

Electroweak processes in few-body systems

Alessandro Baroni



**INT
Fundamental physics
with electroweak probes
July 10, 2018**

Collaborators

Nuclear chiral/ab-initio

L. Girlanda (INFN Lecce, Italy)

L. E. Marcucci (Univ. Pisa, Italy)

A. Kievsky, M. Viviani (INFN Pisa, Italy)

S. Pastore (LANL→WSU, USA)

R. Schiavilla (ODU/JLab, USA)

R. B. Wiringa, S. C. Pieper, A. Lovato (ANL, USA)

M. Piarulli (ANL→WSU, USA)

J. Carlson, S. Gandolfi (LANL, USA)

Lattice QCD

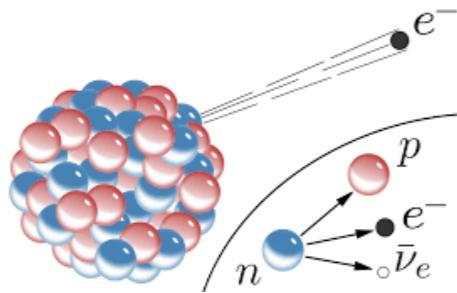
R. A. Briceño (ODU/JLab, USA)

M. T. Hansen (CERN, SUI)

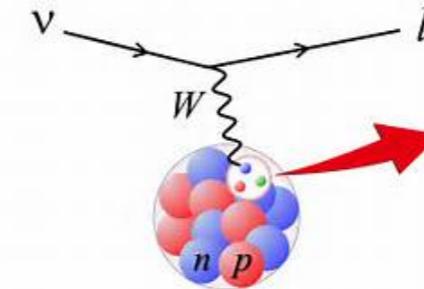
F. Ortega (College of William and Mary, USA)

D. J. Wilson (Trinity College Dublin, IRL)

Nuclear weak processes



- β decays (single and double) important for
 - Precision tests of the Standard Model
 - g_A quenching (implications for $0\nu\beta\beta$)
 - Nuclear astrophysics (Sun chain reaction)



- ν -nucleus scattering important for
 - Neutrino oscillations (SNO, ...)
 - Leptonic CP violation
 - Nuclear astrophysics (Supernovae, ..)

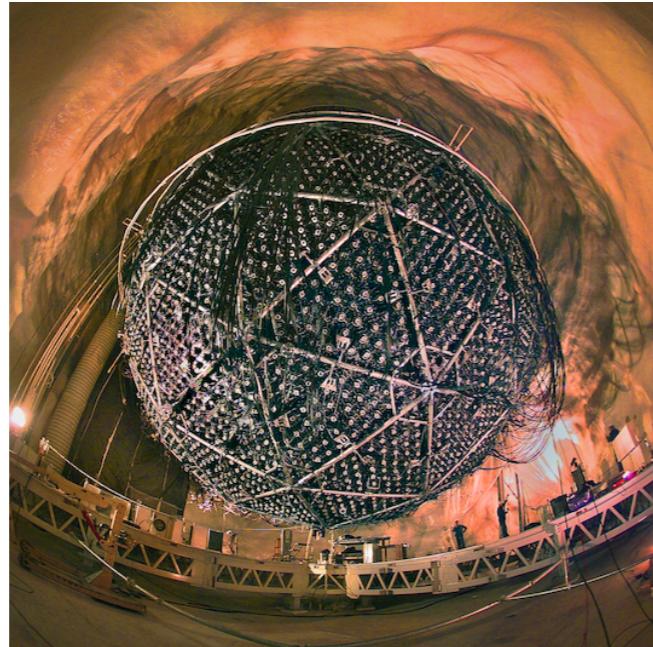
Well - known experimentally
excellent test for the theory

Less - known experimentally
need of theoretical input

SNO

- Solar neutrino problem

$$\Phi_{^8\text{B}}^{\text{Expt.}} \sim \Phi_{^8\text{B}}^{\text{SSM}}$$

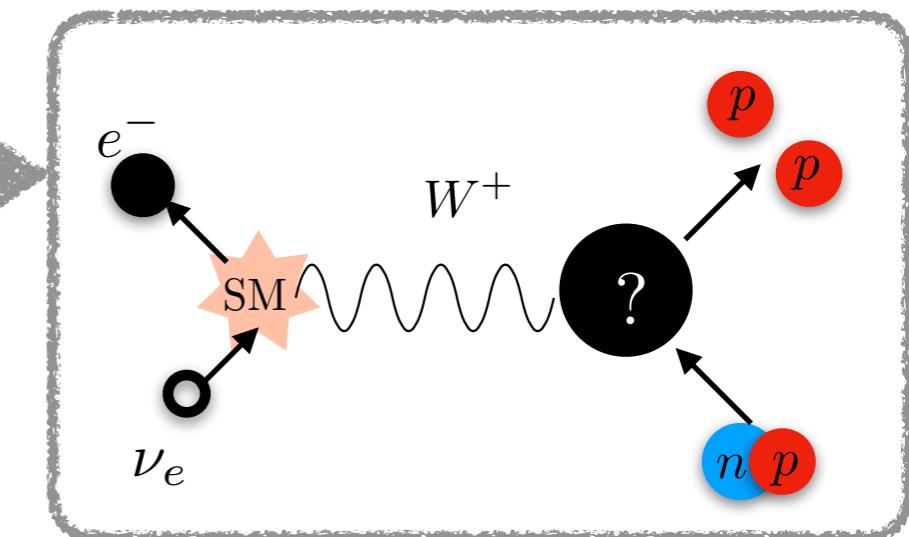


Heavy-water Cherenkov counter
built to study neutrinos coming from
 ${}^8\text{B}$ β - decay (5-15 MeV)

- CC : $\nu_e + d \rightarrow e^- + p + p$

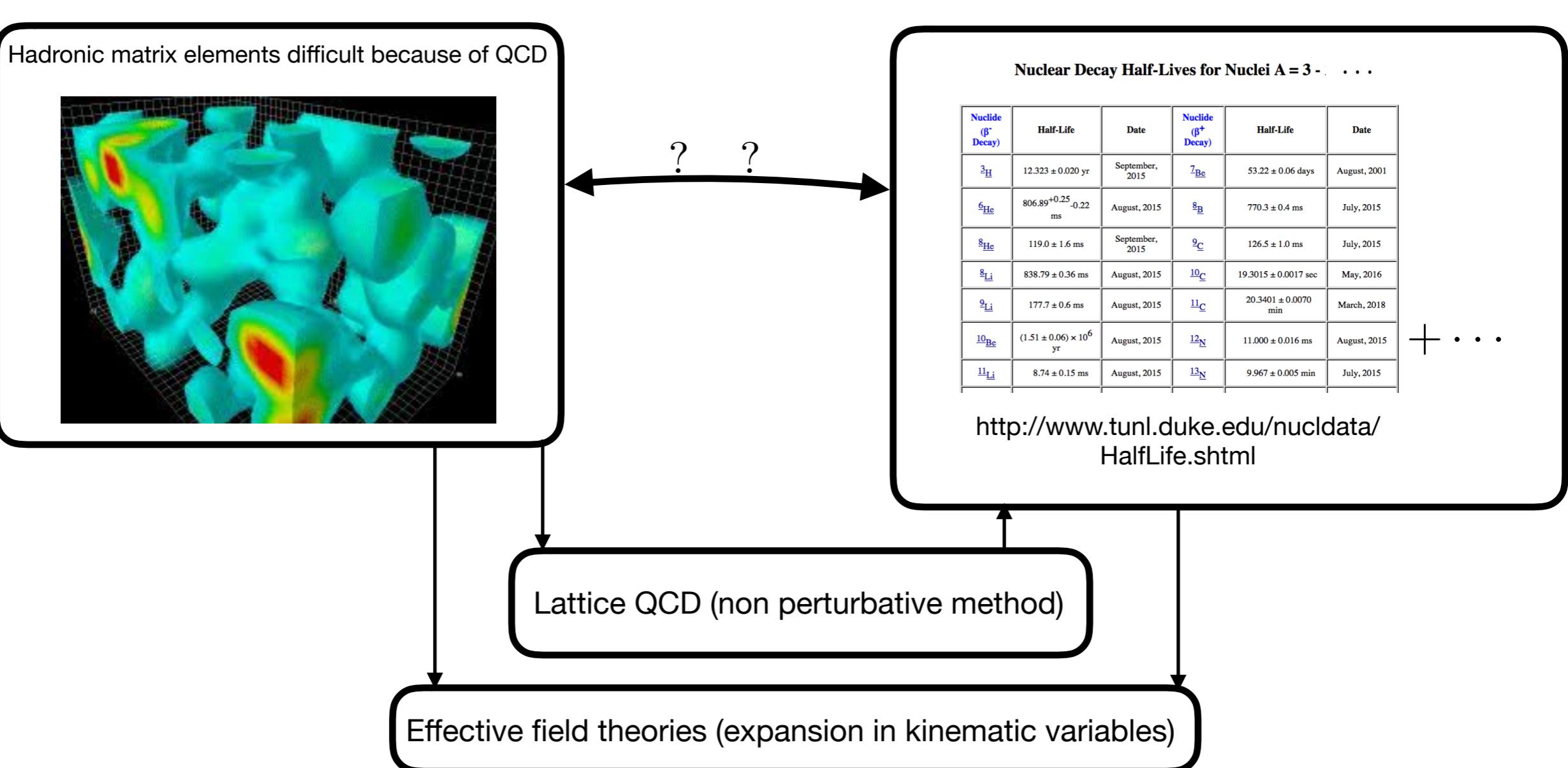
$$\text{NC} : \nu_l + d \rightarrow \nu_l + n + p$$

$$\text{ES} : \nu_l + e^- \rightarrow \nu_l + e^-$$



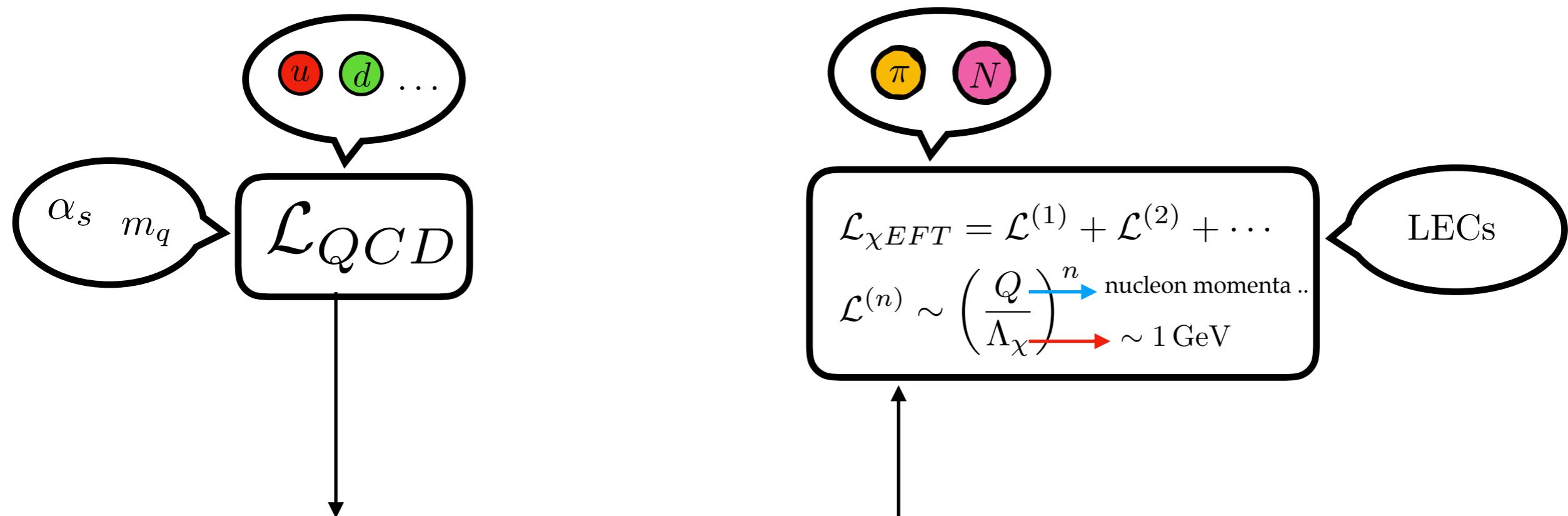
Nuclear electroweak interactions?

Atomic nuclei are a complex quantum-many body systems of **strongly** interacting nucleons



χ EFT

Build the most general Lagrangian with hadronic d.o.f. with the same exact symmetries and approximate symmetries of the underlying theory

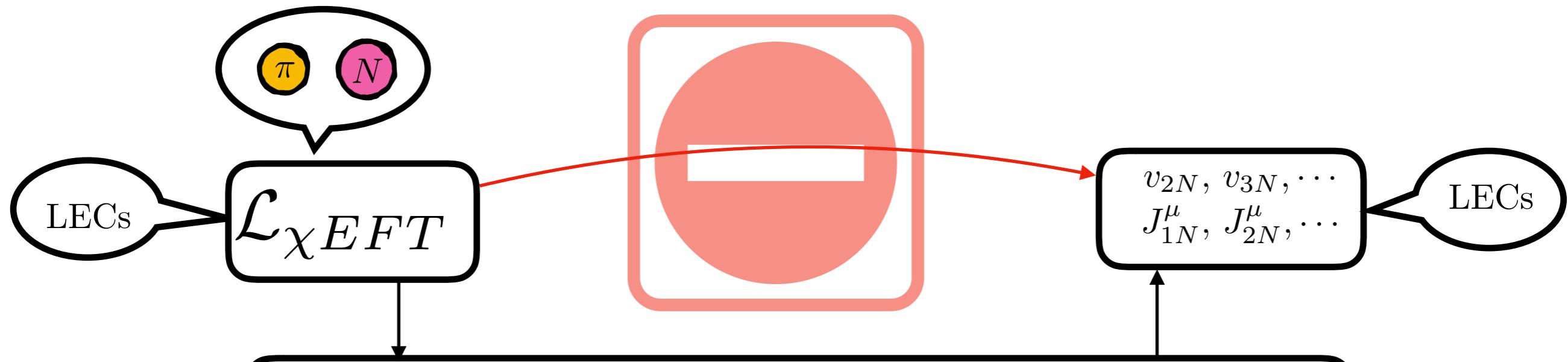


Approximate chiral symmetry requires the pion to couple to other pions and to nucleons by powers of its momentum

- S. Weinberg (1968-1979)

Nuclear χ EFT I

Nuclear bound states cannot be obtained from perturbation theory alone



- Calculate amplitudes (up to a certain order, in the power counting)
- “Prescription” to obtain potentials
- Regularization (cutoff for high momentum components)

More than 25 years of intense work (a very incomplete list of early works)

- S.Weinberg (1990,1991), U.Van Kolck et al. (1994,1996)
- N. Kaiser (1997-200)
- E. Epelbaum et al. (1998-2000), H. Krebs et al. (2007)
- Many many others

Nuclear χ EFT II

Older approach, easier, direct connection with experiments:

Observables for $\pi\pi$, πN , ...

Nuclear observables in two and three body systems

Recent approach, more difficult, connection with QCD, crucial to validate the theory:

LQCD data

Barnea et al. (2015)

Kirscher et al. (2015,2017)

Contessi et al. (2017)

Bansal et al. (2017)

LECs

Regularized operators:

v_{2N}, v_{3N}, \dots
 $J_{1N}^\mu, J_{2N}^\mu, \dots$

Predictions

- Obtain wave functions (*ab initio* methods for A>2: HH, QMC , NCS, CC,)
- Calculate matrix elements
- Assign theoretical uncertainty

Procedure

- Define a weak transition potential $v_5 = A_a^0 \rho_{5,a} - \mathbf{A} \cdot \mathbf{j}_{5,a}$ (similar to EM)
- We require the weak interaction potential to match the on shell scattering amplitude

$$T_5 = v_5 + v_5 \frac{1}{E_i - H_0 + i\epsilon} T_5$$

- Perturbative expansion in powers of the nucleon momenta

$$T_5 = T_5^{\text{LO}} + T_5^{\text{NLO}} + T_5^{\text{N2LO}} + \dots$$

$$v_5 = v_5^{\text{LO}} + v_5^{\text{NLO}} + v_5^{\text{N2LO}} + \dots$$

- Matching order by order

$$\begin{aligned} v_{5,a}^{\text{LO}} &= T_5^{\text{LO}} \\ v_{5,a}^{\text{NLO}} &= T_5^{\text{NLO}} - \left(v_{5,a}^{\text{LO}} \frac{1}{E_i - E_I + i\epsilon} v^{\text{LO}} + \text{permutations} \right) \\ &\dots \end{aligned}$$



$$\begin{aligned} \rho_{5,a} &= \rho_{5,a}^{\text{LO}} + \rho_{5,a}^{\text{NLO}} + \rho_{5,a}^{\text{N2LO}} + \dots \\ \mathbf{j}_{5,a} &= \mathbf{j}_{5,a}^{\text{LO}} + \mathbf{j}_{5,a}^{\text{NLO}} + \mathbf{j}_{5,a}^{\text{N2LO}} + \dots \end{aligned}$$

Procedure

- A subtle point: operators derived are not unique!

$$v_{5,a}^{\text{LO}} = T_5^{\text{LO}}$$
$$v_{5,a}^{\text{NLO}} = T_5^{\text{NLO}} - \left(v_{5,a}^{\text{LO}} \frac{1}{E_i - E_I + i\epsilon} v^{\text{LO}} + \text{permutations} \right)$$

...

