Zero-range EFT for resonant dark matter

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In collaboration with ***Eric Braaten,*** *and* ***Evan Johnson***

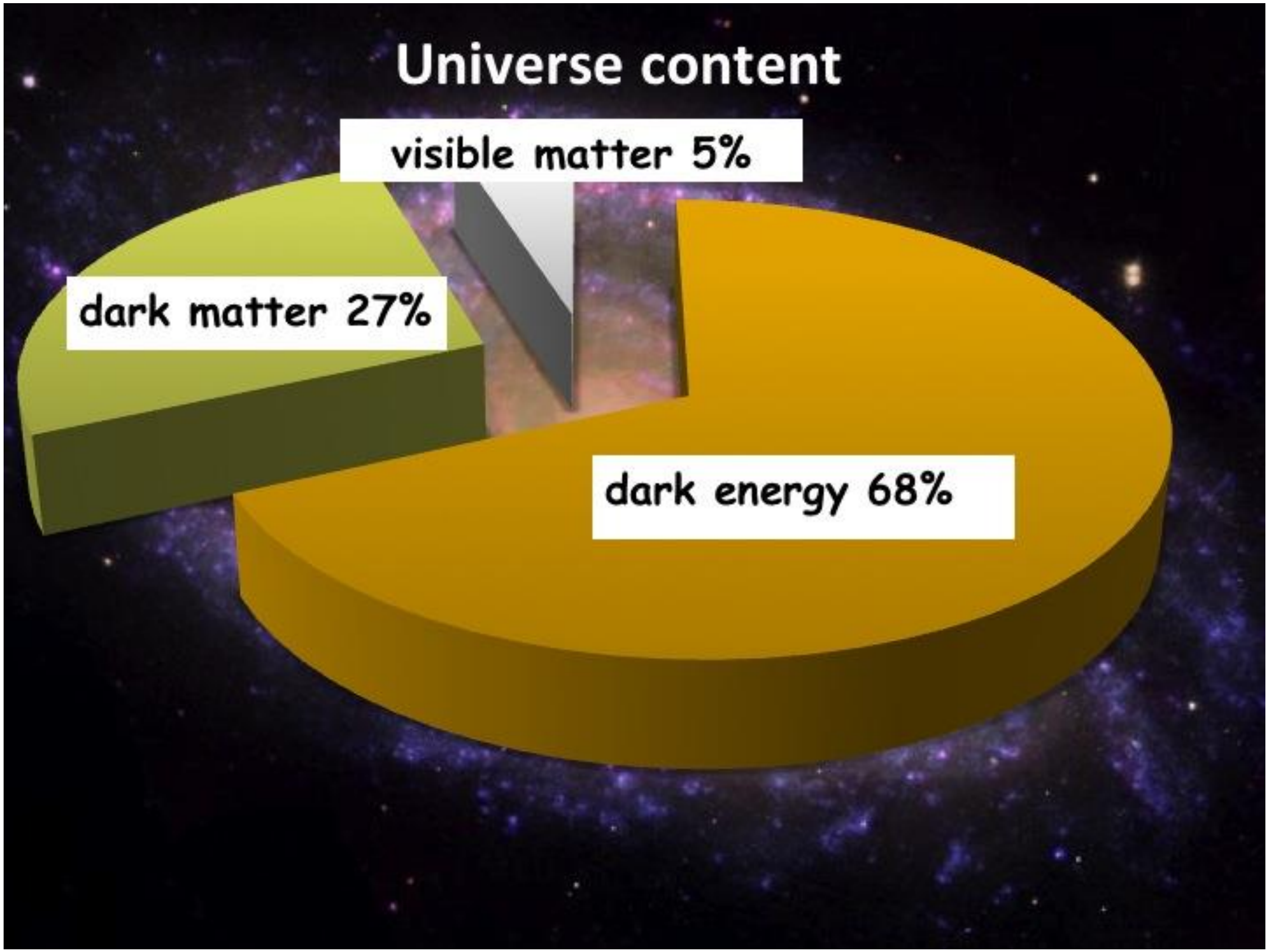
Based on **JHEP 1711 (2017) 108, JHEP 1802 (2018) 150, arXiv:1712.07142**

INT workshop, May 14th, 2018

Outline

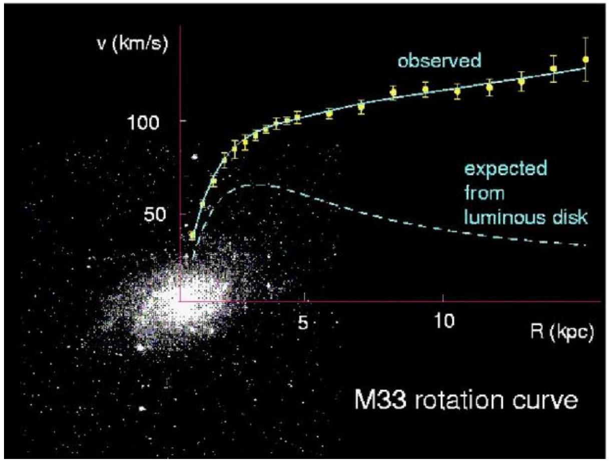
- Introduction
- Framework
- Coulomb Resummation
- Annihilation Effects
- Summary

Energy Budget of Our Universe

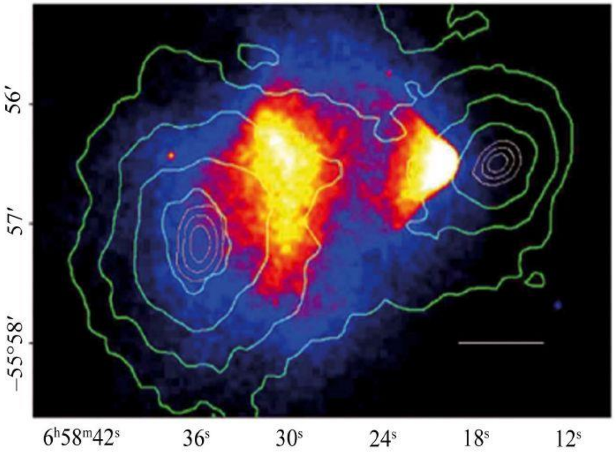


Evidence of Dark Matter

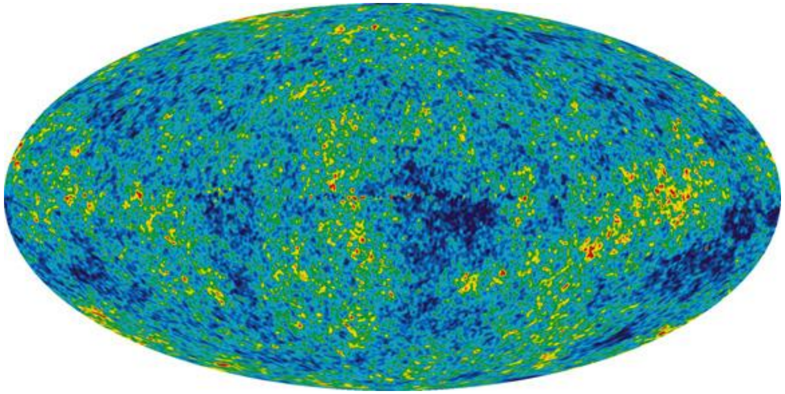
- Rotation Curve



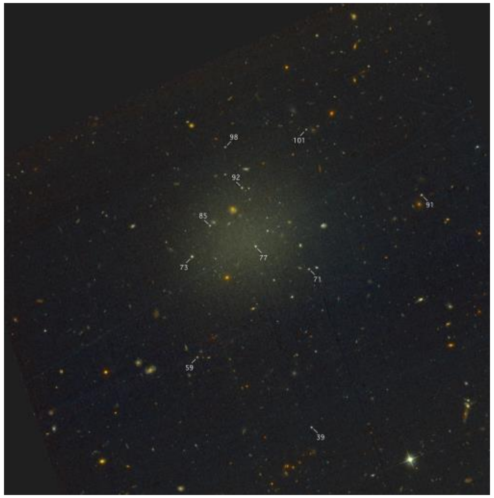
- Bullet Cluster



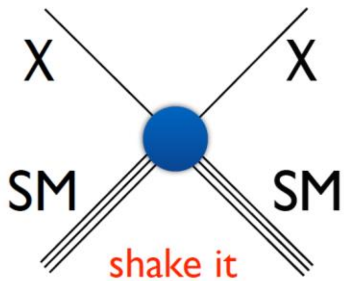
- CMB



- NGC1052–DF2

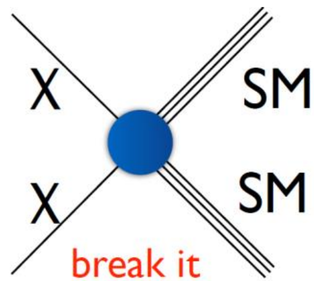


DM Search

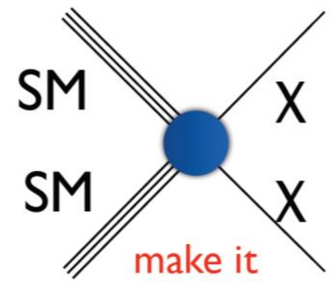


Direct detection

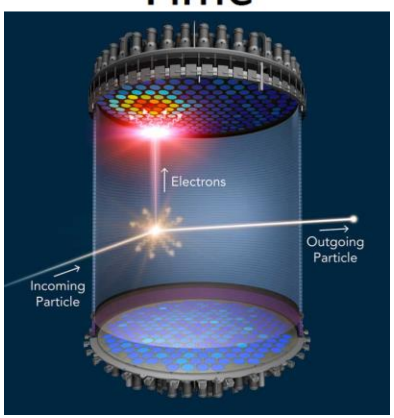
Time

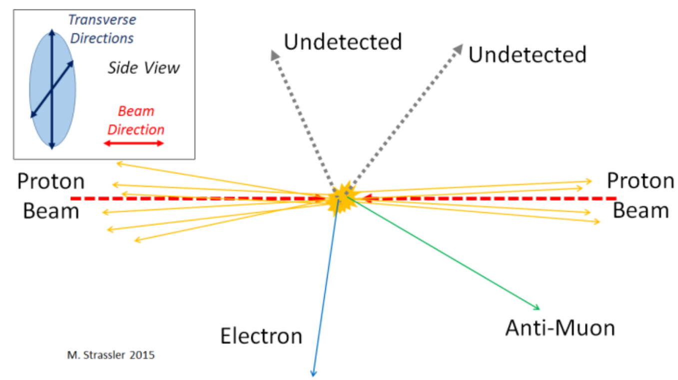


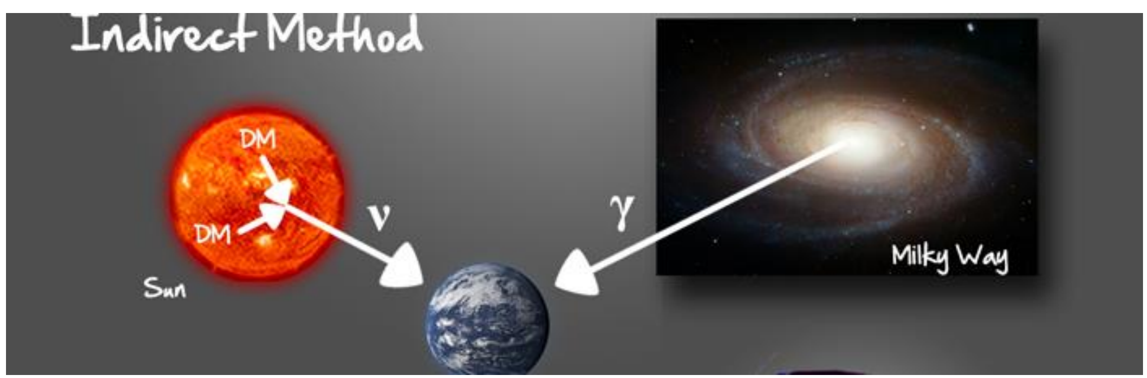
Indirect detection



Collider







Collisionless DM vs. Observations

- The null results of all DM search experiments indicate the DM-SM coupling must be very small, if not zero!

