Quarkonium Free Energy on the lattice and in effective field theories

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(**TUMQCD** collaboration)

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Multi-Scale Problems Using Effective Field Theories,
INT-18-1b, Seattle, 05/21/2018

PRD 93 114502 (2016); **arXiv:1804.10600**

In-medium quarkonia from heavy-ion collisions

- Quarkonium as thermometer of QGP T. Matsui, M. Satz, **PL B178** 416 (1986)
- Oversimplified picture – real-time processes are important.
- First-principle calculation are feasible in an EFT framework

Hierarchies of scales for in-medium quarkonia

- Non-relativistic EFTs with non-relativistic hierarchy of scales

$$M \quad \gg \quad Mv \sim p \sim \frac{1}{r} \quad \gg \quad Mv^2 \sim E$$

- Integrate out heavy scales ⇒ NRQCD and pNRQCD
- The thermal medium introduces the thermal scales

$$T \quad \gg \quad gT \quad \gg \quad g^2 T$$

- Suitable for dimensionally reduced thermal EFTs

→ 4-dimensional with one compact direction ⇒ effectively 3-dimensional

- Many different hierarchies between NR and thermal scales are possible

$$p \sim T \quad , \quad p \sim gT \quad , \quad \ldots$$

EFTs for in-medium quarkonia

- Thermal hierarchies are manifest for asymptotically high temperatures

$$T \to \infty \quad \Rightarrow \quad g(T) \to 0 \qquad \text{where} \quad g = \sqrt{4\pi\alpha_s}$$

- Phenomenologically interesting (HIC): $T < 1\,\text{GeV}$: $\alpha_s \approx 0.4, g \sim 2$
  - Is the weak-coupling approach appropriate for phenomenology?
  - Are the postulated hierarchies actually realized and distinguishable?

- We test the different hierarchies and regimes using realistic lattice QCD simulations. We consider heavy quarks in the static limit.
- We aim at establishing whether the EFT descriptions for quarkonium are suitable for (experimentally) relevant temperatures.

Quarkonium Free Energy on the lattice and in effective field theories

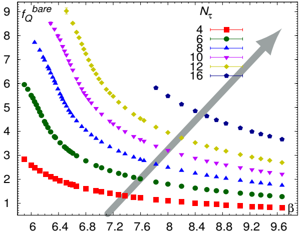
- Overview & Introduction
- Correlators of Polyakov loops and $Q\bar{Q}$ free energy $F_{Q\bar{Q}}$ on the lattice
- Deconfinement and onset of color screening: entropy S_Q
- Comparison to weak-coupling EFTs
- Summary

What is new about the lattices of the TUMQCD study?

$f_Q^{\text{bare}} = -\log \langle P \rangle_T$

Two different volumes and two quark masses:

Controlled **finite volume** and **light quark mass** dependence



TUMQCD collaboration, **arXiv:1604.10600**

Continuum limit with **realistic** quark masses

$N_\tau a(\beta) = 1/T(N_\tau, \beta)$

$T \in [135, 2325]$ MeV

with at least four N_τ

- $N_\tau = 4 - 16$: $12 - 30+$ ens. each, $5.9 \leq \beta \leq 9.67$, $a = 0.0085 - 0.25$ fm.
- HISQ action, errors: $\mathcal{O}(\alpha_s a^2, a^4)$; lattice artefacts are reduced.
- Ensembles: $m_\pi \approx 160$ MeV; $a \geq 0.04$ fm & $m_\pi \approx 320$ MeV; $a \geq 0.025$ fm
- All $N_\tau < 16$, $m_l = \frac{m_s}{5}$: $3 - 5$ ensembles each, $3 - 10 \times 10^4$ **TU** each, $7.03 \leq \beta \leq 8.4$, $a = 0.025 - 0.083$ fm; $T = 0$ lattices available.
  A. Bazavov et al., **PRD 85** 054503 (2012), **PRD 90** 094503 (2014) [HotQCD]
  A. Bazavov et al., **PRD 93** 114502 (2016), **PRD 97** 014510 (2018), arXiv:1804.10600 [TUMQCD]
- r_1 scale for $\beta > 8.4$ from non-perturbative β function **PRD 90** 094503 (2014)

Polyakov loops and free energies of static quark states

- The *Polyakov loop L* is the gauge-invariant expectation value of the traced propagator of a static quark (P) and related to its **free energy**: $L(T) = \langle P \rangle_T = \langle \mathrm{Tr}\, S_Q(x,x) \rangle_T = e^{-F_Q/T}$. L needs renormalization.

