Proton polarisability contribution to the Lamb shift in muonic hydrogen

Mike Birse
University of Manchester

Work done in collaboration with Judith McGovern

Eur. Phys. J. A **48** (2012) 120
arXiv:1708.09341

- Two-photon contribution to the Lamb shift
- Low-energy theorems for doubly-virtual Compton scattering
- Calculation of subtraction term in Chiral EFT

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The proton radius puzzle 1

Lamb shift in muonic hydrogen: $\Delta E_L = E(2p_{\frac{1}{2}}) - E(2s_{\frac{1}{2}}) \simeq +0.2$ eV

Much larger than in electronic hydrogen, dominated by vacuum polarisation and much more sensitive to proton structure , in particular, its charge radius

$$\Delta E_L^{\text{th}} = 206.0668(25) - 5.2275(10)\langle r_E^2 \rangle \text{ meV}$$

Results of many years of effort by Borie, Pachucki, Indelicato, Jentschura and others; collated in Antognini et al, Ann. Phys. **331** (2013) 127

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Includes contribution from two-photon exchange

$$\Delta E^{2\gamma} = 33.2 \pm 2.0\ \mu\text{eV}$$

Sensitive to polarisabilities of proton by virtual photons

Focus of this talk

The University of Manchester

The proton radius puzzle 2

CREMA experiment at PSI: $2p_{\frac{3}{2}} \to 2s_{\frac{1}{2}}$ transitions to both hyperfine $2s$ states

Pohl et al, Nature **466** (2010) 213; Antognini et al, Science **339** (2013) 417

Eliminate hyperfine splitting to get

$$\Delta E_L^{\text{expt}} = 202.3706(23)\ \text{meV}$$

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$$\Delta E_L^{\text{expt}} = 202.3706(23)\ \text{meV}$$

CODATA 2014 value for charge radius, $r_E = 0.8751(61)$ fm (electronic H), gives

$$\Delta E_L^{\text{th}} = 202.064(56)\ \text{meV}$$

Discrepancy: $0.307(56)$ meV ($> 5\sigma$!)

New value for charge radius from muonic H:

$$r_E = 0.84087 \pm 0.00026(\text{exp}) \pm 0.00029(\text{th})\ \text{fm}$$

The proton radius puzzle 3

Solutions:

(a) unexpected new physics?

Hard to find ones that are not excluded by other constraints

eg Carlson and Freid, Phys. Rev. D 92 (2015) 095024; Liu, Cloët and Miller, arXiv:1805.01028

(b) problem with electronic Hydrogen measurements?

Maybe: eH 2S–4P $\rightarrow r_E = 0.8335(95)$ fm

Beyer et al, Science 358 (2017) 79

Or maybe not: 1S–3S $\rightarrow r_E = 0.877(13)$ fm

Fleurbaey et al, Phys Rev Lett 120 (2018) 183001

If so, value of the Rydberg constant will have to change by $> 5\sigma$ (11th digit)

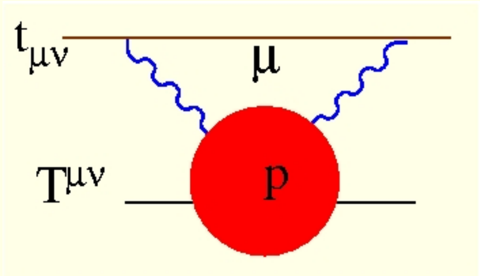
Two-photon exchange 1

In 2010: $\Delta E^{2\gamma} \sim 0.03$ meV was least-well determined contribution to ΔE_L^{th}

But it would need to be 10 times larger to explain the discrepancy

And it still contributes largest single uncertainty

$\rightarrow$ important to determine $\Delta E^{2\gamma}$ and its uncertainty as well as possible



Integral over $T^{\mu\nu}(\nu, q^2)$ – doubly-virtual Compton amplitude for proton

Spin-averaged, forward scattering $\rightarrow$ two independent tensor structures

Common choice:

$$T^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1(\nu, Q^2) + \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2} q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2} q^\nu\right) T_2(\nu, Q^2)$$

multiplied by scalar functions of $\nu = p\cdot q/M$ and $Q^2 = -q^2$

Two-photon exchange 2

Amplitude contains elastic (Born) and inelastic pieces

$$T^{\mu\nu} = T_B^{\mu\nu} + \overline{T}^{\mu\nu}$$

Elastic: photons couple independently to proton (no excitation)