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{DM} + \boxed{\mathcal{L}_{mix}}$$

Collisionless DM vs. Observations

- The null results of all DM search experiments indicate the DM-SM coupling must be very small, if not zero!

$$\mathcal{L} = \mathcal{L}_{SM} + \boxed{\mathcal{L}_{DM}} + \mathcal{L}_{mix}$$

- **DM-DM coupling may not be small!**

- N-body simulations with collisionless DM can not explain some features of observed halo structures

  - Core-Cusp problem
  - Diversity problem
  - Missing Satellites
  - …

For more details, see Tulin and Yu, Phys.Rept. 730 (2018) 1-57 (2018)

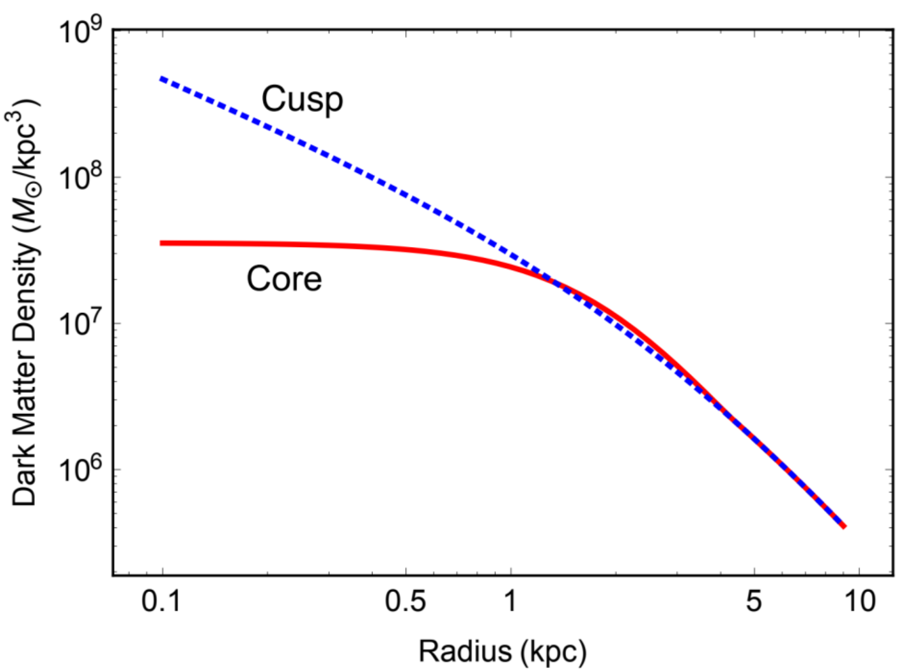
Core-Cusp Problem

- N-body simulations with **collisionless** DM predict the DM density has a **cusp** at the center of halos.

NFW profile: $$\rho(r) = \frac{\rho_s}{(r/r_s)\,(1+r/r_s)^2}$$

Two parameters: ρ_s and r_s

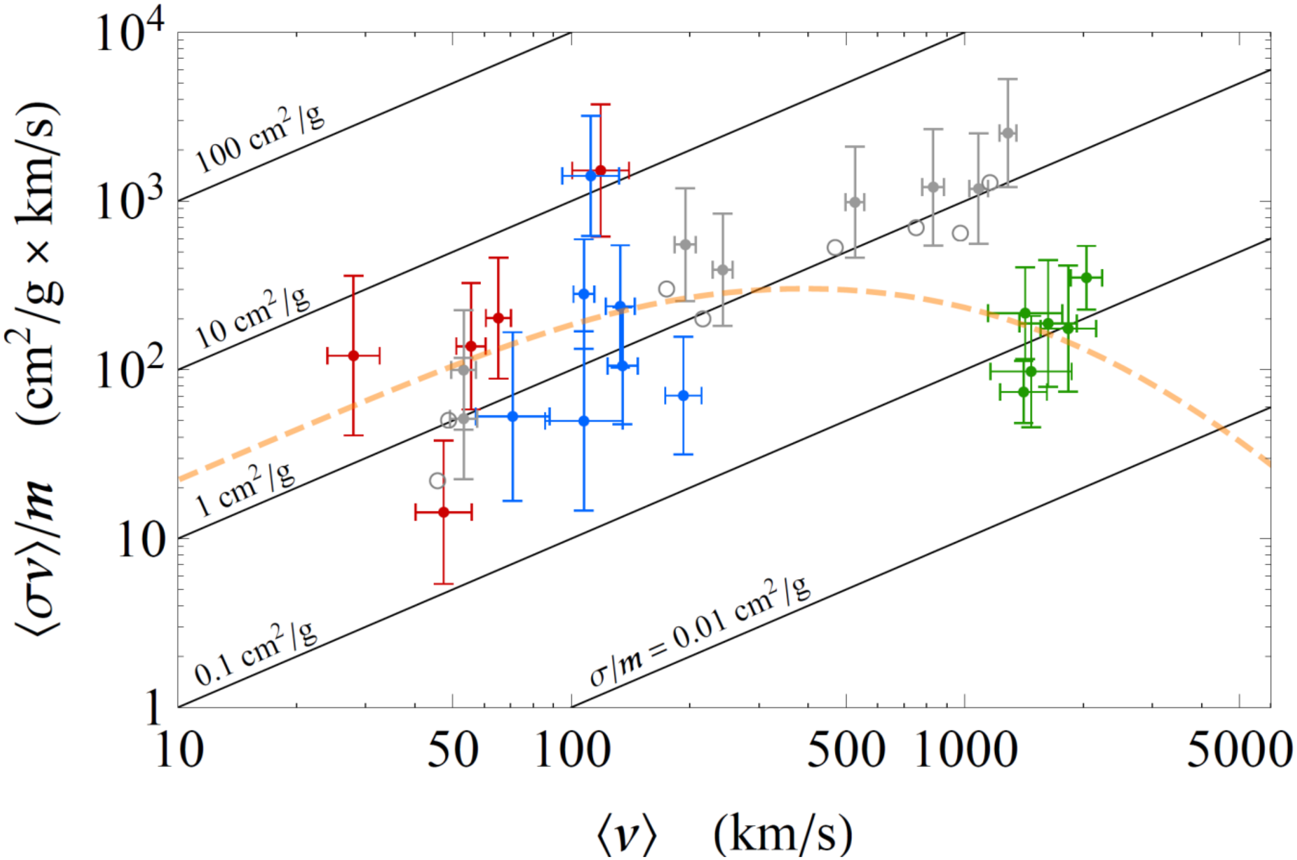
- Observations show a **core** at the center.



A hint of DM self-interaction?

σ_{DM-DM} is Velocity Dependent

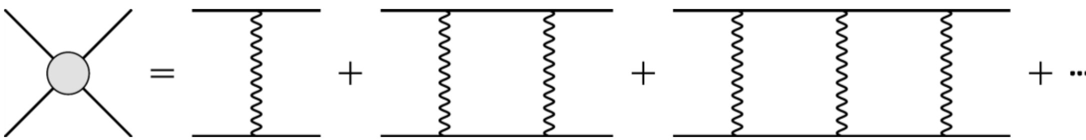
- 12 galaxies and 6 clusters are analyzed.
- Assume σ is **a different constant for each halo**



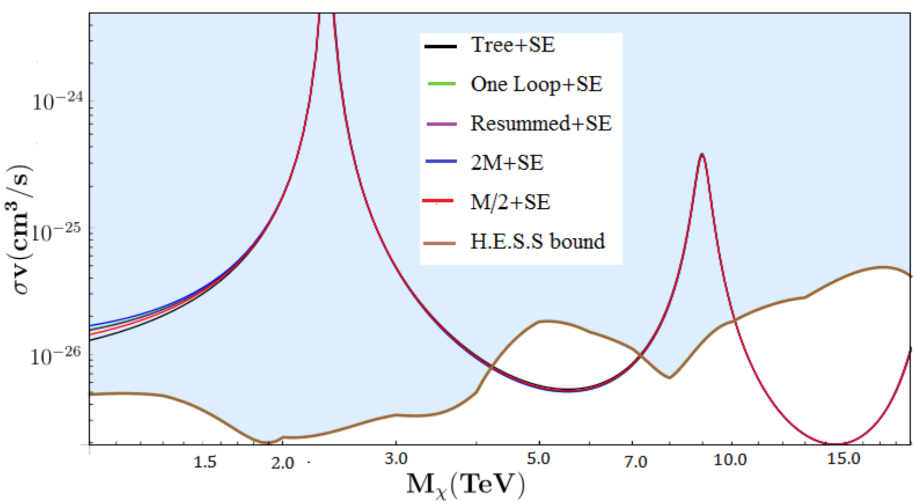
Kaplinghat, Tulin and Yu, PRL (2016)

Sommerfeld and Resonance Effects

- Sommerfeld and resonance effects can naturally introduce v-dep.
- In QFT, these effects are included by summing over the ladder diagram to all orders.



- The ladders are summed by solving Schrödinger equation
- σ and σ_{ann} can be enhanced by several orders of magnitudes



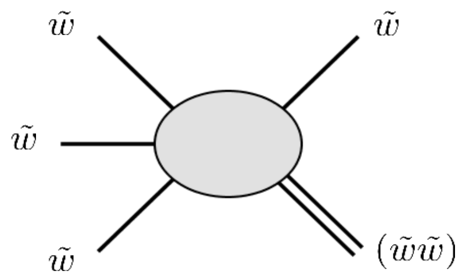
$\sigma_{ann}(w^0 w^0 \to \gamma + X)$

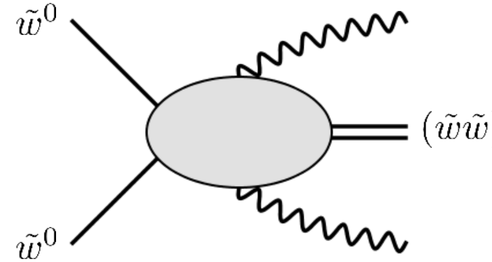
Figure from
Baumgart, et.al. JHEP 1504 (2015) 106

Also see
Beneke, et.al. JHEP 1303 (2013) 148
Hellmann, et.al. JHEP 1308 (2013) 084
Beneke, et.al. JHEP 1505 (2015) 115
Baumgart, et.al. PRL 114 (2015) 211301
Baumgart, et.al. JHEP 1603 (2016) 213
Asadi et.al. JCAP 1702 (2017) 005
Baumgart, et.al. JHEP 1803 (2018) 117

Disadvantages of Current Method

- Disadvantages of solving Schrödinger equation
  - Too many parameters, difficult to be constrained.
  - Difficult to calculate 3-body scattering cross section and bound state formation rate.





  - Exclusive DM annihilation processes (DM-SM coupling) can only be considered perturbatively.

A better framework is needed!

