Proton radius from electron scattering: the experimental side



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## What is "stuff"?

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#### 100 years of protons!

Proton is a composite system. It must have a size!

How big is it?

#### Motivation: "Normal" Hydrogen Spectroscopy



1S — 
$$L_{1S} = 8171.626(4) + 1.5645 \langle r_p^2 \rangle$$
 MHz

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 MHz

#### "Normal" Hydrogen Spectroscopy Results



#### Elastic lepton-proton scattering

Method of choice: Lepton-proton scattering

- Point-like probe
- No strong force
- Lepton interaction "straight-forward"

Measure cross sections and reconstruct form factors.

#### Cross section for elastic scattering

$$\frac{\left(\frac{d\sigma}{d\Omega}\right)}{\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}} = \frac{1}{\varepsilon \left(1 + \tau\right)} \left[ \varepsilon G_{E}^{2} \left(Q^{2}\right) + \tau G_{M}^{2} \left(Q^{2}\right) \right]$$

with:

$$\tau = \frac{Q^2}{4m_p^2}, \quad \varepsilon = \left(1 + 2(1+\tau)\tan^2\frac{\theta_e}{2}\right)^{-1}$$

- Rosenbluth formula
- Electric and magnetic form factor encode the shape of the proton
- Fourier transform (almost) gives the spatial distribution, in the Breit frame

#### How to measure the proton radius

$$\left\langle r_{E}^{2} \right\rangle = -6\hbar^{2} \left. \frac{\mathrm{d}G_{E}}{\mathrm{d}Q^{2}} \right|_{Q^{2}=0} \quad \left\langle r_{M}^{2} \right\rangle = -6\hbar^{2} \left. \frac{\mathrm{d}\left(G_{M}/\mu_{P}\right)}{\mathrm{d}Q^{2}} \right|_{Q^{2}=0}$$



## History of unpolarized electron-proton scattering



#### High-precision p(e,e')p measurement at MAMI

Johannes Gutenberg University of Mainz

Image © 2015 GeoBasis-DE/BKG Image © 2015 DigitalGlobe Image © 2015 AeroWest

Google earth

## High-precision p(e,e')p measurement at MAMI

Mainz Microtron
cw electron beam
10 μA polarized, 100 μA unpolarized
MAMI A+B: 180-855 MeV
MAMI C: 1.6 GeV



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A1 3-spectrometer facility
28 msr acceptance
angle resolution: 3 mrad
momentum res.: 10<sup>-4</sup>

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Google earth

#### Measured settings



#### 1422 settings

JCB et al., Phys. Rev. Lett. 105 (2010) 242001, M. O. Distler, JCB, Th. Walcher, Phys. Lett. B 696, 343 (2011) JCB et al., Phys. Rev. C90 (2014) 015206

#### Cross sections



- Replace electron with muon
- 200 times heavier  $\Longrightarrow$  200 times smaller orbit
- Probability to be "inside" 200<sup>3</sup> higher!



#### The proton radius puzzle



#### From the 2017 Review of Particle Physics

Until the difference between the ep and  $\mu p$  values is understood, it does not make sense to average the values together. For the present, we give both values. It is up to the workers in this field to solve this puzzle.





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  - seems solid
- ep experiments wrong?
  - both scattering and H-spectroscopy wrong?

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  - E.g.: Electrophobic force (Liu, Cloët, Miller arXiv:1805.01028)

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WE NEED MORE DATA

#### Deuteron (arxiv:1607.03165)

- In CODATA, r<sub>d</sub> is correlated strongly to r<sub>p</sub> because it uses "isotope shift" from 1s-2s in both systems.
- But: Can built independent value from deuteron 1s-2s and 2s-8s/8d/12d.
- $\bullet\,$  This gives a difference to muonic deuterium!  $\rightarrow\,$  another puzzle.
- Rydberg from electric Hydrogen and Deuterium in perfect agreement.
- The muonic deuterium  $R_{\infty}$  is in slight disagreement with the muonic hydrogen  $R_{\infty}$  (<3 sigma)

# New hydrogen results: MPQ (A. Beyer et al., Science 358, 79 (2017))



Bydberg constant  $R_{-} = 10.973.731.568.508 \text{ (m}^{-1)}$ 

# New results: Paris (Fleurbaey et al., Phys. Rev. Lett. 120, 183001 (2018))



#### Timeline of proton radius results



#### Comments on some newer scattering results

2010: >0.870 Hill, Paz: old data, z expansion with disp. bounds

 ${\ \bullet\ }$  Bounds on infinite exp.  ${\ \rightarrow\ }$  bounds for truncated exp.?

2012: 0.840(10) Lorenz, Hammer, Meissner: Disp. relation fit.

• Same value but a lot more data. Probably model dominated.

