DM bound states by 3-body recombination

arXiv:1805xxxx, Braaten, DK, Laha

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INT multi-scale problems using EFTs

Dark matter and its bound state

- elementary particle: QCD-like , SUSY …
  composite particle like dark nucleons

- Dark nucleosynthesis by
  radiative fusion : $d+d_N \rightarrow d_{N+1}$ + mediator
  recombination : $3\,d \rightarrow d+d_2$, $d_2+d_N \rightarrow d_{N+1} + d$

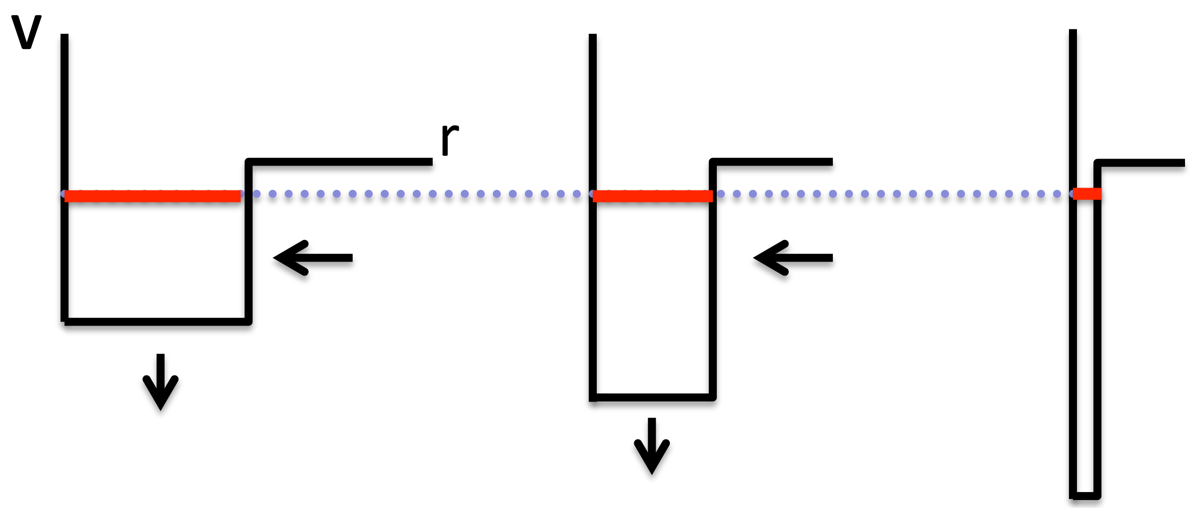
- Without mediator, 3-body recombination forms dark deuteron and is the bottleneck for N-body cluster.
  How many deuterons can be produced?

- Use a simple and very predictive model
  with two parameters mass and scattering length ***a***

Outline

- Universal 2-body physics and small scale structure problem
- 3-body physics and recombination process
- Production of bound state in early universe

Universal physics at low energy



a shallow state remains the same by tuning depth and width simultaneously

1. ***universal regardless of microscopic physics***
2. fine tuned state: large scattering length (a) >> r_e,…
3. point-like interactions can be used.

Large scattering length system

- Quantum Mechanics at low energy

$$f(k) = \frac{1}{-\frac{1}{a} - ik + \frac{r_s}{2}k^2 + \cdots}$$

- At very low energy (k << 1/range), f(k) depends only on scattering length (*a*)
- For large *a* (≥ 1/k), nonperturbative problem!

Large scattering length system

- Cross section

$$\sigma(E) = \frac{8\pi}{1/a^2 + E}$$

~1/E in scaling region (E >>1/a^2)

~a^2 in threshold region (E << 1/a^2)

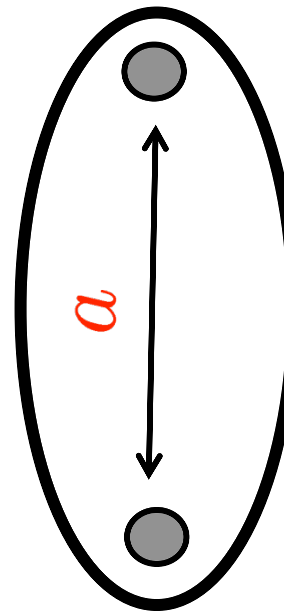
- Molecule (dark deuteron) for positive a>0

  - binding energy

$$E_2 = \frac{1}{a^2}$$

  - size

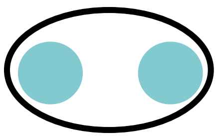
$$\sqrt{\langle r^2 \rangle} = a/\sqrt{2}$$



Scale invariance for $a \to \pm\infty$

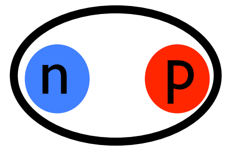
Weakly bound molecules in nature!

He_2

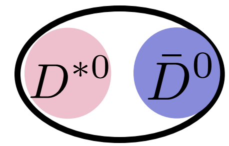


Van der Waals potential (Coulomb)

Deuteron (n+p)



X(3872)*



Nuclear potential (strong force)