Not uniquely defined

- Biblio

S. Weinberg (1990) TOPT; C. Ordonez and U. Van Kolck (1994-1996)

N. Kaiser et al. for nuclear potentials Feynman diagrams (1998)

S. Pastore et al. (2008-2011) for em currents, M. Piarulli et al. (2013) for em currents, TOPT

AB et al. (2016) for axial currents, TOPT

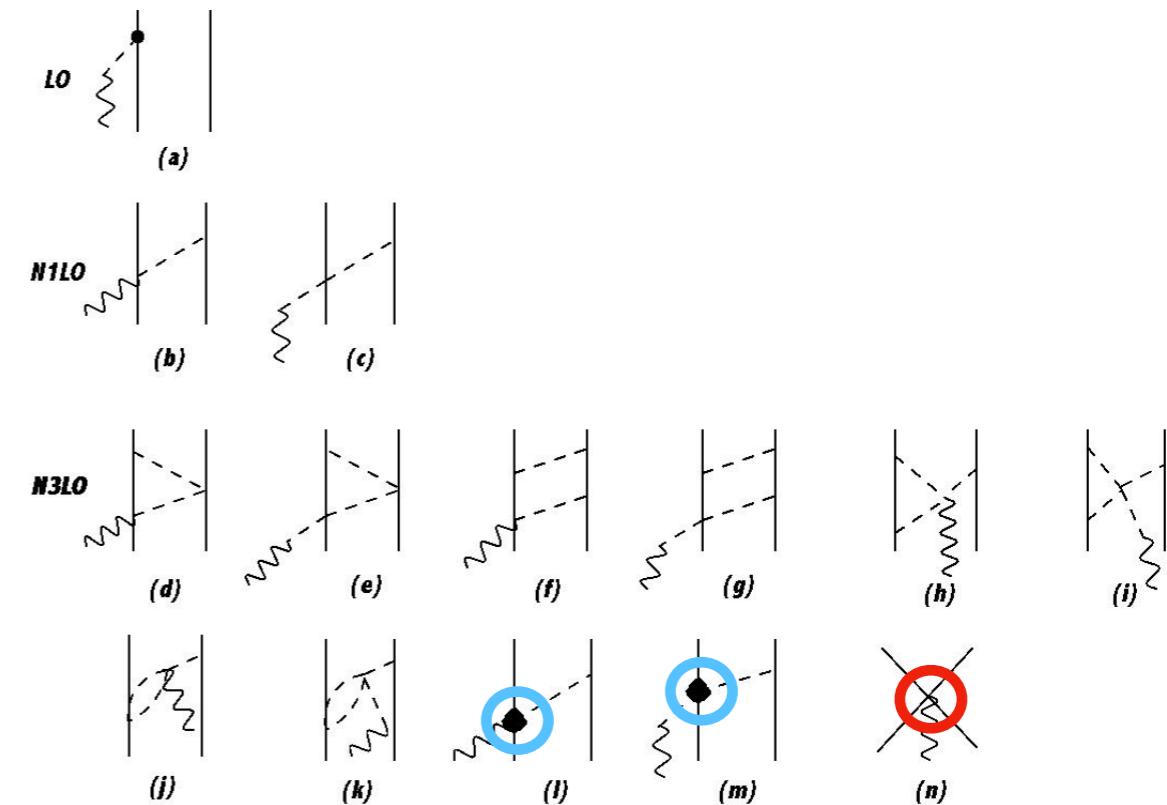
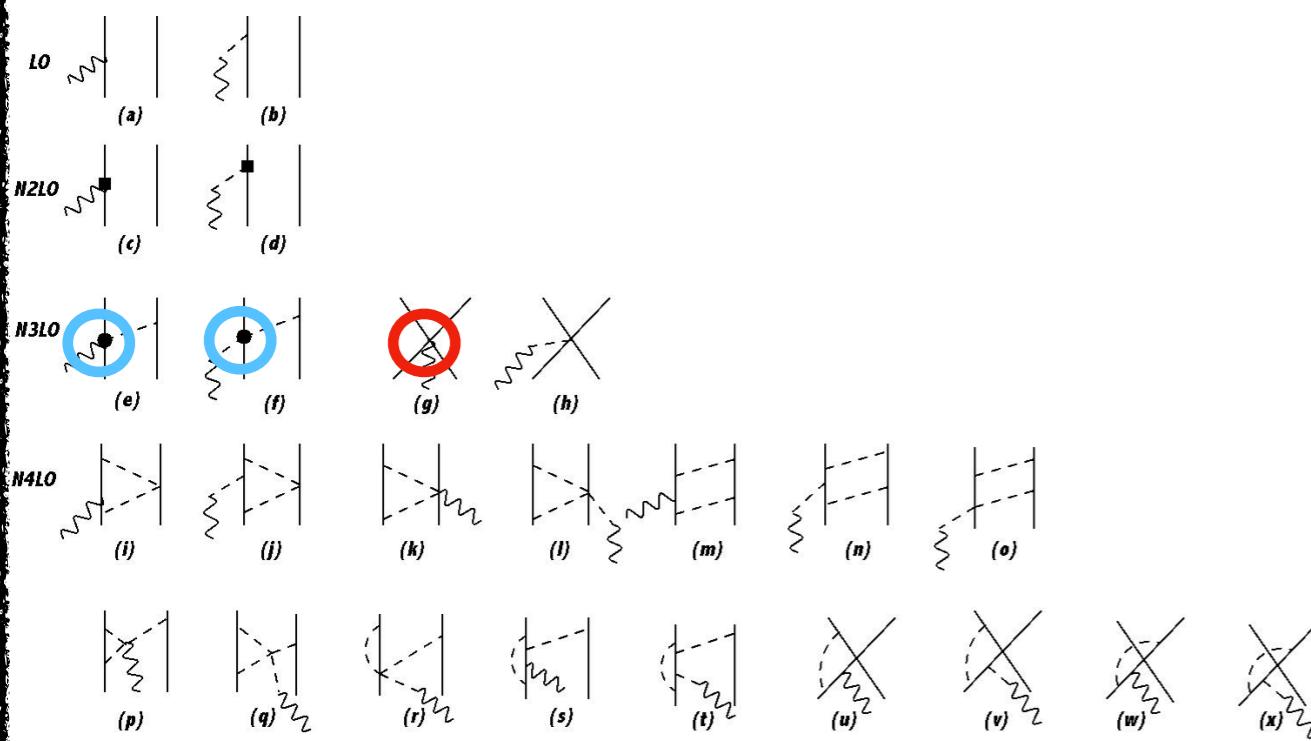
- Alternative approach using unitary transformations:

Epelbaum, Krebs, Meissner, et al. (1998-2017), for nuclear potentials, em and axial currents

Summary

- Axial current and charge derived up to N4LO
- Self consistency checks:
 - ✓ Current conservation in the chiral limit
 $\mathbf{q} \cdot \mathbf{j}_{5,a} = [H, \rho_{5,a}] \longrightarrow$ satisfied order by order
 - ✓ Renormalization of the axial charge (delicate cancellation of divergences)
 - ✓ Independence of the choice of the parametrization of the pion field
- Technical challenges:
 - “New” class of diagrams appear respect to EM currents, formalism had to be adjusted to include them (pion-pole terms)
 - >1000 diagrams in TOPT (no software infrastructure available)
- Difference for some loop topologies with another recent derivation
 - H. Krebs, E. Epelbaum, and Meissner , Unitary transformation

Axial currents



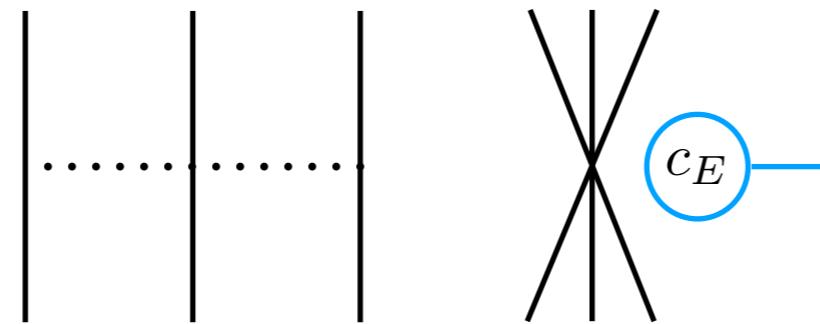
Strong and EM LECs partially known

1+4 “Weak” LECs ??

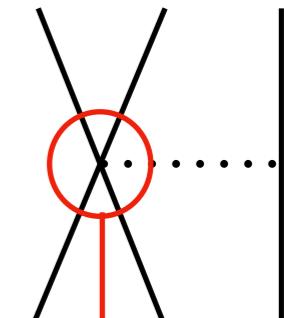
How do we fix them before ?

Actually...

LO chiral 3N force



Fixed to 3N
bindig energies



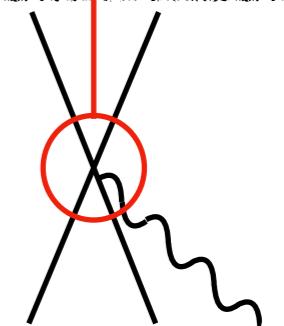
Gardestig and Phillips (2006)

Gazit et al. (2009), Marcucci et al. (2011), AB et al. (2017)

R. Schiavilla private communications (2018) (*correct relation*)

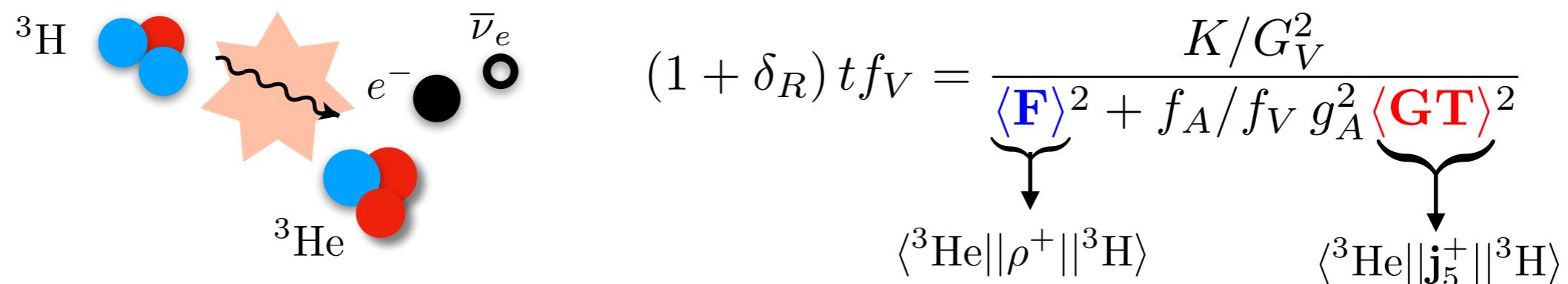
Axial current 2N contact term

Can be fixed with beta decays



Fix the LEC in axial current I

- We look at tritium beta decay rate (simplest beta decay), transition rate well known experimentally



- Wave functions are obtained solving the 3-body Schrödinger equation
[\(Pisa group specialty, Hyperspherical Harmonics, *ab initio* method\)](#)

$$\hat{H}_{\chi\text{EFT}}(c_D) | ^3\text{H}(c_D) \rangle = E_{^3\text{H}} | ^3\text{H}(c_D) \rangle$$

$$\hat{H}_{\chi\text{EFT}}(c_D) | ^3\text{He}(c_D) \rangle = E_{^3\text{He}} | ^3\text{He}(c_D) \rangle$$

- Since the 3N potential depends on 2 unknown LECs we fix c_E to three-nucleon binding energies and we get a family of wave functions

Fix the LEC in the axial current II

- Fitting of the triton GT matrix element using AV18+UIX and N4LO currents

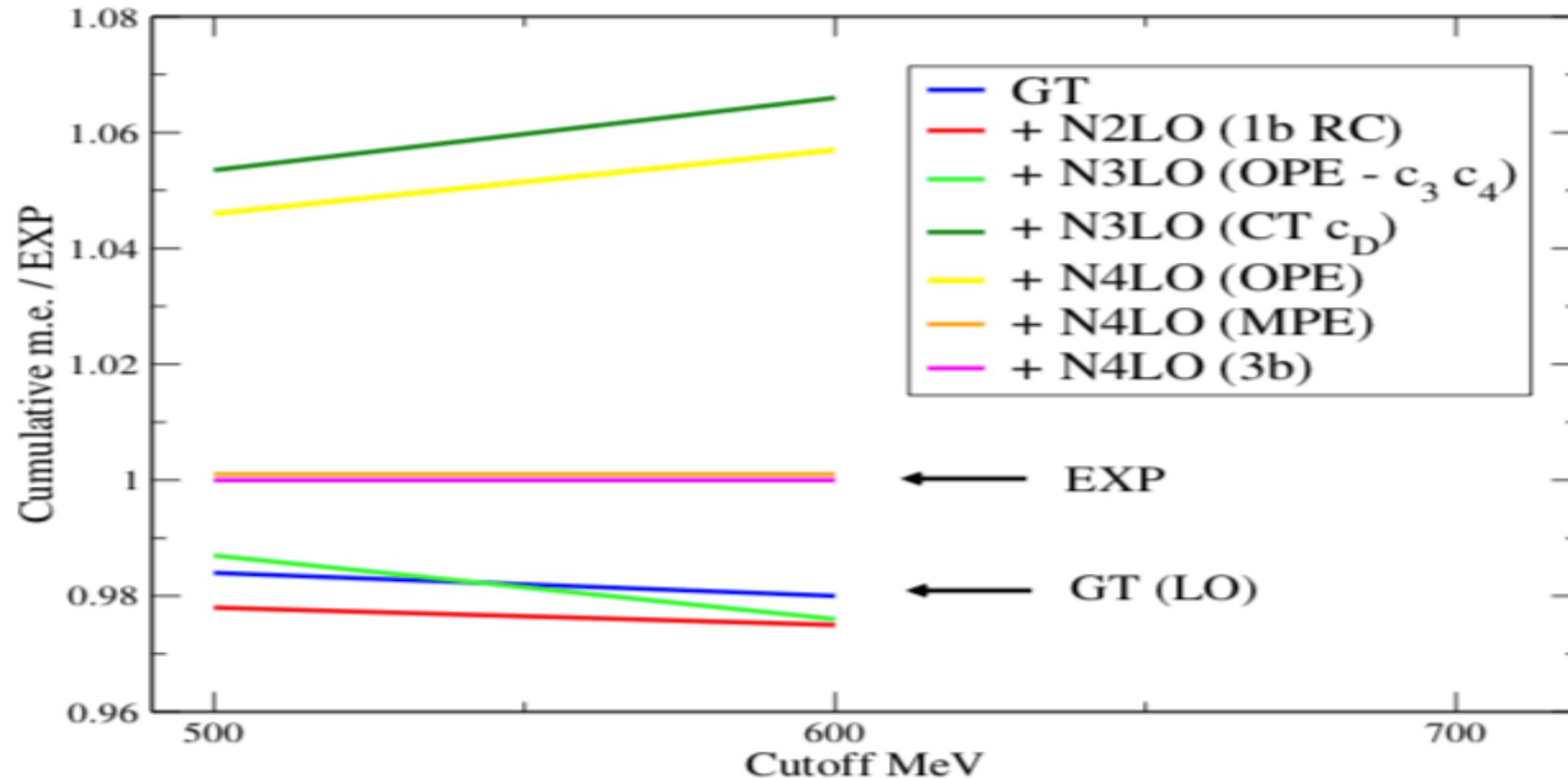


Fig. courtesy of S. Pastore

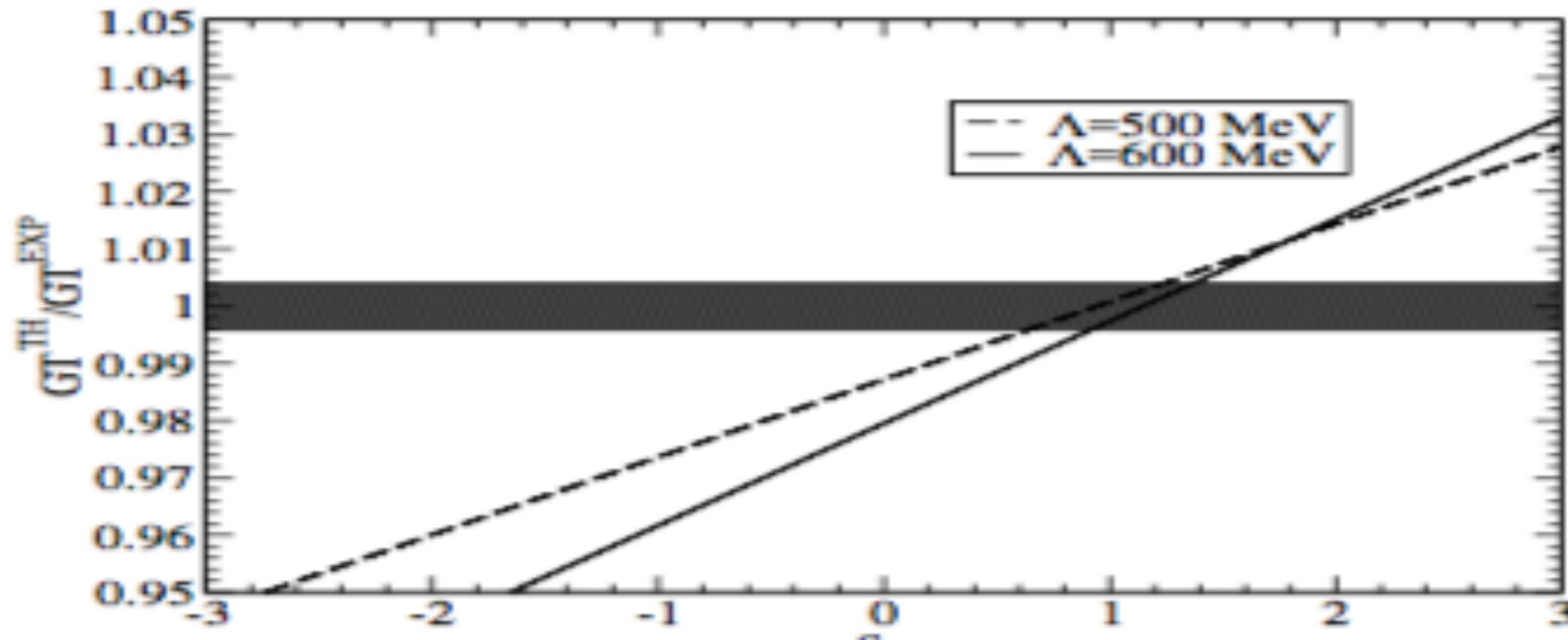
AB, Schiavilla, Marcucci et al. (2016)

Fix the LEC in the axial current III

- Fitting of the triton GT matrix element

$$\langle {}^3\text{He}(c_D) | H_{\chi\text{EFT}}(c_D) | {}^3\text{H}(c_D) \rangle = f(c_D)$$

Wave functions from Entem-Machleidt Chiral potential N3LO currents



	$\Lambda = 500 \text{ MeV}$	$\Lambda = 600 \text{ MeV}$
c_D	0.65 – 1.24	0.92 – 1.37

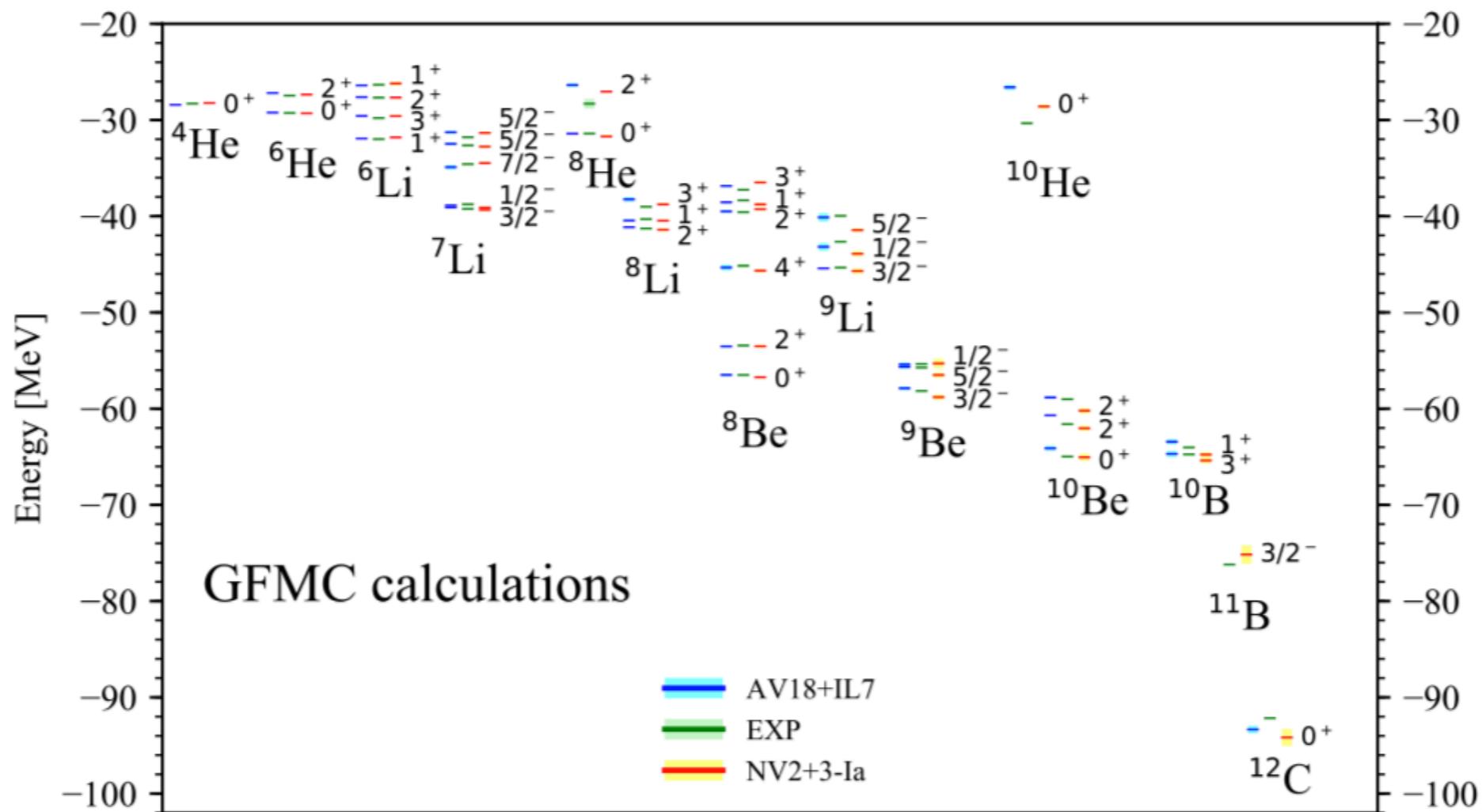
Fig. courtesy of L. E. Marcucci

AB, R. Schiavilla, L.E. Marcucci et al. (2017)
L. Marcucci et al. (2018)

