On-shell effective field theory: a new tool in thermal QFT

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INT Multi-Scale Problems using EFT
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TOPICS

- QCD in extreme conditions and dense nuclear matter
- Baryogenesis and leptogenesis
- Heavy-ion collisions and the Quark-Gluon Plasma
- Quantum fields in and out equilibrium and thermalisation
- Electroweak phase transition beyond the Standard Model
- Early Universe physics and sources for gravitational waves
- Compact stars
- Quantum field dynamics and inflation

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Paolo Bedaque (U. of Maryland)
Szabolcs Borsanyi (U. Wuppertal)
Aleksey Cherman (Washington U., Seattle)
Miguel Angel Escobedo (U. Jyvaskyla)
Jacopo Ghiglieri (CERN)
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Outline

- Breakdown of perturbation theory: HTLs and kinetic theory
- The OSEFT and rationale behind
- OSEFT Lagrangian and propagators
- OSEFT at work: one-loop photon polarization tensor (power corrections to the HTL)
- OSEFT at work: chiral kinetic theory

Breakdown of perturbation theory

Braaten and Pisarski; Frenkel and Taylor, 90'

At high temperature: two relevant scales $\begin{cases} \text{hard} \sim \quad T \\ \text{soft} \sim \quad gT \end{cases}$

$g \ll 1$

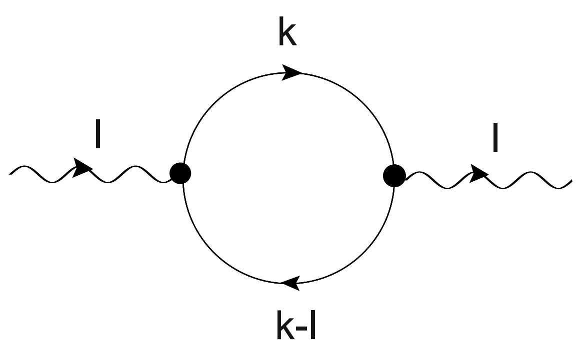
One-loop thermal corrections hard thermal loops (HTLs)

as relevant as the tree amplitudes for soft momenta

(and they arise from hard loop momenta)

and have to be resummed into effective vertices and propagators

In QED, diagrammatically



$$\Pi_{\mathrm{HTL}}(l) \sim g^2 T^2$$

Debye mass

$$\frac{\Pi_{\mathrm{HTL}}(l)}{l^2} \sim 1$$

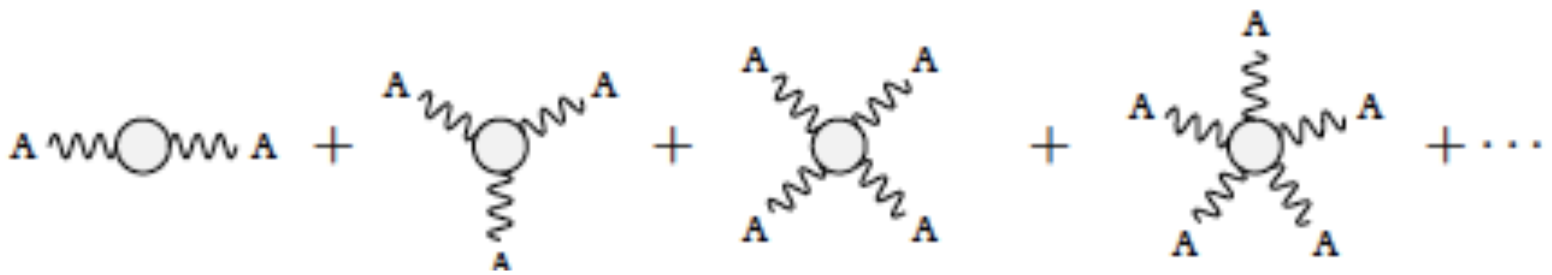
for soft momenta

HTL give account of Debye screening and Landau damping

$$\Pi^{\mu\nu}_{\mathrm{HTL}}(l) = 4e^2 \int \frac{d^3\mathbf{q}}{(2\pi)^3} \left\{ \frac{dn_f}{dq} \left(\delta^{\mu 0}\delta^{\nu 0} - l_0 \frac{v^\mu v^\nu}{v \cdot l} \right) \right\}$$

