Lattice for Quarkonium in Medium

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Introduction: temporal correlators and spectral functions, EFT and potential models

Spatial meson correlation functions:
From light mesons to quarkonia
Bazavov, Karsch, Maezawa, Mukherjee, PP, PRD91 (2015) 054503
PP, Sharma, work in progress

Quarkonium correlators and spectral functions from NRQCD
S. Kim, PP, A. Rothkopf, PRD91 (2015) 054511
S. Kim, PP, A. Rothkopf, work in progress

Summary: a consistent big picture

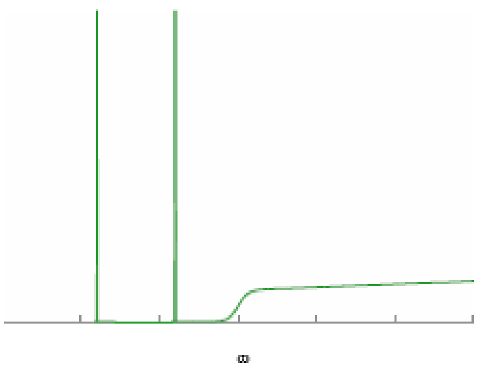
INT Program INT-18-1b , INT, Seattle, May 22, 2018

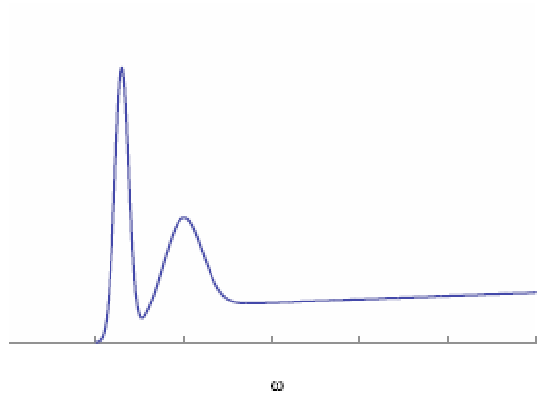
Meson correlators and spectral functions

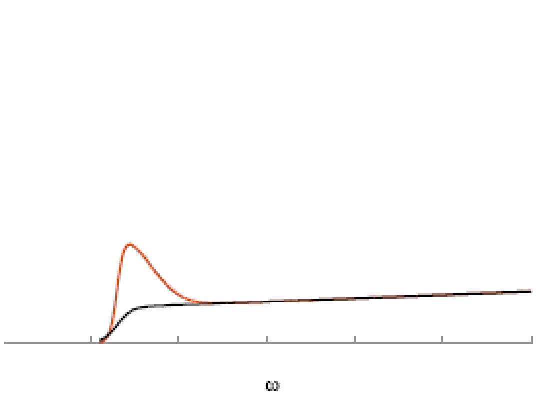
In-medium properties and/or dissolution of mesons are encoded in the spectral functions:

$$\rho(\omega, p, T) = \frac{1}{2\pi}\mathrm{Im}\int_{-\infty}^{\infty} dt e^{i\omega t}\int d^3x e^{ipx}\langle [J(x,t), J(0,0)]\rangle_T$$

Melting is see as progressive broadening and disappearance of the bound state peaks







Due to analytic continuation spectral functions are related to Euclidean time quarkonium correlators that can be calculated on the lattice

$$D(\tau, p, T) = \int d^3x e^{ipx}\langle J(x, -i\tau) J(0,0)\rangle_T$$

$$D(\tau, p, T) = \int_0^{\infty} d\omega \rho(\omega, p, T)\frac{\cosh(\omega \cdot (\tau - 1/(2T)))}{\sinh(\omega/(2T))}$$

MEM ⇒ *σ(ω,p,T)*

1S charmonium survives to *1.6*T_c ??

Umeda et al, EPJ C39S1 (05) 9, Asakawa, Hatsuda, PRL 92 (2004) 01200, Datta, et al, PRD 69 (04) 094507, ...

Temperature dependence of temporal charmonium correlators

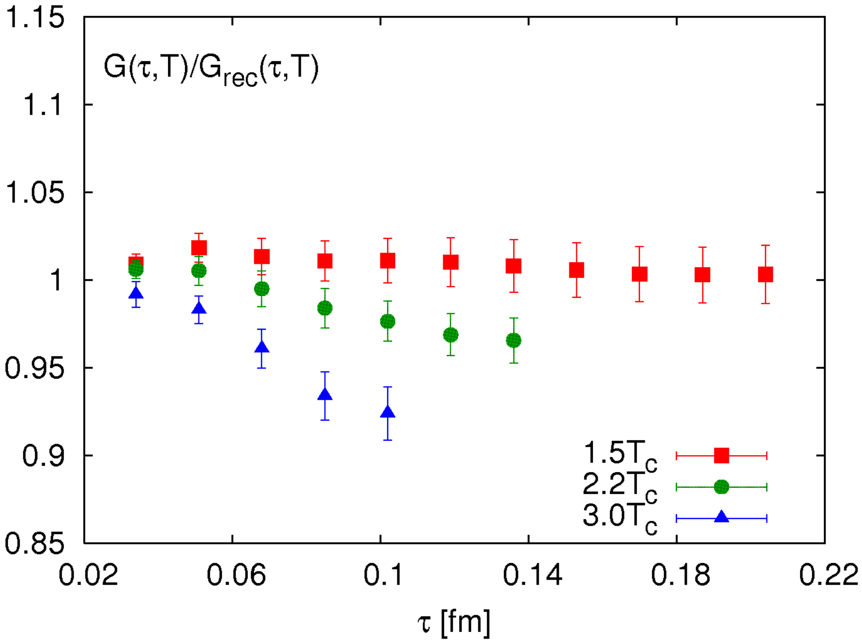
temperature dependene of $D(\tau,T)$

$$D(\tau,T) = \int_0^\infty d\omega \rho(\omega,T) \frac{\cosh(\omega \cdot (\tau - 1/(2T)))}{\sinh(\omega/(2T))}$$

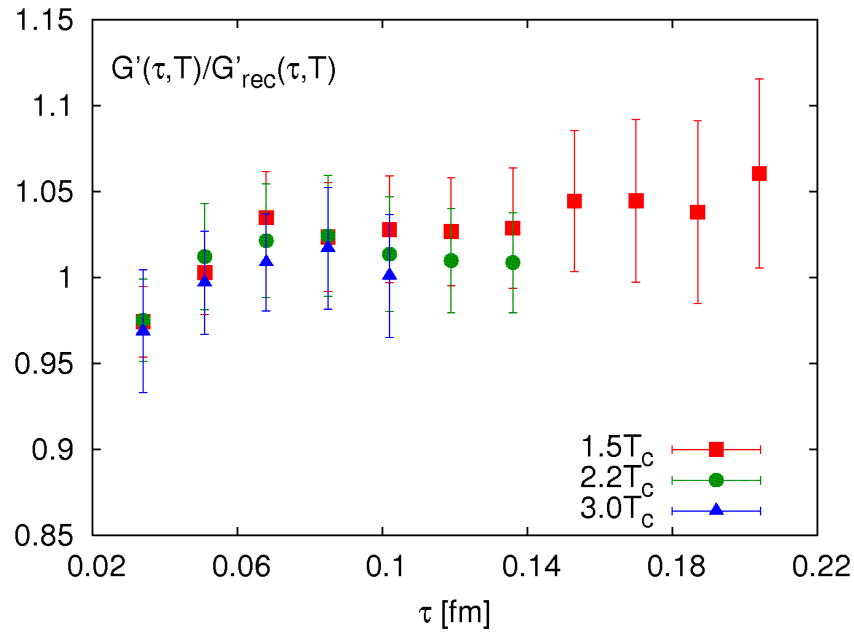
If there is no T-dependence in the spectral function, $D(\tau,T)/D_{rec}(\tau,T)=1$

$$D_{rec}(\tau,T) = \int_0^\infty d\omega \rho(\omega,T=0) \frac{\cosh(\omega \cdot (\tau - 1/(2T)))}{\sinh(\omega/(2T))}$$

Pseudo-scalar $\Leftrightarrow 1S$



Scalar $\Leftrightarrow 1P$

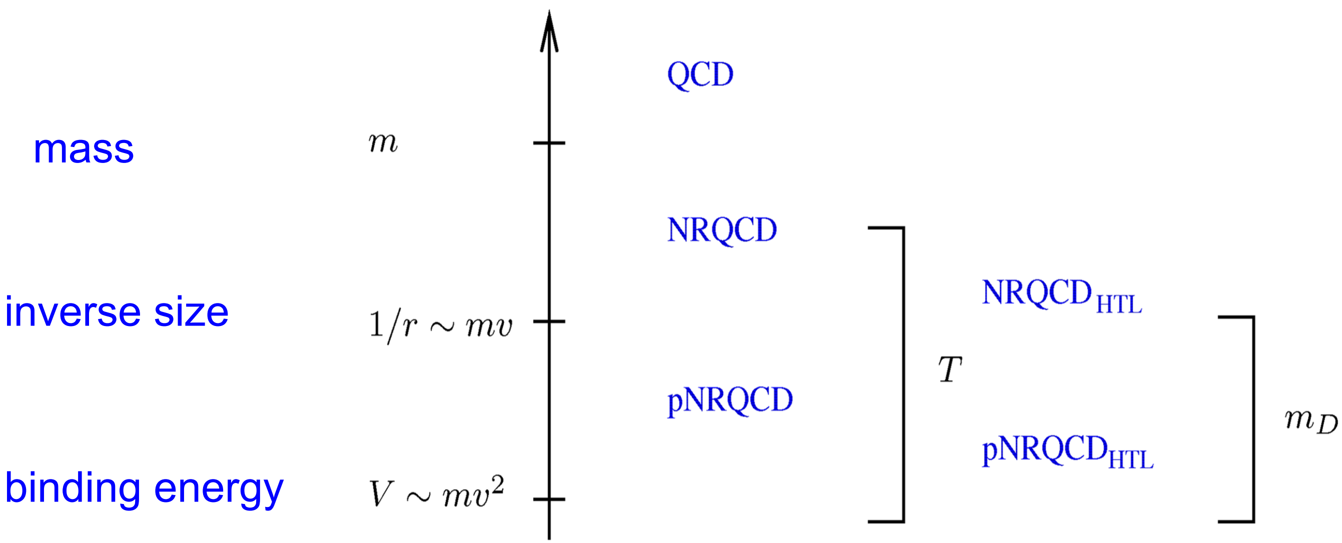


Datta, Karsch, P.P., Wetzorke, PRD 69 (2004) 094507

Effective field theory approach for heavy quark bound states and potential models

The heavy quark mass provides a hierarchy of different energy scales

Brambilla, Ghiglieri, PP, Vairo, PRD 78 (08) 014017



The scale separation allows to construct sequence of effective field theories: NRQCD, pNRQCD

Potential model appears as the tree level approximation of pNRQCD and can be systematically improved in principle
Potential has an imaginary part at *T>0*

