QUARKONIUM IN NUCLEI

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INT Program INT-18-1b

Multi-Scale Problems Using Effective Field Theories
May 7 - June 1, 2018

This talk based on:

G.K., A.W. Thomas & K.Tsushima

— Prog. Part. Nucl. Phys. 100, 161 (2018)

N. Brambilla, G.K., J.Tarrús-Castellà & A.Vairo

— Phys. Rev. D 93, 054002 (2016)

J.Tarrús-Castellà & G.K.

— arXiv: 1803.05412

Motivation

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Nothing really ambitious

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We want to understand matter that amounts to 5% of the mass of the universe

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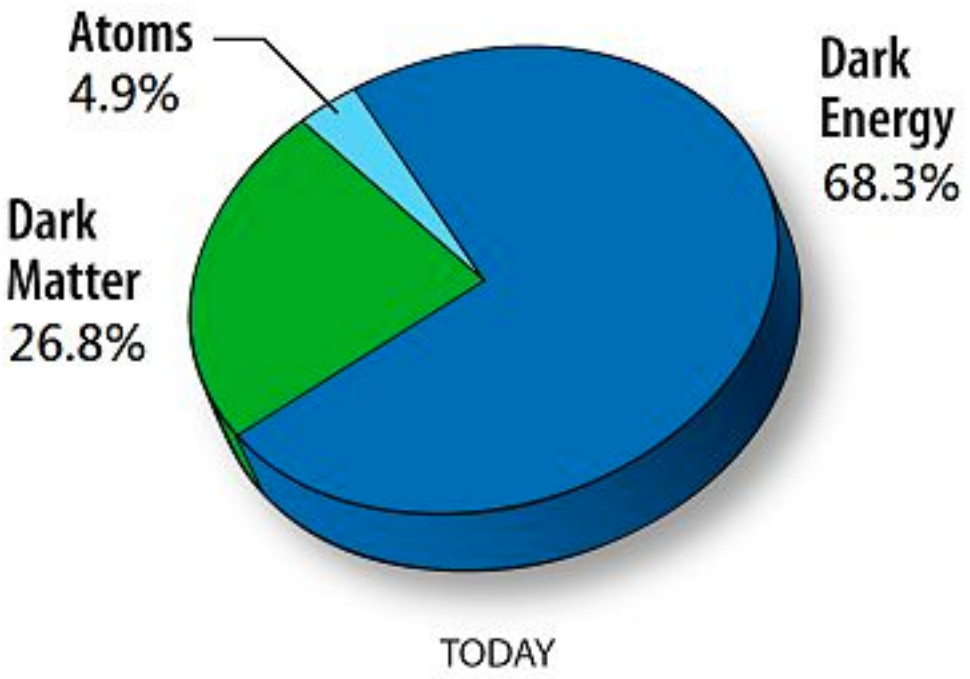

Actually we are even less ambitious than that

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We want to understand the proton mass

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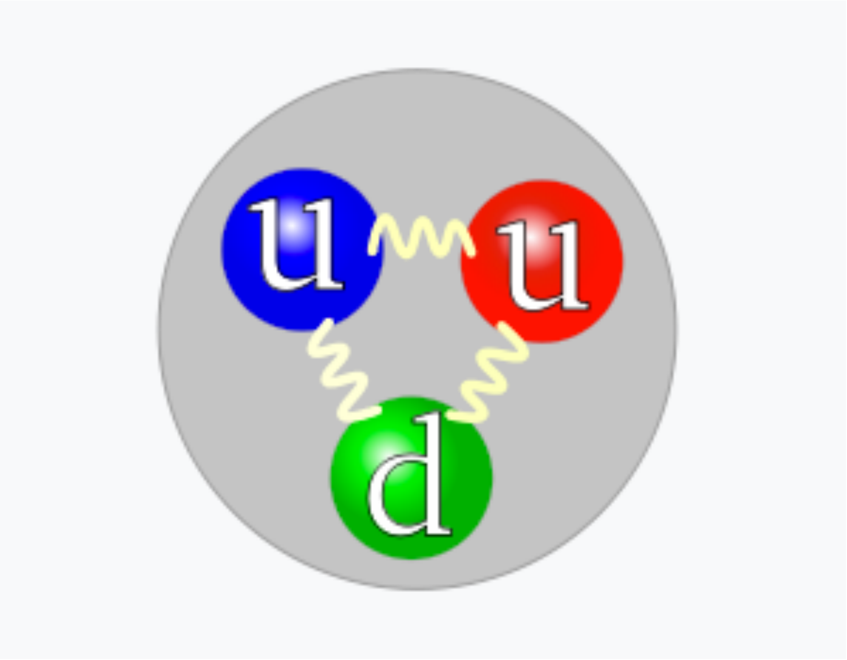
We want to understand the proton mass



Starting point ?

Seems to be

Q C D

Quantum Chromodynamics

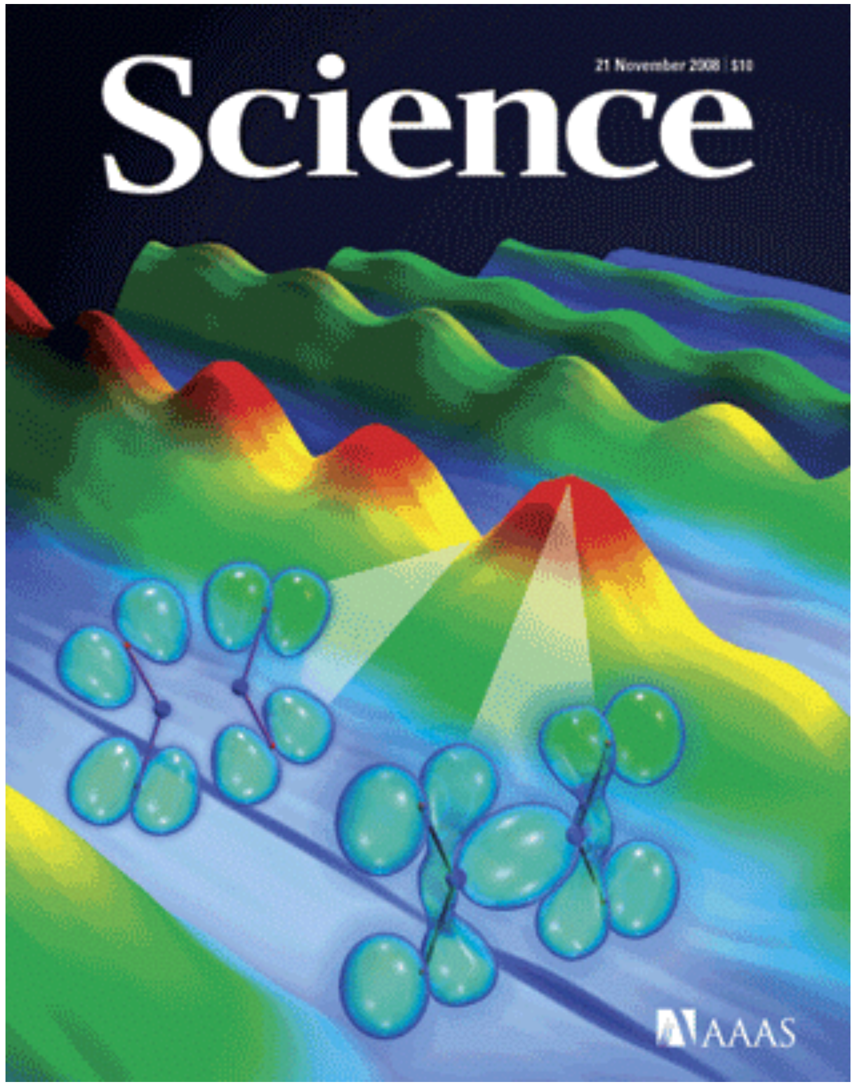
Science

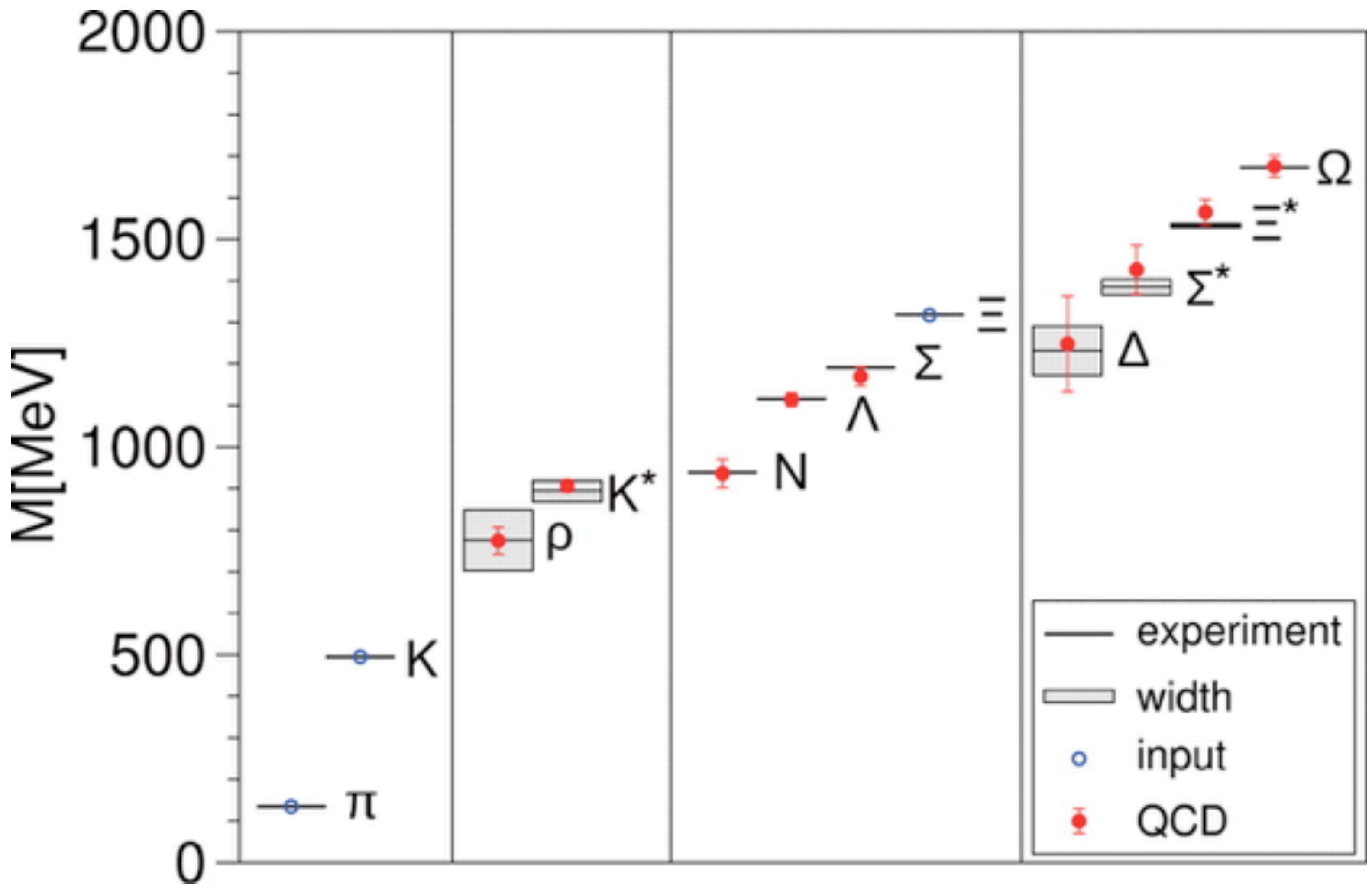
2008

Ab Initio Determination of Light Hadron Masses

S. Dürr, Z. Fodor, J. Frison, C. Hoelbling, R. Hoffmann, S. D. Katz, S. Krieg, T. Kurth, L. Lellouch, T. Lippert, K. K. Szabo and G. Vulvert

Science **322** (5905), 1224-1227.
DOI: 10.1126/science.1163233





Science

2015

Ab initio calculation of the neutron-proton mass difference

Sz. Borsanyi, S. Durr, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, L. Lellouch, T. Lippert, A. Portelli, K. K. Szabo and B. C. Toth

Science **347** (6229), 1452-1455.
DOI: 10.1126/science.1257050

Measuring the legacy of child abuse pp. 1408 & 1480

Overlooked trade-offs in biofuel versus food debates p. 1420

The mass difference between protons and neutrons p. 1452

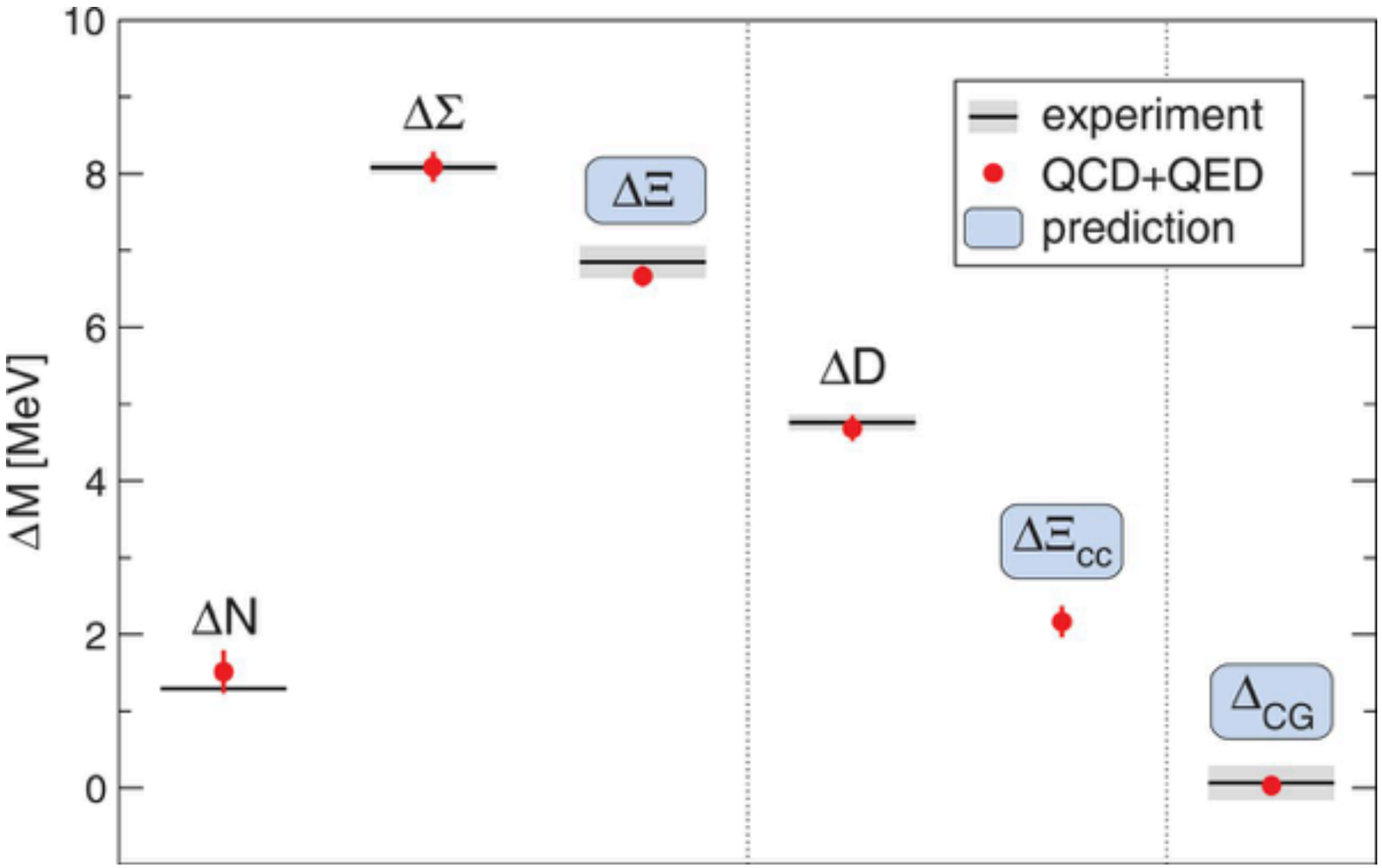
Science

$10
27 MARCH 2015
sciencemag.org

AAAS

Gang of three

How dynactin, dynein, and Bicaudal-D2 motor together
p. 1441



Computation of the masses

Computation of the masses

$h(x):$ hadron interpolating field, e.g. $\pi^+(x) = \overline{u}(x)\gamma_5 d(x)$

$$\langle h(x)h(x+T)\rangle = \frac{\int \left[\mathcal{D}\psi\bar{\psi}A_\mu\right]\, h(x)h(x+T)\, e^{-\int d^4x\, \mathcal{L}_{\rm QCD}}}{\int \left[\mathcal{D}\psi\bar{\psi}A_\mu\right]\, e^{-\int d^4x\, \mathcal{L}_{\rm QCD}}}$$

$$\boxed{\lim_{T\to\infty} \langle h(x)h(x+T)\rangle \sim e^{-M_h\, T}}$$

Great, Impressive …

Great, Impressive …

BUT, how precisely those numbers

come out from

the QCD Lagrangian ?