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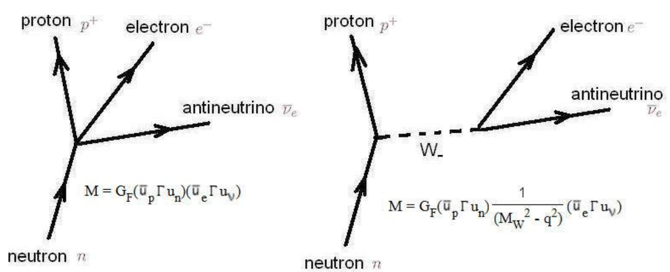
Motivation

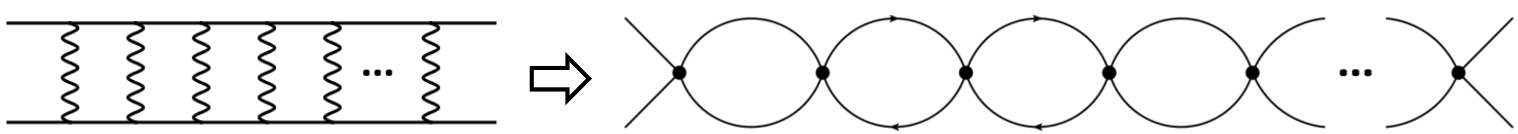
- To avoid messing up nucleosynthesis, the mass of the force-carrier boson must be $> 10\ MeV$
- The kinetic energy of DM now is very small, with $v \sim 10^{-3}c$.

The force-carrier bosons can not be resolved.

- <u>Reminder</u>: Fermi's theory for beta decay



- A ZREFT can be written for DM (similar to pionless EFT)



(Photon is massless and need to be included separately.)

Example: Wino Dark Matter

Non-relativistic EFT

Dark Matter

SM with one additional electroweak triplet $\tilde{w} = (\tilde{w}^+, \tilde{w}^0, \tilde{w}^-)$

- **Masses**: neutral wino $M \sim$ few TeV, charged wino $M + \delta$
  Radiative correction gives $\delta \sim 170$MeV, insensitive to M.

Example: Wino Dark Matter

Non-relativistic EFT

SM with one additional electroweak triplet $\widetilde{w} = \left(\widetilde{w}^{+}, \widetilde{w}^{0}, \widetilde{w}^{-}\right)$

- **Masses**: neutral wino $M \sim$ few TeV, charged wino $M + \delta$
  Radiative correction gives $\delta \sim 170$MeV, insensitive to M.
- **Coupling**: to W/Z boson and photon.
- **Non-relativistic Lagrangian**

$$\mathcal{L}_{\text{kinetic}} = \zeta^{\dagger}\left(i\partial_0 + \frac{\boldsymbol{\nabla}^2}{2M}\right)\zeta + \eta^{\dagger}\left(i\partial_0 + \frac{\boldsymbol{\nabla}^2}{2M} - \delta\right)\eta + \xi^{\dagger}\left(i\partial_0 - \frac{\boldsymbol{\nabla}^2}{2M} + \delta\right)\xi$$

$$H_{\text{potential}} = -\frac{1}{2}\int d^3x \int d^3y \Bigg(\left[\frac{\alpha}{|\boldsymbol{x}-\boldsymbol{y}|} + \frac{\alpha_2 \cos^2\theta_w}{|\boldsymbol{x}-\boldsymbol{y}|} e^{-m_Z|\boldsymbol{x}-\boldsymbol{y}|}\right] \eta^{\dagger}(\boldsymbol{x})\xi(\boldsymbol{y})\,\xi^{\dagger}(\boldsymbol{y})\eta(\boldsymbol{x})$$
$$+\frac{\alpha_2}{|\boldsymbol{x}-\boldsymbol{y}|} e^{-m_W|\boldsymbol{x}-\boldsymbol{y}|}\left[\zeta^{\dagger}(\boldsymbol{x})\zeta^{c}(\boldsymbol{y})\,\xi^{\dagger}(\boldsymbol{y})\eta(\boldsymbol{x}) + \zeta^{c\dagger}(\boldsymbol{x})\zeta(\boldsymbol{y})\,\eta^{\dagger}(\boldsymbol{y})\xi(\boldsymbol{x})\right]\Bigg)$$

(ξ^{+}, ζ, η) annihilates $\left(\widetilde{w}^{+}, \widetilde{w}^{0}, \widetilde{w}^{-}\right)$, respectively.

Non-Perturbative Summation

- Ladder diagrams are summed by solving Schrödinger equation.

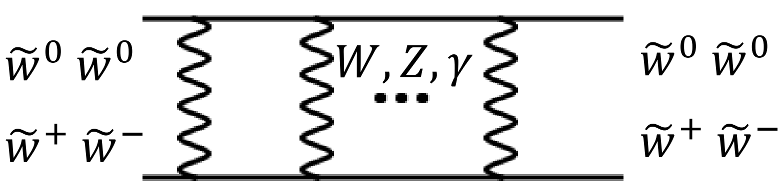
Two scattering channels: $\mathrm{R}_0 = \widetilde{w}^0\widetilde{w}^0$ and $\mathrm{R}_1 = \widetilde{w}^+\widetilde{w}^-$

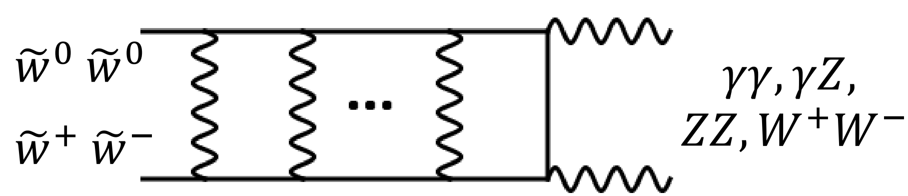
$$\left[-\frac{1}{M}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\left(\frac{d}{dr}\right)^2 + 2\delta\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix} + \boldsymbol{V}(r)\right] r\begin{pmatrix}R_0(r)\\ R_1(r)\end{pmatrix} = E\, r\begin{pmatrix}R_0(r)\\ R_1(r)\end{pmatrix}$$

S-wave scattering potential

$$\boldsymbol{V}(r) = -\alpha_2\begin{pmatrix}0 & \sqrt{2}\,e^{-m_W r}/r\\ \sqrt{2}\,e^{-m_W r}/r & c_w^2\,e^{-m_Z r}/r\end{pmatrix} - \alpha\begin{pmatrix}0 & 0\\ 0 & 1/r\end{pmatrix}$$

- 2-body scattering
- Annihilation to SM particles





- Annihilation is treated pertubatively, which violates unitarity for some values of *M.*

Blum et.al. JCAP 1606 (2016) 021

ZREFT for Wino DM

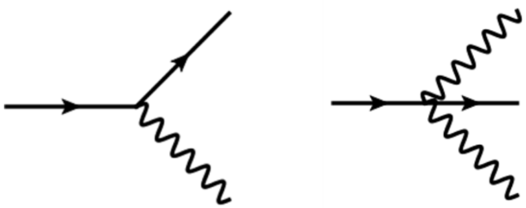
ZREFT Lagrangian

$$\mathcal{L} = \tilde{w}^{0\dagger}\left(i\partial_0 + \frac{\nabla^2}{2M}\right)\tilde{w}^0 + \sum_\pm \tilde{w}^{\pm\dagger}\left(iD_0 + \frac{\boldsymbol{D}^2}{2M} - \delta\right)\tilde{w}^\pm + \mathcal{L}_{\rm EM} + \mathcal{L}_{\rm zero-range}$$

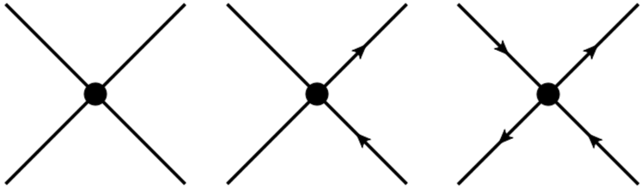
with covariant derivative:

$$D_0\tilde{w}^\pm = (\partial_0 \pm ieA_0)\tilde{w}^\pm \qquad \boldsymbol{D}\tilde{w}^\pm = (\boldsymbol{\nabla} \mp ie\boldsymbol{A})\tilde{w}^\pm$$

- **Coupling to photon**



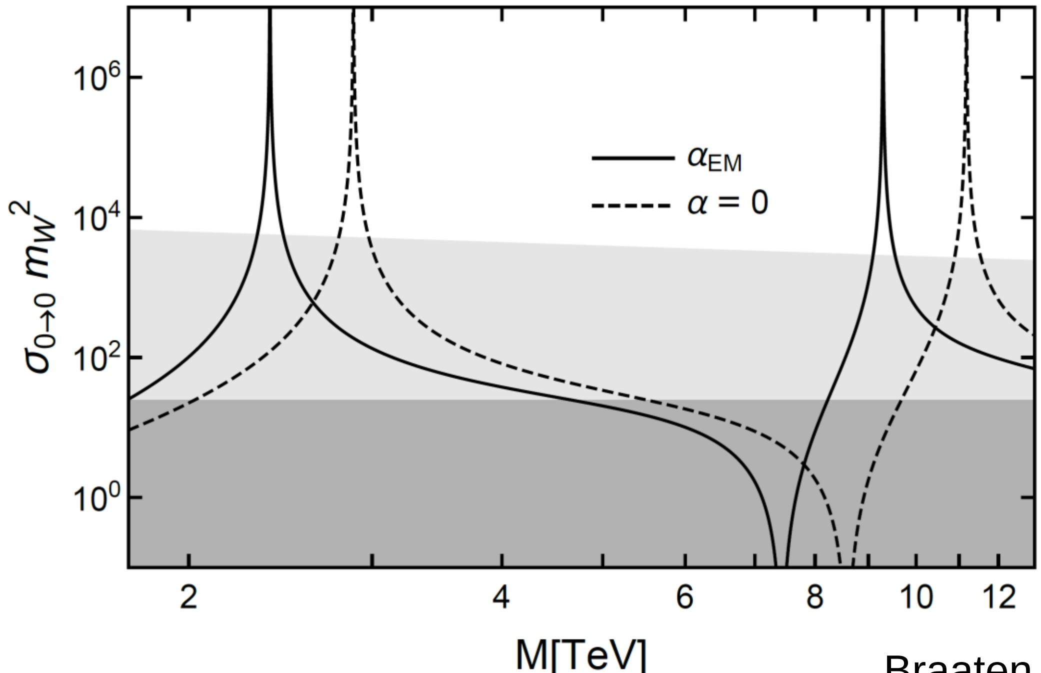
- **Zero-Range coupling**
  (S-wave)



Braaten, Johnson and HZ, 1711 (2017) 108

Number of Channels

- Relevant scales:
  - Weak boson mass: $m_W \sim 80$ GeV
  - Channel transition momentum: $\sqrt{2M\delta}$
  - Inverse scattering length: $1/a_0$
- Two-channel ZREFT is valid if $1/|a_0| << m_W$,
- One-channel ZREFT is valid if $1/|a_0| << \sqrt{2M\delta}$.



We will build two-channel ZREFT for larger region of validity.

Braaten, Johnson and HZ, 1711 (2017) 108

Power Counting of 2-Channel ZREFT

- The renormalization and power counting of the Zero-Range EFT is governed by its RG fixed points

- Three fixed points for a two-channel theory correspond to the number of fine-tuned parameters.