Lattice QCD based potential model

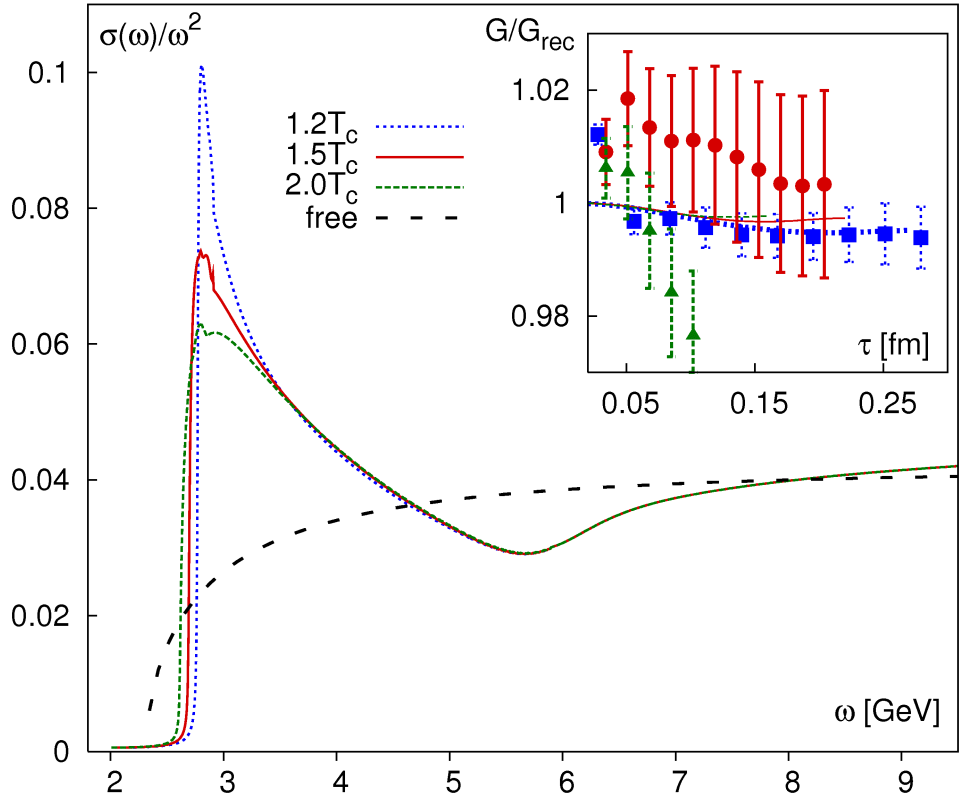
If the octet-singlet interactions due to ultra-soft gluons are neglected :

$$\left[i\partial_0 - \frac{-\nabla^2}{m} - V_s(r,T)\right] S(r,t) = 0 \quad \Rightarrow \quad \sigma(\omega, T) \quad \text{(justified if e.g. } mv^2 \ll T, gT, \Lambda_{QCD})$$

potential model is not a model but the tree level approximation of corresponding EFT that can be systematically improved

Test the approach vs. LQCD : quenched approximation, $F_1(r,T) < \mathrm{Re}V_s(r,T) < U_1(r,T)$, $\mathrm{Im}V(r,T) \approx 0$

Mócsy, P.P., PRL 99 (07) 211602, PRD77 (08) 014501, EPJC ST 155 (08) 101



- resonance-like structures disappear already by *1.2T$_c$*
- strong threshold enhancement above free case

  => indication of correlations

- The correlators do not change significantly despite the melting of the bound states => it is difficult to distinguish bound state from threshold enhancement in lattice QCD

The role of the imaginary part for charmonium

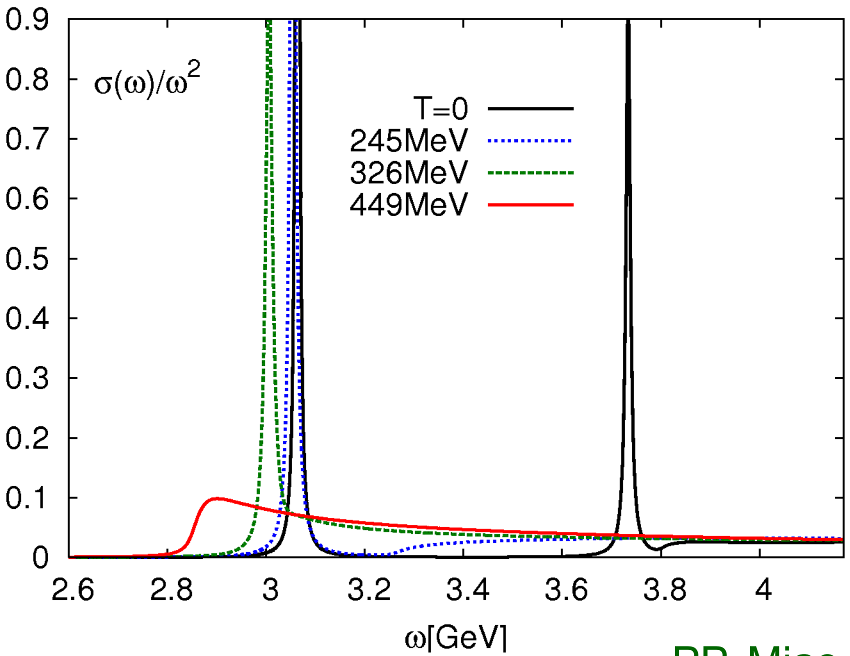
Take the upper limit for the real part of the potential allowed by lattice calculations

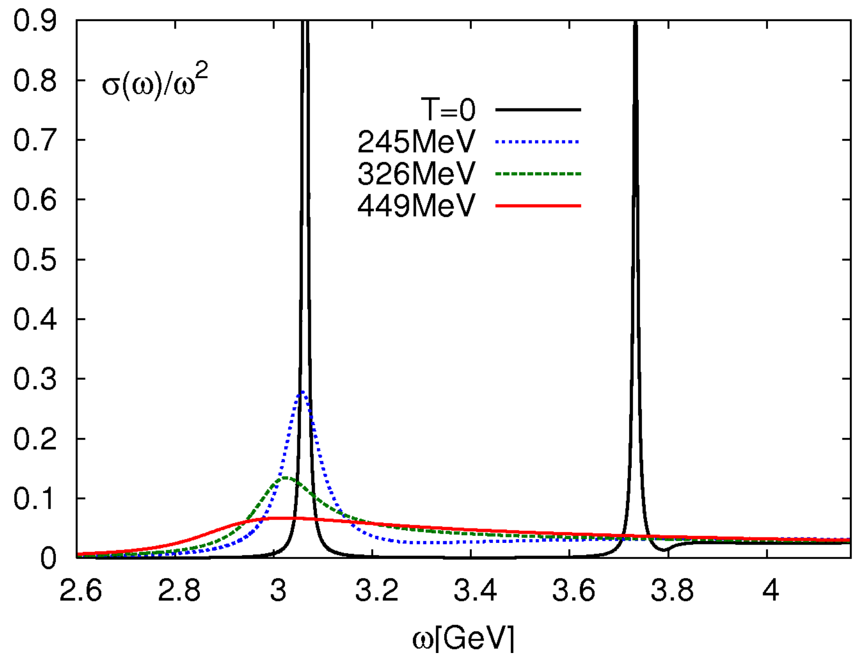
Mócsy, P.P., PRL 99 (07) 211602,

Take the perturbative imaginary part
Burnier, Laine, Vepsalainen JHEP 0801 (08) 043

Im $V_s(r) = 0$:
1S state survives for $T = 330$ MeV

imaginary part of $V_s(r)$ is included :
all states dissolves for $T>245$ MeV





PP, Miao, Mócsy, NPA855 (2011) 125

no charmonium state could survive for $T> 245$ MeV
this is consistent with our earlier analysis of Mócsy, PP, PRL 99 (07) 211602,
Riek and Rapp, arXiv:1012.0019 [nucl-th]

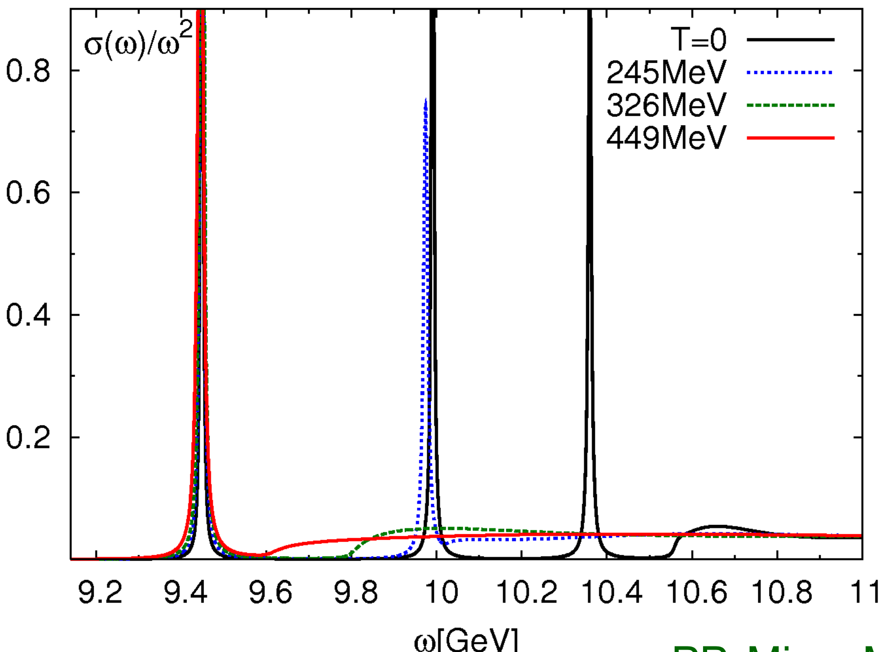
The role of the imaginary part for bottomonium

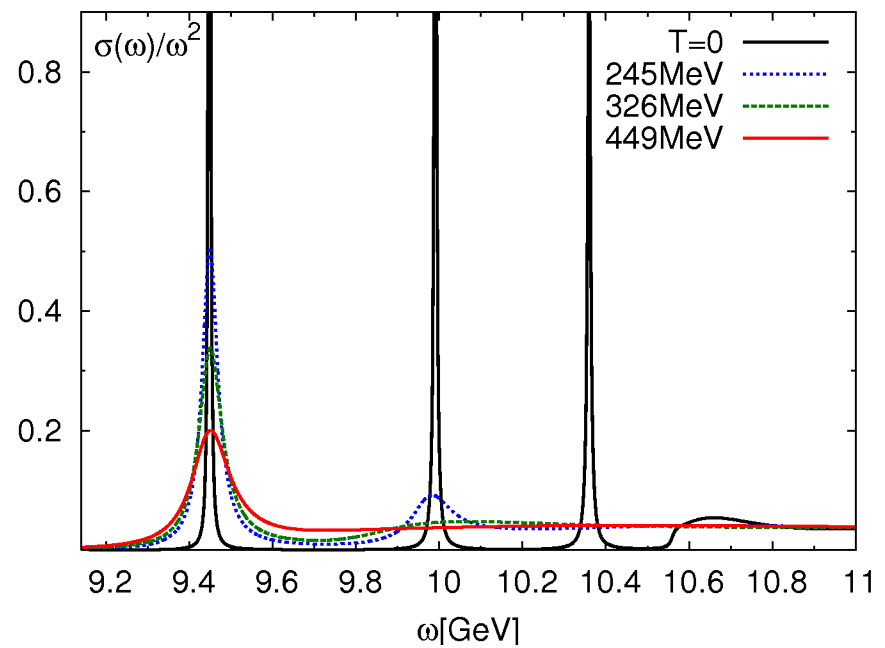
Take the upper limit for the real part of the potential allowed by lattice calculations
Mócsy, PP, PRL 99 (07) 211602,

Take the perturbative imaginary part
Burnier, Laine, Vepsalainen JHEP 0801 (08) 043

Im $V_s(r)$ =0:
2S state survives for *T > 245* MeV
1S state could survive for *T>450* MeV

with imaginary part:
2S state dissolves for *T>245* MeV
1S states dissolves for *T>450* MeV





PP, Miao, Mócsy, NPA855 (2011) 125

Excited bottomonium states melt for *T >245* MeV ; 1S state melts for *T >450* MeV

Use more realistic *T>0* potential calculated on the lattice,
Burnier, Kaczmarek, Rothkopf, PRL 114 (2015) 082001, JHEP12 (2015) 101

Temporal vs spatial meson correlators

Spatial correlation functions can be calculated for arbitrarily large separations $z \to \infty$

$$G(z,T) = \int_0^{1/T} d\tau \int dxdy \langle J(\mathbf{x},-i\tau)J(\mathbf{0},0)\rangle_T, \; G(z\to\infty,T) = Ae^{-m_{scr}(T)z}$$

but related to the same spectral functions $G(z,T) = 2\int_{-\infty}^{\infty} dp e^{ipz} \int_0^{\infty} d\omega \frac{\rho(\omega,p,T)}{\omega}$