Trace anomaly

Take $m_q = 0$ & $m_Q = \infty$

$$x^\mu \to x'^\mu = \lambda\, x^\mu$$

$$q(x) \to q'(x) = \lambda^{3/2}\, q(\lambda x) \qquad A_\mu(x) \to A'_\mu(x) = \lambda\, A_\mu(\lambda x)$$

$$S'_{\rm QCD} = \int d^4x\, \lambda^4\, \mathcal{L}_{\rm QCD}(\lambda x) = \int d^4x'\, \mathcal{L}_{\rm QCD}(x') = S_{\rm QCD}$$

Classical action is invariant

Hadron masses

$|h\rangle$: hadron state $\qquad m_h = \langle h|T^\mu_\mu(x)|h\rangle$

From classical Lagrangian:

$$\frac{\delta S_{\rm QCD}}{\delta\lambda} = -\int d^4x\, T^\mu_\mu(x) = 0$$

$$\boxed{\langle h|T^\mu_\mu|h\rangle = m_h \to 0}$$

Quantum theory

$$\boxed{g = g(\mu)}$$

$$\delta S_{\rm QCD} = \delta\left(-\frac{1}{4\pi\alpha_s}\frac{1}{4}\int d^4x\,\overline{G}^a_{\mu\nu}(x)\overline{G}^{a\mu\nu}(x)\right) = -\frac{2\beta(\alpha_s)}{\alpha_s}S_{\rm QCD}\,\delta\lambda$$

$$\begin{aligned} T^\mu_\mu(x) &= \frac{2\beta(\alpha_s)}{\alpha_s}\frac{1}{4}G^a_{\mu\nu}(x)G^{a\mu\nu}(x) = -\frac{1}{2}b_0\alpha_s\,G^a_{\mu\nu}(x)G^{a\mu\nu}(x) \\ &= -\frac{9}{32\pi^2}g^2G^a_{\mu\nu}(x)G^{a\mu\nu}(x) \end{aligned}$$

— this is the trace anomaly
— no scale invariance
— trace of $T^{\mu\nu}$ is nonzero

$$\boxed{m_h = -\frac{9}{32\pi^2}\langle h|g^2G^a_{\mu\nu}G^{a\mu\nu}|h\rangle}$$

The entire mass comes from gluons

Contribution from quark masses

$$m_h = \frac{\beta(\alpha_s)}{2\alpha_s} G^a_{\mu\nu}(x) G^{a\mu\nu}(x) + \langle h | \bar{q}\, m_q\, q | h \rangle$$

$$\uparrow$$

small

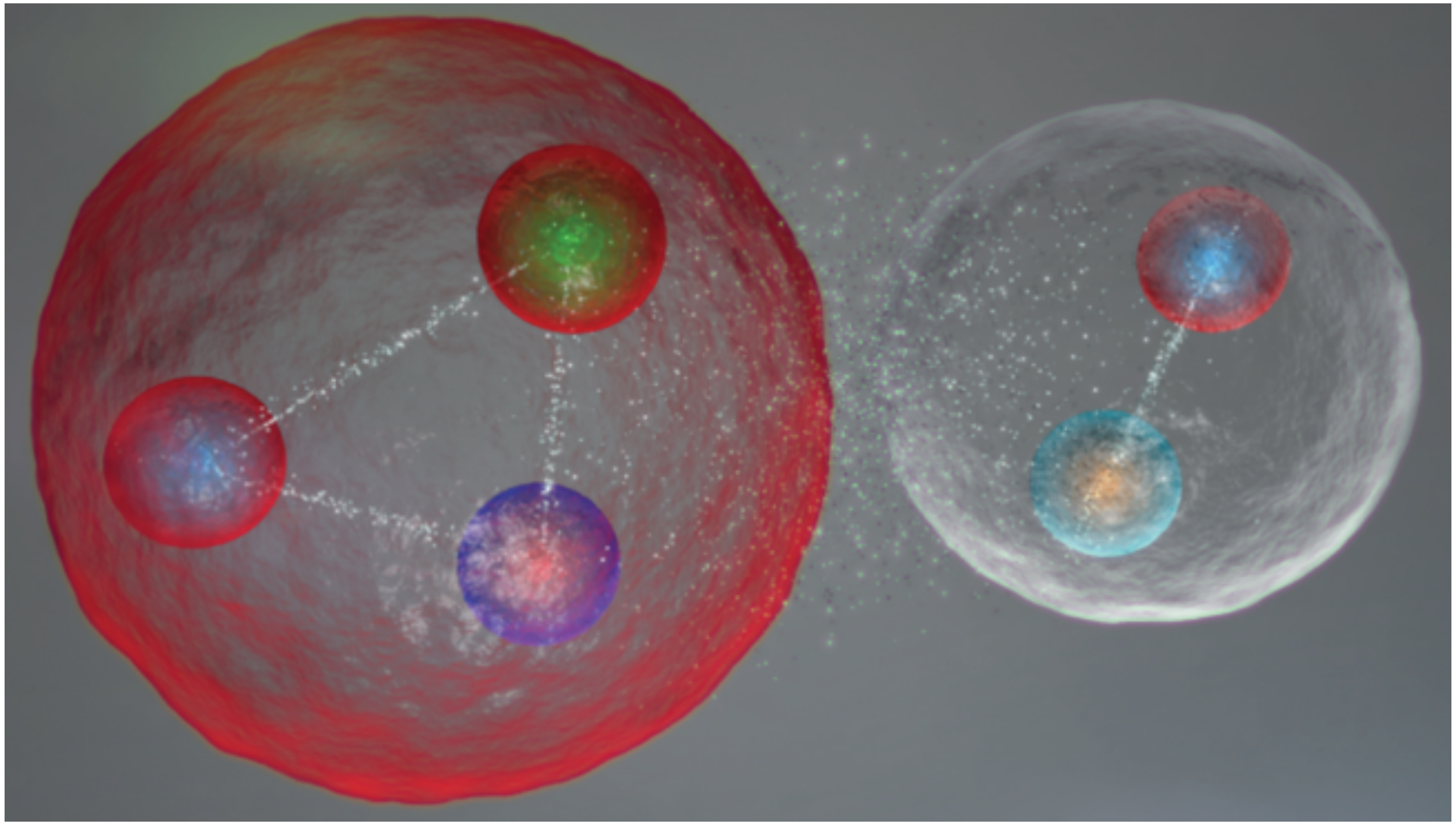
Why is this interesting ?

Because

$$\boxed{\langle h | g^2 G^a_{\mu\nu} G^{a\mu\nu} | h \rangle}$$

contributes to threshold
quarkonium-nucleon scattering

Quarkonium-nucleon



Quarkonium: $\phi(s\bar{s}), \quad \eta_c(c\bar{c}), \quad J/\Psi(c\bar{c}), \quad \eta_b(b\bar{b}), \quad \Upsilon(b\bar{b})$

Quarkonium-nucleon scattering

$$\varphi = \phi(s\bar{s}), \quad \eta_c(c\bar{c}), \quad J/\Psi(c\bar{c}), \quad \eta_b(b\bar{b}), \quad \Upsilon(b\bar{b})$$

Forward amplitude

$$\mathcal{A}_{\varphi N} = \frac{1}{2}\alpha_\varphi \langle N| \left(g\vec{E}\right)^2 |N\rangle$$

α_φ : color polarizability

(property of the quarkonium)

$$\mathcal{A}_{\varphi N} = \frac{1}{2}\alpha_\varphi \langle N| \left(g\vec{E}\right)^2 |N\rangle$$

Measure scattering length:

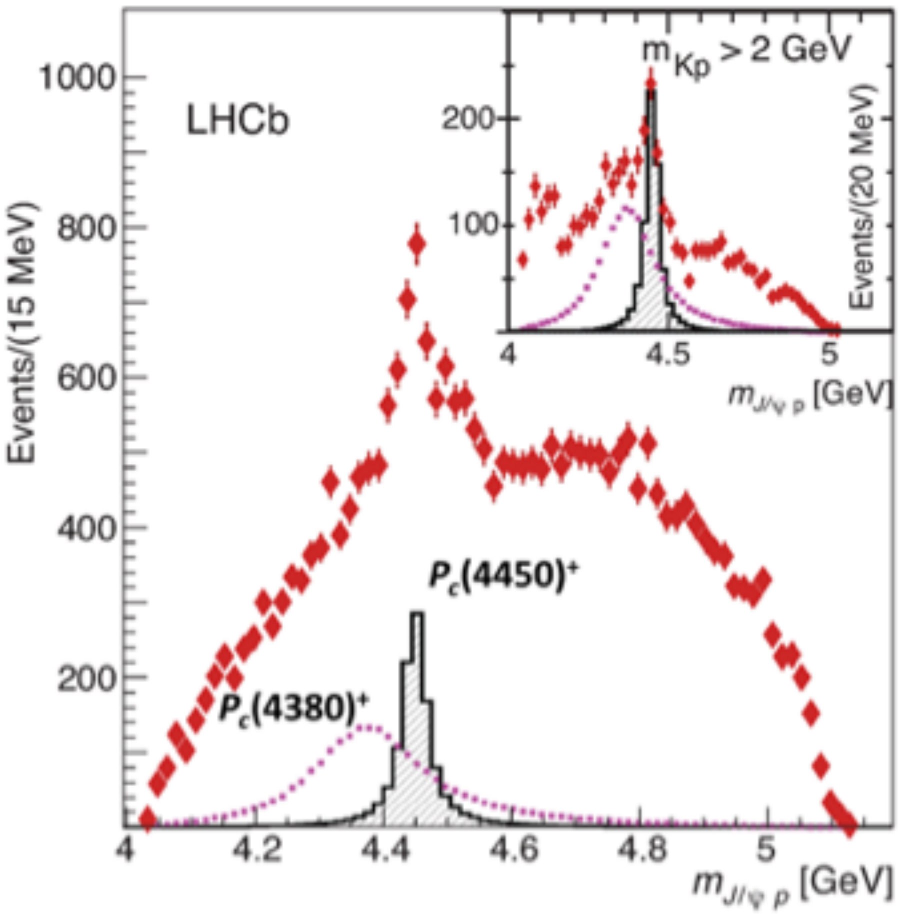
$$a_{\varphi N} = -\left(\frac{\mu_{\varphi N}}{2\pi}\right) \mathcal{A}_{\varphi N} = -\left(\frac{\mu_{\varphi N}}{4\pi}\right) \alpha_\varphi \langle N| \left(g\vec{E}\right)^2 |N\rangle$$

Bound from trace anomaly:

$$\langle N| \left[\left(g\vec{E}\right)^2 - \left(g\vec{B}\right)^2\right] |N\rangle = -\frac{1}{2}\langle N|g^2 G^a_{\mu\nu} G^{a\mu\nu}|N\rangle = \frac{16\pi^2}{9} m_N \leq \langle N| \left(g\vec{E}\right)^2 |N\rangle$$

$$\boxed{a_{\varphi N} \leq -\left(\frac{\mu_{\varphi N}}{4\pi}\right) \frac{16\pi^2}{9} m_N \alpha_\varphi = -\frac{4\pi m_N}{9} \mu_{\varphi N} \alpha_\varphi}$$

Renewed interest in quarkonium-nucleon



Why quarkonium in nuclei?

— scattering amplitude is enhanced

— new exotic nuclear state

— adds a new flavor axis in the nuclear e.o.s.

Scales in nuclei

$\rho \sim \rho_0 = 0.16 \text{ fm}^{-3}$	baryon density (center nucleus)
$d_{\rm av} \sim \rho^{-1/3} \sim 1.8 \text{ fm}$	internucleon distance
$\langle T_{\rm N} \rangle \sim \frac{3}{5} \frac{k_F^2}{2m_N} \sim 20\,\text{MeV} \ll \Lambda_{\rm QCD}$	nucleon kinetic energy

Recall that:

$r_{\rm N} \sim \sqrt{\langle r_{\rm ch}^2 \rangle} \simeq 0.88 \text{ fm} \sim \Lambda_{\rm QCD}^{-1}$	nucleon (charge) radius in free space
$r_{\rm NN}^{\rm hard-core} \sim 0.2\,\text{fm}$	hard-core NN force

Scales in nuclei

$d_{\mathrm{av}} \sim 2\, r_{\mathrm{N}}$ + hard-core NN force

Little (or no) superposition of nucleons in nuclei

\+ Pauli principle (among nucleons)

— Independent-particle model
— Mean-field model
— Nuclear shell model

Gomes, Walecka & Weisskopf

Low-momentum quarkonium in a nucleus

— Quarkonium interacts with light quarks in nucleons by exchanging gluons with wavelengths

$$\lambda \sim r_N$$

— Size of quarkonium

$$r_{J/\Psi} \sim 0.35\,\mathrm{fm}$$

$$\lambda \geq 2\, r_{J/\Psi}$$

⇨

Quarkonium behaves as
a small color dipole
immersed in a uniform gluon field

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$$\langle T_{\mathrm{N}} \rangle \sim \frac{3}{5}\frac{k_F^2}{2m_N} \sim 20\,\mathrm{MeV} \ll \Lambda_{\mathrm{QCD}}$$

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Embbeding quakonium-nucleon into a Nonrelativistic nuclear many-body problem

$$H = H_N + H_{\varphi N}$$

$$\begin{aligned} H_{\varphi N} &= \int d^3r\, \varphi^\dagger(t,\vec{r}) \left(-\frac{1}{2m_\varphi}\nabla^2 \right) \varphi(t,\vec{r}) \\ &+ \int d^3r d^3r'\, N^\dagger(t,\vec{r})\varphi^\dagger(t,\vec{r}\,')\, \underset{\uparrow}{W_{\varphi N}}(\vec{r}-\vec{r}\,')\, \varphi(t,\vec{r}\,')N(t,\vec{r}) \end{aligned}$$

quarkonium-nucleon

G.K., A.W. Thomas & K.Tsushima,
PPNP 100, 161 (2018)

Hartree-Fock equation

— for quarkonium in a nucleus

$$\boxed{-\frac{1}{2m_\varphi}\nabla^2\varphi_\alpha(\vec{r}) + W_{\varphi A}(\vec{r})\varphi_\alpha(\vec{r}) = \epsilon_\alpha\varphi_\alpha(\vec{r})}$$

$$W_{\varphi A}(\vec{r}) = \int d^3r'\, W_{\varphi N}(\vec{r}-\vec{r}')\, \rho_A(\vec{r}')$$

quarkonium-<u>nucleus</u> potential

$$\rho_A(\vec{r}) = \langle A|N^\dagger(\vec{r})N(\vec{r})|A\rangle = \sum_{n=1}^{A} N_n^*(\vec{r})N_n(\vec{r})$$

nuclear density functional

Neglecting back reaction of quarkonium on nucleons,
take density from experiment, no need for a nuclear model

Need quarkonium-nucleon potential

$$W_{\varphi A}(\vec{r}) = \int d^3r'\, W_{\varphi N}(\vec{r}-\vec{r}')\, \rho_A(\vec{r}')$$

From the forward amplitude:

$$W^{\rm pol}_{\varphi N}(\vec{r}) = \frac{4\pi}{2\mu_{\varphi N}}\, a_{\varphi N}\, \delta(\vec{r}) = -\frac{8\pi^2}{9} m_N \alpha_\varphi\, \delta(\vec{r})$$

$$\boxed{W^{\rm pol}_{\varphi A}(\vec{r}) = \frac{4\pi}{2\mu_{\varphi N}}\, a_{\varphi N}\, \rho_A(\vec{r}) = -\frac{8\pi^2}{9} m_N \alpha_\varphi\, \rho_A(\vec{r})}$$

$$k\ \mathrm{cotan}\,\delta(k) = -\frac{1}{a} + \frac{1}{2} r_e k^2 + \cdots$$

J/Ψ in nuclei

— nuclear potentials

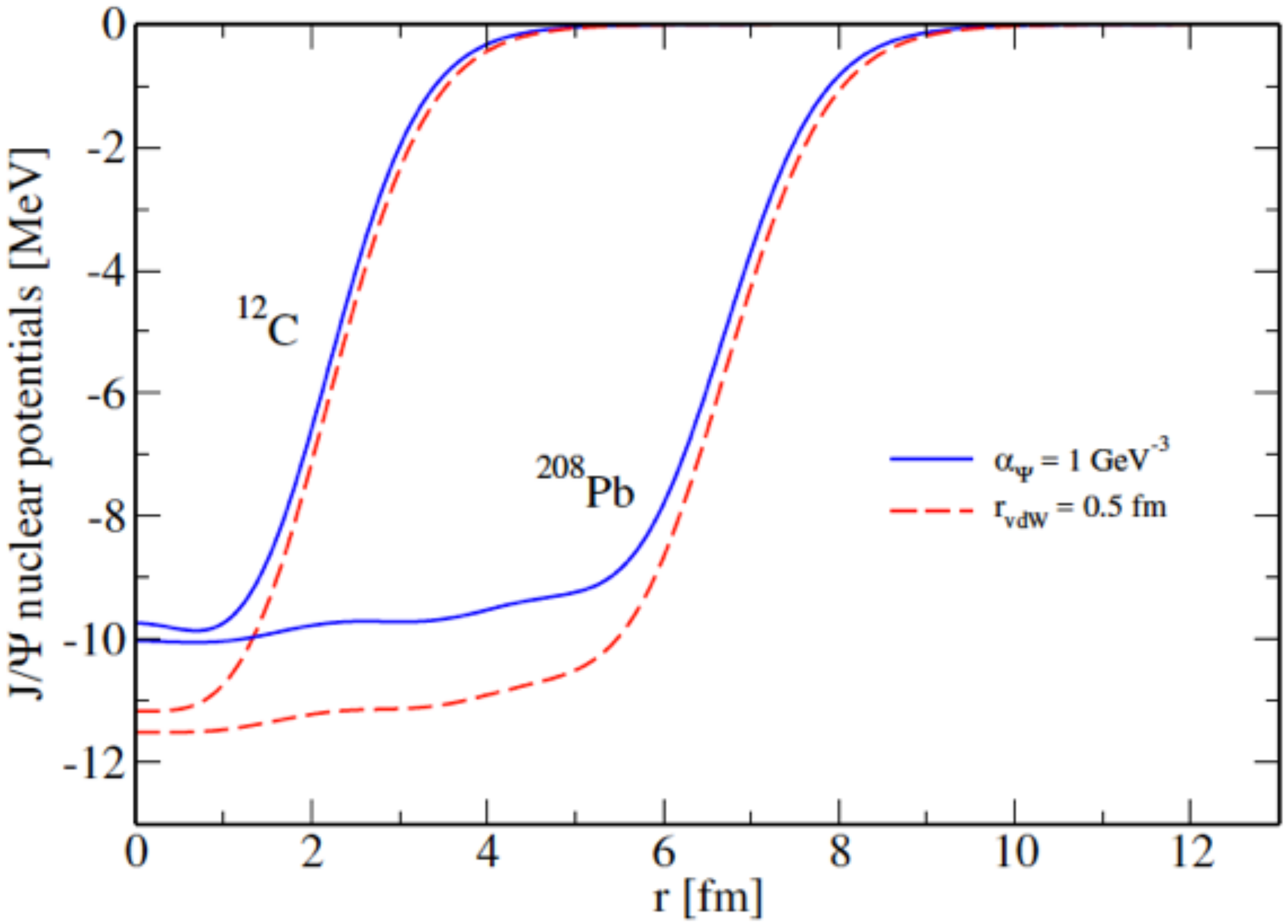


Figure 8: J/Ψ nuclear potentials $W^{\mathrm{pol}}_{J/\Psi A}(\vec{r})$ (solid line) for a polarizability $\alpha_{J/\Psi} = 1\ \mathrm{GeV}^{-3}$ and $W^{\mathrm{latt}}_{J/\Psi A}(\vec{r})$ (dashed line) from a fit to the lattice data with a cutoff $r_{\mathrm{vdW}} = 0.5$ fm.

