Electroweak reactions in “a few-body system”

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Outline

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4. P-wave contribution to the pp fusion reaction
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6. Electromagnetic response functions of the deuteron at large four-momentum transfer
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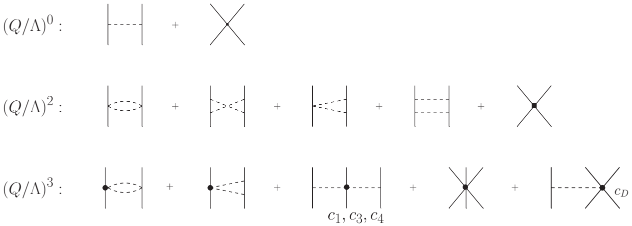
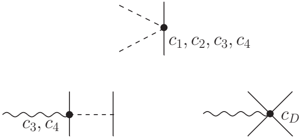
χEFT with external currents

- degrees of freedom: pions and nucleons (and, sometimes, also the delta) plus external (electroweak, dark ...) probes.
- NN forces given by pion exchanges and contact interactions.



- πN processes and nuclear currents that couple to external probes derived from same theory — many LECs are common!



The NNLO$_{\rm sim}$ Interactions [Carlsson et al. (2016)]

- Weinberg counting, Gaussian regulators with cutoff $\Lambda_{\rm EFT} \sim 500$ MeV.

- nonperturbative resummation of all NN interactions up to $(Q/\Lambda)^3$ in the Schrödinger equation.

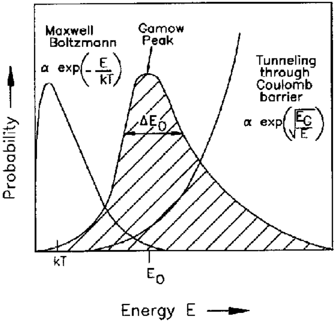
- *simultaneous* fitting of LECs to πN and NN scattering data, E_B and $\langle {r_c}^2 \rangle$ of ^{2,3}H and ^{3}He, $Q(^2\mathrm{H})$, as well as $E_1^A(^3\mathrm{H})$, with both experimental and theoretical uncertainties.

- 42 interactions — 7 values for $\Lambda_{\rm EFT}$ in the range (450, 600) MeV and 6 different truncations of the NN scattering energy in input data — have same long-distance properties.

- covariance matrix of the LECs known for each of the 42 interactions — correlations between *all* LECs taken into account.

- errors in LECs propagate to calculated observables.

$p + p \to d + e^+ + \nu_e$



Picture credit: A. Ray

- Cross section parametrized as S-factor, $S(E) = \sigma(E)\ E\ e^{\pi\sqrt{m_p/E}\ \alpha}$.

- "Gamow window" at $E < 10$ keV. Maclaurin's series useful, $S(E) = S(0) + E\ S'(0) + \ldots$

- lab experiments practically impossible — rigorous analysis of theoretical uncertainty important.

Semi-leptonic weak cross sections and decay rates

- $\sigma,\, \Gamma \sim \sum_{\text{spins}} \int_{\text{phase space}} |\langle f|\mathcal{H}_W|i\rangle|^2$

- Below GeV scale, weak interaction Hamiltonian can be considered a zero-range coupling between leptonic and nucleonic currents,

$$\mathcal{H}_W = \frac{G}{\sqrt{2}} \int \mathrm{d}^3x \left[j_\alpha(\mathbf{x}) J^\alpha(\mathbf{x}) + \text{h.c.} \right] .$$

- Need matrix elements of nucleonic current operator J^α between nucleonic initial and final states.