  A. M. Polyakov, **PL 72B** (1978); L. McLerran, B. Svetitsky, **PRD 24** (1981)

- The *Polyakov loop correlator* is related to *singlet* & *octet free energies*

  $$C_P(r,T) = e^{-F_{Q\bar{Q}}(r,T)} = \tfrac{1}{9} e^{-F_S/T} + \tfrac{8}{9} e^{-F_A/T} = \tfrac{1}{9} C_S(r,T) + \tfrac{8}{9} C_A(r,T).$$

  S. Nadkarni, **PRD 33, 34** (1986)

- *Singlet* & *octet free energies* are **gauge dependent**.
- C_P is related to the **gauge-invariant free energies** $f_{s,o}$ of **pNRQCD**

  $$C_P(r,T) = e^{-F_{Q\bar{Q}}(r,T)} = \tfrac{1}{9} e^{-f_s/T} + \tfrac{8}{9} e^{-f_o/T} + \mathcal{O}(g^6) \text{ for } rT \ll 1.$$

  N. Brambilla et al., **PRD 82** (2010)

Renormalization of free energies

- **Singlet free energy** and **potential** appear to be related for $rm_D \sim 1$:

$$F_S(r,T) = -C_F\alpha_s\left[\frac{e^{-r\,m_D}}{r} + m_D\right] + \mathcal{O}(g^4) = V_S(r) + \mathcal{O}(g^3).$$

N. Brambilla et al., **PRD 82** (2010)

→ F_S and V_S share the same renormalization $2C_Q$, which depends on T only through the lattice spacing: $V_S = V_S^b + 2C_Q \Rightarrow F_S = F_S^b + 2C_Q$.

- Use V_S at $T=0$: fix r_1 scale & determine $2C_Q$ using **static energy**.

A. Bazavov et al., **PRD 85** 054503 (2012), **PRD 90** 094503 (2014) [HotQCD]

- Cluster decomposition theorem: $F_{Q\bar{Q}} = F_S = 2F_Q$ for $r \gg 1/T$.

→ renormalize as $F_{Q\bar{Q}} = F_{Q\bar{Q}}^b + 2C_Q$ and $F_Q = F_Q^b + C_Q$. → **PRD 93 114502 (2016)**

Beyond $C_Q(\beta)$ from $T=0$ lattices – use **direct renormalization** of F_Q
⇒ Infer unknown $C_Q(\beta)$ from known $C_Q(\beta^{\mathrm{ref}})$ using different N_τ, N_τ^{ref}

$$C_Q(\beta) = \left\{C_Q(\beta^{\mathrm{ref}}) + F_Q^{\mathrm{b}}(\beta^{\mathrm{ref}}, N_\tau^{\mathrm{ref}}) - F_Q^{\mathrm{b}}(\beta, N_\tau)\right\}$$

→ S. Gupta et al., **PRD 77** 034503 (2008)

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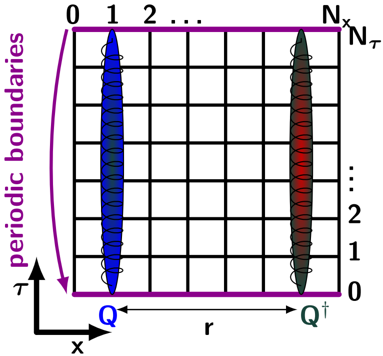
$\Rightarrow$ Infer unknown $C_Q(\beta)$ from known $C_Q(\beta^{\mathrm{ref}})$ using different N_τ, N_τ^{ref}

$$C_Q(\beta) = \left\{C_Q(\beta^{\mathrm{ref}}) + F_Q^{\mathrm{b}}(\beta^{\mathrm{ref}}, N_\tau^{\mathrm{ref}}) - F_Q^{\mathrm{b}}(\beta, N_\tau) + \Delta_{N_\tau, N_\tau^{\mathrm{ref}}}\right\}$$ $\rightarrow$ **PRD 93 114502 (2016)**

Static quark-antiquark correlators



On the lattice **static quarks** are temporal Wilson lines $W = \prod_{\tau/a=1}^{N_\tau} U_0(\tau, \boldsymbol{x})$.

Polyakov loop correlator and $Q\bar{Q}$ free energy

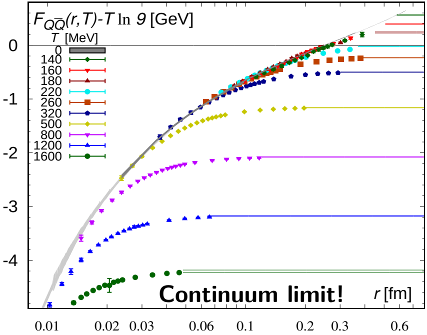
For very small rT and $T \lesssim 500$ MeV

$$F_{Q\bar{Q}} - T\ln 9$$

is very close to the **static energy** V_S of the vacuum.