Quarkonium in nuclei

— use scattering length only

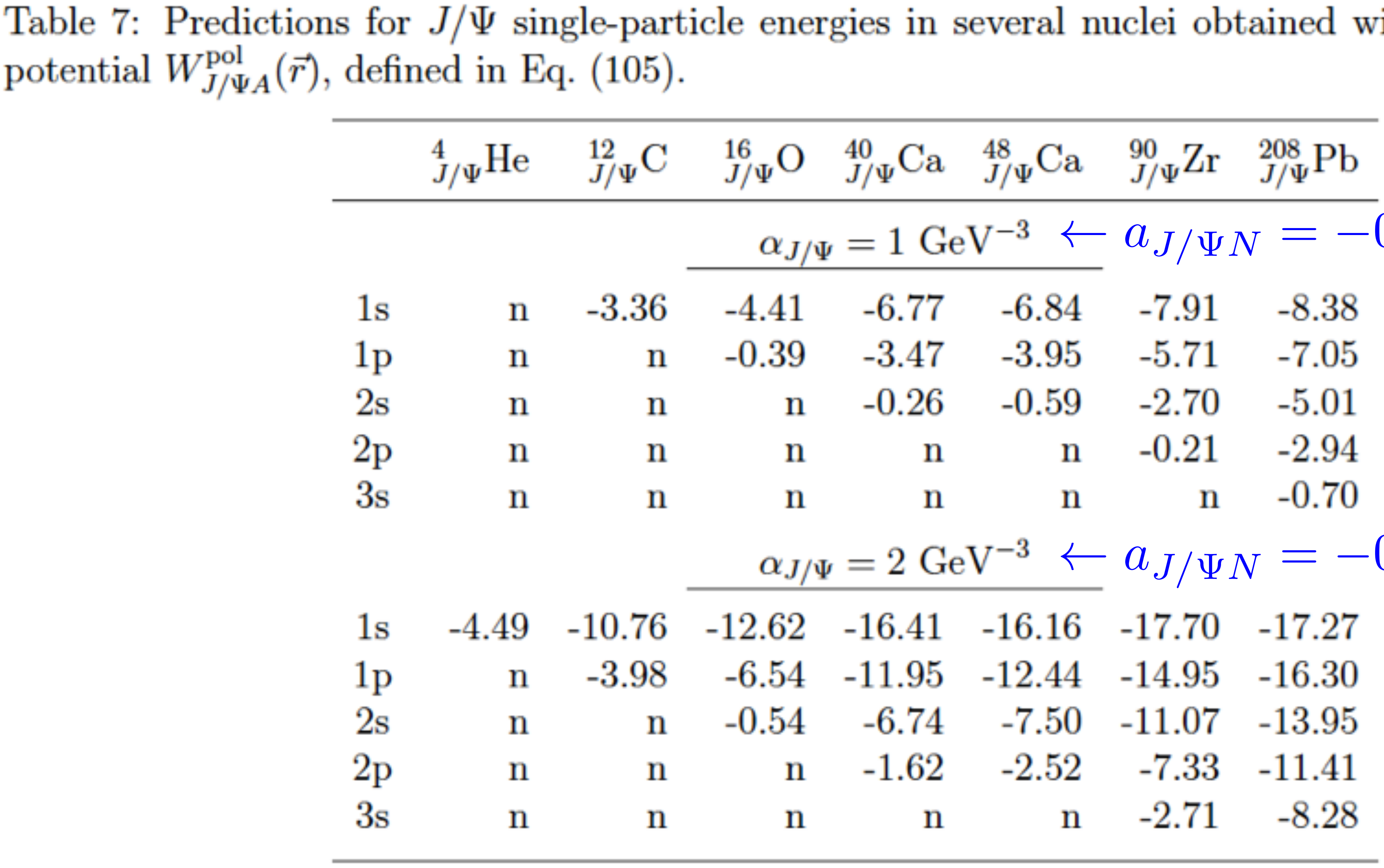
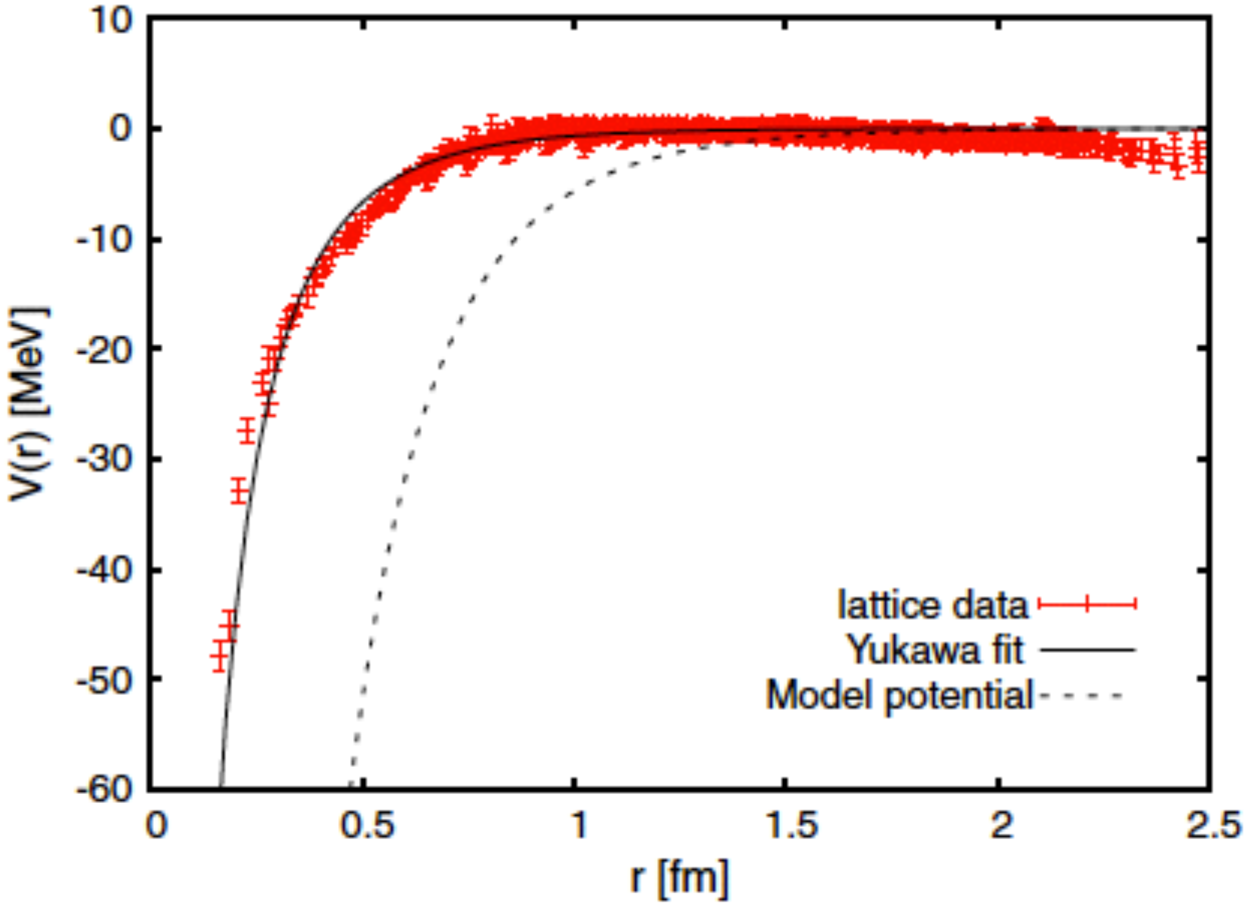


Table 7: Predictions for J/Ψ single-particle energies in several nuclei obtained with the polarization potential $W^{\text{pol}}_{J/\Psi A}(\vec{r})$, defined in Eq. (105).

	${}^{4}_{J/\Psi}\text{He}$	${}^{12}_{J/\Psi}\text{C}$	${}^{16}_{J/\Psi}\text{O}$	${}^{40}_{J/\Psi}\text{Ca}$	${}^{48}_{J/\Psi}\text{Ca}$	${}^{90}_{J/\Psi}\text{Zr}$	${}^{208}_{J/\Psi}\text{Pb}$
			$\alpha_{J/\Psi} = 1\ \text{GeV}^{-3}$ $\leftarrow a_{J/\Psi N} = -0.18\,\text{fm}$				
1s	n	-3.36	-4.41	-6.77	-6.84	-7.91	-8.38
1p	n	n	-0.39	-3.47	-3.95	-5.71	-7.05
2s	n	n	n	-0.26	-0.59	-2.70	-5.01
2p	n	n	n	n	n	-0.21	-2.94
3s	n	n	n	n	n	n	-0.70
			$\alpha_{J/\Psi} = 2\ \text{GeV}^{-3}$ $\leftarrow a_{J/\Psi N} = -0.36\,\text{fm}$				
1s	-4.49	-10.76	-12.62	-16.41	-16.16	-17.70	-17.27
1p	n	-3.98	-6.54	-11.95	-12.44	-14.95	-16.30
2s	n	n	-0.54	-6.74	-7.50	-11.07	-13.95
2p	n	n	n	-1.62	-2.52	-7.33	-11.41
3s	n	n	n	n	n	-2.71	-8.28

Lattice*

— quenched, m$_\pi$ = 640 MeV



Yukawa fit

$$V_{N\eta_c} = -\gamma \, \frac{e^{-\alpha\, r}}{r}$$

$$\gamma = 0.1$$
$$\alpha = 3 \text{ fm}^{-1}$$

T. Kawanay & S. Sasaki, PRD 82, 091501 (2010)

Pion mass dependence

— quenched x unquenched

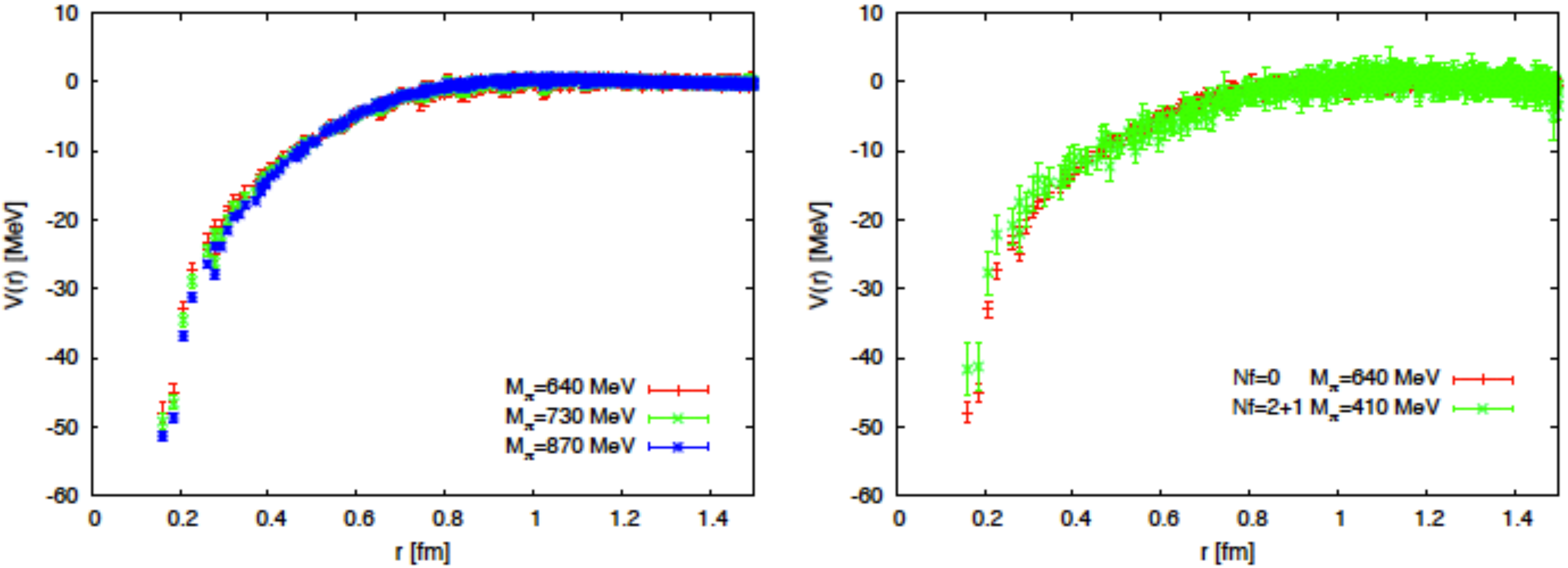


Figure 2: The quark-mass dependence of the η_c-N potential (left) and a comparison between quenched and dynamical simulations (right)

T. Kawanay & S. Sasaki, Pos (Lattice) 2010, 156 (2010)

Scattering length & effective range

— unquenched

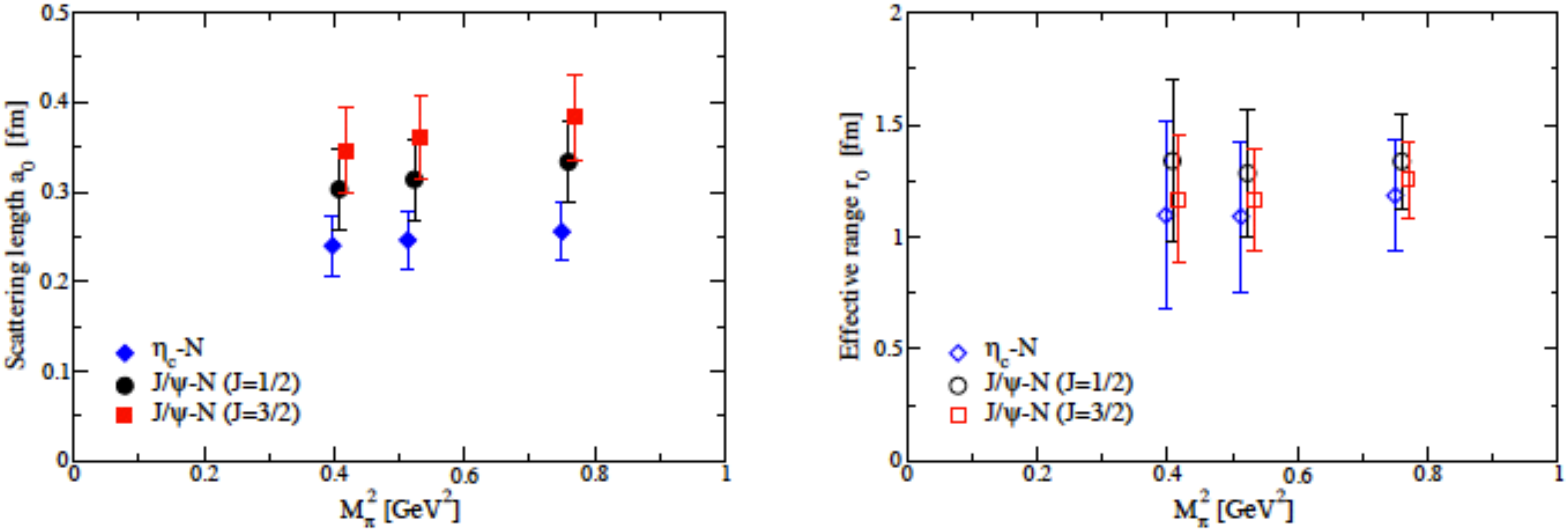


Figure 5: The scattering length a_0 (left) and effective range r_0 (right) as a function of M_π^2. The squared (diamond) symbols have been moved slightly in the plus (minus) x-direction.