  0, 1 or 2 resonances at the scattering threshold
  requiring 0, 1, or 2 fine-tunings

  Lensky and Birse EPJ 47 (2011) 142

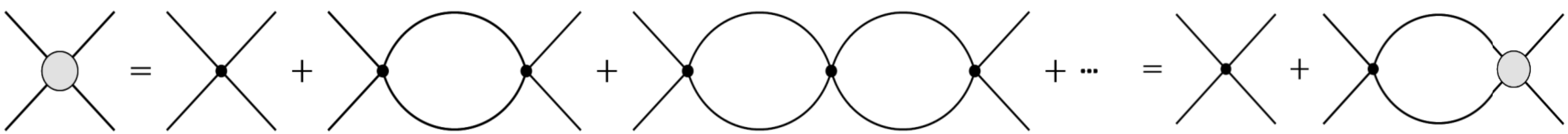
- If only the wino mass M is tuned to its critical value M_*, expect a **single resonance**
  - Single-resonance channel is a linear combination of $\tilde{w}^0\tilde{w}^0$ and $\tilde{w}^+\tilde{w}^-$ with mixing angle ϕ.
  - No scattering in the orthogonal channel.

Braaten, Johnson and HZ, 1711 (2017) 108

Scattering Amplitude at LO

- Ignore the Coulomb interaction for the moment
- The bubble diagrams can be summed with Lippmann-Schwinger equation

2X2 matrix equation



- **Simple and Analytic** elastic scattering amplitude

$$\begin{pmatrix} \mathcal{A}_{00}(E) & \mathcal{A}_{01}(E) \\ \mathcal{A}_{10}(E) & \mathcal{A}_{11}(E) \end{pmatrix} = \frac{4\pi/M}{-\gamma_u + s_\phi^2\sqrt{M(2\delta - E) - i\varepsilon} - ic_\phi^2\sqrt{ME}} \begin{pmatrix} 2c_\phi^2 & \sqrt{2}c_\phi s_\phi \\ \sqrt{2}c_\phi s_\phi & s_\phi^2 \end{pmatrix}$$

with $s_\phi = \sin(\phi)$ and $c_\phi = \cos(\phi)$

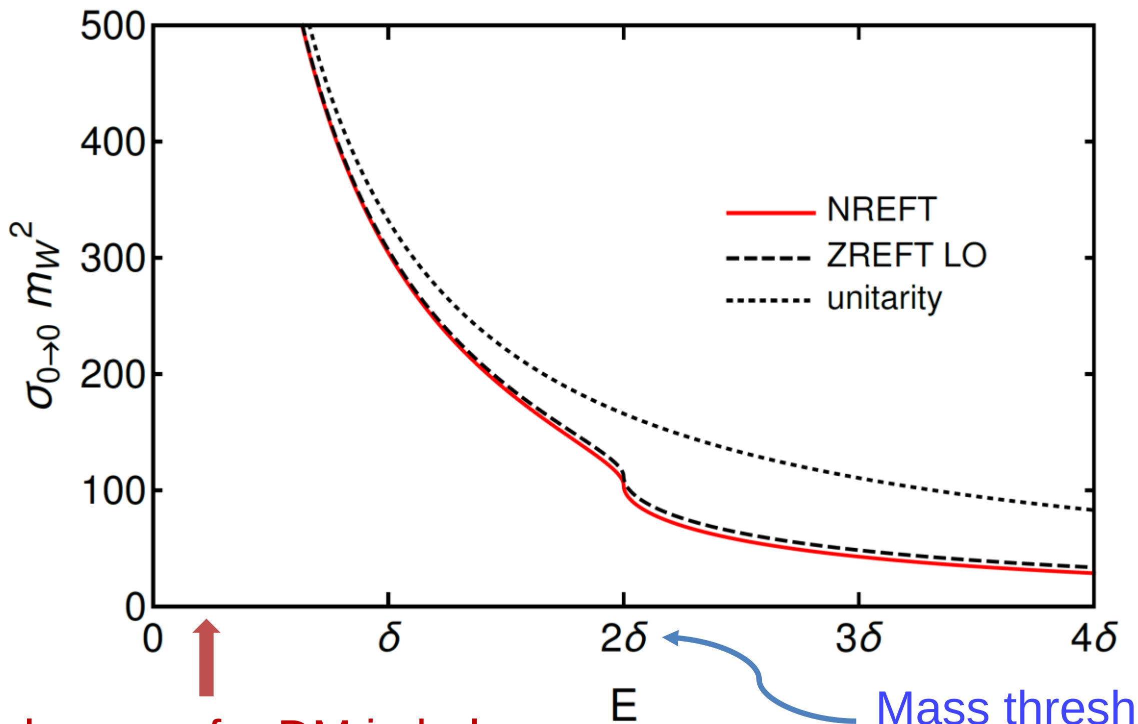
- Values of γ_u and ϕ are fitted by comparing the value and slope of $\mathrm{Re}[1/A_{00}(E)]$ at $E = 0$ with the result from solving Schrödinger equation.

Braaten, Johnson and HZ, 1711 (2017) 108

Comparing LO ZREFT with NREFT

- At first resonant mass $M_* = 2.88$ TeV, we get $\phi = 39.8°$
- $\sigma(E)$ for $\tilde{w}^0\tilde{w}^0 \to \tilde{w}^0\tilde{w}^0$

Braaten, Johnson and HZ, 1711 (2017) 108



$\delta = 170$ MeV

Typical energy for DM in halos

Mass threshold for $\tilde{w}^+\tilde{w}^-$

- ZREFT at LO agrees very well with NREFT below and above the threshold for $\tilde{w}^+\tilde{w}^-$

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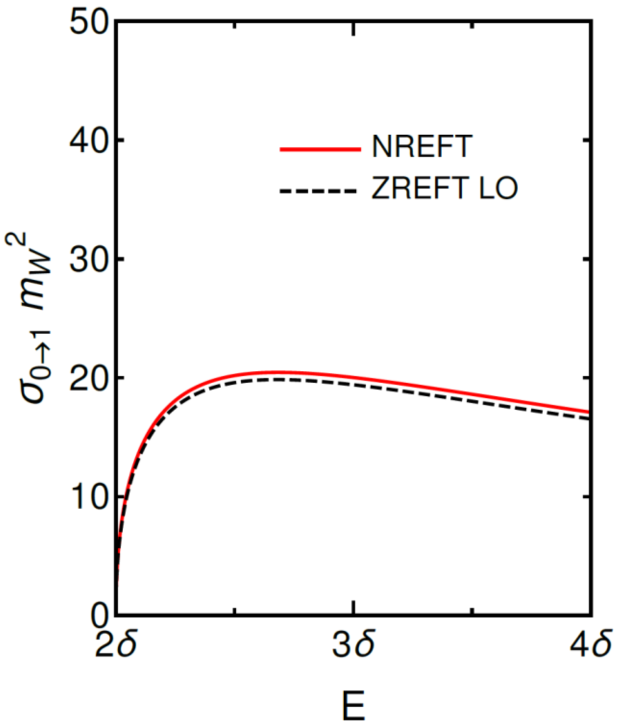
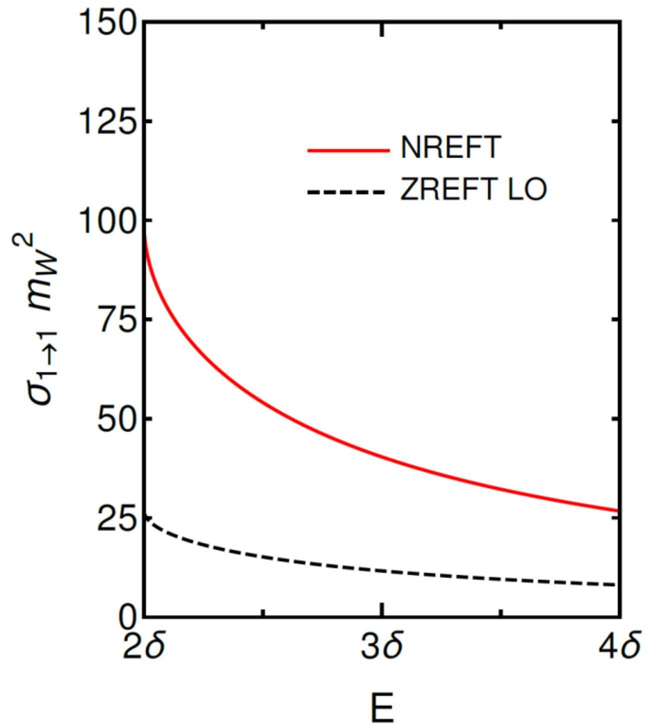
- $\sigma(E)$ for $\tilde{w}^0\tilde{w}^0 \to \tilde{w}^+\tilde{w}^-$



- $\sigma(E)$ for $\tilde{w}^+\tilde{w}^- \to \tilde{w}^+\tilde{w}^-$

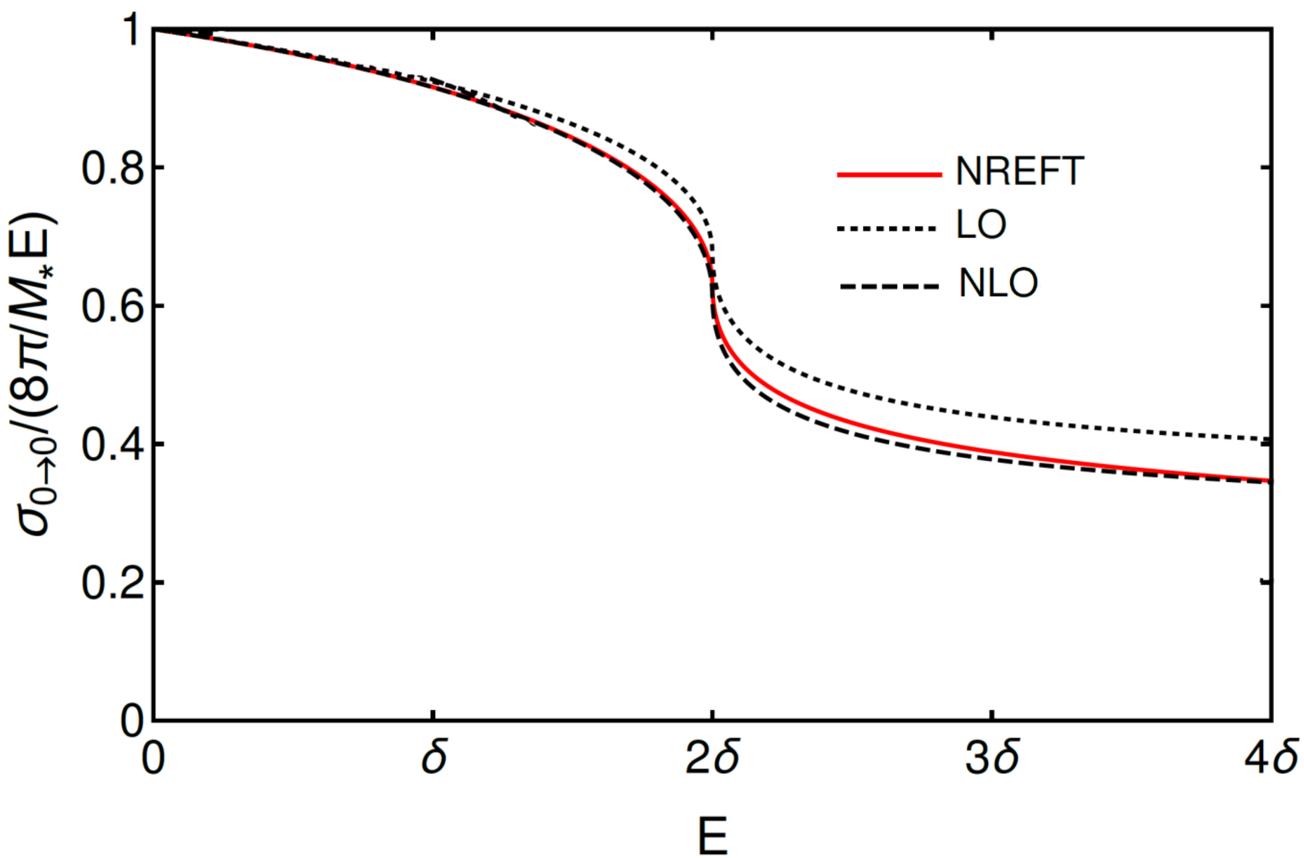


- ZREFT at LO agrees very well with NREFT for $\tilde{w}^0\tilde{w}^0 \to \tilde{w}^+\tilde{w}^-$, but smaller by a factor of three for $\tilde{w}^+\tilde{w}^- \to \tilde{w}^+\tilde{w}^-$.