$\rightarrow$ The relevant **scale hierarchy** is

$$\alpha_s/r \gg T.$$



For large rT and

$$T \gg 200 \text{ MeV}$$

we can explicitly make connection to the **asymptotic** $rT \to \infty$ behavior

$$F_{Q\bar{Q}} \approx 2F_Q.$$

- **Free energy of a $Q\bar{Q}$ pair**, $F_{Q\bar{Q}}$, is also called *color-averaged potential*:

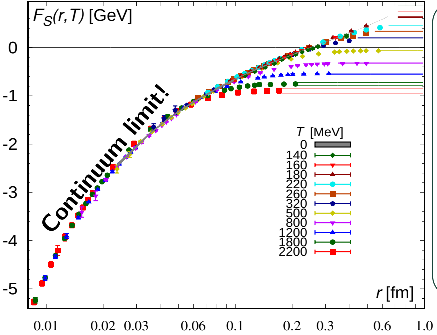
$$C_P(r,T) = \langle P(0)P^\dagger(\boldsymbol{r})\rangle_T^{\text{ren}} = e^{-\frac{F_{Q\bar{Q}}(r,T)}{T}} = \frac{1}{9}e^{-\frac{F_S(r,T)}{T}} + \frac{8}{9}e^{-\frac{F_A(r,T)}{T}}.$$

Static meson correlator and singlet free energy in Coulomb gauge

For $rT \lesssim 0.3$ and $T \lesssim 2.2\,\text{GeV}$ F_S is very close to the **static energy** V_S of the vacuum.

$\rightarrow$ The relevant **scale hierarchy** is

$$\alpha_s/r \gg T.$$



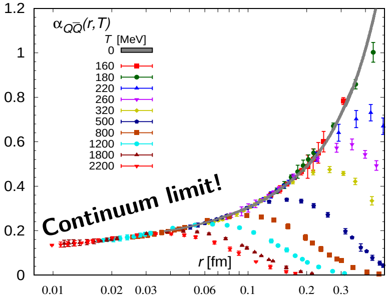
For large rT and $T \gg 200\,\text{MeV}$ we can explicitly make connection to the **asymptotic** $rT \rightarrow \infty$ behavior

$$F_S \approx 2F_Q.$$

- The **singlet free energy** is related to the **gauge-fixed** static meson correlator at $\tau = 1/T$ in **Coulomb gauge**

$$C_S^{\text{ren}}(r,T) = \frac{1}{3}\left\langle \sum_{a=1}^{3} W_a(0) W_a^\dagger(\boldsymbol{r}) \right\rangle_T^{\text{ren}} = e^{-F_S(r,T)/T}.$$

Effective coupling: vacuum-like and screening regimes

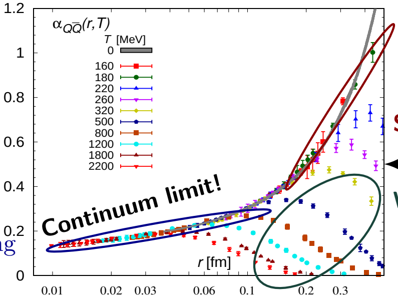


- The **effective coupling** $\alpha_{Q\bar{Q}}(r,T)$ is a suitable proxy for the **force** between the $Q\bar{Q}$ pair and for the QCD coupling α_s running with $1/r$.

$$\alpha_{Q\bar{Q}}(r,T) = \frac{r^2}{C_F}\frac{\partial V_S(r)}{\partial r}$$

- We generalize $\alpha_{Q\bar{Q}}$ with the **singlet free energy** F_S instead of $V_S(r)$.