$$n_f(q) = \frac{1}{e^{q/T} + 1} \qquad v^\mu = (1, \hat{\mathbf{q}})$$

In QCD there are HTLs in all multi gluon amplitudes



- all proportional to the Debye mass
- UV finite, no HTL with ghost legs
- Independent of the gauge
- Depend on external momentum $\sim 1/(v \cdot p)$

Effective action generating HTL diagrams

$$S^{+}_{\rm HTL} = -\frac{m_D^2}{4}\int_{x,v} {\rm Tr}\left(F_{\alpha\mu}\,\frac{v^\alpha v^\beta}{(v\cdot D)^2}\,F_\beta{}^\mu\right)$$

HTLs and transport theory

Blaizot and Iancu, `94
Kelly, Liu, Lucchesi, CM, `94

HTLs can be described with simple transport equations

hard scales → on-shell quasiparticles

soft scales → classical fields

$$n_B(p_0) = \frac{1}{e^{P_0/T} - 1} \sim \frac{T}{p_0}$$

Hot QCD plasmas

$g << 1$

T	hard scale	classical on-shell particles transport equations
$g\,T$	soft scale	classical fields HTL EFT
$g^2T \ln(1/g)$	ultrasoft scale	Langevin type of eqs.
$g^2\,T$	magnetic scale	??

Can we describe with better accuracy the physics described by every perturbative scale in the hot plasma?

ON-SHELL EFFECTIVE FIELD THEORY

an attempt to improve the treatment of the hard scales, and thus also of the soft scales

we are inspired by many successful examples of EFT for QED and QCD: HDET, NRQED/QCD, pNRQED/QCD, SCET, etc,

OSEFT

Physical phenomena dominated by on-shell degrees of freedom

QED, m=0 (but it can be generalized)

OS fermion $p^\mu = p v^\mu$ $v^2 = 0$

Almost OS fermion

$q^\mu = p v^\mu + k^\mu$ $v^\mu = (1, \mathbf{v})$

residual momentum k << p

Almost OS antifermion

$q^\mu = -p \tilde{v}^\mu + k^\mu$ $\tilde{v}^\mu = (1, -\mathbf{v})$

OSEFT Lagrangian

$$\mathcal{L} = \sum_{p,\mathbf{v}} \mathcal{L}_{p,\mathbf{v}} \,, \qquad \mathcal{L}_{p,\mathbf{v}} = \bar{\psi}_{\mathbf{v}} \gamma \cdot iD \psi_{\mathbf{v}} \,, \qquad iD_\mu = i\partial_\mu + eA_\mu$$

$$\psi_{\mathbf{v}} = e^{-ipv\cdot x} \left(P_v \chi_v(x) + P_{\tilde{v}} H^{(1)}_{\tilde{v}}(x) \right) + e^{ip\tilde{v}\cdot x} \left(P_{\tilde{v}} \xi_{\tilde{v}}(x) + P_v H^{(2)}_v(x) \right)$$

with particle/antiparticle projectors

$$P_v = \frac{1}{2} \gamma \cdot v \, \gamma_0 \qquad\qquad P_{\tilde{v}} = \frac{1}{2} \gamma \cdot \tilde{v} \, \gamma_0 .$$

OSEFT Lagrangian

Integrate out the H fields (=solve its classical eqs. of motion)

$$\mathcal{L}_{p,\mathbf{v}} = \chi_v^\dagger(x) \left(i\, v\cdot D \,+ i\not{D}_\perp \frac{1}{2p + i\tilde{v}\cdot D} i\not{D}_\perp \right) \chi_v(x)$$
$$+\xi_{\tilde{v}}^\dagger(x) \left(i\,\tilde{v}\cdot D \,+ i\not{D}_\perp \frac{1}{-2p + i v\cdot D} i\not{D}_\perp \right) \xi_{\tilde{v}}(x)$$

$$P_\perp^{\mu\nu} = g^{\mu\nu} - \frac{1}{2}\left(v^\mu \tilde{v}^\nu + v^\nu \tilde{v}^\mu\right) \qquad D_\perp^\mu = P_\perp^{\mu\nu} D_\nu$$

