DM bound states by 3-body recombination

arXiv:1805xxxx, Braaten, DK, Laha

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INT multi-scale problems using EFTs

Dark matter and its bound state

- \Box elementary particle: QCD-like, SUSY ... composite particle like dark nucleons
- \square Dark nucleosynthesis by radiative fusion : $d+d_N \rightarrow d_{N+1}$ + mediator recombination : $3 d \rightarrow d + d_2$, $d_2 + d_N \rightarrow d_{N+1} + d$
- \Box Without mediator, 3-body recombination forms dark deuteron and is the bottleneck for N-body cluster. How many deuterons can be produced?
- 2 \Box Use a simple and very predictive model with two parameters mass and scattering length *a*

Outline

- \Box Universal 2-body physics and small scale structure problem
- \Box 3-body physics and recombination process
- **QProduction of bound state in early** universe

Universal physics at low energy

a shallow state remains the same by tuning depth and width simultaneously

- 1. *universal regardless of microscopic physics*
- 2. fine tuned state: large scattering length (a) >> r_{a} ,...
- 3. point-like interactions can be used.

Large scattering length system

□ Quantum Mechanics at low energy

$$
f(k) = \frac{1}{-\frac{1}{a} - ik + \frac{rsk^2}{2}k^2 + \cdots}
$$

 \Box At very low energy (k << 1/range), f(k) depends only on scattering length (*a*) \Box For large a (\geq 1/k), nonperturbative problem!

Large scattering length system

q Cross section ~1/E in scaling region $(E \gg 1/a^2)$ $~\sim a^2$ in threshold region (E << $1/a^2$) q Molecule (dark deuteron) for positive *a*>0 $\sigma(E) = \frac{8\pi}{1+2}$ $1/a^2 + E$

^qbinding energy \Box size $E_2 =$ 1 *a*2

Scale invariance for $a \to \pm \infty$

Universal physics with large scattering length

Regardless of microscopic interactions (QED or QCD)*!*

Tunable scattering length in ultracold atoms

Near Feshbach resonance, *a* varies with the B field !

a model system for strongly interacting quantum matter like high T_c superconductor, dense nuclear matter

EFT for identical boson range *r*⁰ and an S-wave scattering length *a* that is much larger than *r*0. The range and the scattering length provide a high energy scale *E*⁰ = ~²*/mr*² elastic cross section begins increasing in accordance with Eq. (2.1). In the scaling **FI TOP IDENTICAI DOSON** ionization \mathbf{A} . The universal low-energy behavior of particles with a large scattering \mathbf{A}

Self interaction: $\sigma_{\rm elastic}(E) = \frac{8\pi}{\sqrt{e^2 + 8\varepsilon}}$ $\frac{1}{a^2 + mE/\hbar^2}$ $E_2 = \hbar^2 / m a^2$ $E_2=\hbar^2/ma^2$ Ω_{τ} Self interaction: $\sigma_{\text{elastic}}(E) = \frac{1}{4(1.8 \text{ F})(1.8 \text{ F})} E_2 = \hbar^2$ $i\mu = \frac{1}{u}$ Ω_{σ} $\sigma_{\text{elastic}}(E) = \frac{1}{4\pi\epsilon_0} E_0 - \frac{1}{4\pi\epsilon_0} E_2 = \hbar^2 / ma^2$ $\frac{1}{u} + \frac{1}{u}$

Annihilation into visible matter: [eb,dk] If the two colliding atoms are distinguishable particles, such as the two spin The universal inelastic scattering cross section for identical bosons is are also decay channels for the dimer. The universal expression for the decay rate is

Annimization into visible matter:

\n
$$
\sigma_{\text{inelastic}}(E) = \frac{8\pi \operatorname{Im}[1/a]}{(mE/\hbar^2)^{1/2}[1/a^2 + mE/\hbar^2]} \qquad \Gamma_2 = \frac{4\hbar \operatorname{Im}[1/a]}{m a} \quad \text{9}
$$