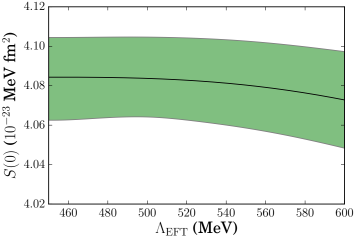
Other calculations

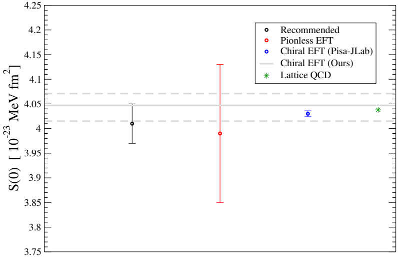
- "realistic" approaches: educated guesses for potentials and currents [Bethe and Critchfield (1938), Bahcall and May (1969), Kamionkowsky and Bahcall (1994), Schiavilla et al. (1998)]
- hybrid approach: current operator derived in EFT sandwiched between phenomenological wave functions. [Park et al. (1998), Park et al. (2003)]
- Effective Field Theories
  - EFT($\not\pi$) [Kong and Ravndal (2001), Butler and Chen (2001), Ando et al. (2008), Chen et al. (2013)]
  - χEFT [Marcucci et al. (2013)]
- Lattice
  - EFT [Rupak and Ravi (2015)]
  - QCD [Savage et al. (2017)]

Our error budget

Theory uncertainty $\sim$ 0.7% from:

- Numerical solution of Schrödinger equation $\sim 0.05\%$
- Polynomial fit to $S(E)$ $\sim 0.005\%$
- Statistical uncertainties in LECs $\sim 0.4\%$
- Energy range of the input NN scattering data $\lesssim 0.2\%$
- Cutoff dependence [$\mathcal{O}(\alpha^2)$ corrections not shown here]:





- The recommended value [Adelberger et al. (2011)] was obtained from "range of values of published calculations".

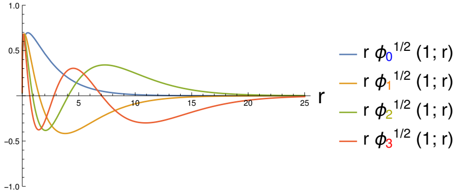
- The error in the Pisa-JLab calculation [Marcucci et al. (2013)] was estimated by using two different values for $\Lambda_{\rm EFT}$, 500 and 600 MeV.

- Pisa-JLab calculation is the only one that includes p-wave pp state. They find a $\sim 0.5\%$ contribution at threshold that grows with E.

Pisa-JLab calculation used a basis of Laguerre polynomials to represent their wave functions,

$$\phi_m^a(\beta; r) \propto e^{-\frac{1}{2}\beta r} L_m^a(\beta r), \tag{1}$$

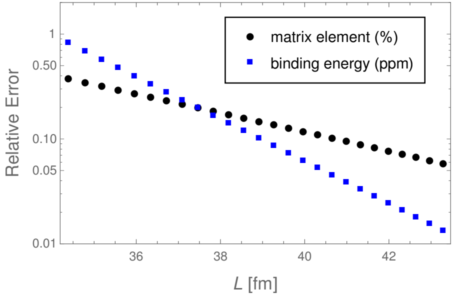
which makes extension to $A > 2$ systems more convenient.



Truncating m at some integer n effectively imposes an UV *and* an IR cutoff. Dirichlet boundary at $r = L$, which scales as

$$L = (4n + 2a + 6)\beta^{-1} \ . \tag{2}$$

For the leading (Gamow-Teller) matrix element at $E = 50$ keV :



- Finite basis is widely used to solve nuclear many-body problems.
- UV truncation error easy to avoid for nuclear interactions but hard to model [König et al. (2014)].
- For IR corrections, equivalent of Lüscher's formula already exists for bound state properties [More et al. (2012)].
- We derive similar analytic formula for finite-volume correction in capture/fusion reactions.

"Lüscher's Formula" for capture matrix elements

$$\mathcal{I}_\lambda(p;\eta;L) = \int_0^L \mathrm{d}r\, u^{(L)}(r)\, r^\lambda\, u_p(r)\ ,$$

- $u^{(L)}(r) \to$ $A_\infty e^{-\gamma_\infty r}\left[1 - e^{-2\gamma_\infty(L-r)}\right] + (1-\delta_{l0})\mathcal{O}(\frac{1}{\gamma_\infty r}) + \mathcal{O}(e^{-\gamma_\infty[2L+r]})$
- $u_p(r) \to \sin\left[pr - \eta_p \log(2pr) + \sigma_l + \delta_l - \frac{\pi l}{2}\right] + \mathcal{O}(\frac{1}{pr})$
- for weak capture, the LO matrix element has $\lambda = 0$
- more generally, $\lambda = 0, 1, \ldots$, and is related to the multipolarity of the transition