Effective coupling: vacuum-like and screening regimes



Running coupling (string tension-like)

Strongly-coupled QGP

←max($\alpha_{Q\bar{Q}}$) for 300 MeV

Weakly-coupled QGP

Effective 3d theory (running of $\alpha_{Q\bar{Q}}$ stalls)

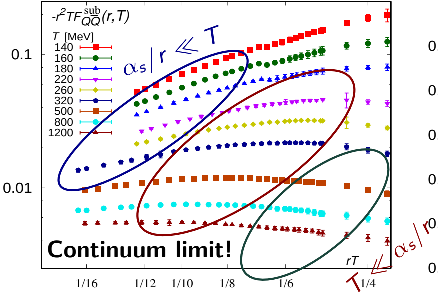
Running coupling (Coulomb-like)

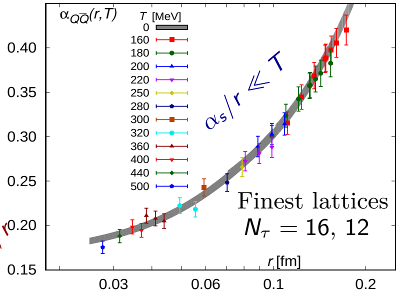
Vacuum-like regime	Screening regime	max($\alpha_{Q\bar{Q}}$)
$rT \lesssim 0.2$	$rT \gtrsim 0.3$	$r_{\max} T \sim 0.4$

- $r_{\max}$ defined through max($\alpha_{Q\bar{Q}}$), which is proxy for the **maximal force**.

→ Weak-coupling approaches may work for $T \gtrsim 300\,\mathrm{MeV}$ ($\alpha_{Q\bar{Q}} \lesssim 0.5$).

Effective coupling: vacuum-like and screening regimes



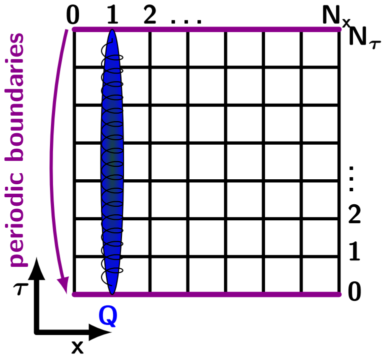


- $F_{Q\bar{Q}}$ has for $rT \ll 1$ two distinct regimes $\alpha_s/r \ll T$ and $T \ll \alpha_s/r$.

Singlet-dominance	Singlet-octet cancellation	Screening regime
$rT \lesssim 0.05, \ldots, 0.15$	$rT \lesssim 0.3$	$rT \gtrsim 0.3$

- We define a running coupling in terms of $F_{Q\bar{Q}}$ in the regime $\alpha_s/r \ll T$.

A single static quark: the Polyakov loop

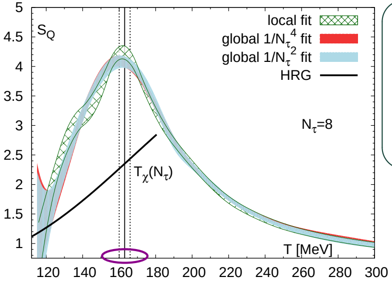


T_χ from chiral observables vs T_S from the peak of the entropy

$F_{Q\bar{Q}}(r \to \infty) = 2F_Q$
$\to$ The **entropy**

$$S_Q = -\frac{dF_Q}{dT}$$

of an isolated quark is independent of the volume and of the renormalization scheme.



Reminder: $aN_\tau = 1/T$
Discretization errors:

$$T_c(N_\tau) = T_c + \mathcal{O}\left(\frac{1}{N_\tau^2}\right)$$

Bazavov et al. [TUMQCD] **PRD 93** 114502 (2016)

- The entropy peaks at $T_S = 153^{+6.5}_{-5}$ MeV in the continuum limit.
- $T_S(N_\tau) \simeq T_\chi(N_\tau)$ for any N_τ Bazavov et al., **PRD 93** 114502 (2016) [TUMQCD], suggests a **tight link between chiral symmetry and deconfinement**.

e.g. as in glueball-sigma mixing scenarios, Y. Hatta, K. Fukushima **PRD 69** 097502 (2004).

N.b. T_χ defined via $O(2)$ scaling of $\chi_{m,l}$ $(O(4)$: $1-3.5$ MeV lower T_χ)

A. Bazavov et al., **PRD 85** 054503 (2012) [HotQCD]

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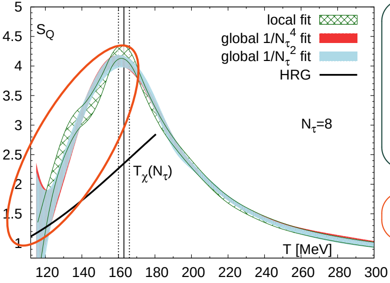

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Bazavov et al. [TUMQCD] **PRD 93** 114502 (2016)

$\frac{dS_Q}{dT} > 0$ for $T < T_c$

- *Hadron resonance gas* (HRG) is limited to only below $T \sim 125$ MeV.

  static HRG results from: A. Bazavov, P. Petreczky, **PRD 87**, 094505 (2013)

- $\frac{dS_Q}{dT} > 0$ for $T < T_c$: the number of bound states of bound states including a static quark increases faster than HRG predictions.
- Large number of **extra states** or **strong thermal modification of (low-lying) states** are needed already substantially below T_c.