E_b=0.11 μeV

$a \approx 20\ r_{VdW}$

E_b=2.2 MeV

$a \approx 3.9\ r_\pi$

E_b=0.3 ± 0.4 MeV

$a \sim 200\ r_\pi$

r_{VdW} = 5 Å

r_π = 1.4 fm

$$E_b \approx 1/a^2, \quad r_{\rm rms} \approx a/\sqrt{2}$$

$$\sigma \approx 4\pi a^2$$

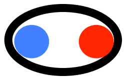
*candidate

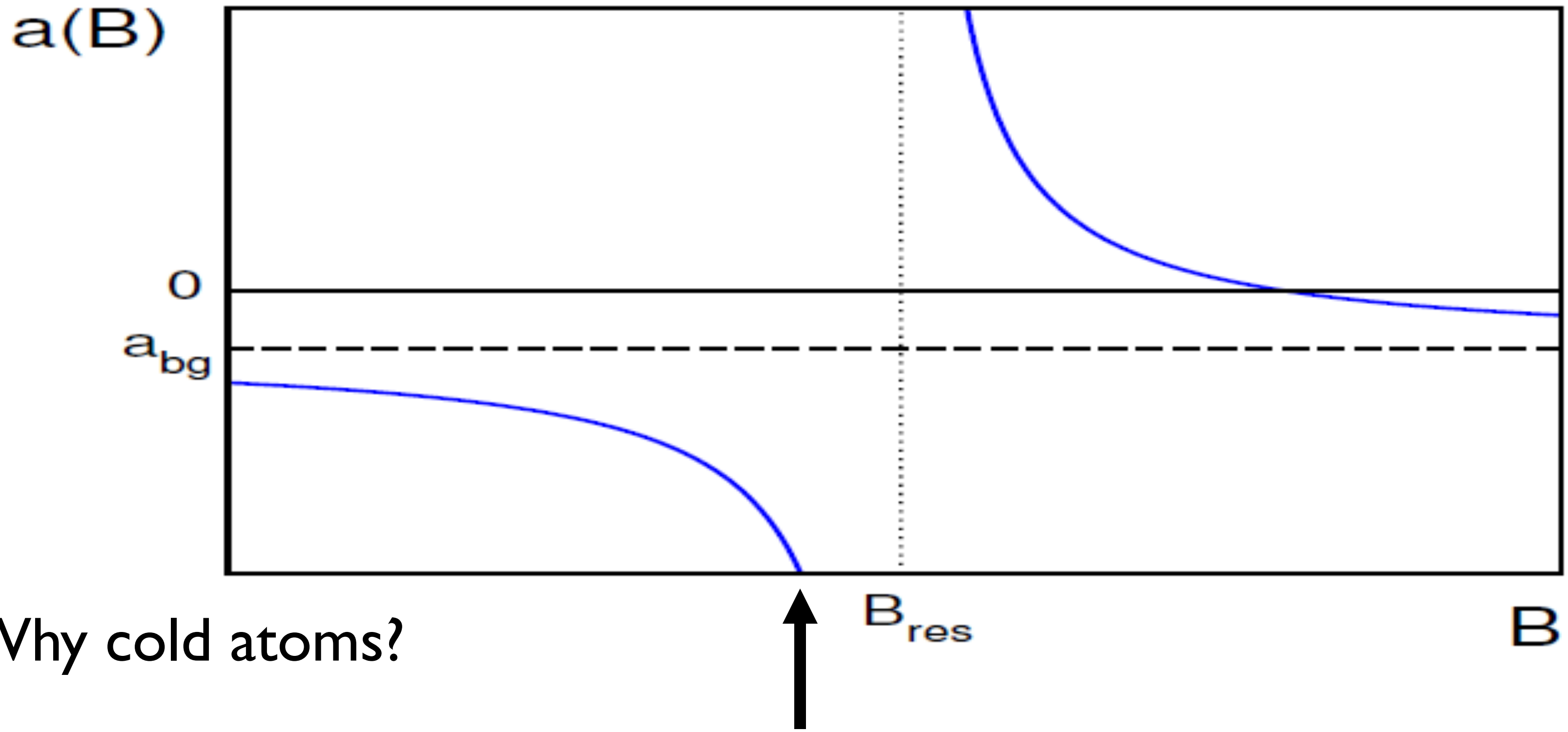
Universal physics with large scattering length

Regardless of microscopic interactions (QED or QCD)*!*

Tunable scattering length in ultracold atoms

Near Feshbach resonance, *a* varies with the B field !

Molecule association in Laboratory. 



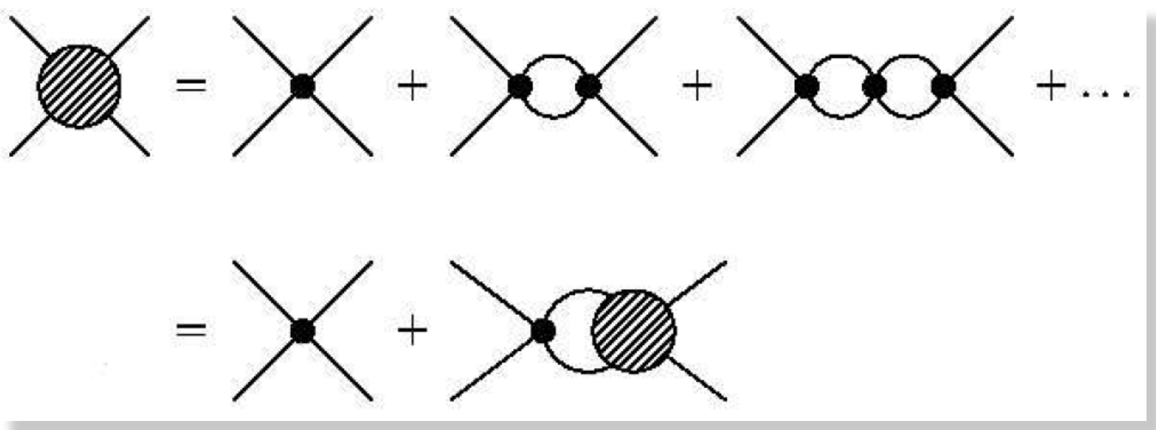
Why cold atoms?

simulating neutron gas (a = -19 fm)

a model system for strongly interacting quantum matter
like high T_c superconductor, dense nuclear matter

EFT for identical boson

NR kinetic term + $\frac{g_2}{4}\left(\psi^\dagger\psi\right)^2$



Renormalization

$$f(k) = -\frac{1}{1/a + ik}$$

$$\frac{1}{a} = \frac{4\pi}{g} + \frac{2}{\pi}\Lambda$$

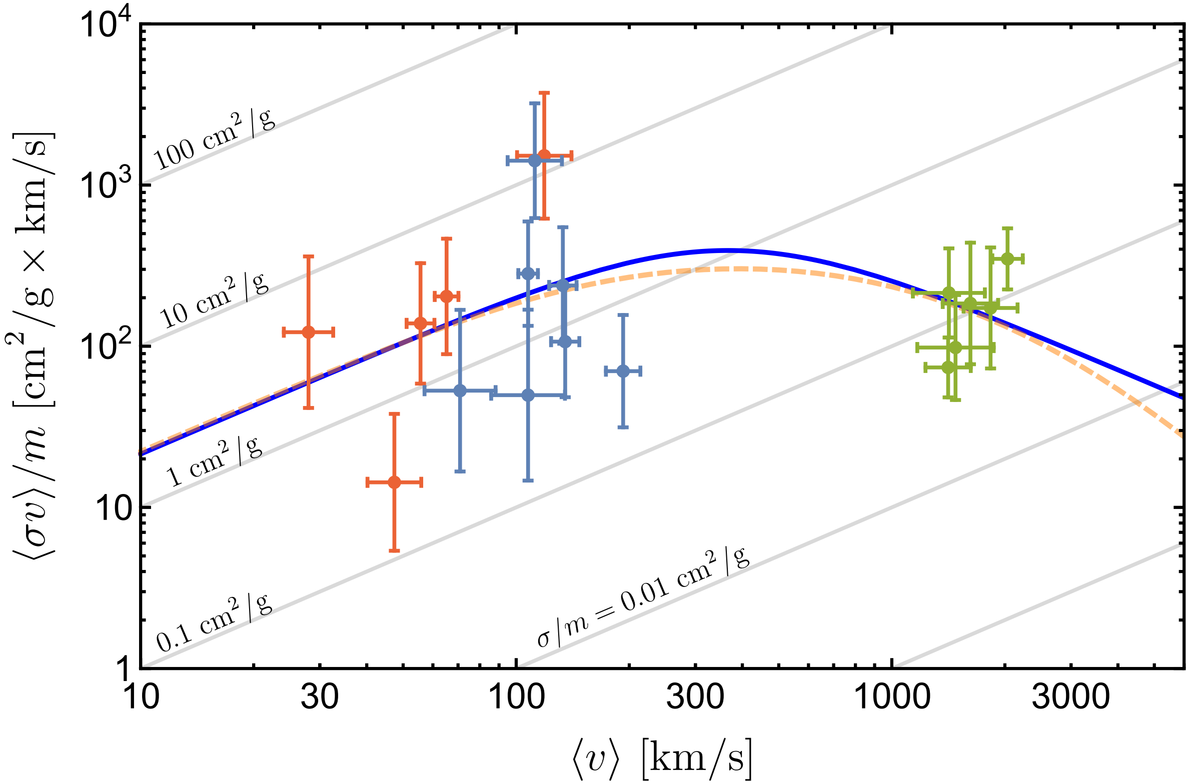
Self interaction: $\sigma_{\text{elastic}}(E) = \dfrac{8\pi}{1/a^2 + mE/\hbar^2}$ $\qquad E_2 = \hbar^2/ma^2$