2014: 0.84 Lorenz, Meissner: z expansion without bounds

• Fit did not converge. In real minimum, large radius is found.

2014: 0.8989(1) Gracyk/Juszczak: Bayesian estimation

Interesting technique, unbelievable? small errors

2016: 0.84? Higinbotham: F-Test to select max. order

• Misunderstood F-test. Absence of proof  $\neq$  proof of absence.

2016: 0.84? Horbatsch/Hessels/Griffioen/Carlson/Maddox... Low-Q

Low-Q fits with low order don't work.

2018: XXX Yan/Higinbotham/...

Small radius fraction finally does bias testing



Mainz data will dominate any fit. Need similar data set to validate!

Have to extrapolate form factor to  $Q^2 = 0$ . Mainz lowest  $Q^2 = 0.0033 \, (\text{GeV/c})^2$ . We use a 10th order polynomial to fit data up to  $1 \, (\text{GeV/c})^2$ . This gets people scared.

Can we fit just a linear term?

#### Can a linear fit work?



#### (Q in units of GeV/c)

We want to measure the radius ( $\sim\sqrt{A}$ ) to within 0.5%, without knowing B. So:

$$B/A \cdot Q^2 \ll 0.01 \longrightarrow Q^2 \ll 0.002 \, (GeV/c)^2$$

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$$B/A \cdot Q^2 \ll 0.01 \longrightarrow Q^2 \ll 0.002 \, (\text{GeV}/c)^2$$

But: Need to measure A to 1%, so measure  $\frac{d\sigma}{d\Omega}$  to  $6 \cdot 0.002 \cdot 0.01 = 0.012\%$ . Good luck.

- Test / fix normalization
   Similar arguments apply, but helpful when dataset contains also higher Q<sup>2</sup>.
- Test for new physics / ultra long range structure Signal can easily, but doesn't have to be undetectable small and still change the radius!
- Measure  $r_M$ Low  $Q^2$  at  $\epsilon = 1$  means lowish  $Q^2$  at  $\epsilon = 0$

$$Q^2 = 4\frac{E}{E}'\sin^2\frac{\theta}{2}$$

- Smaller scattering angle  $\longrightarrow$  PRad
- Lower beam energy  $\rightarrow$  MESA
- Initial State Radiation
# JLAB: PRoton RADius



- High resolution, large acceptance hybrid calorimeter
- Windowless target
- ${\ {\bullet} \ }$  Simultaneous measure ep  ${\ {\rightarrow} \ }$  ep and Møller scattering
- Q<sup>2</sup> range:  $2 \times 10^{-4}$  to  $6 \times 10^{-2}$  (GeV/c)<sup>2</sup>

# JLAB: PRoton RADius



High resolution, large acceptance hybrid calorimeter

#### Status

- Data taken successfully
- Analysis ongoing

# Slide stolen from Weizhi Xiong's talk at CIPANP

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# Form Factor G<sub>E</sub> (Preliminary)



Proton Electric Form Factor G<sub>F</sub>

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# Form Factor G<sub>E</sub> (Preliminary)



Proton Electric Form Factor G<sub>F</sub>

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- Use initial state radiation to reduce effective beam energy
- Have to subtract FSR

- ISR  $\longrightarrow$  small  $E \longrightarrow$  small  $Q^2$
- Extract F.F. from radiative tail
- Or: test radiative tail description





# Target dominant source of uncertainty

# • For Mainz data, systematic errors dominate

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- For Mainz data, systematic errors dominate
  - Background from target walls
  - Acceptance correction for extended target



# Target dominant source of uncertainty

- For Mainz data, systematic errors dominate
  - Background from target walls
  - Acceptance correction for extended target
- Eliminate with jet target
  - point-like
  - o no walls
  - but less density
- Rinse, repeat with D,<sup>3</sup>He,<sup>4</sup>He, ...





### Mainz future plans

- Repeat ISR with new target
- Use new target also for classic approach



Took first data in April! Full MAMI experiment next year, MESA 2021.

<i>r<sub>E</sub></i> (fm)	ep	μp
Spectroscopy	$0.8758 \pm 0.077$	$0.84087 \pm 0.00039$
Scattering	$0.8770 \pm 0.060$	????

Measure radius with muon-proton scattering!

# MUSE - Muon Scattering Experiment at PSI



World's most powerful low-energy  $e/\pi/\mu$ -beam:

Direct comparison of ep and  $\mu p!$ 

- Beam of  $e^+/\pi^+/\mu^+$  or  $e^-/\pi^-/\mu^-$  on liquid  $H_2$  target
  - Species separated by ToF, charge by magnet
- Absolute cross sections for ep and µp
- Ratio to cancel systematics
- Charge reversal: test TPE
- Momenta 115-210 MeV/c  $\Rightarrow$  Rosenbluth  $G_E, G_M$

# Experiment layout



R. Gilman et al., arXiv:1303.2160 (nucl-ex)

- Secondary beam  $\implies$  track beam particles
- Low flux (5 MHz)  $\implies$  large acceptance
- Mixed beam  $\implies$  PID in trigger

 Absolute radius extraction uncertainties similar to current exp's.