T_χ from chiral observables vs T_S from the peak of the entropy

$F_{Q\bar{Q}}(r \to \infty) = 2F_Q$
$\to$ The **entropy**

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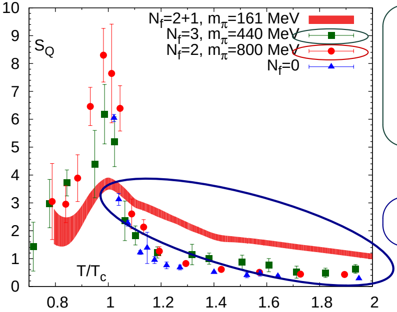
of an isolated quark is independent of the volume and of the renormalization scheme.

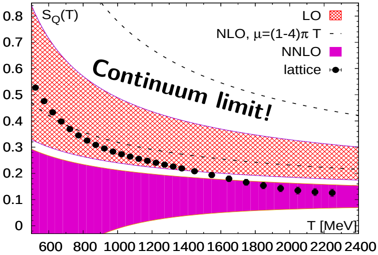


P. Petreczky, K. Petrov
PRD 70 054503 (2004)
O. Kaczmarek, F. Zantow,
hep-lat/0506019 (2005)
Bazavov et al. [TUMQCD]
PRD 93 114502 (2016)

$\frac{dS_Q}{dT} < 0$ for $T > T_c$

- $\frac{dS_Q}{dT} < 0$ for $T > T_c$: the static quark interacts with the **medium only inside its Debye screening radius**, $r \sim 1/m_D \overset{T\to\infty}{\longrightarrow} 0$.
- Deconfinement and **onset of screening** are clearly defined via $S_Q(T_S) = 0$ in the QCD crossover scenario. **MPL A31** no.35, 1630040 (2016)
- The peak is broader and lower for smaller m_{sea} or larger N_f.

Onset of weak coupling in the entropy



- Free energy at leading order $F_Q = -\frac{C_F \alpha_s m_D}{2} + \mathcal{O}(g^4) \overset{m_D \sim gT}{\Rightarrow} S_Q \sim g^3$.

  known to NNLO: M. Berwein, et al., **PRD 93** 034010 (2016)
- Poor convergence of expansion in g – NLO still missing NLO in α_s.
- Continuum results and NNLO agree for $T \gtrsim 10\,T_c$.
- Late onset of weak-coupling behavior: static Matsubara mode is dominant.

  A. Bazavov et al. [TUMQCD] **PRD 93** 114502 (2016)

The vacuum-like regime

- The vacuum-like regime is defined in terms of $rT \ll 1$.
- For $r \ll 1/T$ multipole expansion is appropriate

$\rightarrow$ the appropriate EFT is *pNRQCD*.

- The vacuum-like regime has two sub-regimes:

$$\alpha_s/r \ll T \quad \text{and} \quad \alpha_s/r \gg T$$

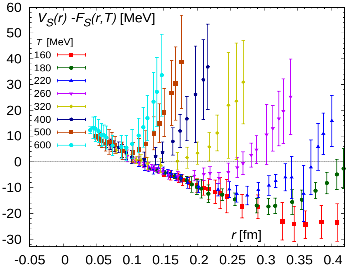
- For $\alpha_s/r \ll T$ weak-coupling calculations are available up to $\mathcal{O}(g^7)$.

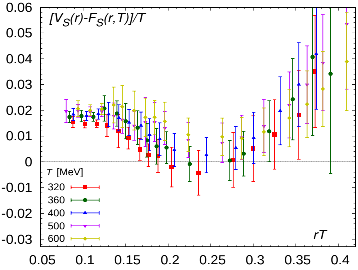
M. Berwein, et al., **PRD 93** 034010 (2016), **PRD 96** 014025 (2017)

- For $\alpha_s/r \gg T$ weak-coupling calculations are not available.
Medium effects are exponentially suppressed as $e^{-(V_o-V_s)/T} \sim e^{-\alpha_s/rT}$.