CONTRIBUTIONS

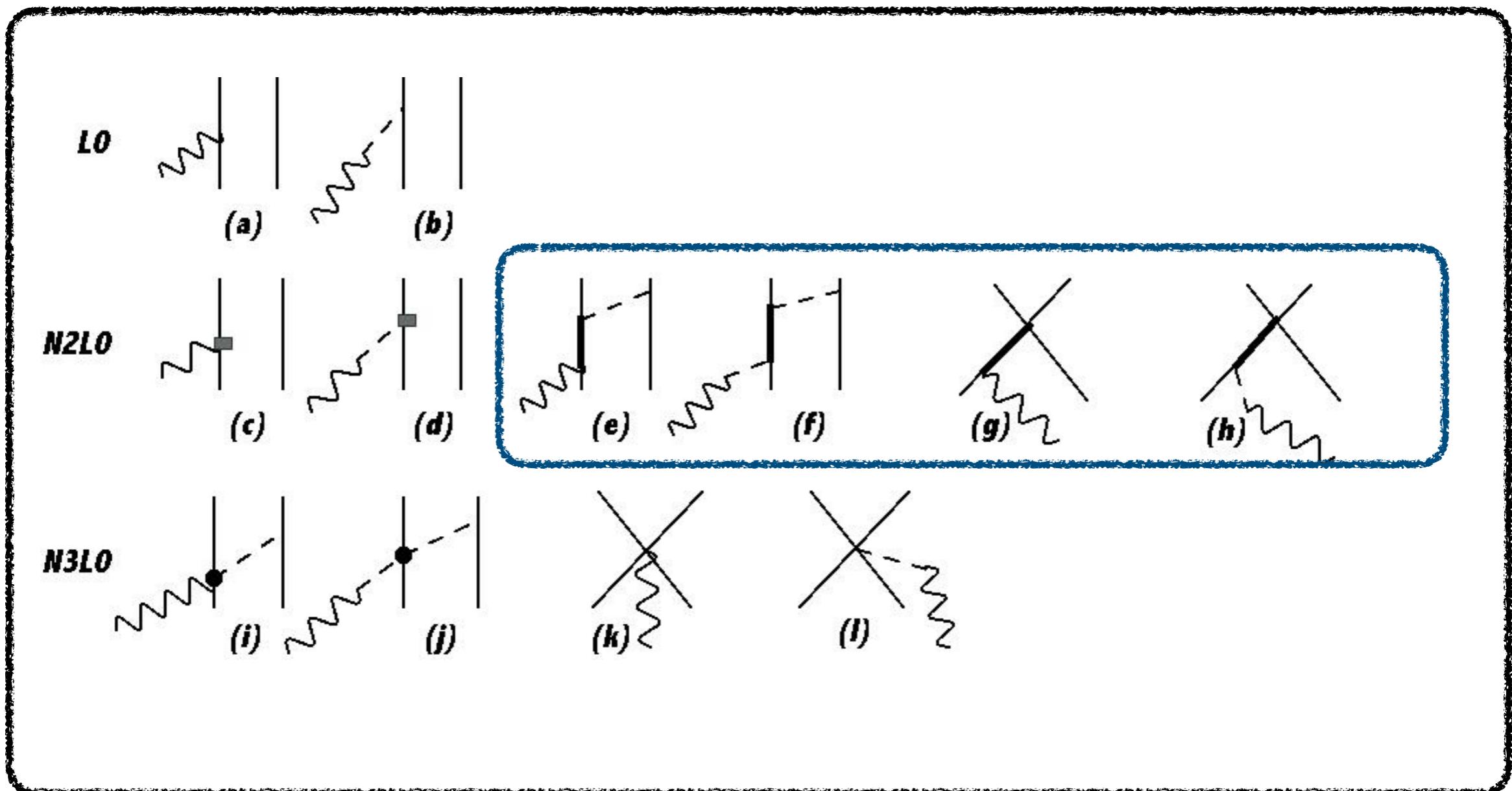
	500 MeV	600 MeV	
LO	0.9363	0.9322	→ Major contribution
N2LO	-0.569×10^{-2}	-0.457×10^{-2}	→ Relativistic correction to 1-body
N3LO(1π)	0.825×10^{-2}	0.043×10^{-2}	→ 2-body tree level, pion range
N4LO(loop)	-0.486×10^{-1}	-0.600×10^{-1}	→ Loop big effect
N4LO(3Bd)	-0.143×10^{-2}	-0.153×10^{-2}	→ 3-body currents, suppressed

Adding Δ 's ? Why?



Δ 's important role in the reproduction of light nuclear spectra
might also be important for electroweak processes in light nuclei

Axial currents with Δ 's excitations

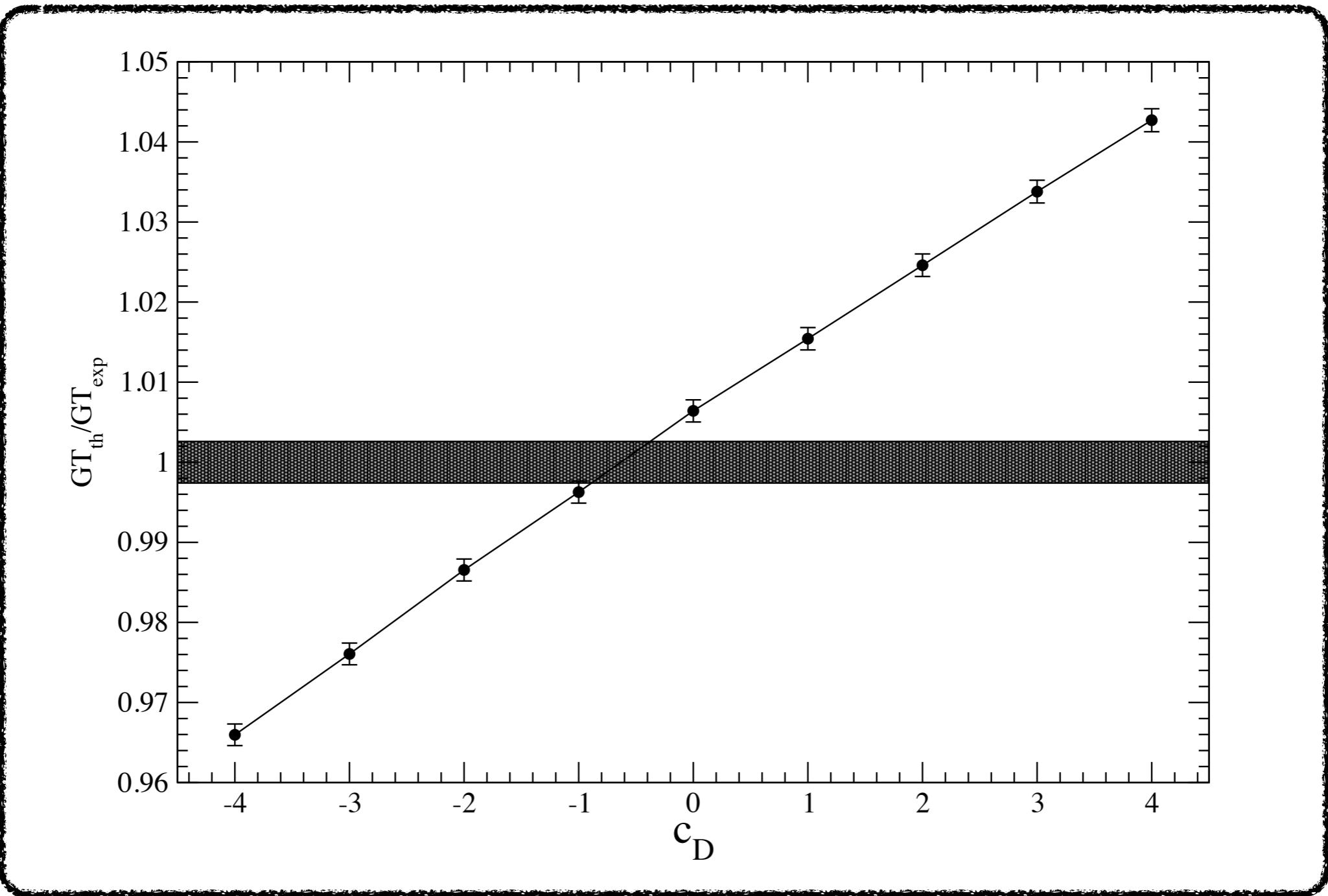


AB, R. Schiavilla, L.E. Marcucci, et al. (2018)

Axial currents with Δ 's excitations

- Derivation is straightforward (only tree level)
- Real challenge adopt consistent regularization with Piarulli et al. potentials
 - Perform Fourier transform and then regularize in r-space
 - Mathematical trickery in AB, R. Schiavilla, L.E. Marcucci, et al. (2018)
(General approach could be applied to other available potentials and currents)
- Only one new LEC h_A (analog of g_A)
- LECs consistent with the potential (only central values, for now..)
- Inclusion of some three-body currents (only irreducible ones)

Axial currents with Δ 's excitations



AB, R. Schiavilla, L.E. Marcucci, et al. (2018)

Axial currents with Δ 's excitations

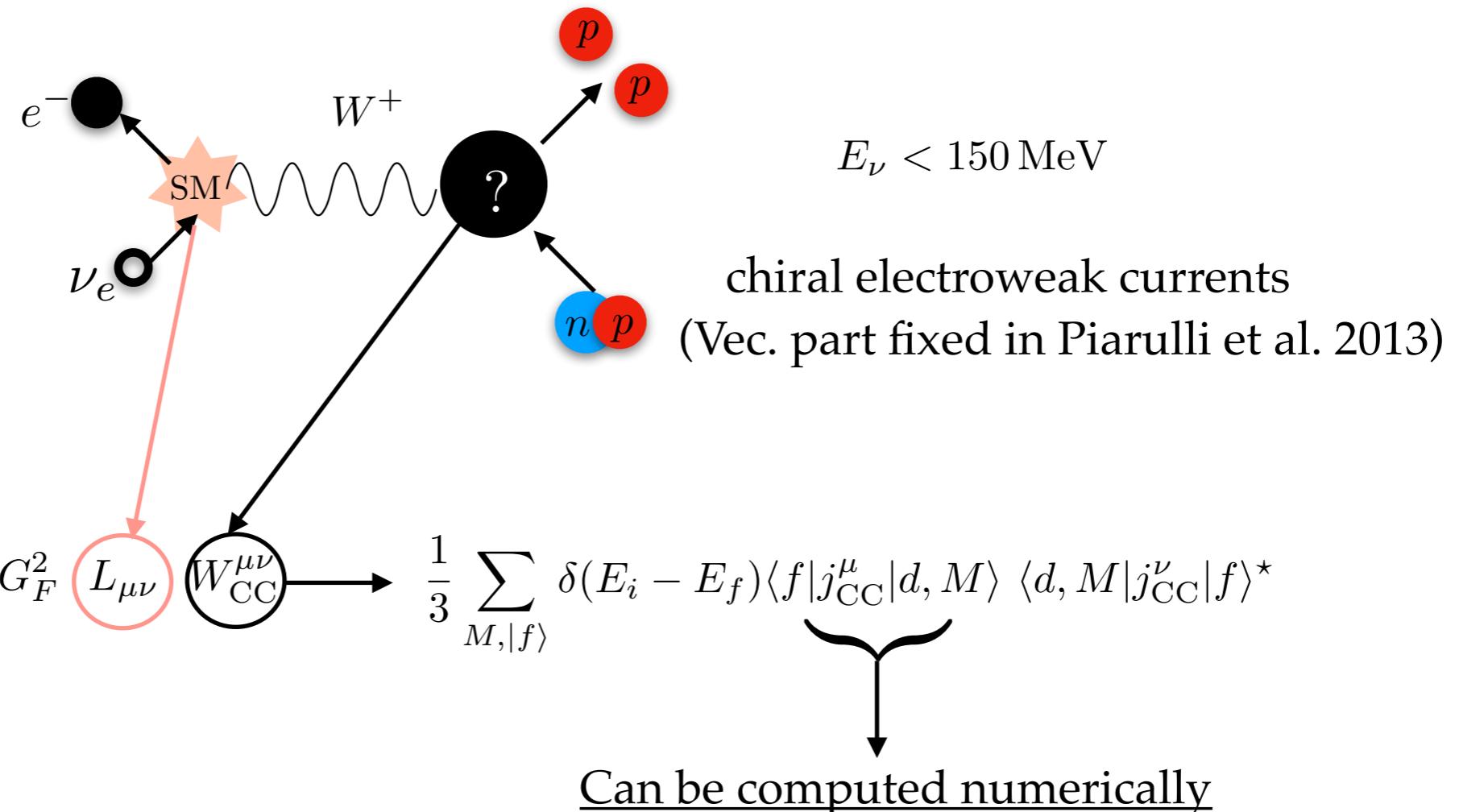
$$B(^3\text{H}) = -8.475 \text{ MeV} \quad B(^3\text{He}) = -7.725 \text{ MeV}$$

	Ia*	Ib*	IIa*	IIb*
c_D^*	(-0.89, -0.38)	(-4.99, -4.42)	(-0.89, -0.33)	(-5.56, -4.94)
c_E^*	(-0.01, -0.17)	(+0.70, +0.40)	(-0.25, -0.45)	(+0.23, -0.13)

Take home message

- LEC in the axial current determined using tritium beta decay
- Loop give important contribution
- Axial current acquires predictive power
- For axial charge as a first step we will assume LECs ~ 1
- Second not trivial application is low energy neutrino deuteron scattering

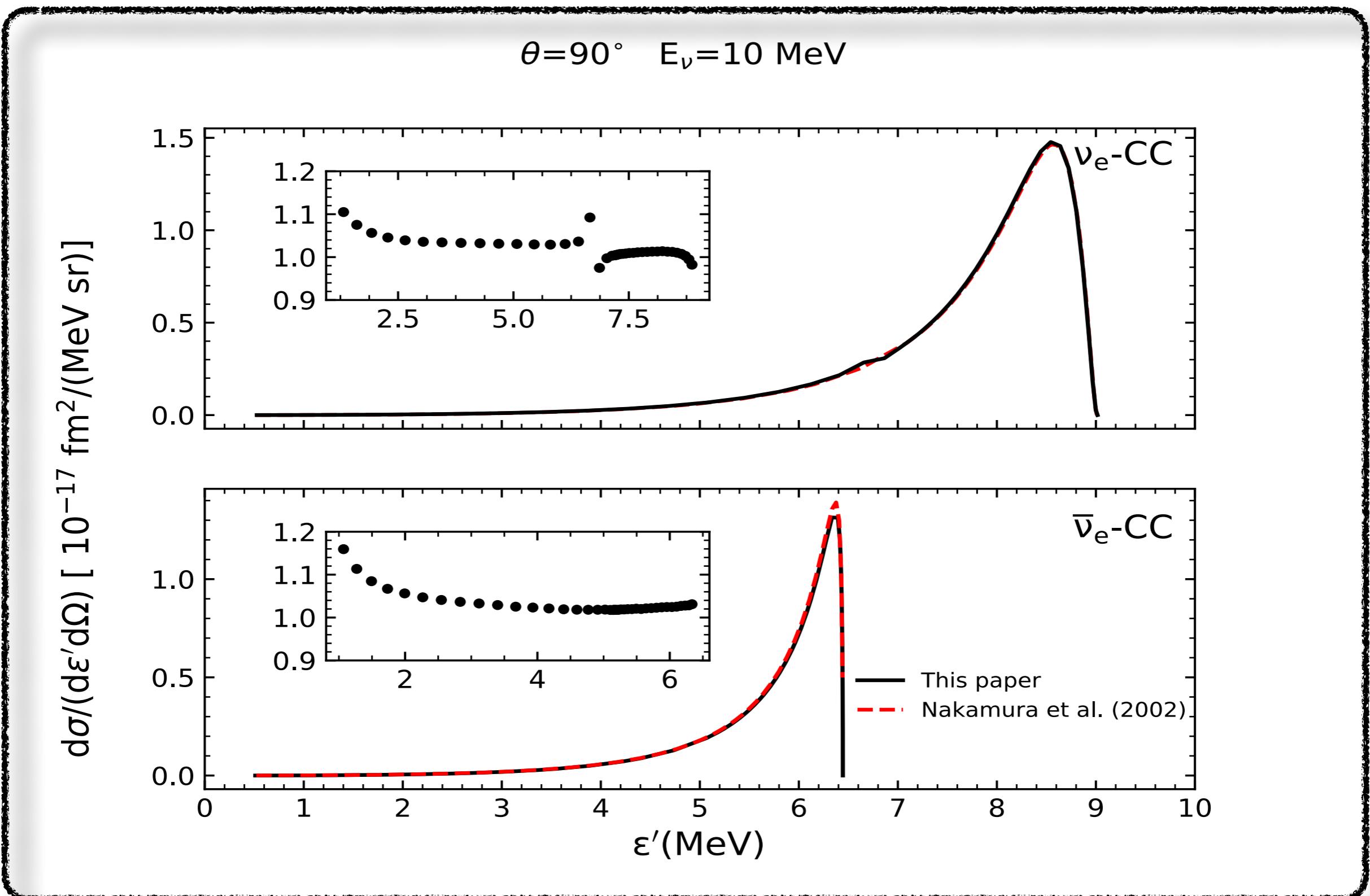
Neutrino deuterium



- Similar for neutral current process

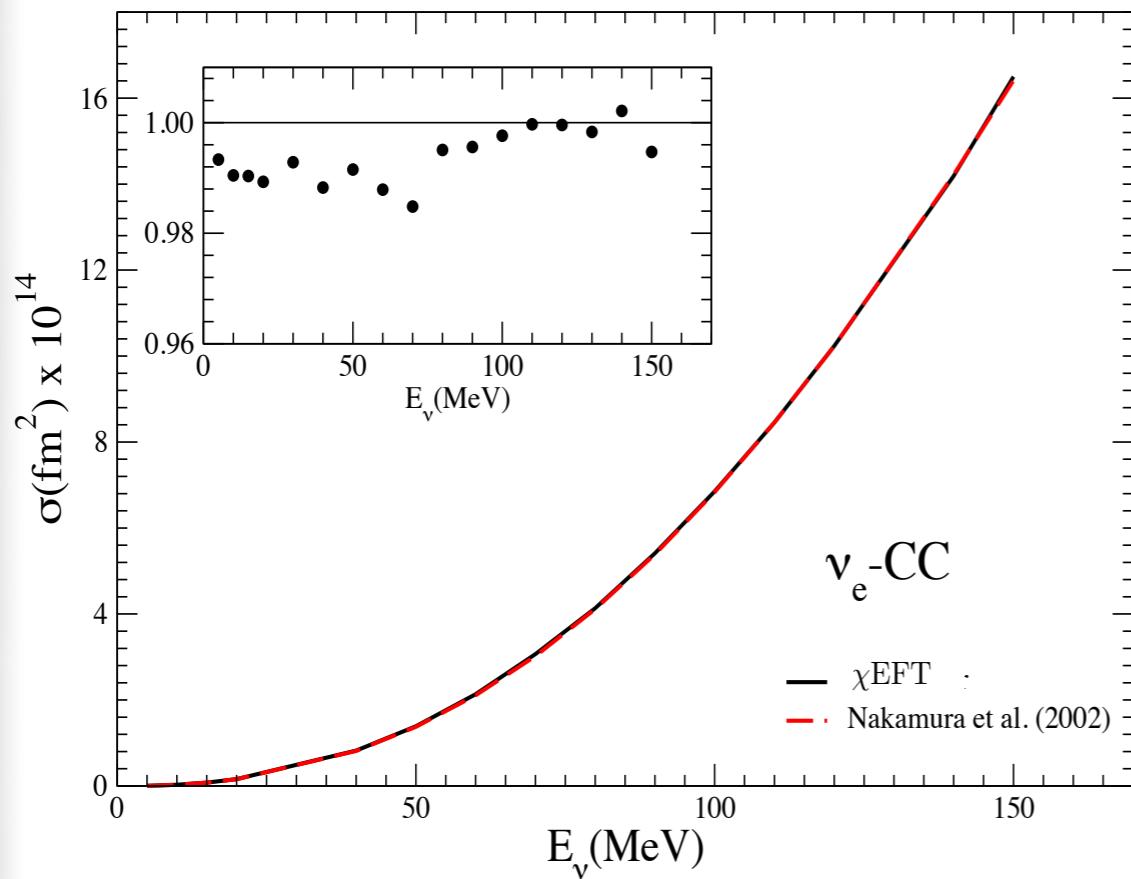
- Nakamura et al. (2002), Phenomenological interactions
- Shen et al. (2011), Phenomenological interactions
- AB and Schiavilla 2017 (first chiral EFT calculation)

