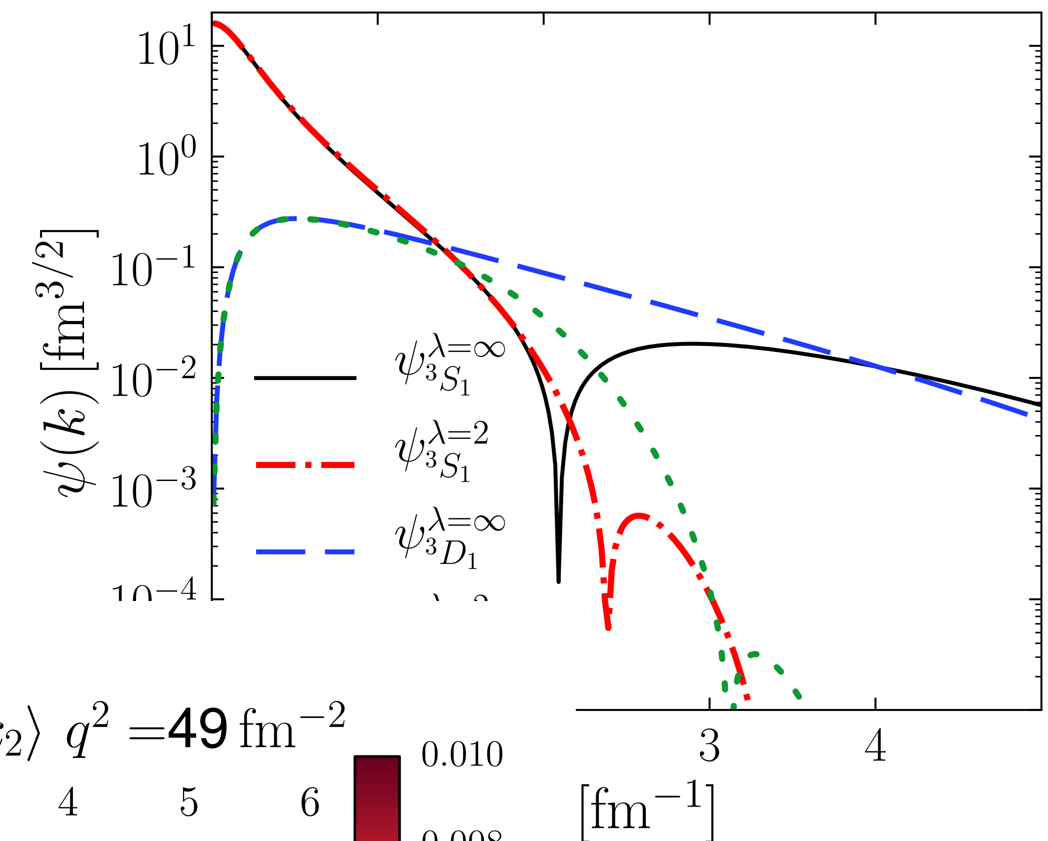
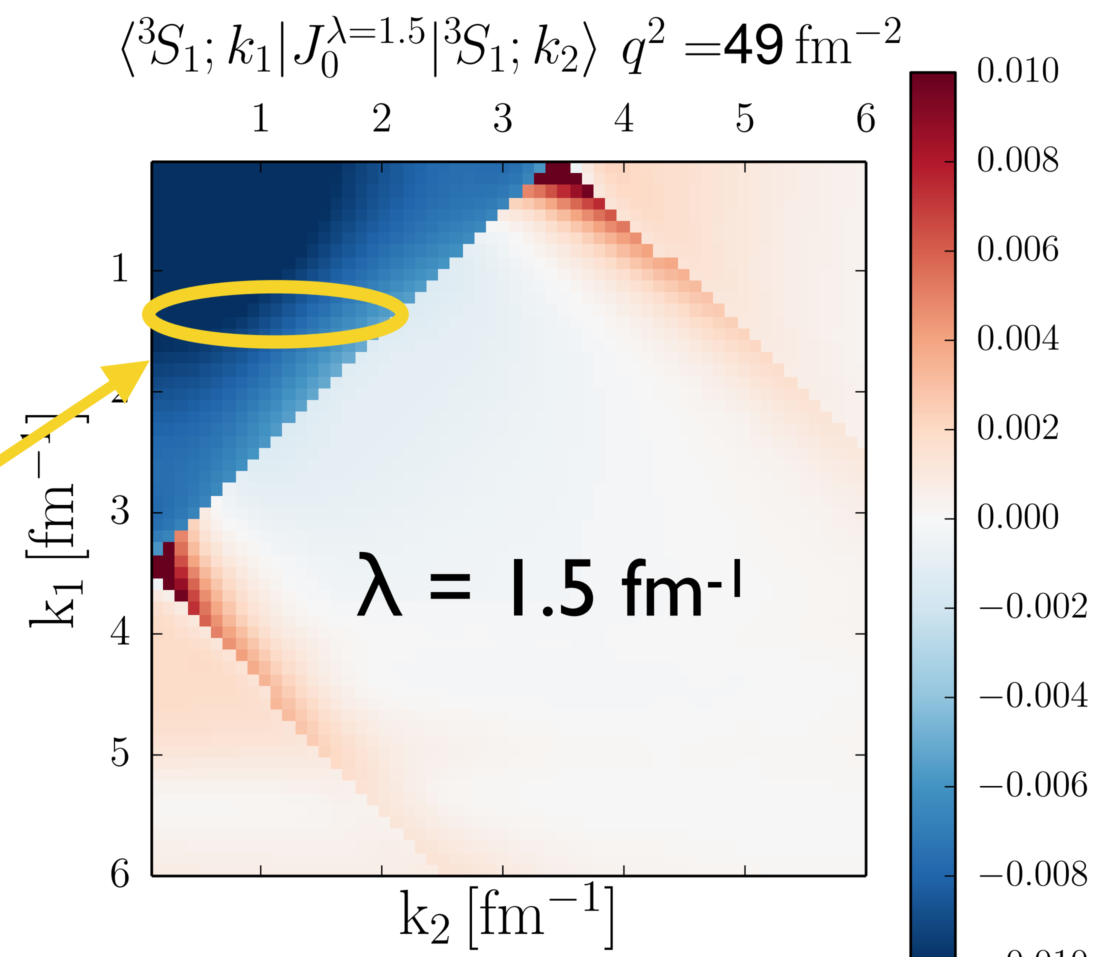




$J^{\lambda}_{\mathbf{q}}(k', k)$
probed by transition

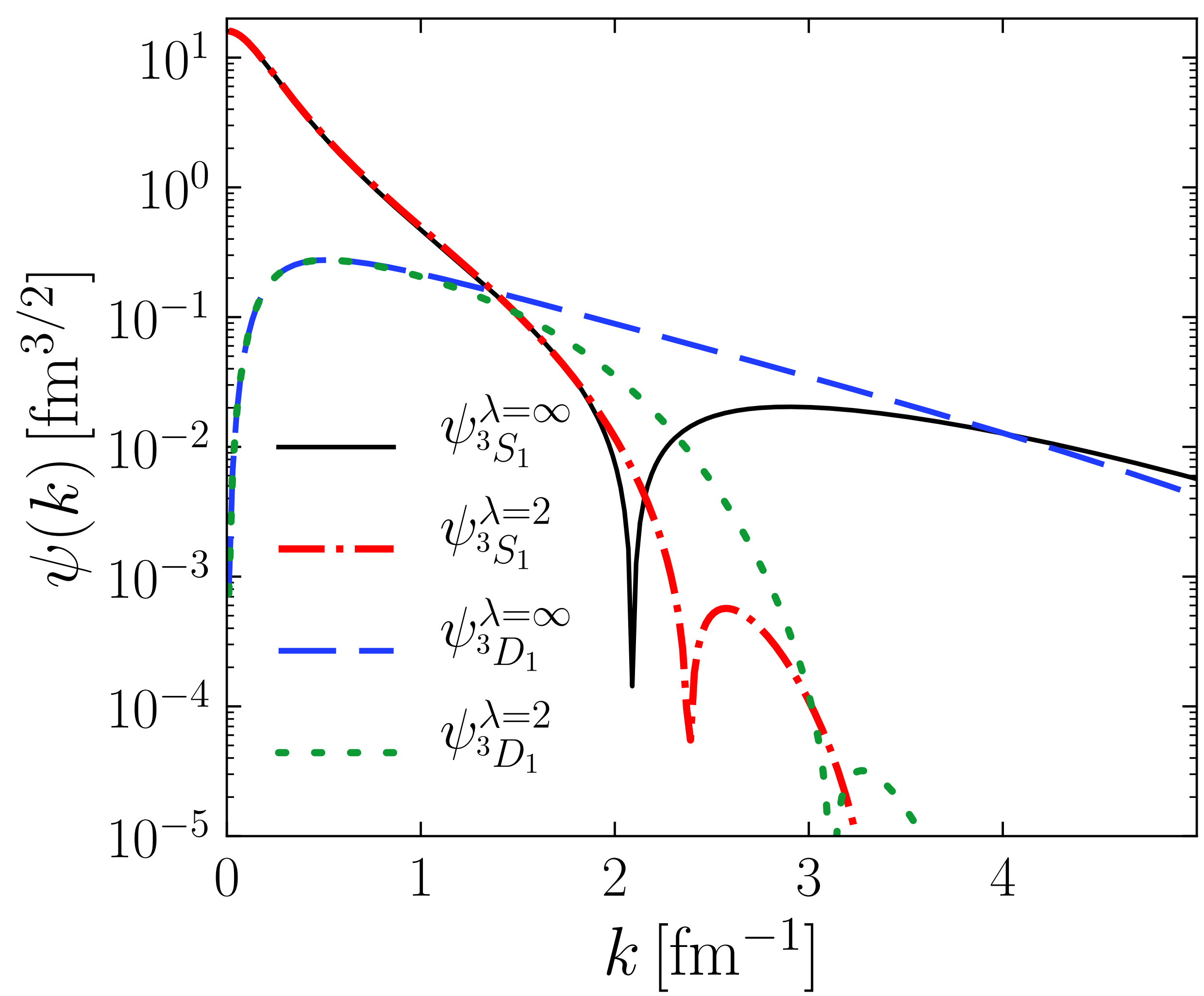
(smooth and non-singular)

∴ FSI ~ T(p',p')

(small!)

λ dependence of interpretations

- Analysis/interpretation of a reaction involves understanding which part of wave functions probed (highly scale dependent!)
- E.g., sensitivity to D-state w.f. in large **q**^{2} processes