Low T limit :

$$\sigma(\omega,p,T) \simeq A_{mes}\delta(\omega^2 - p^2 - M_{mes}^2)$$

$$A_{mes} \sim |\psi(0)|^2 \to m_{scr}(T) = M_{mes}$$

$$G(z,T) \simeq |\psi(0)|^2 e^{-M_{mes}(T)z}$$

High T limit :

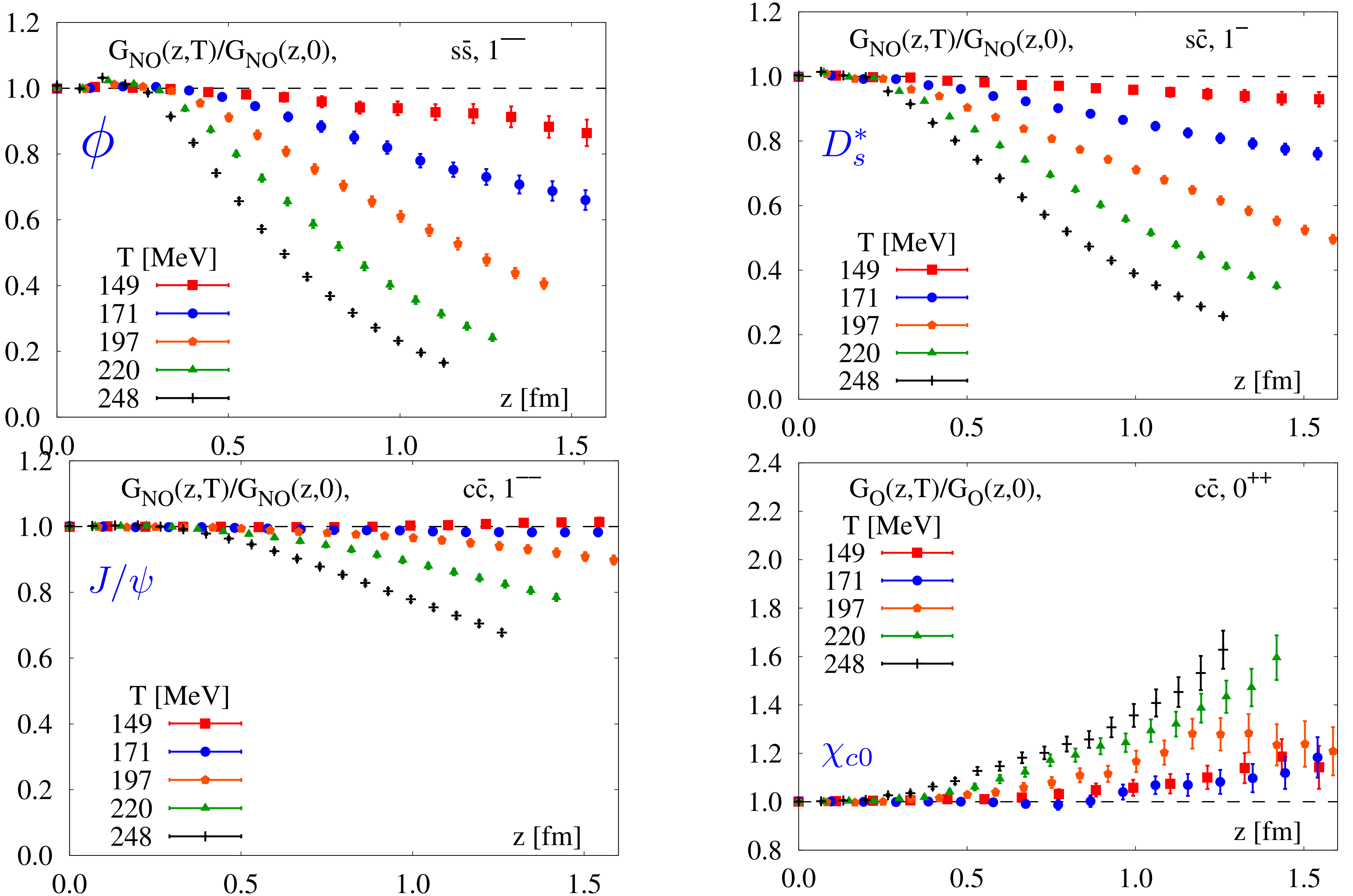
$$m_{scr}(T) \simeq 2\sqrt{m_c^2 + (\pi T)^2}$$

Temporal meson correlator only avalaible for $\tau T < ½$ and thus may not be very sensitive to In-medium modifications of the spectral functions; also require large N_τ (difficult in full QCD)

Spatial correlators can be studied for arbitrarily large separations and thus are more sensitive to the changes in the meson spectral functions; do not require large N_τ (easy in full QCD).

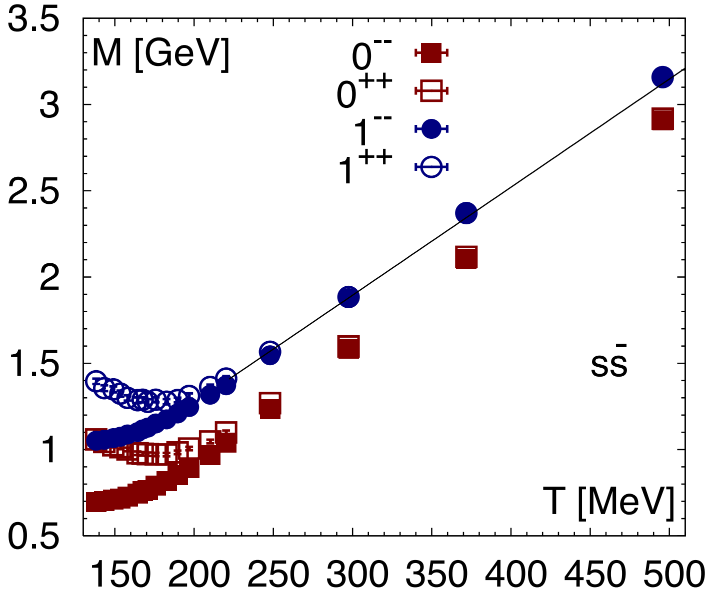
Lattice calculations: spatial meson correlators in 2+1 flavor QCD for ssbar, scbar and ccbar sectors using 48^3x12 lattices and highly improved staggered quark (HISQ) action (HotQCD) , physical m_s and $m_\pi=161$ MeV.

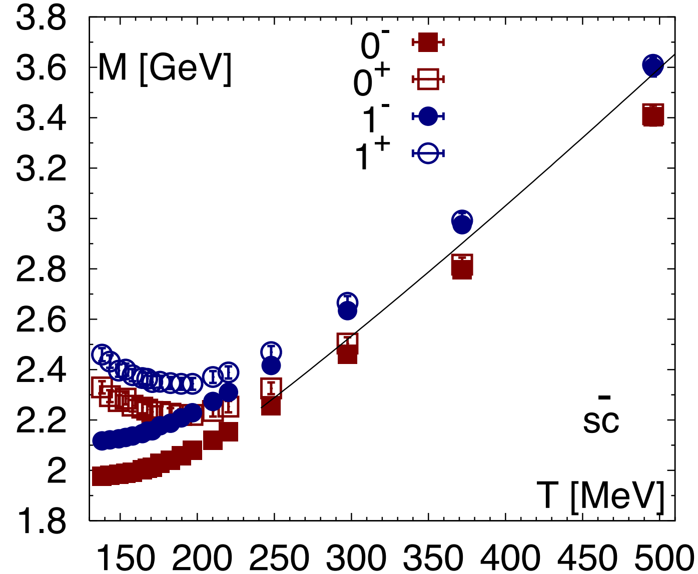
Temperature dependence of spatial meson correlators

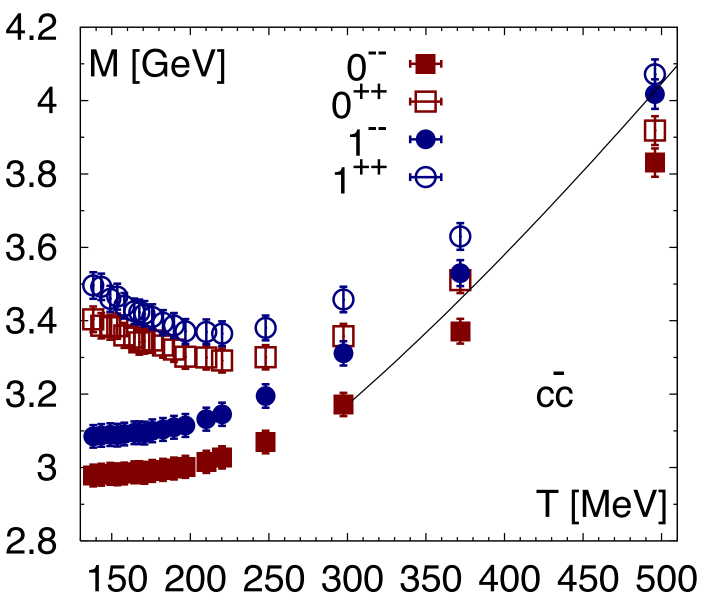


Medium modifications of meson correlators increase with *T*, but decrease with heavy quark content; larger for *1P* charmonium state than for *1S* charmonium state

Temperature dependence of meson screening masses





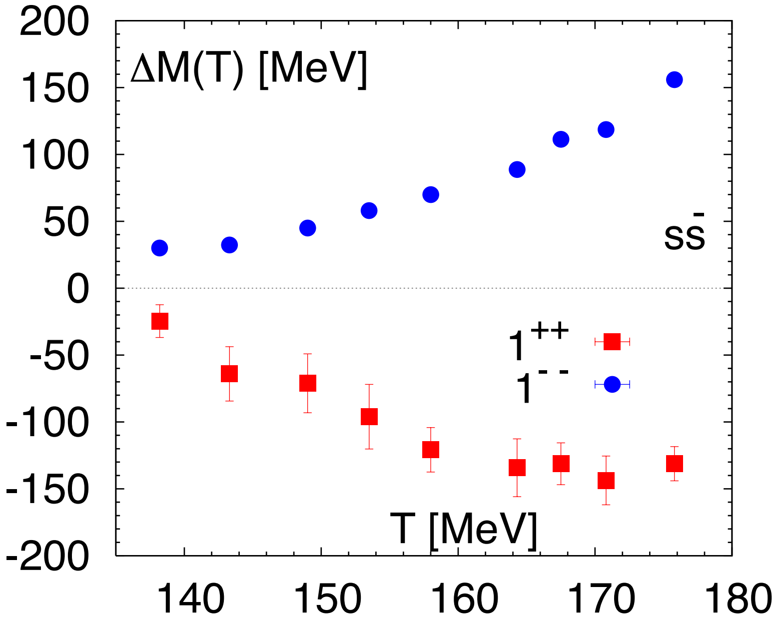


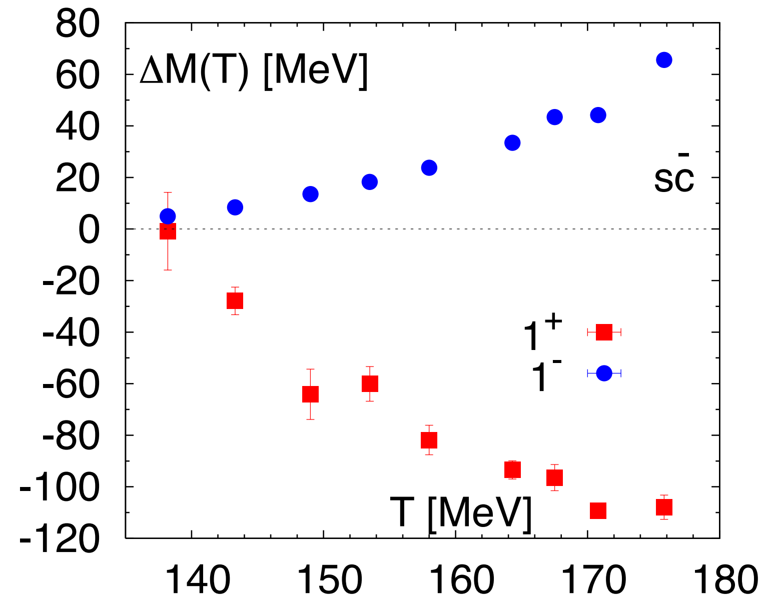
Qualitatively similar behavior of the screening masses for ssbar, scbar and ccbar sectors