Particle/antiparticle fields are totally decoupled,
but there's a symmetry between the particle/antiparticle L

$$v^\mu \Leftrightarrow \tilde{v}^\mu \qquad p \Leftrightarrow -p$$

Expand in powers of 1/p

$$\mathcal{L}^{(0)}_{p,v} = \chi_v^\dagger \, (i\, v \cdot D\,)\, \chi_v$$

$$\mathcal{L}^{(1)}_{p,v} = -\frac{1}{2p}\chi_v^\dagger \left(D_\perp^2 - \frac{e}{2}\sigma_\perp^{\mu\nu} F_{\mu\nu} \right) \chi_v$$

Apply local field redefinitions to eliminate temporal derivatives at higher orders

$$\chi_v \to \chi_v' = \left(1 + \frac{\not{D}_\perp^2}{8p^2} \right) \chi_v$$

$$\mathcal{L}^{(2)}_{p,v} = \frac{1}{8p^2}\chi_v'^\dagger \Big([\not{D}_\perp\,, [i\tilde{v}\cdot D\,, \not{D}_\perp]] - \{(\not{D}_\perp)^2, (iv\cdot D - i\tilde{v}\cdot D)\} \Big) \chi_v'$$

n=3

$$\chi_v \to \chi_v'' = \left(1 - \frac{i}{8p^3} \not{D}_\perp [\tilde{v} \cdot D\,,\, \not{D}_\perp] + \frac{i}{16p^3} \not{D}_\perp^2 (v \cdot D - \tilde{v} \cdot D) - \frac{i}{16p^3} \not{D}_\perp^2 \tilde{v} \cdot D \right) \chi_v'$$

$$\mathcal{L}_{p,v}^{(3)} = \frac{1}{8p^3} \chi_v''^\dagger \left\{ \not{D}_\perp^4 + [\not{D}_\perp, i\tilde{v} \cdot D]^2 - (iv \cdot D - i\tilde{v} \cdot D) \not{D}_\perp^2 (iv \cdot D - i\tilde{v} \cdot D) \right\} \chi_v''$$
$$+ \frac{1}{8p^3} \chi_v''^\dagger \{ (iv \cdot D - i\tilde{v} \cdot D) \not{D}_\perp \, [i\tilde{v} \cdot D, D_\perp] - [i\tilde{v} \cdot D, \not{D}_\perp] \not{D}_\perp (iv \cdot D - i\tilde{v} \cdot D) \} \chi_v''$$

n=4 and higher could be obtained as well

OSEFT Propagators
Real Time Formalism

The momentum p acts as a chemical potential for the fermion quantum fluctuations

from the lowest order Lagrangian

$$S(k) = P_v \gamma_0 \left[\begin{pmatrix} \frac{1}{v\cdot k + i\epsilon} & 0 \\ 0 & \frac{1}{v\cdot k - i\epsilon} \end{pmatrix} + 2\pi i \delta(v\cdot k) \begin{pmatrix} n_f(p+k_0) & n_f(p+k_0) \\ -1 + n_f(p+k_0) & n_f(p+k_0) \end{pmatrix} \right]$$

This propagator might be also deduced from the full propagator, after expanding for large p $q^\mu = p v^\mu + k^\mu$

⇨ this brings an additional p dependence, not contained in the L, of the propagators

OSEFT Propagators

$$S_{R/A} = S_{11} - S_{12/21}\,, \qquad S_S = S_{11} + S_{22}$$

$$S^{R/A}(k) = \frac{P_v \gamma_0}{k_0 \pm i\epsilon - f(\mathbf{k})},$$

$$S^S(k) = P_v \gamma_0 \left(-2\pi i \delta(k_0 - f(\mathbf{k})) \left(1 - 2n_f(p + k_0)\right)\right)$$