Braaten, Johnson and HZ, 1711 (2017) 108

Comparing NLO ZREFT with NREFT

- Two more parameters.
- ZREFT at NLO gives improvement in $\sigma(E)$ for $\tilde{w}^0\tilde{w}^0 \to \tilde{w}^0\tilde{w}^0$.



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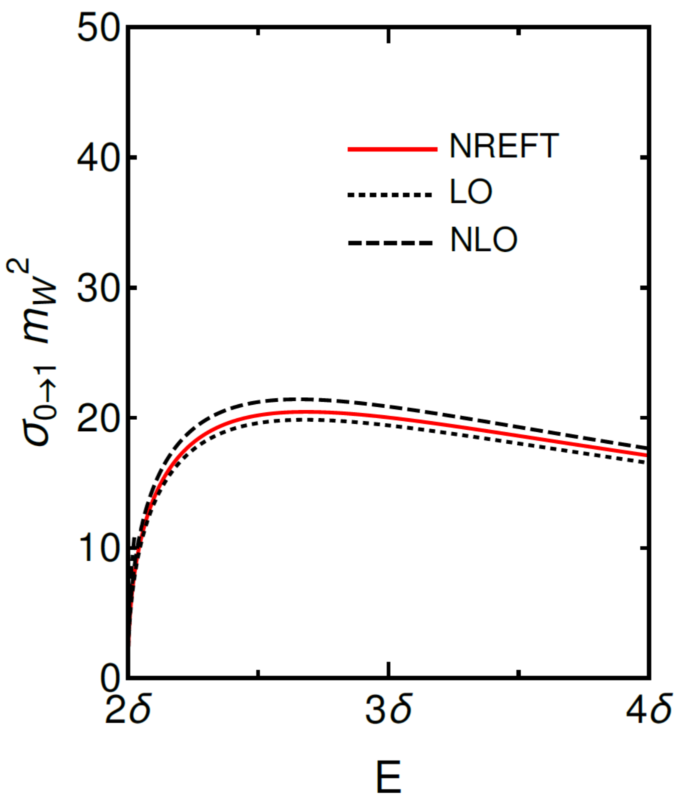
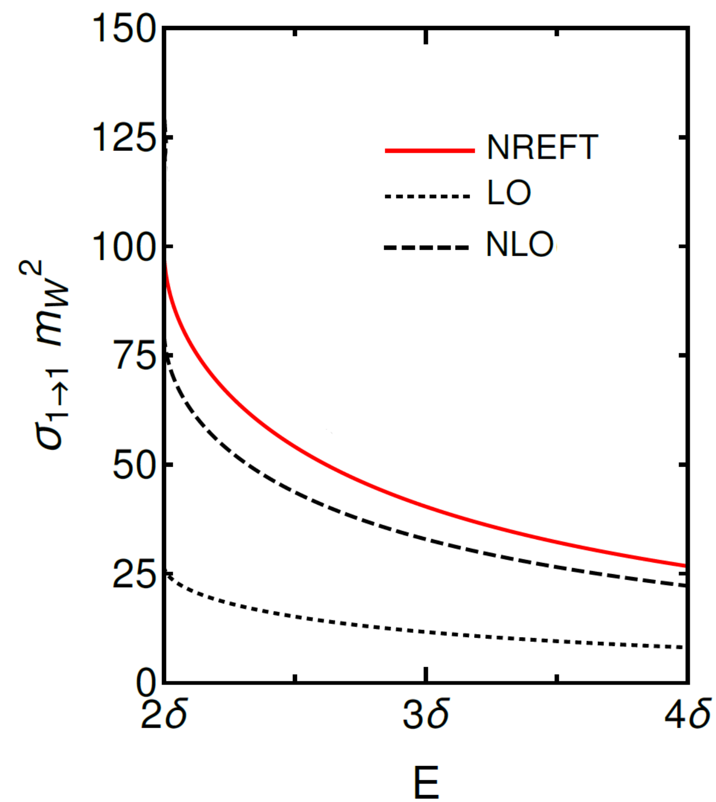
Comparing NLO ZREFT with NREFT

- Two more parameters.

- $\sigma(E)$ for $\tilde{w}^0\tilde{w}^0 \to \tilde{w}^+\tilde{w}^-$



- $\sigma(E)$ for $\tilde{w}^+\tilde{w}^- \to \tilde{w}^+\tilde{w}^-$



- ZREFT at NLO gives significant improvement for $\tilde{w}^+\tilde{w}^- \to \tilde{w}^+\tilde{w}^-$.

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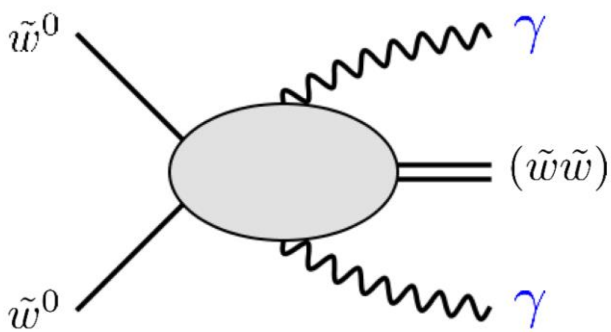
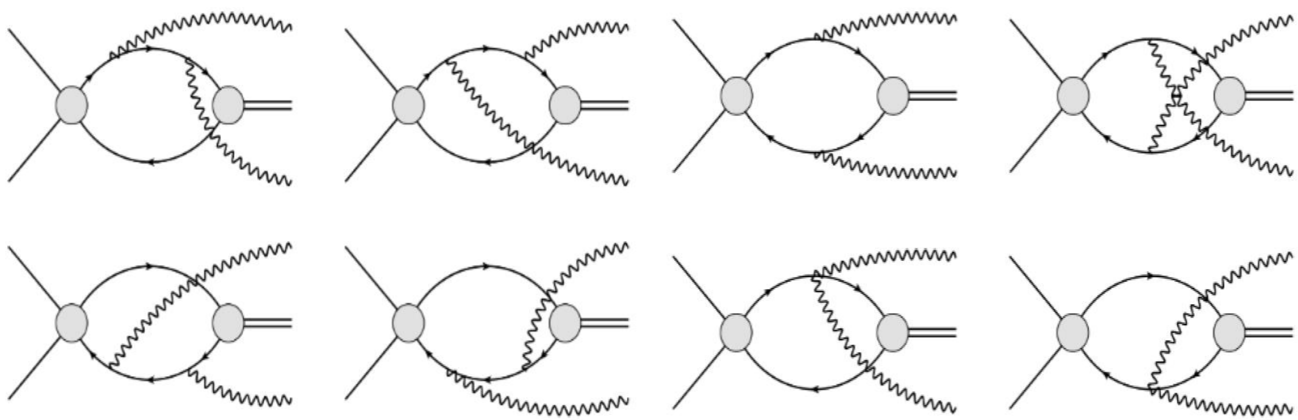
S-wave Bound State Formation

- Form S-wave bound state by emitting two soft photons



- Very complicated calculation with Schrödinger equation
- In ZREFT, only **eight diagrams** to calculate



- Analytical result: $$v\sigma_{\rm ann} \approx \frac{\tan^4\phi\,\alpha^2\,M^2\,\hbar^3}{53760\,a_0\,\delta^5\,c^2}\,(E/Mc^2)^6 \;+\mathcal{O}(1/M^5)$$

Braaten, Johnson and HZ, 1711 (2017) 108

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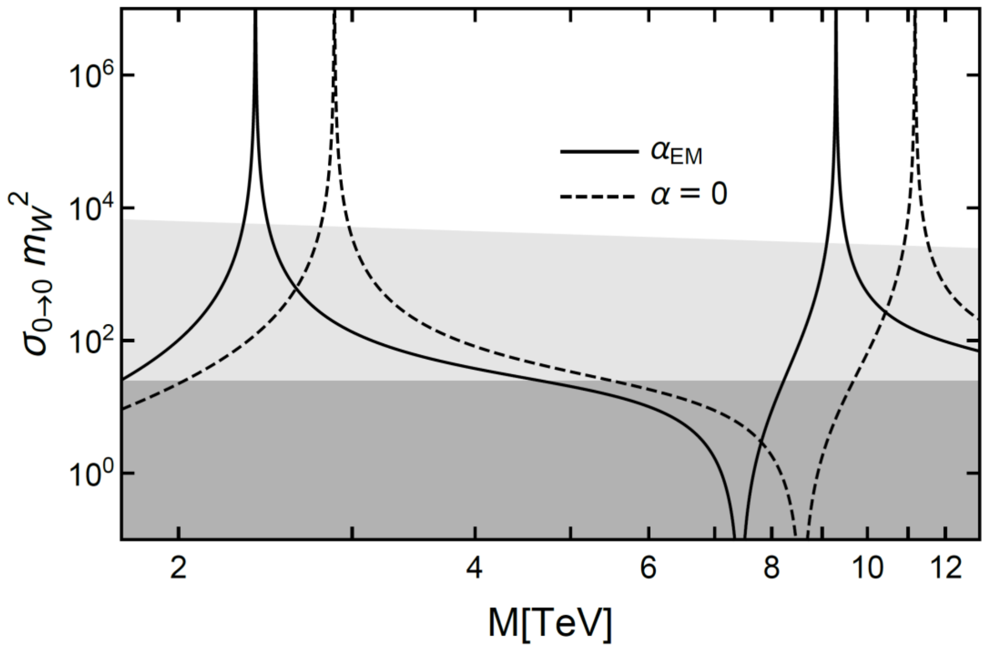
Effect of Coulomb Interaction

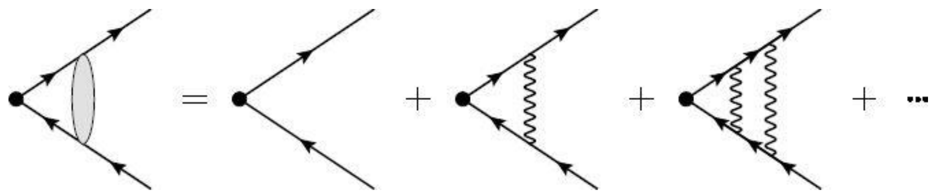
- The photon exchange diagrams needs to be summed to all orders when relative momentum is less than αM
- Extra Coulomb interaction reduces the first resonant mass from $M_* = 2.88$ TeV to $M_* = 2.39$ TeV



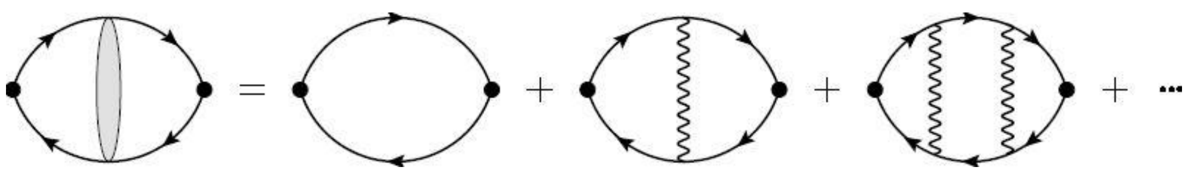
- Coulomb interaction induces infinite number of resonances below $\tilde{w}^+\tilde{w}^-$ threshold (like in the hydrogen atom).