Results I: Differential cross sections

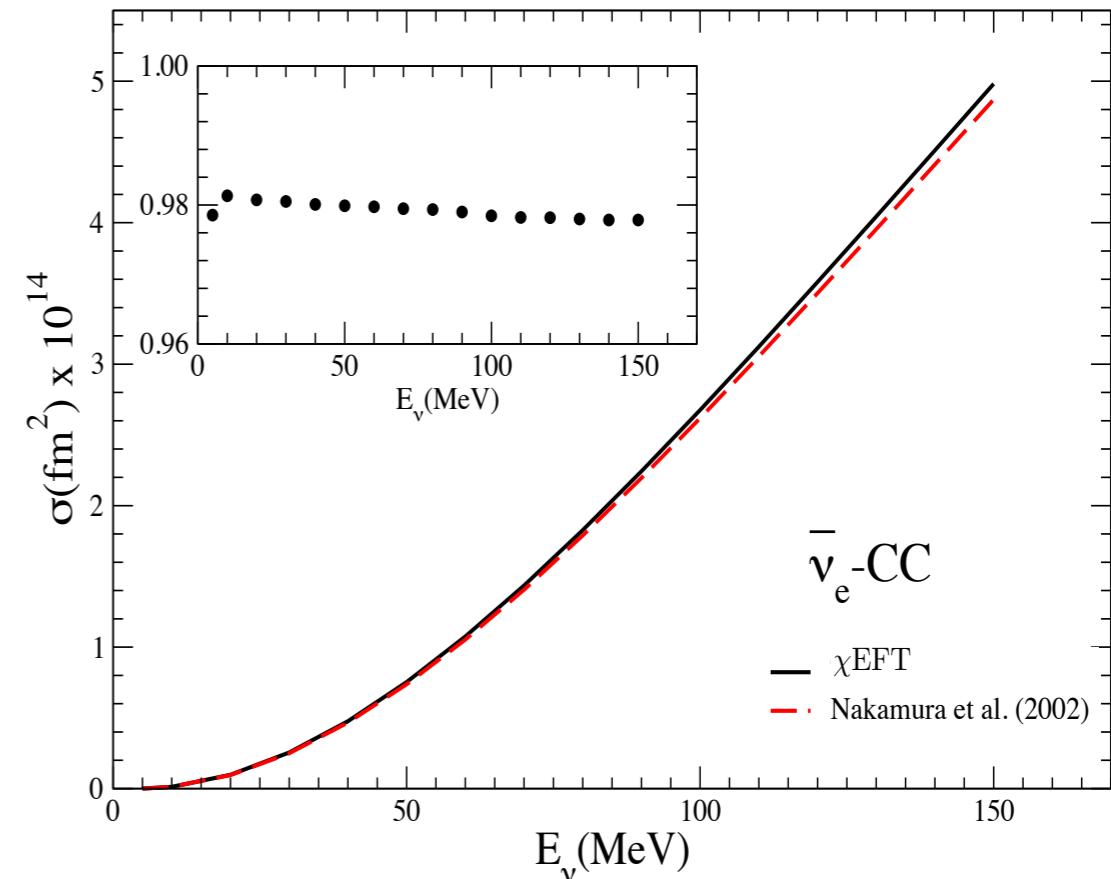


Results II: Total cross sections (CC)

$\Lambda = 500 \text{ MeV}$



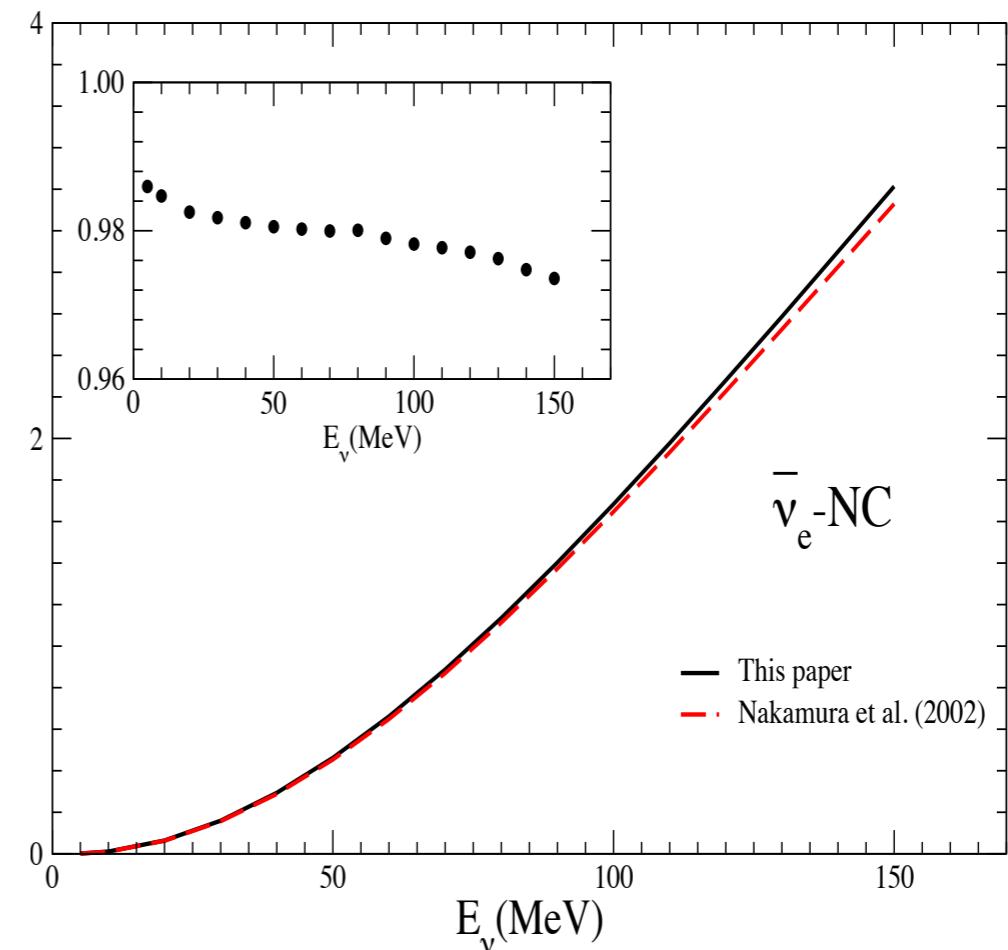
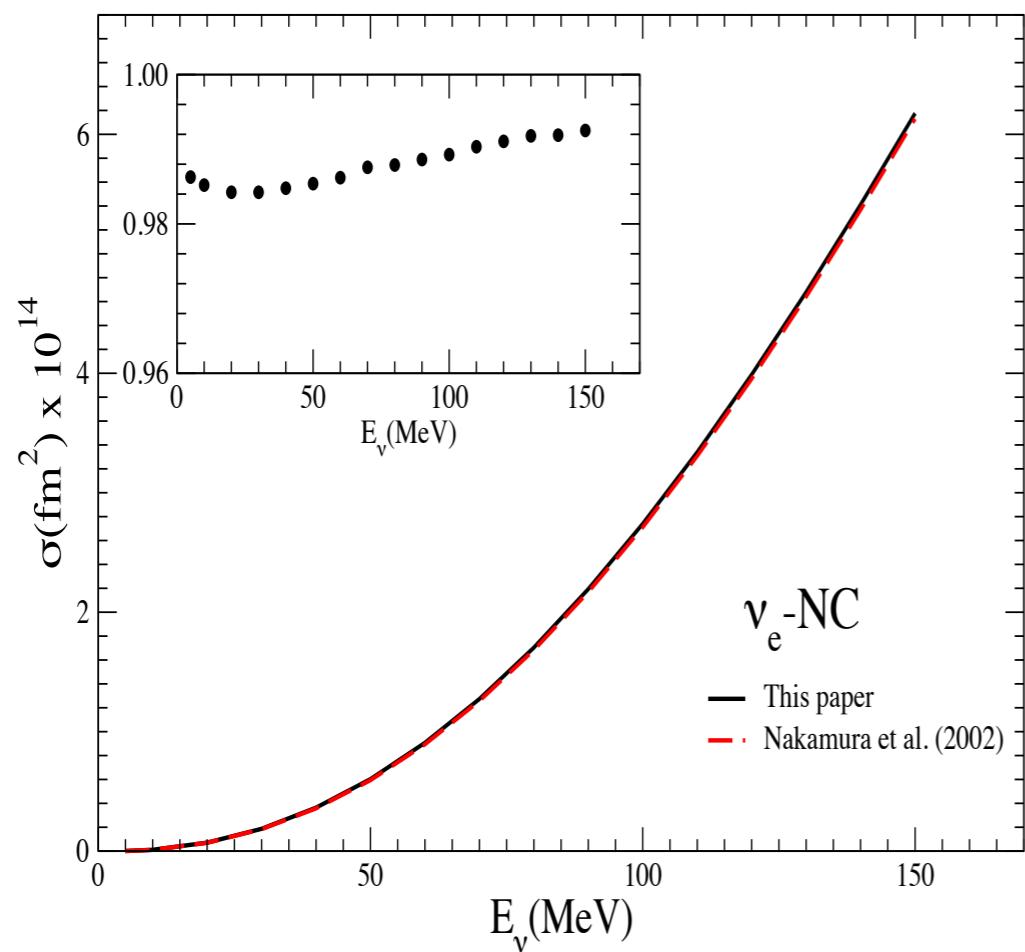
for $\Lambda = 600 \text{ MeV}$ variation $\leq 1\%$



Sign of small theoretical uncertainty

Results II: Total cross sections (NC)

$$\Lambda = 500 \text{ MeV}$$



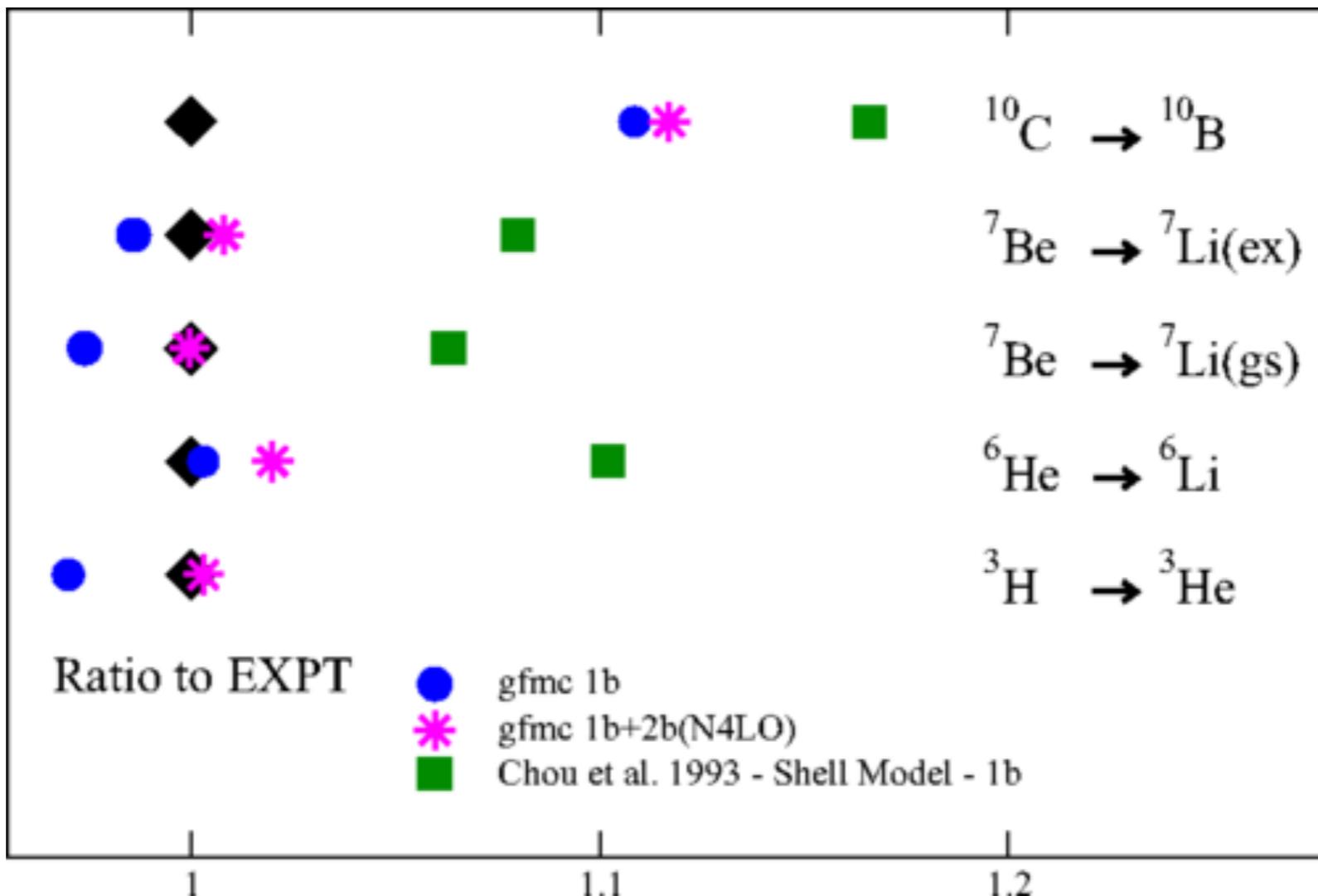
β -decays

Wave functions from AV18+IL7; χ currents TOPT (N4LO), UT (N4LO*)

	${}^6\text{He}$ β -decay	${}^7\text{Be}$ ϵ -capture (gs)	${}^7\text{Be}$ ϵ -capture (ex)	${}^{10}\text{C}$ β -decay
LO	2.168(2.174)	2.294(2.334)	2.083(2.150)	2.032(2.062)
N4LO	$3.73(3.03) \times 10^{-2}$	$6.07(4.98) \times 10^{-2}$	$4.63(4.63) \times 10^{-2}$	$1.61(1.55) \times 10^{-2}$
N4LO*	$3.62(3.43) \times 10^{-2}$	$6.62(5.43) \times 10^{-2}$	$5.31(5.38) \times 10^{-2}$	$1.80(1.00) \times 10^{-2}$
MEC	$6.90(4.57) \times 10^{-2}$	$10.5(10.3) \times 10^{-2}$	$8.88(8.99) \times 10^{-2}$	$5.31(4.28) \times 10^{-2}$
EXP	2.1609(40)	2.3556(47)	2.1116(57)	1.8331(34)

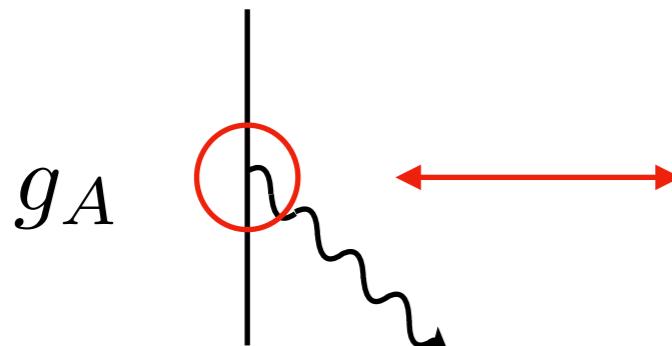
- Small but not negligible contribution from two-body currents
- Small difference between TOPT and UT axial currents to GT
- Phenomenological currents give bigger contribution
(delicate cancellations missing?)
- What will change with fully chiral approach?

β -decays



Electroweak currents, with phenomenological potentials, GFMC
2-body currents play a big role

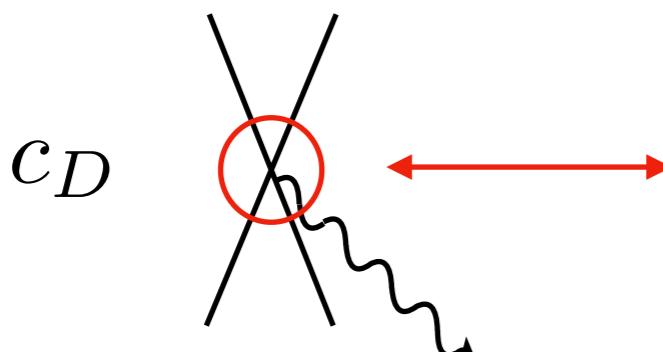
LECs from QCD



$$\langle NN | \mathbf{j}_{5,a}^{(1b)} | NN \rangle$$

precision determination crucial for experiments (neutrino..)