Brambilla et al., **PRD 78** 014017 (2008)

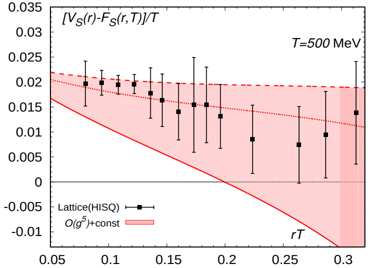
Static energy and singlet free energy on the lattice





- $V_S(T=0) - F_S(T>0)$ up to $\mathcal{O}(\alpha_s^3)$ M. Berwein et al., **PRD 96** 014025 (2017)
- Cancellations in $V_S - F_S$ – smoother for $r/a < 3$, **no renormalization**.
- For $rT \lesssim 0.1$ & $T > 300\,\text{MeV}$: $V_S - F_S \sim 0.02T$, mild N_τ dependence.
- Only mild T dependence up to $rT \lesssim 0.3$.
- For $rT \gtrsim 0.3$ strong medium effects set in rapidly.

Static energy and singlet free energy at weak coupling



Band: NNNLO

Resummation scale

$$\mu = (1-4)\pi T$$

Input QCD scale

$$\Lambda_{\mathrm{QCD}} = 320\,\mathrm{MeV}$$

Bazavov et al., **PRD 90** 074038 (2014)

- Weak-coupling result for hierarchy $\alpha_s/r \ll T$ vanishes for $r \to 0$ as

$$V_S(T=0) - F_S(T>0) \sim \alpha_s^2 rT$$

M. Berwein et al., **PRD 96** 014025 (2017)

- Partial compensations of non-static gluons/quarks by static gluons.
- **Constant term** $\propto \alpha_s^3 T$ in F_S from matching of pNRQCD and NRQCD
- If $\alpha_s/r \gg T \to$ thermal effects exponentially suppressed.

Brambilla et al., **PRD 78** 014017 (2008)

Polyakov loop correlator in pNRQCD

- *pNRQCD*: C_P is given in terms of gauge-invariant **color-singlet** and **color-octet** free energies up to $\mathcal{O}(g^6(rT)^4)$ as N. Brambilla et al., **PRD 82** (2010)

$$C_P(r,T) = e^{-F_{Q\bar{Q}}(r,T)} = \frac{1}{N_c^2} e^{-f_s/T} + \frac{N_c^2-1}{N_c^2} e^{-f_o/T}.$$

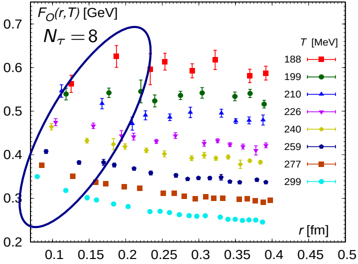
- The decomposition of C_P into gauge-invariant **singlet** and **octet** is defined assuming *weak coupling* – realized for which temperatures?
- For $rT \to 0$ C_P is expressed in terms of **potentials** V_s **and** V_o **at** $T=0$ and of the *adjoint Polyakov loop* L_A at $T>0$ N. Brambilla et al., **PRD 82** (2010)

$$C_P(r,T) = e^{-F_{Q\bar{Q}}(r,T)} = \frac{1}{N_c^2} e^{-V_s/T} + \frac{N_c^2-1}{N_c^2} L_A e^{-V_o/T} + \mathcal{O}(g^6(rT)^0).$$

→ The non-trivial temperature dependence of C_P is mainly due to the interplay and cancellations between **color-singlet** and **color-octet**.

Color octet contribution in the Polyakov loop correlator

repulsive tail



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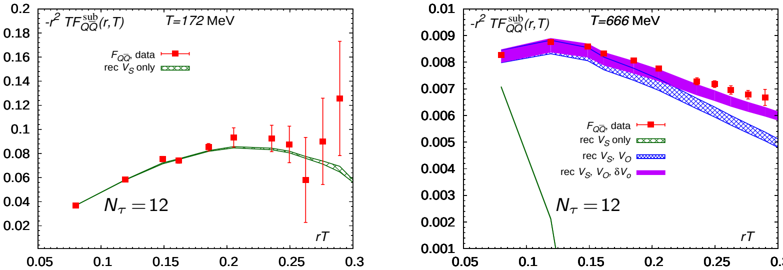
- Use lattice quantities as proxies (static energy V_S for singlet potential V_s) to define an **octet free energy** $e^{-F_O/T} = \frac{9}{8}\left(e^{-F_{Q\bar{Q}}(r,T)} - \frac{1}{9} e^{-V_S/T}\right)$
- F_O decreases rapidly for higher T: the **color-octet** contribution becomes large, the regime $\alpha_s/r \gg T$ is restricted to shorter distances.