Small-scale structure problem **into and Subsetertainty in the constant at an and** *n* **and** *n* **and** *n* **meV. The constant at a 17 MeV. The constant at a 17 MeV. The constant at a 17 MeV. The constant at 27 MeV. The constant at 27 MeV. The constant at 27 M** ↵⁰ = 1*/*137 and fit the parameters *m* and *µ*. Their fitted values are *m* = 15+7 of the mean velocity h*v*i. The data points are from dwarf galaxies (red), low-surfacebrightness galaxies (blue), and galaxy clusters (green) [1]. The curves are the best fit for a

↵⁰ = 1*/*137 and fit the parameters *m* and *µ*. Their fitted values are *m* = 15+7 ⁵ GeV self-interacting dark matter strongly interacting at low energy. diagonal lines are for energy-independent cross sections. This plot follows the same style

and Yu [PRL 16]

A dark photon model needs 3 parameters (dashed curve) A dark photon model needs 3 parameters (dashed curve) nucleons are their mass *m* and the scattering length *a*. The elastic cross section is geus 5 parameters (gasneg curve). The reaction σ

10 Excellent fit (blue curve) with 2 parameters $v \sigma_{\text{elastic}}(v) = \frac{8\pi a^2 v}{1 + (am_\chi/2)^2 v^2}$ $m_{\chi} = 19^{+3}_{-2} \text{ GeV}$ $a = 17 \pm 3 \text{ fm}$ 10 $v \sigma_{\text{elastic}}(v) = \frac{8\pi a^2 v}{1 + \left(\cos \frac{v}{2}\right)^2}$ $\frac{6kx}{1+(am_{\chi}/2)^{2}v^{2}}$

3-body physics: Efimov Trimers *Vitaly Efimov [1970]*

- \Box Infinitely many three-body bound states with accumulation point at 0 binding energy at $a=\pm\infty$
- \Box Energies differ by 22.7² = 515
- □ Sizes differ by 22.7

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Discrete scale invariance in 3-body system as a consequence of broken scale invariance!!

 $s_0 \approx 1.006$

EFT for 3-body sector

Interactions
$$
\frac{g_2}{4} \left(\psi^{\dagger} \psi \right)^2 + \frac{g_3}{36} \left(\psi^{\dagger} \psi \right)^3
$$

 \Box Integral equation for 3-body amplitude

Bedaque, Hammer, and van Kolck [PRL 1999]

 \Box Renormalized 3-body parameter Λ_* is determined up to multiplicative factors of scaling factor exp[2π/s0]=22.72. Λ_*

$$
g_3 = -9\frac{g_2^2}{\Lambda^2}H_{BHvK} \quad H_{BHvK} = -\frac{\sin[s_0 \ln(\Lambda/\Lambda_*) - \arctan(1/s_0)]}{\sin[s_0 \ln(\Lambda/\Lambda_*) + \arctan(1/s_0)]}_{13}
$$

$\sum_{i=1}^{n}$ in a specific angular momentum channel, the STM equation in Eq. (35) reduces to angular momentum channel in Eq. (35) reduces to an Eq. (35) 3-body amplitude integral is sufficiently singular at large q that it is necessary to impose an ultraviolet cutoff in the cutof scattering can be expressed in terms of the phase shifts: $\frac{1}{2}$ (2*J* + 1) The integral is a Mellin transform that can be evaluated analytically. The resulting equation for *s* is 8 sin*(*!*s/*6*)*

□ STM equation for the S-wave q < ^Λ, where ^Λ is much greater than ^p, ^k and (m|E|)¹/². The STM equation for ^J = 0 r the S-way [√]3*^s* cos*(*!*s/*2*)*

$$
\mathcal{A}_0(p,k;E,\Lambda) = \frac{8\pi}{apk} \ln \frac{p^2 + pk + k^2 - mE - i\epsilon}{p^2 - pk + k^2 - mE - i\epsilon} \n+ \frac{2}{\pi} \int_0^{\Lambda} dq \frac{q}{p} \ln \frac{p^2 + pq + q^2 - mE - i\epsilon}{p^2 - pq + q^2 - mE - i\epsilon} \frac{\mathcal{A}_0(q,k;E,\Lambda)}{-1/a + \sqrt{3q^2/4 - mE - i\epsilon}}
$$