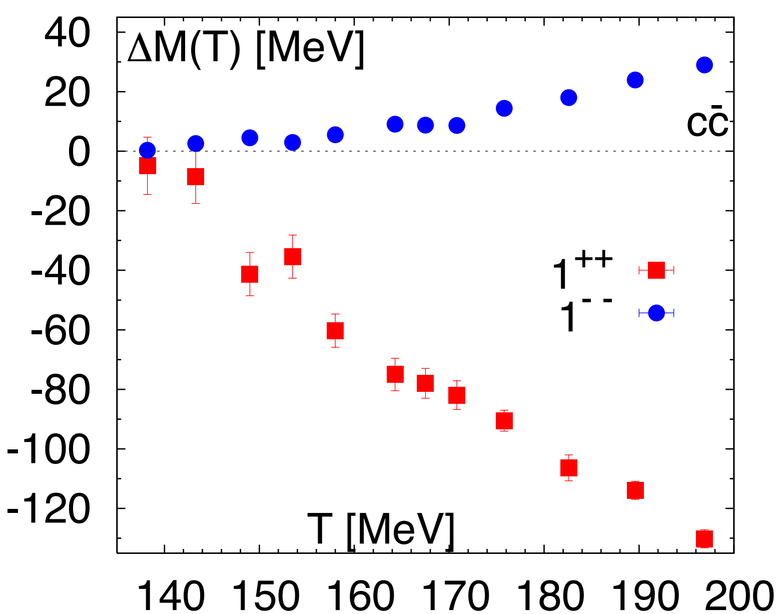
Screening Masses of opposite parity mesons become degenerate at high T
(restoration of chiral and axial symmetry)

Screening masses are close to the free limit $2\,(m_q^{2}+(\pi T)^{2})^{1/2}$ at $T>200$ MeV, $T>250$ MeV, $T>300$ MeV for ssbar, scbar and ccbar sectors, respectively.

Temperature dependence of meson screening masses (cont'd)



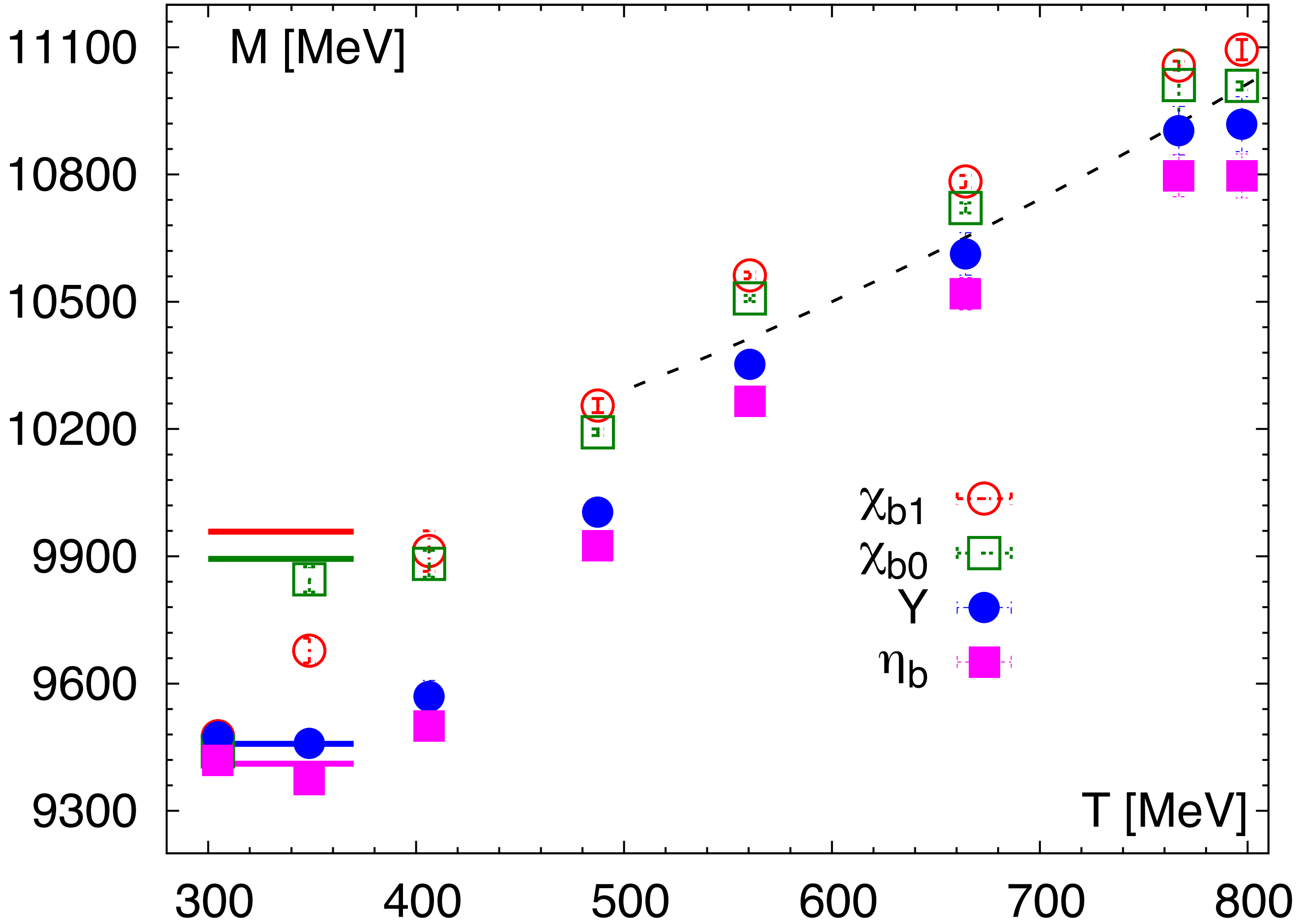




- At low *T* changes in the meson screening Masses $\Delta M = M_{scr}(T) - M_{T=0}$ are indicative of the changes in meson binding energies
- *ΔM* is significant already below T_c
- Above the transition temperature the changes in *ΔM* are comparable to the meson binding energy except for *1S* charmonium (sequential melting)
- For charmonium qualitative agreement with potential models

Bottomonium screening masses

S. Sharma, PP, work in progress



1S bottomonia screening masses agree with the T=0 result for *T<400* MeV

All bottomonia screening masses are similar to the free theory expectations for *T>400* MeV

⇒ Qualitatively consistent picture with potential model expectations

Why NRQCD ?

Quarkonia to a fairly good approximation are non-relativistic bound state

$$p_Q \sim M_Q v \ll M_Q$$

EFT approach: integrate the physics at scale of the heavy quark mass

NRQCD is the EFT at scale << M_Q

Heavy quark fields are non-relativistic Pauli spinors:

$$L_{NRQCD} = \psi^\dagger \left(D_\tau - \frac{D_i^2}{2M_Q} \right) \psi + \chi^\dagger \left(D_\tau + \frac{D_i^2}{2M_Q} \right) \chi + \ldots + \frac{1}{4} F_{\mu\nu}^2 + \bar{q}\gamma_\mu D_\mu q$$

Advantages:

- the large quark is not a problem for lattice calculations, lattice study of bottomonium is feasible (usually $a M_Q << 1$, which is challenging)
- The structure of the spectral function is simpler => more sensitivity to the bound state properties
- Quarkonium correlators are not periodic and can be studied at larger time extent ($=1/T$) => more sensitivity to bound state properties

NRQCD on the Lattice

Inverse lattice spacing provides a natural UV cutoff
for NRQCD provided $a^{-1} \leq 2M_Q$ (lattices cannot be too fine)

Quark propagators are obtained as initial value problem:

$$S_Q(x,\tau+a) = U_4^\dagger(1-\frac{p^2}{2M_Q}\Delta\tau)S_Q(x,\tau),\ \Delta\tau = a/n$$

well behaved if $naM_Q < 3$

Davies, Thacker, PRD 45 (1992) 915

$$D(\tau) = \sum_x \langle O(x,\tau)S_Q(x,\tau)O^\dagger(0,0)S_Q^\dagger(x,\tau)\rangle_T,\quad O(^3S_1;x,\tau)=\sigma_i,\quad O(^3P_1;x,\tau)=\Delta_i\sigma_j-\Delta_j\sigma_i$$

Thacker, Lepage, PRD43 (1991) 196

The energy levels in NRQCD are related to meson masses by a constant
lattice spacing dependent shift, e.g.

$$M_{\Upsilon(1S)} = E_{\Upsilon(1S)} + C_{\text{shift}}(a)$$

Light d.o.f (gluons, u,d,s quarks) are represented by gauge configurations
from HotQCD, $m_s = m_s^{phys}$, $m_{u,d} = m_s/20 \leftrightarrow m_\pi = 161$ MeV
$T > 0$: $48^3 \times 12$ lattices, $T_c = 159$ MeV, the temperature is varied
by varying $a \leftrightarrow \beta = 10/g^2$ Bazavov et al, PRD85 (2012) 054503

$\Rightarrow 140\text{MeV} \leq T \leq 407\text{MeV}$

$2.759 \geq aM_b \geq 0.954$ (ok if $n=2,\ 4$)
$0.757 \geq aM_c \geq 0.427$ (ok if $n \geq 8$)

Bayesian Reconstruction of spectral functions

$$D(\tau) = D(\mathbf{p}=0,\tau) = \sum_{\mathbf{x}} D(\mathbf{x},\tau) = \int_{-2M_q}^{\infty} d\omega\, e^{-\omega\tau}\ \rho(\omega)$$