R. Gupta et al. (2017), C. C. Chang et al. (2018)

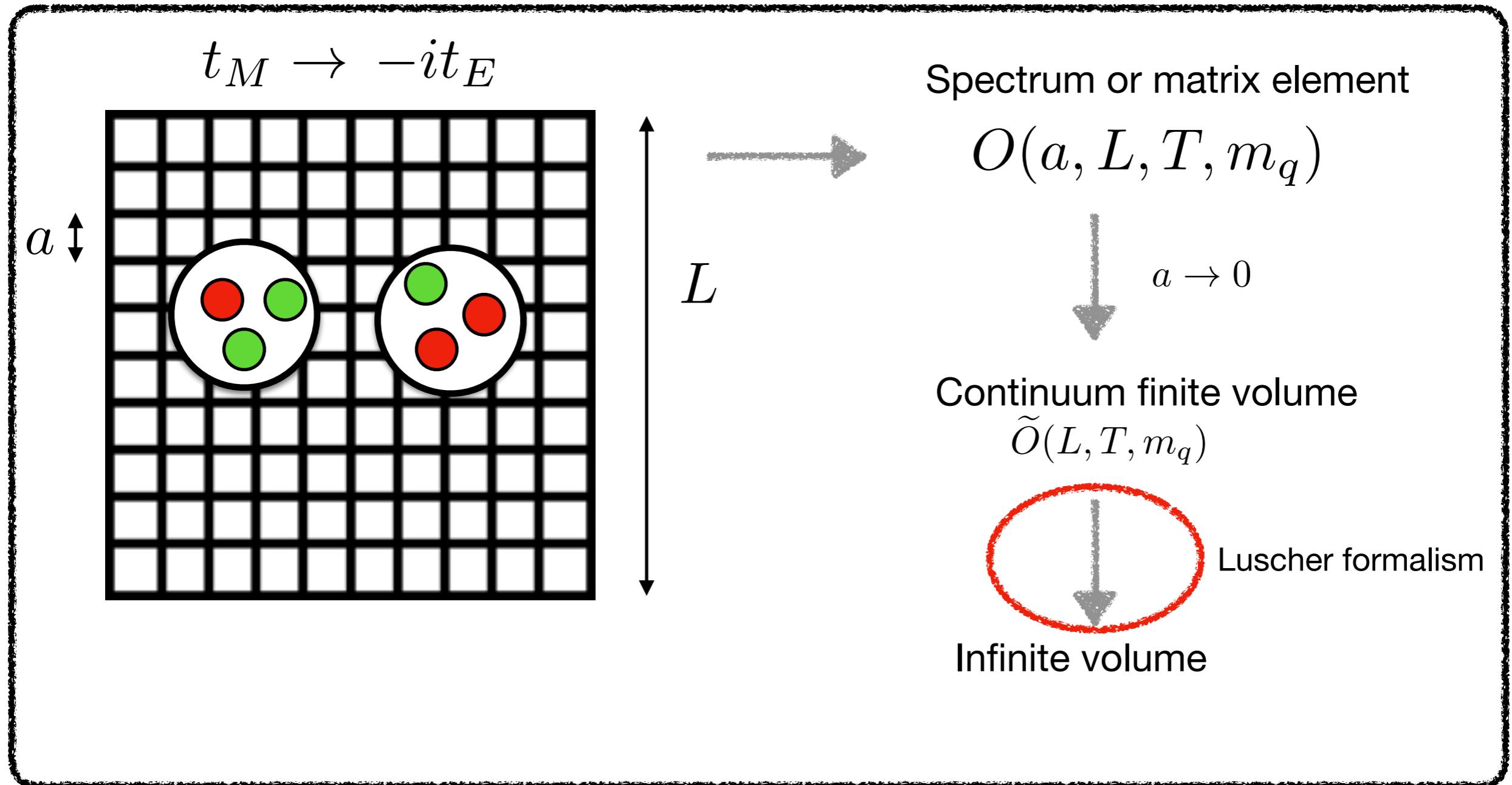


$$\langle NN | \mathbf{j}_{5,a}^{(2b)} | NN \rangle$$

Determination crucial for many nuclear processes and current experiments (neutrino experiment with targets having $A > 1$)

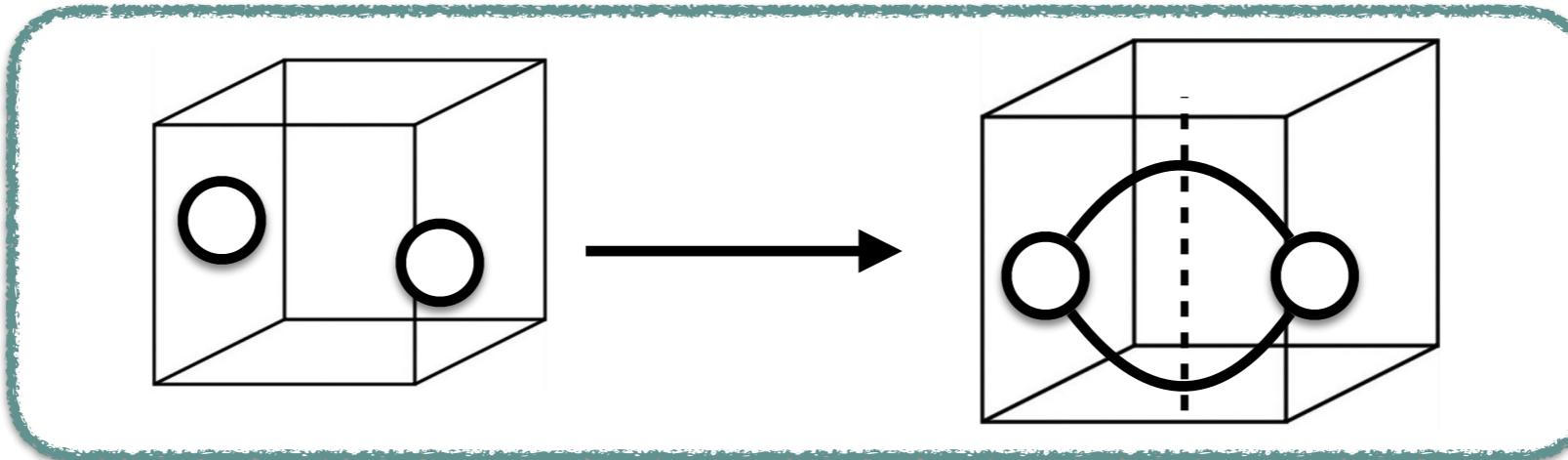
M. J. Savage et al. (2017)

LQCD



Finite volume effects

- Finite volume effects are complicated for matrix elements with multi-hadron states
 - On-shell intermediate states give singularities



- Formalism needed to deal with these effects:
 - Briceño and Hansen (2016) - general, inelastic, relativistic
 - Rusetsky et al. (2012) - EFT dependent, NR
 - Briceño and Davoudi (2012) - EFT dependent, NR

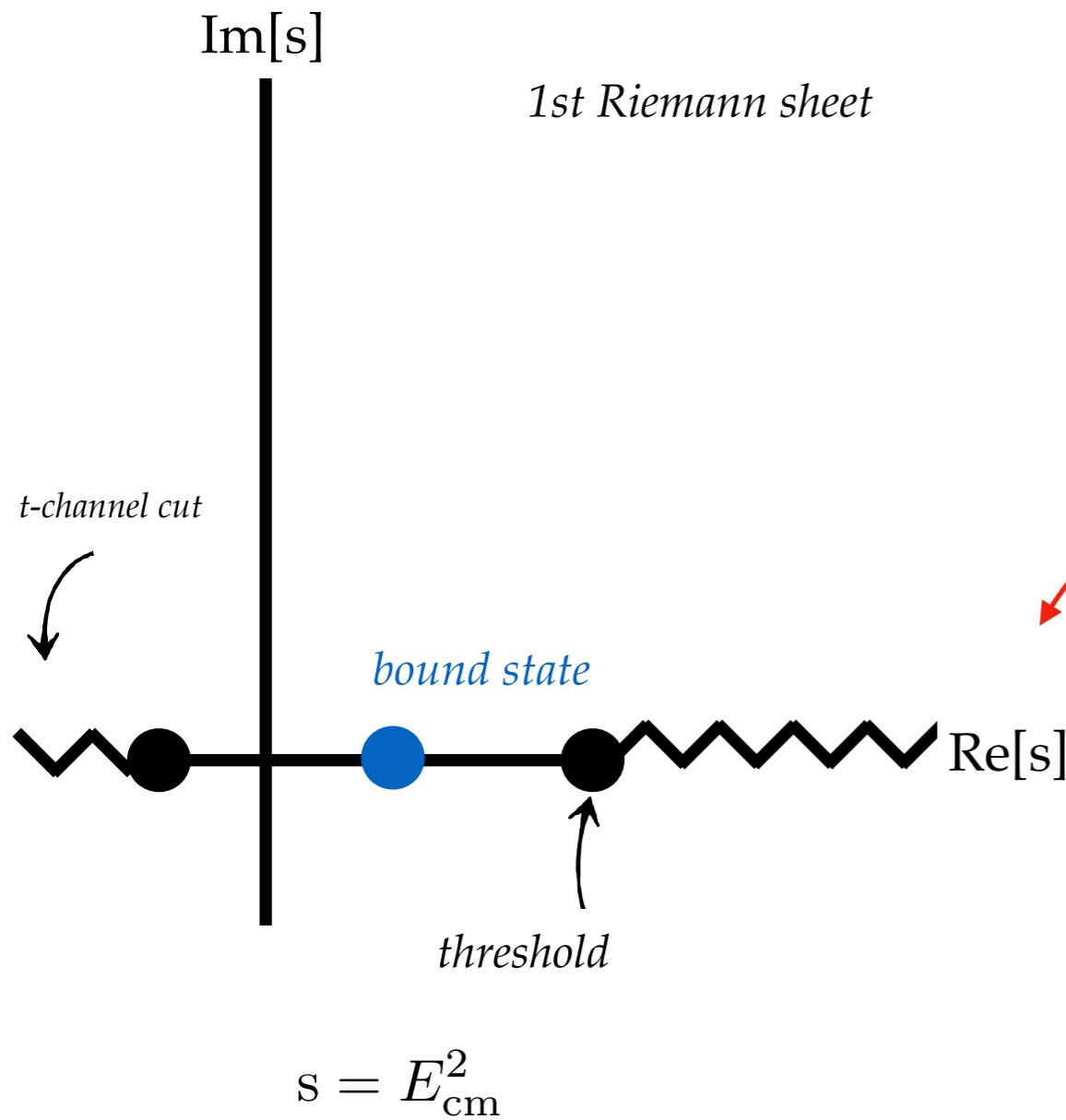


Many works.. not cited here

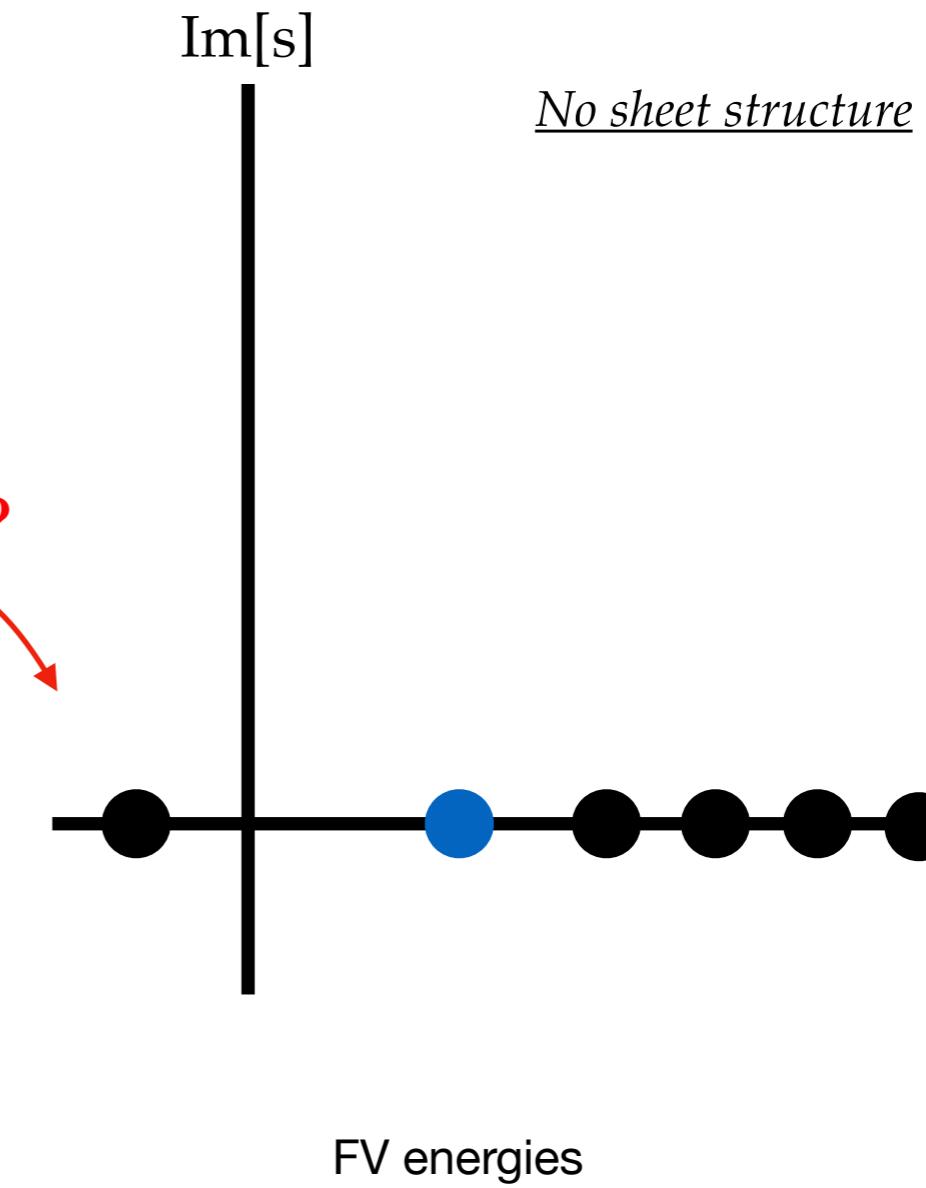
- Luscher formalism (1986, 1991)

Scattering amplitude 101

Infinite volume

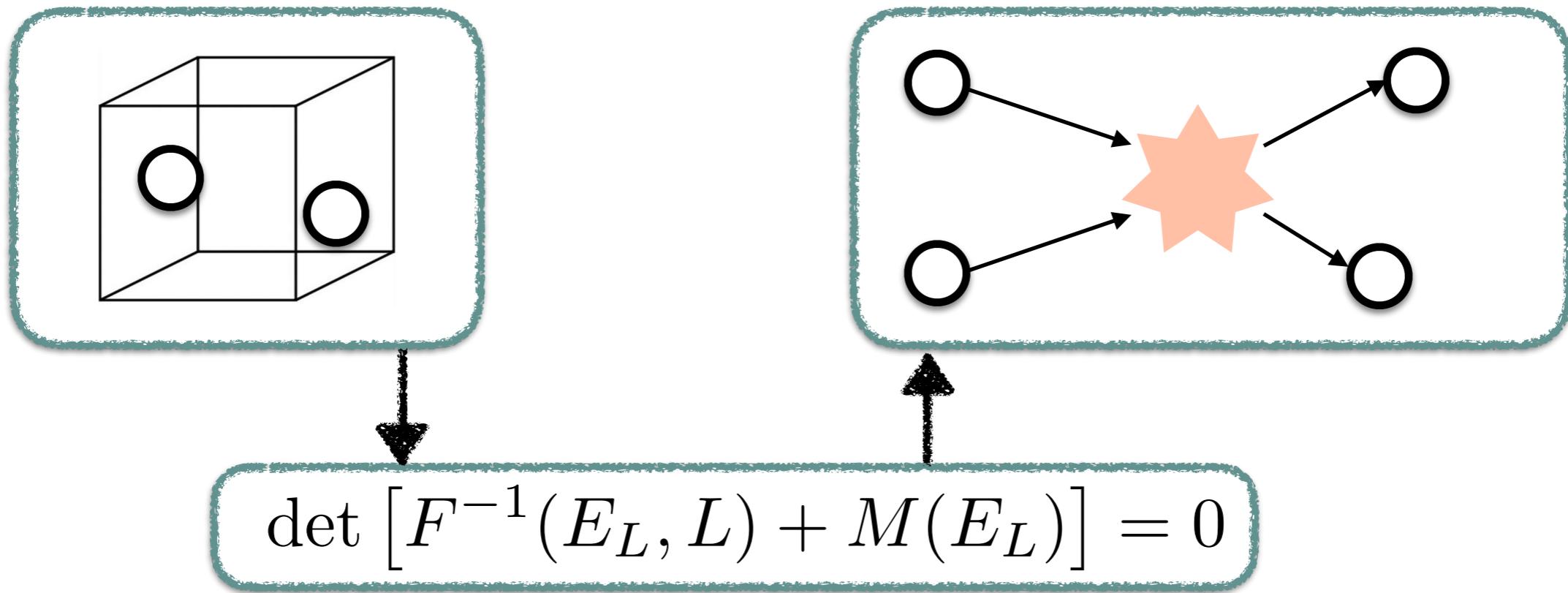


Finite volume



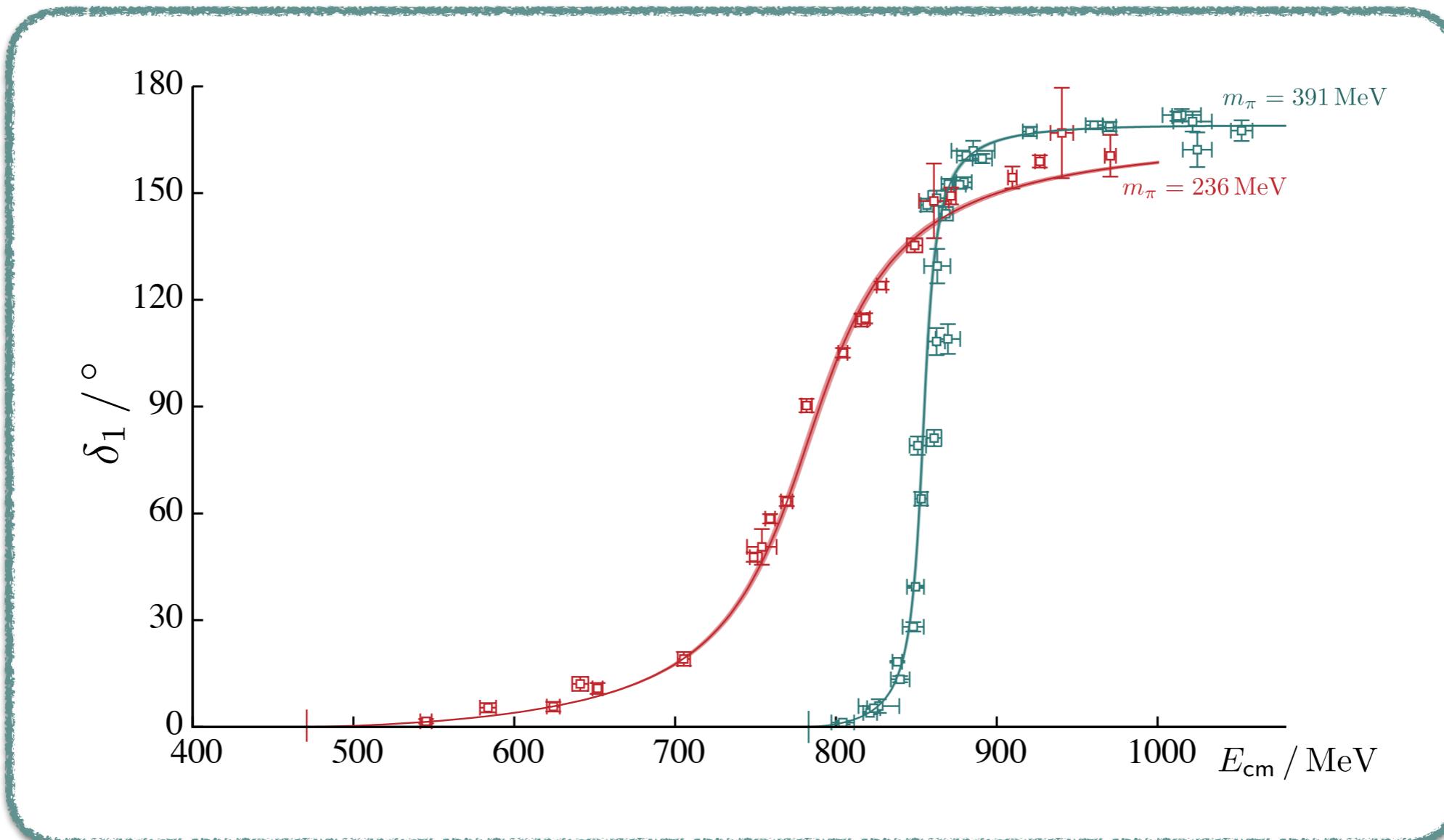
2→2

- FV spectra to infinite volume purely hadronic amplitudes
- Holds for a generic QFT with hadronic d.o.f, up to multi-particle thresholds