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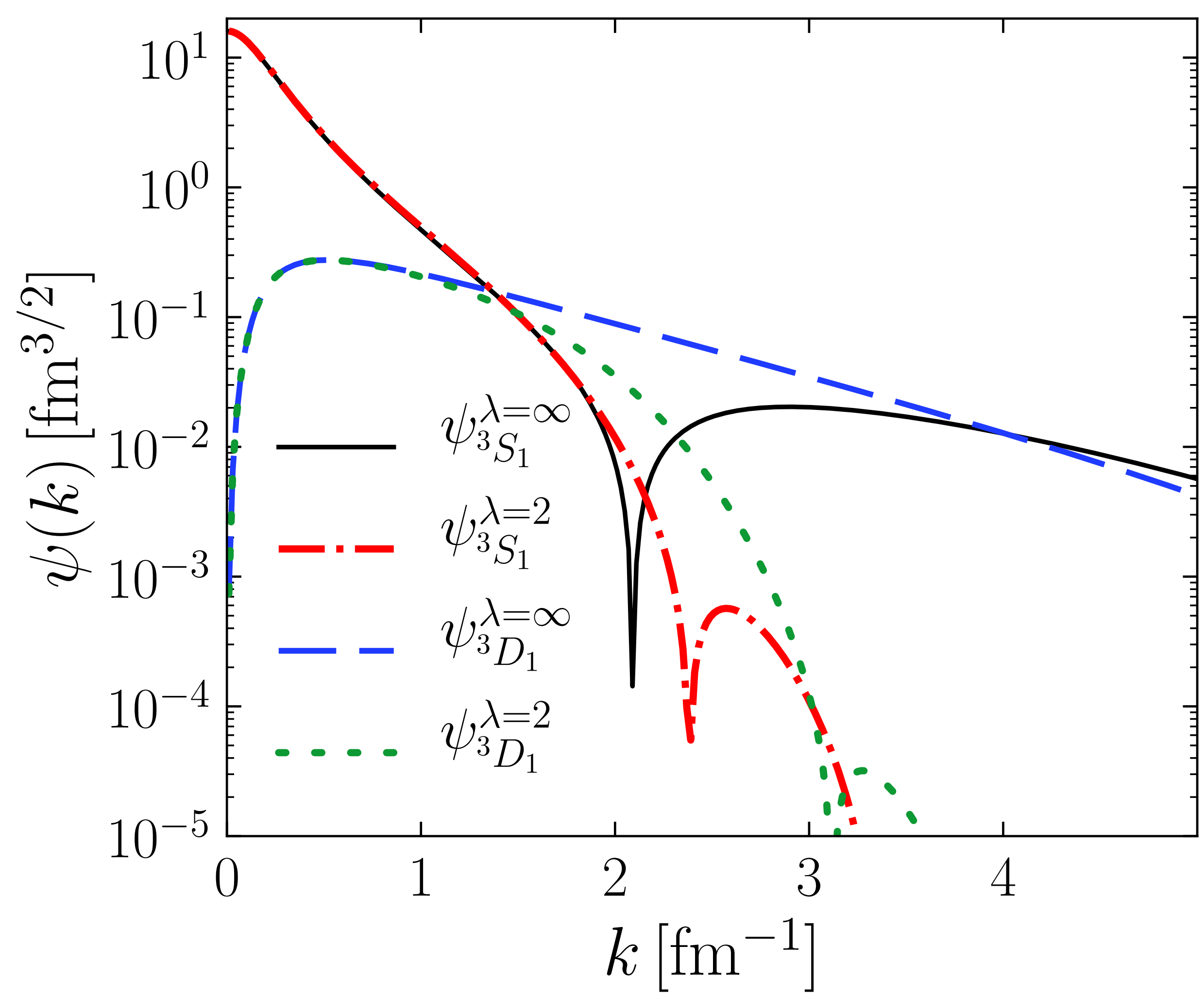

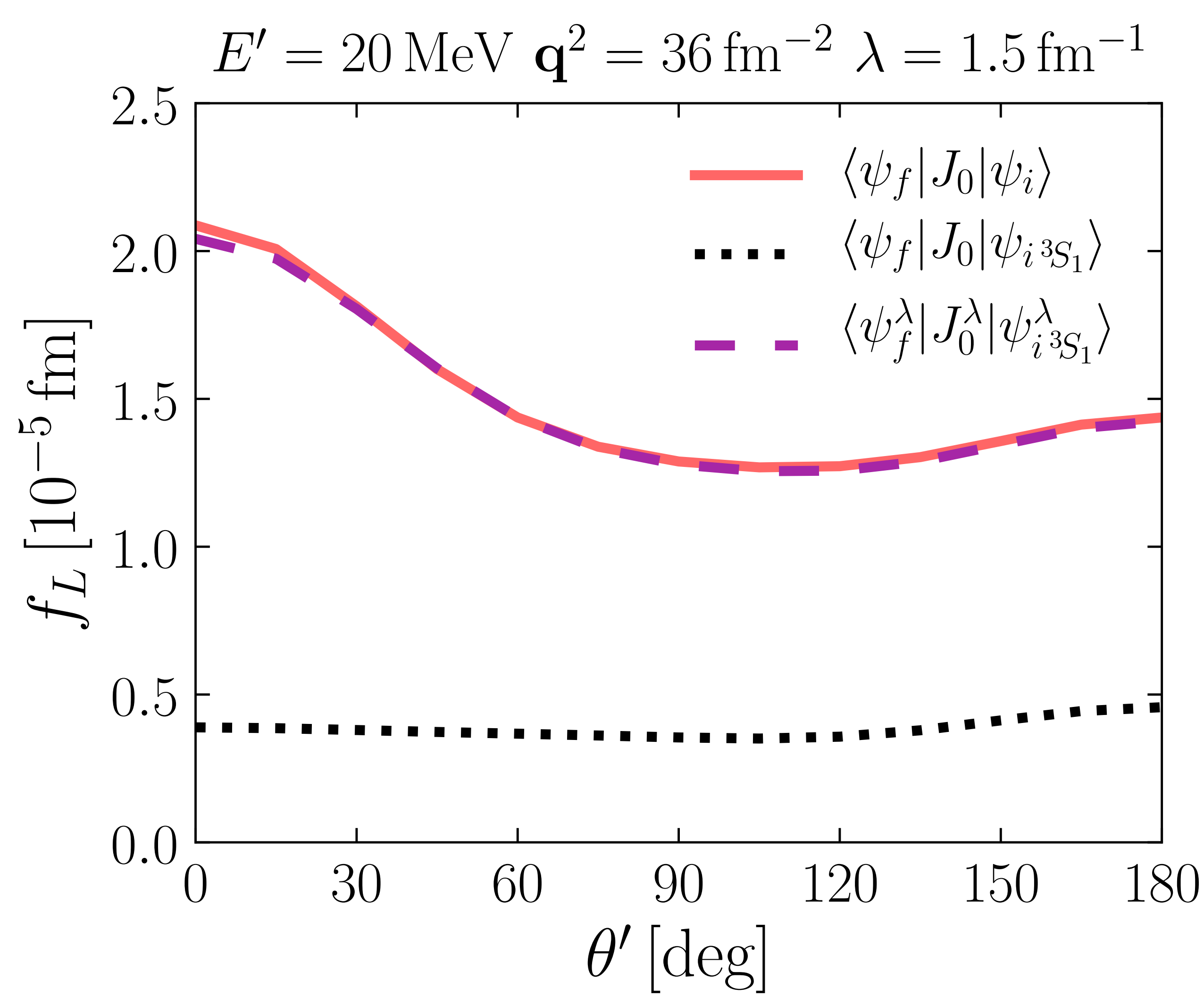

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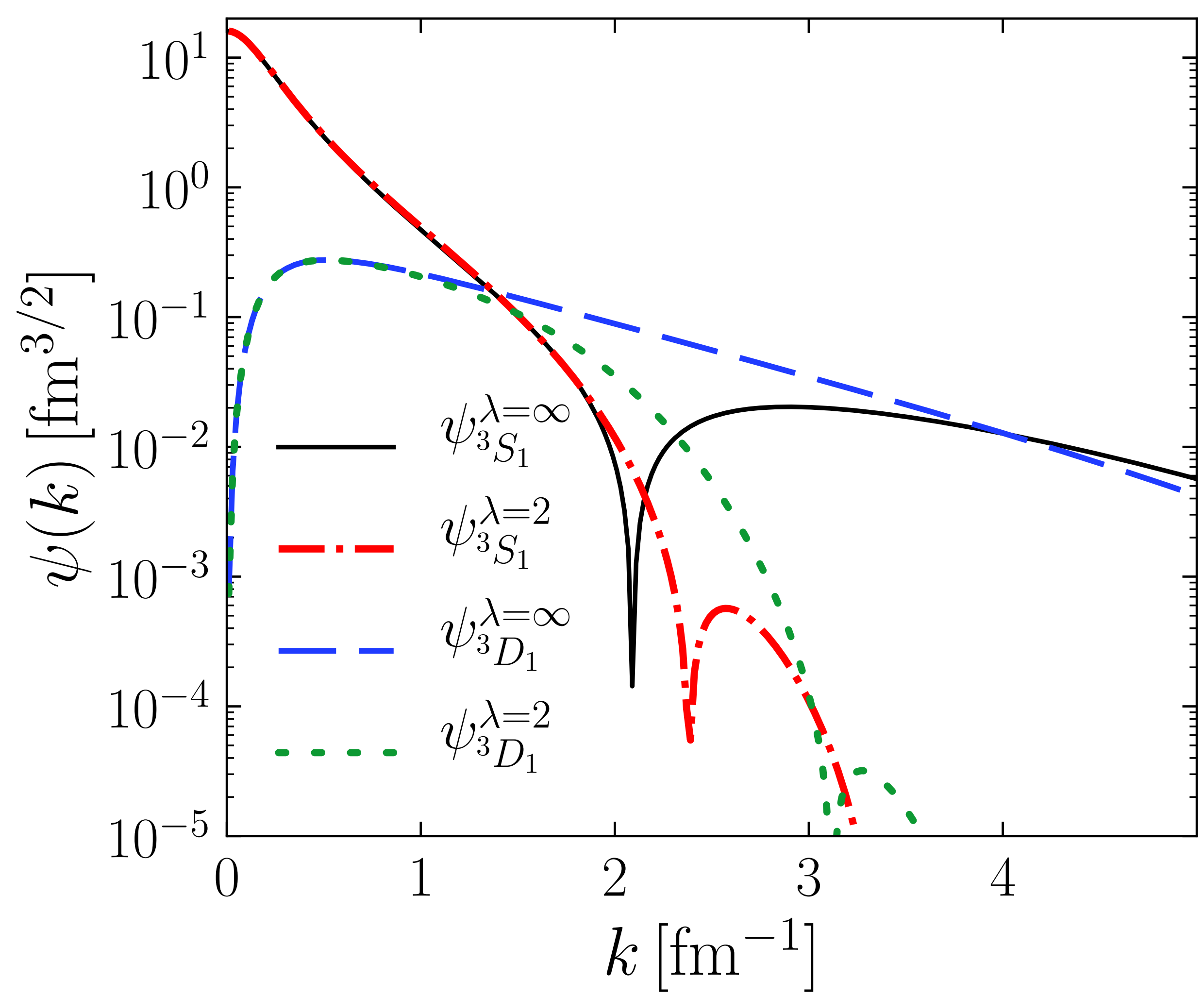