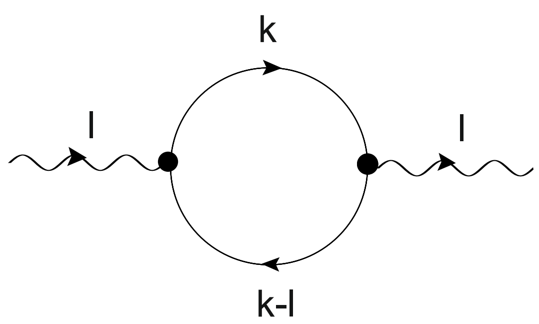
$$f^{(0)}(\mathbf{k}) = k_\parallel\,, \qquad f^{(1)}(\mathbf{k}) = k_\parallel + \frac{\mathbf{k}_\perp^2}{2p}\,, \qquad f^{(2)}(\mathbf{k}) = k_\parallel + \frac{\mathbf{k}_\perp^2}{2p} - \frac{k_\parallel \mathbf{k}_\perp^2}{2p^2}$$

The machinery is (almost) ready for Feynman loop computations!

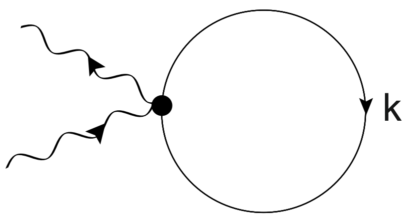
- Interaction vertices ~ $(\text{momentum})^n/p^m$

- Propagators with p dependence (dispersion rules and "chemical potential")

Retarded photon polarization tensor



Bubble



Tadpole

Two topologies: the two are needed to respect gauge invariance at every order (Ward Identity)

Tadpoles: they give account of fermion-photon interactions mediated by an off-shell antifermion in QED

Bubble

$$\Pi_{\rm b}^{\mu\nu}(l) = -\frac{i}{2}\sum_{p,\mathbf{v}}\int\frac{d^4k}{(2\pi)^4}\Big(\mathrm{Tr}[V^\mu\, S_S(k-l)\, V^\nu\, S_R(k)] + \mathrm{Tr}[V^\mu\, S_A(k-l)\, V^\nu\, S_S(k)]\Big)$$

$$- \frac{i}{2}\sum_{p,\mathbf{v}}\int\frac{d^4k}{(2\pi)^4}\Big(\mathrm{Tr}[V^\mu S_A(k-l) V^\nu S_A(k)] + \mathrm{Tr}[V^\mu S_R(k-l) V^\nu S_R(k)]\Big)\ ,$$

Tadpole

$$\Pi_{\rm t}^{\mu\nu}(l) = \frac{i}{2}\sum_{p,\mathbf{v}}\int\frac{d^4k}{(2\pi)^4}\Big(\mathrm{Tr}[W^{\mu\nu}\,(S_S(k)+S_R(k)+S_A(k))]\Big)$$

Perform the k0 integral, and re-express the resulting integral in terms of the original variable

$$q^\mu = p v^\mu + k^\mu$$

$$\sum_{p,\mathbf{v}} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \equiv \int \frac{d^3\mathbf{q}}{(2\pi)^3}$$

$$p = q - k_{\|,\mathbf{q}} + \frac{\mathbf{k}^2_{\perp,\mathbf{q}}}{2q} + \mathcal{O}(\frac{1}{q^2}) \; ,$$

$$\mathbf{v} = \hat{\mathbf{q}} - \frac{\mathbf{k}_{\perp,\mathbf{q}}}{q} - \frac{\hat{\mathbf{q}}\mathbf{k}^2_{\perp,\mathbf{q}} + 2k_{\|,\mathbf{q}}\mathbf{k}_{\perp,\mathbf{q}}}{2q^2} + \mathcal{O}(\frac{1}{q^3})$$