- Lüscher (1986, 1991) [elastic scalar bosons]
- Rummukainen & Gottlieb (1995) [moving elastic scalar bosons]
- Kim, Sachrajda, & Sharpe / Christ, Kim & Yamazaki (2005) [QFT derivation]
- Feng, Li, & Liu (2004) [inelastic scalar bosons]
- Hansen & Sharpe / Briceño & Davoudi (2012) [moving inelastic scalar bosons]
- Briceño (2014) [general 2-body result]

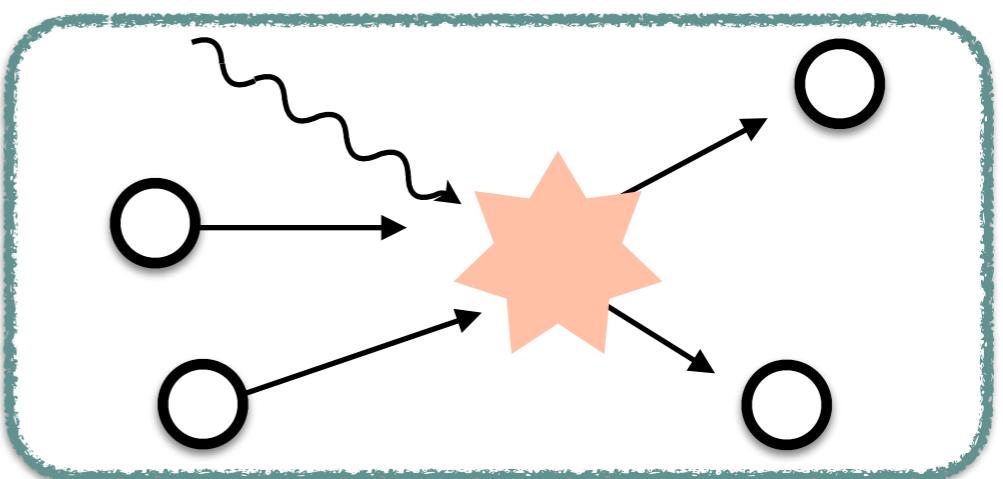
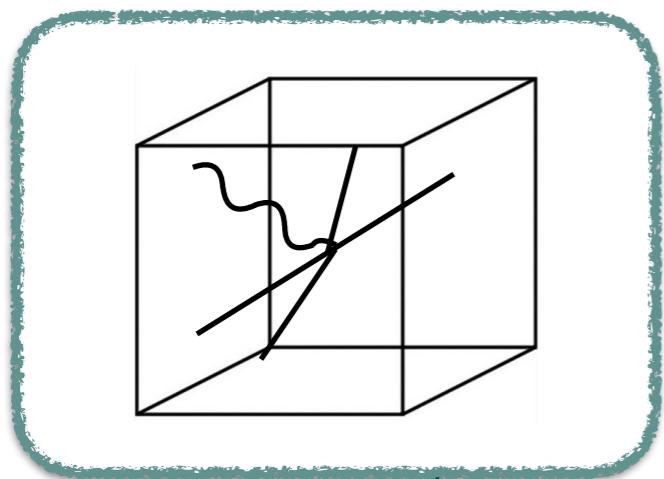
$2 \rightarrow 2$ (pion sector)



Wilson, Briceño, Dudek, Edwards, and Thomas (2015)

$2 + \mathcal{J} \rightarrow 2$

- FV matrix elements to infinite volume electroweak amplitudes



$$\langle 2 | \mathcal{J} | 2 \rangle |_L^2 = \frac{1}{L^6} \text{Tr} [R(E_L, L) W_{L,\text{df}} R(E_L, L) W_{L,\text{df}}]$$

Briceño & Hansen (2016)

Briceño & Davoudi (2013)

Detmold & Savage (2004)

$2 + \mathcal{J} \rightarrow 2$ (some details)

$$\langle 2 | \mathcal{J} | 2 \rangle |_L^2 = \frac{1}{L^6} \text{Tr} [R(E_L, L) W_{L,\text{df}} R(E_L, L) W_{L,\text{df}}]$$

$$W_{L,\text{df}} = W_{\text{df}} + M G(L, w) M$$

Current challenge

$$W_{\text{df}} = \text{Diagram with blue dot} = \text{Diagram with black dot} - \text{Diagram with black dot and loop} - \text{Diagram with black dot and loop} - \dots$$

$$w = \text{Diagram with wavy line}$$

$$G(L, w) = \text{Diagram with } V - \text{Diagram with } \infty$$

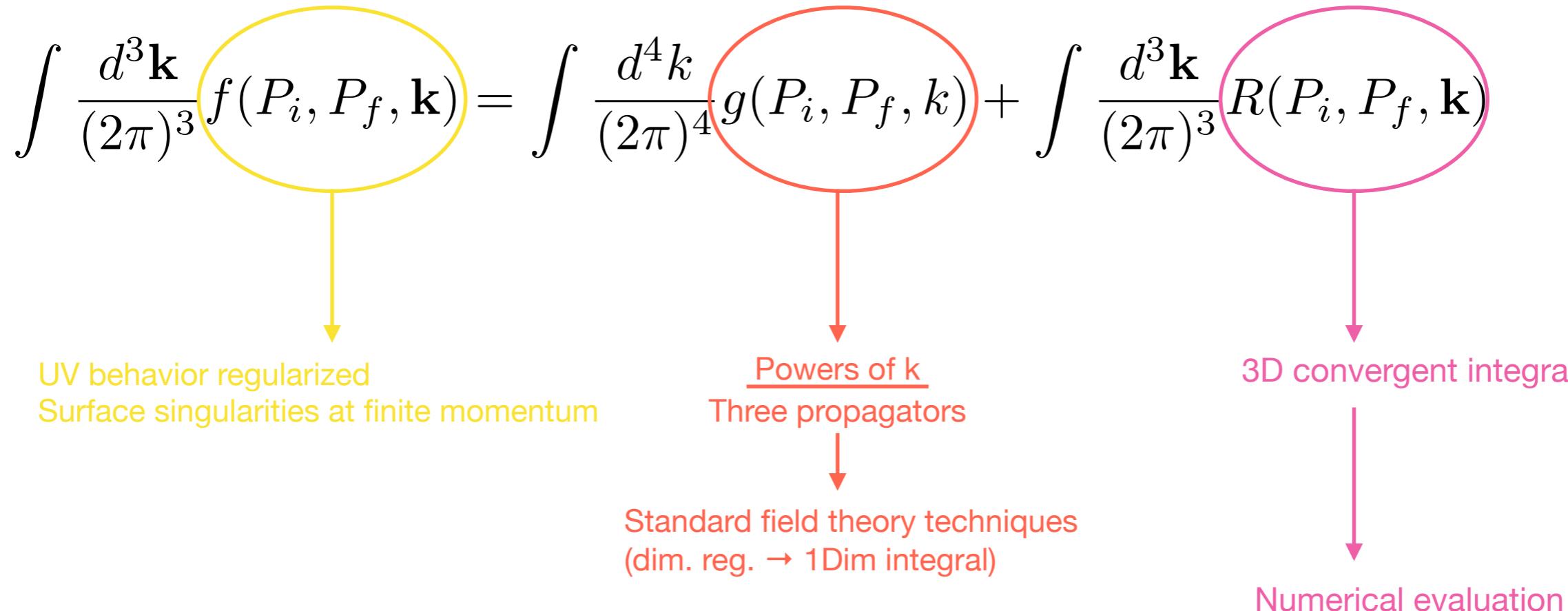
$$M = \text{Diagram with black dot and four lines}$$

G function evaluation I

$$G(P_i, P_f, L) = \left(\frac{1}{L^3} \sum_{\mathbf{k}} - \int_{\mathbf{k}} \right) f(P_i, P_f, \mathbf{k})$$

- The sum is “easy”
- The integral is highly not trivial (spectator particle goes on-shell)
 - integrand singularities are two surfaces in three-dimension
 - standard principal value techniques in one dimension fail
 - techniques from other fields are not suitable
 - using mathematical trickery we can isolate the singularities, treat them with standard field theory techniques, and be left with a 3D **smooth** integral

G function evaluation II (Sketch)



For all important details

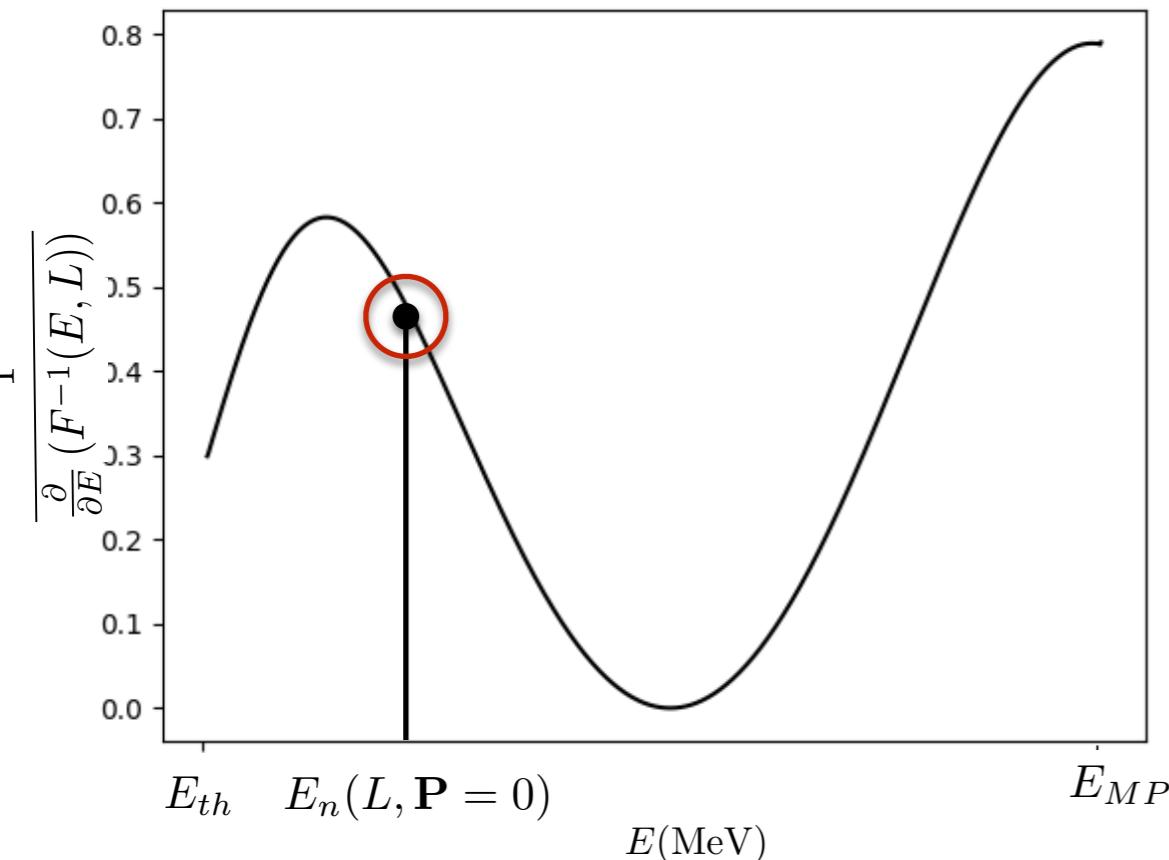


AB, R. A. Briceño, M. T. Hansen, F. Ortega, D.J. Wilson (2018)
To appear (soon)

Kinematic functions

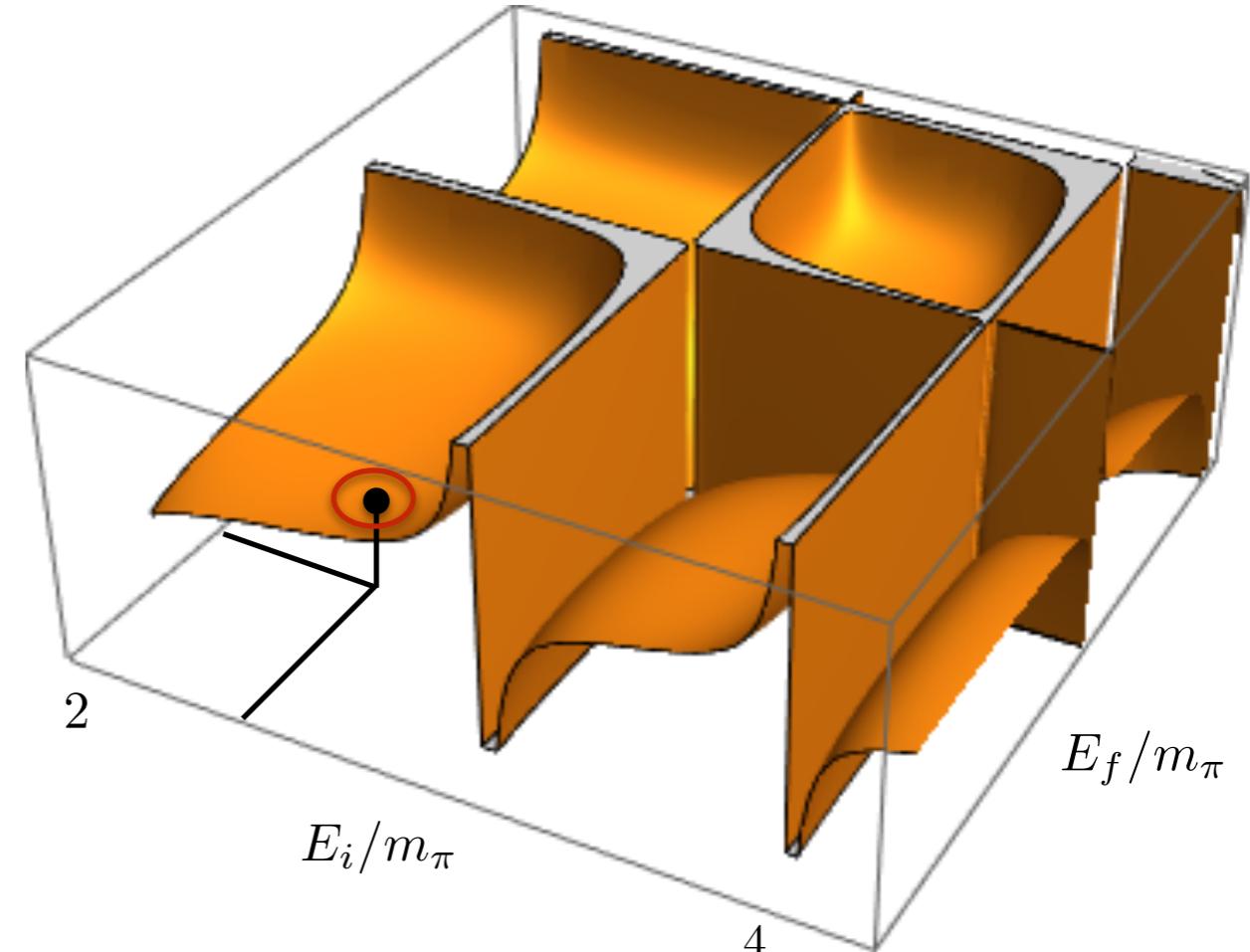
$$R(E_L, L) = \frac{1}{\frac{\partial}{\partial E} (F^{-1}(E, L) + M(E))} \Big|_{E=E_L}$$

$$G(E_i, E_f, L) = \left[\frac{1}{L^3} \sum_{\mathbf{k}} - \int d\mathbf{k} \right] (\dots\dots)$$



E_{th} = threshold energy

E_{MP} = multiparticle states energy



Singularities at free particle energies

Take home message

-  General, relativistic, EFT independent, finite volume formalism to treat processes relevant for many electroweak processes (nuclear and not only) derived
-  A crucial ingredient the new kinematic function is closed to the general complete numerical implementation
-  Test this framework with a simple example (pions are “easier”)
-  Use this FV formalism in an actual LQCD calculation

THANK YOU!