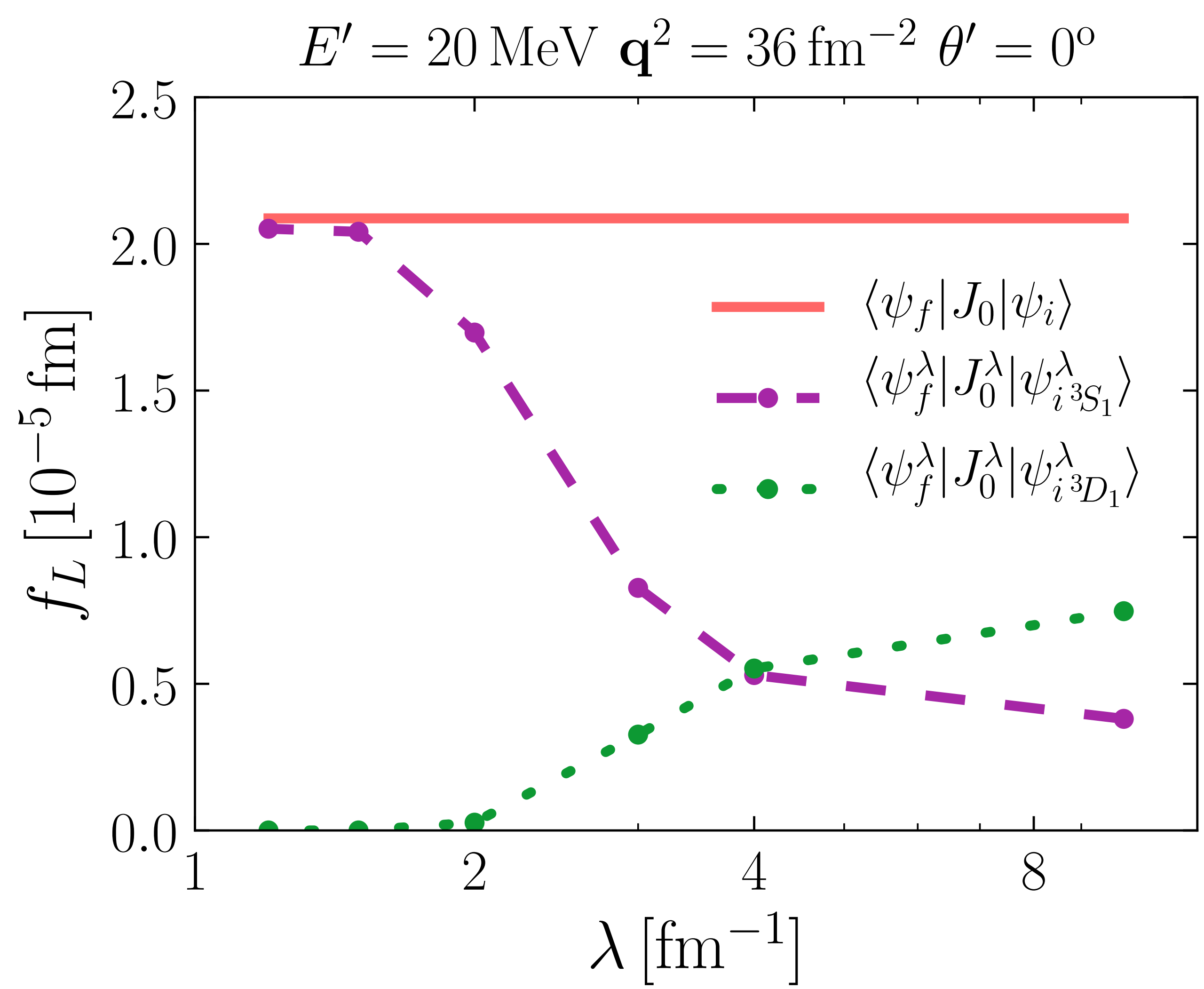

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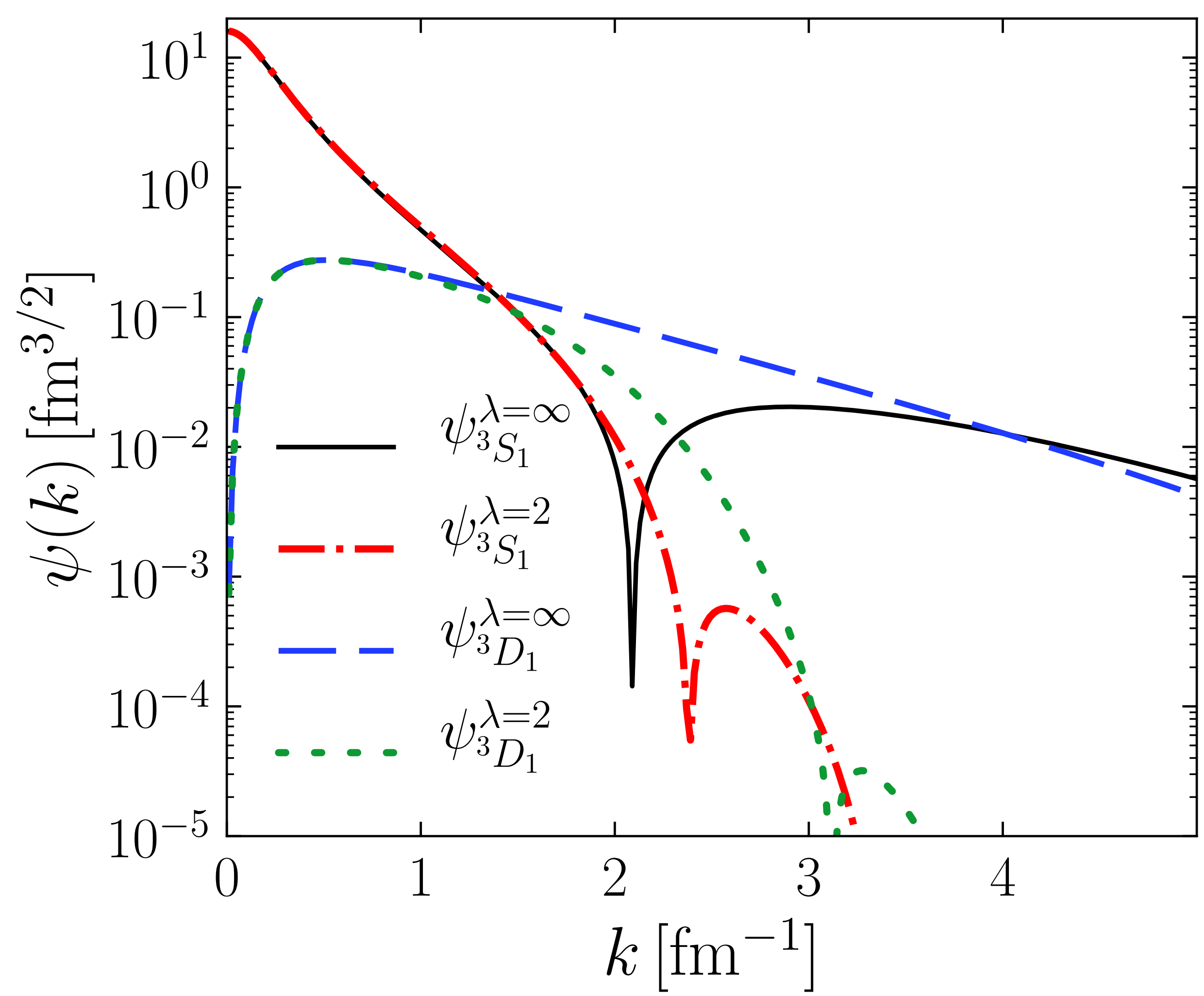
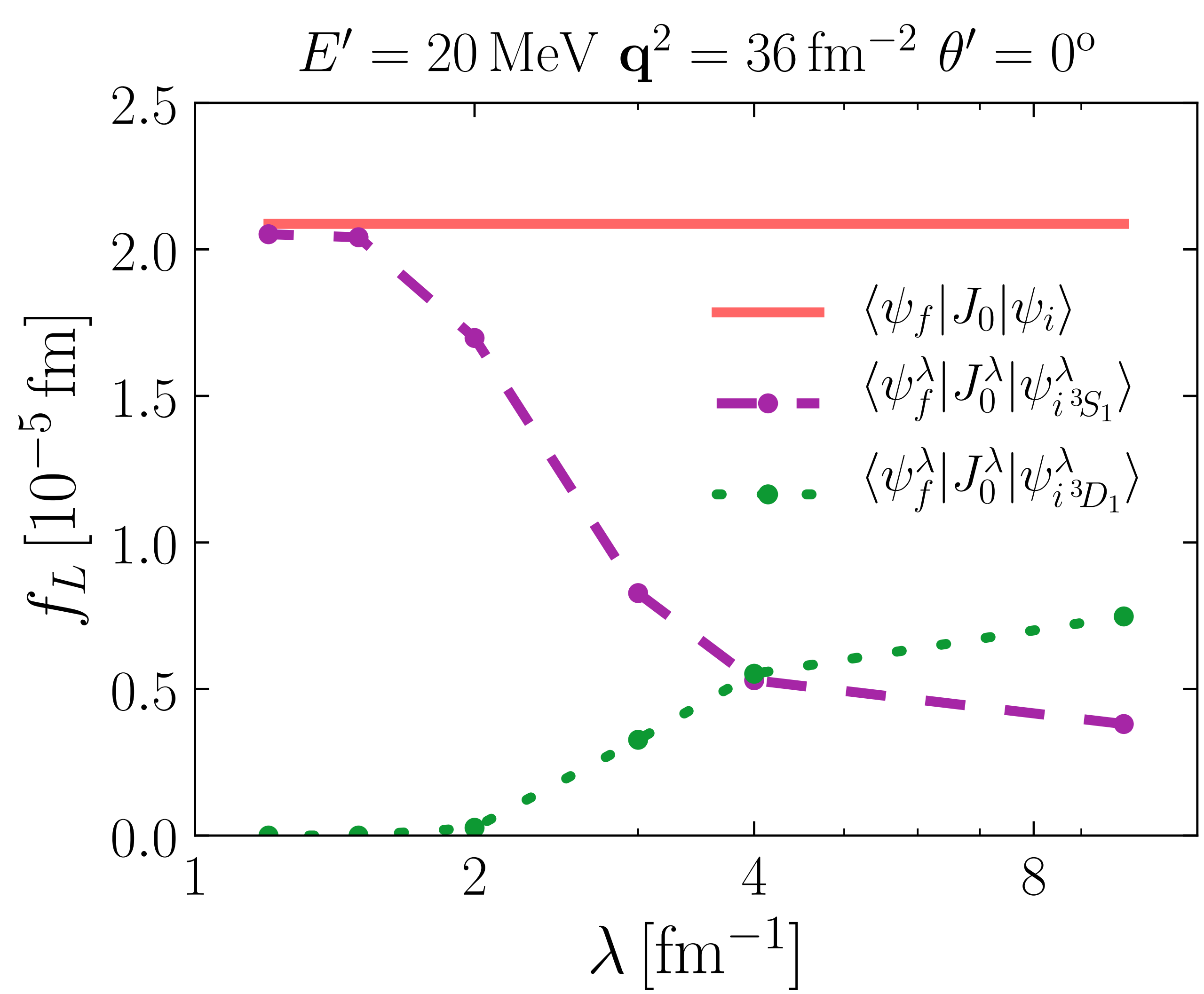


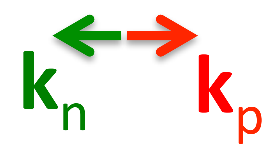


λ dependence of SRC interpretation

- SRC interpretations of hard knockout processes are scale dependent
- Consider large q^2 near threshold (small p′) for θ=0 in **high-resolution** picture (COM frame of outgoing np)

Before

After



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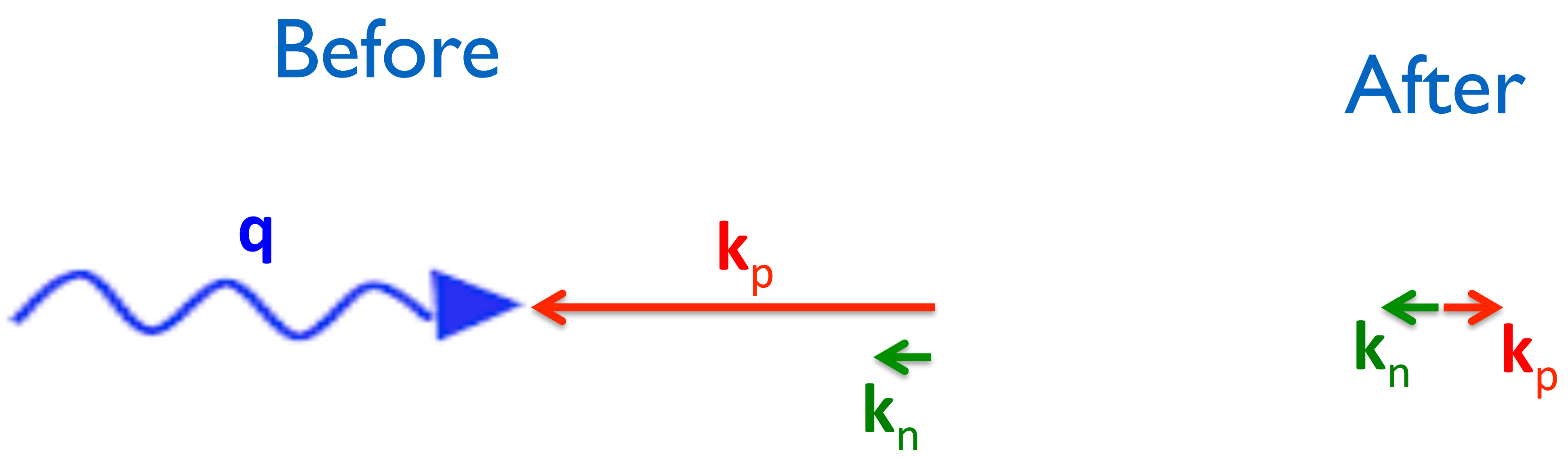

photon only couples to proton

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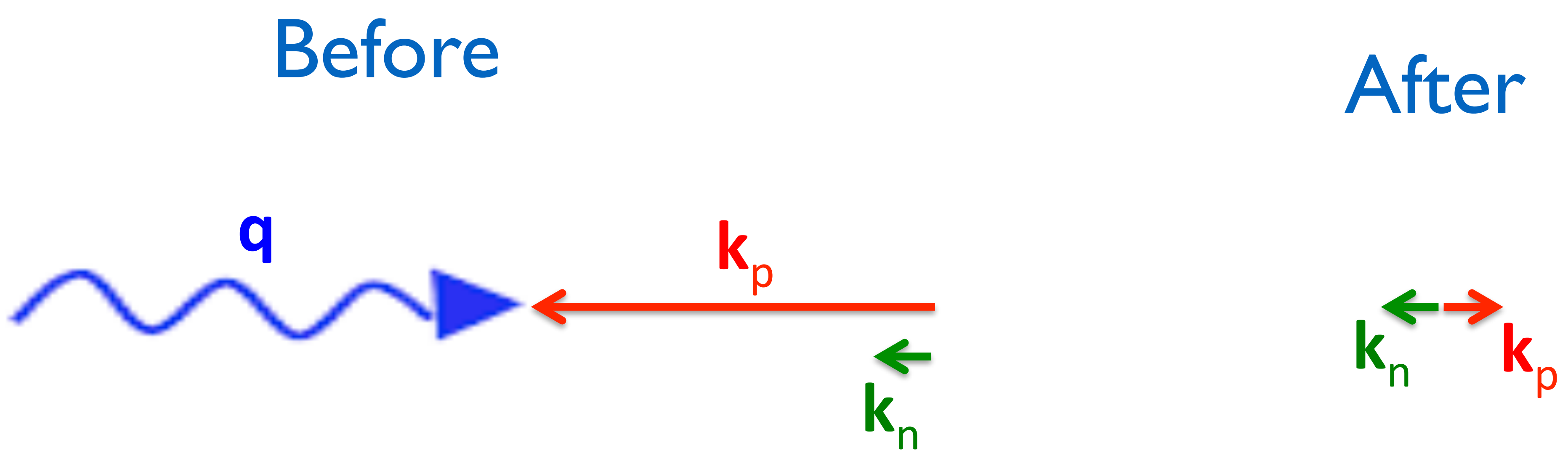