Discretize the integral $D_i^\rho = \sum_{l=1}^{N_\omega} \exp[-\omega_l \tau_i]\ \rho_l\ \Delta\omega_l$ and find ρ_l

using Bayesian approach, i.e. maximizing

$$P[\rho|D,I] \propto P[D|\rho,I]P[\rho|I]$$

Likelihood:

$$P[D|\rho I] = \exp(-L)$$

$$L[\rho] = \frac{1}{2}\sum_{ij}(D_i - D_i^\rho)C_{ij}(D_j - D_j^\rho)$$

Prior probability:

$$P[\rho|I] = \exp[S]$$

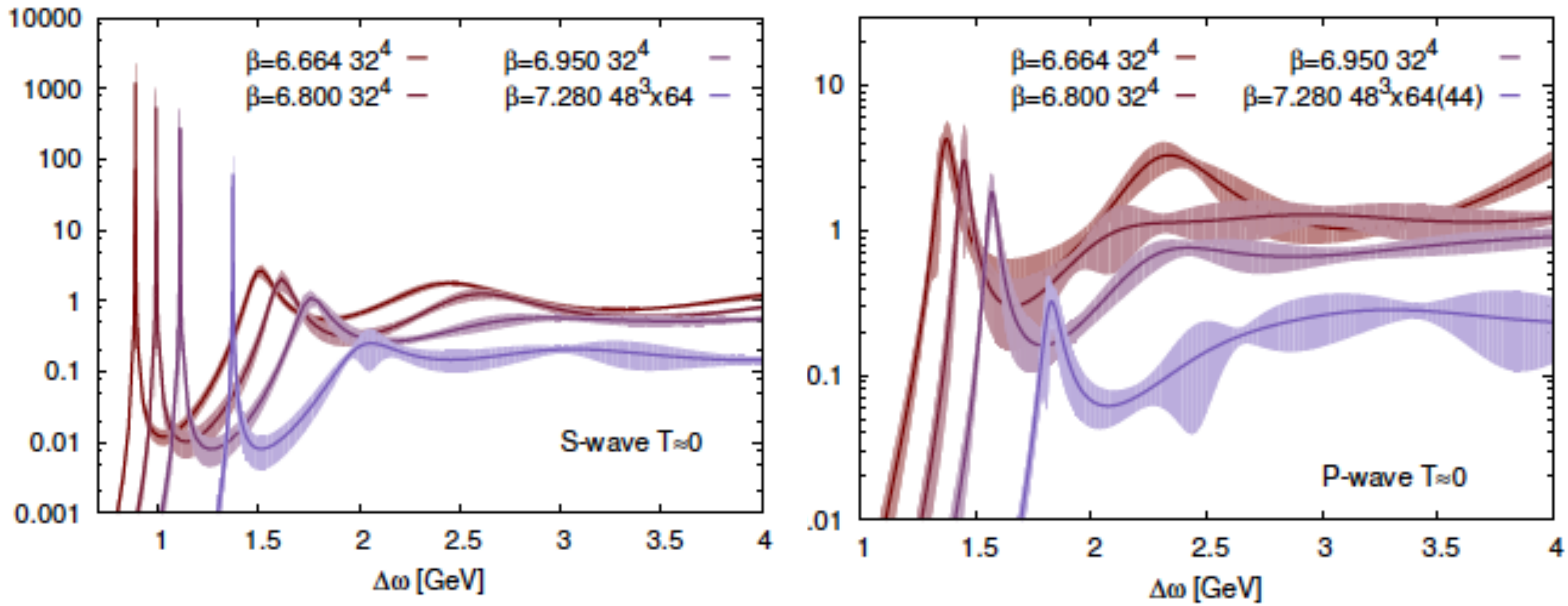
$$S[\rho] = \alpha \sum_l \left(1 - \frac{\rho_l}{m_l} + \log\left[\frac{\rho_l}{m_l}\right]\right)\Delta\omega_l.$$

Different from MEM !

no restriction on the search space
no flat directions

Burnier Rothkopf, PRL 111 (2013) 182003

Bottomonium spectral functions at T=0



Well resolved Υ ground state peak   Acceptable resolution for χ_b state

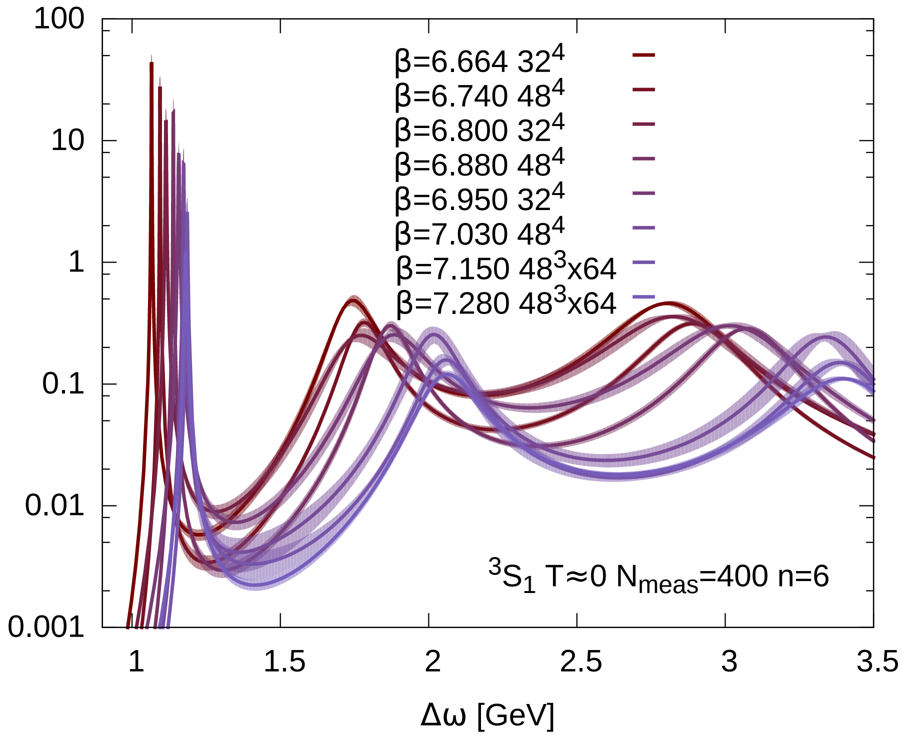
But excited states, Υ', Υ'', χ_b' cannot be resolved well

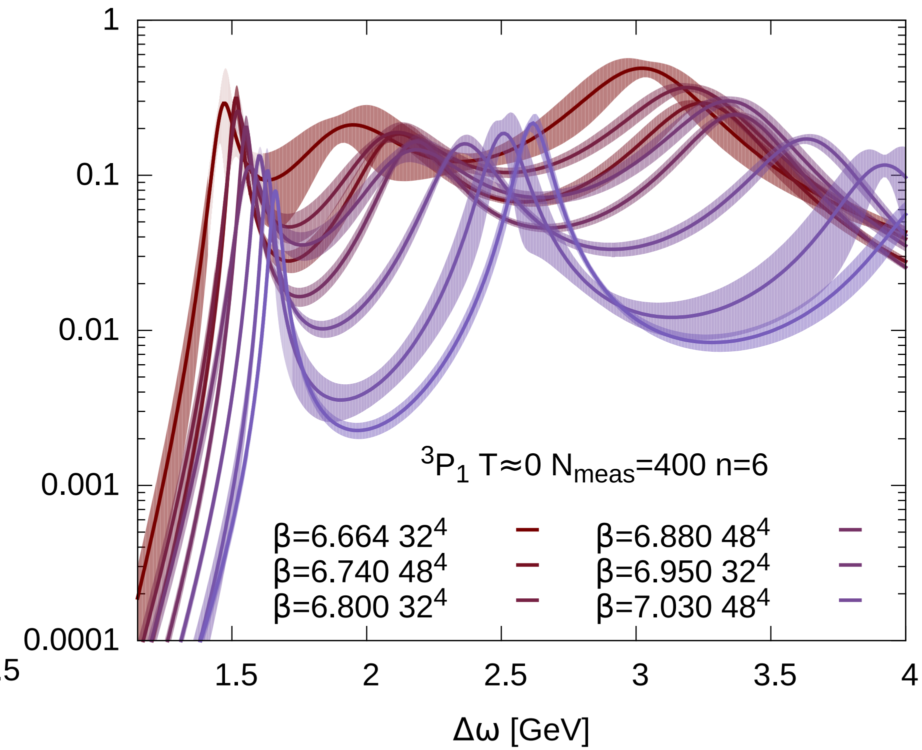
Define the NRQCD energy shift $C_{\rm shift}(a)$ by fixing the Υ peak to PDG

$$E_\Upsilon + C_{\rm shift}(a) = 9.46030\ \mathrm{GeV}$$

$\Rightarrow$ prediction for mass of other states: η_b, χ_{b0}, χ_{b1}, h_b

Charmonium spectral functions at T=0





Well resolved J/ψ peak

Only the position of the χ_{c1} state can be reliably determined

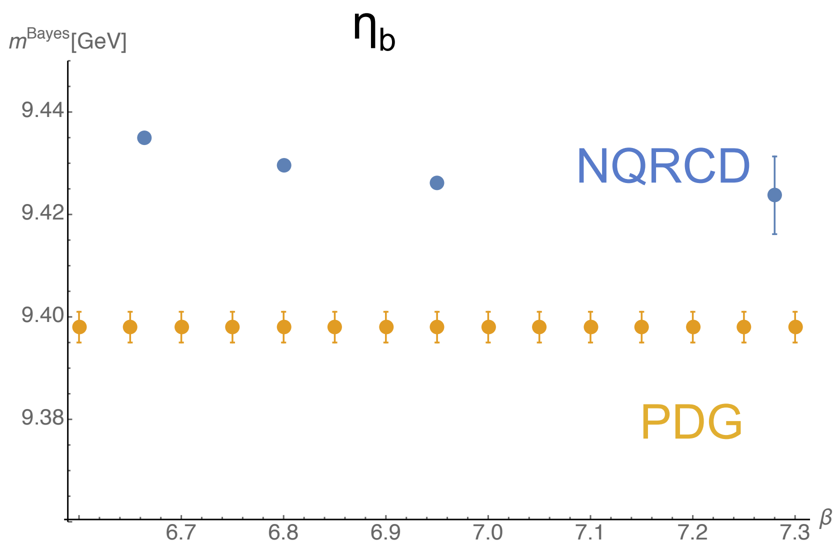
Excited states cannot be determined, artifacts at $\Delta\omega > 3$ GeV

Define the NRQCD energy shift $C_{\mathrm{shift}}(a)$ by fixing the J/ψ peak to PDG

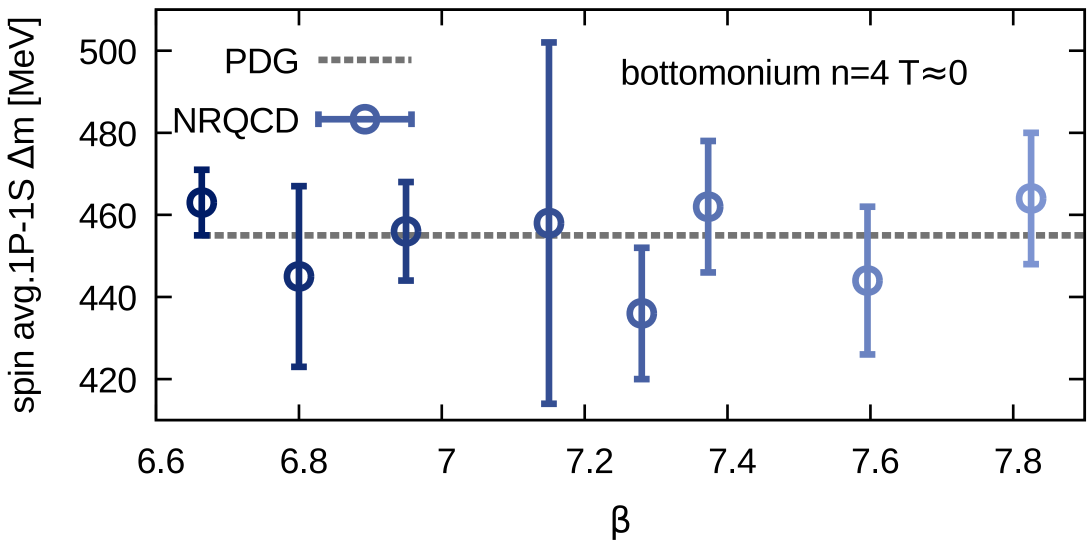
$$E_{J/\psi} + C_{\mathrm{shift}}(a) = 3.097 \text{ GeV}$$

$\Rightarrow$ prediction for mass of other states: η_c, χ_{c0}, χ_{c1}, h_c

How Well NQRCD Works for Bottomonium ?

