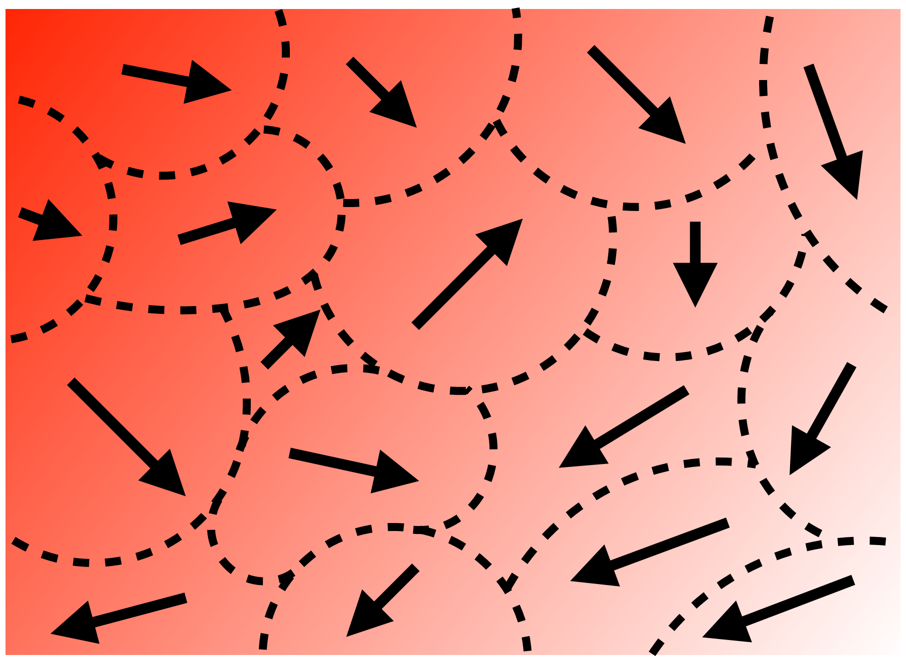
Hydrodynamics as optimized/renormalized perturbation theory



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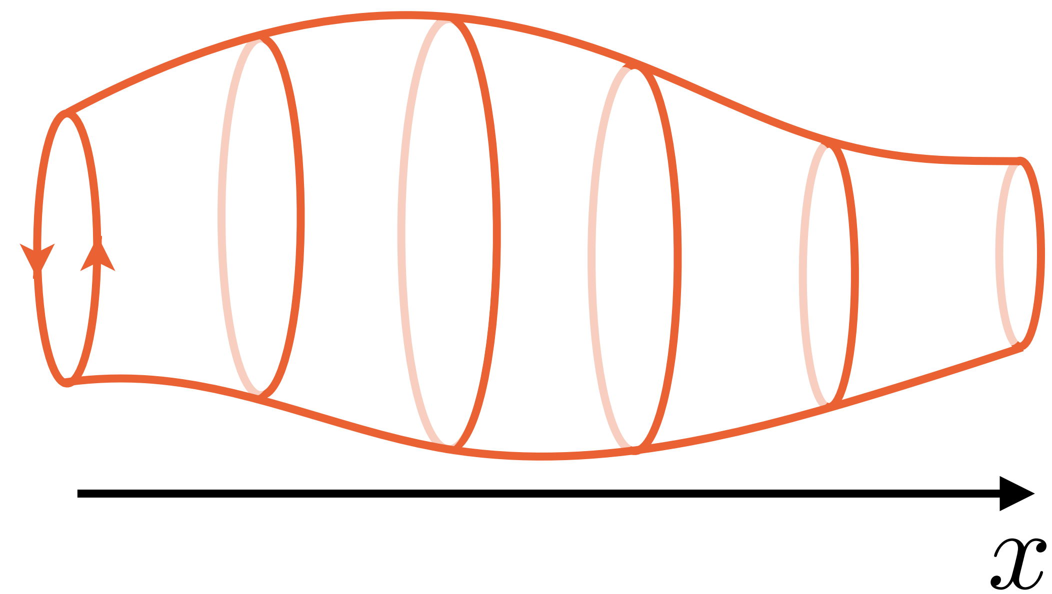


Masaru Hongo

RIKEN iTHEMS Program

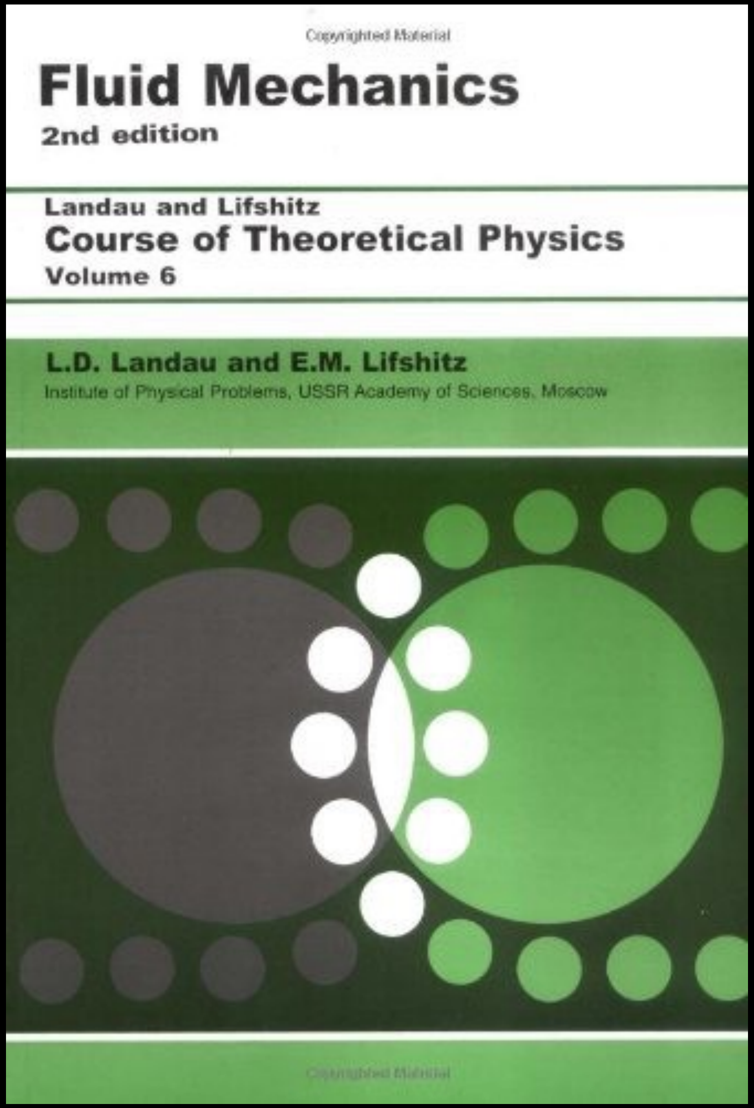
"Multi-Scale Problems Using Effective Field Theories", INT , 2018/5/11

Based on Hayata-Hidaka-**MH**-Noumi PRD (2015), **MH** Ann.Phys (2017)

Today's main Question

Q. Why $T^{\mu\nu} = (e+p)u^\mu u^\nu + pg^{\mu\nu} + \cdots$?

Answer 1.



Answer2. My talk

Outline

MOTIVATION:

Relativistic hydrodynamics
from quantum field theory?

APPROACH:

QFT for initial local Gibbs distribution

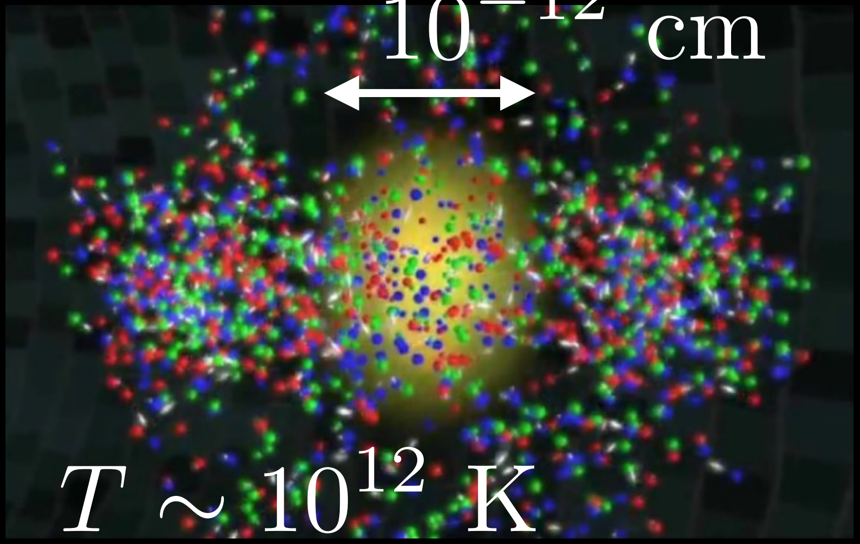
RESULTS:

Derivation of Navier-Stokes eq.
& anomaly-induced transports

Hydrodynamics is

- **Effective theory** for **macroscopic dynamics**
- **Universal description**, not depending on details
- **Only conserved quantity**

Quark-Gluon Plasma | **Hydro:** $\{\beta(x),\ \vec{v}(x)\}$ | **Neutron Star**



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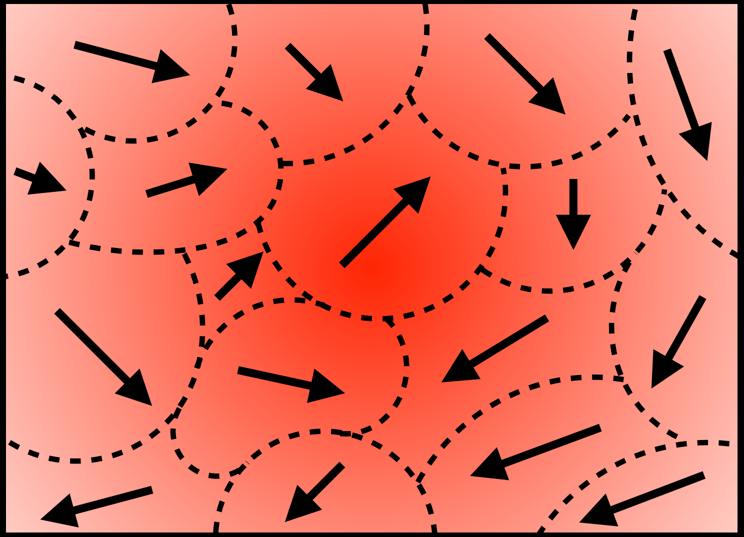