Q Analytic solution in scaling limit $A_+ p^{-1+1S_0} + A_- p^{-1-1S_0}$ **a** Analytic solution in scaling limit $A_+ p^{-1+is_0} + A_- p^{-1-is_0}$ **E** Analytic solution in scaling limit $A_+ p^{-1+1S_0} + A_- p^{-1-1S_0}$ obtained from In the universal regime where the energy *E* is much smaller than the energy scale Λ in scaling limit $A_+ \, p^{-1+\mathrm{i} s_0} + A_- \, p^{-1-\mathrm{i} s_0}$

a Phase shift $A_J(k_E, k_E; E) = \frac{3\pi}{1 - \frac{2\pi}{1 - \frac{2$ $\mathcal{L} = \frac{A D \sqrt{3}}{A D \sqrt{3}}$ $\mathcal{A}_J(k_E,k_E;E) = \frac{3\pi}{\frac{1}{k}\cos\left(\frac{\mathcal{S}(J)}{2}\right)}$ $k_E\cot\delta_{AD}^{(J)}(E)-ik_E$ a_n **a a** shift f_n is a different α of α dimensionless α **d rnase snite** $A_J(k_E, k_E; E) = \frac{1}{k_E \cot \delta_{AD}^{(J)}(E) - ik_E}$ in the form \Box $\mathbf{F_L} = \mathcal{A}_J(k_E, k_E; E) = \frac{3\pi}{\sqrt{D_E}}$ $k_E \cot \delta_{AD}^{\sim}(E) - i k_E$

□ analytic log-periodic dependence in the S-wave By demanding that the solution of the integral (336) has a well-defined limit as " → ∞, Bedaque, Hammer, and

! "

$$
\exp\left(2i\delta_0(E)\right) = s_{22}(x) + \frac{s_{12}(x)^2 \exp[2is_0 \log(a/a_+)]}{1 - s_{11}(x) \exp[2is_0 \log(a/a_+)]}
$$

 a_+ = 3-body param a_+ is a higher-partial waves depend on x only a_{\parallel} = 3-hody param a_+ = 3-body param a_{+} = 3-body param

□ higher-partial waves depend on x only. **Let nigher-partial waves depend on x only.** $x = E/E_2$ and the set of the set of $x = E/E_2$ and $x = E$ ⊔ nigher-partial waves depend o

(A⁺ *^q*+i*s*⁰ ⁺ *^A*−*q*−i*s*⁰ *)*. (353)

3-body recombination 3-hody re \overline{C} 3~²*k*² *,* (3.2) *{*eq:Ecm*}*

n Defined by rate equation for homogeneous distribution: *n*3 1: *J*=0 *^k* cot *^J* (*k*) *ikP^J* (cos ✓)*.* (3.3) *{*eq:fktheta*}*

$$
\frac{d}{dt}n_2 = +K_3(T)n_1^3 \qquad \frac{d}{dt}n_1 = -2K_3(T)n_1^3
$$

n_i: nucleon density

□ Boltzmann averaged and hyper-angular averaged n²: deuteron density
Reltement exerced and byper angular exerced a_2 : a_3 recombination rate. Alternatively it can be recombined in the three-body in the th _{be expressive} and the dimer-angular averaged and sensity and $\frac{1}{2}$. Boltzmann averaged and hyper-angular averaged

$$
K_3(T) = \frac{16\sqrt{3}\,\pi\hbar^3}{m^2(kT)^3} \int_0^\infty dE \, e^{-E/kT} \left(E_2 + E \right) \sigma_{\text{breakup}}(E_2 + E)
$$