Coulomb Resummation in ZREFT

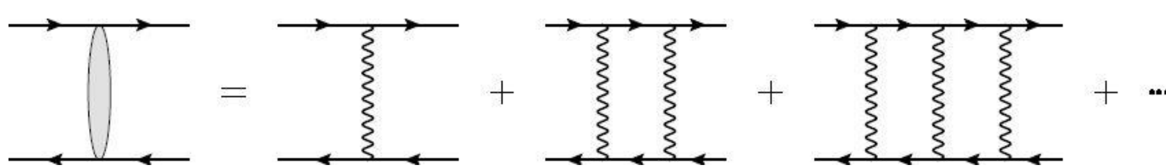
- External charged winos can exchange photons:



- Intermediate changed winos can also exchange photons:



- There is additional pure-Coulomb scattering for charged winos:



Braaten, Johnson and HZ, 1802 (2018) 150

Coulomb Resummation

- No extra parameter (except $\alpha = 1/137$).
- **Analytic** elastic scattering amplitude

Braaten, Johnson and HZ, 1711 (2017) 108

$$\begin{pmatrix} \mathcal{A}_{00}(E) & \mathcal{A}_{01}(E) \\ \mathcal{A}_{10}(E) & \mathcal{A}_{11}(E) \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & \mathcal{A}_C(E) \end{pmatrix} + \frac{4\pi}{M\,L_u} \begin{pmatrix} 1 & 0 \\ 0 & W_1(E) \end{pmatrix} \begin{pmatrix} 2c_\phi^2 & \sqrt{2}c_\phi s_\phi \\ \sqrt{2}c_\phi s_\phi & s_\phi^2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & W_1(E) \end{pmatrix}$$

With $L_u(E) = -\gamma_u + s_\phi^2 K_1(E) - i c_\phi^2 \sqrt{ME}$, $s_\phi = \sin(\phi)$ and $c_\phi = \cos(\phi)$

- $\mathcal{A}_C(E)$ $W_1(E)$ and $K_1(E)$ are analytic functions which are known exactly.

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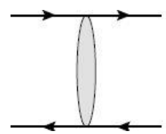
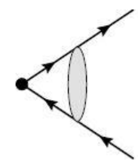
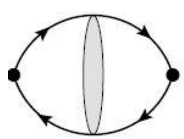




$$\begin{pmatrix} \boldsymbol{\mathcal{A}}_{00}(E) & \boldsymbol{\mathcal{A}}_{01}(E) \\ \boldsymbol{\mathcal{A}}_{10}(E) & \boldsymbol{\mathcal{A}}_{11}(E) \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & \boxed{\mathcal{A}_C(E)} \end{pmatrix} + \frac{4\pi}{M\,L_u} \begin{pmatrix} 1 & 0 \\ 0 & \boxed{W_1(E)} \end{pmatrix} \begin{pmatrix} 2c_\phi^2 & \sqrt{2}c_\phi s_\phi \\ \sqrt{2}c_\phi s_\phi & s_\phi^2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \boxed{W_1(E)} \end{pmatrix}$$

With $L_u(E) = -\gamma_u + s_\phi^2 \boxed{K_1(E)} - ic_\phi^2\sqrt{ME}$, $s_\phi = \sin(\phi)$ and $c_\phi = \cos(\phi)$



- Compare to the result without Coulomb effect

$$\begin{pmatrix} \mathcal{A}_{00}(E) & \mathcal{A}_{01}(E) \\ \mathcal{A}_{10}(E) & \mathcal{A}_{11}(E) \end{pmatrix} = \frac{4\pi/M}{-\gamma_u + s_\phi^2\sqrt{M(2\delta - E) - i\varepsilon} - ic_\phi^2\sqrt{ME}} \begin{pmatrix} 2c_\phi^2 & \sqrt{2}c_\phi s_\phi \\ \sqrt{2}c_\phi s_\phi & s_\phi^2 \end{pmatrix}$$

Comparing LO ZREFT with NREFT

- At first resonant mass $M_* = 2.39$ TeV, we get $\phi = 41°$
- $\sigma(E)$ for $\tilde{w}^0\tilde{w}^0 \to \tilde{w}^0\tilde{w}^0$

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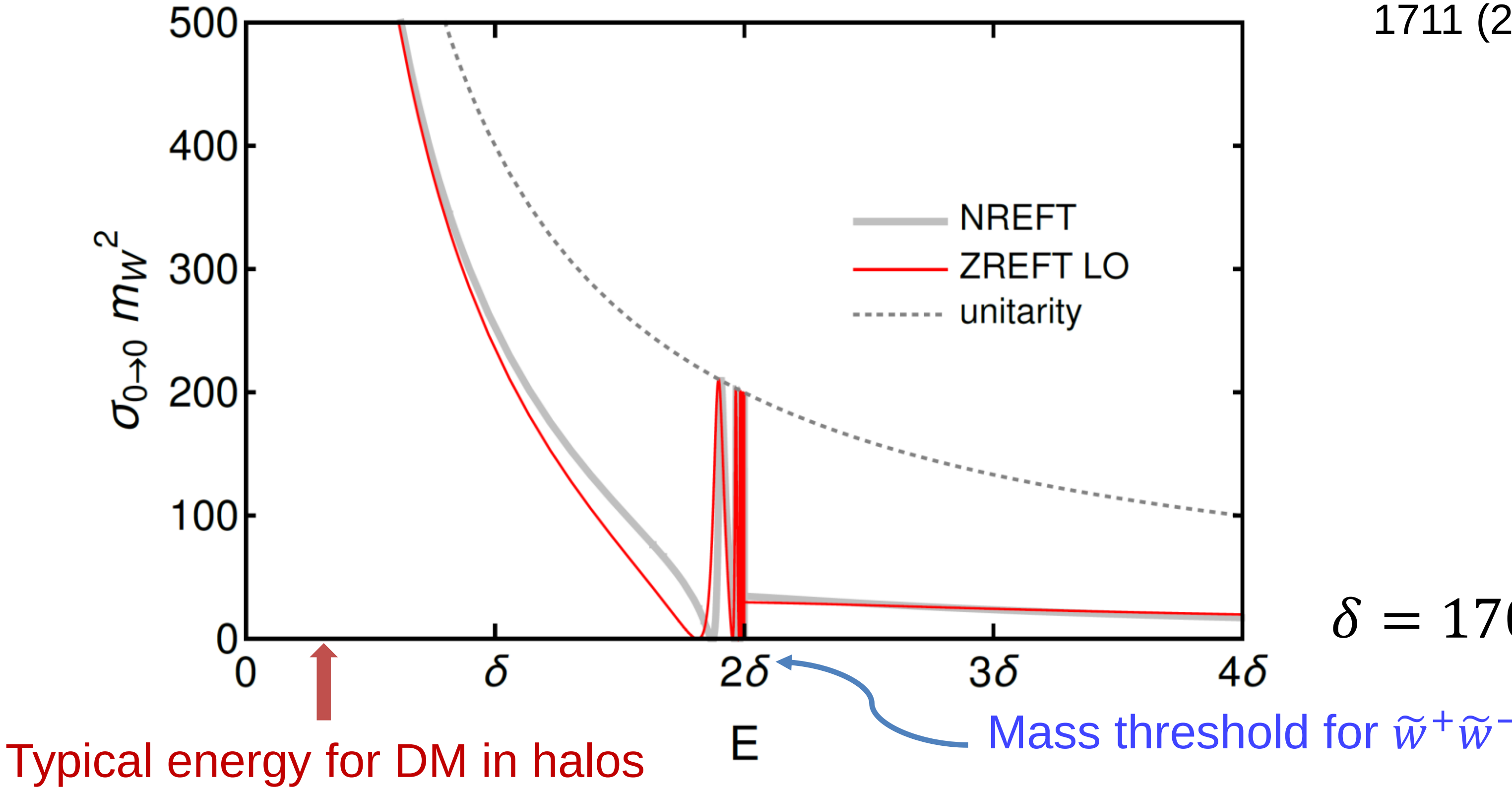

$\delta = 170$ MeV

- ZREFT at LO agrees reasonably well with NREFT below and above the threshold for $\tilde{w}^+\tilde{w}^-$, even in the resonance region.

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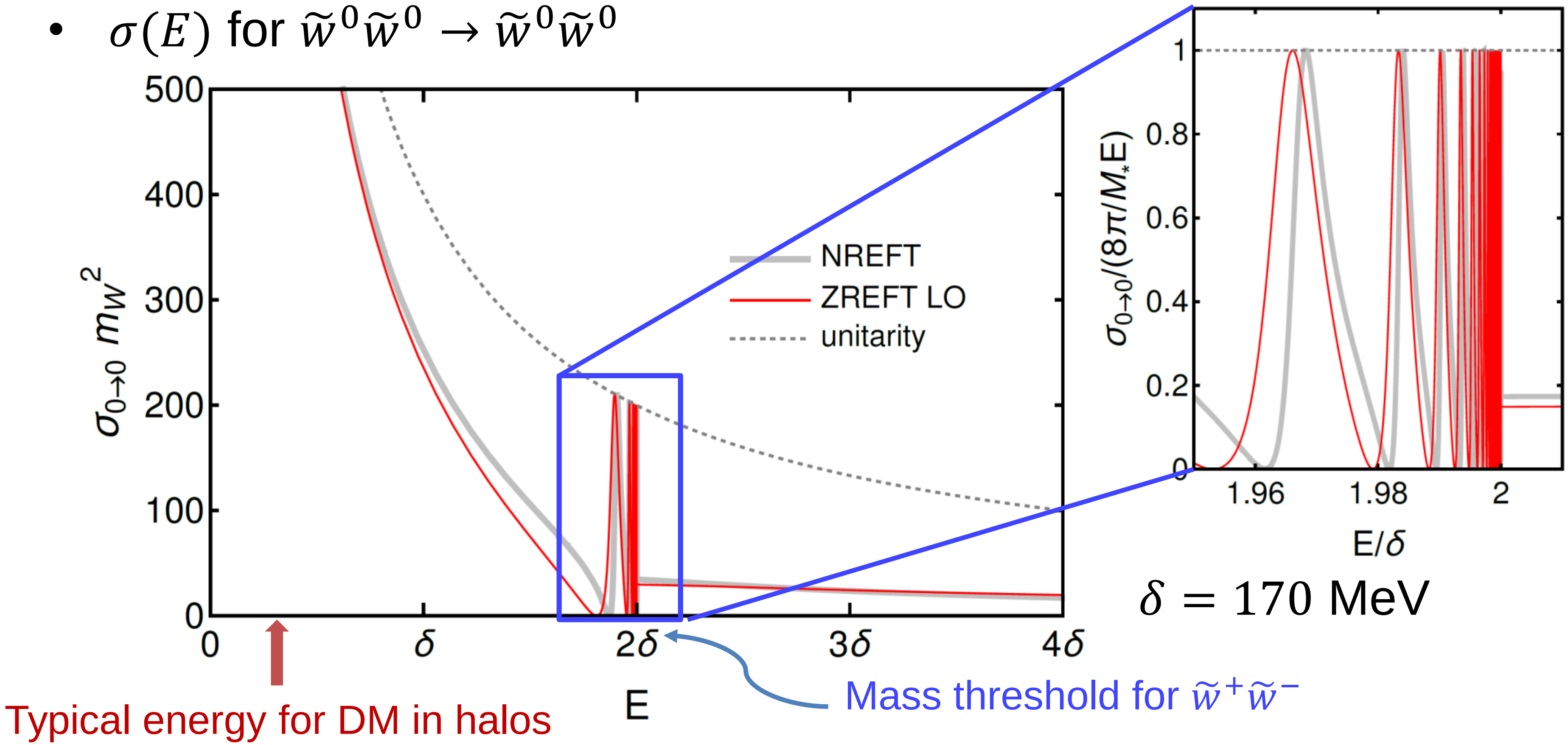