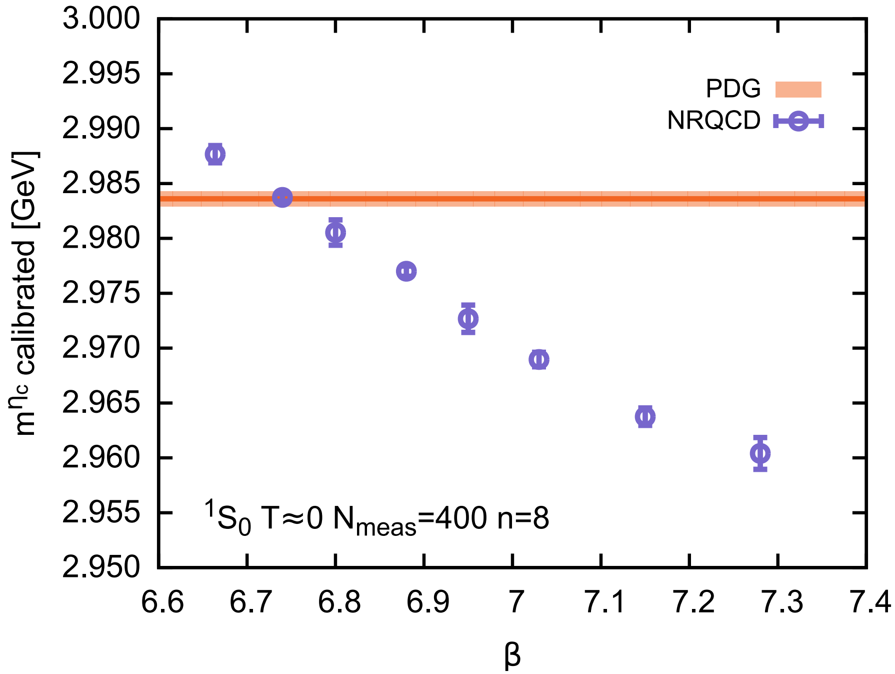


NRQCD can reproduce the hyperfine splitting in bottomonium with accuracy < *20-40* MeV depending on the lattice spacing

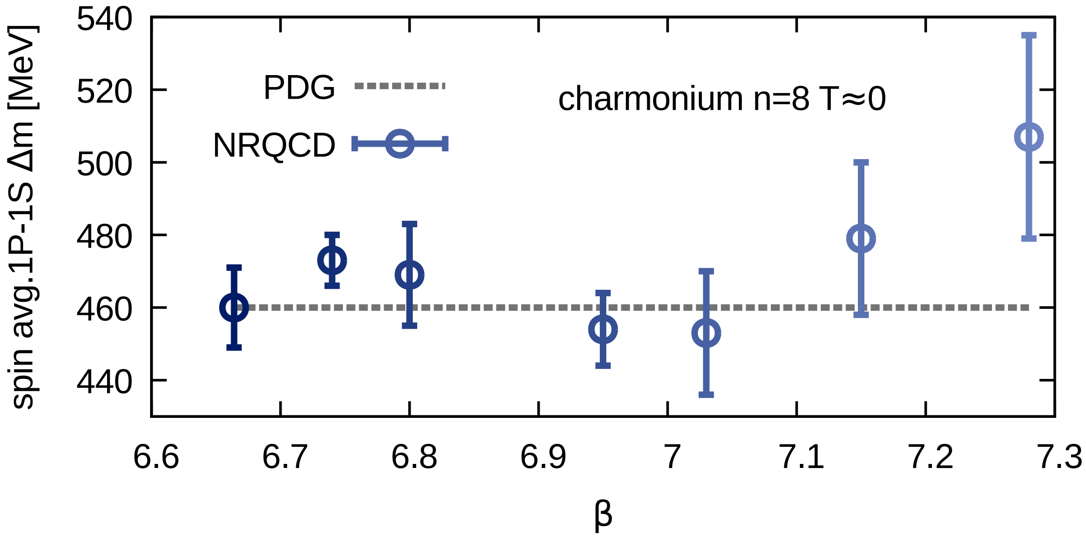


NRQCD can reproduce the spin averaged *1P-1S* splitting in bottomonium

How Well NQRCD Works for Charmonium ?

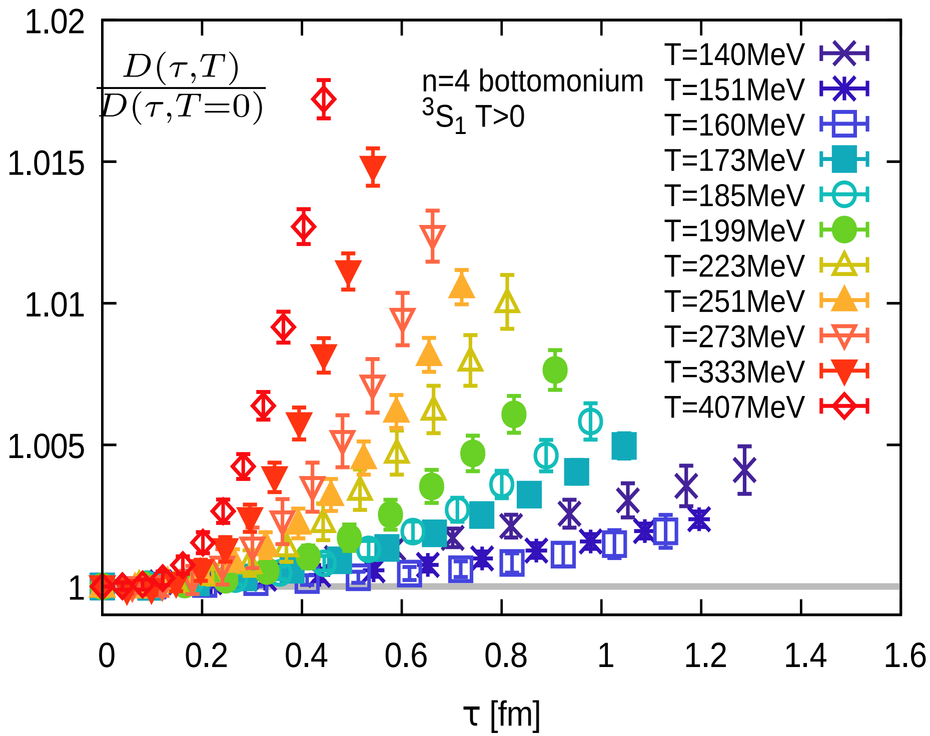


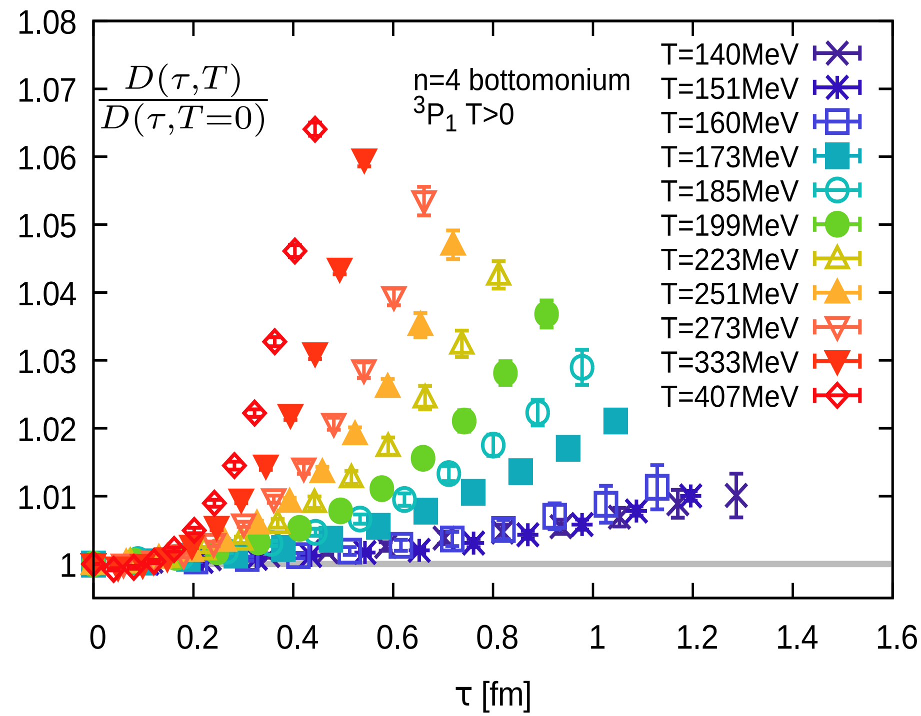
NRQCD can reproduce the hyperfine splitting in charmonium with an accuracy < *40* MeV



NRQCD can reproduce the spin averaged *1P-1S* splitting in charmonium well for lattice spacing a < 0.09fm

Temperature dependence of the bottomonium correlators



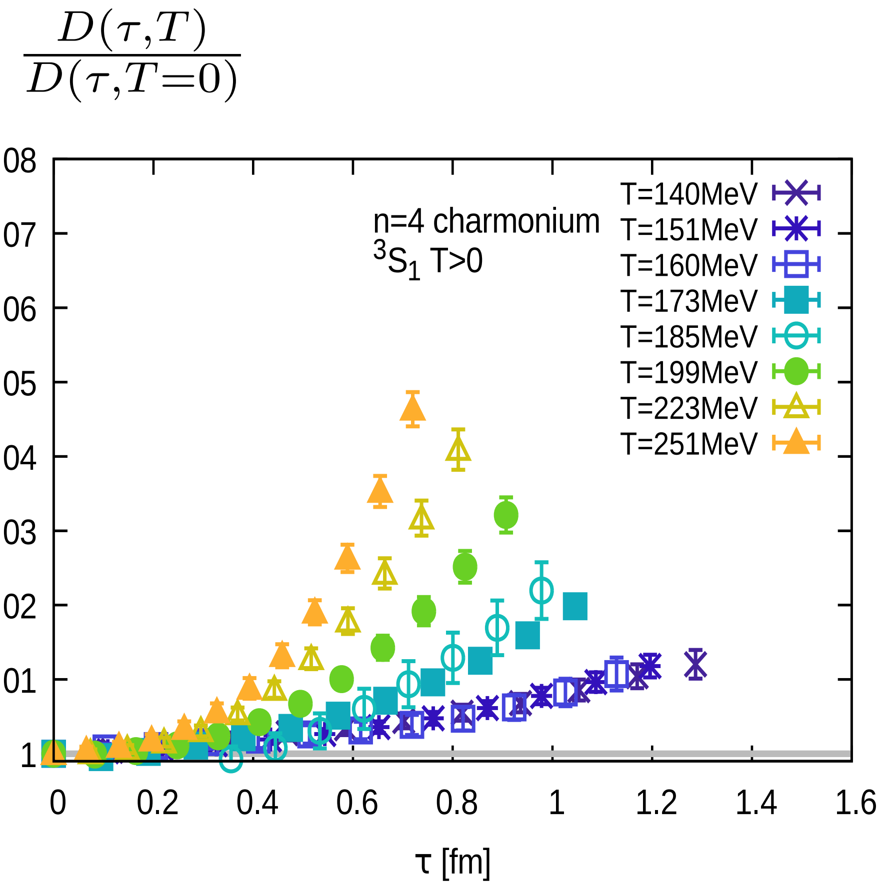


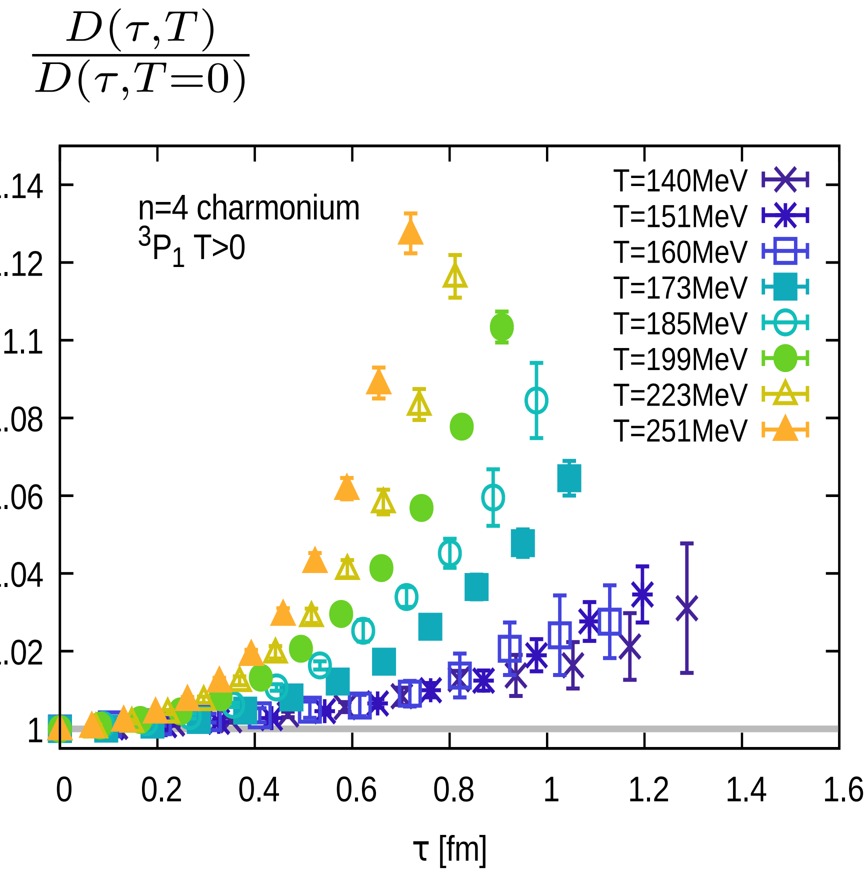
change in Υ correlator $< 2\%$

change in χ_{b1} correlator $< 7\%$

$\Rightarrow$ hints for sequential melting pattern: stronger medium modification of χ_{b1} spectral function than for Υ spectral function