• need to remove terms already accounted for in Lamb shift (iterated Coulomb, leading dependence on $\langle r_E^2 \rangle$)

$\rightarrow$ leaves "third Zemach moment" with relativistic corrections

Inelastic: proton excited $\rightarrow$ polarisation effects

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Inelastic: proton excited $\rightarrow$ polarisation effects

Elastic amplitude from Dirac nucleon with Dirac and Pauli form factors

K. Pachucki, Phys. Rev. A **60** (1999) 3593

$$\Gamma^\mu = F_D(q^2)\gamma^\mu + iF_P(q^2)\frac{\sigma^{\mu\nu} q^\nu}{2M}$$

("Sticking in form factors": Hill and Paz, Phys. Rev. D 95 (2017) 094017)

Doubly-virtual Compton scattering

Gives elastic amplitude

$$T_1^B(\nu,Q^2) = \frac{e^2}{M}\left[\frac{Q^4\left(F_D(Q^2)+F_P(Q^2)\right)^2}{Q^4-4M^2\nu^2} - F_D(Q^2)^2\right]$$

$$T_2^B(\nu,Q^2) = \frac{4e^2MQ^2}{Q^4-4M^2\nu^2}\left[F_D(Q^2)^2+\frac{Q^2}{4M^2}F_P(Q^2)^2\right]$$

On-shell intermediate nucleon states $\rightarrow$ poles at $\nu = \pm Q^2/2M$

- residues given unambiguously by elastic form factors

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On-shell intermediate nucleon states $\to$ poles at $\nu=\pm Q^2/2M$

- residues given unambiguously by elastic form factors

Final term in T_1: no pole corresponding to on-shell intermediate nucleon

But this depends on choice of tensor basis (energy-dependent tensors)

cf Walker-Loud et al, Phys Rev Lett **108** (2012) 232301; Gasser et al, Eur Phys J C 75 (2015) 375

Also parts of this term are required by low-energy theorems

Thomson limit at $O(1)$, Dirac radius at $O(q^2)$

$\to$ choose to keep it as part of Born amplitude

Low-energy theorems

V^2CS not directly measurable, but constrained by LETs

Expand in tensor basis without kinematic singularities $(1/q^2)$

Tarrach, Nuov Cim **28A** (1975) 409

$\rightarrow$ two independent tensors of order q^2: correspond to polarisabilities $\alpha+\beta$ and β from real Compton scattering

$$\begin{aligned}\overline{T}_1(\omega,Q^2) &= 4\pi Q^2\beta+4\pi\omega^2(\alpha+\beta)+O(q^4)\\ \overline{T}_2(\omega,Q^2) &= 4\pi Q^2(\alpha+\beta)+O(q^4)\end{aligned}$$

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Nonpole term in Born amplitude T_1^B contains piece $\propto Q^2$, fixed by LET:

$$F_D(Q^2)^2 = 1 - \left[\frac{1}{3}\langle r_E^2\rangle - \frac{\kappa}{2M^2}\right]Q^2 + O(Q^4)$$

Moving this to inelastic amplitude would modify LET for $\overline{T}_1$

(if β defined in usual way from real Compton scattering)

All these LETs automatically built into EFTs at 4th order (NRQED, HBChPT)

eg Hill and Paz, Phys Rev Lett **107** (2011) 160402

Dispersion relations

Information on forward V^{2}CS away from $q = 0$ from structure functions $F_{1,2}(\nu, Q^2)$ via dispersion relations

$$\overline{T}_2(\nu, Q^2) = -\int_{\nu_{th}^2}^{\infty} d\nu'^2 \, \frac{F_2(\nu', Q^2)}{\nu'^2 - \nu^2}$$

– integral converges since $F_2 \sim 1/\nu$ at high energies

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But $F_1 \sim \nu$ so need to use subtracted dispersion relation

$$\overline{T}_1(\nu, Q^2) = \overline{T}_1(0, Q^2) - \nu^2 \int_{\nu_{th}^2}^{\infty} \frac{\mathrm{d}\nu'^2}{\nu'^2} \, \frac{F_1(\nu', Q^2)}{\nu'^2 - \nu^2}$$

$F_{1,2}(\nu, Q^2)$ well determined from electroproduction experiments at JLab

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Subtraction function $\overline{T}_1(0,Q^2)$ not experimentally accessible