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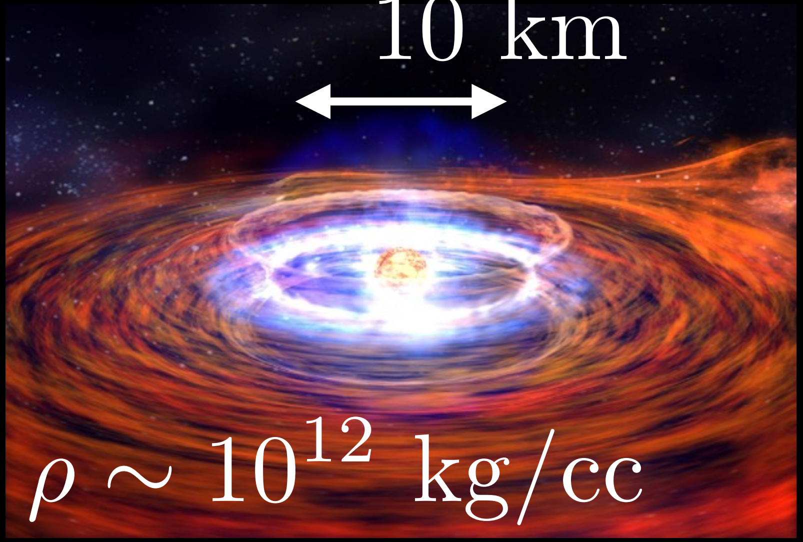


http://www.bnl.gov/rhic/news2/news.asp?a=1403&t=pr

http://newsoffice.mjitugenn.edu/2012/model-bursting-star-0302

Motivation

Microscopic

$$\mathcal{L}_{\mathrm{QCD}}$$

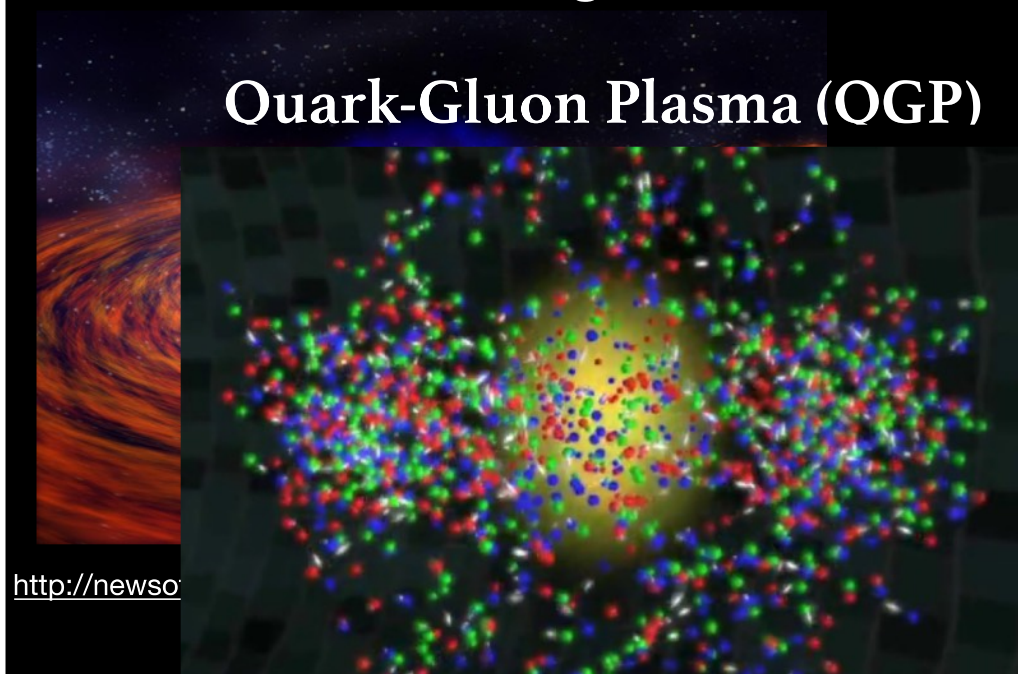
QFT

d.o.f.

Quark, Gluon

Neutron Star (Magnetar)

Quark-Gluon Plasma (QGP)



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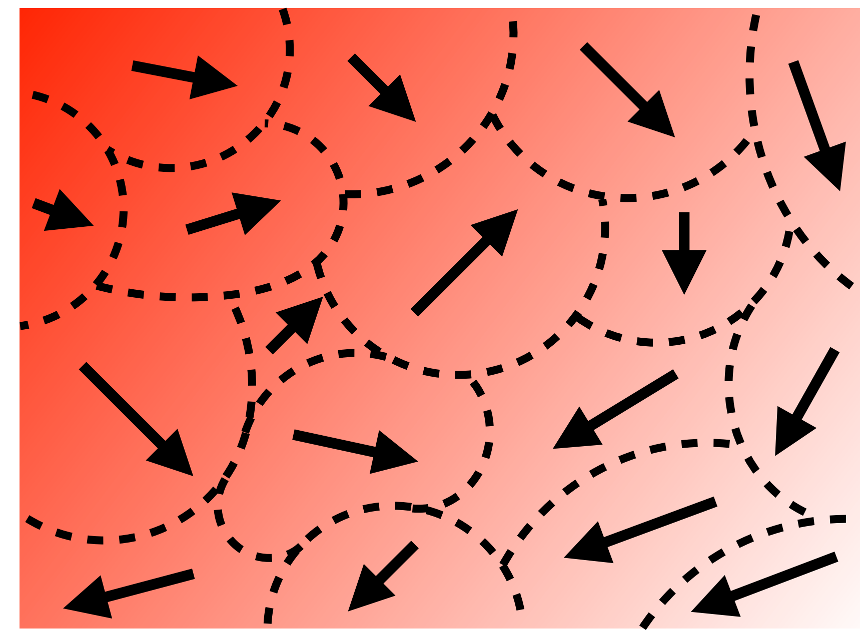
?

Question.

How to bridge the gap between micro and macro?

Macroscopic



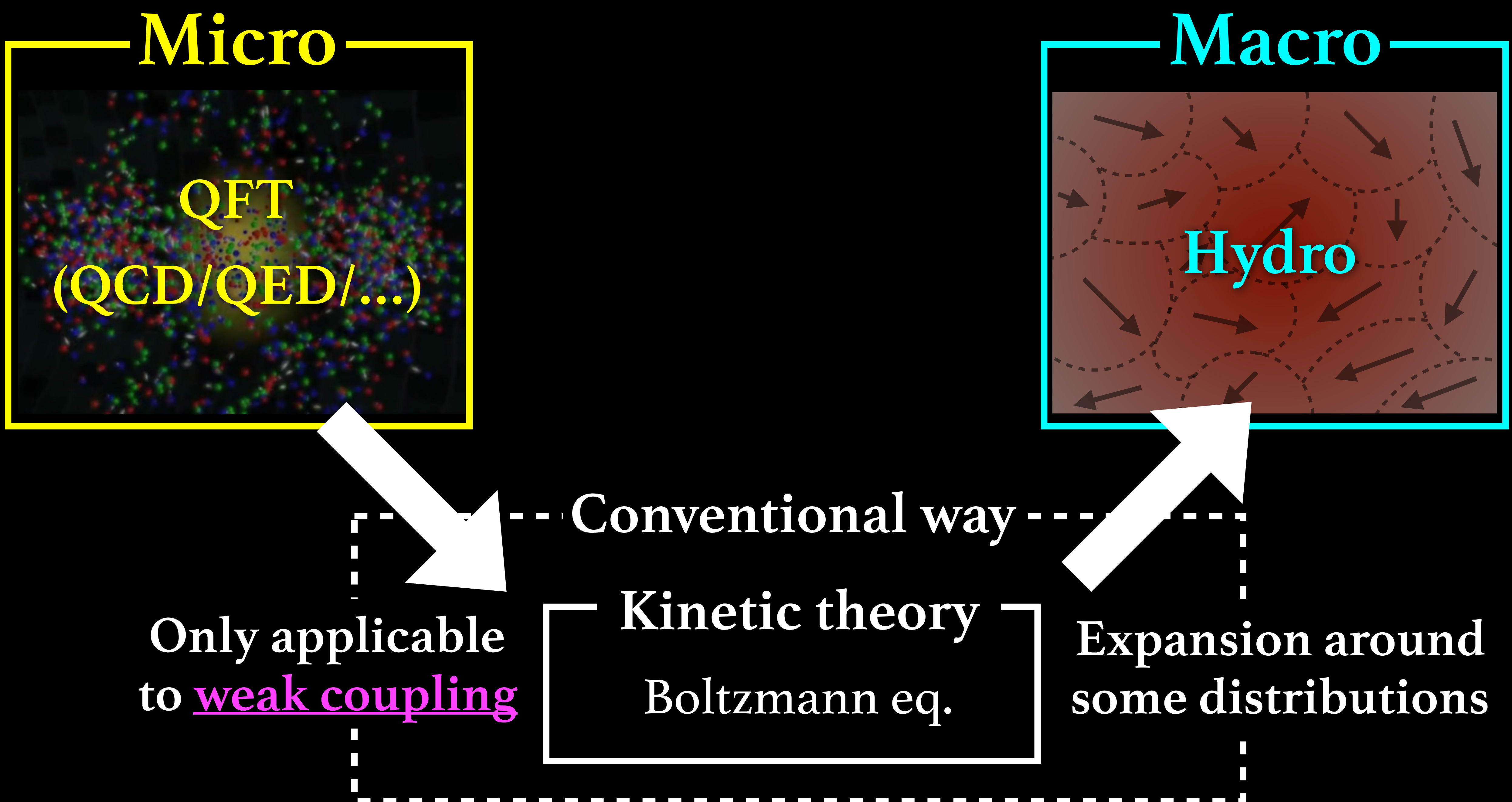
Hydrodynamics

- Haehl et al. (2015)
- Harder et al. (2015)
- Crossley et al. (2015)