T. Kawanay & S. Sasaki, Pos (Lattice) 2010, 156 (2010)

Fit to lattice results

Reproduce scattering length,
leave effective range free

$$W^{\mathrm{latt}}_{\varphi N}(r) = -W_0\,[1 - f(r, r_{\mathrm{vdW}})] + V^{\mathrm{fit}}_{\eta_c N}(r)\, f(r, r_{\mathrm{vdW}})$$

$$f(r, r_{\mathrm{vdW}}) = \frac{1}{1 + (r_{\mathrm{vdW}}/r)^{10}}.$$

$$\boxed{W^{\mathrm{latt}}_{\varphi A}(\vec{r}) = \int d^3r'\, W^{\mathrm{latt}}_{\varphi N}(\vec{r} - \vec{r}\,')\rho_A(\vec{r}\,').}$$

Fit to lattice results

$$(a_{J/\Psi N})_{\mathrm{SAV}} \sim 0.35\ \mathrm{fm} > a_{\eta_c N} \sim 0.25\ \mathrm{fm}$$

$$r_e \sim 1.0\ \mathrm{fm}$$

	$\eta_c N$		$J/\Psi N$	
r_{vdW}	W_0	r_e	W_0	r_e
0.3	252	1.4	288	1.2
0.5	74	1.7	95	1.4

J/Ψ in nuclei

— nuclear potentials

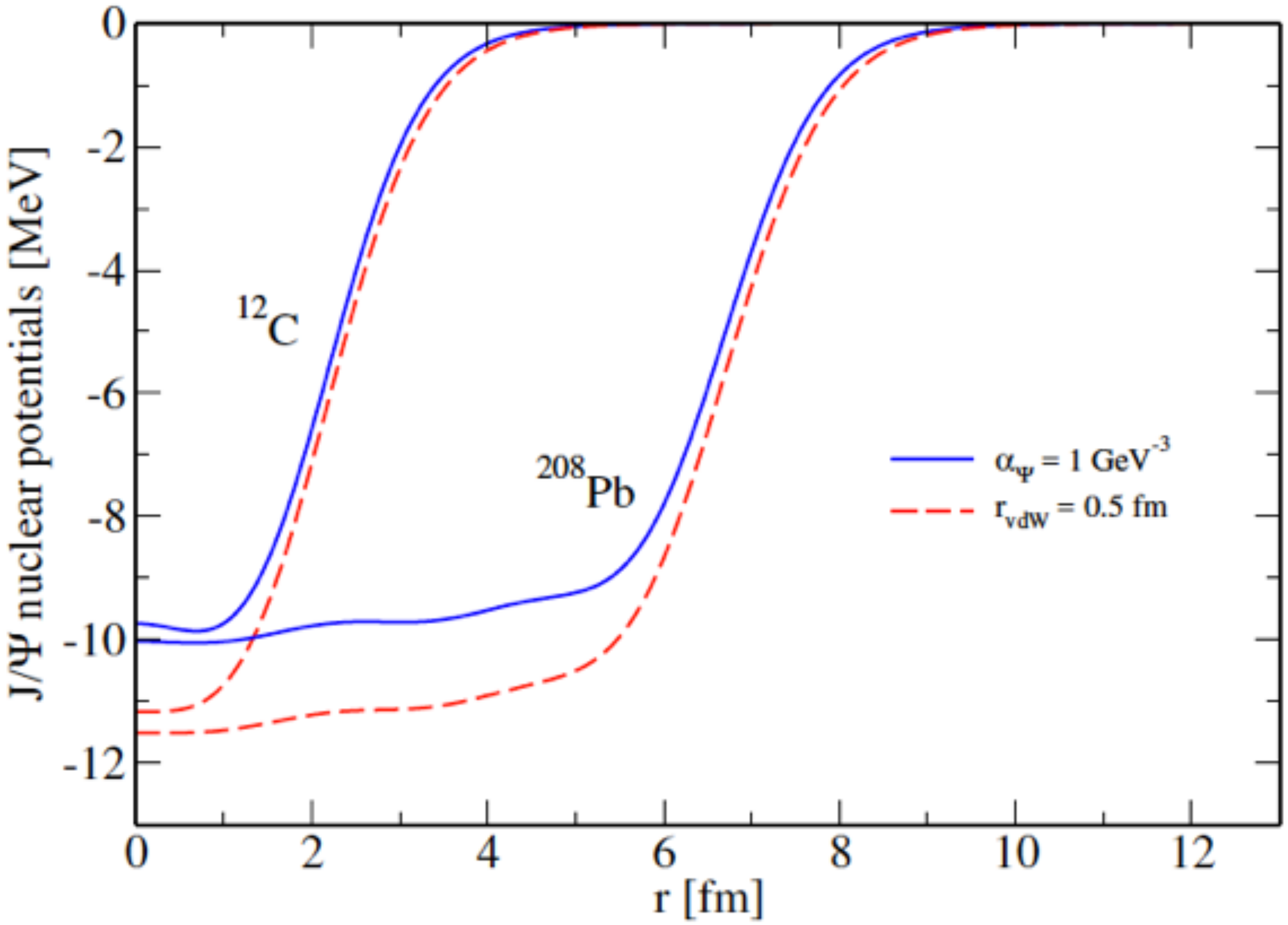


Figure 8: J/Ψ nuclear potentials $W^{\rm pol}_{J/\Psi A}(\vec{r})$ (solid line) for a polarizability $\alpha_{J/\Psi} = 1\ \mathrm{GeV}^{-3}$ and $W^{\rm latt}_{J/\Psi A}(\vec{r})$ (dashed line) from a fit to the lattice data with a cutoff $r_{\rm vdW} = 0.5$ fm.

Quarkonium in nuclei

— use of lattice potential

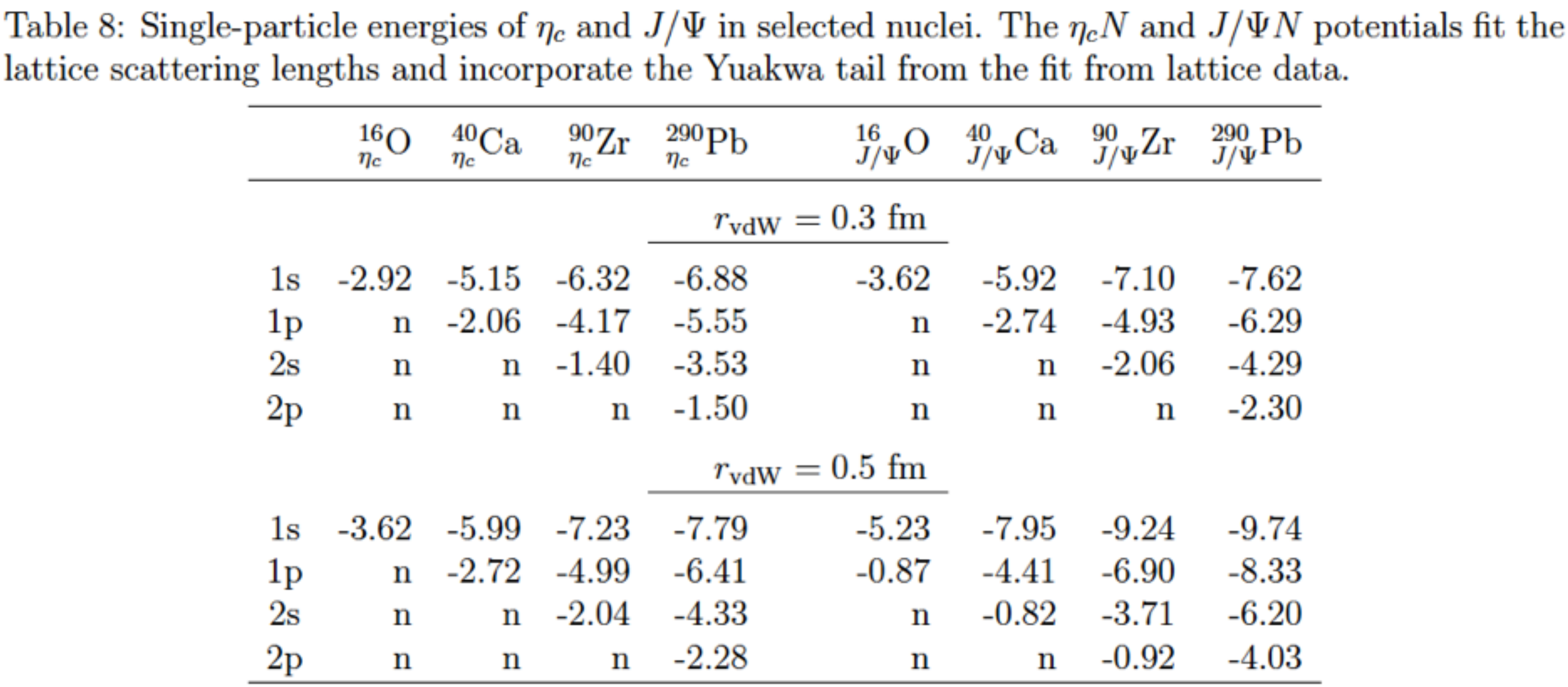


Table 8: Single-particle energies of η_c and J/Ψ in selected nuclei. The $\eta_c N$ and $J/\Psi N$ potentials fit the lattice scattering lengths and incorporate the Yuakwa tail from the fit from lattice data.

	$^{16}_{\eta_c}$O	$^{40}_{\eta_c}$Ca	$^{90}_{\eta_c}$Zr	$^{290}_{\eta_c}$Pb	$^{16}_{J/\Psi}$O	$^{40}_{J/\Psi}$Ca	$^{90}_{J/\Psi}$Zr	$^{290}_{J/\Psi}$Pb
				$r_{\rm vdW} = 0.3$ fm				
1s	-2.92	-5.15	-6.32	-6.88	-3.62	-5.92	-7.10	-7.62
1p	n	-2.06	-4.17	-5.55	n	-2.74	-4.93	-6.29
2s	n	n	-1.40	-3.53	n	n	-2.06	-4.29
2p	n	n	n	-1.50	n	n	n	-2.30
				$r_{\rm vdW} = 0.5$ fm				
1s	-3.62	-5.99	-7.23	-7.79	-5.23	-7.95	-9.24	-9.74
1p	n	-2.72	-4.99	-6.41	-0.87	-4.41	-6.90	-8.33
2s	n	n	-2.04	-4.33	n	-0.82	-3.71	-6.20
2p	n	n	n	-2.28	n	n	-0.92	-4.03

PHYSICAL REVIEW D **91,** 114503 (2015)

Quarkonium-nucleus bound states from lattice QCD

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(Received 9 November 2014; published 11 June 2015)

Quarkonium-nucleus systems are composed of two interacting hadronic states without common valence quarks, which interact primarily through multigluon exchanges, realizing a color van der Waals force. We present lattice QCD calculations of the interactions of strange and charm quarkonia with light nuclei. Both the strangeonium-nucleus and charmonium-nucleus systems are found to be relatively deeply bound when the masses of the three light quarks are set equal to that of the physical strange quark. Extrapolation of these results to the physical light-quark masses suggests that the binding energy of charmonium to nuclear matter is $B_{\rm phys}^{\rm NM} \lesssim 40$ MeV.

DOI: 10.1103/PhysRevD.91.114503 PACS numbers: 11.15.Ha, 12.38.Gc, 13.40.Gp

Models

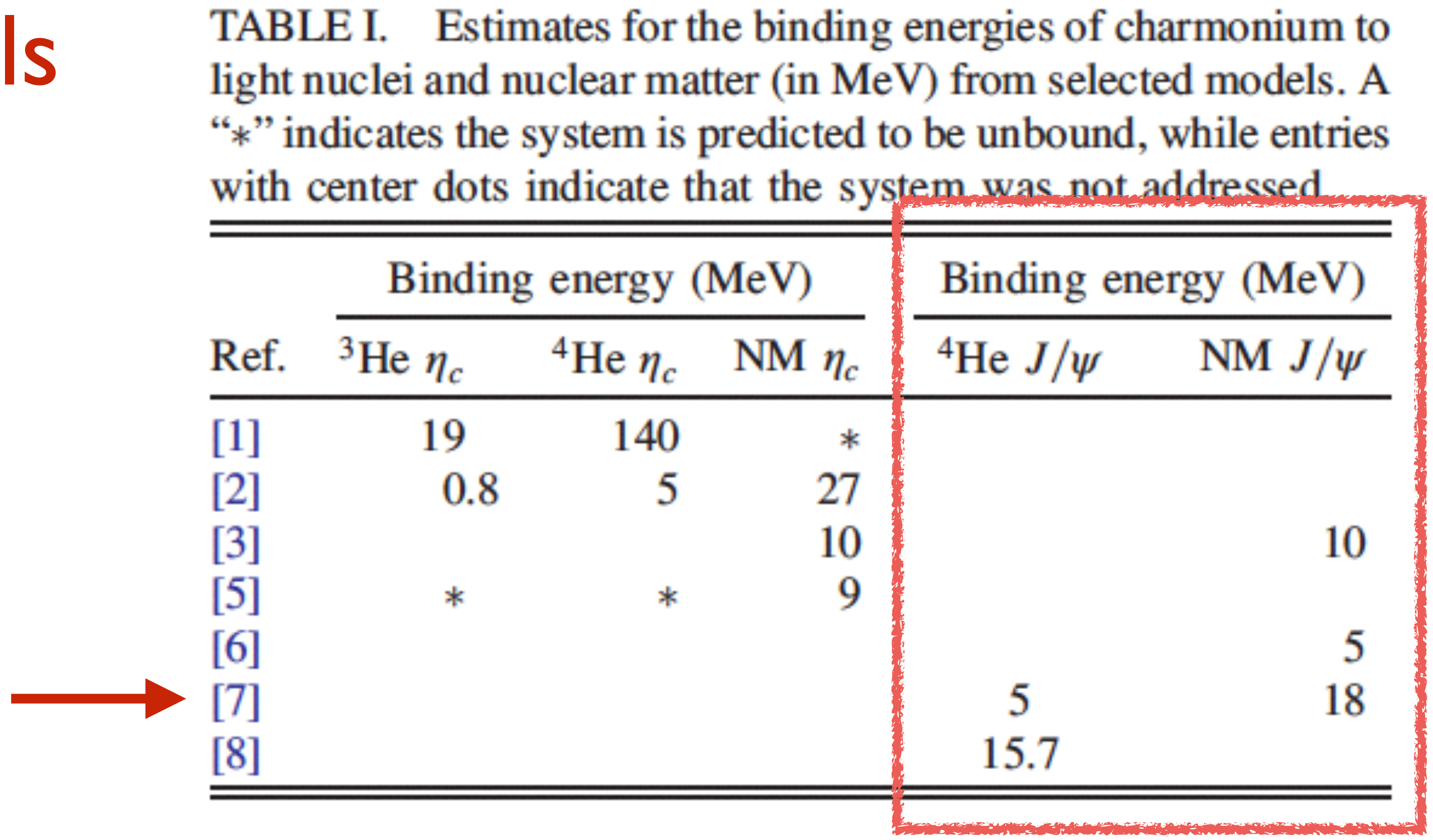


TABLE I. Estimates for the binding energies of charmonium to light nuclei and nuclear matter (in MeV) from selected models. A "$*$" indicates the system is predicted to be unbound, while entries with center dots indicate that the system was not addressed.

	Binding energy (MeV)			Binding energy (MeV)	
Ref.	^{3}He η_c	^{4}He η_c	NM η_c	^{4}He J/ψ	NM J/ψ
[1]	19	140	$*$		
[2]	0.8	5	27		
[3]			10		10
[5]	$*$	$*$	9		
[6]					5
[7]				5	18
[8]				15.7	

NPLQCD

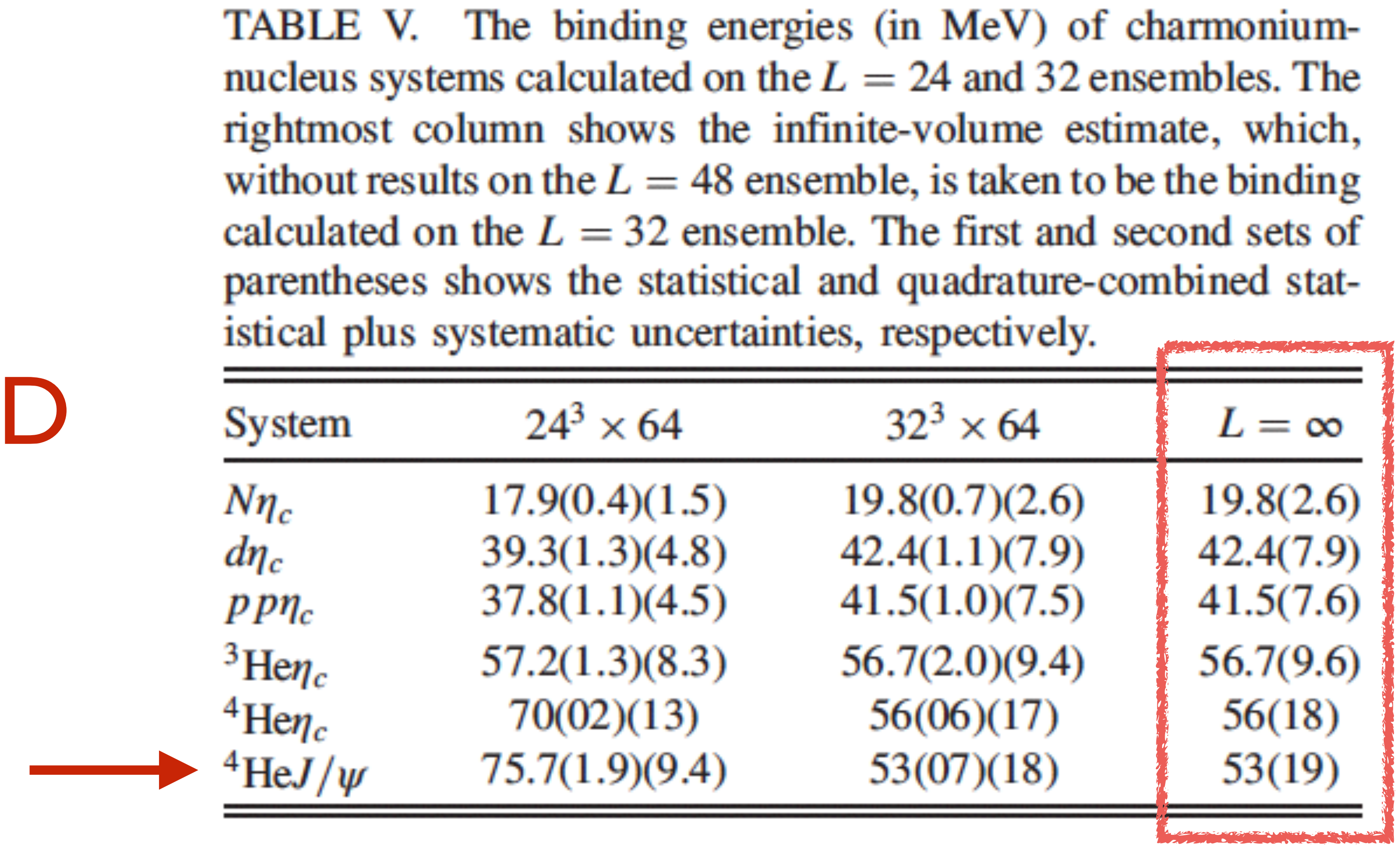
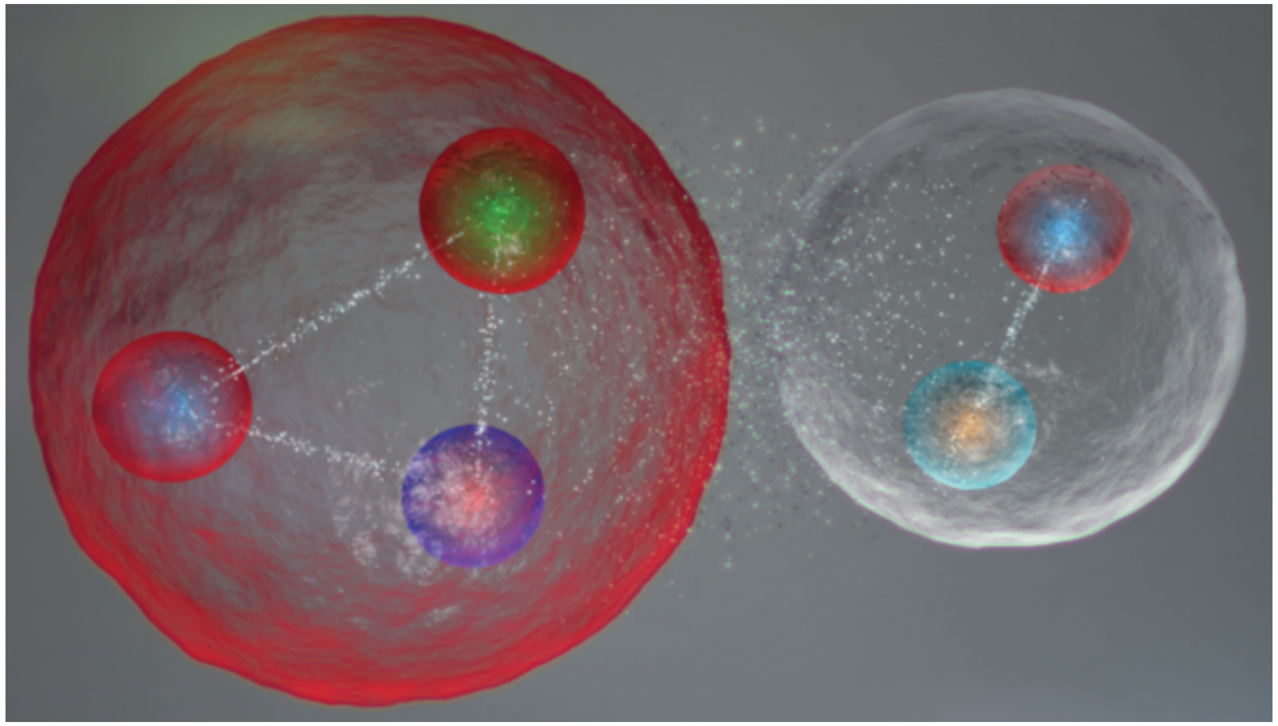