Temperature dependence of the charmonium correlators





change in J/ψ correlator $< 5\%$

change in χ_{c1} correlator $< 12\%$

$\Rightarrow$ hints for sequential melting pattern:
changes in the J/ψ correlator are about the same as in the χ_b correlator (same size); changes in the χ_c correlators are factor of two larger

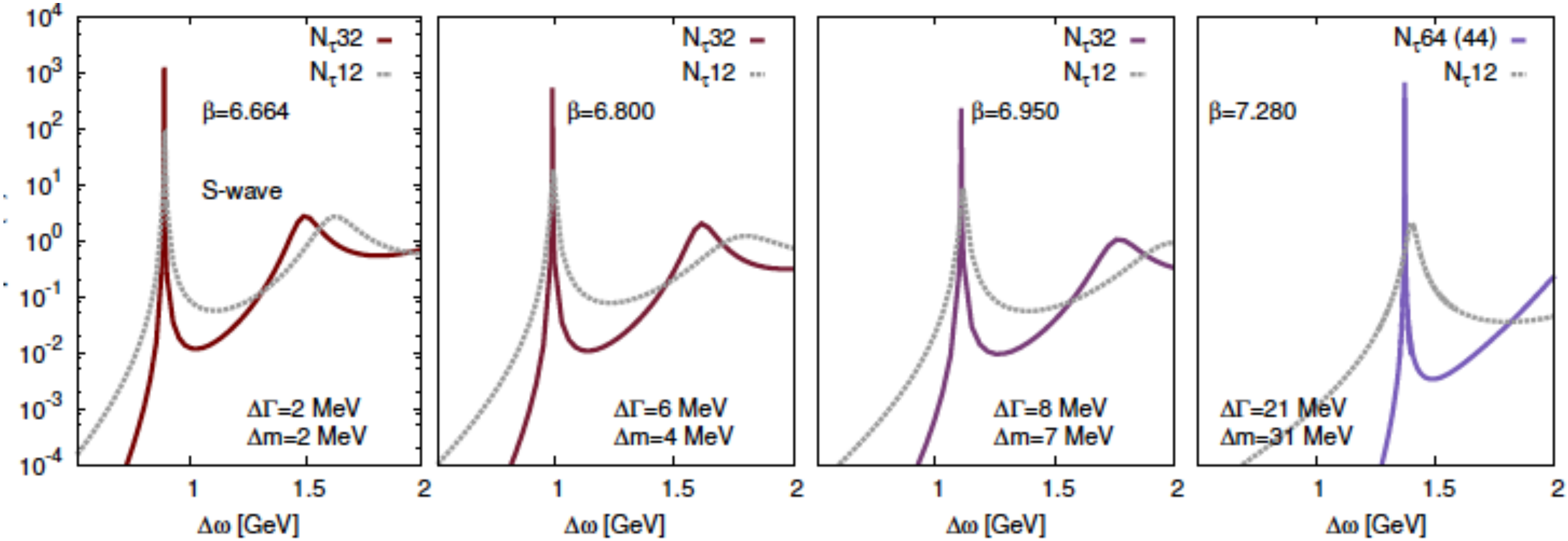
Reconstructing Spectral Functions at T>0

Two main problems:

1) $\tau < 1/T \Rightarrow$ limited temporal extent at high T

2) relatively small number of time slices ($N_\tau = 12$ in our study)

Study these effects at $T = 0$ by using only the first 12 data points:

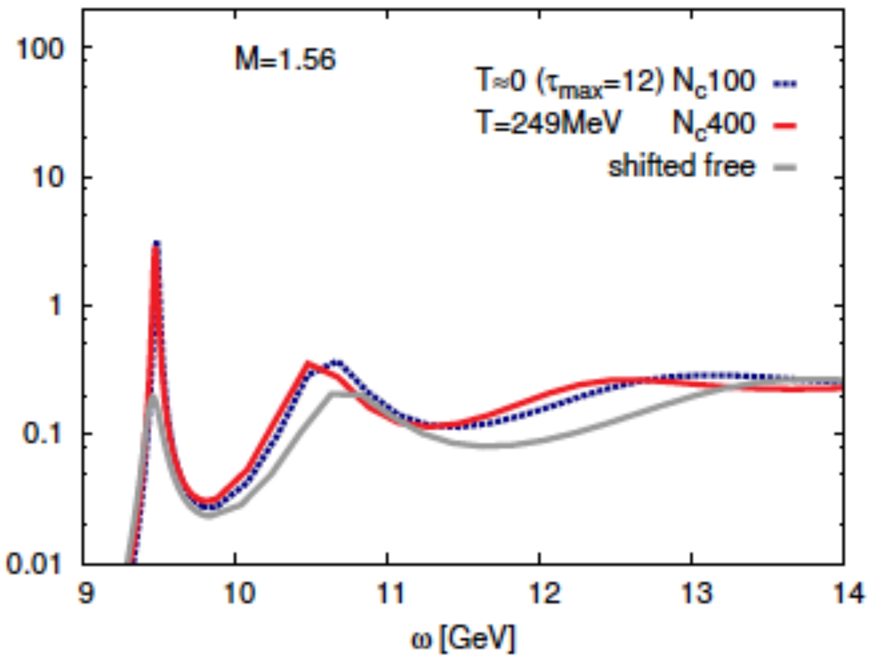


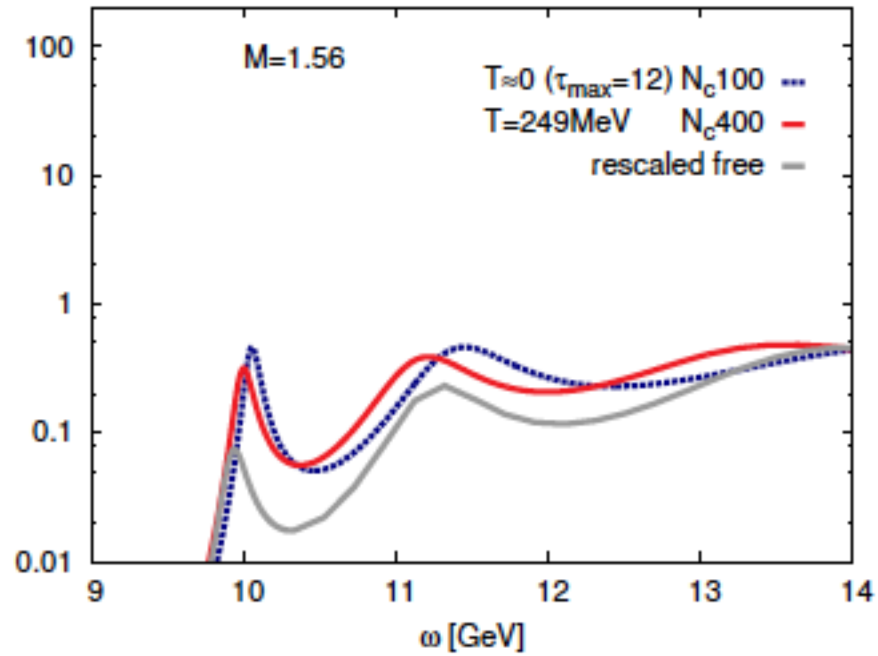
Decreasing $\tau_{max} = 1/T$ leads to broadening of the bound state peak

(to be taken into account in comparison $T = 0$ and $T > 0$ spectral functions)

Bottomonium Spectral Functions at T>0

Compare $T = 0$, $T > 0$ and free spectral functions reconstructed using the same systematics ($\tau_{max} = 1/T$ and $N_{data} = 12$)

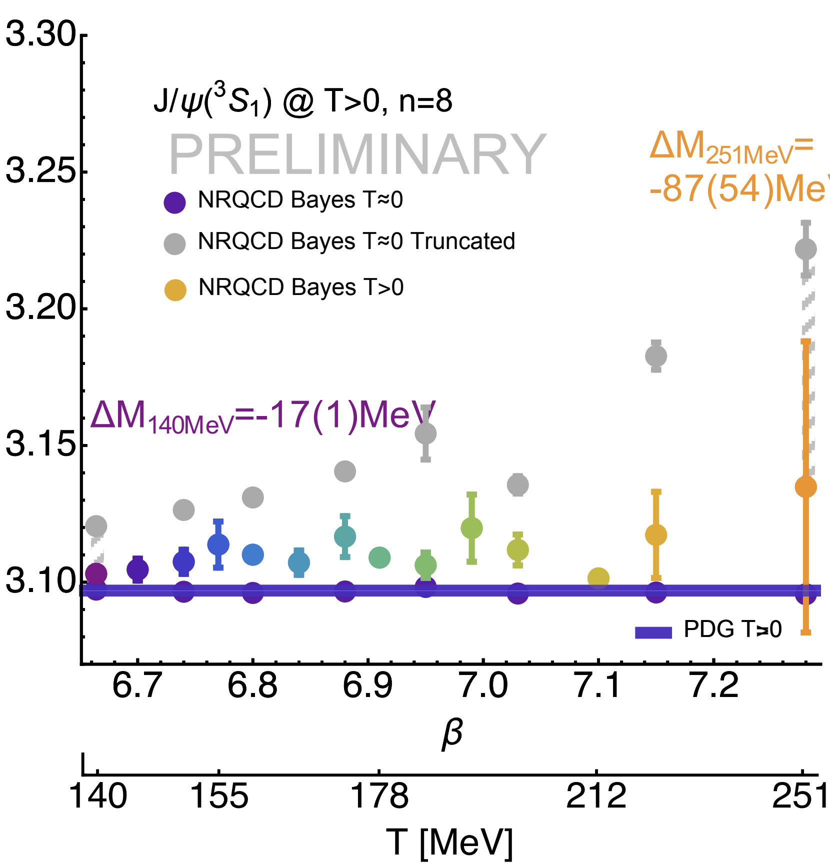


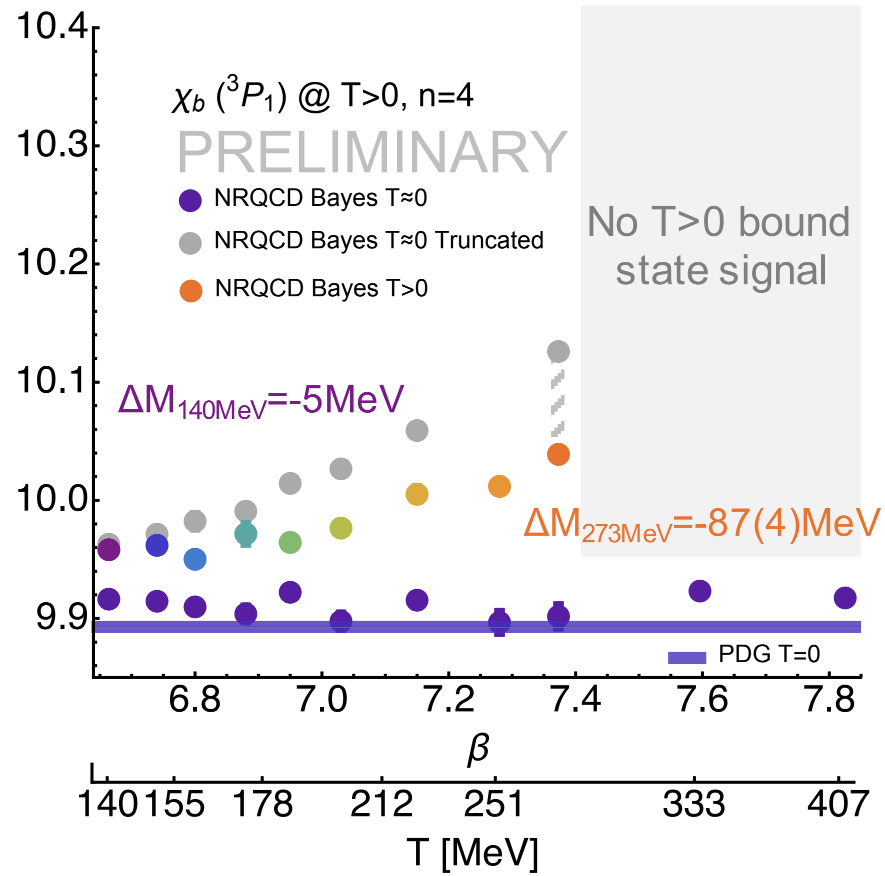


Both Υ and χ_b survive up to temperature $T > 249$ MeV

Onia masses at T>0

Onia masses from the peak positions:

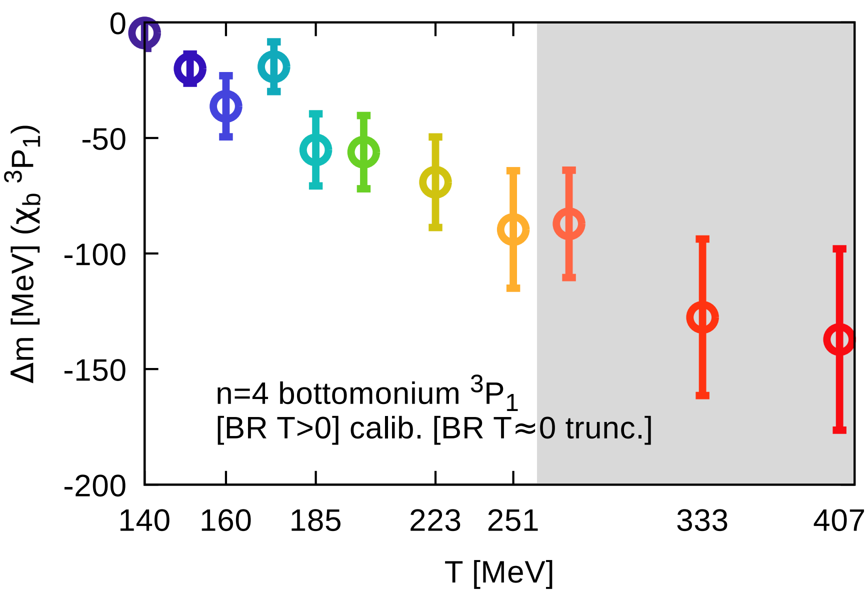


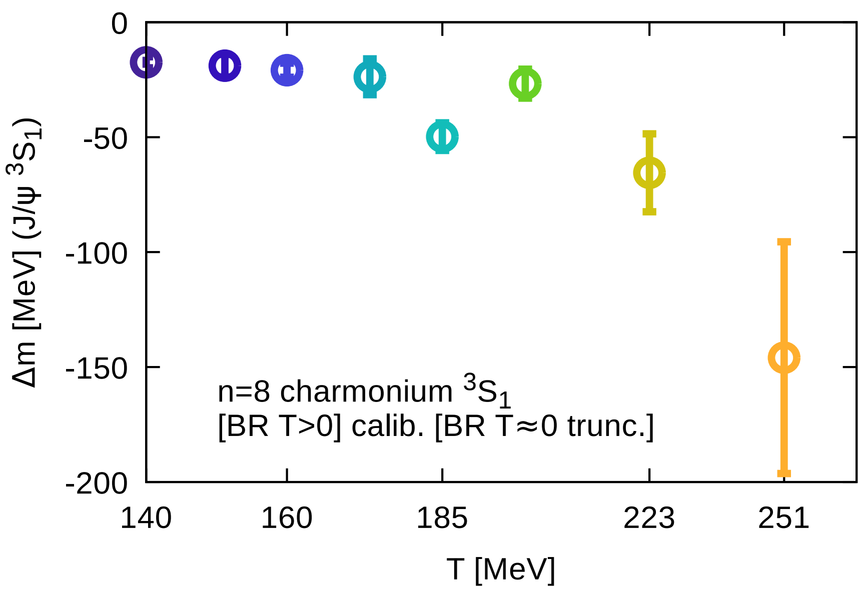


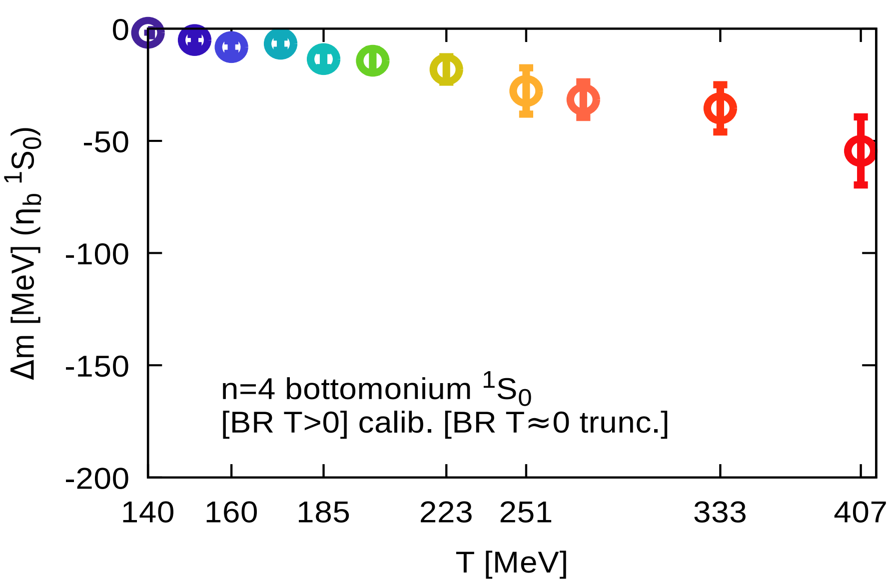
Shifts in the peak location is smaller at *T>0* than in the vacuum for the same temporal extent ➔ the actual onia masses decrease with increasing temperature

Kim, PP, Rothkopf, arXiv:1704.05221

Onia masses at T>0







Similar in medium mass shifts for J/ψ and χ_b

Much smaller mass shift for Υ
(for $T < 407$ MeV)

Summary

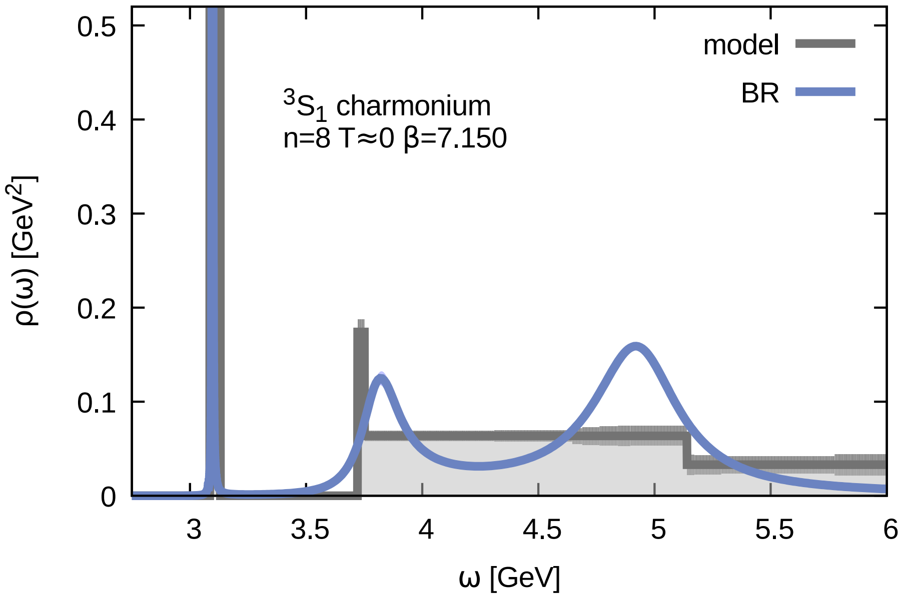
- Temporal meson correlators are not very sensitive to the changes in the spectral functions because of the limited time extent temporal extent at *T>0*

- Spatial meson correlators and NRQCD correlators are sensitive to the temperature to the changes in the spectral functions and are consistent with sequential melting picture: smaller onia dissolve at higher temperatures, e.g. and *1S* onia survive till higher temperature than *1P* onia

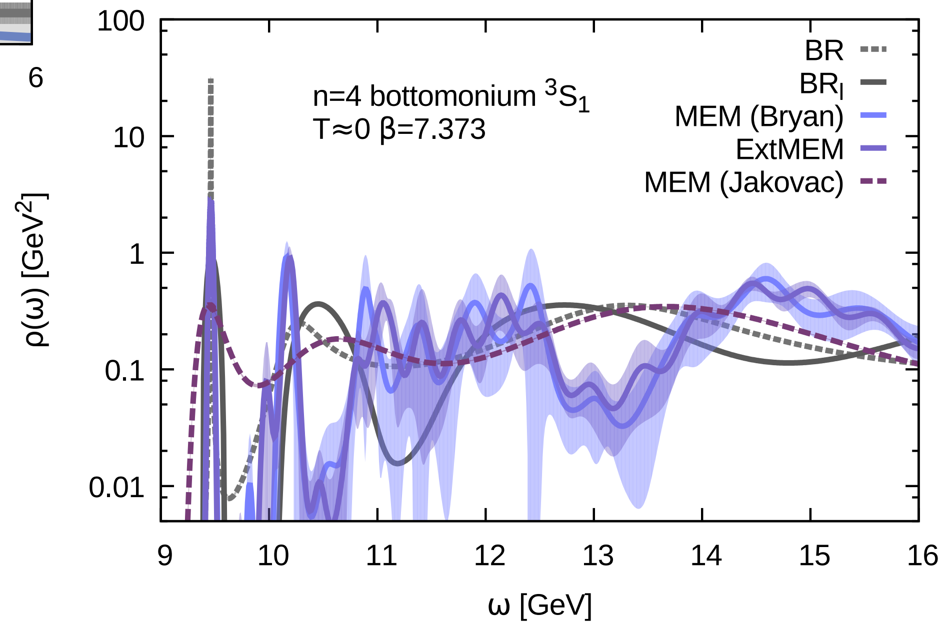
The behavior of the correlation functions is consistent with the potential model findings

To obtain information about the quarkonia properties from Bayesian analysis systematic effects have to be taken into account: so far only information that can be obtained is the decrease in the quarkonium masses

Back-up:



model
BR
$^3S_1$ charmonium
n=8 T≈0 β=7.150
ρ(ω) [GeV^2]
ω [GeV]
0
0.1
0.2
0.3
0.4
0.5
3
3.5
4
4.5
5
5.5
6



BR
BR_l
MEM (Bryan)
ExtMEM
MEM (Jakovac)
n=4 bottomonium $^3S_1$
T≈0 β=7.373
ρ(ω) [GeV^2]
ω [GeV]
100
10
1
0.1
0.01
9
10
11
12
13
14
15
16