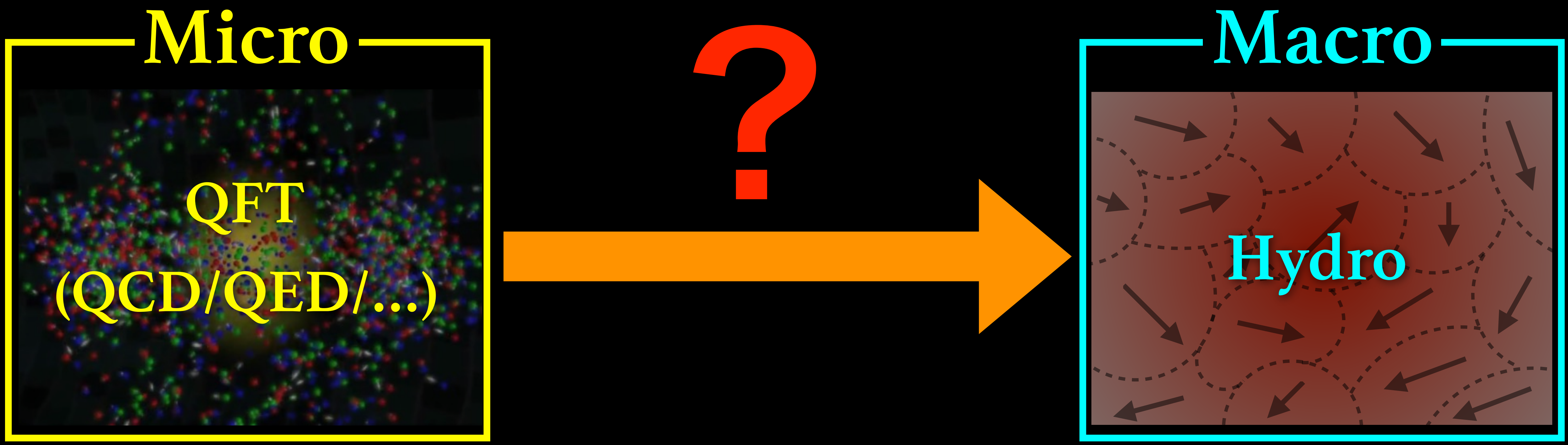
d.o.f.

$T(x),\ \vec{v}(x),\ \mu(x)$

How to construct hydrodynamics



How to construct hydrodynamics

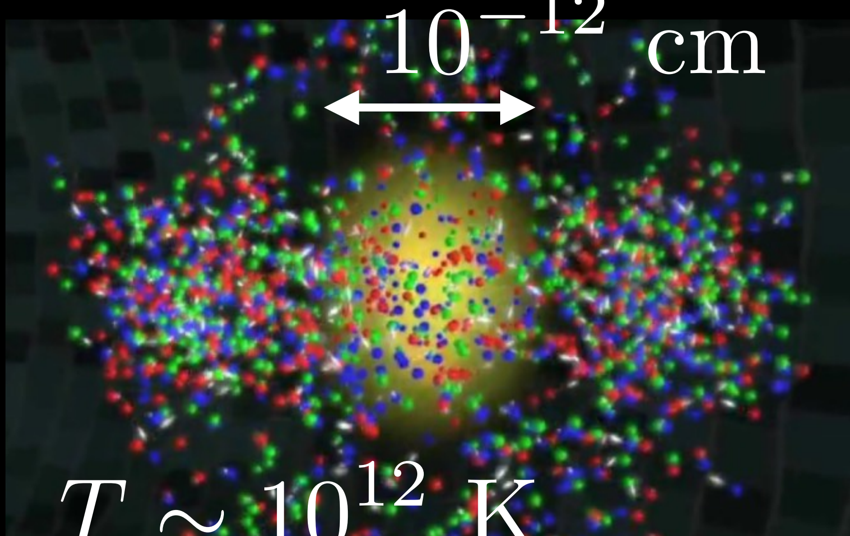


Another motivation

Hydrodynamics is

- **Effective theory** for **macroscopic dynamics**
- **Universal description**, not depending on details
- **Only conserved quantity**

Quark-Gluon Plasma | **Hydro:** $\{\beta(x),\ \vec{v}(x)\}$ | **Neutron Star**



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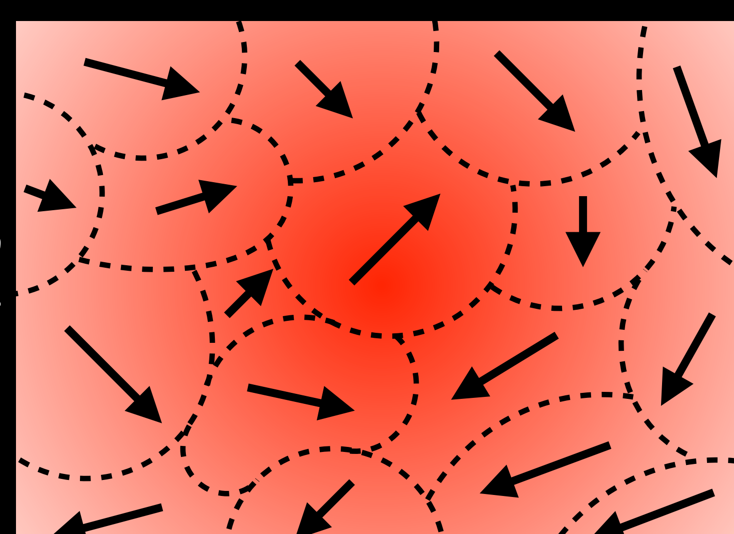

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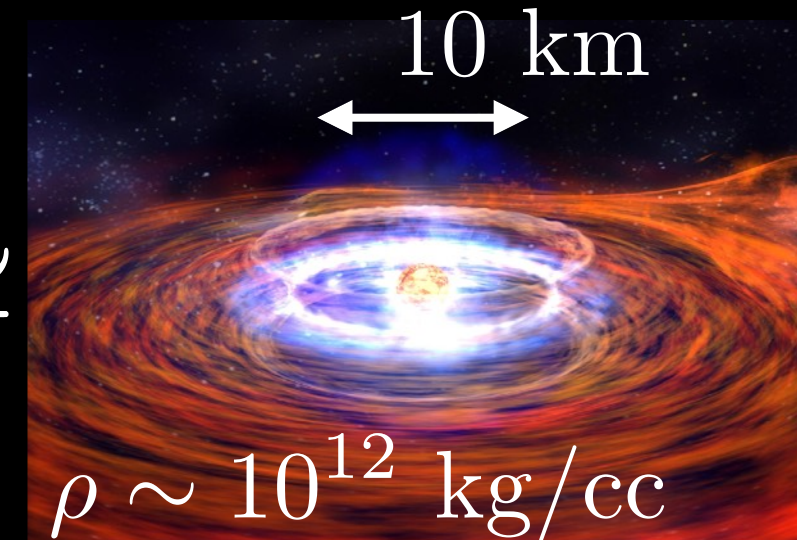

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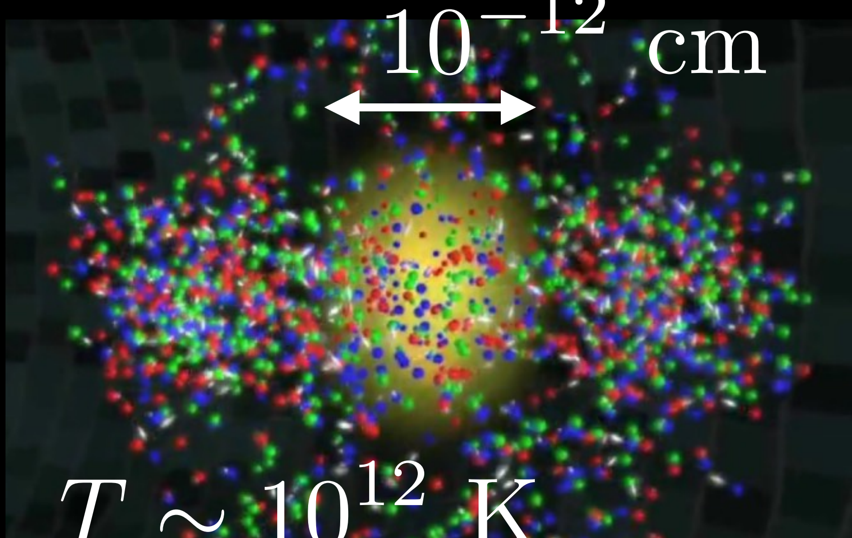
http://newsoffice.mjitugenn.edu/2012/model-bursting-star-0302

Hydrodynamics is

- **Effective theory** for **macroscopic dynamics**
- **Universal description**, not depending on details
- **Only conserved quantity ~ ~~symmetry~~ of system**