At $\gamma_\infty L \gg 1, \lambda, \eta$,

$$\Delta\mathcal{I}_\lambda(p;\eta_p;L) = \frac{2A_\infty\gamma_\infty}{\gamma_\infty^2 + p^2} L^\lambda\, e^{-\gamma_\infty L}\, \sin\left(\delta_l + \sigma_l - \frac{\pi l}{2} + pL - \eta_p \log 2pL\right).$$

“Lüscher’s Formula” for capture matrix elements

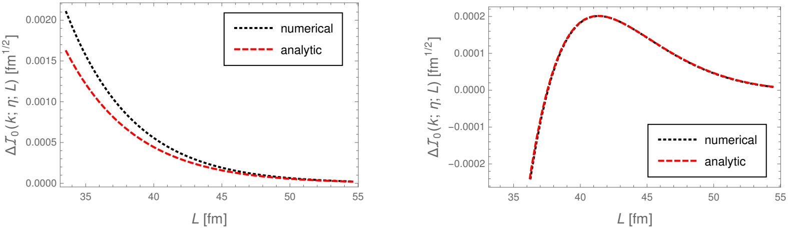


Figure: The IR correction to the radial matrix element of the LO (Gamow-Teller) operator for *pp* fusion at $E = 50$ keV (left) and 1 MeV (right). The numerical results were calculated using Harmonic oscillator bases of varying dimensionality.

“Lüscher’s Formula” for capture matrix elements

- depends on the type of basis functions used only through L.
- contains no fit parameters at $A = 2$.
- should work for $A > 2$ nuclei. However, one might have to use our formula as an *extrapolant* with γ_∞, A_∞ and δ_l fit to numerical data at several L values.
- gets better at larger L and higher energies, i.e. smaller η_p.
- needs improvement/extension for application to the rp-process regime.

Proton-Proton Weak Capture in Chiral Effective Field Theory

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The astrophysical S factor for proton-proton weak capture is calculated in chiral effective field theory over the center-of-mass relative-energy range 0–100 keV. The chiral two-nucleon potential derived up to next-to-next-to-next-to leading order is augmented by the full electromagnetic interaction including, beyond Coulomb, two-photon and vacuum-polarization corrections. The low-energy constants entering the weak current operators are fixed so as to reproduce the $A = 3$ binding energies and magnetic moments and the Gamow-Teller matrix element in tritium β decay. Contributions from S and P partial waves in the incoming two-proton channel are retained. The S factor at zero energy is found to be $S(0) = (4.030 \pm 0.006) \times 10^{-23}$ MeV fm^2, with a P-wave contribution of 0.020×10^{-23} MeV fm^2. The theoretical uncertainty is due to the fitting procedure of the low-energy constants and to the cutoff dependence.

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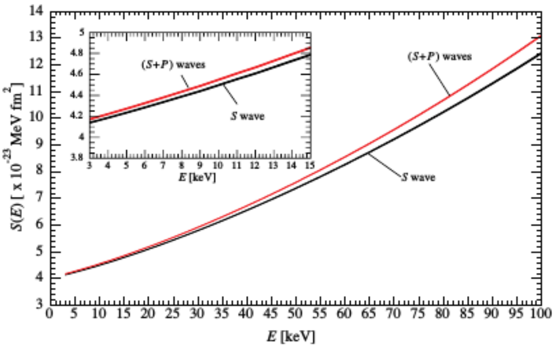
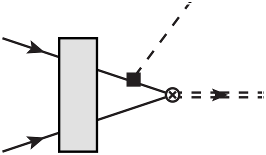


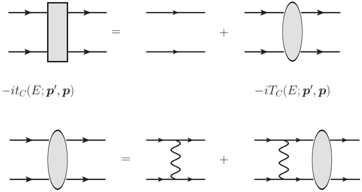
P-wave contribution to pp fusion in pionless EFT