□ Computed from breakup cross section $\overline{}$ \blacksquare puted from breakup cross section *k*2 *J*=0

$$
\sigma_{\text{breakup}}(E) = \frac{\pi}{k^2} \sum_{J=0}^{\infty} (2J+1) \left(1 - \left| e^{2i\delta_J(k)} \right|^2 \right)
$$

3-body recombination $\overline{\mathbf{3}}$ ody recombination **combination** *limit*, where *kT* is much smaller than the energy scale *E*² = ~²*/ma*² set by the

Q Defined by rate equation for homogeneous distribution: number density *n*² of dimers increases from 3-body recombination is proportional to r homog *C*3(*a/a*+) eou:
} *istributior* **Principle and the sense of the property of the sense of the sense** ◆³ ~*a*

$$
\frac{d}{dt}n_2 = +K_3(T)\,n_1^3 \qquad \qquad \frac{d}{dt}n_1 = -2\,K_3(T)n_1^3 \qquad \qquad \substack{\mathsf{n}_\textup{l}: \textup{nucleon density}\\ \mathsf{n}_\textup{l}: \textup{nucleon density}\\ \mathsf{n}_\textup{2}: \textup{deuteron density}}}
$$

1ensity n_2 : deuteron density

I low T limit: S-wave dominant and constant of T n₂: deuteron de
E2 Iow T limit: S-wave dominant and constant of T

$$
K_3(T) \longrightarrow C_3(a/a_+) \frac{\hbar a^4}{m}
$$
 $C_3(a/a_+) \approx 67.1 \sin^2[s_0 \log(a/a_+)]$

 $\sqrt{ }$

T

a high T limit: power-law scaling *F* $\frac{1}{2}$ $: L \rightleftharpoons L$ high T limit: power-law scaling

$$
K_3(T) \longrightarrow c_{\sigma} \frac{4\sqrt{3}}{\pi} \frac{\hbar \lambda_T^4}{m} \qquad \lambda_T \propto 1/\sqrt{T}
$$

$K₃$ at T≈0 in ultracold atom We can obtain a simple and α

a positive *a* : a⁴ scaling and destructive $\left\{\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ \uparrow & \uparrow & \uparrow \end{array}\right\}$ interferences $K_3(T) \longrightarrow C_3(a/a_+)$ $\hbar a^4$ *m* $\bigcup_{10^{-22}}$ is dominated by the S-wave contribution. The limiting behavior of the limiting beha *C*3(*a/a*+) $\frac{d}{d}$ \int in Eq. (3.15). It can be approximated with an error of less than $\hbar a^4$

> ^q negative *a* : a⁴ scaling and trimer resonances p rim ~
44 *m*

□ Rate equation in homogeneous distribution:

$$
\frac{d}{dt}n_2 = -K_2(T)n_1n_2 \qquad \frac{d}{dt}n_1 = +2K_2(T)n_1n_2
$$

Definition to K₃
$$
K_2(T) = \frac{e^{-E_2/kT}}{2\sqrt{2}\lambda_T^3}K_3(T)
$$

 low T: suppressed by Boltzmann factor high T: $\ \lambda_T \propto 1/\sqrt{T}$ scaling $\sqrt{ }$ *T*

Temperature dependence

 \square S-wave dominant at low T but suppressed at high T \square Bands due to all possible values of 3-body parameter a_+ \Box K₃: constant at low T and T⁻² at high T \Box K₂: exponentially small at low T and T^{-1/2} at high T □ Recombination is more efficient at low T 19

Evolution in early universe

 \square Evolution starts

after DM decoupling from visible matter: very small or zero coupling to matter

- □ Initial condition: pure dark nucleons, bound state broken immediately at high T_{dc} >>E₂
- \Box Evolution ends in early universe before DM captured by gravitational potential of galaxies

□ Primary interest is deuteron number or fraction: including recombination K_3 and breakup K_2 but omitting 3-, more-body cluster formations 4-body recombination (4d \rightarrow d₃ d) suppressed by n_d^4 2-body reaction (2d₂ \rightarrow d₃ d) needed for many d₂'s