Annihilation into visible matter:

$$\sigma_{\text{inelastic}}(E) = \frac{8\pi\,\text{Im}[1/a]}{(mE/\hbar^2)^{1/2}\left[1/a^2 + mE/\hbar^2\right]} \qquad \Gamma_2 = \frac{4\hbar\,\text{Im}[1/a]}{m\,a}$$

Small-scale structure problem

self-interacting dark matter strongly interacting at low energy.



Kaplinghat, Tuling, and Yu [PRL 16]

A dark photon model needs 3 parameters (dashed curve)

Excellent fit (blue curve) with 2 parameters

$$v\,\sigma_{\text{elastic}}(v) = \frac{8\pi a^2 v}{1 + (a m_\chi/2)^2 v^2}$$

$$m_\chi = 19^{+3}_{-2}\ \text{GeV} \qquad a = 17 \pm 3\ \text{fm}$$

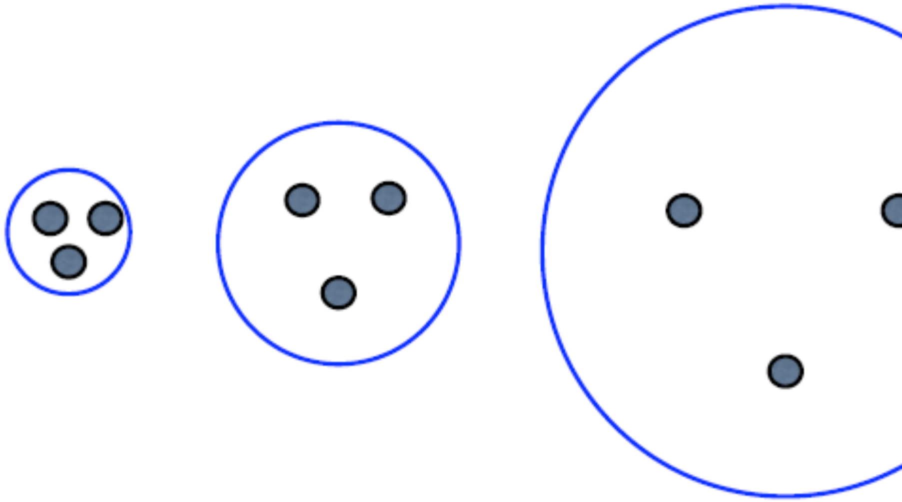
3-body physics: Efimov Trimers

Vitaly Efimov [1970]

- Infinitely many three-body bound states with accumulation point at 0 binding energy at $a = \pm\infty$
- Energies differ by $22.7^2 = 515$
- Sizes differ by 22.7

$$e^{\pi/s_0} \approx 22.7$$

$$s_0 \approx 1.006$$



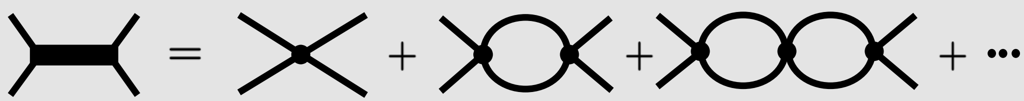
Discrete scale invariance in 3-body system

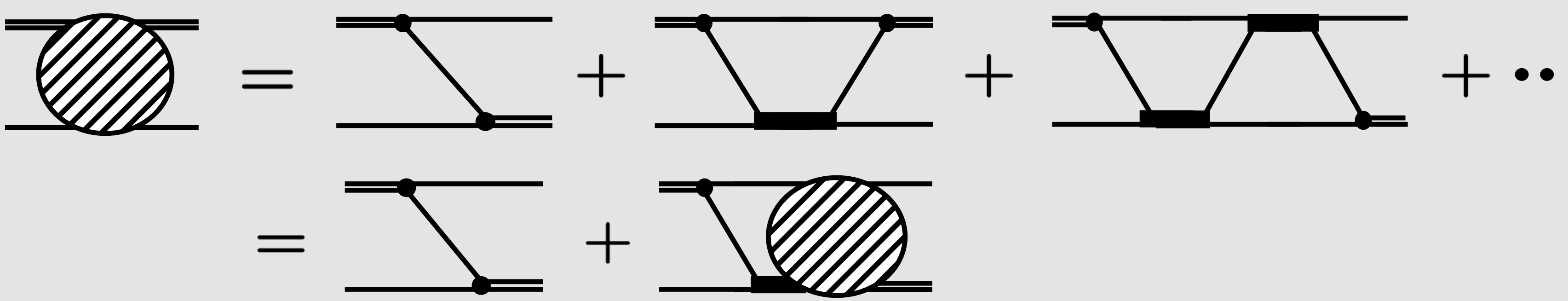
as a consequence of broken scale invariance!!

EFT for 3-body sector

- Interaction $\frac{g_2}{4}\left(\psi^\dagger\psi\right)^2$

- Integral equation for 3-body amplitude





$$\propto \frac{\cos[s_0 \ln(k/\Lambda)]}{k}$$

Log-periodic function!