Anticipated uncertainty $\sim$ larger of $\dfrac{Q}{m_\pi} \lesssim \dfrac{1}{20}$ and $\dfrac{\gamma}{m_\pi} \sim \dfrac{1}{3}$



$$\mathcal{L}_W^{(\mathrm{LO})} = -\frac{G_V}{\sqrt{2}}\left(l_+^0 N^\dagger \tau^- N + g_A \mathbf{l}_+ \cdot N^\dagger \boldsymbol{\sigma} \tau^- N \right)$$

only contributes for nonzero recoil momentum (k). $k/\gamma \lesssim 0.01$



$$\mathcal{L}_W^{(\mathrm{NLO})} = -\frac{G_V}{\sqrt{2}}\left[-g_A l_+^0 N^\dagger \boldsymbol{\sigma} \cdot \frac{i\overleftrightarrow{\nabla}}{2m} \tau^- N - \mathbf{l}_+ \cdot \left(N^\dagger \frac{i\overleftrightarrow{\nabla}}{2m} \tau^- N \right) \right]$$

$Q/m \lesssim 0.01$

Anticipated size of the p-wave contribution to S-factor $\sim \mathcal{O}(10^{-4})$.

Results so far ...

$$\overline{|\mathcal{M}^{(\mathrm{NLO})} + \mathcal{M}^{(\mathrm{rec})}|^2} = 16\left(\frac{G_V}{\sqrt{2}}\right)^2 \Big\{ (E_\nu E_e + \mathbf{p}_\nu \cdot \mathbf{p}_e)\left(3|T_{\mathrm{rec}}|^2 + 2{g_A}^2 \mathbf{T}_{\mathrm{NLO}} \cdot \mathbf{T}^*_{\mathrm{NLO}}\right)$$
$$+ 6\,(\mathbf{p}_\nu \cdot \mathbf{T}_{\mathrm{NLO}})\,(\mathbf{p}_e \cdot \mathbf{T}^*_{\mathrm{NLO}}) + 3\,(E_\nu E_e - \mathbf{p}_\nu \cdot \mathbf{p}_e)\,\mathbf{T}_{\mathrm{NLO}} \cdot \mathbf{T}^*_{\mathrm{NLO}}$$
$$- (6 + 4g_A^2)\,(E_\nu \mathbf{p}_e + E_e \mathbf{p}_\nu) \cdot \mathbf{T}_{\mathrm{NLO}}\, T^*_{\mathrm{rec}} + 2g_A^2\,(3E_\nu E_e - \mathbf{p}_\nu \cdot \mathbf{p}_e)\,|\,T_{\mathrm{rec}}|^2 \Big\},$$

where

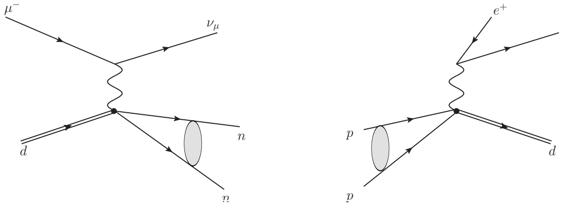
$$T_{\mathrm{rec}} = -\sqrt{8\pi\gamma}\, e^{i\sigma_1}\, e^{-\pi\eta_p/2}\,(\mathbf{p}_\nu + \mathbf{p}_e)\cdot\mathbf{p}\,|\Gamma(2 + i\eta_p)|\,\frac{1}{(\gamma^2 + p^2)^2}\, e^{2\eta_p \arctan p/\gamma}\,, \qquad (3)$$

and

$$\mathbf{T}_{\mathrm{NLO}} = \sqrt{8\pi\gamma}\, e^{i\sigma_1}\, e^{-\pi\eta_p/2}\,\frac{\mathbf{p}}{m}|\Gamma(2 + i\eta_p)|\,\frac{1}{p^2 + p^2\eta_p^2}\left[1 + \frac{p^2 + 2p\eta_p\gamma - \gamma^2}{\gamma^2 + p^2}\, e^{2\eta_p \arctan p/\gamma}\right].$$

The MuSun experiment at PSI

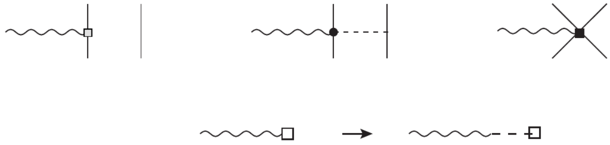
The MuSun experiment aims at measuring $\mu^- + d \to \nu_\mu + n + n$ capture rate with 1.5% precision.