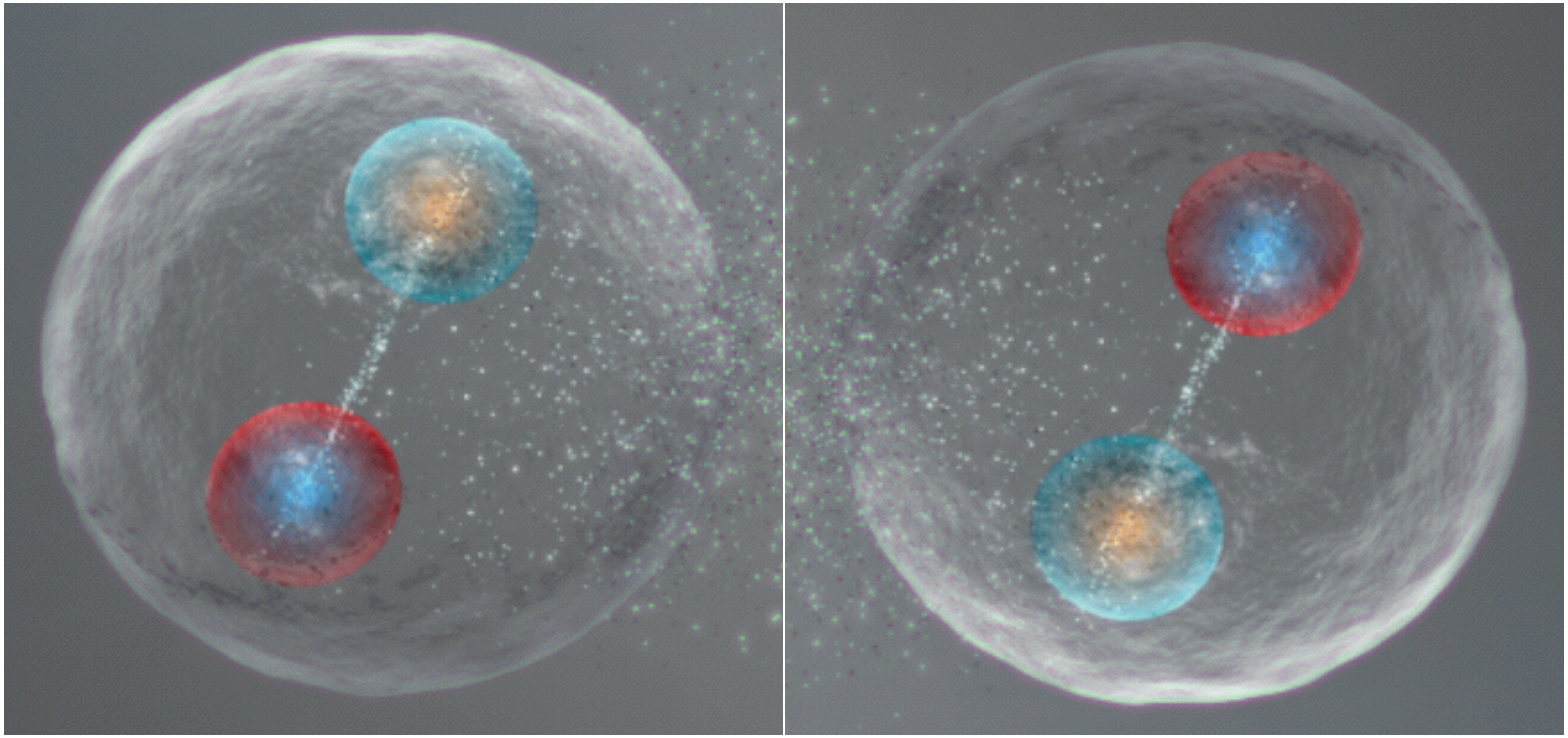
TABLE V. The binding energies (in MeV) of charmonium-nucleus systems calculated on the $L = 24$ and 32 ensembles. The rightmost column shows the infinite-volume estimate, which, without results on the $L = 48$ ensemble, is taken to be the binding calculated on the $L = 32$ ensemble. The first and second sets of parentheses shows the statistical and quadrature-combined statistical plus systematic uncertainties, respectively.

System	$24^3 \times 64$	$32^3 \times 64$	$L = \infty$
$N\eta_c$	17.9(0.4)(1.5)	19.8(0.7)(2.6)	19.8(2.6)
$d\eta_c$	39.3(1.3)(4.8)	42.4(1.1)(7.9)	42.4(7.9)
$pp\eta_c$	37.8(1.1)(4.5)	41.5(1.0)(7.5)	41.5(7.6)
^{3}Heη_c	57.2(1.3)(8.3)	56.7(2.0)(9.4)	56.7(9.6)
^{4}Heη_c	70(02)(13)	56(06)(17)	56(18)
^{4}HeJ/ψ	75.7(1.9)(9.4)	53(07)(18)	53(19)



Can one do better?

$\eta_b - \eta_b$



N. Brambilla, GK, J. Tarrús-Castellà, A. Vairo, PRD 93, 054002 (2016)

Chromopolarizability & color van der Waals forces

— an EFT perspective

Interactions between color neutral objects:

Via creation of instantaneous color dipole moments &
gluon transitions in virtual color-octet intermediate state

— Polarizability—

EFT approach

— Chromopolarizability of 1S bottomonium;

use pNRQC (potential Nonrelativistic QCD)

— van der Waals force between two bottomonia;

use QCD trace anomaly to match pNRQC to a chiral EFT

$$\begin{tikzcd}
\text{QCD} \arrow[r] & \text{NRQCD} \arrow[r] & \text{pNRQCD} \arrow[d] & & \\
& & \text{gWEFT} \arrow[r] & \chi\text{EFT} \arrow[r] & \text{WEFT}
\end{tikzcd}$$

Polarizability

van der Waals

Scales

m : bottom mass, v : relative velocity

$$m \gg mv \gg mv^2 \gg \Lambda_{\rm QCD}$$

QCD ⟶ NRQCD ⟶ pNRQCD ⟶ gWEFT

m_ϕ: mass bottomonium, $r_{\phi\phi} \sim 1/m_\pi$: relative distance

$$\mathbf{k}_{\phi\phi}^2/m_\phi = m_\pi^2/m_\phi \ll m_\pi$$

gWEFT ⟶ χEFT ⟶ WEFT

pNRQCD*

— obtain e.g. bound states

S,*O*: singlet, octet $Q\overline{Q}$ states

$$\begin{aligned}\mathcal{L}_{\mathrm{pNRQCD}} &= \mathcal{L}_{\mathrm{light}} + \int d^3r \left\{ \mathrm{Tr}\left[\mathrm{S}^\dagger \left(i\partial_0 - h_{\mathrm{s}} \right) \mathrm{S} + \mathrm{O}^\dagger \left(iD_0 - h_{\mathrm{o}} \right) \mathrm{O} \right] \right. \\ &\left. + gV_A(r)\, \mathrm{Tr}\left[\mathrm{O}^\dagger (\vec{r}\cdot\vec{E})\mathrm{S} + \mathrm{S}^\dagger (\vec{r}\cdot\vec{E})\mathrm{O} \right] + \frac{g}{2} V_B(r)\, \mathrm{Tr}\left[\mathrm{O}^\dagger (\vec{r}\cdot\vec{E})\mathrm{O} + \mathrm{O}^\dagger \mathrm{O} (\vec{r}\cdot\vec{E}) \right] \right\}\end{aligned}$$

$$h_{\mathrm{s}} = -\frac{\nabla_r^2}{m_Q} - \frac{\nabla_R^2}{4m_Q} + V_{\mathrm{s}}(r)$$

$$h_{\mathrm{o}} = -\frac{\nabla_r^2}{m_Q} - \frac{D_R^2}{4m_Q} + V_{\mathrm{o}}(r)$$

$$V_s(r) = -C_F \frac{\alpha_s(r)}{r},$$

$$V_o(r) = \left(\frac{C_A}{2} - C_F \right) \frac{\alpha_s(r)}{r},$$

$$V_A(r) = 1,$$

$$V_B(r) = 1,$$

$$C_A = N_c = 3,\ C_F = (N_c^2 - 1)/(2N_c) \text{ and } T_F = 1/2$$

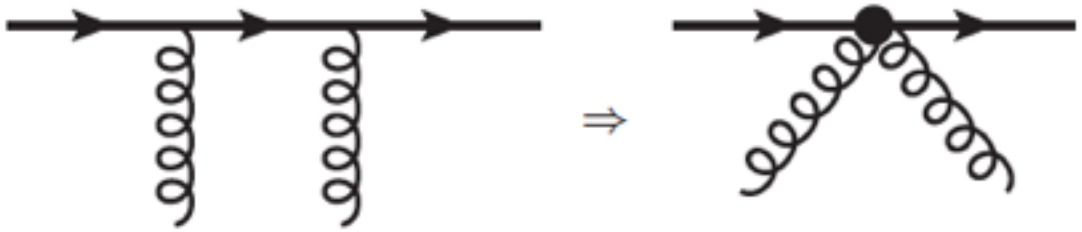
---

* A. Pineda and J. Soto, Nucl. Phys. B, Proc. Suppl. 64, 428 (1998)
N. Brambilla, A. Pineda, J. Soto, and A. Vairo, Nucl. Phys. B 566, 275 (2000)

gWEFT

— hadronization, chromopolarizability

pNRQCD ⟶ gWEFT



$$\mathcal{L}_{\mathrm{gWEFT}} = \mathcal{L}_{\mathrm{light}} + \varphi^{\dagger}(t,\vec{R})\left(i\partial_0 - E_{\varphi} + \frac{\nabla_{\vec{R}}^2}{4m} + \frac{1}{2}\alpha_{\varphi} g^2 \vec{E}^2 + \cdots\right)\varphi(t,\vec{R})$$

Chromopolarizability

$$\alpha_{\varphi} = -\frac{2V_A^2 T_F}{3N_c}\langle\varphi|r^i \frac{1}{E_{\varphi} - h_o} r^i|\varphi\rangle$$

Results: polarizability

$$E_\varphi = -m_Q \frac{(C_F \alpha_s)^2}{4} = -\frac{1}{m_Q a_0}$$

$$m_\varphi = 9.4454 \text{GeV} \left\{ \begin{array}{l} \text{average of} \\ \eta_b \ \& \ \Upsilon_b(1S) \end{array} \right.$$

$$m = 5 \text{ GeV}$$

$$\alpha_s(1 \text{ GeV}) \approx 0.5$$

$$\alpha_s(1.5 \text{ GeV}) \approx 0.35$$

$$\alpha_{\eta_b} = 0.50^{+0.42}_{-0.38} \text{ GeV}^{-3}$$

$$\alpha_s(2 \text{ GeV}) \approx 0.3$$

$$\Upsilon(2S) \to \Upsilon(1S)\pi\pi$$

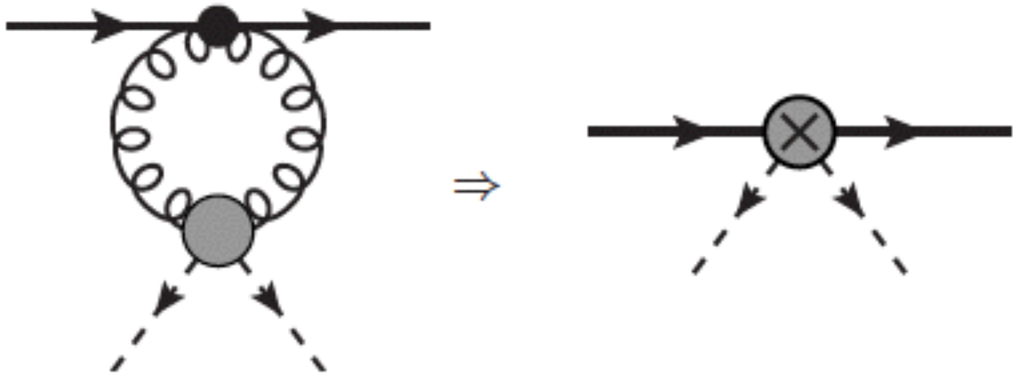
$$\beta_{\Upsilon-\Upsilon'} = 0.66 \text{ GeV}^{-3}$$

M.B. Voloshin, Mod. Phys. Lett A 19, 665 (2004)

van der Waals force

gWEFT $\longrightarrow$ χEFT

Nonperturbative matching



QCD trace anomaly

$$g^2\langle\pi^+(p_1)\pi^-(p_2)|\boldsymbol{E}_a^2|0\rangle = \frac{8\pi^2}{3b}\left((p_1+p_2)^2\kappa_1 + m_\pi^2\kappa_2\right)$$

$$\kappa_1 = 1 - 9\kappa/4,\ \kappa_2 = 1 - 9\kappa/2 \qquad b = \frac{11}{3}N_c - \frac{2}{3}N_f$$

$$\kappa = 0.186 \pm 0.003 \pm 0.006 \quad \longleftarrow \quad \psi' \to J/\psi\pi^+\pi^-$$

Matching gWEFT $\longrightarrow$ χEFT

$$\mathcal{L}^{\phi}_{\chi\mathrm{EFT}} = \phi^\dagger \left(i\partial_0 - \frac{\boldsymbol{\nabla}^2}{2m_\phi} \right) \phi$$

$$\mathcal{L}^{\pi}_{\chi\mathrm{EFT}} = \frac{F^2}{4} \left(\langle \partial_\mu U \partial^\mu U^\dagger \rangle + \langle \chi^\dagger U + \chi U^\dagger \rangle \right)$$

$$U = e^{i\phi/F} = u^2 , \quad \phi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$$

$$\chi = 2B\hat{m}\mathbf{1} \qquad F = F_\pi = 92.419\,\mathrm{MeV}$$

$$\mathcal{L}^{\phi-\pi}_{\chi\mathrm{EFT}} = \phi^\dagger \phi \frac{F^2}{4} \left(c_{d0} \langle \partial_0 U \partial_0 U^\dagger \rangle + c_{di} \langle \partial_i U \partial^i U^\dagger \rangle + c_m \langle \chi^\dagger U + \chi U^\dagger \rangle \right)$$

$$c_{d0} = -\frac{4\pi^2 \alpha_\phi}{b} \kappa_1 \qquad c_{di} = -\frac{4\pi^2 \alpha_\phi}{b} \kappa_2 \qquad c_m = -\frac{12\pi^2 \alpha_\phi}{b}$$

van der Waals force

$r_{\phi\phi} \sim 1/m_\pi$ $\mathbf{k}^2_{\phi\phi}/m_\phi = m^2_\pi/m_\phi \ll m_\pi$

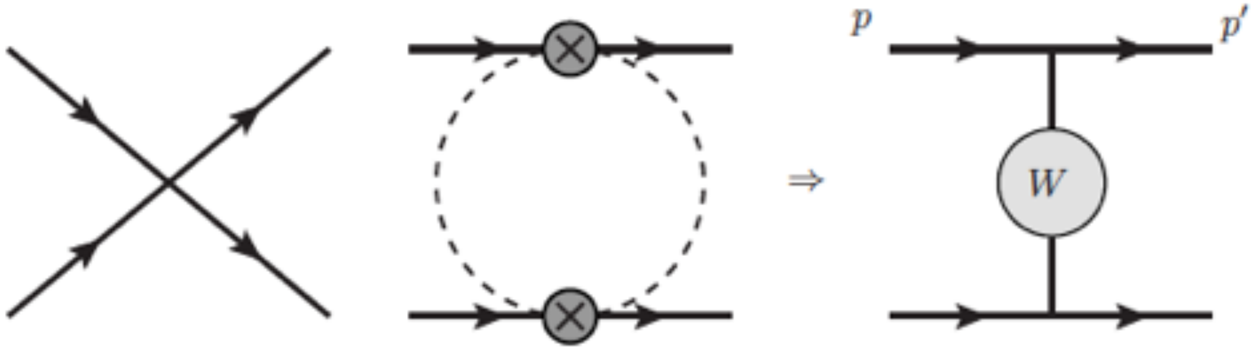
Relative motion at energies lower than pion mass