Rate equation that must be overcome by $3-$ body recombination in order to form the larger dark α *n*dark(*t*) = *n*1(*t*)+2*n*2(*t*)*.* (5.1) *{*eq:dn12*}* [eb,dk] The time evolution equations for *n*1(*t*) and *n*2(*t*) obtained from the Boltzmann 2 *n*_{te} 2 *n*_{1(*ation*) $\frac{1}{2}$} [eb,dk] The time evolution equations for *n*1(*t*) and *n*2(*t*) obtained from the Boltzmann

a Including up to n_1^3 and up to n_2 and n_1 : nucleon densite in the data of n_2 : deuteron densite is an area of n_1 : deuteron densite is an area of n_2 : deuteron densite is an area of n_1 : deuteron densi \int_0^{π} ncluding up to n₁³ and up to n₂

 n_1 : nucleon density n_2 : deuteron density

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$$
\left(\frac{d}{dt} + 3H\right)n_1 = -2K_3(T)n_1^3 + 2K_2(T)n_1n_2 - 2K_1(T)n_1^2
$$

$$
\left(\frac{d}{dt} + 3H\right)n_2 = K_3(T)n_1^3 - K_2(T)n_1n_2 - \Gamma_2 n_2
$$

 \Box Hubble expansion H, annihilation via d-d scattering K₁ and via d_2 decay $\overline{\Gamma}_2$ \Box ignoring K_1 and $\mathsf{\Gamma}_2$: upper bound for n_2 \Box mubble expansion m, annimiation via d-d scattering N_1 and density \Box *n*₂ and *n*₂ (*t*₁ and *n*₂ (*t*₁ moder bound for n₂ baryon number are negligible, so the total dark baryon number density is [eb,dk] where *^H* is the Hubble function, *^K*3(*T*), *^K*2(*T*), and *^K*1(*T*) are temperature- *{*eq:dn1n2*}* dependent expansion ri, animination via d-d stattering is and via Hubble function *H*(*t*) depends on time, being determined by the scale factor *a*(*t*) of $\frac{u_1}{d}$ is $\frac{u_1}{d}$ and $\frac{u_2}{d}$. upper bound for $\frac{u_2}{d}$ \square Hubble expansion H, annihilation via d-d scattering K₁ and via \Box ignoring K, and Γ : upper bound for n the universe: *H* = *d* ln(*a*)*/dt*. The rate coecients are functions of the temperature n¹ Hubble expension H ennibiletion via d d centtering V 3-body recombination term in Eq. (3.23 a). Since we ignore the annumation of the annual since we ignore the annihilation of the annual since we ignore the annual since $\frac{1}{2}$ on the dark matter interactions; it is just diluted by the Hubble expansion. Ibbic expansion is, animination via a a scattering is and via

a Rate for total density is simply the Hubble expansion. $n_{\text{dark}}(t) = n_1(t) + 2n_2(t)$, $n_{\text{dark}}(z) = \frac{p_{\text{cdm}}}{m} (1+z)^3$ $\left(\frac{u}{u} + 3H(t)\right) n_{\text{dark}}(t) = 0$ *T*(*t*) of the dark matter, which also depends on time. \Box nace for coal definity is simply crie fractic expansion: $\begin{array}{ccc} \Delta & & & \Delta \\ \hline \end{array}$ $\int \frac{dt}{dt} + 3H(t) \int n_{\text{dark}}(t) = 0$ αu $dt = -H^{-1} \ln(1 + z)$ **The Rate for total density is simply the Hubble expansion.** $p_{\text{down}}(t) = p_1(t) + 2p_2(t)$ $n_1(t) \geq \frac{p_{\text{cdm}}}{(1 + z)^3}$ m_{χ} ² $\left(\begin{array}{cc} d & qH(t) \end{array}\right)_{\infty}$ (t) $=0$ z: redshift $\left(dt + \frac{\partial \mathbf{H}(\mathbf{e})}{\partial t}\right)$ redack $\left(\mathbf{e}\right)$ of the analyzing the annihilation of the two \mathbf{H} $\alpha v = \mathbf{H} \mathbf{m}(\mathbf{I} + \mathbf{z})$ $n_{\text{dark}}(t) = n_1(t) + 2n_2(t).$ $\int d$ $\frac{d}{dt} + 3H(t)$ ◆ $\left(\frac{a}{dt} + 3H(t)\right) n_{\text{dark}}(t) = 0$ 2.1 **c**ds me $n_{\text{dark}}(z) = \frac{\rho_{\text{cdm}}}{z}$ m_χ $(1+z)^3$ $au = -11$ matter in $\frac{1}{2}$ z: redshift