Subtraction term 1

Satisfies LET: $\overline{T}_1(0,Q^2)/Q^2 \to 4\pi\beta$ as $Q^2 \to 0$

But Lamb shift requires integral over all Q^2

Define form factor

$$\overline{T}_1(0,Q^2) = 4\pi\beta\, Q^2\, F_\beta(Q^2)$$

Large Q^2: operator-product expansion gives $Q^2 F_\beta(Q^2) \propto Q^{-2}$

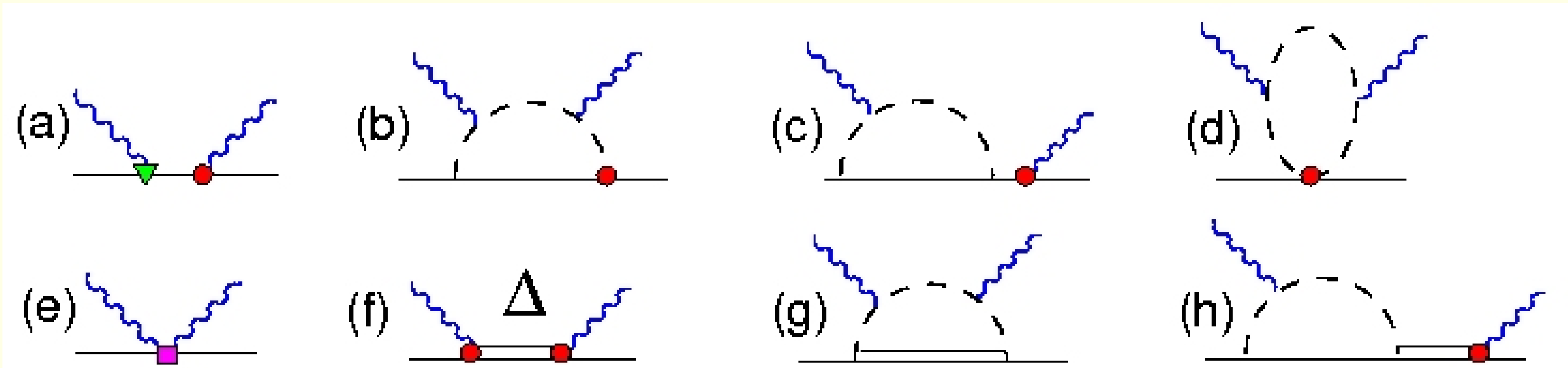
Collins, Nucl Phys B 149 (1979) 90; Hill and Paz, Phys. Rev. D 95 (2017) 094017

Small Q^2: use chiral effective field theories to calculate $F_\beta(Q^2)$

Subtraction term 2

HBChPT at 4th order, plus leading effect of $\gamma N\Delta$ form factor

- same diagrams as for real Compton scattering McGovern et al, Eur. Phys. J. A 49 (2013) 12



- minor modifications for different kinematics
- leading Δ pole (f) absorbed by LEC in seagull diagram (e)
- $\pi\Delta$ loops like (g) effectively absorbed by LECs in tadpoles like (d)
- leading contribution of $\gamma N\Delta$ form factor given by (h)
- subtract elastic contribution calculated to this order (pole + nonpole)

Subtraction term 3

3rd order EFTs give $F_\beta(Q^2)$ that can be integrated to give Lamb shift

But do not reproduce observed β

(and hence have incorrect slope for subtraction term at $Q^2 = 0$)

And single order gives no way to estimate convergence of chiral expansion

Alarcón et al, Eur Phys J C 74 (2014) 2852; Peset and Pineda, Eur Phys J A 51 (2015) 32

4th order EFTs contain LEC needed to reproduce experimental β

(and one to satisfy Dirac radius LET)