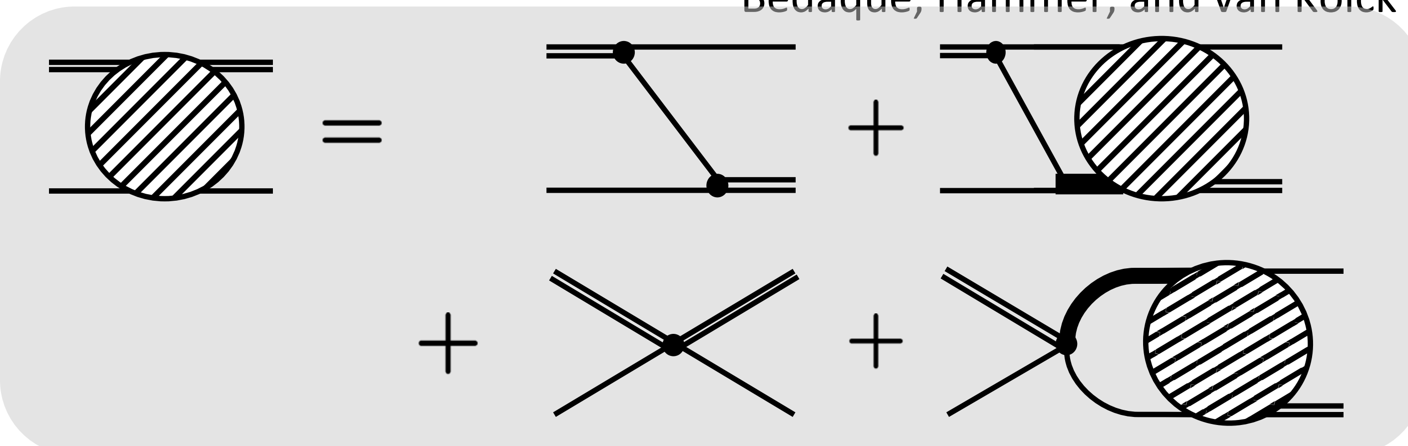
$s_0 \approx 1.006$

EFT for 3-body sector

- Interactions $\frac{g_2}{4}\left(\psi^\dagger\psi\right)^2 + \frac{g_3}{36}\left(\psi^\dagger\psi\right)^3$

- Integral equation for 3-body amplitude

Bedaque, Hammer, and van Kolck [PRL 1999]



- Renormalized 3-body parameter Λ_* is determined up to multiplicative factors of scaling factor exp[2π/s0]=22.7[2].

$$g_3 = -9\frac{g_2^2}{\Lambda^2}H_{BHvK} \qquad H_{BHvK} = -\frac{\sin[s_0 \ln(\Lambda/\Lambda_*) - \arctan(1/s_0)]}{\sin[s_0 \ln(\Lambda/\Lambda_*) + \arctan(1/s_0)]}$$

3-body amplitude

- STM equation for the S-wave

$$\mathcal{A}_0(p,k;E,\Lambda) = \frac{8\pi}{apk}\ln\frac{p^2+pk+k^2-mE-i\epsilon}{p^2-pk+k^2-mE-i\epsilon} + \frac{2}{\pi}\int_0^\Lambda dq\,\frac{q}{p}\ln\frac{p^2+pq+q^2-mE-i\epsilon}{p^2-pq+q^2-mE-i\epsilon}\,\frac{\mathcal{A}_0(q,k;E,\Lambda)}{-1/a+\sqrt{3q^2/4-mE-i\epsilon}}$$

- Analytic solution in scaling limit $A_+\,p^{-1+is_0} + A_-\,p^{-1-is_0}$

- Phase shift $\mathcal{A}_J(k_E,k_E;E) = \dfrac{3\pi}{k_E\cot\delta_{AD}^{(J)}(E) - ik_E}$

  - analytic log-periodic dependence in the S-wave

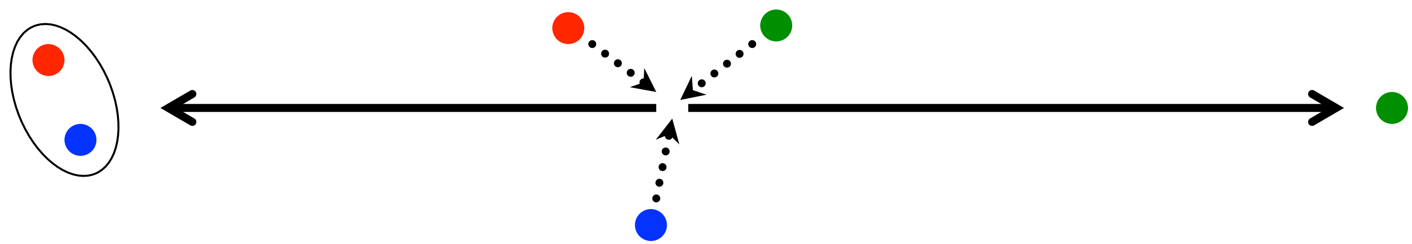
$$\exp\left(2i\delta_0(E)\right) = s_{22}(x) + \frac{s_{12}(x)^2\exp[2is_0\log(a/a_+)]}{1-s_{11}(x)\exp[2is_0\log(a/a_+)]}$$

  - higher-partial waves depend on x only.

a_+ = 3-body param

$x = E/E_2$

3-body recombination



- Defined by rate equation for homogeneous distribution:

$$\frac{d}{dt}n_2 = +K_3(T)\, n_1^3 \qquad \frac{d}{dt}n_1 = -2\, K_3(T) n_1^3$$

n_1: nucleon density
n_2: deuteron density

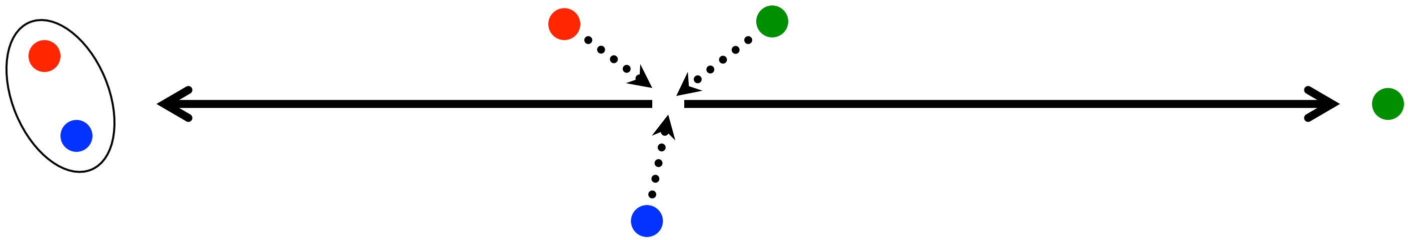
- Boltzmann averaged and hyper-angular averaged

$$K_3(T) = \frac{16\sqrt{3}\,\pi\hbar^3}{m^2(kT)^3}\int_0^\infty dE\, e^{-E/kT}\,(E_2+E)\,\sigma_{\text{breakup}}(E_2+E)$$

- Computed from breakup cross section

$$\sigma_{\text{breakup}}(E) = \frac{\pi}{k^2}\sum_{J=0}^{\infty}(2J+1)\left(1-\left|e^{2i\delta_J(k)}\right|^2\right)$$

3-body recombination



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n_1: nucleon density
n_2: deuteron density