BACKUP SLIDES

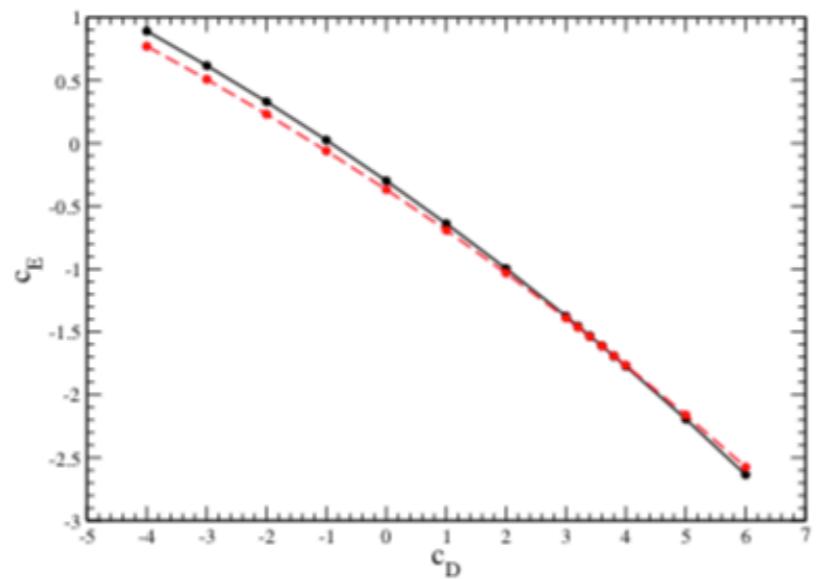
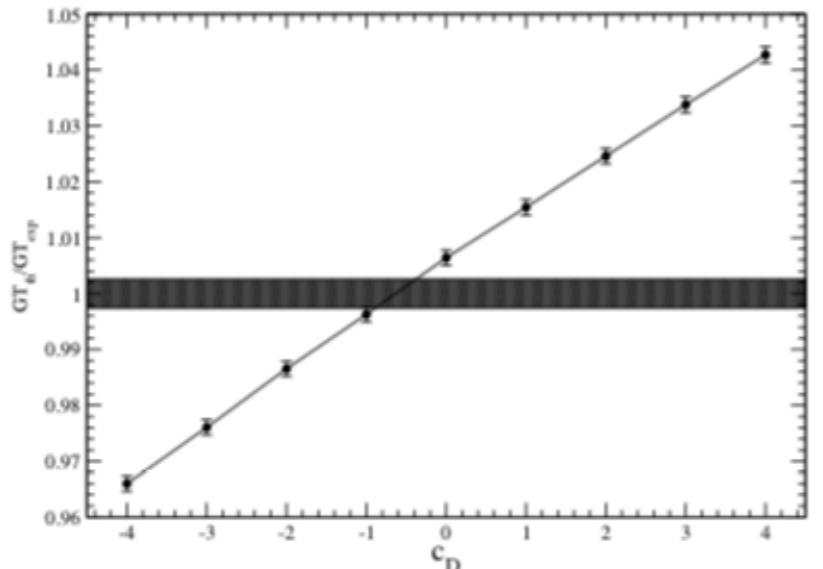
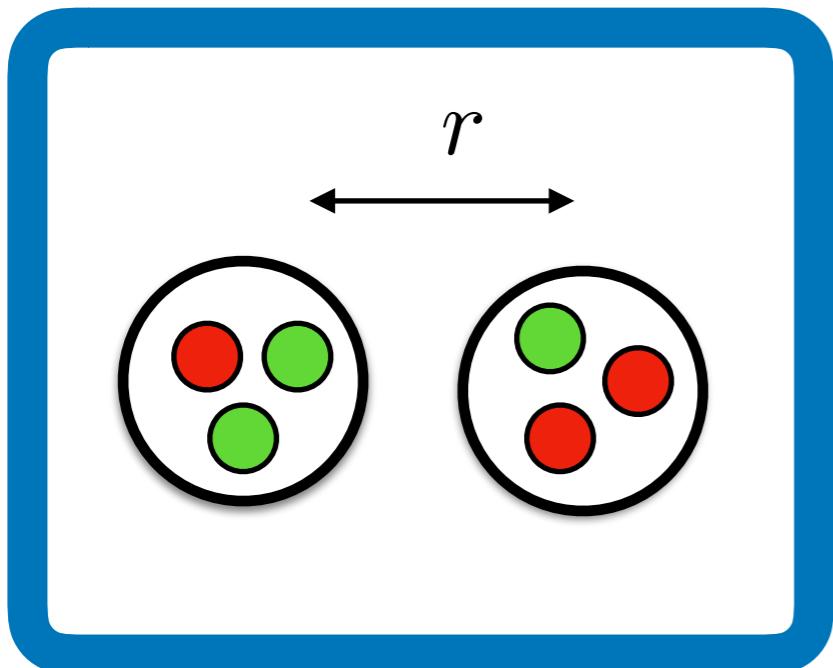


FIG. 2. Upper panel: The calculated ratio GT_{th}/GT_{exp} as function of c_D (solid line; each point on this line reproduces the trinucleon binding energies). Lower panel: The c_D - c_E trajectories obtained by fitting the experimental trinucleon binding energies (solid line) and nd doublet scattering length (dashed line) (the intercept of these two lines gives the c_D and c_E values that reproduce these two observables simultaneously). The NV2+3-Ia chiral interactions are used here for illustration. The values of 8.475 MeV and 7.725 MeV, and 0.645 ± 0.010 fm [42] are used for the 3H and 3He binding energies, and nd scattering length, respectively. Note that these energies have been corrected for the small contributions (+7 keV in 3H and -7 keV in 3He) due to the $n-p$ mass difference [43]. The band (left panel) results from experimental uncertainty GT_{EXP} , which has conservatively been doubled.

Outlook

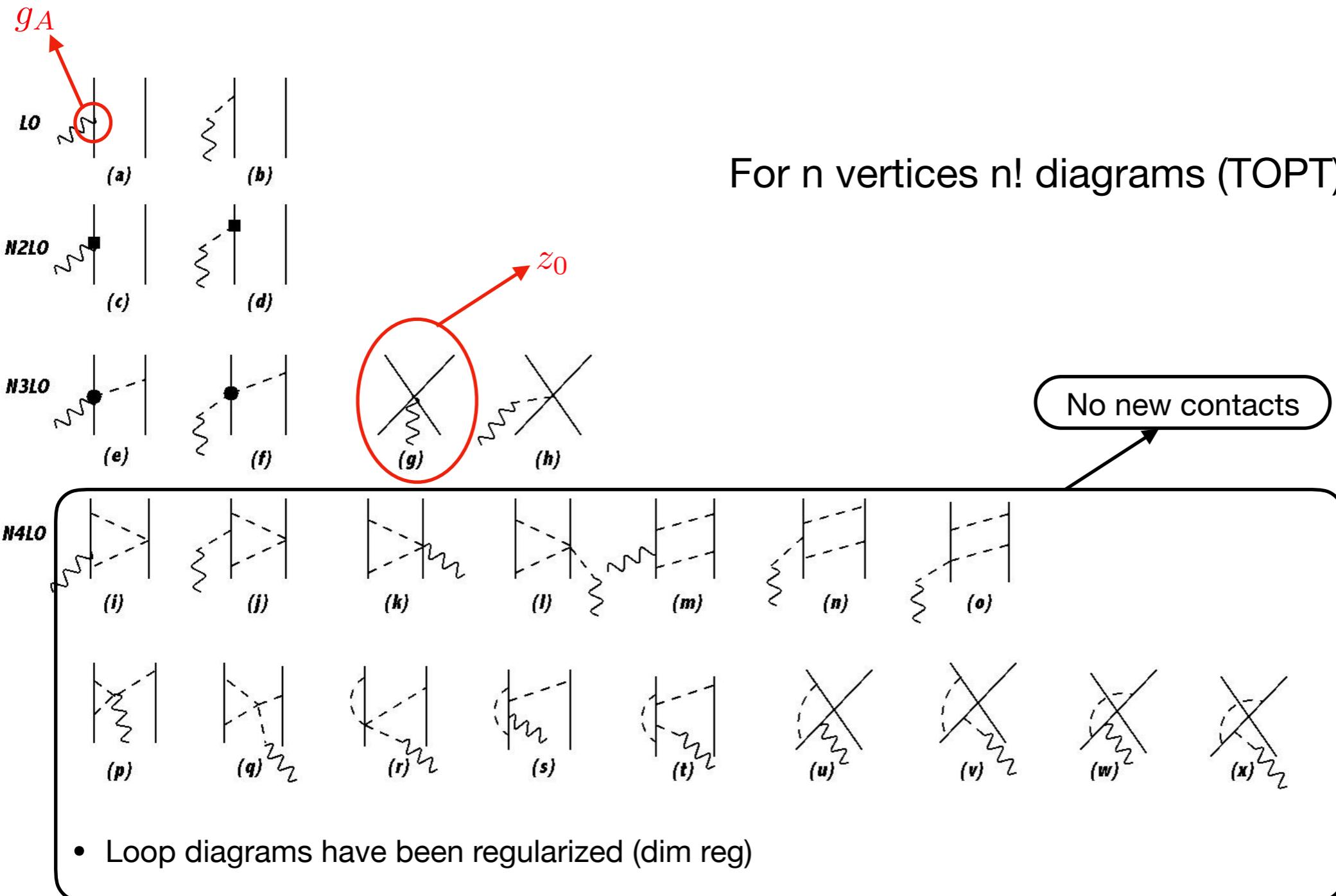
- Currents derived up to N4LO
- LEC in the axial current fixed with experimental GT matrix element
- Prediction for neutrino deuteron → confirm phenomenological approaches
- Hybrid calculations in beta decays denote big effect of two-body currents
- Systematic study of theoretical uncertainty
- LQCD to determine the LEC in the axial current ([validation](#))
(Savage et al. 2017)
- Refine calculations for beta decays/include delta in the currents (?)
(Goity et al. 2012)

An example (no currents)



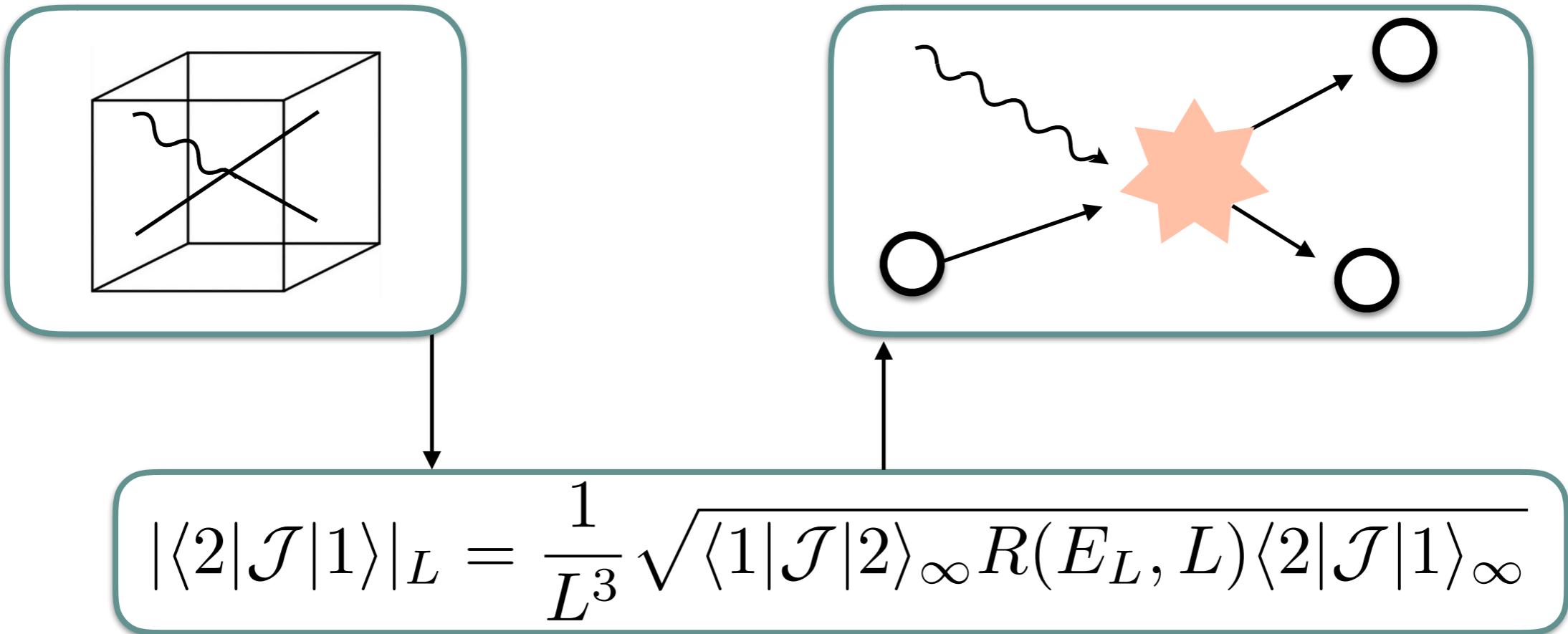
FV effects for $L \gg r$ (no mirror images)
Infrared artifact

Axial current



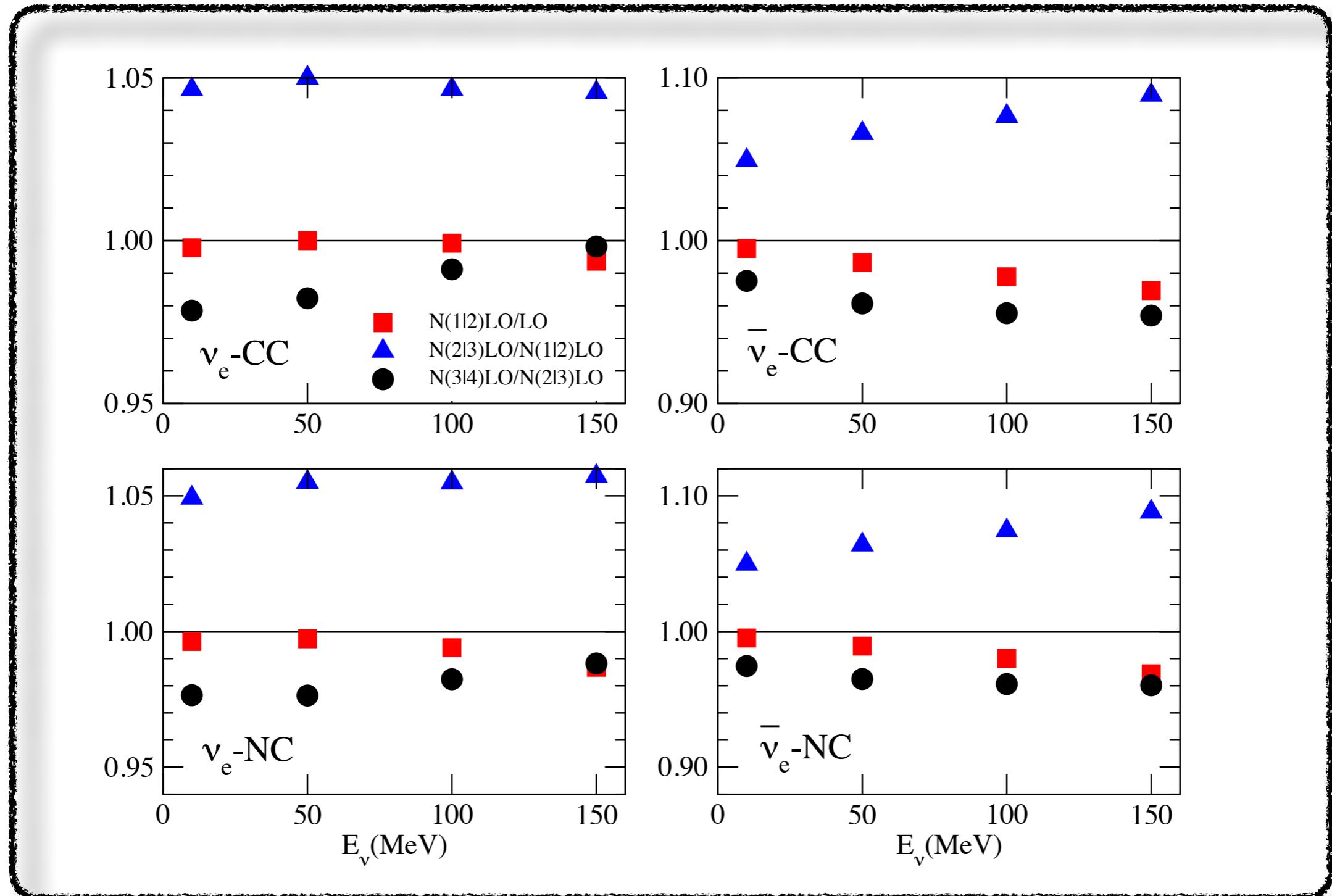
$1 + \mathcal{J} \rightarrow 2$

- FV matrix elements to infinite volume electroweak amplitudes



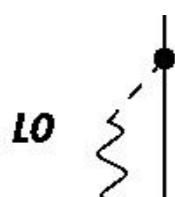
- Lellouch & Lüscher (2000) [K-to- $\pi\pi$ at rest]
- Kim, Sachrajda, & Sharpe / Christ, Kim & Yamazaki (2005) [moving K-to- $\pi\pi$]
- Hansen & Sharpe (2012) [D-to- $\pi\pi$ /KK]
- Briceño, Hansen Walker-Loud / Briceño & Hansen(2014-2015)[general 1-to-2]

Convergence pattern



Axial charge

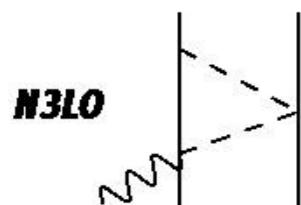
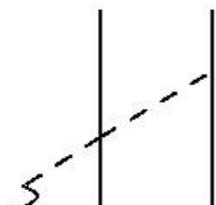
Many thousands of diagrams for n vertices $n!$ diagrams (TOPT)



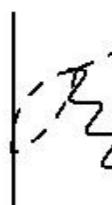
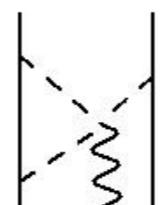
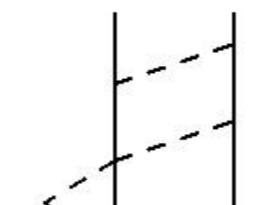
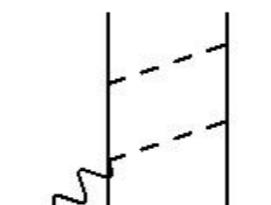
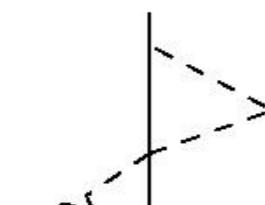
(a)



(b)



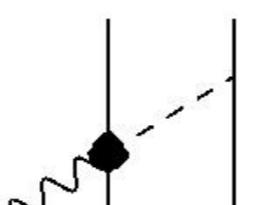
(d)



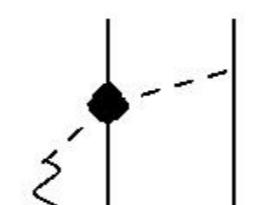
(j)



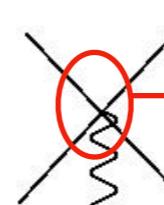
(k)



(l)



(m)

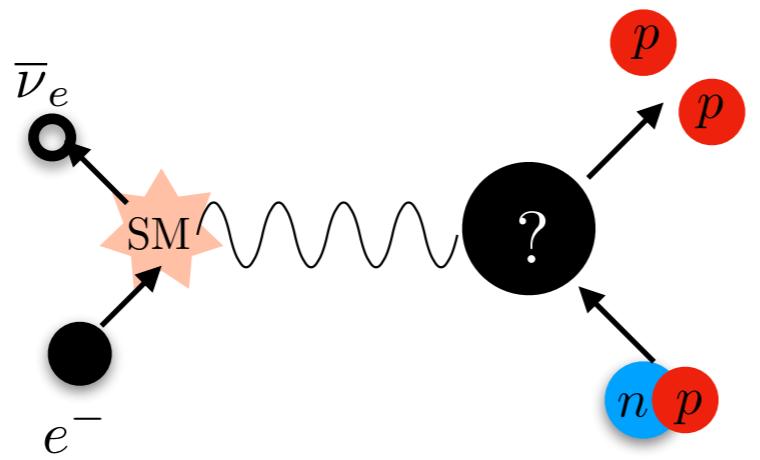


(n)

Loop diagrams have been regularized (dim reg) divergences are reabsorbed by contact terms and higher order πN couplings