Difference between 3rd and 4th orders can be used to estimate errors

But give a form factor $F_\beta(Q^2)$ that cannot be integrated for large Q^2

Could be renormalised by μp contact interaction, fit to Lamb shift

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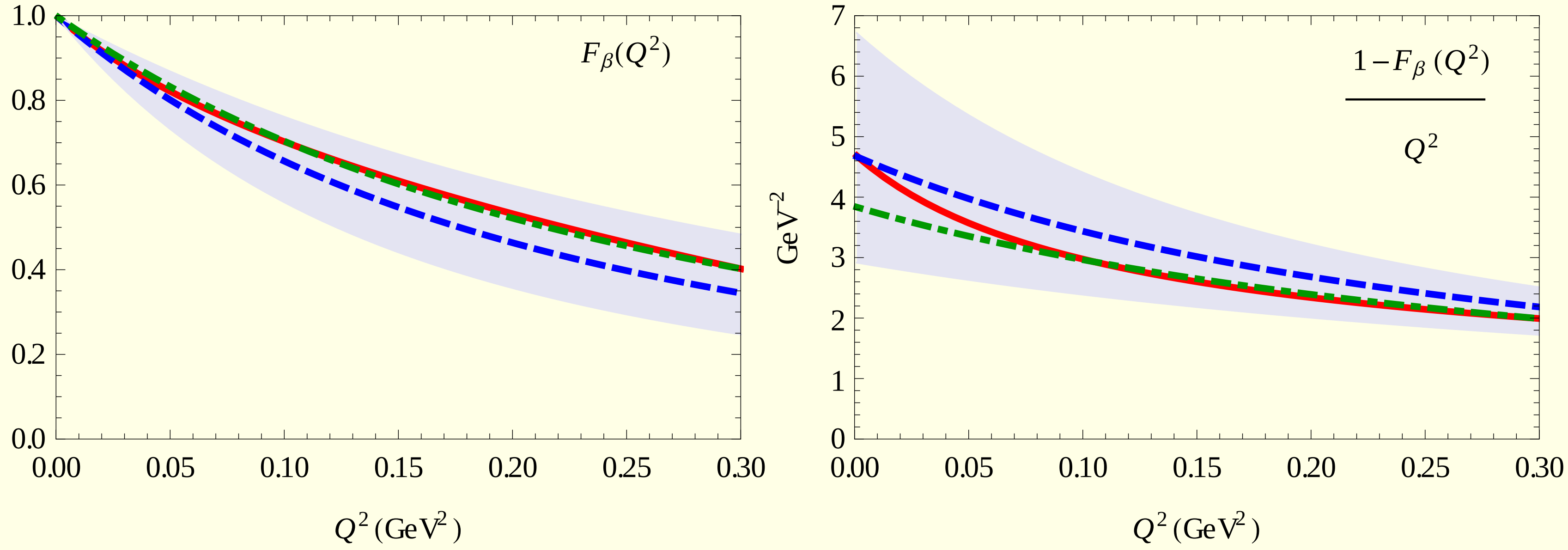
Could be renormalised by μp contact interaction, fit to Lamb shift

Here: estimate of uncertainty from difference between 3rd and 4th orders
with allowance for possible slower convergence of Δ contributions
And extrapolate to higher Q^2 by matching EFT onto dipole form of OPE

$$F_\beta(Q^2) \sim \frac{1}{(1 + Q^2/2M_\beta^2)^2}$$

Form factor 1



EFT calculation

Dipole matched at $Q^2 = 0 \to M_\beta = 462$ MeV; at $Q^2 \sim m_\pi^2 \to M_\beta = 510$ MeV

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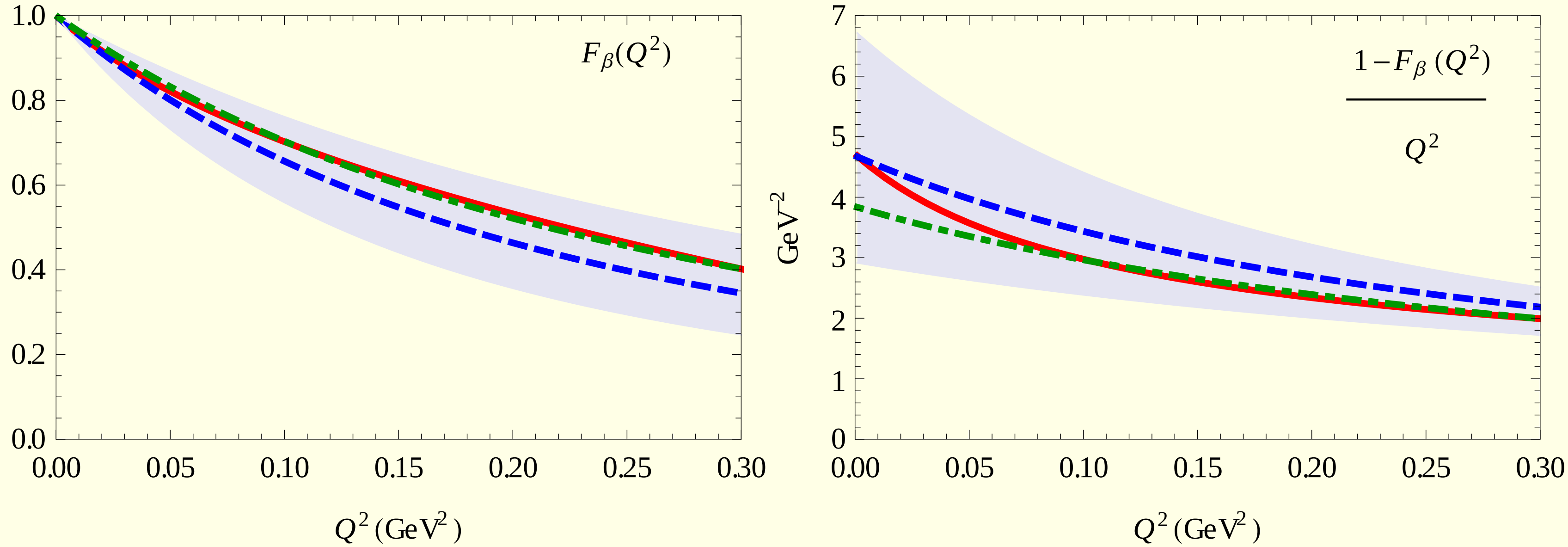