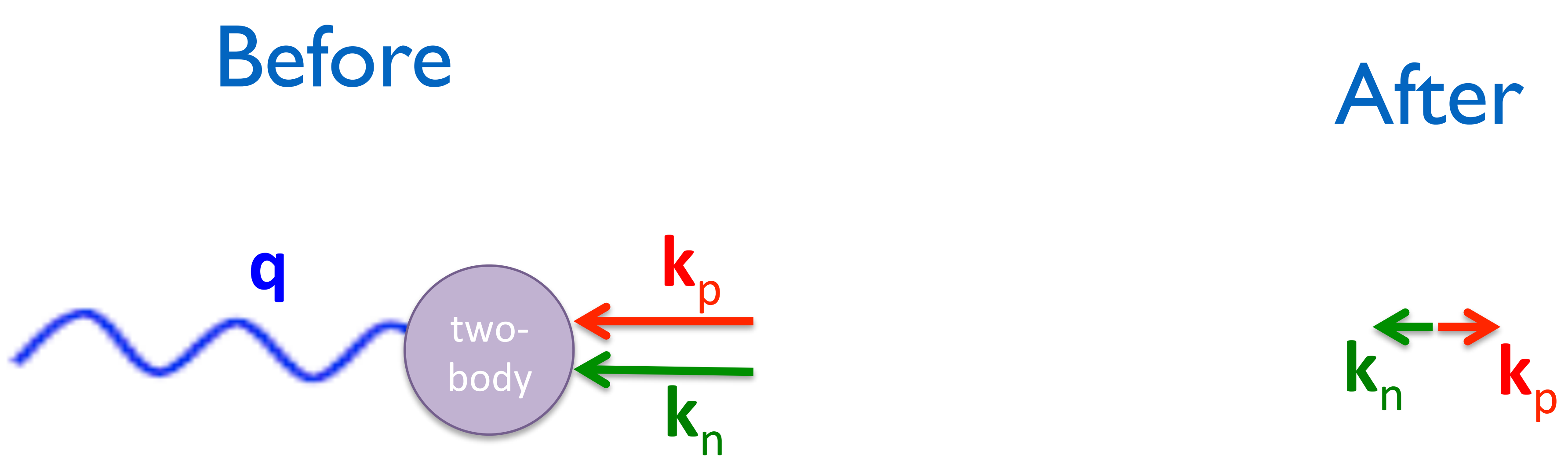
photon only couples to proton

∴ proton has large momentum => initial large relative momentum

(i.e., SRC pair)

λ dependence of SRC interpretation

- SRC interpretations of hard knockout processes are scale dependent
- Consider large q^2 near threshold (small p′) for θ=0 in **low-resolution** picture (COM frame of outgoing np)



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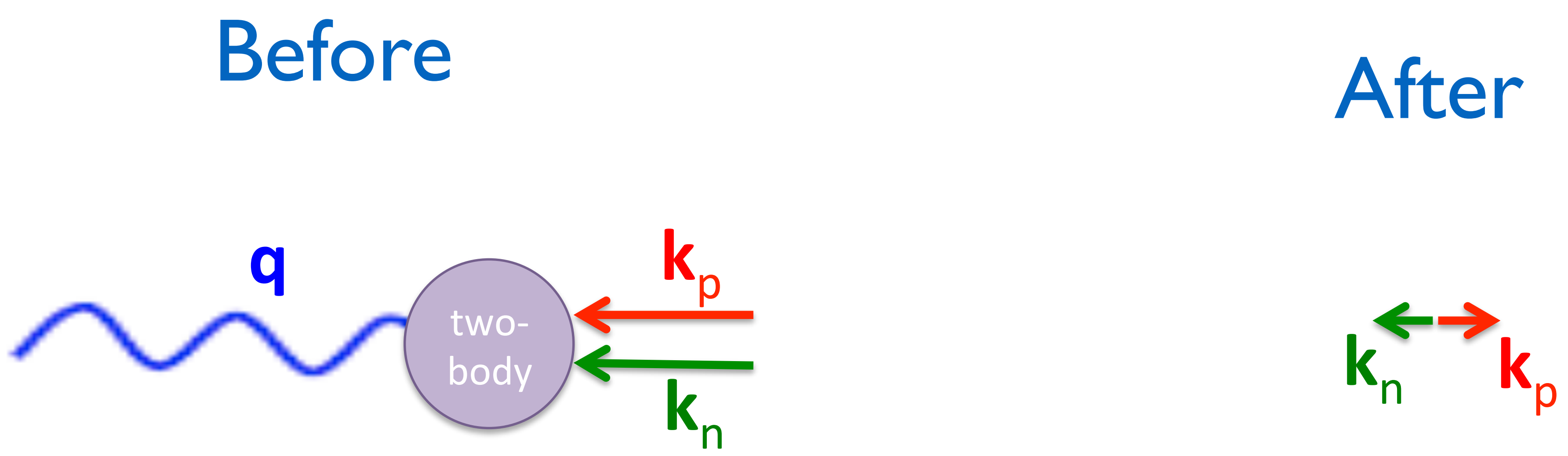
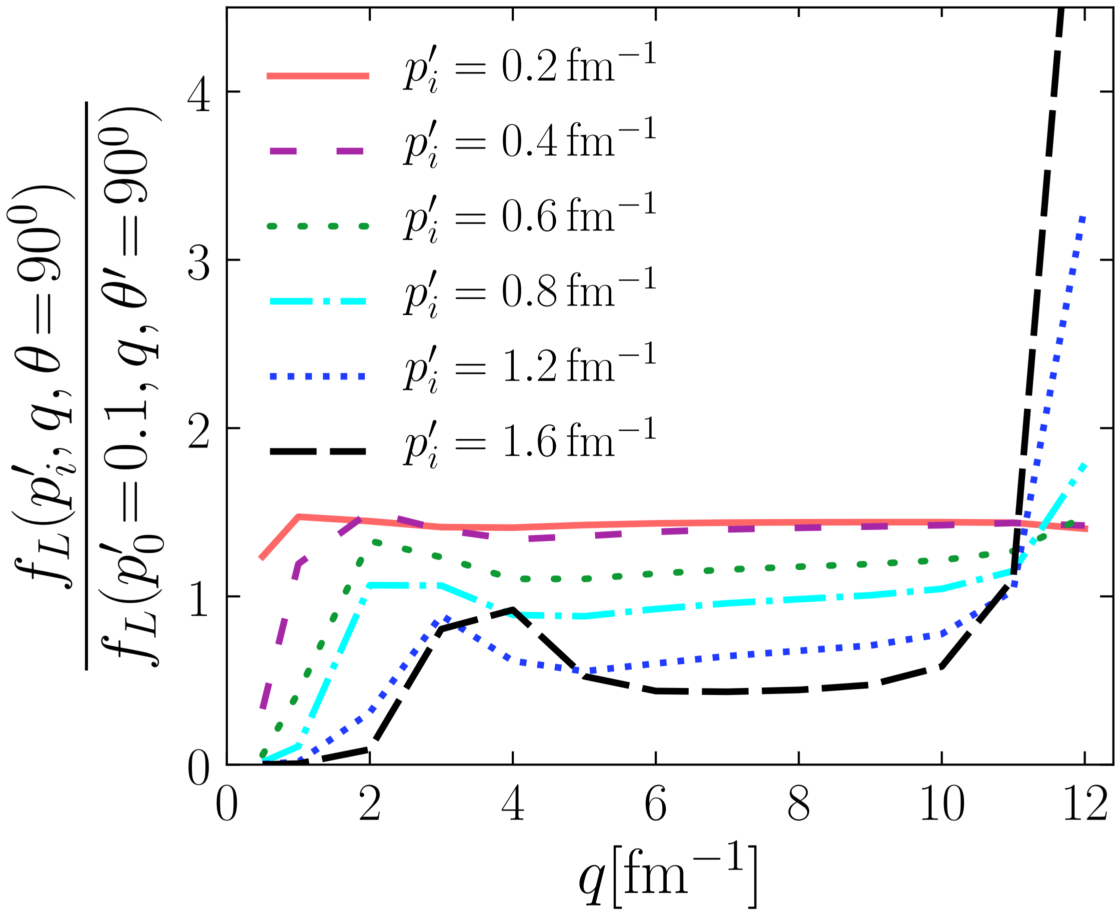


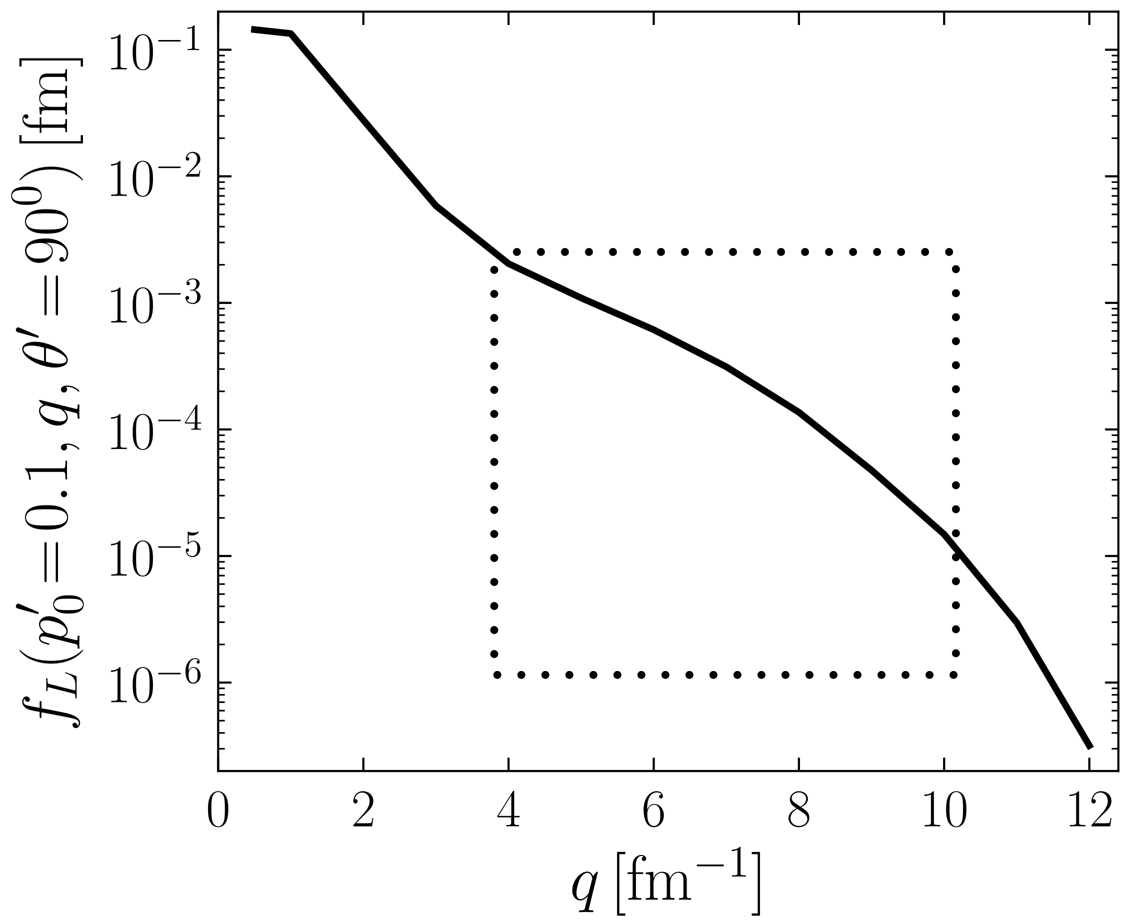
no large relative momentum in evolved deuteron wf