In term of redshift *T*(*z*) ⇡ *T*cmb **In term of redshift** 1 + *z*dc $\int_{\mathbb{R}^2} f(x) dx$ of the photons is proportional to the photons i momentum of Leasing changes the momentum of a particle by a particle by a particle by a particle by a factor o $\sqrt{2}$ other hand, the temperature *T*(*z*) of the photons is proportional to their average the three-body parameter *a*+. For the mass and the scattering length, we use values that can solve structure problems of the universe. The values that give the universe. The values that give the values that give the universe. The values of the value

□ Temperature of DM (NR) and photon (relativistic): by gravitational potential wells and *T*cmb = 2*.*73 K is the present temperature of \mathbf{D} The decouperature of DM (NR) and photon (relativistic): decoupling of dark matter and ordinary matter ordinary matter of α *ⁿ*dark(*t*) = *ⁿ*dark(0) ✓*a*(0) i **c**): *T*(*z*) ⇡ *T*(0) (1 + *z*) **Q** Temperature of DM (NR) and photon (relation *,* (5.7a) *{*eq:Tdark*}* **a** and the log-periodic parameter α multiplicative factors of ⁰ ⇡ 22*.*69, we consider eight values that are equally spaced

equilibrium: *T*(*z*dc) = *T*(*z*dc), where *z*dc is the redshift at decoupling. We are not $\alpha(z) \approx I(0) (1+z)^{-1}$ $I_{\gamma}(z) \approx I_{\rm cmb} (1+z)$ decoupling redshift can be determined by the condition that the temperature of the temperature of π (1 iii) $T(z) \approx T(0) (1+z)^2$ $T_{\gamma}(z) \approx T_{\text{cmb}} (1+z)$ \mathcal{C} $(1 + \alpha)$ $T \sim$

Typical decoupling temp T_{dc} =m/20 defines redshift z_{dc} at T_{dc} : 1_{deconl} $\mathbf{t} \cdot \mathbf{c}$ parameter is larger than the e \mathbf{r} reas multiplativistic degrees of $z_{\text{dc}} \approx \frac{m_{\chi/20}}{T_{\text{crbs}}} \approx 4 \times 10^{12}$ $m_\chi/20$ $T_{\rm cmb}$ **dshift** z_{dc} **at** \int_{dc} : $z_{dc} \approx \frac{m_{\chi}/20}{T_{\chi}} \approx 4 \times 10^{12}$ $uping$ term H_dc = cm zo T_{A} = m/20 $f(2)$ emb

determines T(0): $T(z) \approx T_{\text{cmb}}$ $(1 + z)^2$ $1 + z_{\text{dc}}$ $\lim_{x \to \infty} \frac{1 + z^2}{(1 + z)^2}$ *f*2(*z*)=2 *n*2(*z*)*/n*dark(*z*)*.* (5.10) *{*eq:dfraction*}* 1 + $(1 + z)^2$ $=$ $\frac{2}{3}$ $\frac{2}{3}$ $\frac{2}{3}$ $\frac{2}{3}$ $\frac{d}{dx}$ $\frac{d}{dx}$ $\frac{d}{dx}$ $\frac{d}{dx}$

redshift $z_2=10^{10}$ at $E_2=7$ keV $\int \mathbf{F} \cdot d\mathbf{r} = \int \mathbf{F} \cdot d\mathbf{r}$ *T* the theory is the theory μ_2 due μ_2 matter ordinary matter ordinary matter ordinary matter ordinary matter ordinary matter the microscopy of local chemical ch \inf z_2 = 10¹⁰ at E₂=7 keV $\mathbf{10} \quad \mathbf{r} \quad \mathbf{71} \quad \mathbf{M}$ $f(u)$ at $f_2 - r$ key