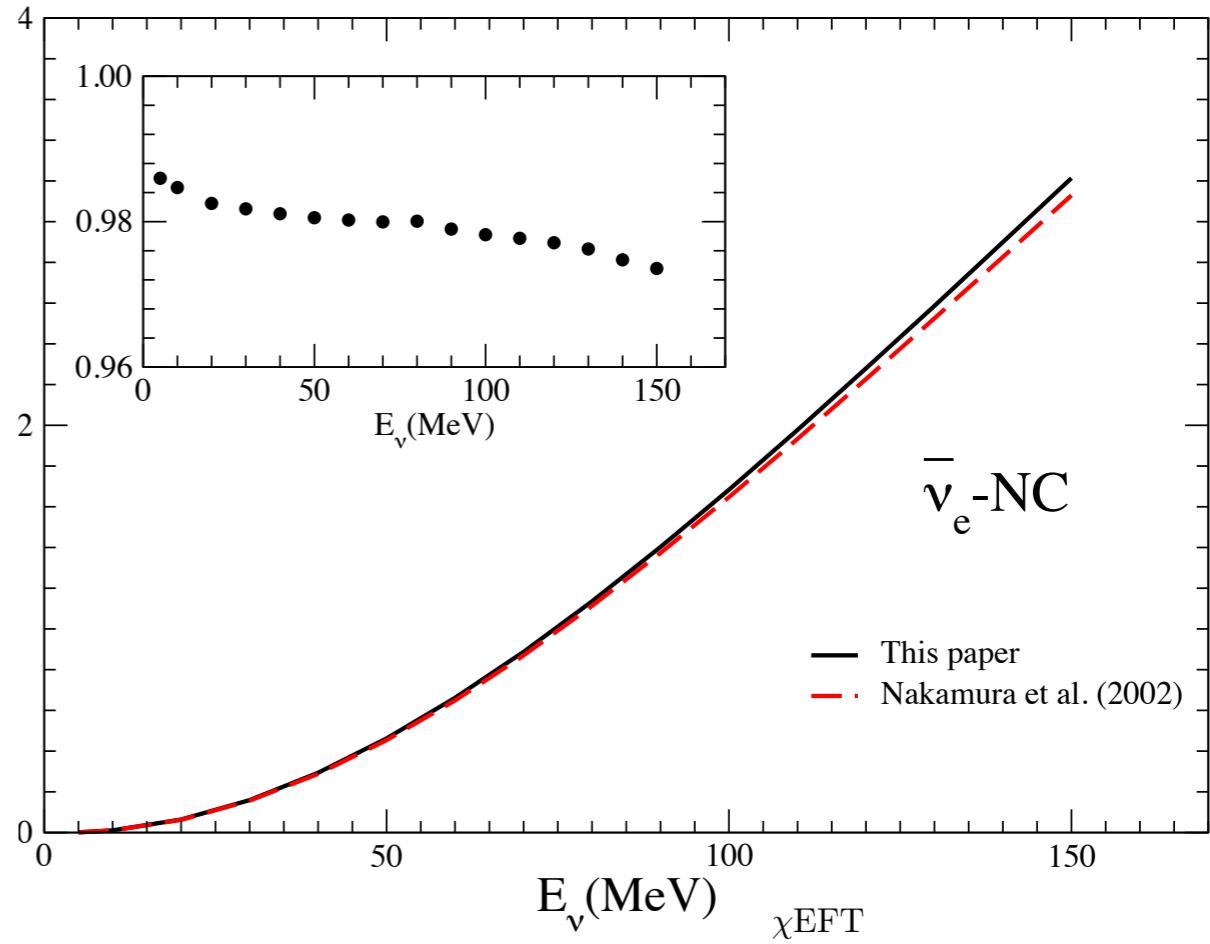
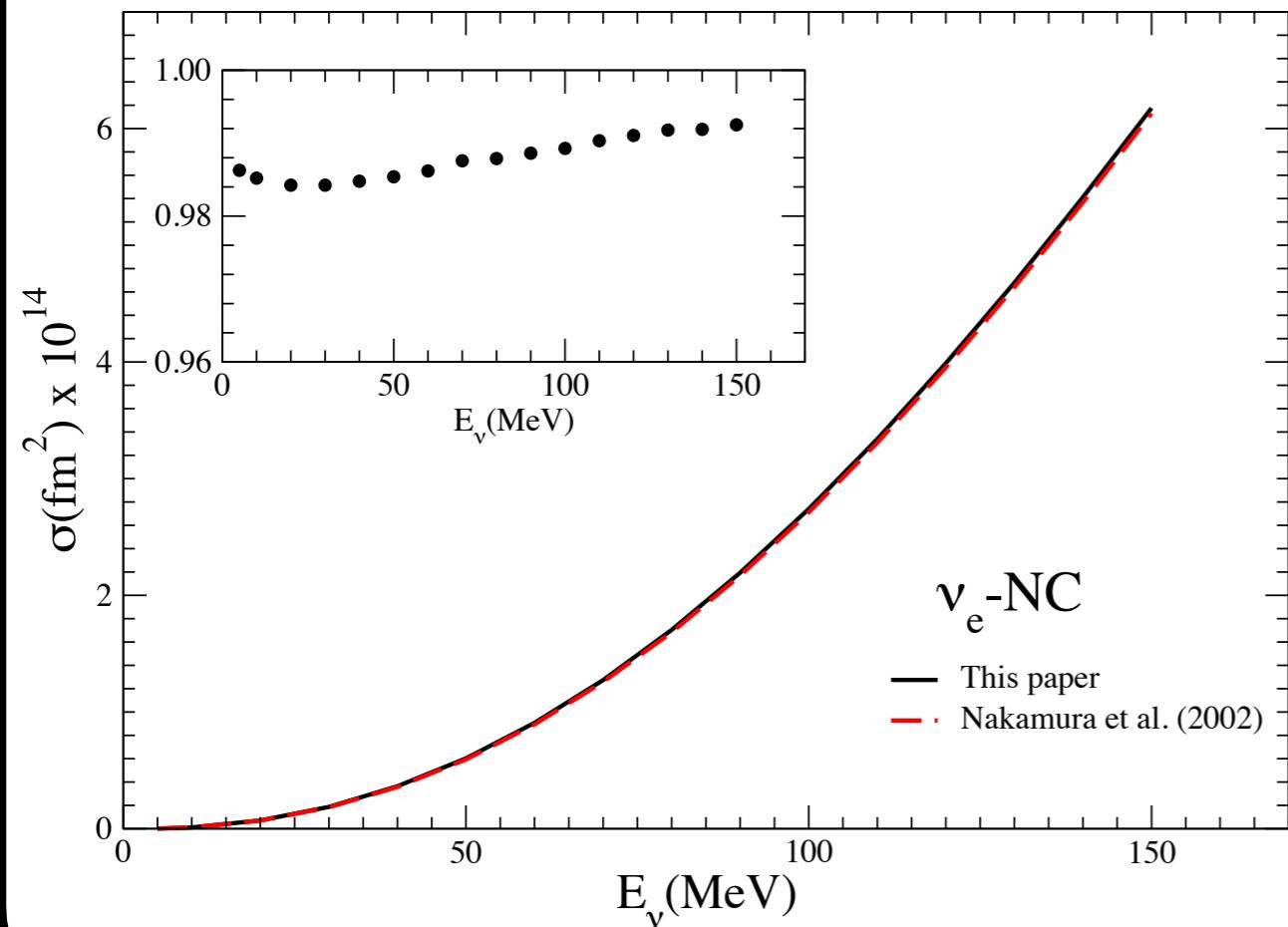
Comparison with others ?

\beta decays Saori

 ^3He

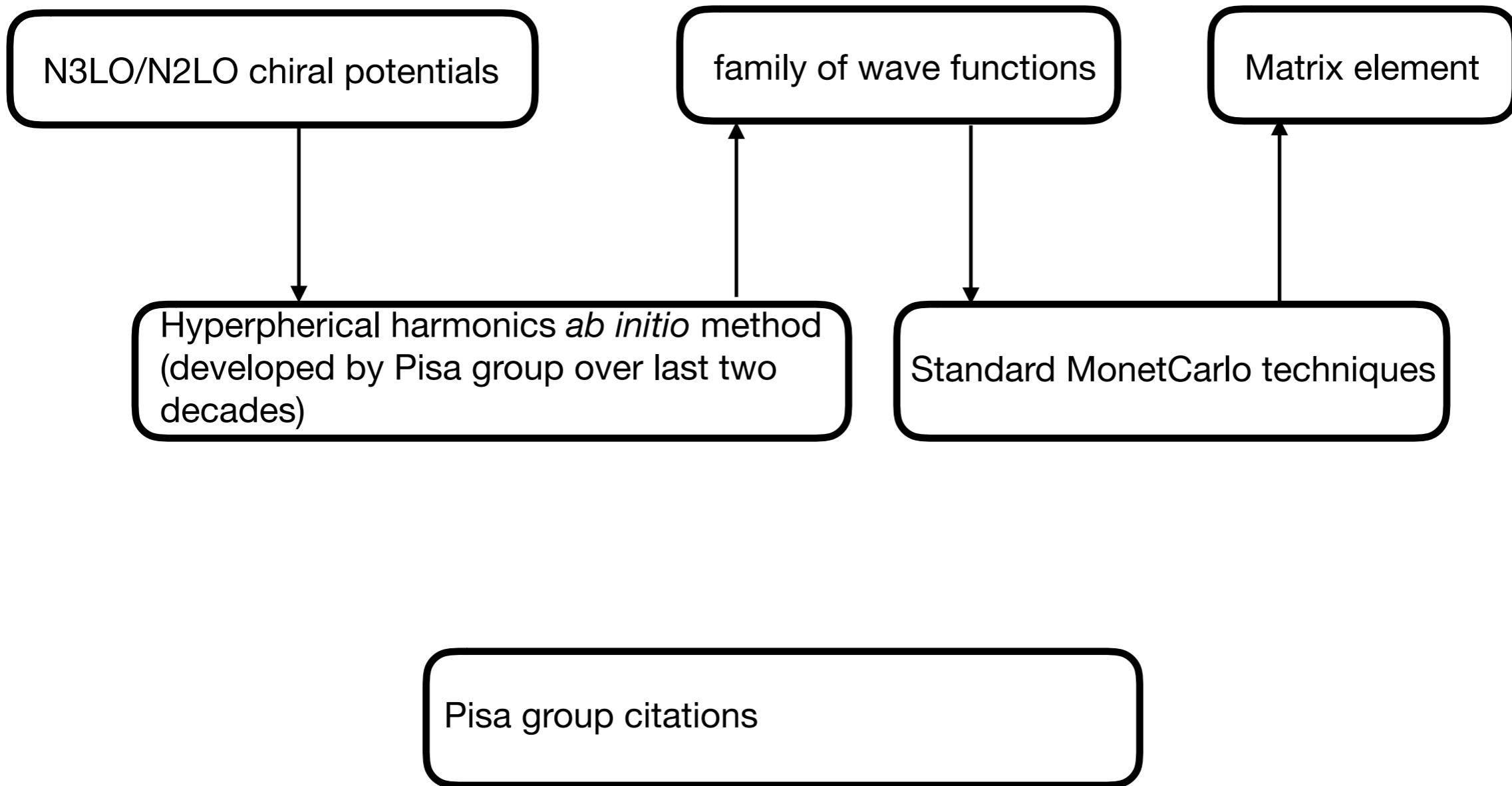
Results: Neutral currents

$\Lambda = 500 \text{ MeV}$



for $\Lambda = 600 \text{ MeV}$ variation $\leq 1\%$

Triton calculation



Λ	500 MeV	600 MeV
LO	0.9363 (0.9224)	0.9322 (0.9224)
N2LO	-0.569(-0.844) $\times 10^{-2}$	-0.457(-0.844) $\times 10^{-2}$
N3LO(OPE)	0.825(1.304) $\times 10^{-2}$	0.043(7.517) $\times 10^{-2}$
N4LO(Loop)	-0.486(-0.650) $\times 10^{-1}$	-0.600(-0.852) $\times 10^{-1}$
N4LO(3Bd)	-0.143(-0.183) $\times 10^{-2}$	-0.153(-0.205) $\times 10^{-2}$

- N3LO/N2LO full chiral
- AV18/UIX hybrid



- Loop give not negligible contribution
- Preliminary three-body currents seem negligible

500 MeV

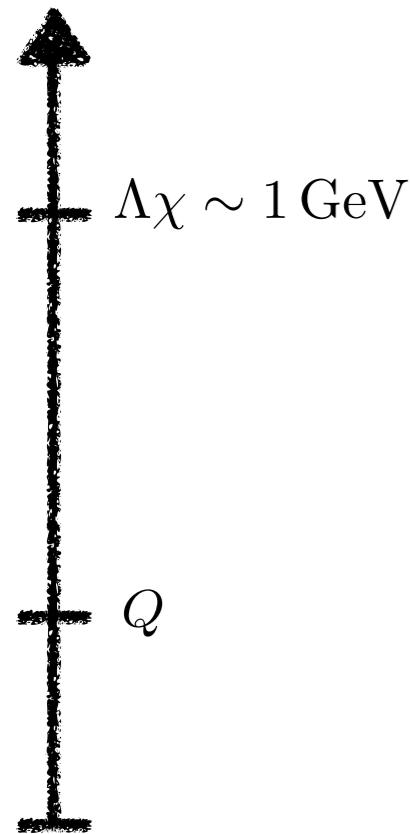
Axial currents: from amplitudes to nuclear operators II

- Matching holds for on shell scattering amplitude
- Matching is not unique → Nuclear operators are not unique
 - iterations of LS depend on the off-the-energy-shell extension of lower order currents and potentials
 - Not unique operators **should** be related by a unitary transformation ([no general proof at the moment](#))

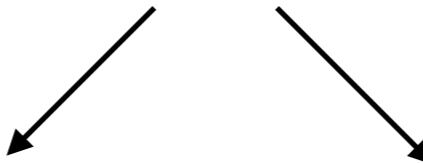
Outlook

Backup slides

χ Effective field theory



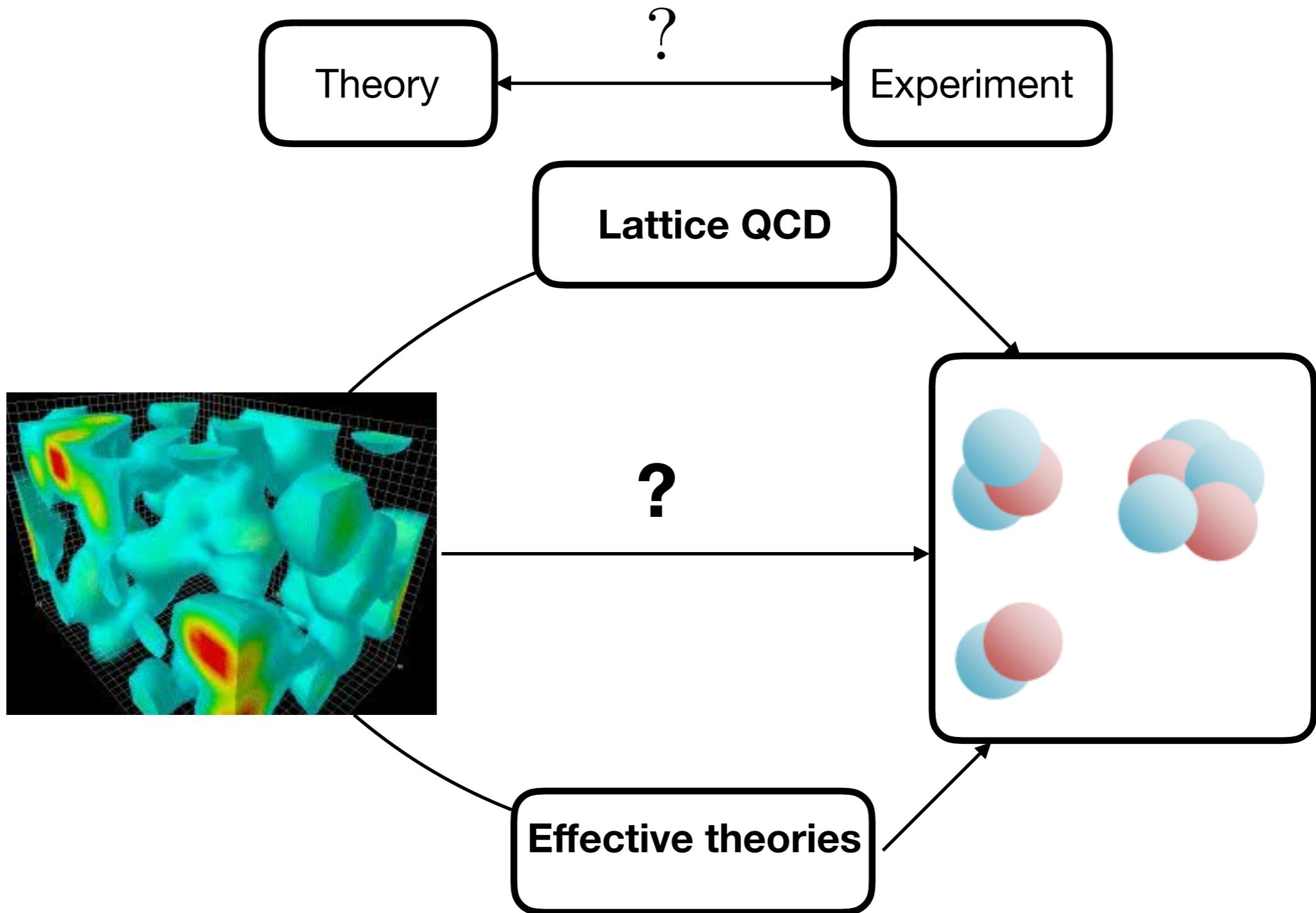
- Pion and nucleons degrees of freedom
- Exact
- Lagrangian is an expansion in powers of $Q/\Lambda\chi$
$$\mathcal{L}_{\chi\text{EFT}} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \dots$$
- Low energy constants (encode our ignorance)



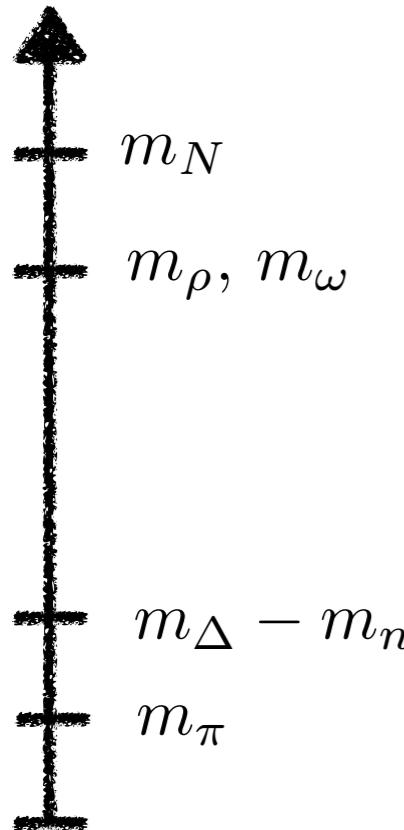
Experiments (past and present)

Lattice QCD (near future)

Theory approaches



Effective field theories

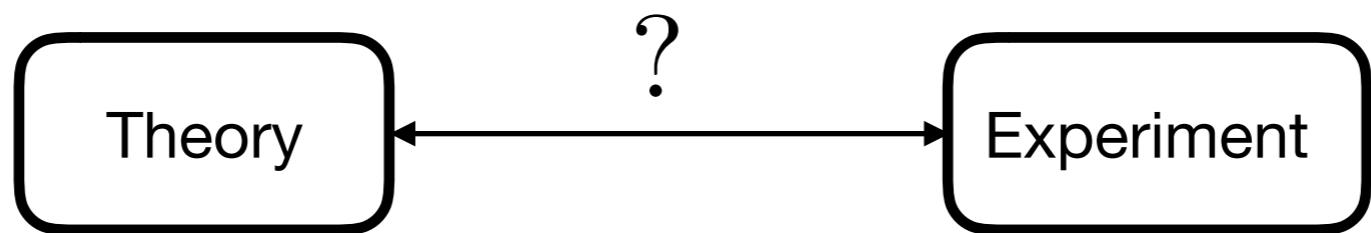


Low energy approximations of an underlying theory

- Exploit separation of scales
- Build the most general Lagrangian consistent with the symmetries of the underlying theory

- Weinberg 1979

Strategy



Pipeline