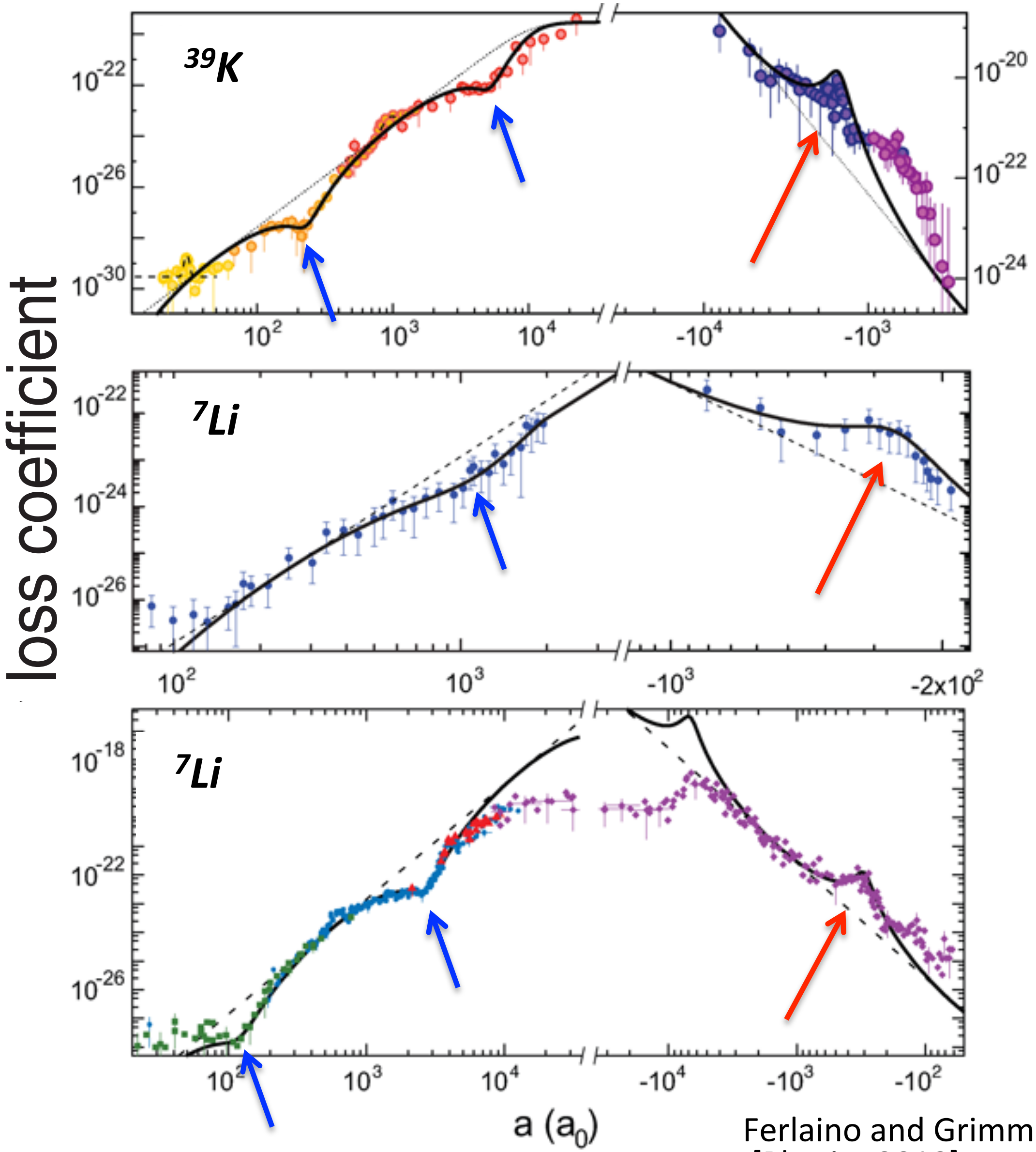
- low T limit: S-wave dominant and constant of T

$$K_3(T) \longrightarrow C_3(a/a_+)\,\frac{\hbar a^4}{m} \qquad C_3(a/a_+) \approx 67.1 \sin^2[s_0 \log(a/a_+)].$$

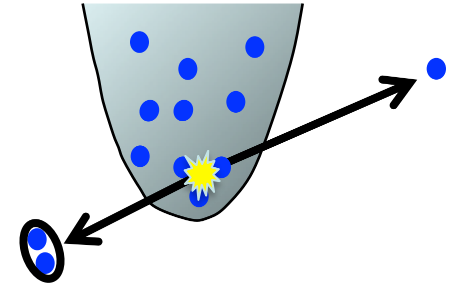
- high T limit: power-law scaling

$$K_3(T) \longrightarrow c_\sigma \frac{4\sqrt{3}}{\pi}\,\frac{\hbar\lambda_T^4}{m} \qquad \lambda_T \propto 1/\sqrt{T}$$

K_3 at T≈0 in ultracold atom



Ferlaino and Grimm [Physics 2010]



- positive *a* :
  a^4 scaling and destructive interferences

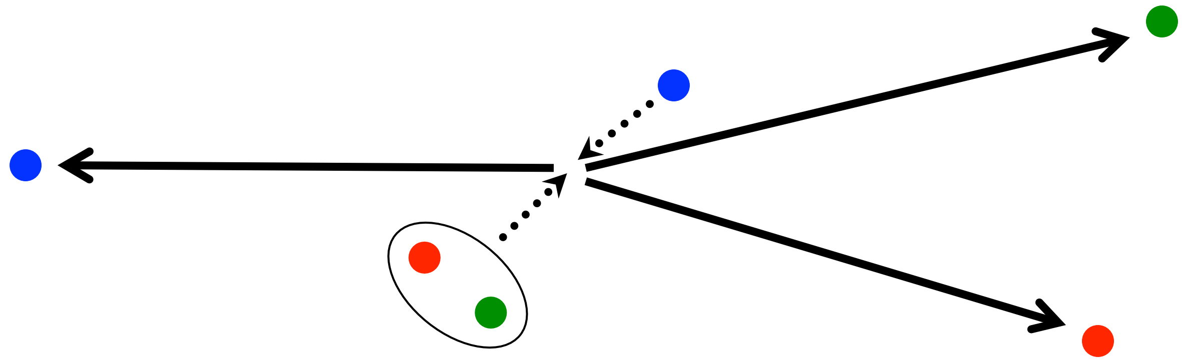
$$K_3(T) \longrightarrow C_3(a/a_+)\,\frac{\hbar a^4}{m}$$

$$C_3(a/a_+) \approx 67.1 \sin^2[s_0 \log(a/a_+)]$$

- negative *a* :

a^4 scaling and trimer resonances

Breakup



- Rate equation in homogeneous distribution:

$$\frac{d}{dt}n_2 = -K_2(T)n_1n_2 \qquad \frac{d}{dt}n_1 = +2\,K_2(T)n_1n_2$$

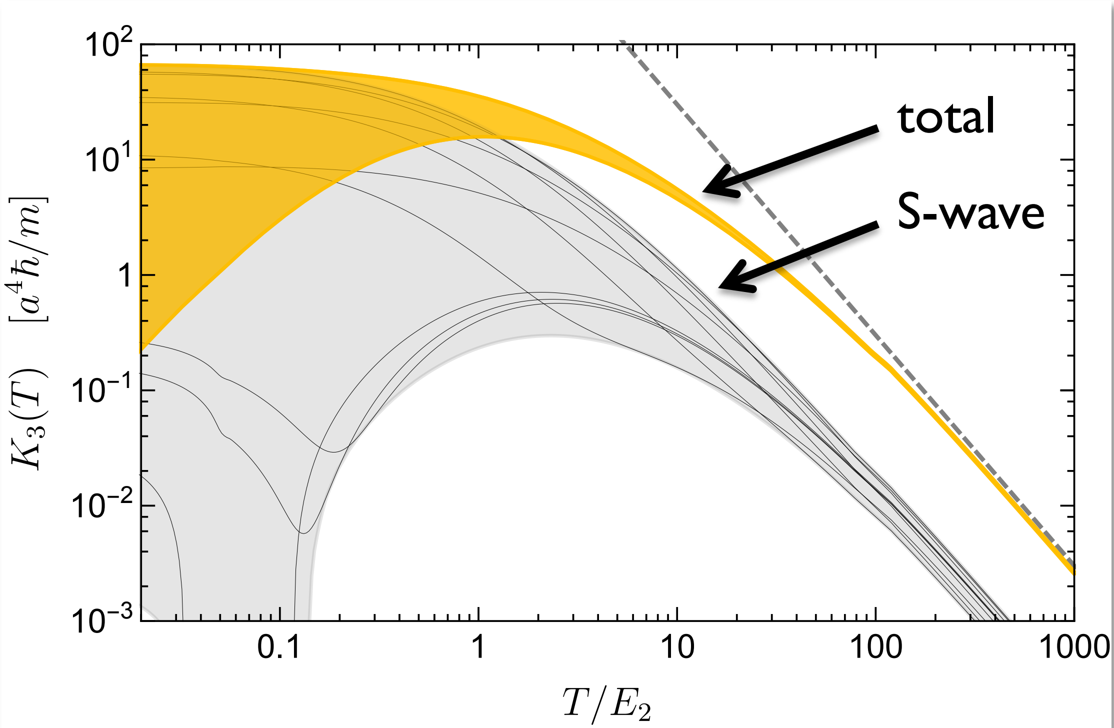
- Relation to K_3

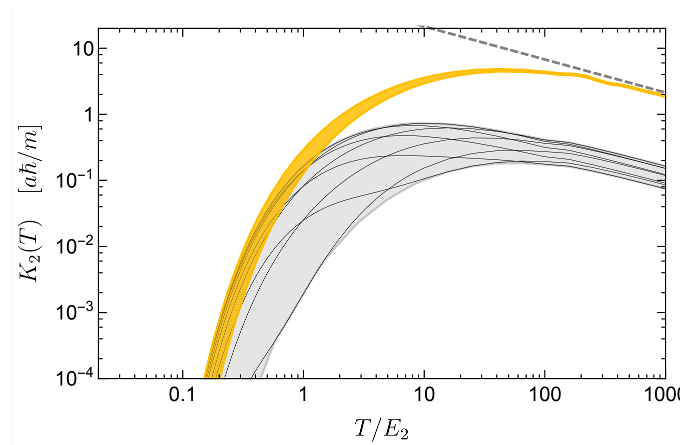
$$K_2(T) = \frac{e^{-E_2/kT}}{2\sqrt{2}\lambda_T^3}K_3(T)$$

low T: suppressed by Boltzmann factor

high T: $\lambda_T \propto 1/\sqrt{T}$ scaling

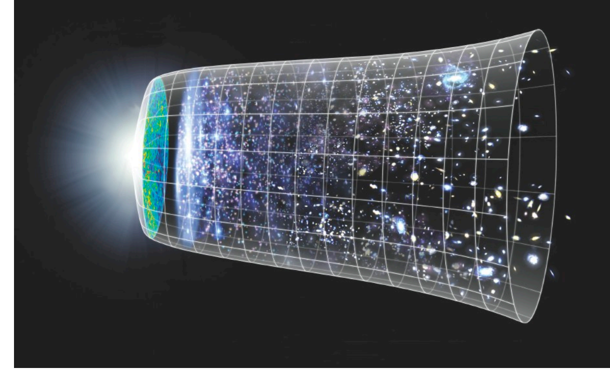
Temperature dependence





- S-wave dominant at low T but suppressed at high T
- Bands due to all possible values of 3-body parameter a_+
- K_3: constant at low T and T^{-2} at high T
- K_2: exponentially small at low T and $T^{-1/2}$ at high T
- Recombination is more efficient at low T

Evolution in early universe



- Evolution starts
  after DM decoupling from visible matter: very small or zero coupling to matter
- Initial condition: pure dark nucleons,
  bound state broken immediately at high $T_{dc} >> E_2$
- Evolution ends in early universe
  before DM captured by gravitational potential of galaxies
- Primary interest is deuteron number or fraction:
  including recombination K_3 and breakup K_2
  but omitting 3-, more-body cluster formations
  4-body recombination ($4d \rightarrow d_3\, d$) suppressed by n_d^4
  2-body reaction ($2d_2 \rightarrow d_3\, d$) needed for many d_2's

Rate equation

n_1: nucleon density
n_2: deuteron density

- Including up to n_1^3 and up to n_2

$$\left(\frac{d}{dt} + 3H\right) n_1 = -2K_3(T)\, n_1^3 + 2K_2(T)\, n_1 n_2 - 2K_1(T)\, n_1^2$$

$$\left(\frac{d}{dt} + 3H\right) n_2 = K_3(T)\, n_1^3 - K_2(T)\, n_1 n_2 - \Gamma_2\, n_2$$

  - Hubble expansion H, annihilation via d-d scattering K_1 and via d_2 decay Γ_2
  - ignoring K_1 and Γ_2 : upper bound for n_2

- Rate for total density is simply the Hubble expansion.

$$n_{\rm dark}(t) = n_1(t) + 2n_2(t). \qquad n_{\rm dark}(z) = \frac{\rho_{\rm cdm}}{m_\chi}\,(1+z)^3$$

$$\left(\frac{d}{dt} + 3H(t)\right) n_{\rm dark}(t) = 0$$

z: redshift

$$dt = -H^{-1}\ \ln(1+z)$$

In term of redshift

- Temperature of DM (NR) and photon (relativistic):

$$T(z) \approx T(0)\,(1+z)^2 \qquad T_\gamma(z) \approx T_{\rm cmb}\,(1+z)$$

Typical decoupling temp T_{dc}=m/20

defines redshift z_{dc} at T_{dc}: $$z_{\rm dc} \approx \frac{m_\chi/20}{T_{\rm cmb}} \approx 4\times10^{12}$$

determines T(0): $$T(z) \approx T_{\rm cmb}\,\frac{(1+z)^2}{1+z_{\rm dc}}$$

redshift $z_2=10^{10}$ at E_2=7 keV

- Hubble function from PDG $$H(z) = H_0\left[\Omega_\gamma z^4 + \Omega_m z^3 + \Omega_\Lambda\right]^{1/2}$$