Quark-Gluon Plasma | **Hydro:** $\{\beta(x),\ \vec{v}(x)\}$ | **Neutron Star**



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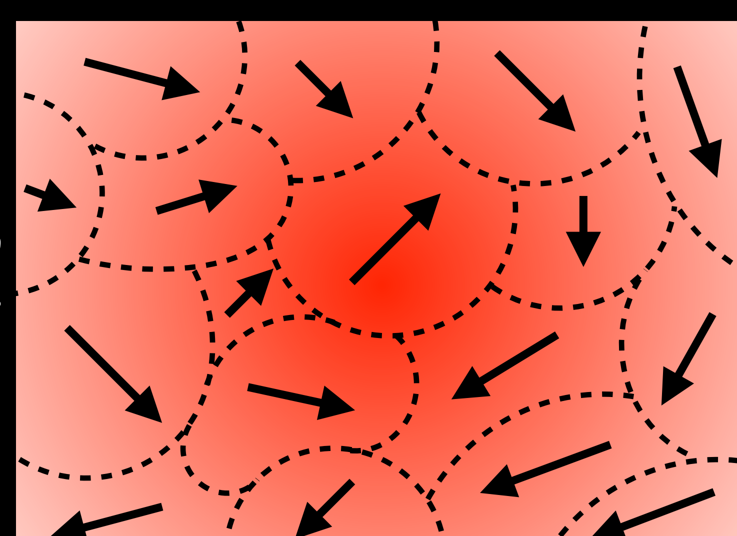

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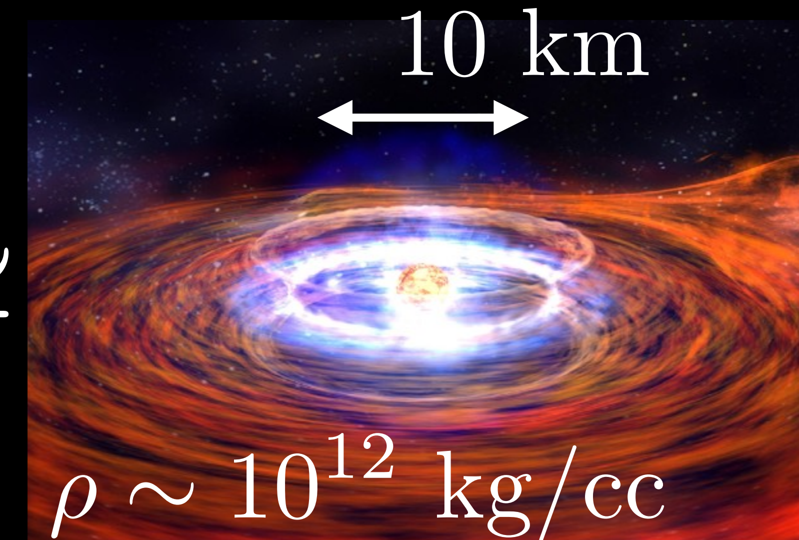

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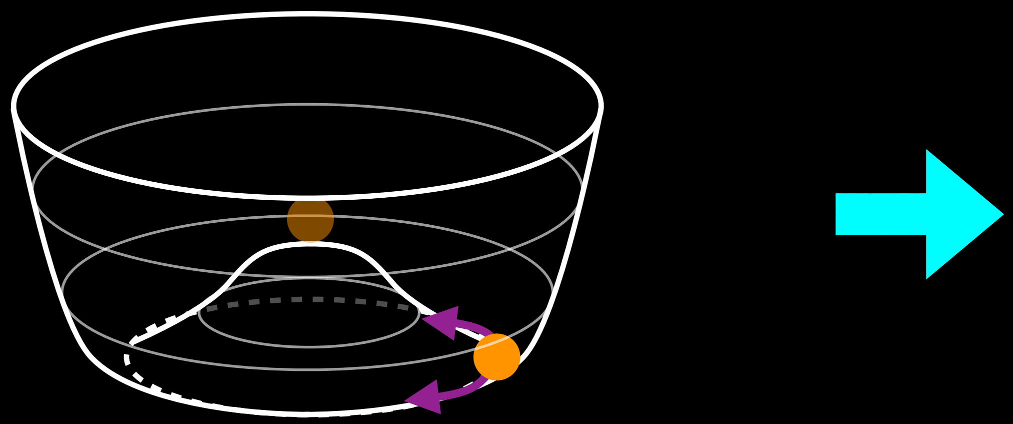
http://newsoffice.mjitugenn.edu/2012/model-bursting-star-0302

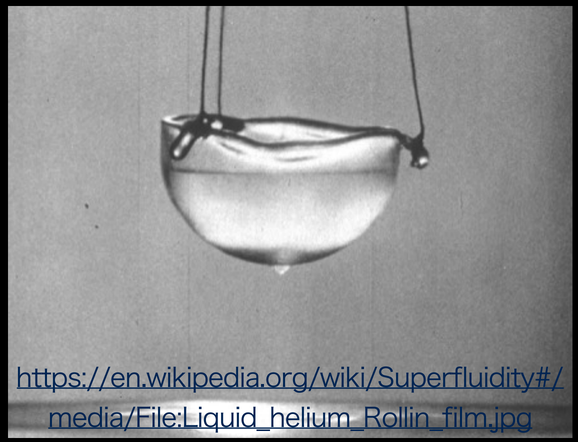
Symmetry breaking & Hydro

◆ Spontaneous symmetry breaking

Micro：Selecting vacuum

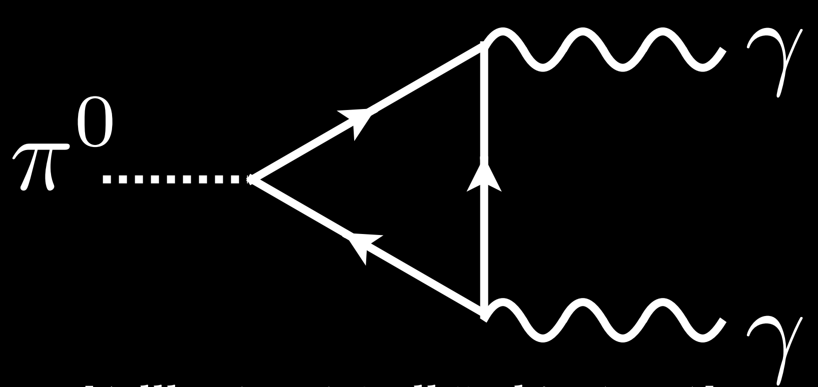


Macro：Superfluid



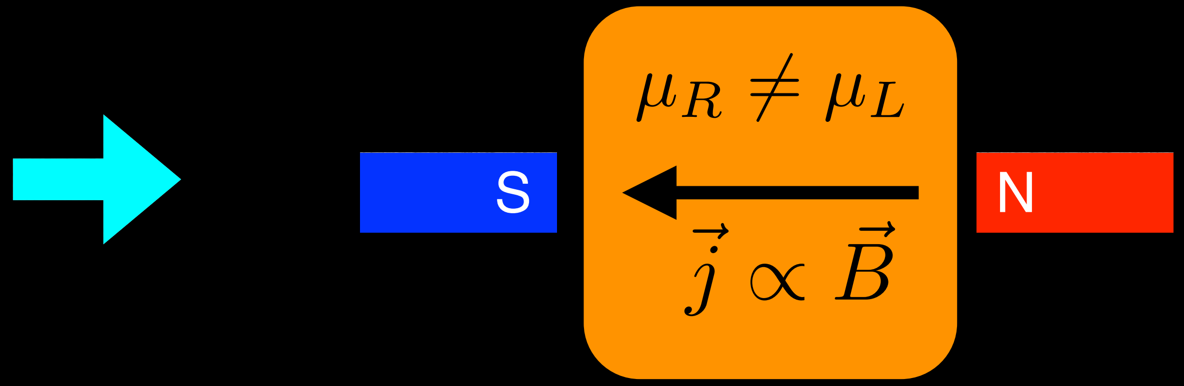
◆ Symmetry breaking by quantum anomaly

Micro：π⁰ decay



[Adller (1969), Bell-Jackiw (1969)]

Macro：Anomalous transport



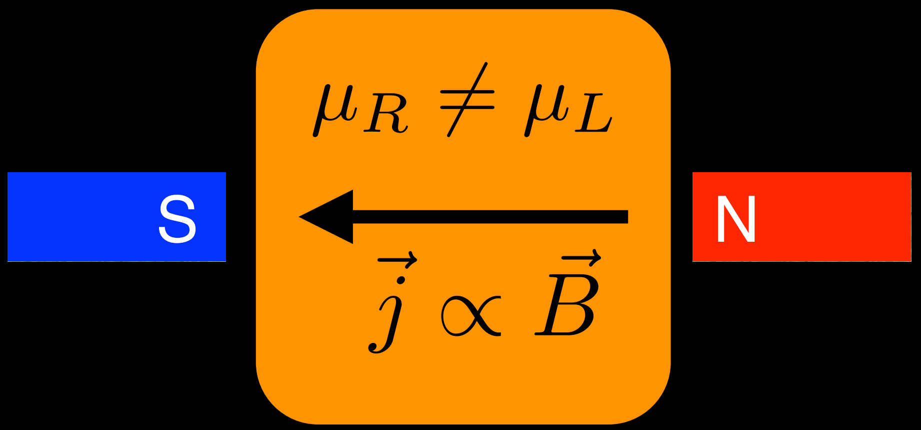
[Erdmenger et al. (2008), Son-Surowka (2009)]

Anomaly-induced transport

◆ **Chiral Magnetic Effect (CME)**

[Fukushima et al.(2008), Vilenkin (1980)]

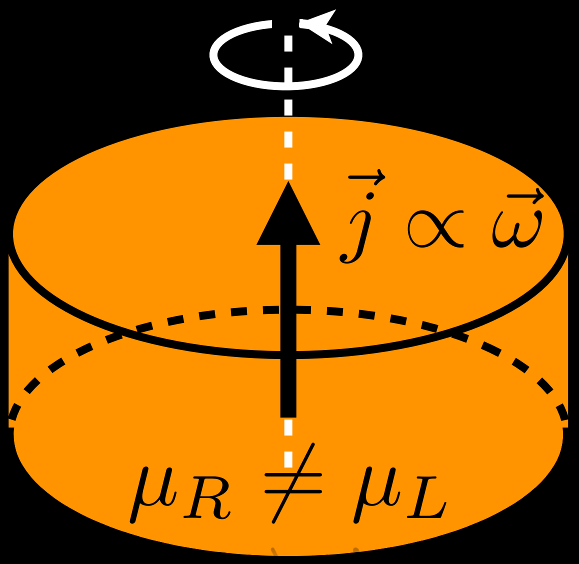
$$\vec{j} = \frac{e\mu_5}{2\pi^2}\vec{B}$$