EFT calculation

Dipole matched at $Q^2 = 0 \rightarrow M_\beta = 462$ MeV; at $Q^2 \sim m_\pi^2 \rightarrow M_\beta = 510$ MeV

Form-factor mass

$$M_\beta = 485 \pm 100 \pm 40 \pm 25 \text{ MeV}$$

Uncertainties from:

- higher-order effects and uncertainties in input (shaded)
- $\beta = (3.1 \pm 0.5) \times 10^{-4}$ fm^3 Griesshammer *et al*, Prog Part Nucl Phys **67** (2012) 841
- matching uncertainty

Form factor 2

Extended and corrected OPE calculation gives coefficient of Q^{-2} for large Q^2

Hill and Paz, Phys. Rev. D 95 (2017) 094017

$$\frac{Q^2 T_1(0,Q^2)}{4\pi\alpha_{\rm EM} M} \sim 0.27 - 0.37$$

Our extrapolation: 0.2–23

Central value too high by factor of 3 to 4

But wide uncertainty band covers OPE result

And Lamb shift integral is heavily weighted to small Q^2

$\rightarrow$ interpolation from EFT to OPE will not shift result outside our error band

Born subtraction: pole?

Alternative dispersion relation for full amplitude including Born terms

Hill and Paz, Phys. Rev. D 95 (2017) 094017

Subtraction term for $T_1(\nu, Q^2)$ has slope for $Q^2 \to 0$

$$\frac{T_1(0,Q^2) - T_1(0,0)}{Q^2} = -\frac{4\pi\alpha_{\rm EM}}{3M}(1+\kappa)^2\langle r_M^2\rangle + \frac{4\pi\alpha_{\rm EM}}{3M}\langle r_E^2\rangle - \frac{2\pi\alpha_{\rm EM}}{M^3}\kappa + 4\pi\beta$$

- first term: Born pole, -3.93 ± 0.39 GeV^{-3}
- second and third terms: Born nonpole, 0.54 ± 0.01 GeV^{-3}
- final term: polarisability, 0.41 ± 0.06 GeV^{-3}

Born pole gives large slope with large uncertainty (from magnetic radius r_M)
Subtraction term with this slope multiplying poorly-known form factor $F_\beta(Q^2)$
$\to$ unnecessarily inflated error
Pole: well-defined structure, Q^2 dependence of residue given by elastic form factors

- can be extracted unambiguously from amplitude, DR applied to remainder

Born subtraction: nonpole?