^q Hubble function from PDG 22 decoupling redshift can be determined by the decoupling $U(z) = U \left[\Omega \ z^4 + \Omega \ z^3 + \Omega \ z^1 \right]^{1/2}$ ble function from PDG $\frac{H(z) = H_0 \left[\frac{3L\gamma}{2} + \frac{3L_m z}{2} + \frac{3L_A}{2} \right]}{22}$ *dz ^f*² ⁼ ² *^K*3(*T*) *ⁿ*² dark (1 *f*2) ³ ⁺ *^K*2(*T*) *ⁿ*dark *^f*2(1 *^f*2) $H(z) = H_0 \left[\Omega_\gamma z^4 + \Omega_m z^3 + \Omega_\Lambda \right]^{1/2}$ v_1 /0.13 p_0 $\left[\Omega_\gamma z^\ast + \Omega_m z^\beta + \Omega_\Lambda\right]^{-1}$

Deuteron mass fraction have the limiting behaviors given in Eqs. (4.9) and (4.16). They scale as *^T* and ⁴ teron r 2 \boldsymbol{n} ass fractio *.* (5.13) *{*eq:f2equilibrium*}*

Q Definition $f_2(z) = 2 n_2(z)/n_{\rm dark}(z)$. $1 - f_2 = n_1(z)/n_{\rm dark}(z)$ $\sum_{i=1}^{n} f(x) \cdot f(x) = 2n(x)/n$ (5.2) **d** Demind $J^2(\cdot)$ = $J^2(\cdot)$ add $J^2(\cdot)$ $I - f_2 = r$ *m* $\frac{1}{2}$ **I** Definition $f_2(z) = 2 n_2(z)/n_{\text{dark}}(z)$ is the approximated by the approximated by the set of $\sum_{i=1}^n f_i(z)$ $1 - f_2 = n_1(z)/n_{\text{dark}}(z)$

$$
\frac{d}{dz}f_2 = \frac{1}{(1+z)H} \Big[-2 K_3(T) n_{\text{dark}}^2 (1-f_2)^3 + K_2(T) n_{\text{dark}} f_2(1-f_2) \Big]
$$

□ High T (z>>z₂):

\nsame scaling behavior in z on RHS and constant f₂

\n
$$
\frac{f_2}{(1-f_2)^2} = 4\sqrt{2} \left(\frac{2\pi(1+z_{\text{dc}})}{m_\chi k T_{\text{cmb}}} \right)^{3/2} \frac{\rho_{\text{cdm}}}{m_\chi} \sim 10^{-11}
$$

 \Box Low T (z<<z₂): K_2 exponentially small and quartic approach to $f_2(z=0)$ 23 U LOW V ($Z < z_2$): K_2 exponentially small and quartic approach to $I_2(z-v)$ $2H_0\Omega^{2/2}_\gamma m_\gamma^2$ $f_2(z) = f_2(0) - C_3(a/a_+) \frac{a \mu_{\text{cdm}}}{a \pi \pi a^{1/2}} z^4$ $\frac{211036\gamma}{\chi}$ 110 $\frac{1}{\chi}$ 23 $V = \frac{1}{2}$. I_n the low-temperature region is the rate coefficient in the rate coefficient coefficient $\frac{4}{3}$ $f_2(z) = f_2(0) - C_2(a/a_1)\frac{a^2 \rho_{\rm cdm}}{z^4}z^4$ and $2H_0 \Omega_\gamma^{1/2} m_\chi^3$ and 23 K₂ exponentially small and quartic a $f_2(z) = f_2(0) - C_3(a/a_+) \frac{a^4 \rho_c^2}{2H_1 \Omega^1}$ cdm $2H_0 \Omega_\gamma^{1/2} m_\chi^3$ *z*4