Deuteron mass fraction

- Definition $f_2(z) = 2\, n_2(z)/n_{\rm dark}(z)$ $\quad 1 - f_2 = n_1(z)/n_{\rm dark}(z)$

$$\frac{d}{dz} f_2 = \frac{1}{(1+z)H}\Big[- 2\, K_3(T)\, n_{\rm dark}^2\, (1-f_2)^3 + K_2(T)\, n_{\rm dark}\, f_2 (1-f_2) \Big]$$

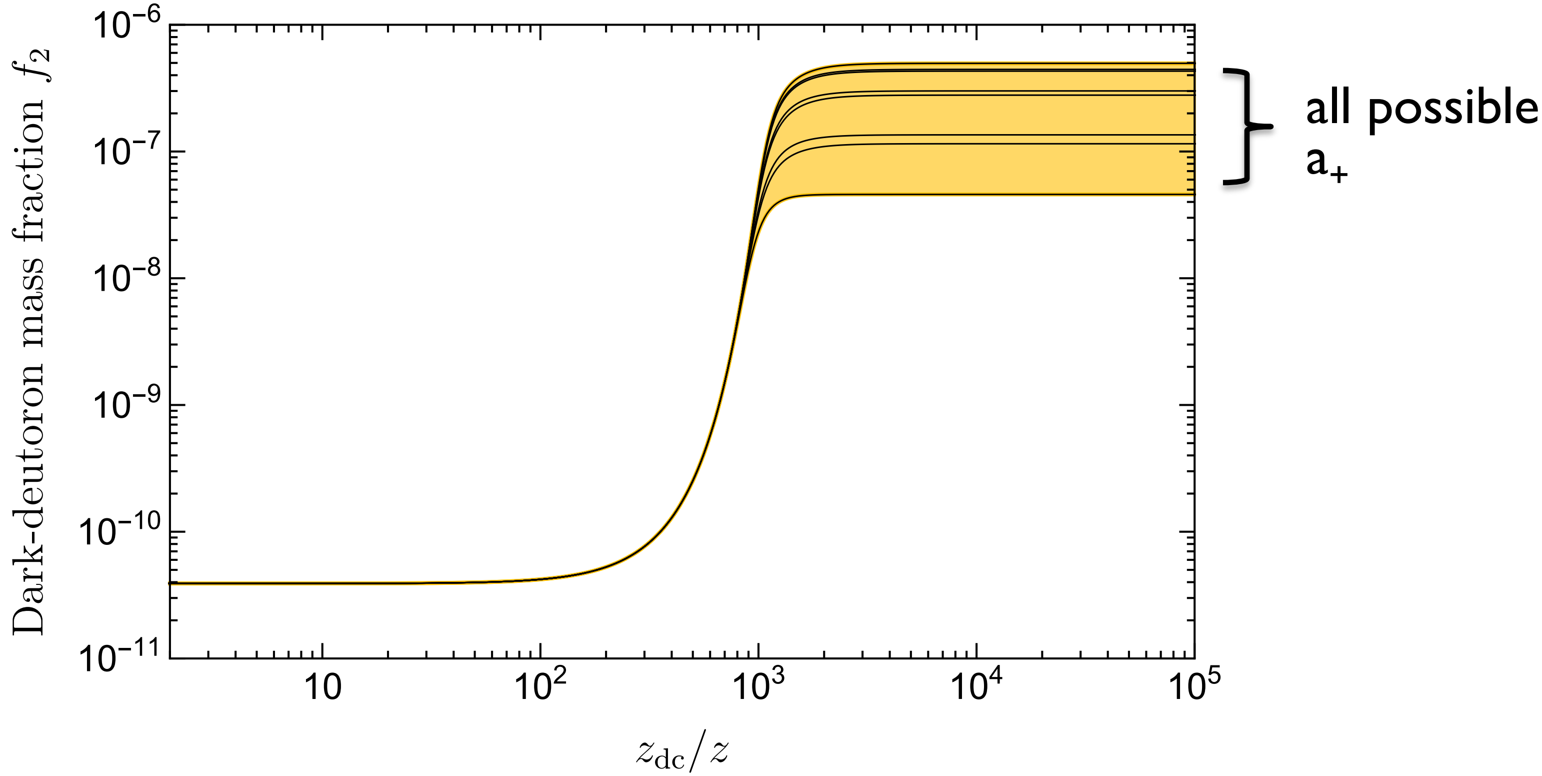
- High T ($z \gg z_2$):
  same scaling behavior in z on RHS and constant f_2

$$\frac{f_2}{(1-f_2)^2} = 4\sqrt{2} \left(\frac{2\pi(1+z_{\rm dc})}{m_\chi\, kT_{\rm cmb}} \right)^{3/2} \frac{\rho_{\rm cdm}}{m_\chi} \ \sim 10^{-11}$$

- Low T ($z \ll z_2$):
  K_2 exponentially small and quartic approach to $f_2(z{=}0)$

$$f_2(z) = f_2(0) - C_3(a/a_+)\, \frac{a^4 \rho_{\rm cdm}^2}{2 H_0 \Omega_\gamma^{1/2} m_\chi^3}\, z^4$$

Numerical result for m=19 GeV & a= 17 fm



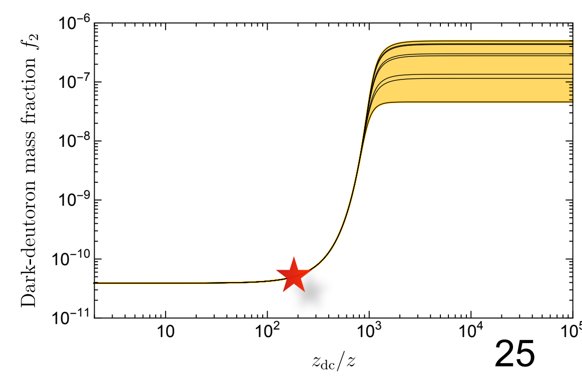
- a plateau 4×10^{-11} due to equilibrium between K_3 and K_2
- dramatic increase by $10^{3\text{-}4}$ near z_2 ($T \sim E_2$) due to enhanced K_3 and suppressed K_2
- *$f_2(0)$ is bounded between 5x10^{-8} and 5x10^{-7}*

Small and negligible?