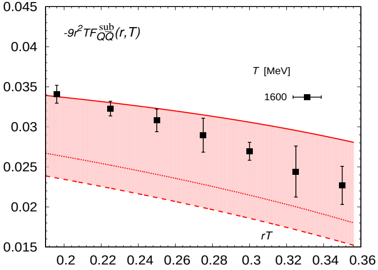
Test of pNRQCD for the Polyakov loop correlator



$$C_P(r,T) = e^{-F_{Q\bar{Q}}(r,T)} = \frac{1}{N_c^2} e^{-V_s/T} + \frac{N_c^2-1}{N_c^2} L_A e^{-V_o/T} + \mathcal{O}(g^6 (rT)^0).$$

- Low $T = 172\,$MeV: **color-singlet, i.e.** V_S, is enough for reconstructing C_P (no sensitivity to **color-octet**). Data are in the regime $\alpha_s/r \gg T$.
- High $T = 666\,$MeV: cancellation between **color-singlet** and **-octet** leads to $1/r^2$ behavior in $F_{Q\bar{Q}}$. Data are in the regime $\alpha_s/r \ll T$.
- **Casimir scaling violation** $8V_o + V_s = 3\frac{\alpha_s^3}{r}[\frac{\pi^2}{4} - 3]$ B. Kniehl et al., **PLB 607** (2005)

Direct comparison for the Polyakov loop correlator



Band: NNNLO

Resummation scale

$\mu = (1-4)\pi T$

Input QCD scale

$\Lambda_{\rm QCD} = 320\,\mathrm{MeV}$

Bazavov et al., **PRD 90** 074038 (2014)

$$C_P(r,T) = e^{-F_{Q\bar{Q}}(r,T)} = \frac{1}{N_c^2} e^{-V_s/T} + \frac{N_c^2-1}{N_c^2} L_A e^{-V_o/T} + \mathcal{O}(g^6(rT)^0).$$

- A recent calculation of C_P at NNNLO using pNRQCD in the regime $\alpha_s/r \ll T$ up to order g^7. M. Berwein et al., arXiv:1704.07266
- Both results agree, although the uncertainty of the weak-coupling result is large even at $T \gtrsim 10\ T_c$.
- For $rT \ll 0.2$ the hierarchy $\alpha_s/r \ll T$ eventually breaks down.

The screening regime

- The screening regime is defined in terms of $r \gtrsim 1/m_D$.
- Hierarchy is automatically built into dimensionally-reduced EFT.

→ The appropriate EFT is *EQCD*.

- The screening regime has two sub-regimes:

$$r \sim 1/m_D \quad \text{and} \quad r \gg 1/m_D$$

- In the electric screening regime, $r \ll 1/m_D$, chromo-electric fields are important. Weak-coupling calculations are available up to $\mathcal{O}(g^5)$.

S. Nadkarni, **PRD 33** (1986)

M. Laine et al., **JHEP 0703** 054 (2007)

M. Berwein, et al., **PRD 96** 014025 (2017)

- In the asymptotic screening regime, $r \gg 1/m_D$, Chromo-magnetic fields are dominant. Non-perturbative methods are required.

M. Laine, M. Vepsalainen, **JHEP 0909** 023 (2009)

Weak-coupling prediction for F_S in the screening regime

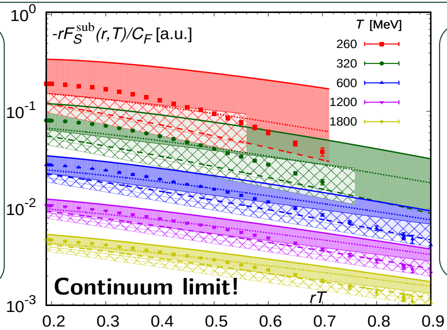
Leading order

$$F_S^{\rm sub}|_{\rm LO} = -\frac{C_F \alpha_s e^{-rm_D}}{r}$$

Using the NLO Debye mass

E. Braaten, A. Nieto

PRD 53 3421 (1996)



Hashed bands: LO
Solid bands: NLO

Resummation scale

$$\mu = (1-4)\pi T$$

Input QCD scale

$$\Lambda_{\rm QCD} = 320\,\text{MeV}$$

Bazavov et al.,

PRD 90 074038 (2014)

- NLO singlet free energy (two-gluon exchange is deferred to NNLO)
  $$F_S^{\rm sub}|_{\rm NLO} = F_S^{\rm sub}|_{\rm LO}\left(1+\alpha_s N_c rT[2-\ln(2x)-\gamma_E+e^{2x}E_1(x)]\right),\ x = 2rm_D$$
- Correction due to field renormalization:
  $$\delta F_S^{\rm sub} = F_S^{\rm sub}|_{\rm LO}\left(1-\frac{rm_D}{2}\delta Z_1\right)$$
  M. Berwein, et al., **PRD 96** 014025 (2017)
- F_S in the electric screening regime is controlled by the parameter m_D.