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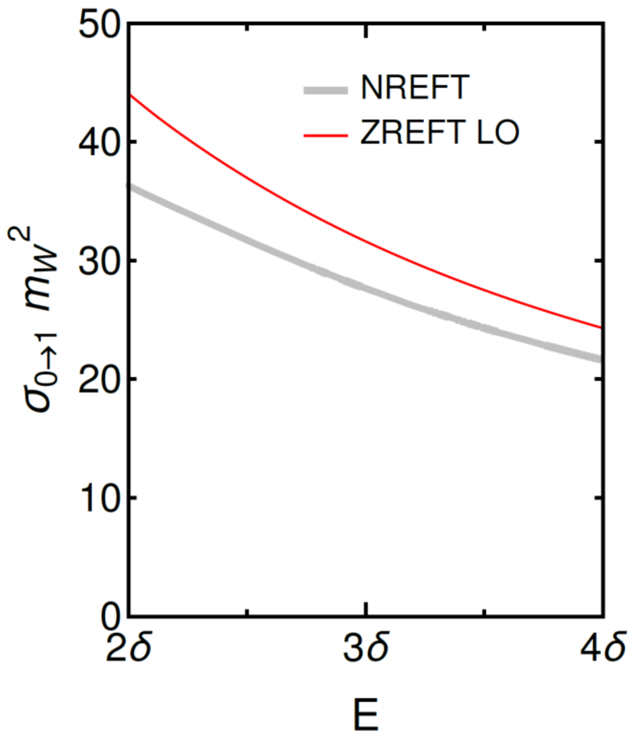
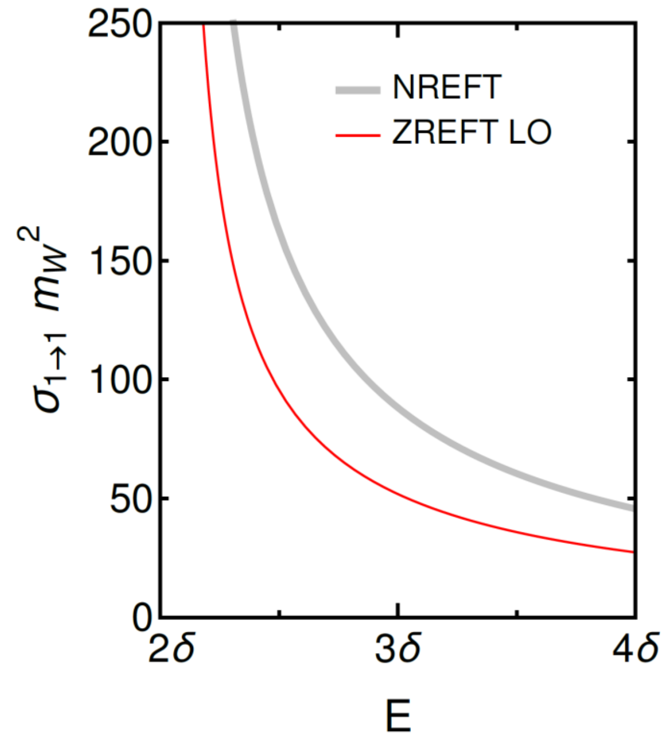


- $\sigma(E)$ for $\tilde{w}^+\tilde{w}^- \to \tilde{w}^+\tilde{w}^-$



- ZREFT at LO agrees reasonably well with NREFT for both $\tilde{w}^0\tilde{w}^0 \to \tilde{w}^+\tilde{w}^-$ and $\tilde{w}^+\tilde{w}^- \to \tilde{w}^+\tilde{w}^-$.

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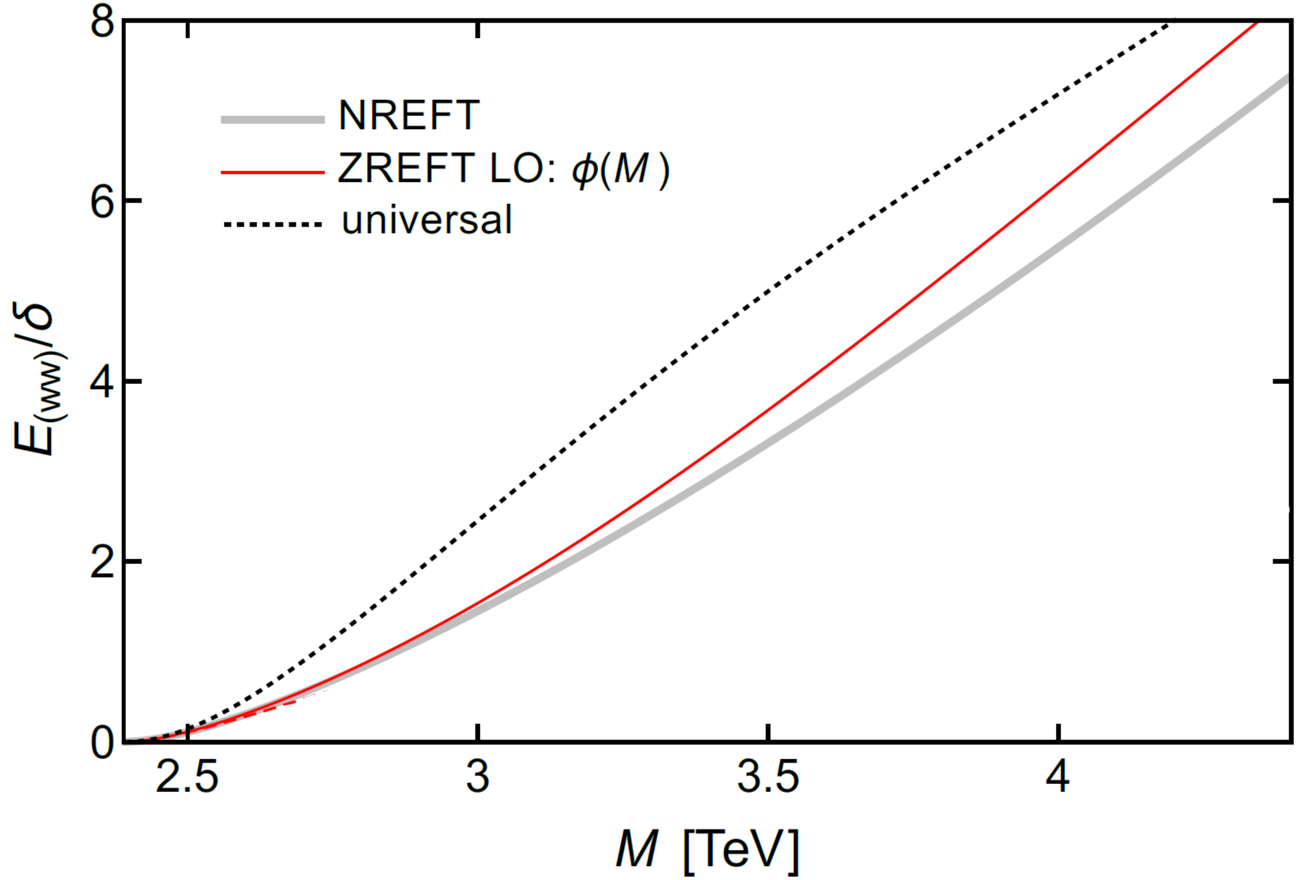
ZREFT for S-wave Bound State

- With ϕ fitted from the scattering process, ZREFT also gives good description of the binding energy.



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Outline

- ✓ Introduction
- ✓ Framework
- ✓ Coulomb Resummation
- Annihilation Effects
- Summary

Wino Annihilation

- The annihilation effect can be included by adding an imaginary part in the Hamiltonian

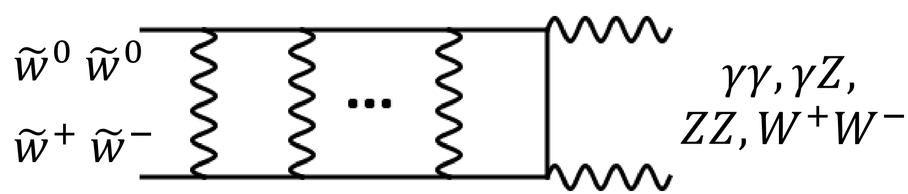
$$\left[-\frac{1}{M}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\left(\frac{d}{dr}\right)^2+\begin{pmatrix}0 & 0\\ 0 & 2\delta\end{pmatrix}+\boldsymbol{V}(r)-i\frac{\delta(r)}{2\pi r^2}\boldsymbol{\Gamma}\right]r\begin{pmatrix}R_0(r)\\ R_1(r)\end{pmatrix}=E\,r\begin{pmatrix}R_0(r)\\ R_1(r)\end{pmatrix}$$

- Real potential for scattering

$$\boldsymbol{V}(r)=-\alpha_2\begin{pmatrix}0 & \sqrt{2}\,e^{-m_W r}/r\\ \sqrt{2}\,e^{-m_W r}/r & c_w^2\,e^{-m_Z r}/r\end{pmatrix}-\alpha\begin{pmatrix}0 & 0\\ 0 & 1/r\end{pmatrix}$$

- **Small** imaginary potential for annihilation into SM particles

$$\boldsymbol{\Gamma}=\frac{\pi\alpha_2^2}{2M^2}\begin{pmatrix}2 & \sqrt{2}\\ \sqrt{2} & 3\end{pmatrix}$$



Annihilation Rate

- The annihilation effect can be included by adding an imaginary part in the Hamiltonian

$$\left[-\frac{1}{M}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\left(\frac{d}{dr}\right)^2+\begin{pmatrix}0 & 0\\ 0 & 2\delta\end{pmatrix}+\boldsymbol{V}(r)-i\frac{\delta(r)}{2\pi r^2}\boldsymbol{\Gamma}\right]r\begin{pmatrix}R_0(r)\\ R_1(r)\end{pmatrix}=E\,r\begin{pmatrix}R_0(r)\\ R_1(r)\end{pmatrix}$$

- Conventional method:

Solve the equation with real potential. Then treat the imaginary part as perturbation.

- ➢ Relatively easy in numerical calculation,
- ➢ Can obtain partial annihilation rate.
- ➢ Violates unitarity in the regions where M is close to the resonant values.

Annihilation Cross Section

- The annihilation effect can be included by adding an imaginary part in the Hamiltonian

$$\left[-\frac{1}{M}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\left(\frac{d}{dr}\right)^2+\begin{pmatrix}0 & 0\\ 0 & 2\delta\end{pmatrix}+\boldsymbol{V}(r)-i\frac{\delta(r)}{2\pi r^2}\boldsymbol{\Gamma}\right]r\begin{pmatrix}R_0(r)\\ R_1(r)\end{pmatrix}=E\,r\begin{pmatrix}R_0(r)\\ R_1(r)\end{pmatrix}$$

- We can also replace the delta function by a narrow Gaussian and solve the full equation carefully.
- The annihilation cross section is obtained using optical theorem ($T_{10}=0$ for $E<2\delta$)

$$\sigma_{0,\mathrm{ann}}(E)=\frac{2\pi}{M^2v_0(E)^2}\left(2\,\mathrm{Im}T_{00}(E)-\left|T_{00}(E)\right|^2-\left|T_{10}(E)\right|^2\right)$$

$$\sigma_{1,\mathrm{ann}}(E)=\frac{\pi}{M^2v_1(E)^2}\left(2\,\mathrm{Im}T_{11}(E)-\left|T_{01}(E)\right|^2-\left|T_{11}(E)\right|^2\right)$$

➢ Does not violate unitarity

➢ Can not obtain partial annihilation rate.