1-body current makes no contribution

∴ 2-body current mostly stops the low-relative momentum np pair

Factorization of q-dependence





Low-resolution picture gives natural explanation of factorized q-dependence for $p' \ll \lambda \ll q$

$$\Delta J_0^\lambda(\mathbf{q}) \approx g_{\mathbf{q}} \delta(\mathbf{r})$$

q-dependence factorized into Wilson coefficient

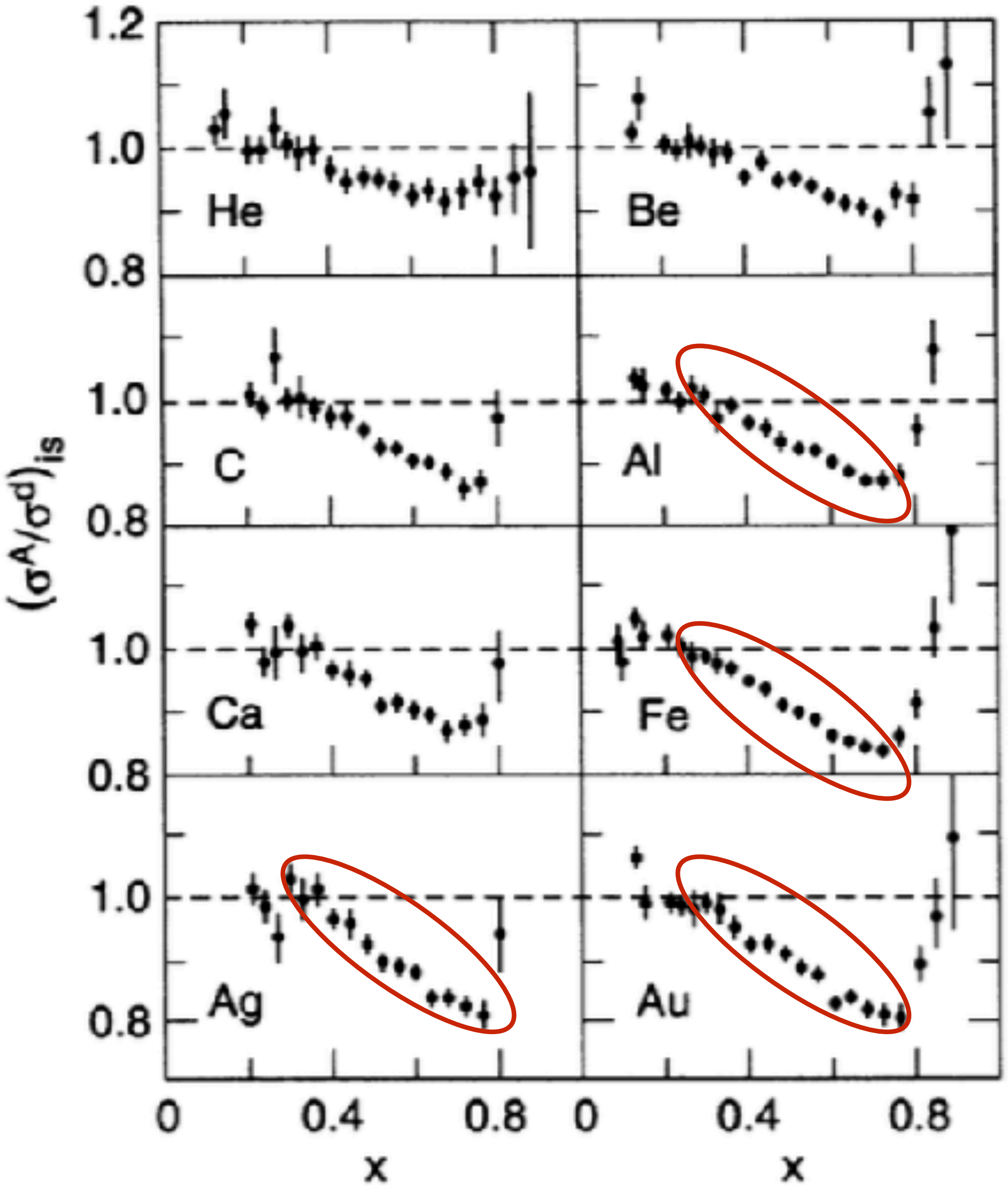
Running summary part2

- Interpretations (SRCs, components of nuclear wf’s probed, etc.) of knockout reactions are necessarily scale-dependent
- Using RG to minimize FSI for given kinematics worth exploring
- Low-resolution theories can describe hard knockout reactions with surprising simplifications (smaller FSI, natural explanation of factorized q-dependence,…)

Scale dependence of short range correlations in medium-mass nuclei

EMC Effect

electron-nucleus DIS



- Non-interacting limit ratio should be 1
- BE/A ~ 1% of M_N, Q
- 20% deviations from 1 = EMC effect

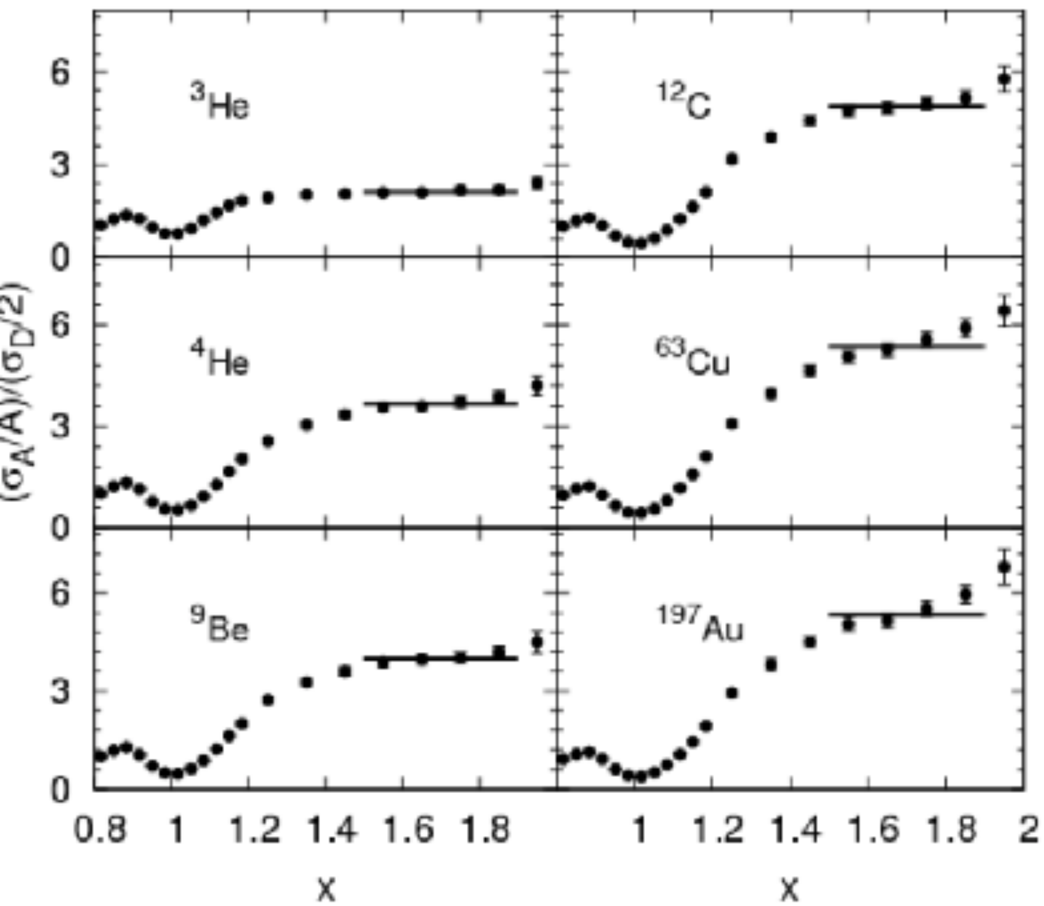
nucleon structure modified in-medium

Slope used to quantify size of effect

European Muon Collaboration, Phys. Lett. B 123 (1983)

Short-range correlations

Quasi-elastic (e,e'2N)



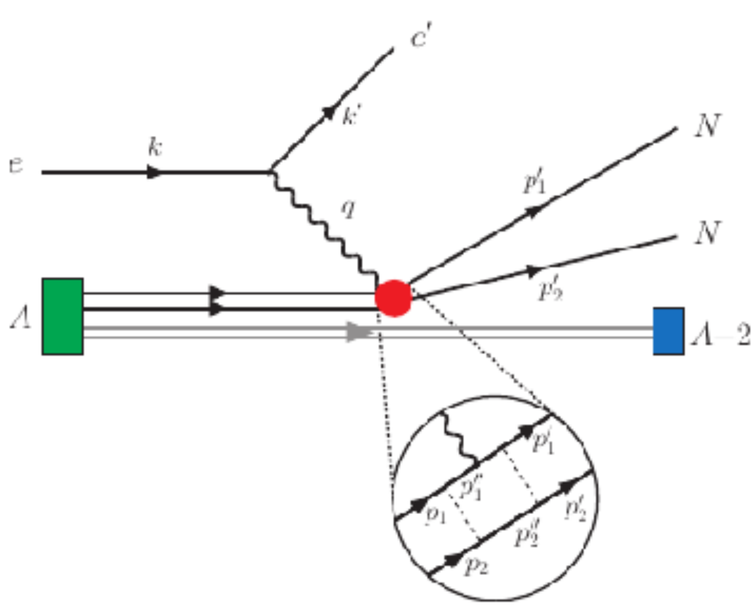
Fomin et al., Phys. Rev. Lett. 108 (2012)

plateaus for 1.5< x <2.0

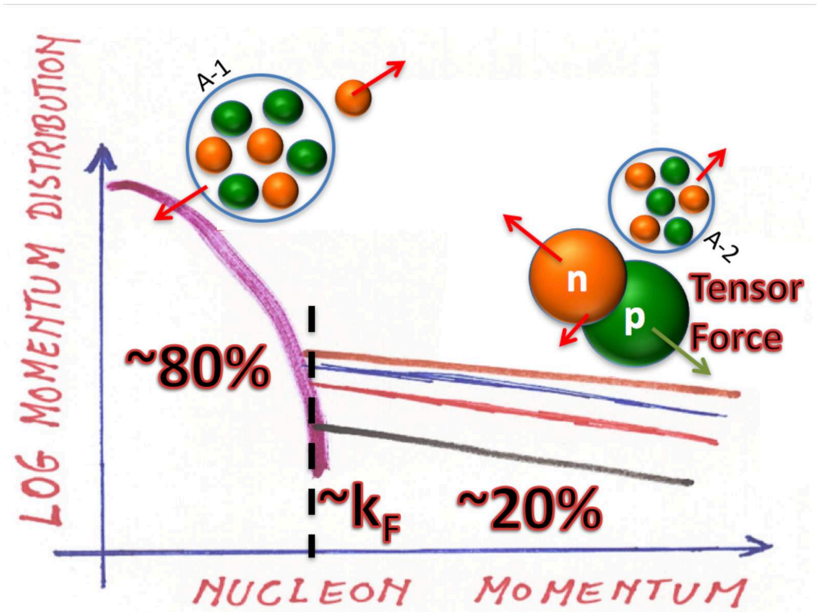
SRC interpretation:

NN interaction scatters pair $p_1, p_2 < k_F$ to intermediate-state momenta >> k_F which are then knocked out by photon

$$a_2(A) = \frac{2}{A} \frac{\sigma_A(x_B, Q^2)}{\sigma_d(x_B, Q^2)} \approx \frac{n_A(\mathbf{q} > k_F)}{n_d(\mathbf{q} > k_F)}$$

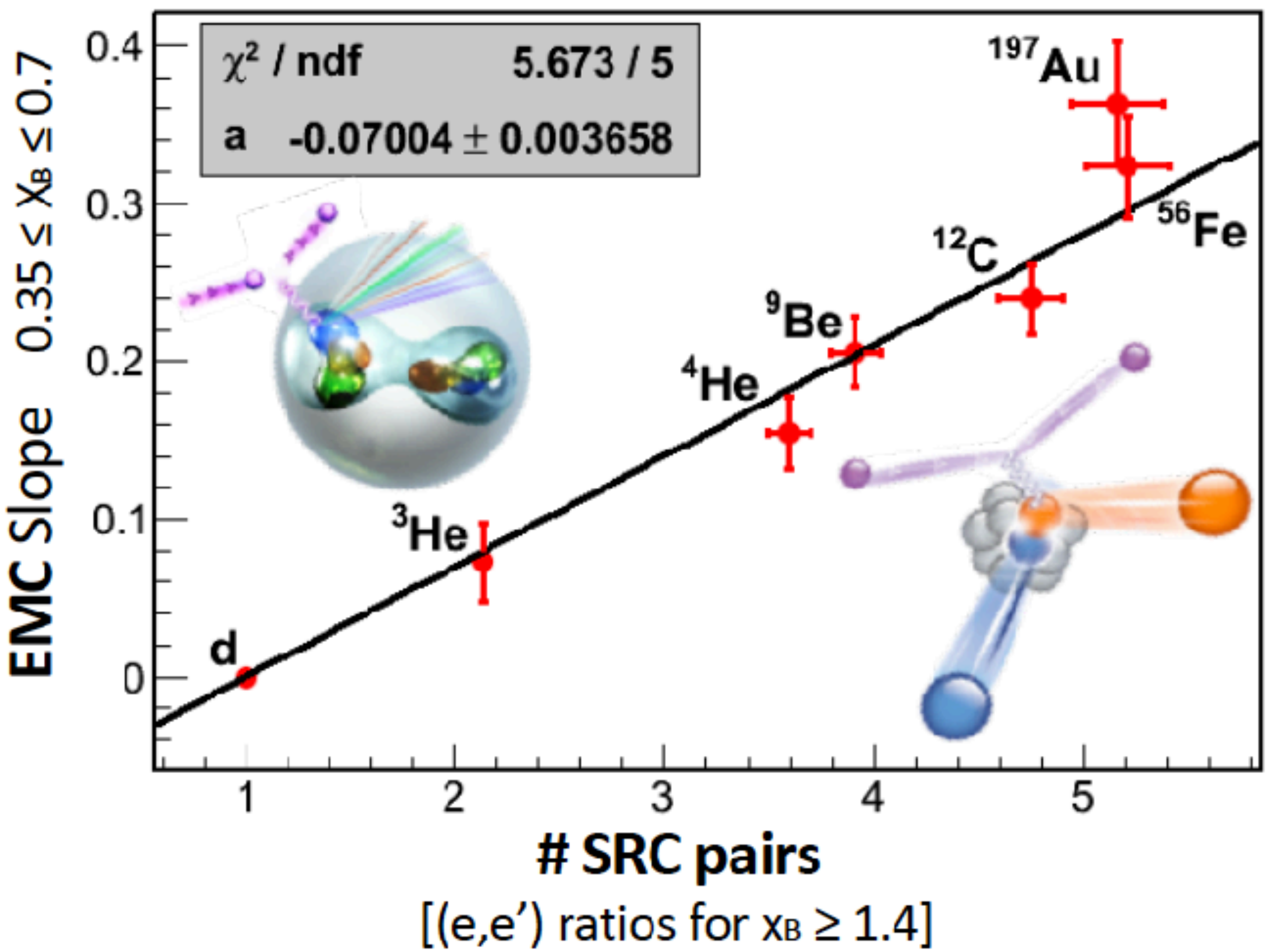


Hen et al., Rev. Mod. Phys. 89 (2017)



Empirical correlation of EMC effect

Hen et al., RMP (2017); Hen et al., IJMPE (2013); Hen et al., PRC (2012); Weinstein, Piasetzky, Higinbotham, Gomez, Hen, and Shneor, PRL (2011).

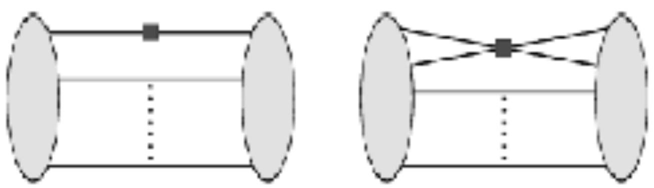


Why should 2 seemingly unrelated processes be linearly related?

EFT explanation

Chen and Detmold Phys. Lett. B 625 (2005)
Chen, et al., Phys. Rev. Lett. 119 (2017)

- Match isoscalar twist-2 quark operators to LO nucleon operators



$$\Longrightarrow \quad \langle x^n \rangle_A(Q) \approx \langle x^n \rangle_N(Q) \left[A + \alpha_n(\Lambda) \langle A| : (N^\dagger N)^2 : |A\rangle \right] + \cdots$$

DIS: $F_2^A(x, Q^2)/A \approx F_2^N(x, Q^2) + g_2(A, \Lambda) f_2(x, Q^2, \Lambda)$

$$g_2(A, \Lambda) = \frac{1}{2A} \langle A| : (N^\dagger N)^2 : |A\rangle_\Lambda$$

QES: $\sigma_A/A \approx \sigma_N + g_2(A, \Lambda)\sigma_2(\Lambda)$

$$a_2(A, 1 < x < 2) \approx \frac{g_2(A, \Lambda)}{g_2(2, \Lambda)}$$

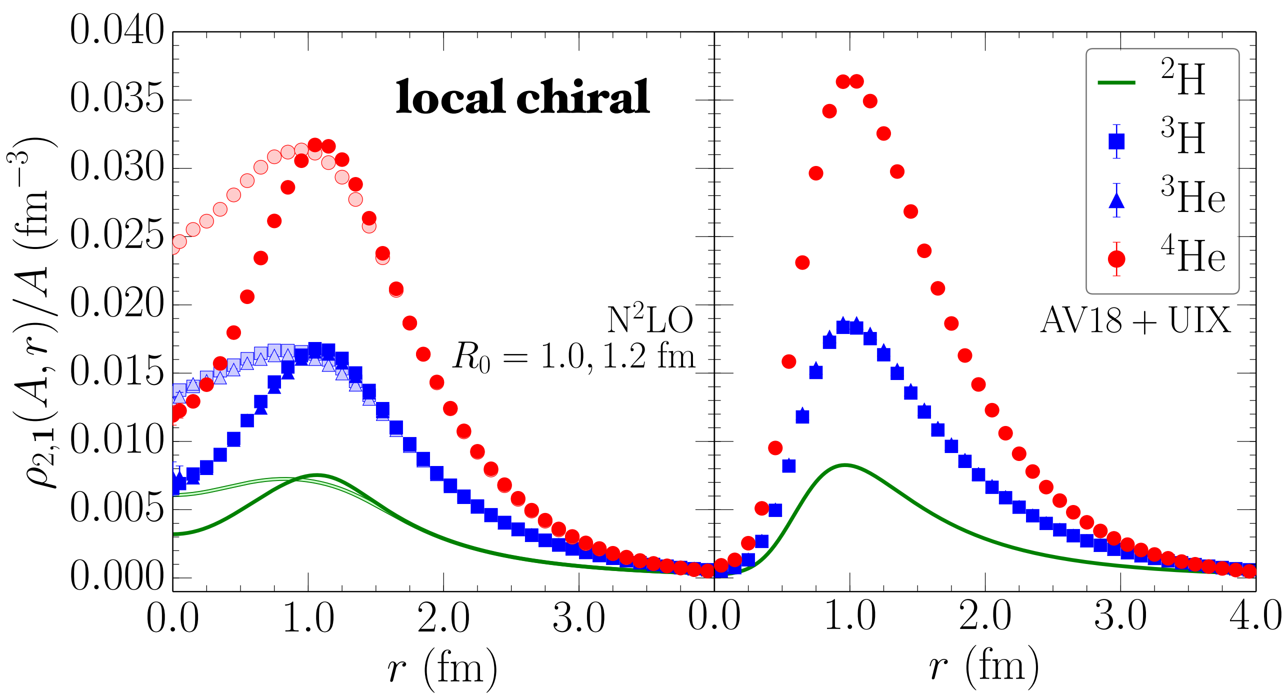
$$\frac{dR_{EMC}(A, x)}{dx} \approx C(x) \left[a_2(A) - 1 \right]$$

scale dependence cancels in ratio!

linear relation between EMC/SRC!

QMC results for a_2

Chen, et al., Phys. Rev. Lett. 119 (2017)

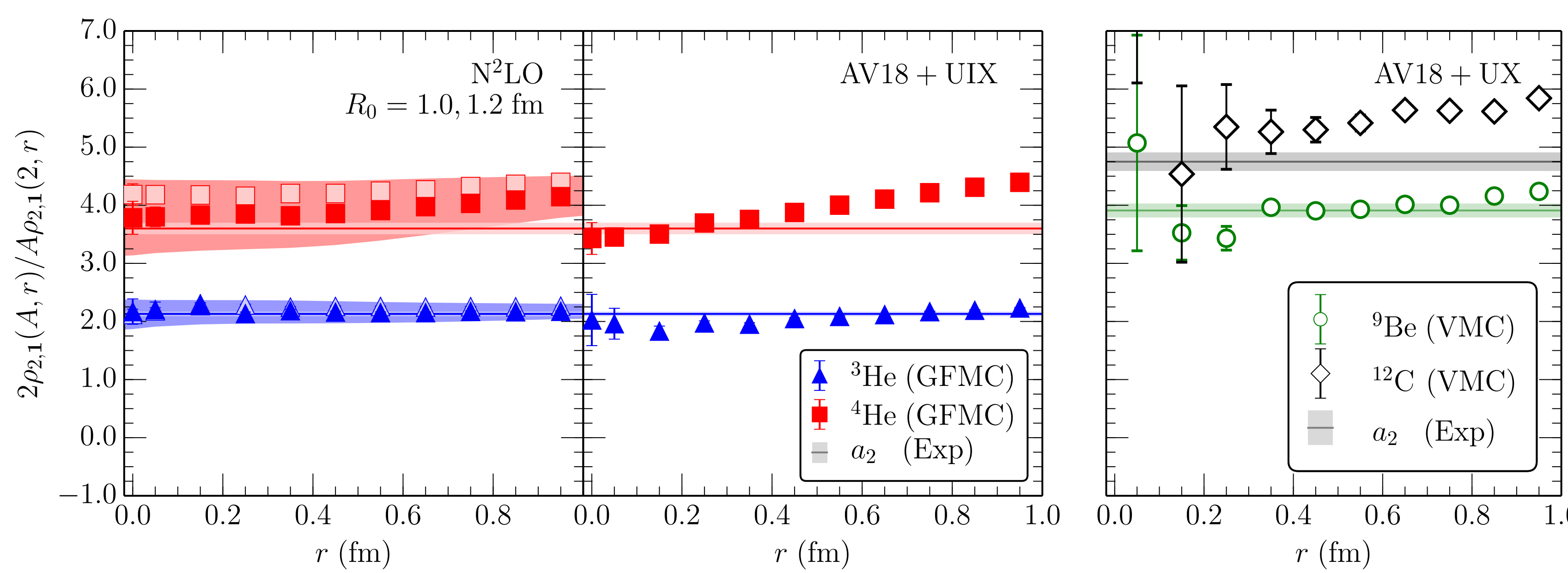


$$\rho_{2,1}(A,r) = \frac{1}{4\pi r^2}\langle\Psi|\sum_{i<j}^{A}\delta(r - |\mathbf{r}_i - \mathbf{r}_j|)|\Psi\rangle$$

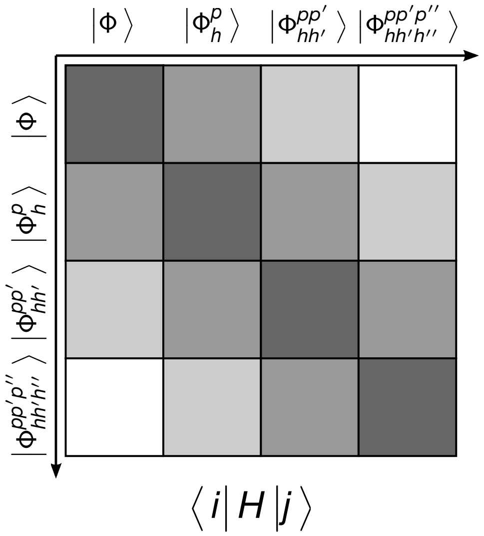
Scale and scheme dependent

…But ratio is ~ independent of scale and scheme!!