Weak-coupling prediction for $F_{Q\bar{Q}}$ in the screening regime

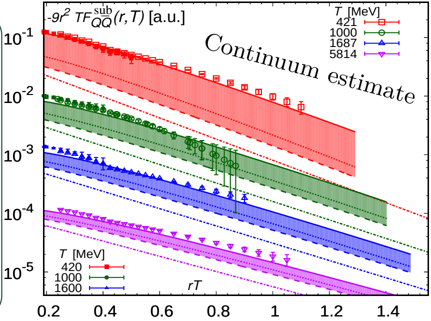
Leading order

$$F^{\rm sub}_{Q\bar{Q}}|_{\rm LO} = -\frac{N_c^2-1}{8} \times \left[\frac{\alpha_s e^{-rm_D}}{N_c rT}\right]^2$$

at scale $\mu = 4\pi T$

Using the NLO Debye mass

E. Braaten, A. Nieto **PRD 53** 3421 (1996)



Dash-dotted line: LO
Solid bands: NLO

Resummation scale

$$\mu = (1-4)\pi T$$

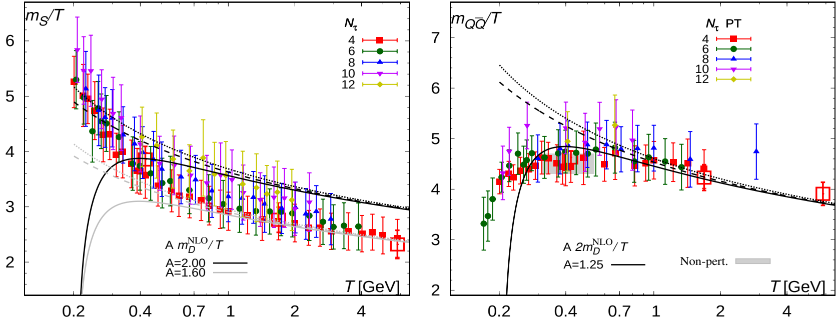
Input QCD scale

$$\Lambda_{\rm QCD} = 320\,{\rm MeV}$$

Bazavov et al., **PRD 90** 074038 (2014)

- Strong signal-to-noise problem → calculation requires larger volumes.
- Use data for $N_\tau = 4$ with (estimated) correction for cutoff effects.
- We compare to the full $\mathcal{O}(g^5)$ result. M. Berwein, et al., **PRD 96** 014025 (2017)
- Previous EFT calculations had been missing important pieces.

Asymptotic screening regime



- Severe signal-to-noise problem → no continuum limit. Cutoff effects are mild for $N_\tau \geq 8$, but require estimates of asymptotic behavior.
- Asymptotic screening mass factor $1.6 - 2$ larger than m_D for F_S
- Asymptotic screening mass only slightly larger than $2m_D$ for $F_{Q\bar{Q}}$
- Good agreement with results from direct EQCD simulations.

A. Hart, et al., **NPB 586** 443 (2000)

Summary

- We study color screening and deconfinement using the renormalized Polyakov loop correlator and related observables.
- We extract the continuum limit of static quark correlators in $N_f = 2{+}1$ QCD up to $T \sim 2\,\mathrm{GeV}$ and down to $r \sim 0.01\,\mathrm{fm}$.
- Static quark correlators are **vacuum-like up to** $rT \lesssim 0.3$ and are well-described by pNRQCD for $T > 300\,\mathrm{MeV}$.
- In C_P we find numerical evidence for the distinction between the regimes of **singlet dominance,** $\alpha_s/r \gg T$, and **singlet-octet cancellaton,** $\alpha_s/r \ll T$**.** For **singlet dominance** we can define an effective coupling.
- Static quark correlators have an **electric screening regime up to** $0.3 \lesssim rT \lesssim 0.6$ and are well-described by EQCD for $T > 300\,\mathrm{MeV}$. The perturbative Debye mass controls this regime.
- We identify in the entropy $S_Q = -\frac{dF_Q}{dT}$ crossover behavior at $T \sim T_c$ and extract $T_S = 153^{+6.5}_{-5}$ MeV from the entropy, in agreement with $T_\chi = 160(6)$ MeV (chiral susceptibilities, O(2) scaling fits, $\frac{m_l}{m_s} = \frac{1}{20}$).
- S_Q becomes weakly coupled only at very high temperatures, $T \gtrsim 10\,T_c$.