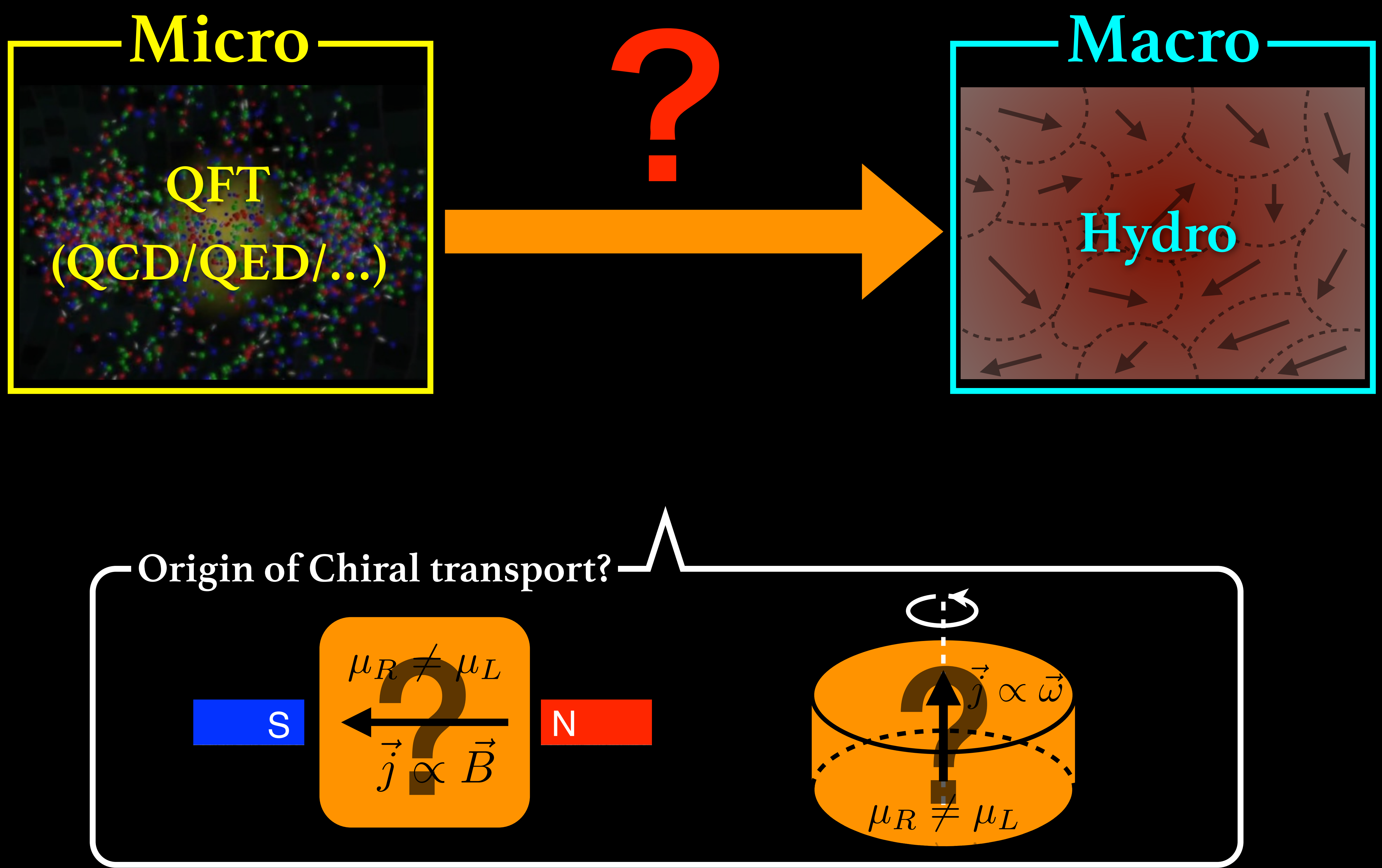
◆ **Chiral Vortical Effect (CVE)**

[Erdmenger et al. (2008), Son-Surowka (2009)]

$$\vec{j} = \frac{\mu\mu_5}{2\pi^2}\vec{\omega}$$



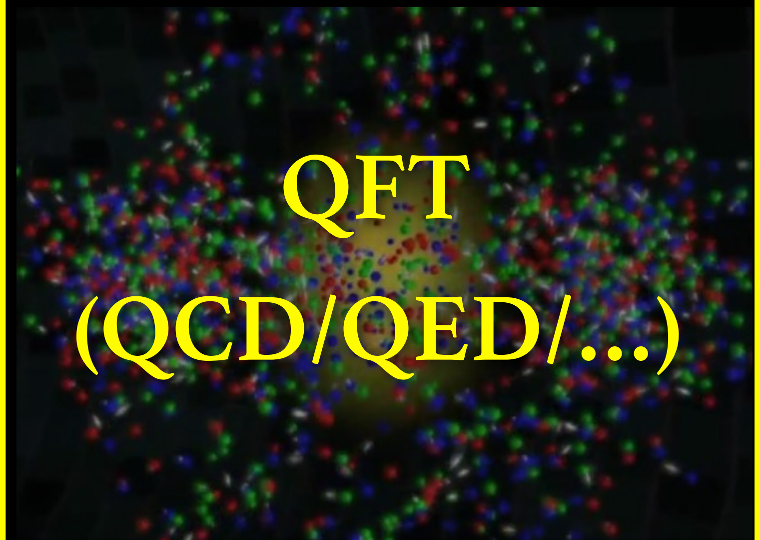
Motivation: How to construct hydro?



Motivation: How to construct hydro?

Nakajima (1957), Mori (1958), McLennan (1960)
Zubarev et al. (1979), Becattini et al. (2015)
Hayata-Hidaka-MH-Noumi (2015)

Micro



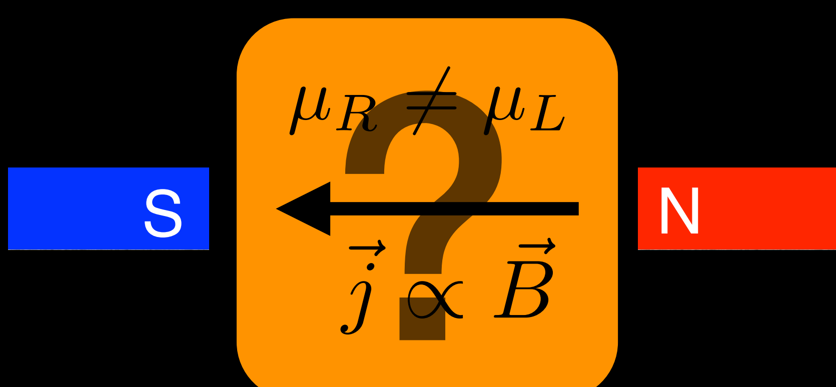
Local Thermal equil.
+ Small deviation

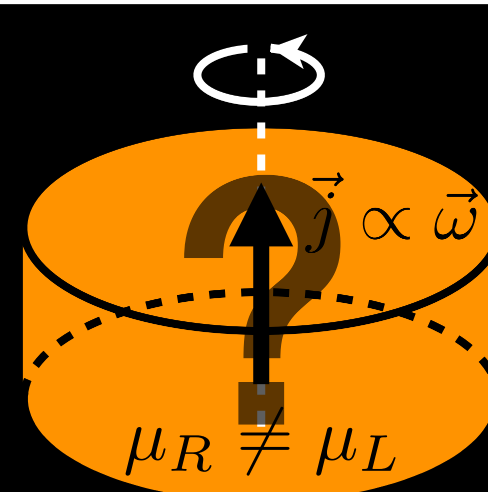
Also applicable to
<u>strong coupling</u>

<u>Physical Properties</u>
EOS, Kubo formula, ...

Macro

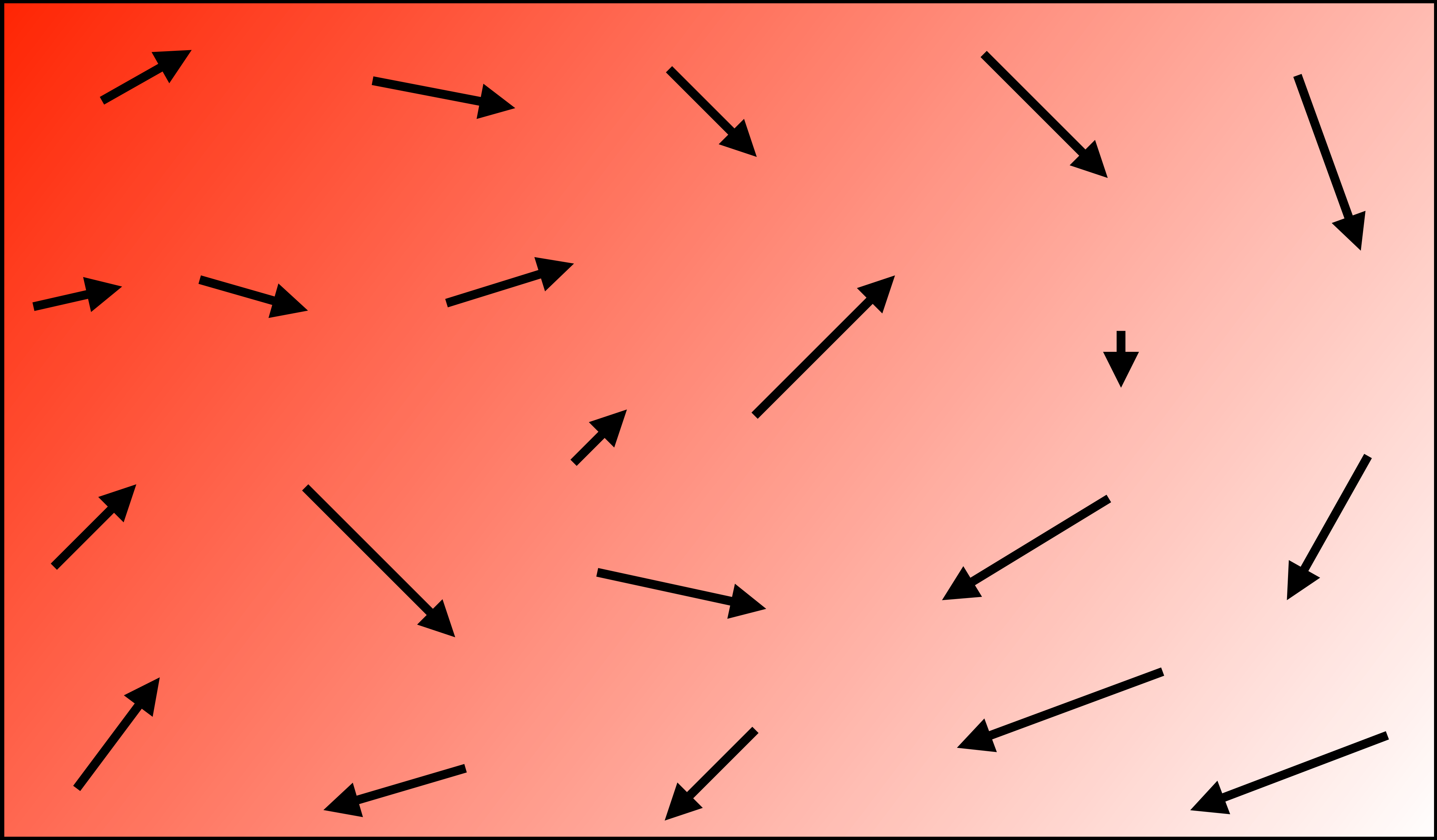
Origin of Chiral transport?