$$n_f(p) = n_f(q) + \frac{dn_f}{dq}\left(-k^{\mathbf{q}}_{\|} + \frac{\mathbf{k}^2_{\perp,\mathbf{q}}}{2q}\right) + \frac{1}{2}\frac{d^2 n_f}{dq^2} k^2_{\|,\mathbf{q}} +$$

n=1, HTLs are recovered

$$\Pi^{\mu\nu}_{(1)}(l) = 4e^2 \int \frac{d^3\mathbf{q}}{(2\pi)^3} \left\{ \frac{dn_f}{dq} \left(\delta^{\mu 0}\delta^{\nu 0} - l_0 \frac{v^\mu_\mathbf{q} v^\nu_\mathbf{q}}{v_\mathbf{q}\cdot l} \right) + \mathcal{O}(\frac{1}{q^2}) \right\}$$

$$v^\mu_\mathbf{q} \equiv (1, \hat{\mathbf{q}})$$

n=2, both tadpoles and bubble vanish after angular integration!

$$\Pi^{\mu\nu}_{b,(2)}(l) = e^2 \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{1}{q} \frac{dn_f}{dq} \left\{ l_{\parallel,\mathbf{q}} \left(l^\mu_{\perp,\mathbf{q}} v^\nu_\mathbf{q} + l^\nu_{\perp,\mathbf{q}} v^\mu_\mathbf{q} \right) \frac{1}{v\cdot l} \right.$$
$$\left. + v^\mu_\mathbf{q} v^\nu_\mathbf{q} \left(\frac{\mathbf{l}^2_{\perp,\mathbf{q}} - 2l^2_{\parallel,\mathbf{q}}}{v_\mathbf{q}\cdot l} + \frac{\mathbf{l}^2_{\perp,\mathbf{q}} l_{\parallel,\mathbf{q}}}{(v_\mathbf{q}\cdot l)^2} \right) + \mathcal{O}(\frac{1}{q}) \right\}$$

We have carried out the same computation in QED to the same accuracy to match our OSEFT results and check the consistency of the approach

The computation in QED requires to expand for large internal loop the integrand of the Feynman diagram:

we recognize the structures seen in the OSEFT computation

counterterms to eliminate UV, and also to reproduce finite local pieces of QED

$$\mathcal{L}_{c.t.} = -\frac{Z(\alpha,\epsilon)C(\alpha,\mu)}{2}F_{0i}F^{0i} - \frac{Z'(\alpha,\epsilon)C'(\alpha,\mu)}{4}F_{ij}F^{ij}$$

$$Z = Z' = Z_{QED} = 1 - \frac{2}{3\epsilon}\frac{\alpha}{\pi}$$

$$C = 1 + \frac{\alpha}{\pi}C^{(1)} \quad , \quad C' = 1 + \frac{\alpha}{\pi}C'^{(1)}$$

$$C^{(1)} = 0$$

$$C'^{(1)} = \frac{2}{3}\left(\ln\frac{\sqrt{\pi}T}{2\mu} - \frac{\gamma}{2} - 1\right)$$

In the MS scheme, for $\mu = \frac{\sqrt{\pi}}{2} T e^{-1-\gamma/2}$

$$\Pi^L_{\text{total},(3)}(l_0,\mathbf{l}) = \frac{\alpha}{\pi}\left[\mathbf{l}^2 - \frac{1}{3}l_0^2 + \frac{1}{6}\frac{l_0}{|\mathbf{l}|}\left(l_0^2 - 3\mathbf{l}^2\right)\left(\ln\left|\frac{l_0+|\mathbf{l}|}{l_0-|\mathbf{l}|}\right| - i\pi\,\Theta(|\mathbf{l}|^2 - l_0^2)\right)\right],$$