— integrate out the pion

χEFT $\longrightarrow$ WEFT

Matching

χEFT ⟶ WEFT



$$L_{\varphi\varphi} = \int d^3R\, \varphi^\dagger(t,\vec{R}) \left(i\partial_0 + \frac{\nabla^2}{2m_\varphi} \right) \varphi(t,\vec{R})$$
$$-\frac{1}{2} \int d^3R d^3R'\, \varphi^\dagger(t,\vec{R})\varphi^\dagger(t,\vec{R}')\, W_{\varphi\varphi}(\vec{R},\vec{R}')\, \varphi(t,\vec{R}')\varphi(t,\vec{R})$$

vdW potential

$$
\begin{aligned}
\widetilde{W}(\mathbf{k}^2) \quad = \quad & \text{contact terms} \\
& -\frac{3}{8}\left(\mathbf{k}^2 c_{di} + 2m_\pi^2\left(c_{di} - c_m\right)\right)^2 \, B\left[m_\pi^2, -\mathbf{k}^2\right] \\
& -\frac{3}{2}\left(c_{d0} - c_{di}\right)\left(\mathbf{k}^2 c_{di} + 2m_\pi^2\left(c_{di} - c_m\right)\right) \, C_1\left[m_\pi^2, -\mathbf{k}^2\right] \\
& -\frac{3}{2}\left(c_{d0} - c_{di}\right)^2 \, C_2\left[m_\pi^2, -\mathbf{k}^2\right]
\end{aligned}
$$

$$B\left[m_\pi^2\,, -\mathbf{k}^2\right] = \frac{1}{16\pi^2}\left(\lambda + 1 - \log\frac{m_\pi^2}{\nu^2} + \sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}\,\log\left[\frac{\sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}-1}{\sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}+1}\right]\right)$$

$$B\left[m_\pi^2\,, -\mathbf{k}^2\right] = \frac{1}{16\pi^2}\left(\lambda + 1 - \log\frac{m_\pi^2}{\nu^2} + \sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}\log\left[\frac{\sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}-1}{\sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}+1}\right]\right)$$

$$\begin{aligned} C_2\left[m_\pi^2 \;- \mathbf{k}^2\right] \quad = \quad & \frac{31\mathbf{k}^4 + 280\mathbf{k}^2 m_\pi^2 + 705 m_\pi^4}{19200\pi^2} + \frac{1}{1280\pi^2}\Bigg(\left(\mathbf{k}^4 + 10\mathbf{k}^2 m_\pi^2 + 30 m_\pi^4\right) \\ & \times\left(\lambda - \log\frac{m_\pi^2}{\nu^2}\right) + \mathbf{k}^4\left(1+\frac{4m_\pi^2}{\mathbf{k}^2}\right)^{5/2}\log\left[\frac{\sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}-1}{\sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}+1}\right]\Bigg) \end{aligned}$$

$$B\left[m_\pi^2\,,-\mathbf{k}^2\right]=\frac{1}{16\pi^2}\left(\lambda+1-\log\frac{m_\pi^2}{\nu^2}+\sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}\log\left[\frac{\sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}-1}{\sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}+1}\right]\right)$$

$$C_2\left[m_\pi^2\;-\mathbf{k}^2\right]\;=\;\frac{31\mathbf{k}^4+280\mathbf{k}^2m_\pi^2+705m_\pi^4}{19200\pi^2}+\frac{1}{1280\pi^2}\left(\left(\mathbf{k}^4+10\mathbf{k}^2m_\pi^2+30m_\pi^4\right)\right.$$
$$\left.\times\left(\lambda-\log\frac{m_\pi^2}{\nu^2}\right)+\mathbf{k}^4\left(1+\frac{4m_\pi^2}{\mathbf{k}^2}\right)^{5/2}\log\left[\frac{\sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}-1}{\sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}+1}\right]\right)$$

$$C_1\left[m_\pi^2\;-\mathbf{k}^2\right]\;=\;\frac{5\mathbf{k}^2+24m_\pi^2}{576\pi^2}+\frac{1}{192\pi^2}\left(\left(\mathbf{k}^2+6m_\pi^2\right)\left(\lambda-\log\frac{m_\pi^2}{\nu^2}\right)\right.$$
$$\left.+\mathbf{k}^2\left(1+\frac{4m_\pi^2}{\mathbf{k}^2}\right)^{3/2}\log\left[\frac{\sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}-1}{\sqrt{1+\frac{4m_\pi^2}{\mathbf{k}^2}}+1}\right]\right)$$

vdW potential

$$W(r) = \frac{1}{2\pi^2 r}\int_{2m_\pi}^{\infty} d\mu\,\mu\,e^{-\mu r}\,\mathrm{Im}\left[\widetilde{W}(\epsilon - i\mu)\right]$$

$$= -\frac{3\pi\beta^2 m_\pi^2}{8b^2 r^5}\Bigl[\Bigl(4\left(\kappa_2+3\right)^2\left(m_\pi r\right)^3 + \left(3\kappa_1^2 + 43\kappa_2^2 + 14\kappa_1\kappa_2\right) m_\pi r\Bigr) K_1(2m_\pi r)$$

$$+2\left(2\left(\kappa_2+3\right)\left(\kappa_1+5\kappa_2\right)\left(m_\pi r\right)^2 + \left(3\kappa_1^2 + 43\kappa_2^2 + 14\kappa_1\kappa_2\right)\right) K_2(2m_\pi r)\Bigr]$$

↓ asymptotic

$$W(r) = -\frac{3(3+\kappa_2)^2\pi^{3/2}\beta^2}{4b^2}\frac{m_\pi^{9/2}}{r^{5/2}}e^{-2m_\pi r}$$

Completed the calculation of
H. Fujii and D. Kharzeev, PRD 60, 114039 (1999)

Numerical result

— vdW potential

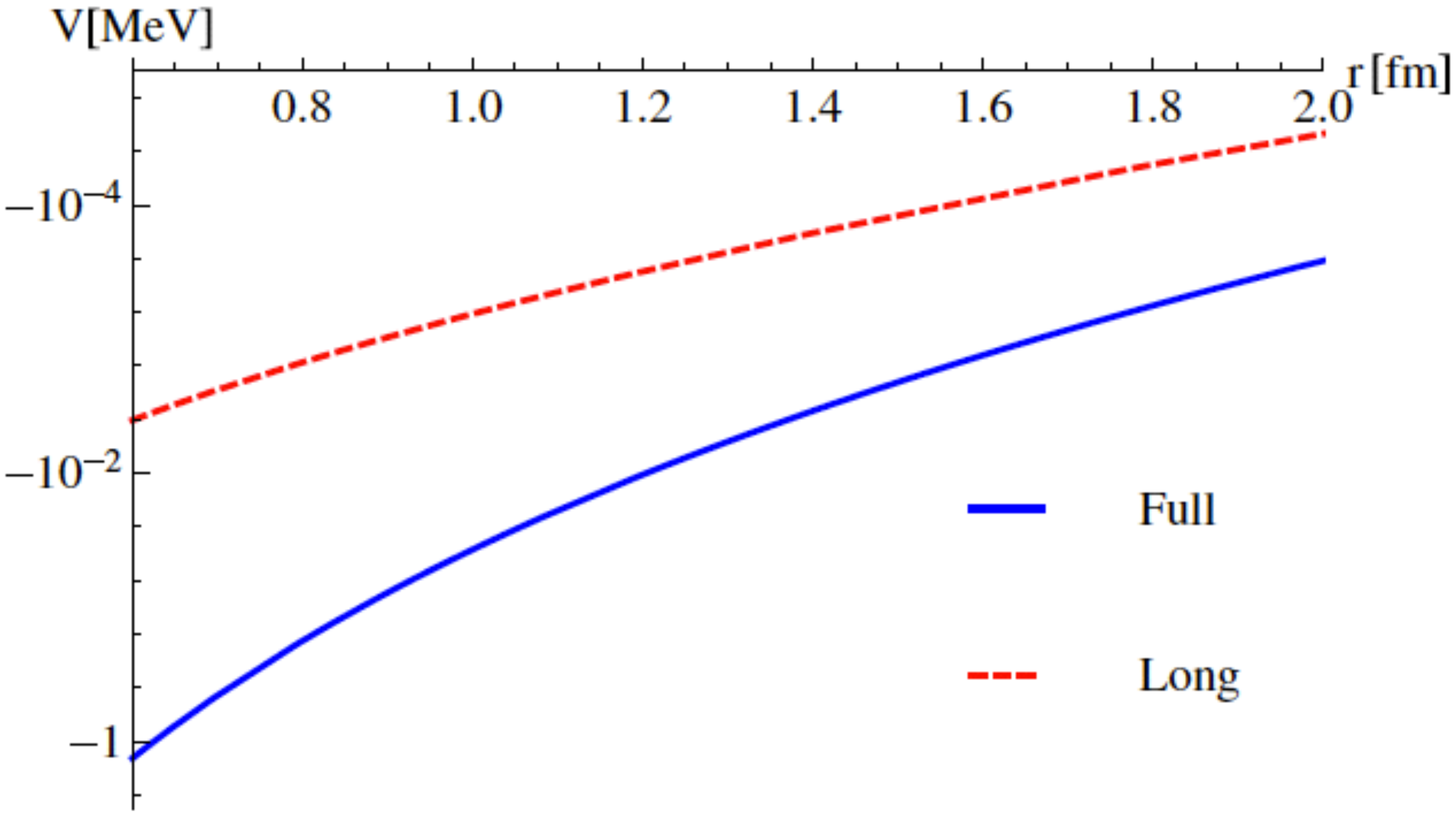


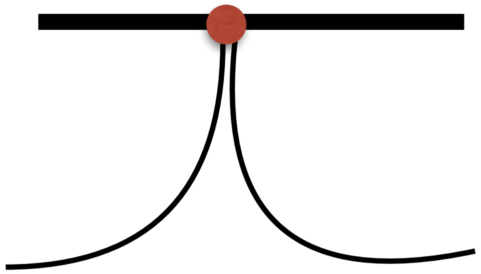
FIG. 9. Comparison of the van der Waals potential (40) (blue line) with its long-range expansion (41) (red line) for $\beta = 0.92\ \mathrm{GeV}^{-3}$ and other parameters like in Fig. 8.

Are there $\eta_b \, \eta_b$ bound-states?

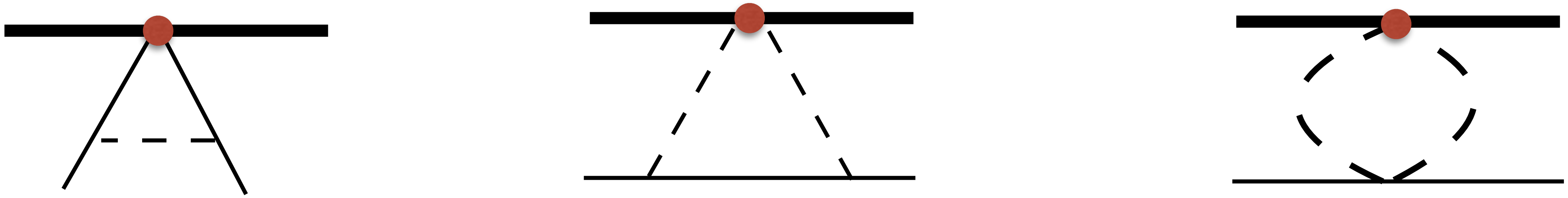
— It is likely, but depends somewhat
on the medium- and short-range pieces

Quarkonium-nucleon*

Lattice



Chiral EFT



J.T. Castellà, GK
arXiv: 1803.05412

Degrees of freedom & Scales & Power counting

DOF: nucleons, quarkonia, pions

Scales: $E_N, E_\phi \sim m_\pi \ll \Lambda_\chi \sim 1\mathrm{GeV}$

Power counting: terms of the effective Lagrangian organized in powers of $\frac{m_\pi}{\Lambda_\chi}$

(~ Weinberg for NN)

Loops: dimensional regularization

Quarkonium-nucleon EFT
— QNEFT

$$\boxed{u^2 = U = e^{i\Phi/F}\,, \quad \Phi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}}$$

Nucleon-pion

$$\mathcal{L}^N = N^\dagger\left(iD_0 + \frac{\boldsymbol{D}^2}{2\hat{m}_N}\right)N - \frac{g_A}{2}N^\dagger \boldsymbol{u}\cdot\boldsymbol{\sigma}N$$

$$u_\mu = i\left\{u^\dagger, \partial_\mu u\right\} \qquad D_\mu N = \partial_\mu N + \Gamma_\mu N \qquad \Gamma_\mu = \tfrac{1}{2}\left[u^\dagger, \partial_\mu u\right]$$

Quarkonium-nucleon EFT
— QNEFT

Quarkonium-Nucleon

$$\mathcal{L}^{\phi-N} = - c_0 N^\dagger N \phi^\dagger \phi - d_m \langle \chi_+ \rangle N^\dagger N \phi^\dagger \phi - d_1 \boldsymbol{\nabla} \left(N^\dagger N\right) \cdot \boldsymbol{\nabla} \left(\phi^\dagger \phi\right)$$

$$- d_2 \left(N^\dagger \overleftrightarrow{\boldsymbol{D}} N\right) \cdot \left(\phi^\dagger \overleftrightarrow{\boldsymbol{\nabla}} \phi\right) - d_3 \boldsymbol{D} N^\dagger \cdot \boldsymbol{D} N \phi^\dagger \phi$$

$$- d_4 N^\dagger N \boldsymbol{\nabla} \phi^\dagger \cdot \boldsymbol{\nabla} \phi$$

$$\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u \qquad \chi = 2B\hat{m}\mathbb{1} \qquad m_u = m_d \equiv \hat{m}$$

Low-energy quarkonium-nucleon dynamics

Quarkonium-nucleon dynamics,
e.g bound to nucleus occurs at energies

$$E_{\phi N} \sim \frac{k_{\phi N}^2}{2\mu_{\phi N}} < \frac{m_\pi^2}{\Lambda_\chi} \ll m_\pi$$

Integrate out the pion

Low-energy quarkonium-nucleon dynamics

$$\langle T_{\mathrm{N}} \rangle \sim \frac{3}{5} \frac{k_F^2}{2m_N} \sim 20\,\mathrm{MeV} \ll \Lambda_{\mathrm{QCD}}$$

Quarkonium-nucleon dynamics,
e.g bound to nucleus occurs at energies

$$E_{\phi N} \sim \frac{k_{\phi N}^2}{2\mu_{\phi N}} < \frac{m_\pi^2}{\Lambda_\chi} \ll m_\pi$$

Integrate out the pion

Low-energy quarkonium-nucleon dynamics

$$\boxed{\langle T_{\rm N}\rangle \sim \frac{3}{5}\frac{k_F^2}{2m_N} \sim 20\,{\rm MeV} \ll \Lambda_{\rm QCD}}$$

Quarkonium-nucleon dynamics,
e.g bound to nucleus occurs at energies

$$E_{\phi N} \sim \frac{k_{\phi N}^2}{2\mu_{\phi N}} < \frac{m_\pi^2}{\Lambda_\chi} \ll m_\pi$$

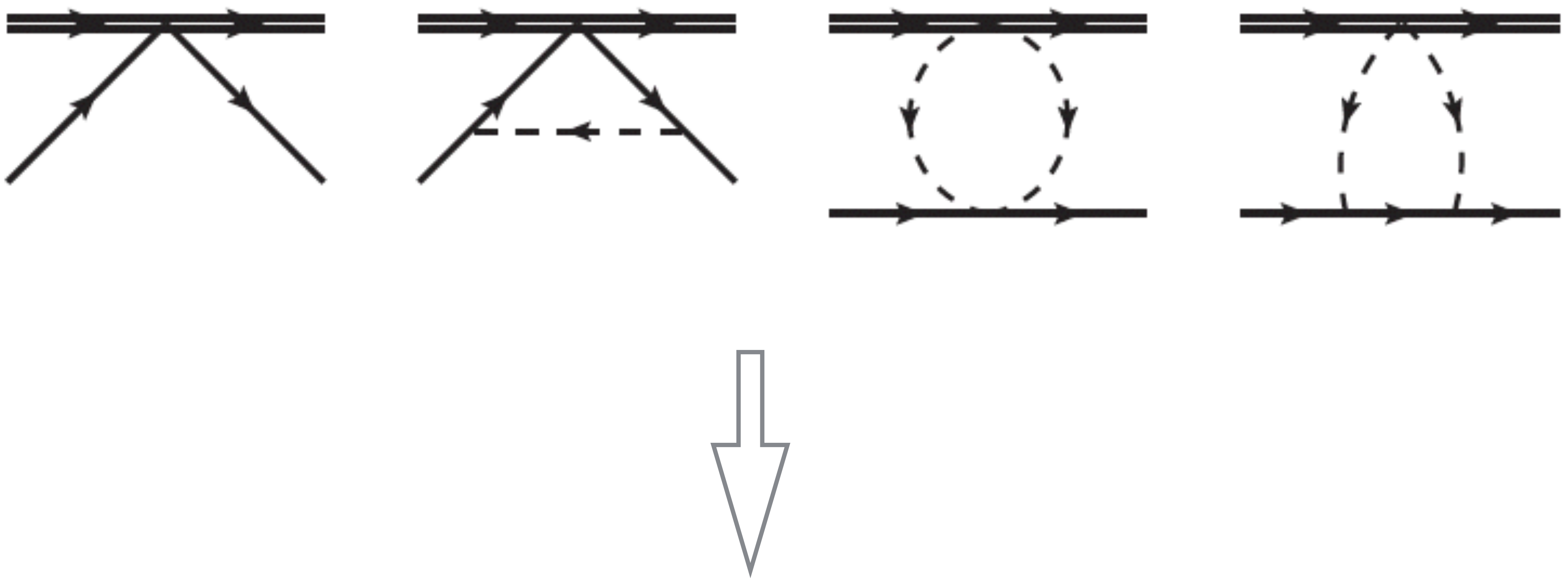
Integrate out the pion

Quarkonium-nucleon potential — pQNEFT

Integrate out the pion

$$
\begin{aligned}
\mathcal{L}^{\text{pQNEFT}} =& N^\dagger \left(i\partial_0 + \frac{\boldsymbol{\nabla}^2}{2m_N} \right) N + \phi^\dagger \left(i\partial_0 + \frac{\boldsymbol{\nabla}^2}{2m_\phi} \right) \phi \\
& - C_0 N^\dagger N \phi^\dagger \phi - D_1 \boldsymbol{\nabla} \left(N^\dagger N \right) \cdot \boldsymbol{\nabla} \left(\phi^\dagger \phi \right) \\
& - D_2 \left(N^\dagger \overleftrightarrow{\boldsymbol{\nabla}} N \right) \cdot \left(\phi^\dagger \overleftrightarrow{\boldsymbol{\nabla}} \phi \right) - D_3 \boldsymbol{\nabla} N^\dagger \cdot \boldsymbol{\nabla} N \phi^\dagger \phi - D_4 N^\dagger N \boldsymbol{\nabla} \phi^\dagger \cdot \boldsymbol{\nabla} \phi \\
& - \int d^3 r \, N^\dagger N(t, \boldsymbol{x}_1) V(\boldsymbol{x}_1 - \boldsymbol{x}_2) \phi^\dagger \phi(t, \boldsymbol{x}_2)
\end{aligned}
$$