More on
“What is hydro?”

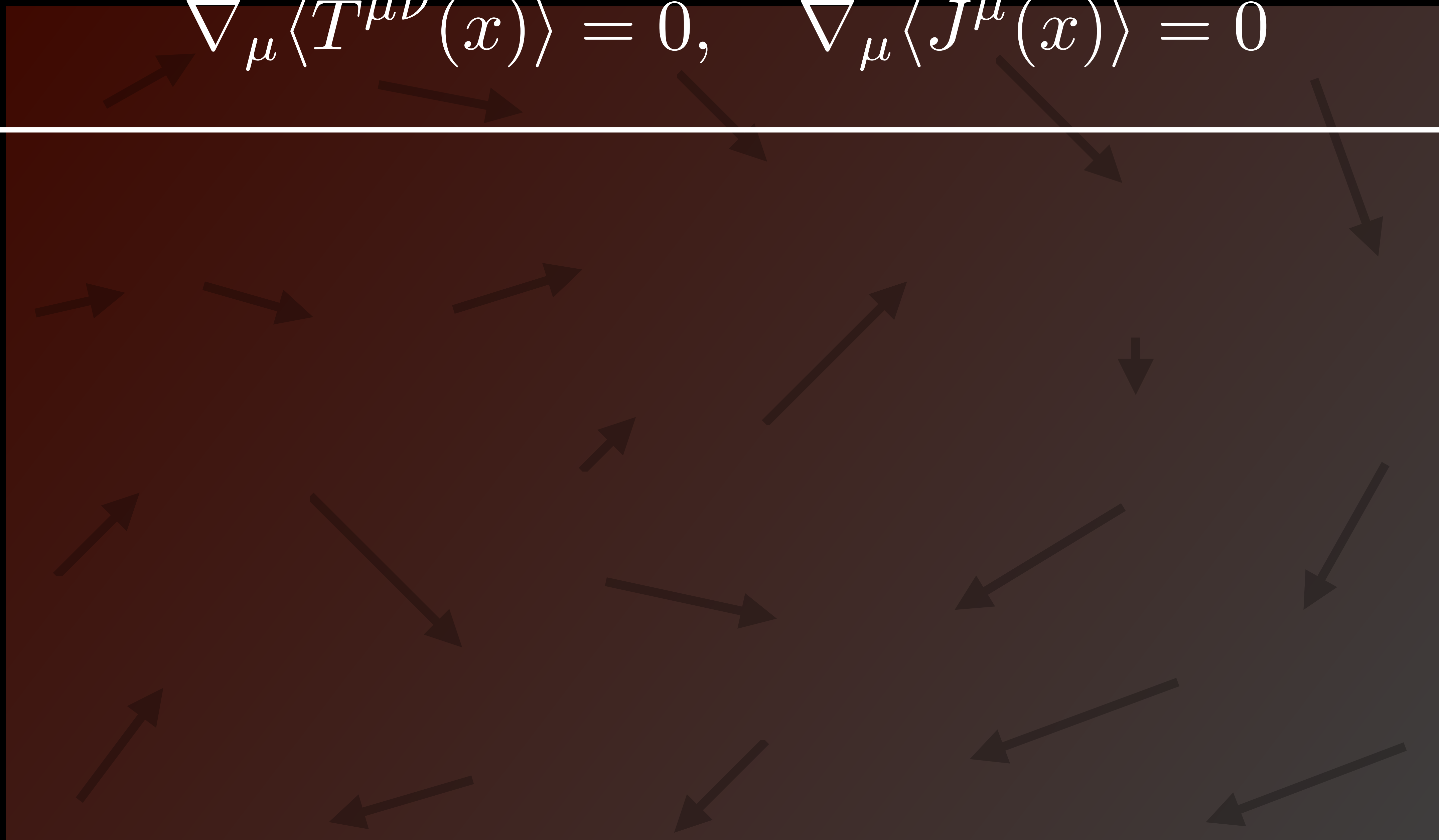
Hydrodynamic equation?



Hydrodynamic equation?

Conservation laws

$$\nabla_\mu \langle \hat{T}^{\mu\nu}(x) \rangle = 0, \quad \nabla_\mu \langle \hat{J}^{\mu}(x) \rangle = 0$$



Simple case: Diffusion equation

Conservation law

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

Constitutive relation

$$\vec{j} = -D\vec{\nabla} n$$

Physical properties

Value of diffusion constant D

$$\frac{\partial n}{\partial t} - D\nabla^2 n = 0$$

Hydrodynamic equation?

Given

Conservation laws

$$\nabla_\mu \langle \hat{T}^{\mu\nu}(x) \rangle = 0, \quad \nabla_\mu \langle \hat{J}^{\mu}(x) \rangle = 0$$

Constitutive relations

Avg. of current op. is determined by charge density!

$$\langle \hat{T}^{\mu\nu}(x) \rangle = T^{\mu\nu}[T^{0\nu}, J^0] = T^{\mu\nu}[\beta^\nu, \nu]$$

$$\langle \hat{J}^{\mu}(x) \rangle = J^{\mu}[T^{0\nu}, J^0] = J^{\mu}[\beta^\nu, \nu]$$

Physical properties

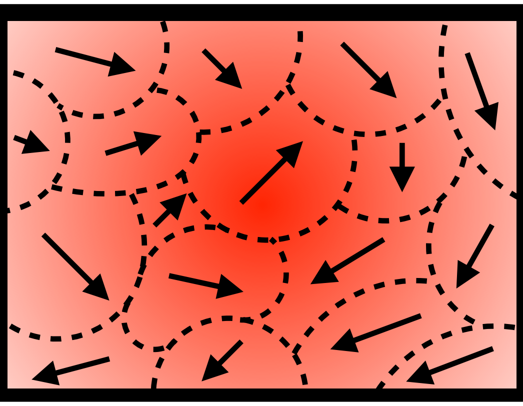
Equation of State (static): $p = p[T^{0\nu}, J^0] = p[\beta^\nu, \nu]$

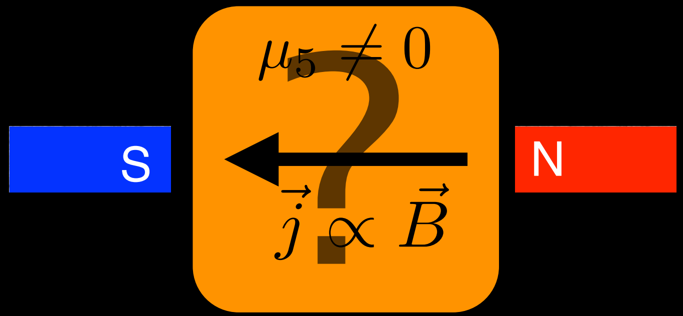
Trasport coeff. (dynamic): $L_i = L_i[T^{0\nu}, J^0] = L_i[\beta^\nu, \nu]$

Outline

MOTIVATION:

Relativistic hydrodynamics
from **quantum field theory?**





APPROACH:

QFT for **initial local Gibbs distribution**

RESULTS:

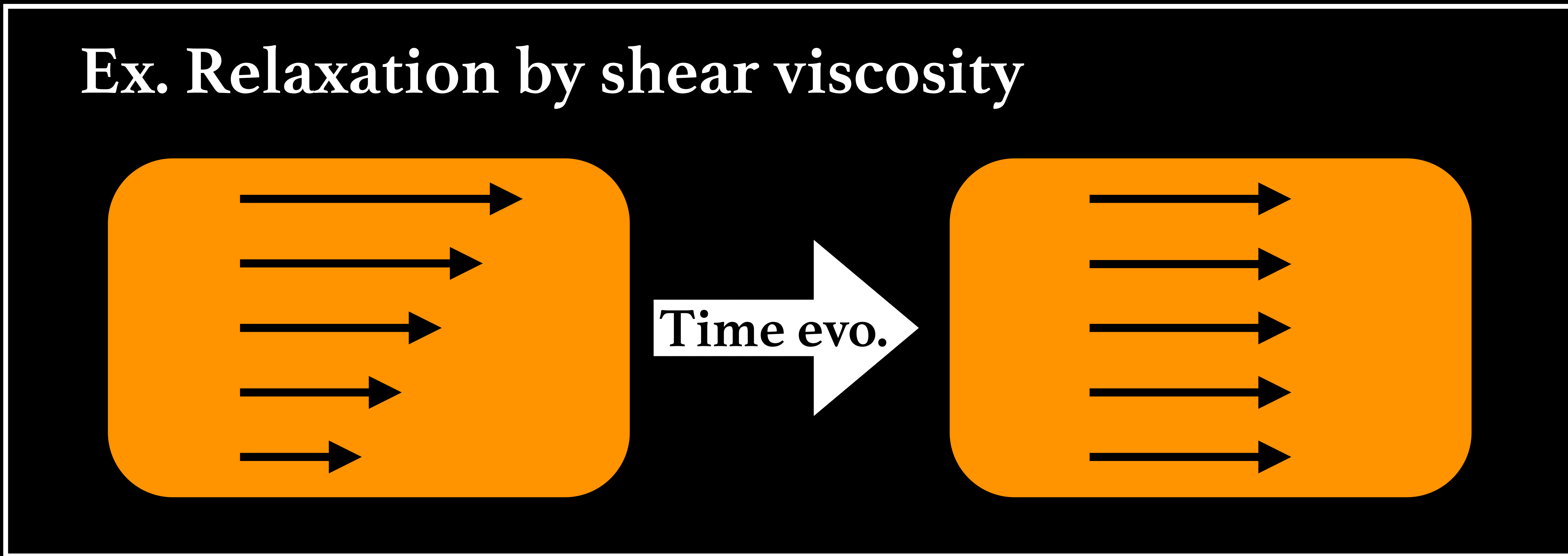
Derivation of Navier-Stokes eq.
& **anomaly-induced transports**

Non-equilibrium

Statistical Mechanics in a Nutshell

(4 pages)

How to treat dissipation?



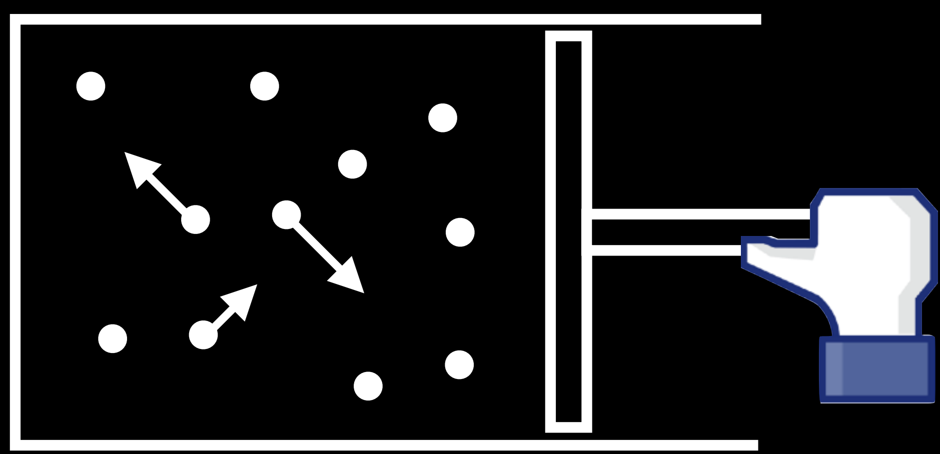
Friction-like phenomena between velocity gradient

From the 2nd law **Entropy** ↗

How the 2nd law is derived for equilibrium?

2nd law: Kelvin's principle

$\nu(t)$



$$\Gamma = (\vec{r}_1, \cdots, \vec{r}_N, \vec{p}_1, \cdots, \vec{p}_N)$$

$$\mathcal{H}(\Gamma) = \sum_i \frac{\vec{p}_i^{\,2}}{2m} + V(\vec{r}_1, \cdots \vec{r}_N; \nu)$$

Consider a operation s.t.

$$\nu(t): \quad \underbrace{\nu(0)}_{\nu_0} = \underbrace{\nu(\tau)}_{\nu_1}$$

When the system evolves as $\Gamma \to \Gamma_t$ with the Hamilton EoM,

work defined by $W(\Gamma) \equiv H_{\nu_1}(\Gamma_t) - H_{\nu_0}(\Gamma)$

satisfies $\langle W(\Gamma) \rangle \geq 0$ for equilibrium state Γ

Derivation of Kelvin's principle

For **equilibrium state** Γ $\quad \langle W(\Gamma)\rangle \geq 0$ is satisfied

For canonical ensamble $\ni \Gamma$

$$\int d\Gamma \frac{1}{Z} e^{-\beta \mathcal{H}_{\nu_0}(\Gamma)} W(\Gamma) \geq 0$$

Proof.

$$\frac{1}{Z_{\beta,\nu_0}} \int d\Gamma e^{-\beta W(\Gamma)} e^{-\beta \mathcal{H}_{\nu_0}(\Gamma)} = \frac{1}{Z_{\beta,\nu_0}} \int d\Gamma e^{-\beta(H_{\nu_1}(\Gamma_t) - H_{\nu_0}(\Gamma))} e^{-\beta \mathcal{H}_{\nu_0}(\Gamma)}$$

$$= \frac{1}{Z_{\beta,\nu_0}} \int d\Gamma_t \left| \frac{d\Gamma_t}{d\Gamma} \right| e^{-\beta H_{\nu_1}(\Gamma_t)} = \frac{Z_{\beta,\nu_1}}{Z_{\beta,\nu_0}} = 1$$

$$\langle e^{-\beta W(\Gamma)} \rangle = 1 \geq 1 - \beta \langle W(\Gamma) \rangle \quad \Rightarrow \quad \boxed{\langle W(\Gamma) \rangle \geq 0}$$

Supplement: Jarzynski equality

Recalling $Z_{\beta,\nu} = e^{-\beta F(\beta,\nu)}$ leads to

[Jarzynski, 1997]

$$\langle e^{-\beta W(\Gamma)} \rangle = e^{-\beta(F(\beta,\nu_1)-F(\beta,\nu_0))}$$

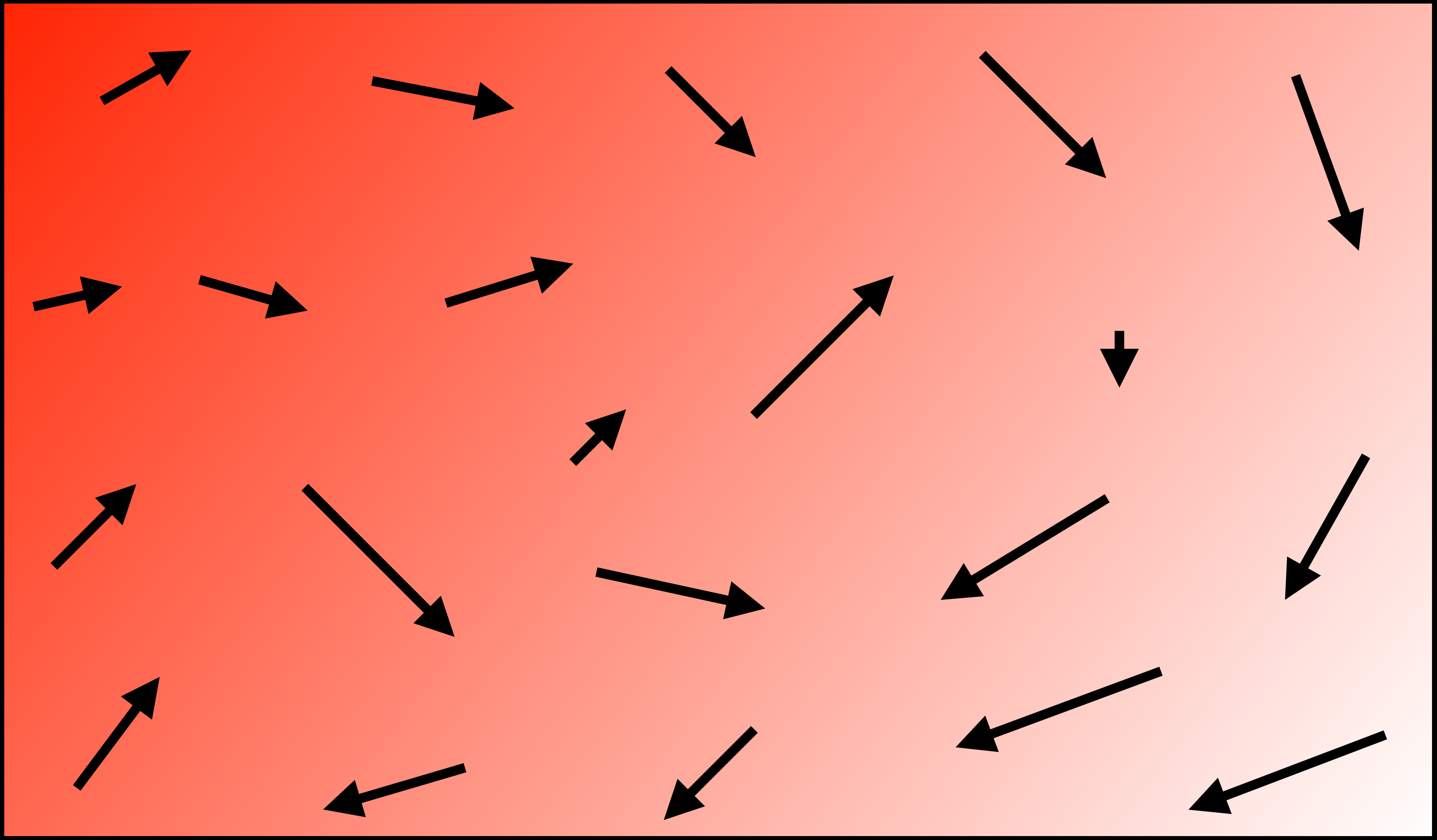
Proof.