- Absolute radius extraction uncertainties similar to current exp's.
- Difference: Common uncertainties cancel!
- → factor two more sensitivity



#### MUSE can verify $7\sigma$ effect with similar significance!

- Proton radius puzzle persists since 2010
- We need new data to resolve it
- A lot of data incoming in the next years, but pretty hard limit on achievable errors
- MUSE, with electron and muon scattering, will test
  - existing radius value
  - Iepton universality
  - two photon exchange / proton polarizability

The most exciting phrase to hear in science, the one that heralds new discoveries, is not "Eureka!" but "That's funny ..."

— Isaac Asimov

It's a common theme that a polynomial fit is related to a Taylor expansion around 0, sharing important traits mainly radius of convergence.

- "We will fit ... a simple Taylor series expansion." R.J. Hill and G. Paz, Phys. Rev. D 82, 113005 (2010)
- "correct inclusion of the lowest singularity" I. Lorenz and U.G-Meißner, Phys. Lett. B 737, 57 (2014)
- "Maclaurin fits", D. W. Higinbotham et al., Phys.Rev. C93, 055207 (2016)
- "We do not advocate using polynomial fits.... since convergence ... is not assured..." K. Griffioen et al., arxiv:1509.06676

This is wrong.

# Traits of Taylor, Weierstrass, Fits

#### Taylor expansion

- Is correct in all order (to truncated order) at  $x_0$ .
- Converges on a radius up to the next pole.

• Error is 
$$R_k = \frac{f^{(k+1)}(\xi_c)}{k!} (x - \xi_c)^k (x - x_0)$$

#### Weierstrass theorem

- Any function continuos over [a, b] can be approximated with a polynomial in that range.
- The convergence is uniform:  $\forall \epsilon > 0, \exists \text{ poly., so that } ||f(x) - p(x)||_{\infty} < \epsilon, x \in [a, b]$

#### Polynomial fit

- Minimizes L2-norm over the points:  $||f(x) p(x)||_2$
- Will converge to the function, NOT to the Taylor expansion of the function

# We have no choice

#### Taylor expansion

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- Let's fit perfect pseudo data
- Compare with Taylor expansion
- Input function: dipole, i.e. pole at  $Q^2 = -0.71 \, (\text{GeV}/c)^2$

# Fit results



Fit within 40 ppm over data range, better than expansion for  ${\cal Q}^2 > 0.15\,({\rm GeV}/c)^2$ 

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Fit within 40 ppm over data range, better than expansion for  $Q^2 > 0.15\,(\text{GeV}/c)^2$ 

• remaps flexibility:

- a lot of flexibility to small Q<sup>2</sup>: Gap is 2.2% of data range instead of 0.4%
- not enough at high Q<sup>2</sup>
- harder to fit: many local minima

# Failures to fit conformal mapping polynomials

I. Lorenz and U.G. Meißner, "Reduction of the proton radius discrepancy by 3  $\sigma$ ", Phys. Lett. B 737, 57 (2014)



### Finding better minima via random search



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Griffioen et al. "Are Electron Scattering Data Consistent with a Small Proton Radius?", arxiv:1509.06676 advocate a fit up to  $0.02 (\text{GeV}/c)^2$ . They find:

- Linear fit:  $r_e = 0.835(3)$  fm.
- Quadratic fit:  $r_e = 0.850(15)$  fm.

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$$r_e = 0.835(3)$$
 fm.

• Quadratic fit:  $r_e = 0.850(15)$  fm.

#### Questions

- Why 0.02? What happens at 0.01? 0.03?
- What is the bias of this method?

- Use two input parametrizations
  - Our 10th order polynomial fit
  - 10th order polynomial fit with radius forced to 0.841 fm.
- Generate 1000 pseudo data sets each
- Fit
- Look at extracted  $r_e$  as function of cut off.
- Compare with known input radius











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### Results on data



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- Horbatsch and Hessels, "Evaluation of the strength of electron-proton scattering data for determining the proton charge radius", Phys. Rev. C 93 015204 compare conformal mapping fits (large radius) and dipole fits (small radius) with varying cut-off.
- We already know that dipole fit to the whole range has a large bias.
- But what about smaller range?