Matching



renormalization of couplings + van der Waals

Renormalized couplings

$$\begin{aligned} C_0 &= c_0 + 4m_\pi^2 d_m + \frac{9g_A^2 m_\pi^2 c_0}{64\pi^2 F^2}\left(\log\frac{m_\pi^2}{\nu^2} + \frac{2}{3}\right) + \frac{3g_A^2 m_\pi^3}{64\pi F^2}\left(5c_{di} - 3c_m\right) \\ D_1 &= d_1 + \frac{g_A^2 m_\pi}{256\pi F^2}\left(23c_{di} - 5c_m\right) \\ D_j &= d_j \quad \text{for} \quad j = 2, 3\,\text{and}\,4 \end{aligned}$$

Long-distance part of QN potential — vdW force

$$V(r) = \frac{3g_A^2 m_\pi^3}{128\pi^2 F^2 r^6}\, e^{-2m_\pi r} \{c_{di}\left[6 + m_\pi r(2 + m_\pi r)(6 + m_\pi r(2 + m_\pi r))\right]$$

$$+c_m m_\pi^2 r^2 \left(1 + m_\pi r\right)^2\}$$

No free parameters here:

— trace anomaly

— chiral physics

First, model-independent derivation of a quarkonium-nucleon van der Waals force

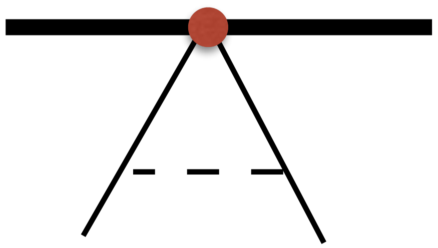
For $r \gg \frac{1}{2m_\pi}$:

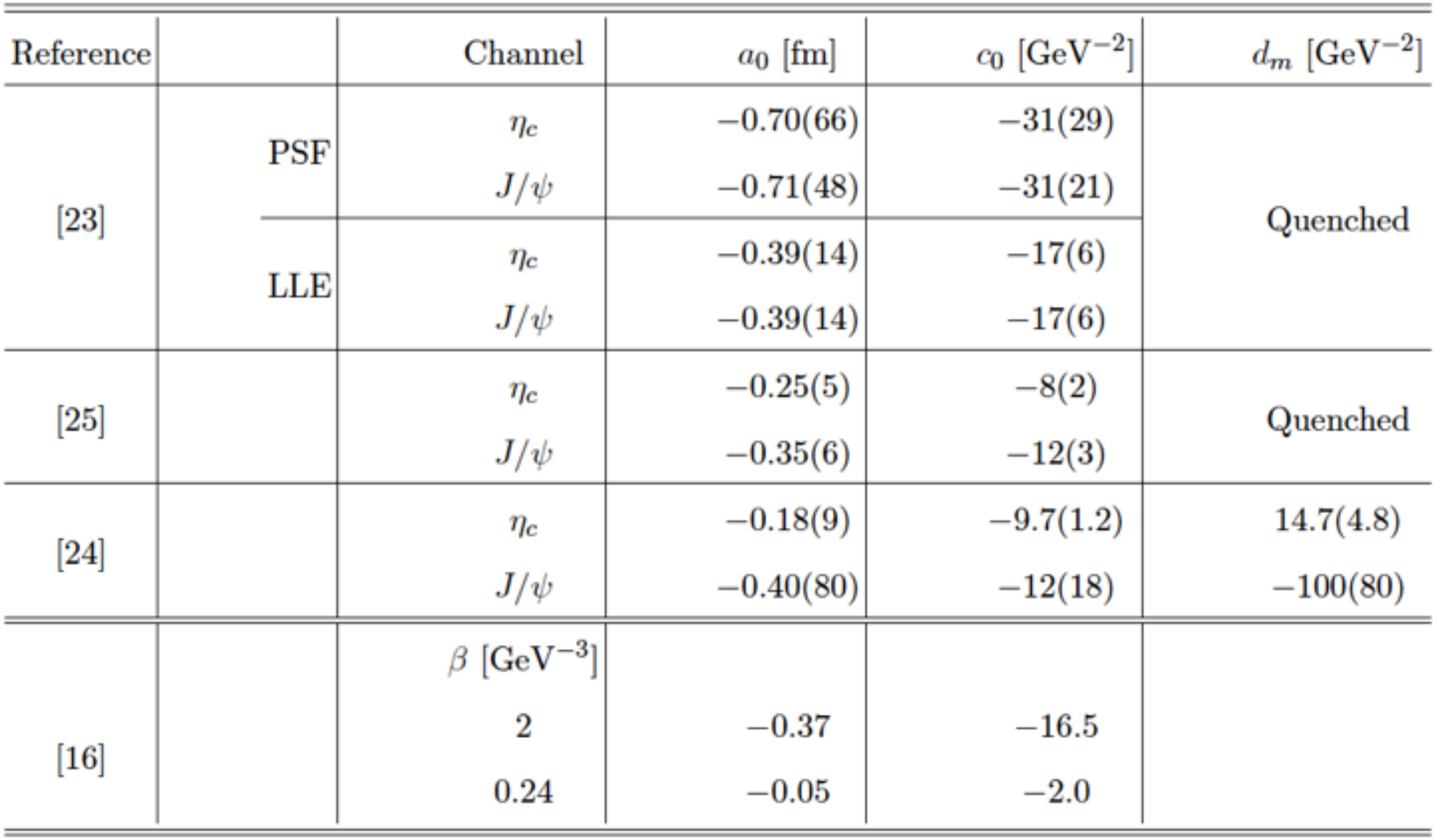
$$V(r) = \frac{3g_A^2 m_\pi^4 \left(c_{di} + c_m\right)}{128\pi^2 F^2} \frac{e^{-2m_\pi r}}{r^2}$$

To extrapolate lattice data to physical quark masses, need:

$$\begin{aligned} m_N &= \hat{m}_N - 4c_1 m_\pi^2 - \frac{3g_A^2 m_\pi^3}{32\pi F^2} \\ m_\phi &= \hat{m}_\phi - F^2 c_m m_\pi^2 \end{aligned}$$

Unknown contact couplings — get them from lattice QCD



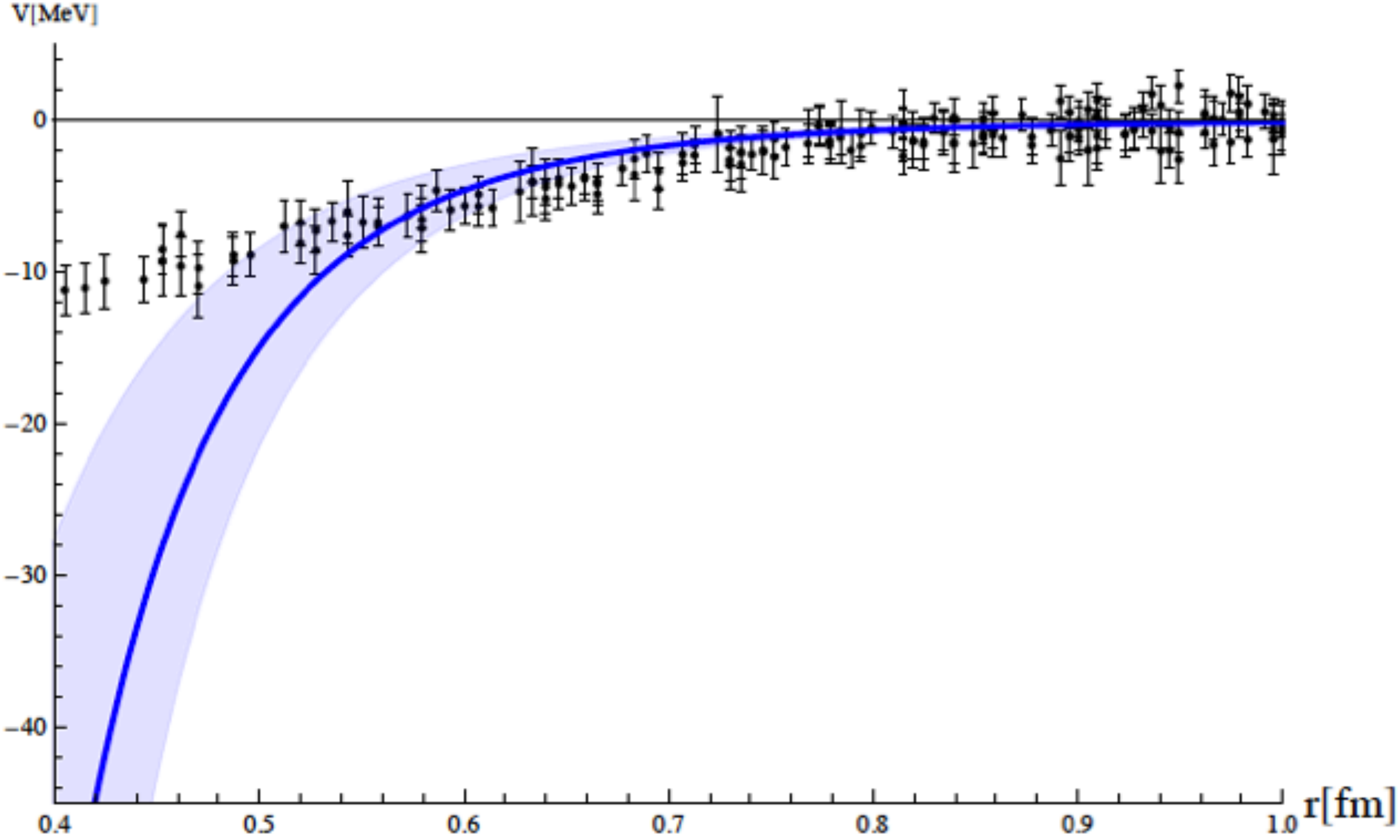


Reference		Channel	a_0 [fm]	c_0 [GeV^{-2}]	d_m [GeV^{-2}]
[23]	PSF	η_c	−0.70(66)	−31(29)	Quenched
		J/ψ	−0.71(48)	−31(21)	
	LLE	η_c	−0.39(14)	−17(6)	
		J/ψ	−0.39(14)	−17(6)	
[25]		η_c	−0.25(5)	−8(2)	Quenched
		J/ψ	−0.35(6)	−12(3)	
[24]		η_c	−0.18(9)	−9.7(1.2)	14.7(4.8)
		J/ψ	−0.40(80)	−12(18)	−100(80)
[16]		β [GeV^{-3}]			
		2	−0.37	−16.5	
		0.24	−0.05	−2.0	

Comparing long distance part with HAL lattice potential

vdW force

$$\boxed{\eta_c - N}$$

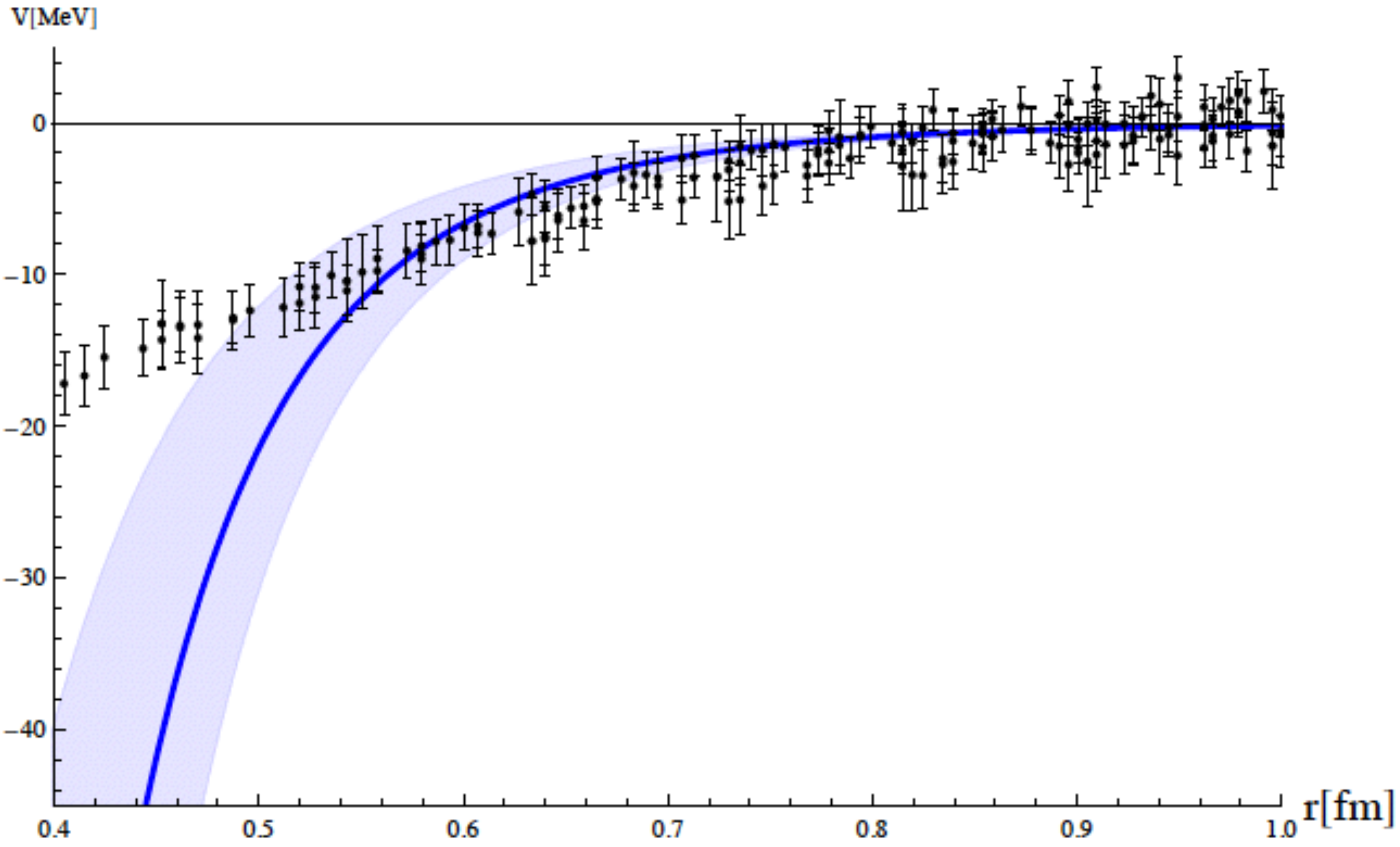


Lattice:

T. Kawanay & S. Sasaki, Pos (Lattice) 2010, 156 (2010)

vdW force

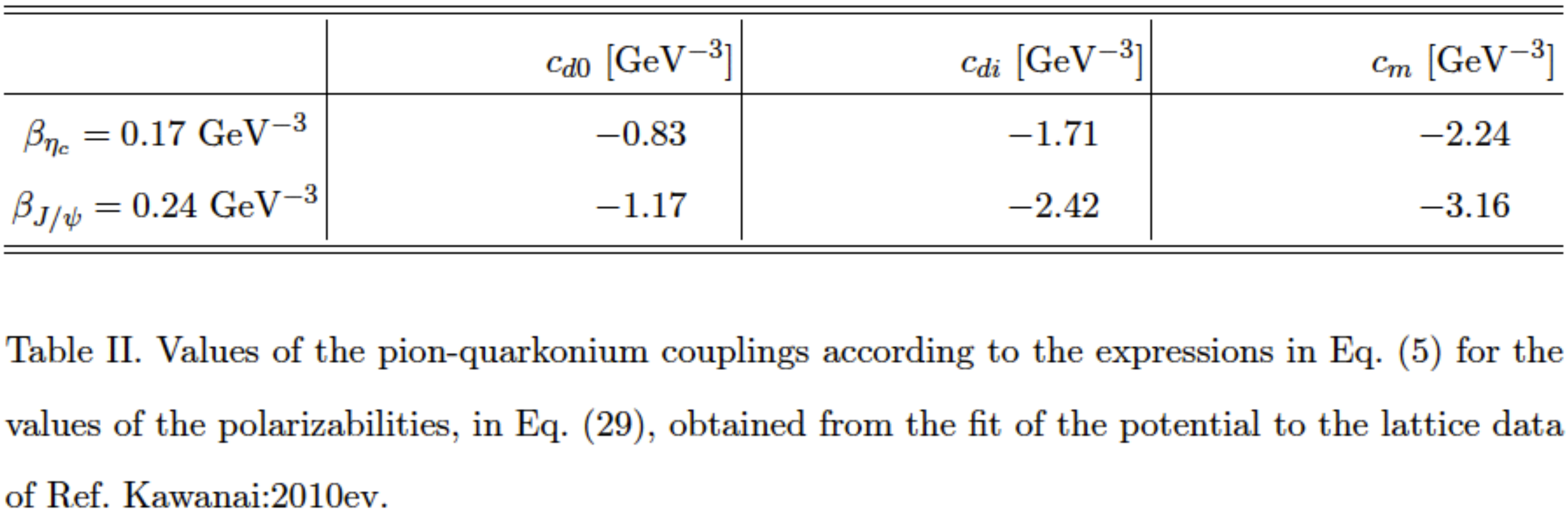
$$\boxed{J/\Psi - N}$$



Lattice:

T. Kawanay & S. Sasaki, Pos (Lattice) 2010, 156 (2010)

Fits of the polarizabilites



	c_{d0} [GeV^{-3}]	c_{di} [GeV^{-3}]	c_m [GeV^{-3}]
$\beta_{\eta_c} = 0.17$ GeV^{-3}	-0.83	-1.71	-2.24
$\beta_{J/\psi} = 0.24$ GeV^{-3}	-1.17	-2.42	-3.16

Table II. Values of the pion-quarkonium couplings according to the expressions in Eq. (5) for the values of the polarizabilities, in Eq. (29), obtained from the fit of the potential to the lattice data of Ref. Kawanai:2010ev.