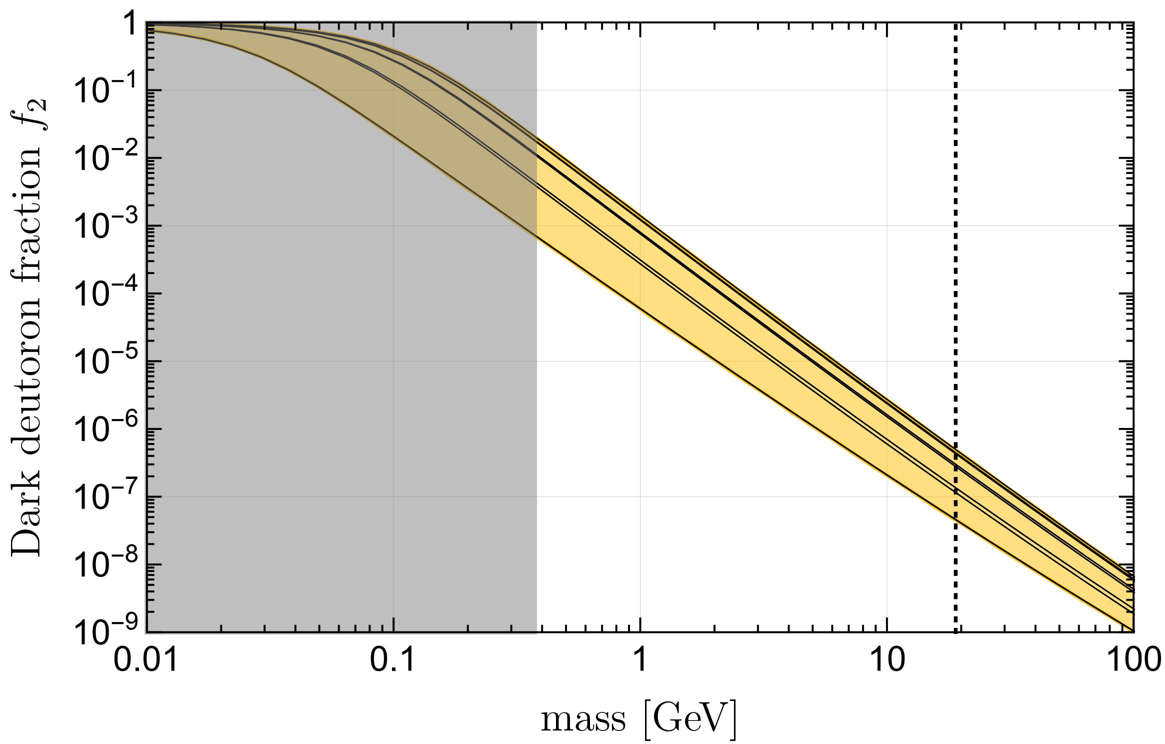
- The tiny fraction $\sim 10^{-7}$ suppresses cluster formations
  but NOT annihilation into visible matter.
- Annihilation rate $$\frac{d}{dt} n_{SM} = K_1(T) n_1^2 + \Gamma_2 n_2$$

$$K_1(T) = \left(32\pi\, g(kT/E_2) \frac{E_2}{kT}\right) \frac{\hbar\,\mathrm{Im}[a]}{m} \qquad \Gamma_2 = \frac{4\hbar\,\mathrm{Im}[1/a]}{m\,a}$$

- Their ratio is insensitive to coupling to visible matter and tells which process is more efficient.
- ratio =1 for $z_{dc}/z = 160$ ($f_2 \sim 5\mathrm{x}10^{-11}$)
  It easily reach $10^{4\sim5}$ at z_{dc}/z=1000.

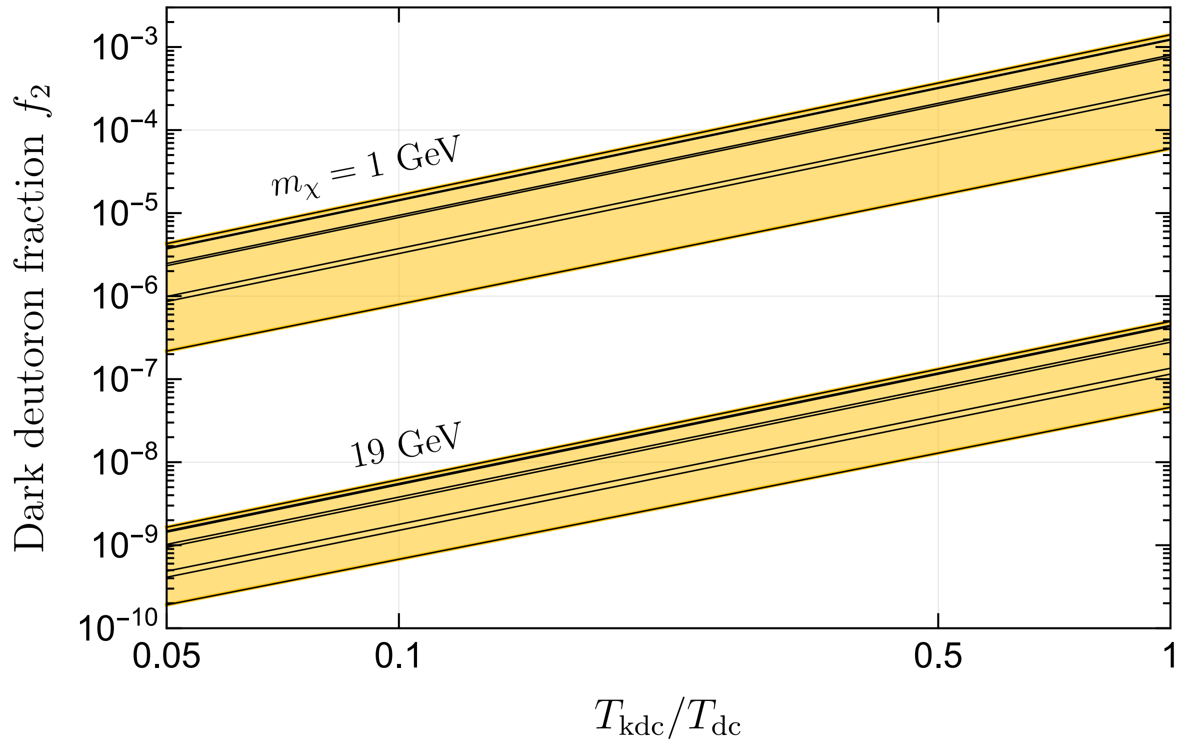


Mass dependence



- Relaxed constraint: $\sigma_{\rm elastic}/m = 2\ {\rm cm}^2/{\rm g}.$ *a* is known for given mass.
- The fraction scales like $m^{-2.5}$
- Gray region violates $T_{dc} >> E_2$. (initial condition n_2=0 invalid)
- The fraction can be as large as 10^{-3} at mass 1 GeV
- 40% fraction at 0.1 GeV requires huge rate $110\ {\rm cm}^2/{\rm g}$

Kinetic decoupling



- Kinetic equilibrium between DM and ordinary matter can be maintained longer $T_{kdc} < T_{dc.}$
- The fraction decreases and scaling like $(T_{kdc}/T_{dc})^{-1.9}$
- Introducing T_{kdc} is essentially same as lowering $T_{dc.}$

Summary

- Self-interacting dark matter can solve the small-scale structure problem
  - large scattering length system fits to self-interacting rates: m= 19 GeV, a=17 fm
- Production of bound state in early universe by 3-body recombination (bottleneck for large nuclei).
  - $f_2 \sim 10^{-11}$ at high T >>E_2
  - f_2 is enhanced by $10^{3\sim4}$ and approaches $10^{-7\sim-8}$ at low T << E_2
- With relaxed constraint, the fraction can go up to 10^{-3} at 1 GeV.

Thank you!