## Results dipole fit to (pseudo) data



## F-test to determine fit order

- Nested models!
- Compare two hypothesis:
  - H0: The true model has order j
  - H1: The true model has order j+k (or any order>j)

$$F = \frac{\chi_{H0}^2 - \chi_{H1}^2}{\chi_{H1}^2} \frac{N - j}{k}$$

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#### Fisher-Snedecor

- F follows a Fisher-Snedecor distribution if H0 is true
- Otherwise: Non-central Fisher-Snedecor distribution (best case)



- Can rule out H0 at a given CL if  $F > F_{crit}$ 
  - Type I error: If H0 is falsely rejected, H0 is true.
    - $\Longrightarrow$  F is Fisher-Snedecor distributed
    - $\implies$  Can calculate how often  $F > F_{crit}$  by random chance
- Can NOT rule out H1 at same CL if  $F < F_{crit}$ 
  - Type II error: Have to assume H1 is correct!
    - $\Longrightarrow$  F NOT Fisher-Snedecor distributed
    - $\implies$  Small F can reject BOTH H0 and H1
  - This is what D. Higinbotham et al. do wrong
  - James does it (semi) correct in explanation, but wrong in example

## Remarks

- One can disprove H0 without assuming H1 to be right!
- Science: We can disprove a theory (because a prediction is off), we can not prove one.
- Other tests: similar story
  - Akaike Information Criterion (AIC) tells you if data can disprove that a certain model is "enough"
- This does not touch the problem of bias!

## Remarks

- One can disprove H0 without assuming H1 to be right!
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  - Akaike Information Criterion (AIC) tells you if data can disprove that a certain model is "enough"
- This does not touch the problem of bias!
- Here:
  - We know that the form factor  $\rightarrow 0$  for  $Q^2 \rightarrow \infty$ .
  - Any finite polynomial goes to  $\pm\infty$
  - Neither H0 nor any H1 can be true
- Everything Should Be Made as Simple as Possible, But Not Simpler (Einstein, probably)
- Let's pretend we are John Snow and know nothing.

#### F-test results for low-order polynomials



## F-test /AIC results for full data range



#### F-test /AIC results for full data range



## F-test /AIC results for full data range



- Fitting is hard
- Taylor and polynomial fits are unrelated
- Have to balance between bias and overfitting
  - Cutting data set to small Q<sup>2</sup> makes balance HARDER.
- Statistical tests do not tell you about bias.
- Statistical tests, done right, support our analysis.
- Test your method on pseudo data!

   ⇒ If you want to disprove large radius, show that you
   can replicate the large radius!
- The resolution of the puzzle can not be found in refitting of the data
- For more info: arXiv:1606.02159

# Poly/Taylor possible $Q_0^2$



## PSI setup (CREMA)



## Muonic Hydrogen Spectroscopy Results



## Muonic Hydrogen Spectroscopy Results



#### Many images from arXiv:1707.09749

 $^{8}Be$  is special: two narrow, highly energetic states which can decay to ground state via E/M



## Decay modes of $^{8}Be(18.15)$



Hadronic, electromagnetic and through internal pair conversion

## The Atomkin experiment



1.04 MeV proton beam on <sup>7</sup>Li to <sup>8</sup>Be(18.15) +  $\gamma$ . Followed by decay. Looked at  $e^{\pm}$  pairs from internal conversion.

## The beryllium anomaly



- This model has  $\chi^2/d.o.f.$  of 1.07, significance of 6.8 $\sigma$
- Bump, not last bin effect
- Rises/falls when scanning through proton energies around resonance
- Excess only happens for symmetric-energy pairs
- Preliminary reports of same excess in <sup>8</sup>Be(17.6) (same group)

- Group has a history of finding peaks
- $\bullet\,$  IIUC, the detector acceptance has a minimum at  $140^\circ\,$
- DM boson interpretation is proto-phobic to evade NA48/2 limits

• Actually:  $\frac{\epsilon_p}{\epsilon_n}$  coupling below  $\pm 8\%$ . Z<sup>0</sup> is  $\sim 7\%$ 

In DarkLight, production is via Bremsstrahlung, predominantly ISR off the electron. We can look at  $e^{-}Ta \rightarrow e^{-}TaX$ , followed by  $X \rightarrow e^{-}e^{+}$ Irreducible background:  $e^{-}Ta \rightarrow e^{-}Ta\gamma^{*} \rightarrow e^{-}Tae^{+}e^{-}$  In DarkLight, production is via Bremsstrahlung, predominantly ISR off the electron. We can look at  $e^{-}Ta \rightarrow e^{-}TaX$ , followed by  $X \rightarrow e^{-}e^{+}$ Irreducible background:  $e^{-}Ta \rightarrow e^{-}Ta\gamma^{*} \rightarrow e^{-}Tae^{+}e^{-}$ Best kinematics:

- highest production rate if X takes all electron energy. CS rise beats all
- with limited out-of-plane acceptance, symmetric angle optimal

#### Reach