Annihilation Effect in ZREFT

- With annihilation effect, the two parameters γ_u and ϕ in LO ZREFT are **complex**

$$\begin{pmatrix} \mathcal{A}_{00}(E) & \mathcal{A}_{01}(E) \\ \mathcal{A}_{10}(E) & \mathcal{A}_{11}(E) \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & \mathcal{A}_C(E) \end{pmatrix} + \frac{4\pi}{M\,L_0} \begin{pmatrix} 1 & 0 \\ 0 & W_1(E) \end{pmatrix} \begin{pmatrix} 2 & \sqrt{2}t_\phi \\ \sqrt{2}t_\phi & t_\phi^2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & W_1(E) \end{pmatrix}$$

With $L_0(E) = -\gamma_0 + t_\phi^2\left[K_1(E) - K_1(0)\right] - i\sqrt{ME}$ and $t_\phi = \tan(\phi)$

- The values of these two complex parameters are obtained by fitting the (complex) value and the slope of $8\pi/(M\,A_{00}) + i\sqrt{ME}$

$$2v_0\sigma_{0,\mathrm{ann}}(E) = \frac{16\pi/M}{|L_0(E)|^2}\mathrm{Im}\left[\gamma_0 - \left(t_\phi^2 - |t_\phi^2|\right)\left[K_1(E) - K_1(0)\right]\right]$$

$$2v_1\sigma_{1,\mathrm{ann}}(E) = \frac{(8\pi/M)C^2(E)}{|L_0(E)|^2}\mathrm{Im}\left[(t_\phi^2)^*\gamma_0 - \left(t_\phi^2 - |t_\phi^2|\right)^*\kappa_0(E)\right]$$

Braaten, Johnson and HZ, arXiv:1712.07142

Comparing LO ZREFT with NREFT

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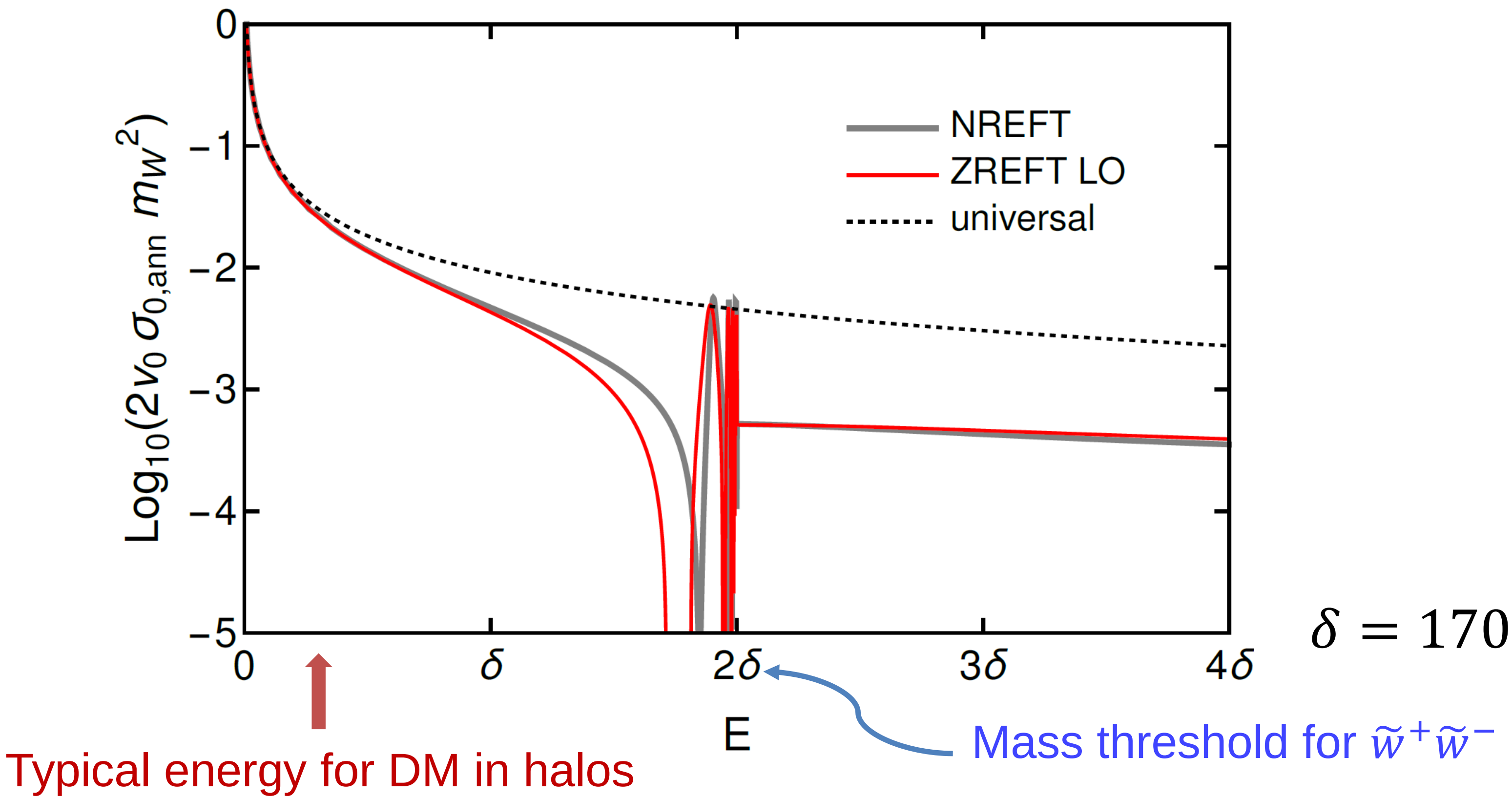
- $\sigma_{ann}(E)$ for $\widetilde{w}^0\widetilde{w}^0 \to SM$



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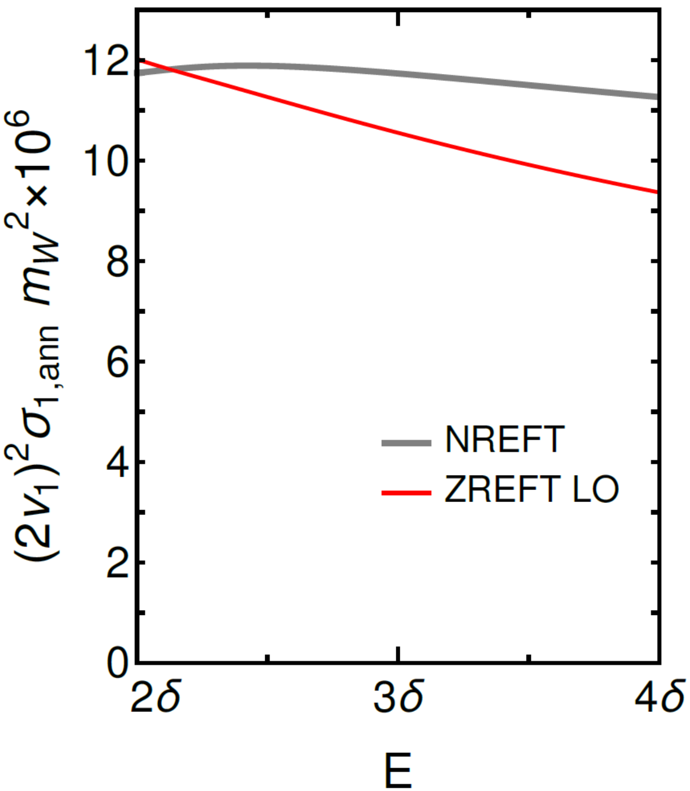
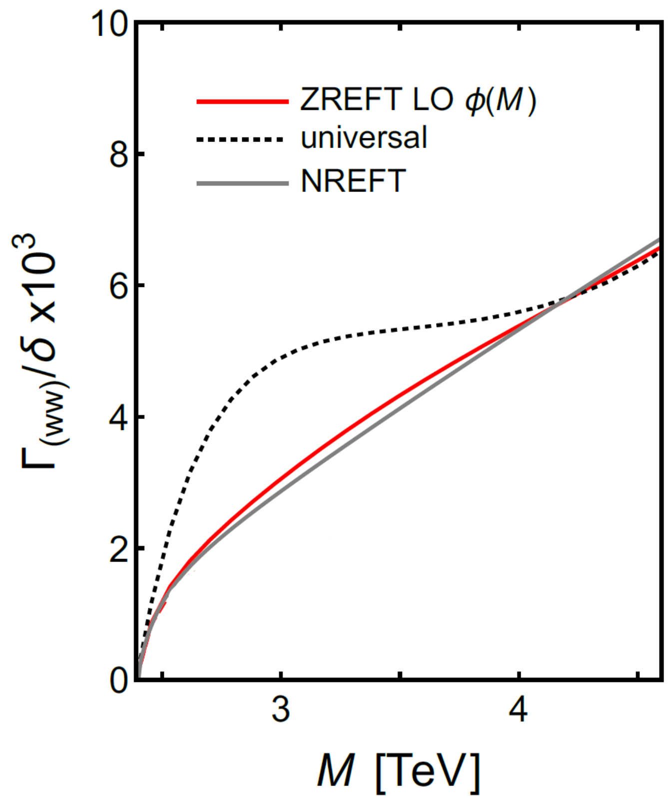


- Bound state width as function of M



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Partial Ann. Rate in ZREFT

- ZREFT provides an elegant way to calculate partial annihilation rate while not violating the unitarity

$$2v_0\sigma_{0,\mathrm{ann}}(E) = \frac{16\pi/M}{|L_0(E)|^2}\mathrm{Im}\left[\gamma_0 - \left(t_\phi^2 - |t_\phi^2|\right)\left[K_1(E) - K_1(0)\right]\right]$$

$$2v_1\sigma_{1,\mathrm{ann}}(E) = \frac{(8\pi/M)C^2(E)}{|L_0(E)|^2}\mathrm{Im}\left[(t_\phi^2)^*\gamma_0 - \left(t_\phi^2 - |t_\phi^2|\right)^*\kappa_0(E)\right]$$

With $L_0(E) = -\gamma_0 + t_\phi^2\left[K_1(E) - K_1(0)\right] - i\sqrt{ME}$ and $t_\phi = \tan(\phi)$

- The partial annihilation rate is obtained by using in L_0 the γ_0 and t_ϕ fitted from solving Schrödinger equation with full imaginary potential, and using in the numerators the γ_0 and t_ϕ fitted from solving Schrödinger equation with partial imaginary potential.

Braaten, Johnson and HZ, arXiv:1712.07142

Summary

- Zero-Range EFT (ZREFT) is well-suited for low-energy dark matter in halos if the dark matter mass is near a S-wave resonance.
- Zero-Range EFT describes low energy wino scattering very well
  - **Two parameters** at LO reproduces the results from conventional method of solving the Schrödinger equation
  - systematically improvable with two more parameters at NLO
  - provides an elegant method to calculate the partial annihilation rate without violating unitarity.
- ZREFT can also be used to calculate more complicated processes, such as multi-body scattering and bound state formation, which are very difficult in conventional method.

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Thank you!