$$\Pi^T_{\text{total},(3)}(l_0,\mathbf{l}) = \frac{\alpha}{\pi}\left[\frac{1}{2}l_0^2 - \frac{2}{3}\mathbf{l}^2 + \frac{1}{6}\frac{l_0^4}{\mathbf{l}^2} - \frac{1}{12}\frac{l_0^3}{|\mathbf{l}|^3}\left(l_0^2 + 2\mathbf{l}^2 - 3\frac{\mathbf{l}^4}{l_0^2}\right)\left(\ln\left|\frac{l_0+|\mathbf{l}|}{l_0-|\mathbf{l}|}\right| - i\pi\,\Theta(|\mathbf{l}|^2 - l_0^2)\right)\right]$$

which corrects the HTL result

$$\Pi^L_{\text{total},(1)}(l_0,\mathbf{l}) = m_D^2\left(\frac{l_0}{2|\mathbf{l}|}\left(\ln\left|\frac{l_0+|\mathbf{l}|}{l_0-|\mathbf{l}|}\right| - i\pi\,\Theta(|\mathbf{l}|^2 - l_0^2)\right) - 1\right)$$

$$\Pi^T_{\text{total},(1)}(l_0,\mathbf{l}) = -m_D^2\,\frac{l_0^2}{2|\mathbf{l}|^2}\left[1 + \frac{1}{2}\left(\frac{|\mathbf{l}|}{l_0} - \frac{l_0}{|\mathbf{l}|}\right)\left(\ln\left|\frac{l_0+|\mathbf{l}|}{l_0-|\mathbf{l}|}\right| - i\pi\,\Theta(|\mathbf{l}|^2 - l_0^2)\right)\right]$$

New pieces: perturbative corrections to the soft propagation.

For soft momentum $l \sim eT$

$$\Pi^{(1-loop)} \sim \alpha T^2 + \alpha l^2$$

HTL

new piece!

competes with 2-loops coming from hard scales for soft momenta

$$\Pi^{(2-loops)} \sim \alpha^2 T^2$$

Nothing new: loop expansion =/= perturbative expansion

Effective Lagrangian

Carignano, CM, Soto, '18

$$\mathcal{L}^{(1)}_{\rm HTL} = \frac{e^2}{2}\int \frac{d^3q}{(2\pi)^3}\left\{ \frac{2n_F(q)}{q}\left(F_{\rho\alpha}\frac{v^\alpha v^\beta}{(v\cdot\partial)^2}F^{\rho}{}_{\beta}\right) - \frac{2(n_F(q)+n_B(q))}{q}\left(\bar\psi\frac{v\cdot\gamma}{(iv\cdot D)}\psi\right)\right\}$$

In d spatial dimensions

$$v^\mu = q^\mu/|\mathbf{q}|$$

$$\mathcal{L}^{(3)\gamma}_{\rm HTL} = \frac{e^2\nu^{3-d}}{4}\int \frac{d^dq}{(2\pi)^d}\frac{1-2n_F(q)}{q^3}\left\{F_{\rho\alpha}\frac{v^\alpha v^\beta}{(v\cdot\partial)^4}\partial^4 F_{\beta}{}^{\rho}\right\}$$

$$\mathcal{L}^{(3)\,\psi}_{\rm HTL} = \frac{e^2\nu^{3-d}}{4}(d-1)\left[\int \frac{d^dq}{(2\pi)^d}\frac{n_F(q)+n_B(q)}{q^3}\left\{\bar\psi\, D^2\frac{v\cdot\gamma}{(iv\cdot D)^3}D^2\,\psi\right\}\right.$$

$$\left. + \int \frac{d^dq}{(2\pi)^d}\frac{1+2n_B(q)}{2q^3}\left\{\bar\psi\left(D^2(iD\cdot\gamma)\frac{1}{(iv\cdot D)^2} + +\frac{1}{(iv\cdot D)^2}(iD\cdot\gamma)D^2\right)\psi\right\}\right] + \mathcal{O}(e^3)$$