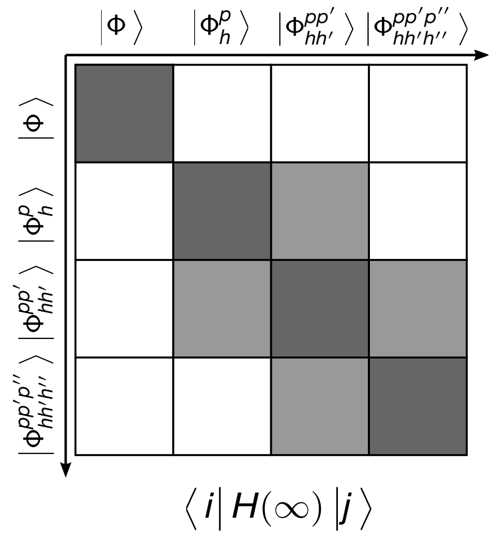
$$a_2(A) = \frac{2}{A}\frac{\rho_{2,1}(A,0)}{\rho_{2,1}(2,0)}$$



IM-SRG(2) calculations of a_2



$$\frac{dH(s)}{ds} = [\eta(s), H(s)]$$



$$\frac{d\hat{\rho}_{21}(s)}{ds} = [\eta(s), \hat{\rho}_{21}(s)]$$

$$\hat{\rho}_{21}(s=0) = \frac{1}{2}\sum_{\alpha,\beta}\int d\mathbf{R}\, N_\alpha^\dagger N_\beta^\dagger N_\beta N_\alpha$$

$$\langle \Psi_0|\hat{\rho}_{21}|\Psi_0\rangle = \langle \Phi|\hat{\rho}_{21}(\infty)|\Phi\rangle$$

Opportunities

access heavier nuclei (here up to A = 40)

access wider range of interactions**

(not limited to local interactions)

Challenges

QMC cleanly extrapolates to r = 0

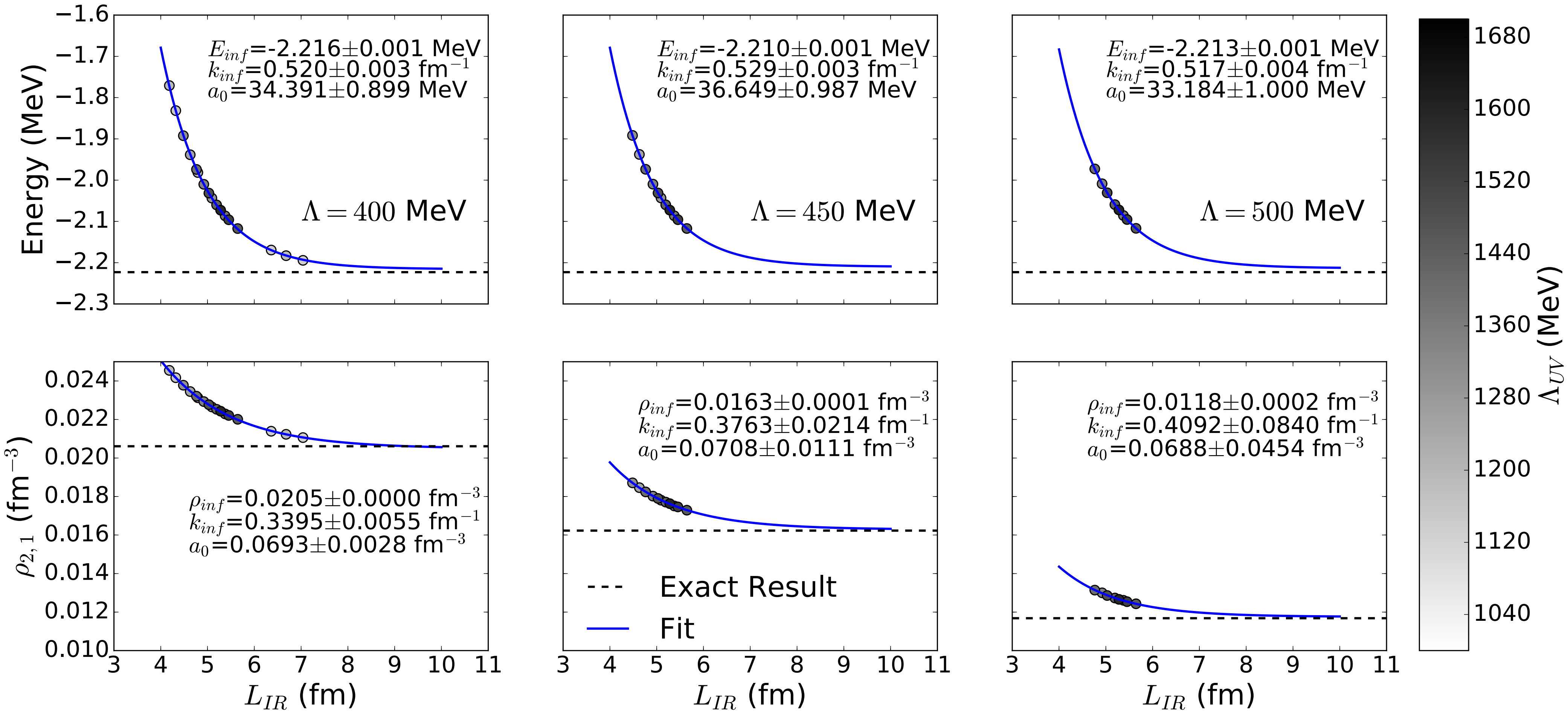
(vs. implicit smearing due to truncated HO basis)

impact of IM-SRG(2) truncation errors?

** here we use the semi-local n4lo NN interaction of Reinert, Krebs, and Epelbaum

arXiv:1711.08821

IR extrapolations of $\rho_{2,1}$



Truncated HO basis ==> IR cutoff (box size L_{IR}) and UV cutoff

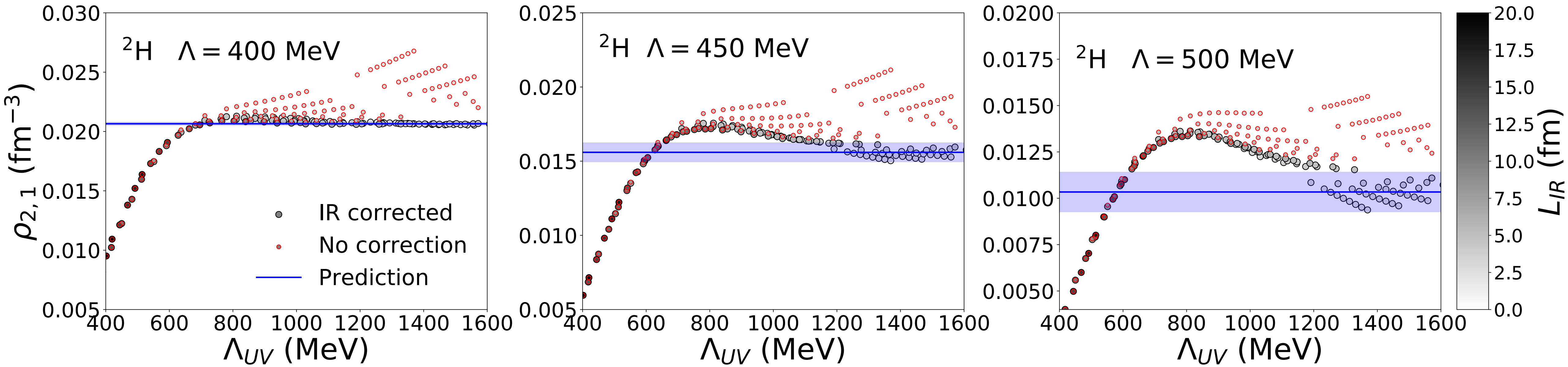
$$L_{IR} \sim \sqrt{2(2n+l)_{\max}+3}\; b$$

$$\rho_{2,1}(L_{IR}) = \rho_{2,1}(\infty) + a_0 e^{-k_\infty L_{IR}}$$

$$\Lambda_{UV} \sim \sqrt{2(2n+l)_{\max}+3}\; b^{-1}$$

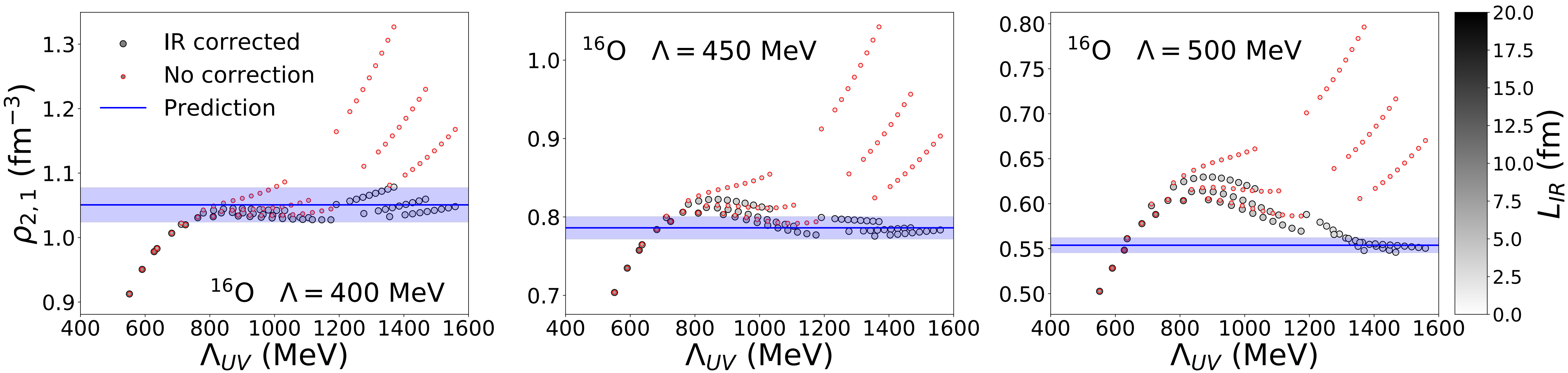
"motivated" by energy/radii formulae More et al. PRC87 (2013)