Numerical result for m=19 GeV & a= 17 fm

a a plateau 4×10^{-11} due to equilibrium between K₃ and K₂

- dramatic increase by 10³⁻⁴ near z_2 (T ~ E₂) due to enhanced K₃ and suppressed $K₂$
- \Box $f_2(0)$ is bounded between 5×10^{-8} and 5×10^{-7}

S *mall and negligible? dtn*¹ ⁼ 2*K*1(*T*) *ⁿ*²

 \square The tiny fraction \sim 10⁻⁷ suppresses cluster formations but NOT annihilation into visible matter. **The tiny fraction ~TU
but NOT ensibilation** $\frac{d}{dx}$ into visible matter raccion we coppied to the imaginary part of 1*/a* is timed to the real part of 1*/a* is timed to the real part of real part of α is the real par a_l, in which case the interest in the interest in the numerator part can be in the numerator of α

$$
\textbf{a\text{ Annihilation rate}} \quad \frac{d}{dt} n_{SM} = K_1(T) n_1^2 + \Gamma_2 n_2
$$

$$
K_1(T) = \left(32\pi g (kT/E_2) \frac{E_2}{kT}\right) \frac{\hbar \operatorname{Im}[a]}{m} \qquad \Gamma_2 = \frac{4\hbar \operatorname{Im}[1/a]}{m a}
$$

- **Q Their ratio is insensitive to coupling** to visible matter and tells **and the industry of 1***0***⁻¹⁰⁻⁰ cancels in the ratio of 1⁰⁻⁰ cancels in the ratio of the ratio of 11/₃⁻¹⁰⁻⁰ cancels in the ratio of 11/2** which process is more efficient. The figure is the decay of $\begin{array}{|c|c|c|c|c|}\hline \text{and} & \text{if} & \text{$ [dk] where the dimensionless function *g*(*t*) is 1(⇡*/t*)¹*/*²*e*¹*/t*[1erf(1*/* \square Their ratio is insensitive to coupling \square in the contract of \square
	- □ ratio =1 for $z_{dc}/z = 160$ (f₂ ~5x10⁻¹¹) It easily reach 10^{4-5} at $z_{dc}/z=1000$.

Mass dependence nitude. This feature is expected from the exponential suppression of the breakup process in Eq. (4.6) and *mass* dependence at α

Q Relaxed constraint: $\sigma_{\text{elastic}}/m = 2 \text{ cm}^2/\text{g}$. *a* **is known for given mass.** \Box The fraction scales like m^{-2.5}

- Gray region violates T_{dc} >> E₂. (initial condition n₂=0 invalid)
- \Box The fraction can be as large as 10^{-3} at mass 1 GeV
- \Box 40% fraction at 0.1 GeV requires huge rate $110\ {\rm cm^2/g}$

Kinetic decoupling

- ^q Kinetic equilibrium between DM and ordinary matter can be maintained longer T_{kdc} < T_{dc}
- \Box The fraction decreases and scaling like $(T_{kdc}/T_{dc})^{-1.9}$
- \Box Introducing T_{kdc} is essentially same as lowering T_{dc}

Summary

 \Box Self-interacting dark matter can solve the small-scale structure problem

 \Box large scattering length system fits to selfinteracting rates: $m=19$ GeV, $a=17$ fm

 \Box Production of bound state in early universe by 3-body recombination (bottleneck for large nuclei).

$$
\Box f_2 \sim 10^{-11} \text{ at high T} >> E_2
$$

 \Box f₂ is enhanced by 10^{3~4} and approaches10^{-7~-8} at $low T << E₂$

28 \Box With relaxed constraint, the fraction can go up to 10^{-3} at 1 GeV.