Nonpole Born term different

- analytic in ν (in standard tensor basis)
- follows from Lorentz invariance (eg by "sticking form factors" into Dirac equation)
- but only terms up to order Q^2 fixed by LETs

(at higher orders: new LECs in V^{2}CS)

We choose to extract it from the subtraction term
and evaluate it using empirical form factors

- terms beyond order Q^2 contain contributions beyond order of our EFT
- effects of this choice should fall within our error estimate

Muonic H energy shift 1

$$\Delta E^{2\gamma}_{\rm sub}(2p-2s) = \frac{\alpha_{\rm EM}\,\phi(0)^2}{4\pi m}\int_0^\infty {\rm d}Q^2\,\frac{\overline{T}_1(0,Q^2)}{Q^2}\left[1+\left(1-\frac{Q^2}{2m^2}\right)\left(\sqrt{\frac{4m^2}{Q^2}+1}-1\right)\right]$$

- with dipole form, 90% comes from $Q^2 < 0.3$ GeV^2
- rather insensitive to value of M_β
- main source of error: β

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- with dipole form, 90% comes from $Q^2 < 0.3$ GeV^2
- rather insensitive to value of M_β
- main source of error: β

Result:

$$\Delta E^{2\gamma}_{\rm sub} = -4.2 \pm 1.0\,\mu{\rm eV}$$

Comparable to previous, model-based results Pachucki, Phys. Rev. A **60** (1999) 3593; Carlson and Vanderhaeghen, Phys. Rev. A **84** (2011) 020102

But with errors under much better control

Muonic H energy shift 2

Combined with results of Carlson and Vanderhaeghen

Carlson and Vanderhaeghen, Phys. Rev. A **84** (2011) 020102

- elastic (with nonpole term reinstated): $\Delta E^{2\gamma}_{\rm el} = 24.7 \pm 1.3\ \mu$eV
- inelastic (dispersive): $\Delta E^{2\gamma}_{\rm inel} = 12.7 \pm 0.5\ \mu$eV

$\rightarrow$ total: $\Delta E^{2\gamma} = 33.2 \pm 2.0\ \mu$eV

Main sources of uncertainty:

- magnetic polarisability β
- elastic form factors

Muonic deuterium 1

Lamb shift

$$\Delta E_L^{\text{th}} = 230.468(20) - 6.1103(3)\langle r_d^2 \rangle \text{ meV}$$

theory collated by Krauth et al, Ann Phys 366 (2016) 168

Two-photon exchange picks up both nuclear and hadronic contributions

Nuclear polarisability dominated by electric dipole term

Full contribution to shift from high-quality NN potentials: $\Delta E_{\text{nuc}}^{2\gamma} = 1.6615(103)$ meV

based on work of Hernandez et al, Phys Lett B 736 (2014) 344; Pachucki and Wienczek, Phys Rev A 91 (2015) 040503; updated in: Hernandez et al, Phys Lett B 778 (2018) 377

But omitted higher-order atomic effects $\rightarrow$ error on dipole contribution may be underestimated Pachucki

Muonic deuterium 2

Single proton elastic: from CV, rescaled by $\xi = (m_r^{\mu d}/m_r^{\mu p})^3$: $\Delta E_{\rm el}^{2\gamma} = 0.0289(15)$ meV

Single neutron elastic neglected

Inelastic contribution from DRs with deuteron structure functions: $\Delta E_{\rm inel}^{2\gamma} = 0.028(2)$ meV

Carlson et al, Phys Rev A 89 (2014) 022504

Subtraction term: take proton value, assume isoscalar (cf magnetic polarisabilities)
and rescale for μD: $\Delta E_{\rm sub}^{2\gamma} = 0.010(10)$ meV (taking 100% error)
$\rightarrow$ total $\Delta E^{2\gamma} = 1.7091(146)$ meV

CREMA: three hyperfine transitions Pohl et al, Science 6300 (2016) 669

$$\Delta E_L^{\rm expt} = 202.202.8785(34)\ \text{meV}$$

Gives deuteron radius $r_d = 2.12562(78)$ fm

Also $> 5\sigma$ below CODATA 2014, $r_d = 2.1413(25)$ fm

Uncertainty completely dominated by two-photon exchange

The University of Manchester

Summary

μH Lamb shift: subtraction term in two-photon-exchange contribution calculated using chiral EFT at 4th order, with Δ contribution

$$\Delta E^{2\gamma}_{\rm sub} = -4.2 \pm 1.0\ \mu\text{eV}$$

Complete two-photon exchange contribution

$$\Delta E^{2\gamma} = 33 \pm 2\ \mu\text{eV}$$

- factor 10 too small to explain proton radius puzzle (330 μeV)
- dominant error on μH Lamb shift
- main sources of uncertainty: β (subtraction) and form factors (elastic)

μD Lamb shift: hadronic subtraction term only crudely estimated

- significant additional sources of uncertainty
- along with nuclear structure piece
- and higher-order atomic corrections to it