- first precise measurement of weak reaction on NN system.
- simplest possible test of semileptonic nuclear calculations.
- input data to constrain *pp* fusion rate without contaminating it with NNN physics.

χEFT calculation of muon capture by deuteron

- Higher-energy process than *pp* fusion — pion-pole diagrams need to be added:



- Combining with the recent Roy-Steiner extraction of $c_i{}'$s [Hoferichter et al. (2015, 2016), Siemens et al. (2017)], with remaining relevant LECs fitted NN data, allows us to fix c_D from ${}^1S_0$ capture rate.

- Upto $\mathcal{O}(Q^3)$, predicts NN electroweak currents and pion-range part of NNN interaction, c_D.

- Important test of *q*-dependence of nuclear matrix elements and of the single-nucleon axial form factor.

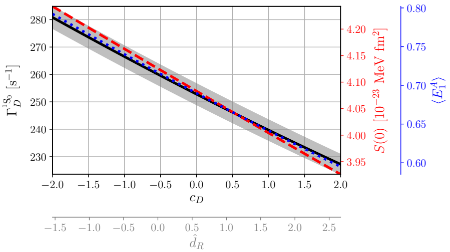
Muon capture: Results

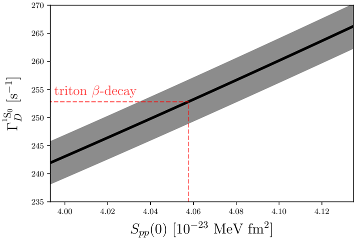
$$\text{NNLO}_{\text{sim}} \quad : \quad \Gamma_D^{(^1S_0)} = 252.4^{+2.5}_{-2.1}\ s^{-1}\,.$$

$$\text{NNLO}_{\text{RS}}\ ^1 : \quad \Gamma_D^{(^1S_0)} = 252.8 \pm 4.6\ s^{-1}\,.$$

Additional uncertainty from nucleon axial radius [2] : $\Delta\Gamma_D^{(^1S_0)} \sim 3.9\ s^{-1}$.

Including higher partial waves : $\Gamma_D = 397.8\ s^{-1}$.





[1] With NN contacts fitted to $T_{lab} < 200$ MeV phaseshifts from Granada PWA, and ^{2}H binding and radius.

[2] From variation of r_A^2 in the 1-σ interval (0.24,0.68) fm^2.

The electromagetic response functions

The longitudinal response function

$$R_L(q,\omega) = \sum_f |\langle\psi_f|\rho(q)|\psi_i\rangle|^2\,\delta(\omega + m_d - E_f) \qquad (4)$$