UV convergence of $\rho_{2,1}$



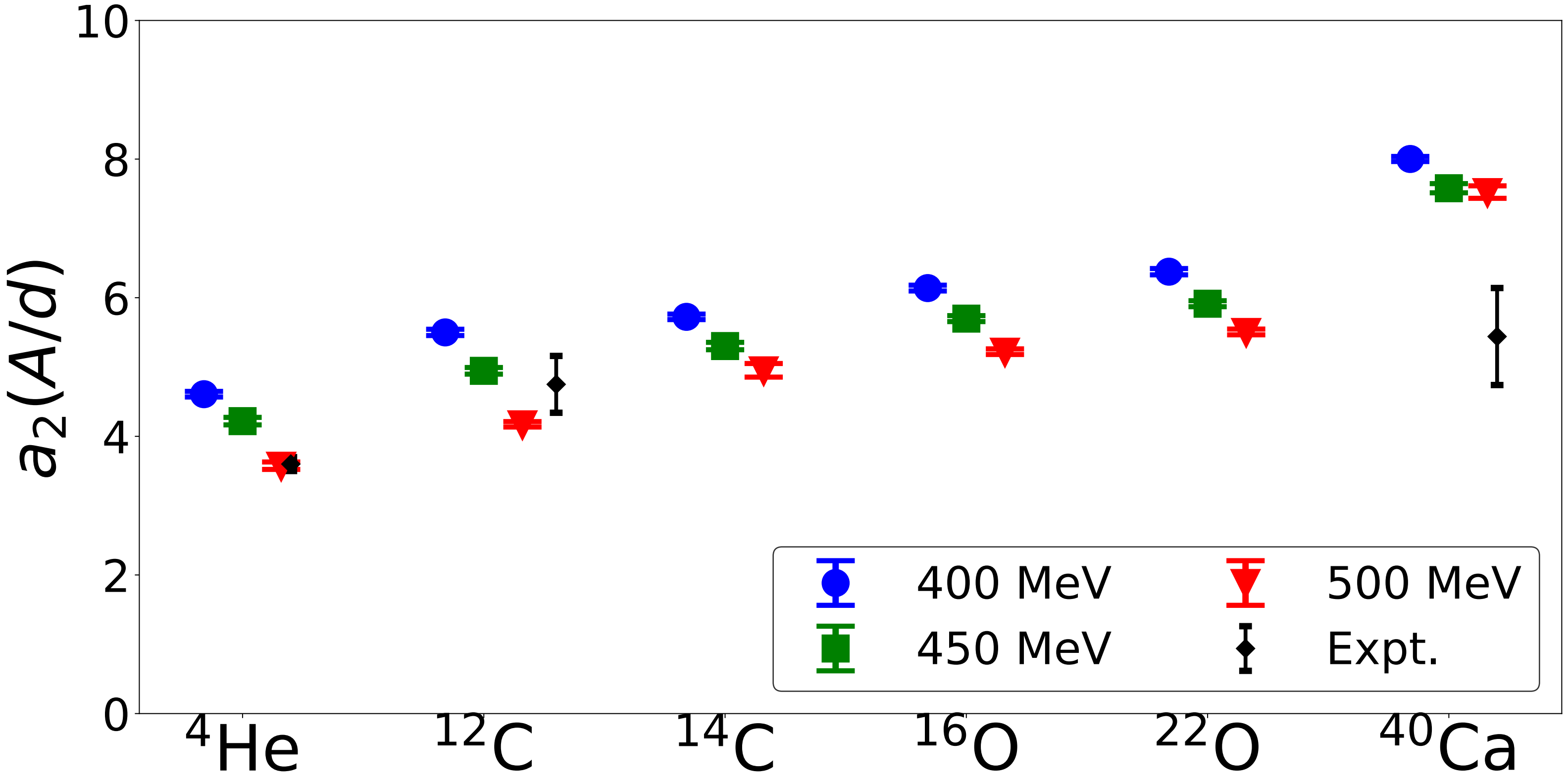
- No well-founded UV extrapolation formula
- UV convergence reasonable (w/IR correction added) for Λ= 400, 450 MeV
- 500 MeV convergence not so nice

UV convergence of $\rho_{2,1}$



- No well-founded (yet) UV extrapolation formula
- UV convergence not as clean for A > 2, not fully understood

Results for a_2



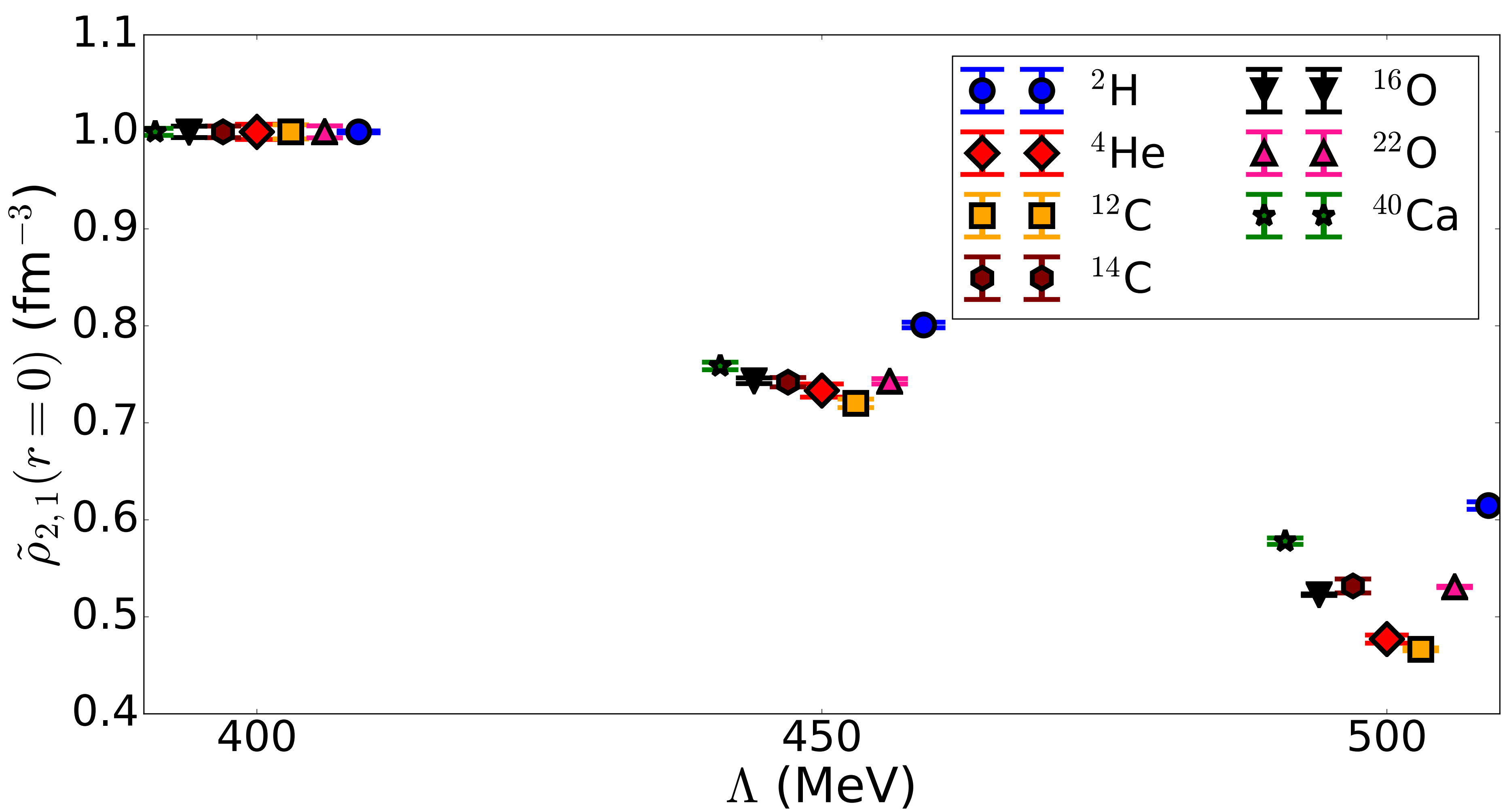
has IM-SRG(2) truncation errors

$$a_2(A/d) = \frac{2}{A}\frac{\rho_{21}(A)}{\rho_{21}(2)}$$

no many-body truncation errors (calculated in FCI)

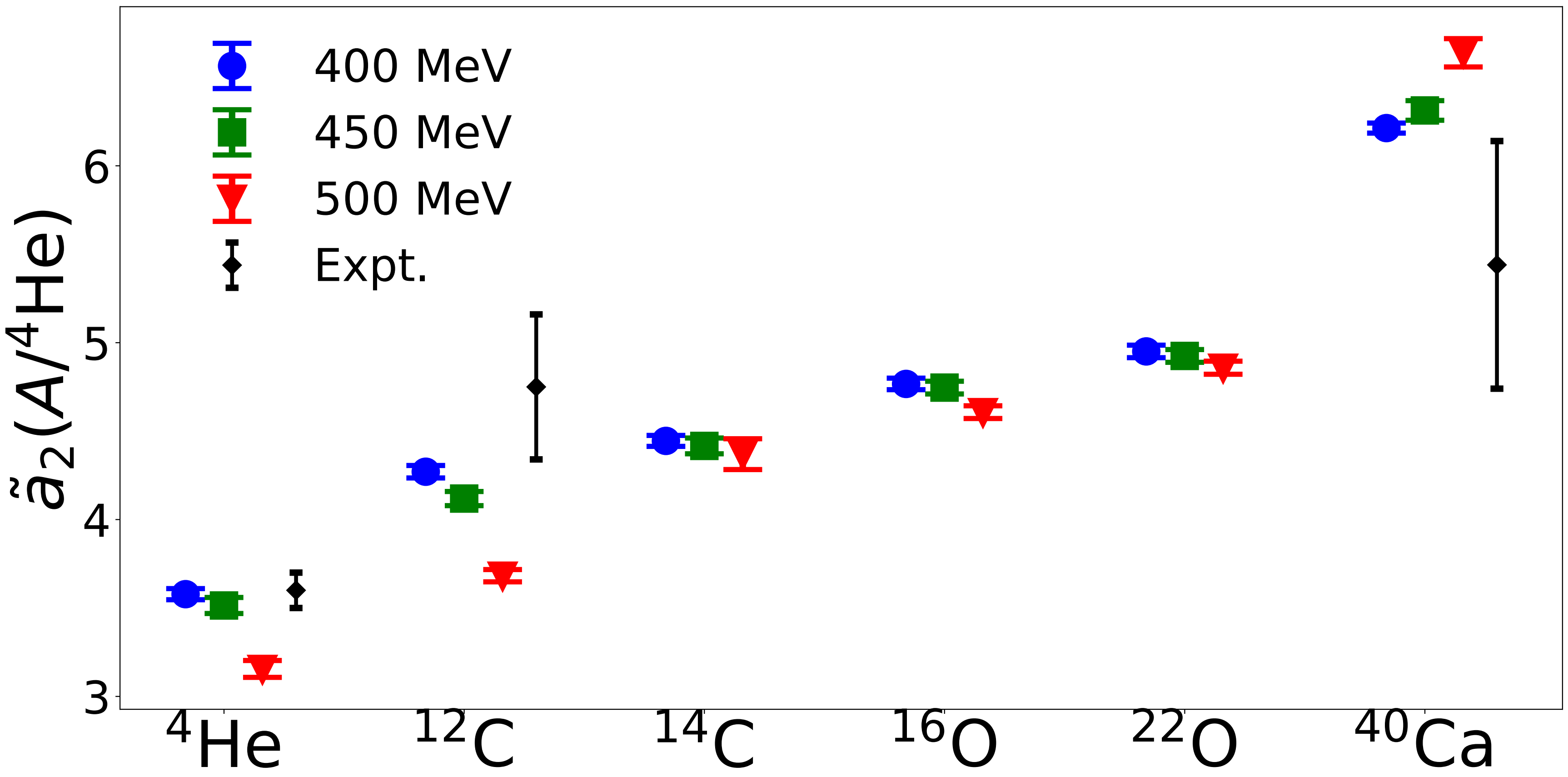
Need to disentangle UV/IR convergence and IM-SRG(2) truncation errors (and estimate EFT truncation error) before concluding if a_2 scale-independent

One possible hint



- ρ_{21} normalized to Λ=400. Different A-values should be equal
- A=2 systematically off from other A-values
- A=2 done in FCI, while A>2 in IM-SRG(2)
- Maybe better to normalize a_2 to ^{4}He instead ?

Results for a_2 (normalized to A=4)



Looks a little more systematic…

… but need better control on IR/UV convergence, IM-SRG(2) truncation errors, etc. to say more

Conclusions

- First IM-SRG calculations of SRC factor $a_2(A)$ carried out for closed-shell systems thru ^{40}Ca
- Results “not crazy” [reasonably scale-independent and close to experimental values], but much more work needed to disentangle systematics of IR/UV convergence and IM-SRG(2) truncation errors
- Near term todo list:
    - other EFT interactions (local, semi-local, non-local, different chiral orders, different cutoffs, etc.) and SRG-evolved ones to access wider range of resolution scales