Are there quarkonium-nucleon bound states at this order in pQNEFT?

Scattering amplitude (s-wave)

$$\mathcal{A} = \frac{2\pi}{\mu_{\phi N}} \frac{1}{p \cot \delta - ip} = \frac{2\pi}{\mu_{\phi N}} \frac{1}{-\frac{1}{a_0} + \frac{1}{2} r_0 p^2 + \ldots}$$

$$a_0 = \frac{\mu_{\phi N}}{2\pi} \left[c_0 + 4 d_m m_\pi^2 + \frac{9 g_A^2 m_\pi^2 c_0}{64 \pi^2 F^2} \left(\log \frac{m_\pi^2}{\nu^2} + \frac{2}{3} \right) + \frac{3 g_A^2}{64 \pi F^2} m_\pi^3 \left(5 c_{di} - 3 c_m \right) \right]$$

$$r_0 = \frac{8\pi}{\mu_{\phi N} c_0^2} \left[(d_1 + d_2) + \frac{g_A^2}{256 \pi F^2} m_\pi \left(23 c_{di} - 5 c_m \right) \right]$$

No quarkonium-nucleon bound states within the applicability of the present calculation:

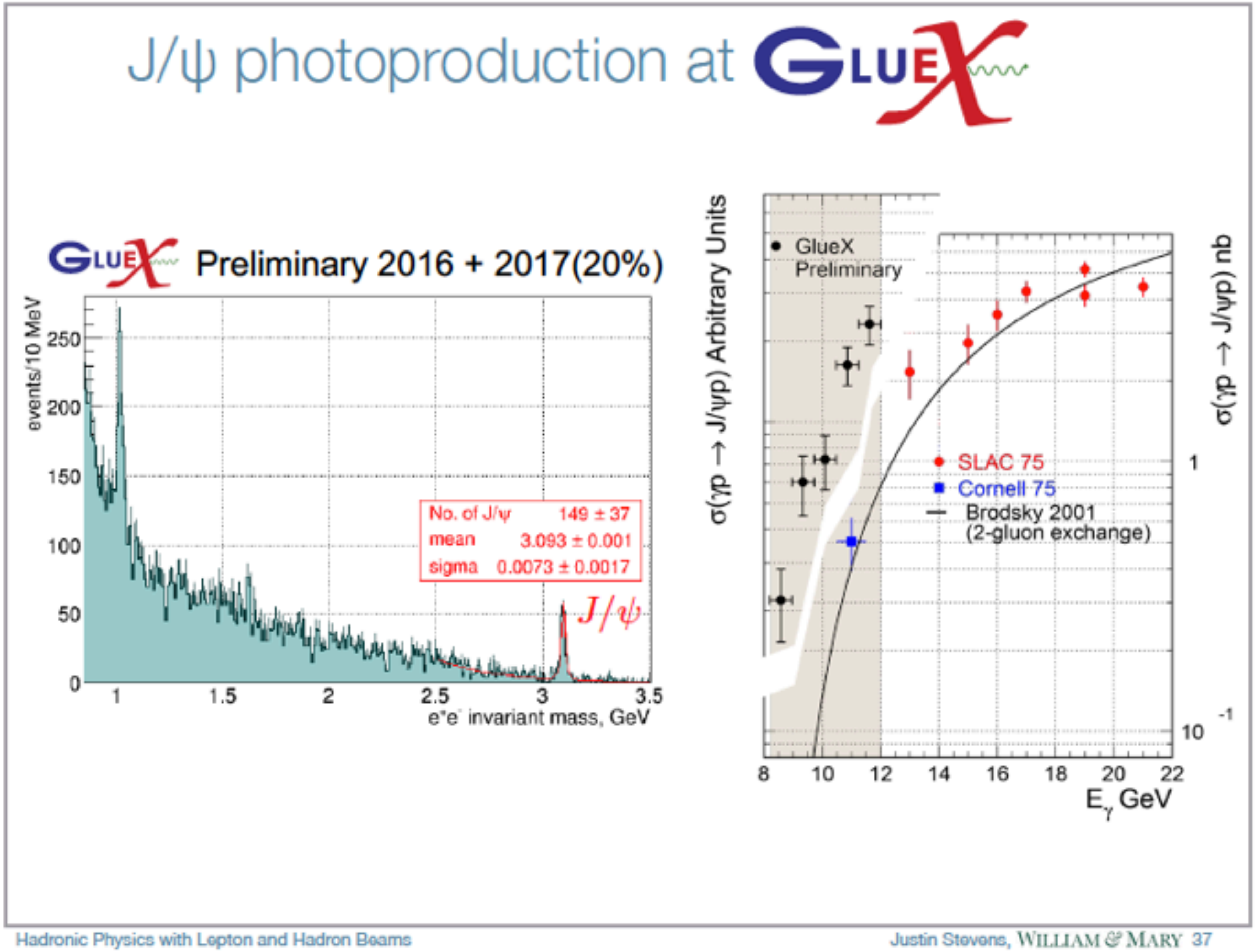
$$|p_{\phi N}| \leq m_\pi$$

Are there quarkonium-nucleus bound states at this order in pQNEFT?

YES, for sufficiently large nuclei

Experiments

— JLab

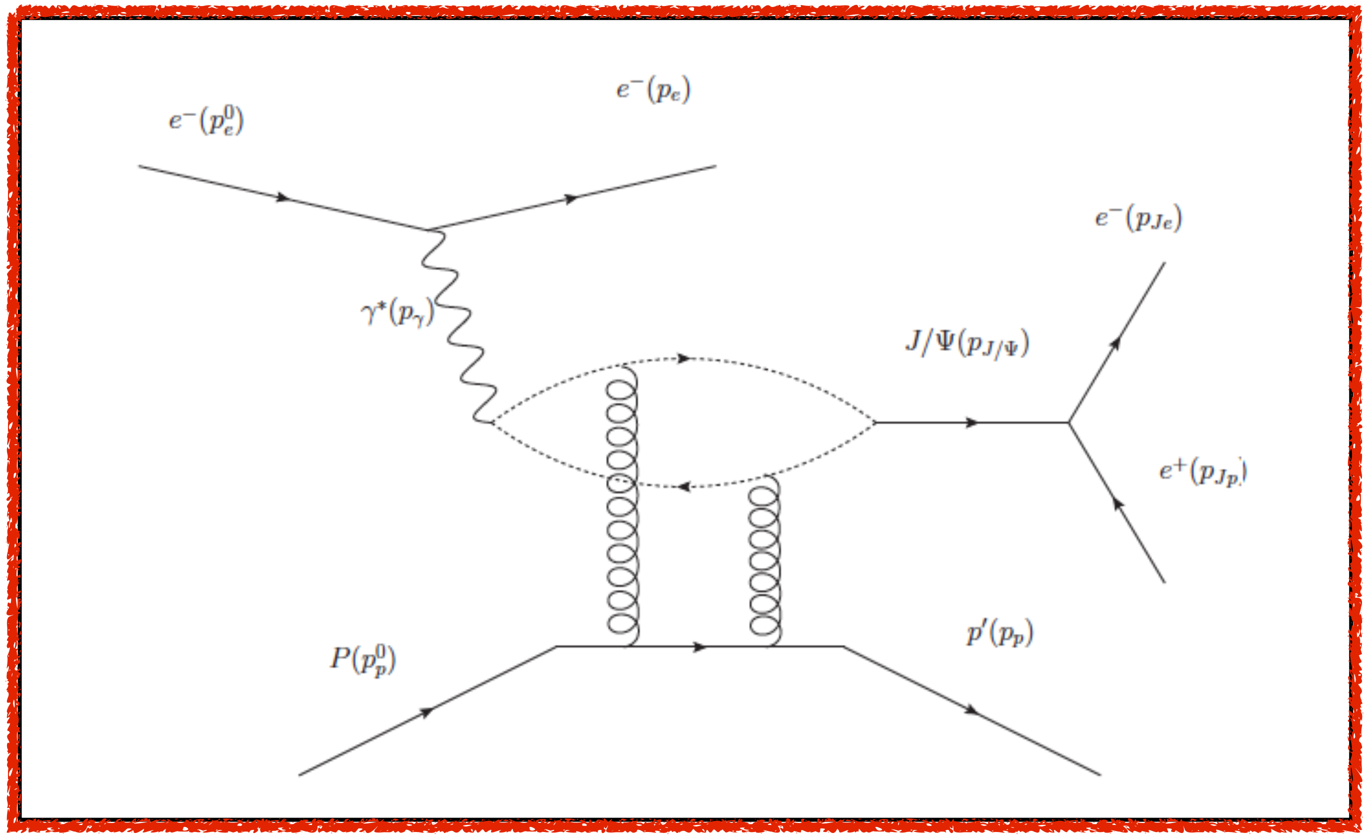


J/ψ photoproduction at GlueX

Hadronic Physics with Lepton and Hadron Beams

Justin Stevens, WILLIAM & MARY 37

ATHENNA* collaboration JLab @ 12 GeV



Hall A — E12-12-006
K. Hafidi, Z.-E. Meziani, N. Sparveris, Z.W. Zhao

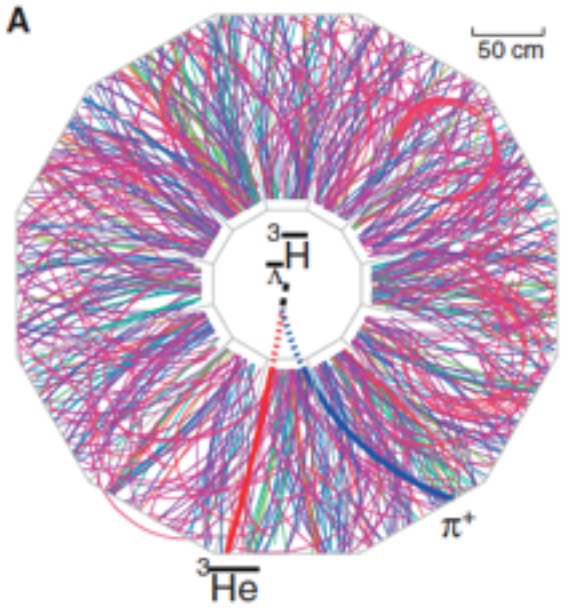
*A J/Ψ THreshold Electroproduction on the Nucleon and Nuclei Analysis

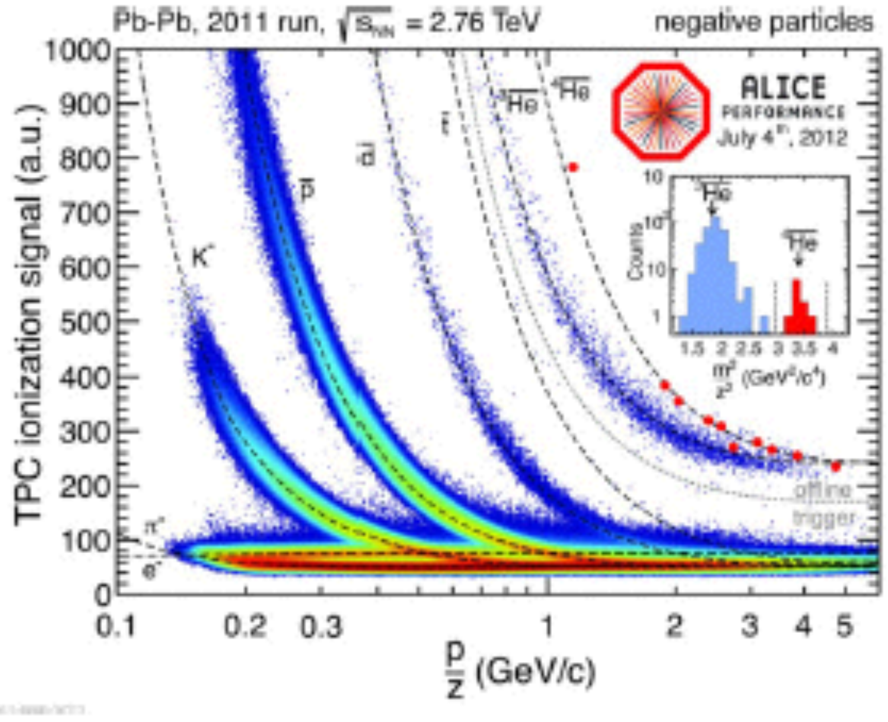
Hall C — E-12-16-007 (Pentaquarks)
Z.-E. Meziani, S. Joosten, et al.

How About coalescence at the LHC?

— Chances of a charmed hadron meeting one or two nucleons not smaller than of two antinucleons and one antihyperon meeting to form an antihypernucleus

Observation of an Antimatter Hypernucleus
The STAR Collaboration
Science **328**, 58 (2010);
DOI: 10.1126/science.1183980

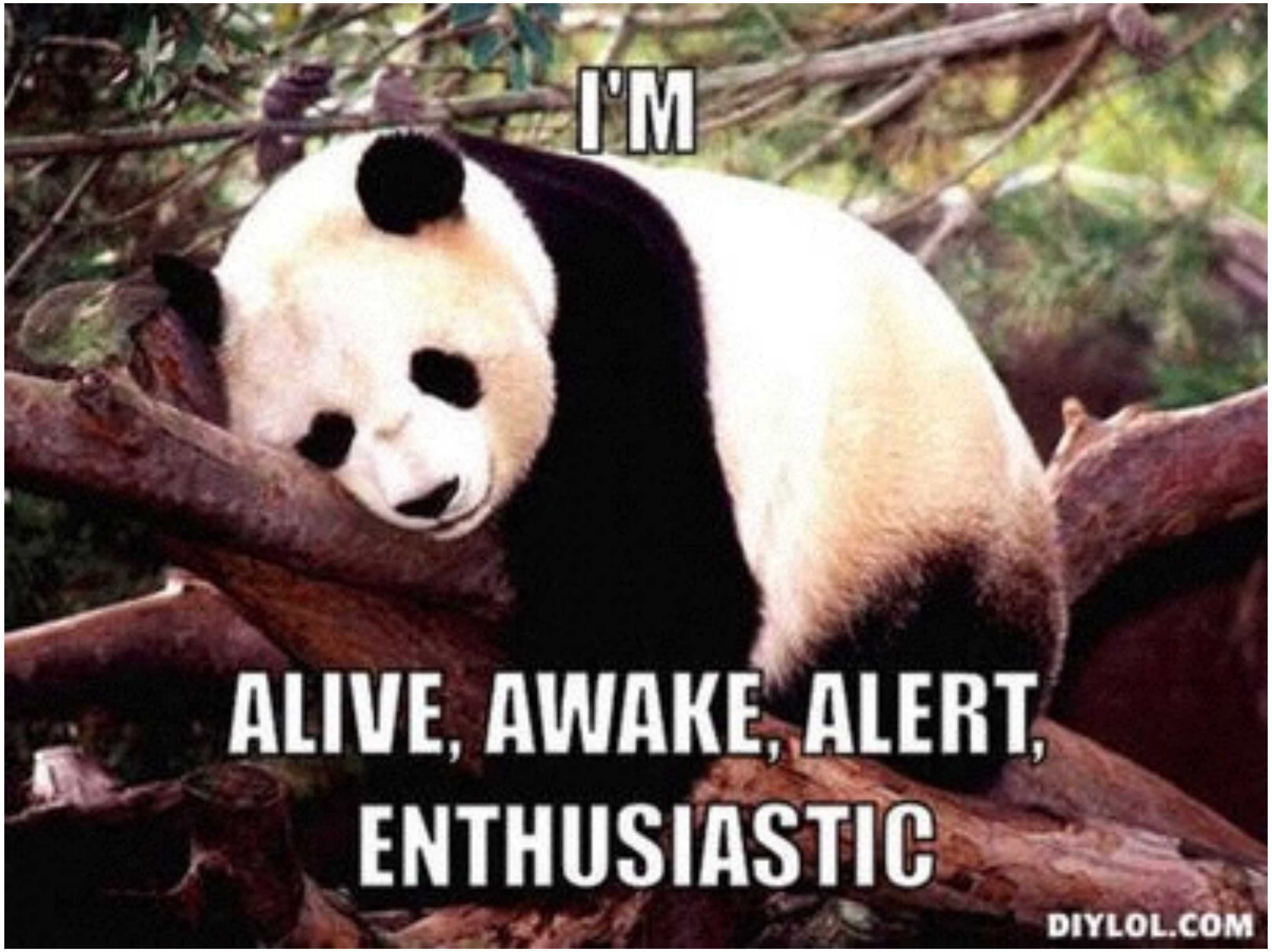




Need to detect in coincidence the decay products

Experiments

P̄ANDA @ FAIR



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