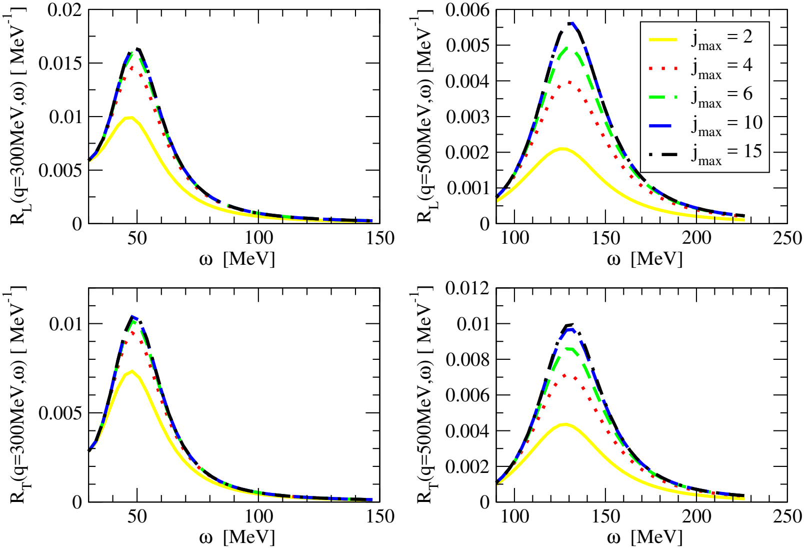
and the transverse response function

$$R_T(q,\omega) = \sum_f |\langle\psi_f|\mathbf{j}_T(q)|\psi_i\rangle|^2\,\delta(\omega + m_d - E_f) \qquad (5)$$

are related to the cross section in the one-photon exchange limit by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega\mathrm{d}\omega} = \sigma_{\mathrm{Mott}}\left(\left[\frac{q^\mu q_\mu}{\mathbf{q}^2}\right]^2 R_L - [\frac{q^\mu q_\mu}{2\mathbf{q}^2} - \tan^2(\theta/2)]\,R_T\right)\,. \qquad (6)$$

EM responses of the deuteron in the impulse approximation



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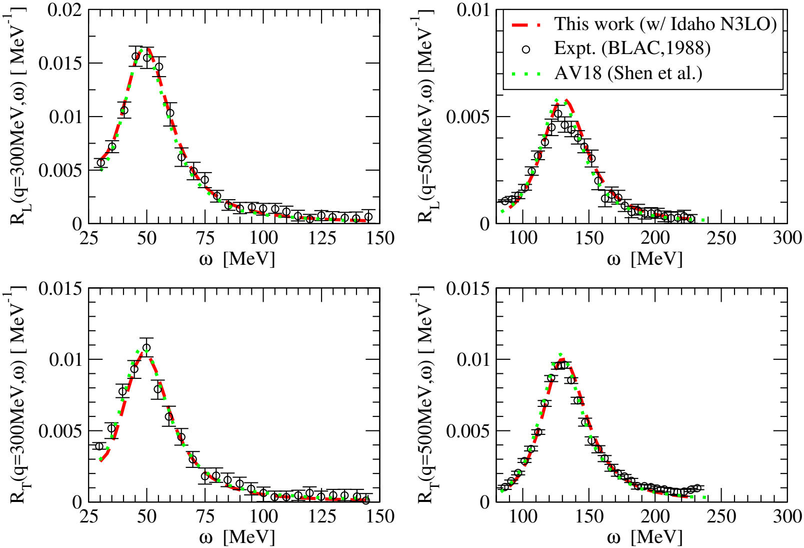
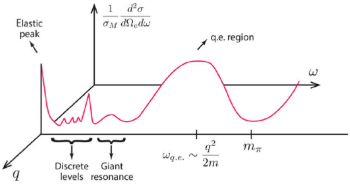


Next up: νd and ν-nucleus scattering



Picture credit: Bacca and Pastore

- ν experiments use event generators that need nuclear physics input for ν-nucleus cross section.
- The *ab initio* approach, with chiral EFT interactions, can provide important benchmark for models that go into these generators.
- Inclusive ν-induced breakup of $^2\mathrm{H}$.
- Extend to ν-induced one/two-nucleon knockout from $^4\mathrm{He}$, $^{12}\mathrm{C}$, $^{16}\mathrm{O}$, ..., $^{40}\mathrm{Ar}$ using nuclear many-body methods.

Summary

1. Calculated *pp* fusion cross section. Result agrees with another χEFT calculation once their result is corrected for basis truncation error.

2. Derived analytic expressions for *p*-wave contributions to *pp* fusion reaction in pionless EFT. Numbers due soon!

3. Performed uncertainty analysis of the $^1S_0$ μd capture rate and obtained correlation with the *pp* fusion *S*-factor.

4. Upcoming experimental μ-d capture rate value, combined with Roy-Steiner determination of c_i's, fixes electroweak currents and pion-exchange part of NNN force completely from πN and NN sectors.

5. Better determination of nucleon axial form factor is required for more precise calculations of finite-q^μ nuclear weak processes.

6. Will calculate ν-nucleus cross sections starting from ν-d.

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