Chiral Transport Equation

Son and Yamamoto, '12; Stephanov and Yin, '12, CM and J Torres-Rincon, '14

$$\frac{\partial f_p}{\partial t} + (1+e\hbar\mathbf{B}\cdot\mathbf{\Omega})^{-1}\left\{\left[\tilde{\mathbf{v}} + e\hbar\,\tilde{\mathbf{E}}\times\mathbf{\Omega} + e\hbar\,\mathbf{B}(\tilde{\mathbf{v}}\cdot\mathbf{\Omega})\right]\cdot\frac{\partial f_p}{\partial \mathbf{r}} + e\left[\tilde{\mathbf{E}} + \tilde{\mathbf{v}}\times\mathbf{B} + e\hbar\mathbf{\Omega}\,(\tilde{\mathbf{E}}\cdot\mathbf{B})\right]\cdot\frac{\partial f_p}{\partial \mathbf{p}}\right\} = 0$$

$$\mathbf{\Omega} = \frac{\mathbf{p}}{2p^3}$$ Berry curvature

where

$$\tilde{\mathbf{E}} = \mathbf{E} - \frac{1}{e}\frac{\partial \epsilon^+_{\mathbf{p}}}{\partial \mathbf{r}}$$

$$\tilde{\mathbf{v}} = \frac{\partial \epsilon^+_{\mathbf{p}}}{\partial \mathbf{p}} \qquad \epsilon^{\pm}_{\mathbf{p}} = \pm p\left(1 - e\hbar\,\lambda\frac{\mathbf{B}\cdot\mathbf{p}}{2p^3}\right)$$

helicity

Quantum modifications of density and current

$$n = \int \frac{d^3p}{(2\pi\hbar)^3}(1 + e\hbar\mathbf{B}\cdot\mathbf{\Omega})f_p$$

$$\mathbf{j} = -\int \frac{d^3p}{(2\pi\hbar)^3}\left[\epsilon_p \frac{\partial f_p}{\partial \mathbf{p}} + e\hbar\mathbf{\Omega}\cdot\frac{\partial f_p}{\partial \mathbf{p}}\epsilon_p\mathbf{B} + \hbar\epsilon_p\mathbf{\Omega}\times\frac{\partial f_p}{\partial \mathbf{r}} - e\hbar f_p\mathbf{E}\times\mathbf{\Omega}\right]$$

One can reproduce the chiral anomaly equation

$$\frac{\partial n}{\partial t} + \nabla\cdot\mathbf{j} = -e^2\hbar\int\frac{d^3p}{(2\pi\hbar)^3}\left(\mathbf{\Omega}\cdot\frac{\partial f_p}{\partial \mathbf{p}}\right)\mathbf{E}\cdot\mathbf{B}$$

$$\partial_\mu j^\mu_A = \frac{e^2}{2\pi^2\hbar^2}\mathbf{E}\cdot\mathbf{B} \qquad \partial_\mu j^\mu_V = 0$$

Foldy-Wouthuysen Diagonalization

- The Dirac eq. for a free fermion mixes particles and antiparticles d.o.f.

- It is possible to diagonalize the Dirac equation as an expansion in $\hbar$

CM and J Torres-Rincon, '14

exact for the free theory

approx. for an interacting theory

OSEFT: EFT counterpart of the FW diagonalization in QM

Covariant formulation of the chiral transport equation??

soon to appear …

Hidaka, Pu, Yang '15 …

Peculiar properties
under Lorentz symmetry of massless with spin
(and then also of chiral kinetic theory)

side jumps

Chen, Son Stephanov, `15
Chen, Son Stephanov,, Yee, Yin `15

OSEFT in an arbitrary frame

- Introduce a frame vector

$$p \to u \cdot p \qquad \gamma_0 \to u \cdot \gamma \qquad u^\mu = \frac{v^\mu + \tilde{v}^\mu}{2}$$

$$v^2 = \tilde{v}^2 = 0 \,, \qquad v \cdot \tilde{v} = 2 \qquad u \cdot v = 1 \,, \qquad u^2 = 1$$

$$\mathcal{L} = \bar{\chi}_v(x) \left(i\, v \cdot D + i \not{D}_\perp \frac{1}{2E + i\tilde{v} \cdot D} i \not{D}_\perp \right) \frac{\not{\tilde{v}}}{2} \chi_v(x) + \bar{\xi}_{\tilde{v}}(x) \left(i\, \tilde{v} \cdot D + i \not{D}_\perp \frac{1}{-2E + iv \cdot D} i \not{D}_\perp \right) \frac{\not{v}}{2} \xi_{\tilde{v}}(x)$$

$$P_v = \frac{1}{2} \not{v} \not{u} = \frac{1}{4} \not{v} \not{\tilde{v}} \qquad P_{\tilde{v}} = \frac{1}{2} \not{\tilde{v}} \not{u} = \frac{1}{4} \not{\tilde{v}} \not{v}$$

Sum over velocity directions of SCET L!

Reparametrization Invariance

same as in SCET Manohar et al, 2002

Apparent breaking of Lorentz Invariance $\{v_\mu M^{\mu\nu}, \tilde{v}_\mu M^{\mu\nu}\}$

$$\psi_{v,\tilde{v}}(x) = \psi'_{v',\tilde{v}'}(x)$$

type I	type II	type III
$v^\mu \to v^\mu + \lambda_\perp^\mu$	$v^\mu \to v^\mu$	$v^\mu \to (1+\alpha)v^\mu$
$\tilde{v}^\mu \to \tilde{v}^\mu$	$\tilde{v}^\mu \to \tilde{v}^\mu + \epsilon_\perp^\mu$	$\tilde{v}^\mu \to (1-\alpha)\tilde{v}^\mu$

$$v \cdot \lambda_\perp = v \cdot \epsilon_\perp = \tilde{v} \cdot \lambda_\perp = \tilde{v} \cdot \epsilon_\perp = 0.$$

On-shell and residual parts of the momenta change

$$\delta_{(\mathrm{I})}\mathcal{L}_{p,v} = \delta_{(\mathrm{II})}\mathcal{L}_{p,v} = \delta_{(\mathrm{III})}\mathcal{L}_{p,v} = 0$$

OSEFT used to derive CKT

Derive a transport equation in the EFT

$$S_{E,v}(x,y) = -\langle \bar{\chi}_v(y)\chi_v(x)\rangle$$

$$S = \sum_{\chi=\pm} P_\chi J^\rho_\chi \gamma_\rho \ , \qquad P_\chi = \frac{1+\chi\gamma_5}{2}$$

$$J^\rho_\chi = v^\rho G_\chi$$

$$G_{E,v}(x,y) = \langle \bar{\chi}_v(y)\frac{\tilde{\not{v}}}{2}\chi_v(x)\rangle$$

Wigner transform (gauge cov. modified)

$$X = \frac{1}{2}(x+y) \ , \quad s = x-y$$

$$\bar{G}_v(X,k) = \int d^4 s e^{ik\cdot s} U\left(X, X+\frac{s}{2}\right) G_v(X+\frac{s}{2}, X-\frac{s}{2}) U\left(X-\frac{s}{2}, X\right)$$

- Deduce both on-shell conditions and dynamical eqs at every order in the 1/E expansion

- Perform the Wigner transformation and a gradient expansion $\partial_X \ll \partial_s$

- Project over chiralities

- Return to the original variables

$$G^{\chi}_{E,v}(X,k) = a\delta_+(K^{\chi}) f^{\chi}_{E,v}(X,k)$$

dispersion law at n=1

$$k_0 = k_{\parallel} + \frac{\mathbf{k}_{\perp} - \lambda \mathbf{B} \cdot \mathbf{v}}{2p} \qquad q_0 = \epsilon_q = p + k_0$$

$$\epsilon_q = q - \lambda \frac{\mathbf{B} \cdot \mathbf{q}}{2q^2}$$

Outlook

- OSEFT: tool to treat systematically the hard degrees of freedom in hot plasmas - with all the advantages of the EFTs
- OSEFT also helpful for transport theory