$$\frac{1}{Z_{\beta,\nu_0}} \int d\Gamma e^{-\beta W(\Gamma)} e^{-\beta \mathcal{H}_{\nu_0}(\Gamma)} = \frac{1}{Z_{\beta,\nu_0}} \int d\Gamma e^{-\beta(H_{\nu_1}(\Gamma_t)-H_{\nu_0}(\Gamma))} e^{-\beta \mathcal{H}_{\nu_0}(\Gamma)}$$

$$= \frac{1}{Z_{\beta,\nu_0}} \int d\Gamma_t \left| \frac{d\Gamma_t}{d\Gamma} \right| e^{-\beta H_{\nu_1}(\Gamma_t)} = \frac{Z_{\beta,\nu_1}}{Z_{\beta,\nu_0}}$$

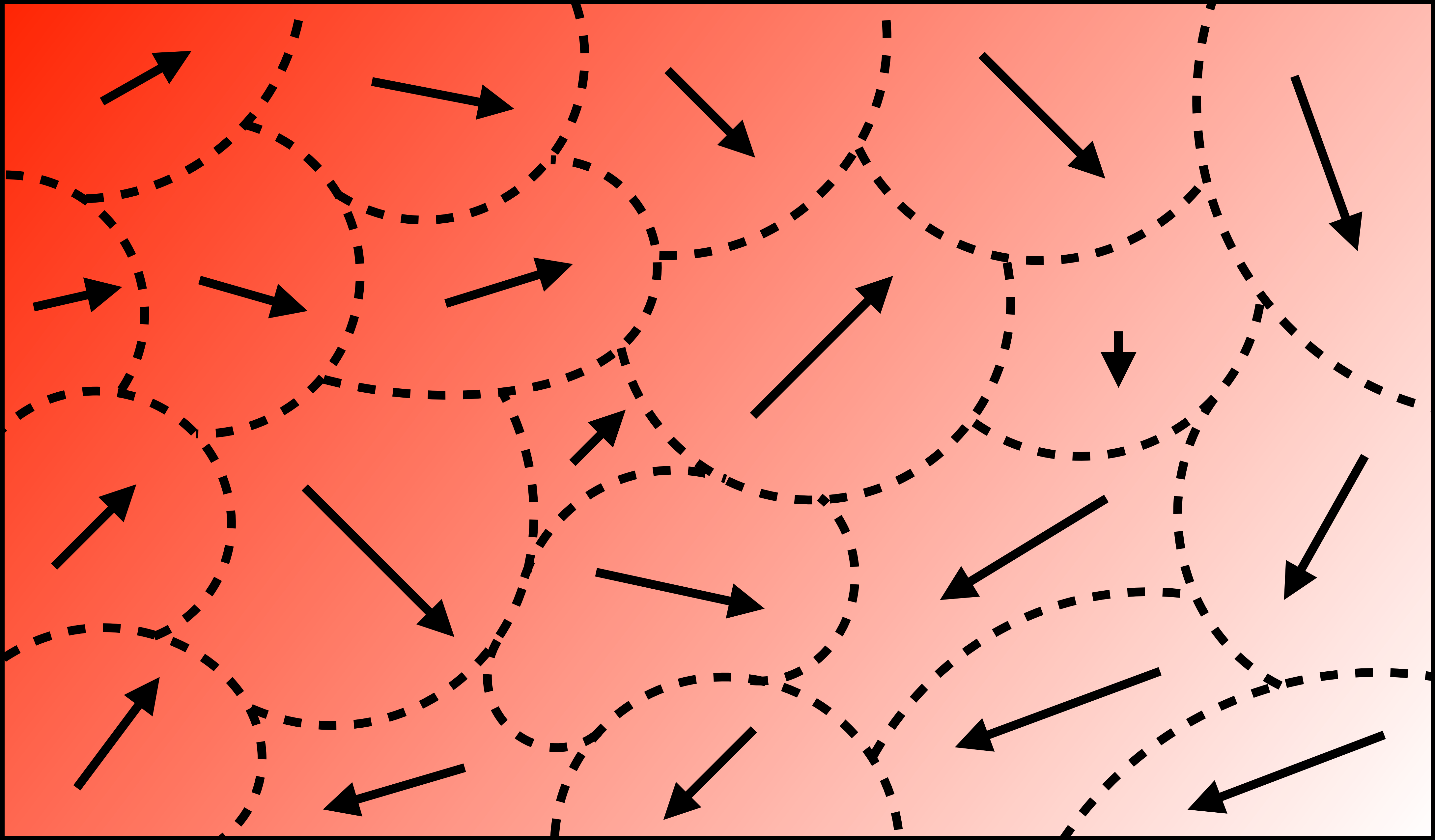
Lesson from simple exercise

Prepare an appropriate initial ensamble
which describes local thermodynamics



Setup

Local thermal equilibrium



Determined only by **local temperature, local velocity…** at that time

How to describe local thermal equil.

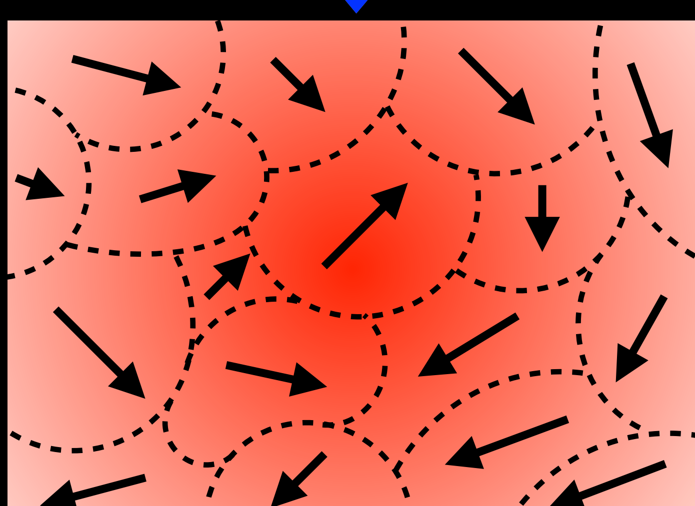
$$T = \text{const.}$$

Global thermal equilibrium:

Gibbs distribution:

$$\hat{\rho}_G = e^{-\beta \hat{H} - \Psi[\beta]}, \quad \Psi[\beta] \equiv \log \mathrm{Tr} e^{-\beta \hat{H}}$$

Localize



$\{\beta(x),\ \vec{v}(x)\}$

Local thermal equilibrium:

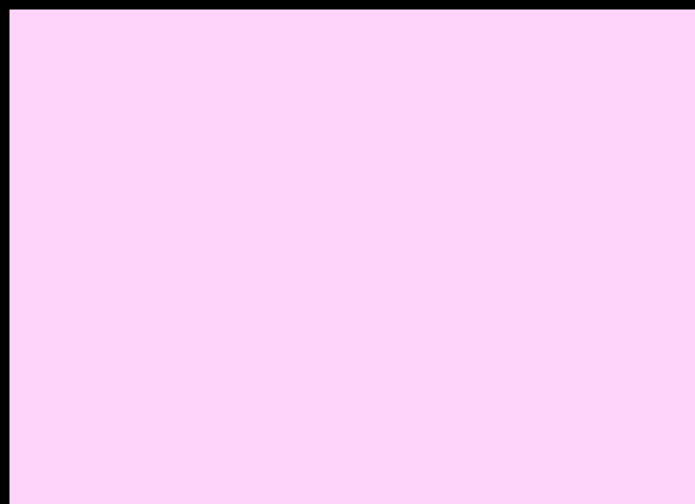
Local Gibbs (LG) distribution:

$$\hat{\rho}_{LG} = e^{-\hat{K} - \Psi[\beta^\mu(x), \nu(x)]}$$

$$\hat{K} = -\int d^3x \left(\beta^\mu(\boldsymbol{x}) \hat{T}^0{}_\mu(\boldsymbol{x}) + \nu(\boldsymbol{x}) \hat{J}^0(\boldsymbol{x}) \right)$$

What is Local Gibbs distribution?

Gibbs distribution



What is the state with maximizing information entropy: $S(\hat{\rho}) = -\mathrm{Tr}\hat{\rho}\log\hat{\rho}$

under constraints:

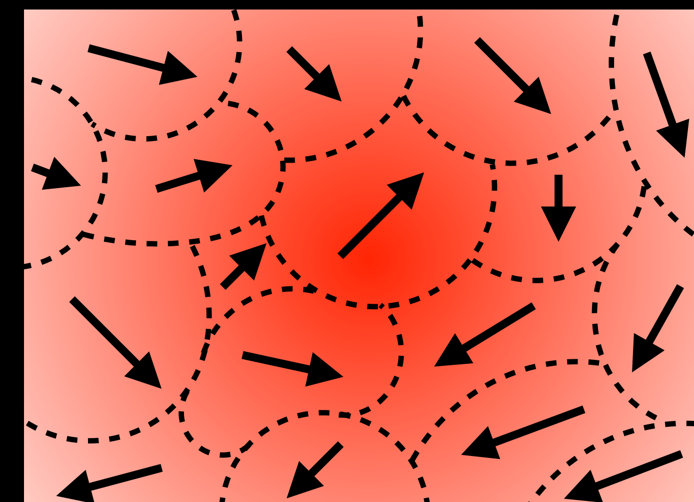
$$\langle \hat{H} \rangle = E = \text{const.}, \; \langle \hat{N} \rangle = N = \text{const.}$$

Answer:

$$\hat{\rho}_{\mathrm{G}} = e^{-\beta \hat{H} - \nu \hat{N} - \Psi[\beta,\nu]}$$

Lagrange multipliers: $\Lambda^a = \{\beta, \nu = \beta\mu\}$

Local Gibbs distribution



What is the state with maximizing information entropy: $S(\hat{\rho}) = -\mathrm{Tr}\hat{\rho}\log\hat{\rho}$

under constraints:

$$\langle \hat{T}^0{}_\mu(x) \rangle = p_\mu(x), \; \langle \hat{J}^0(x) \rangle = n(x)$$

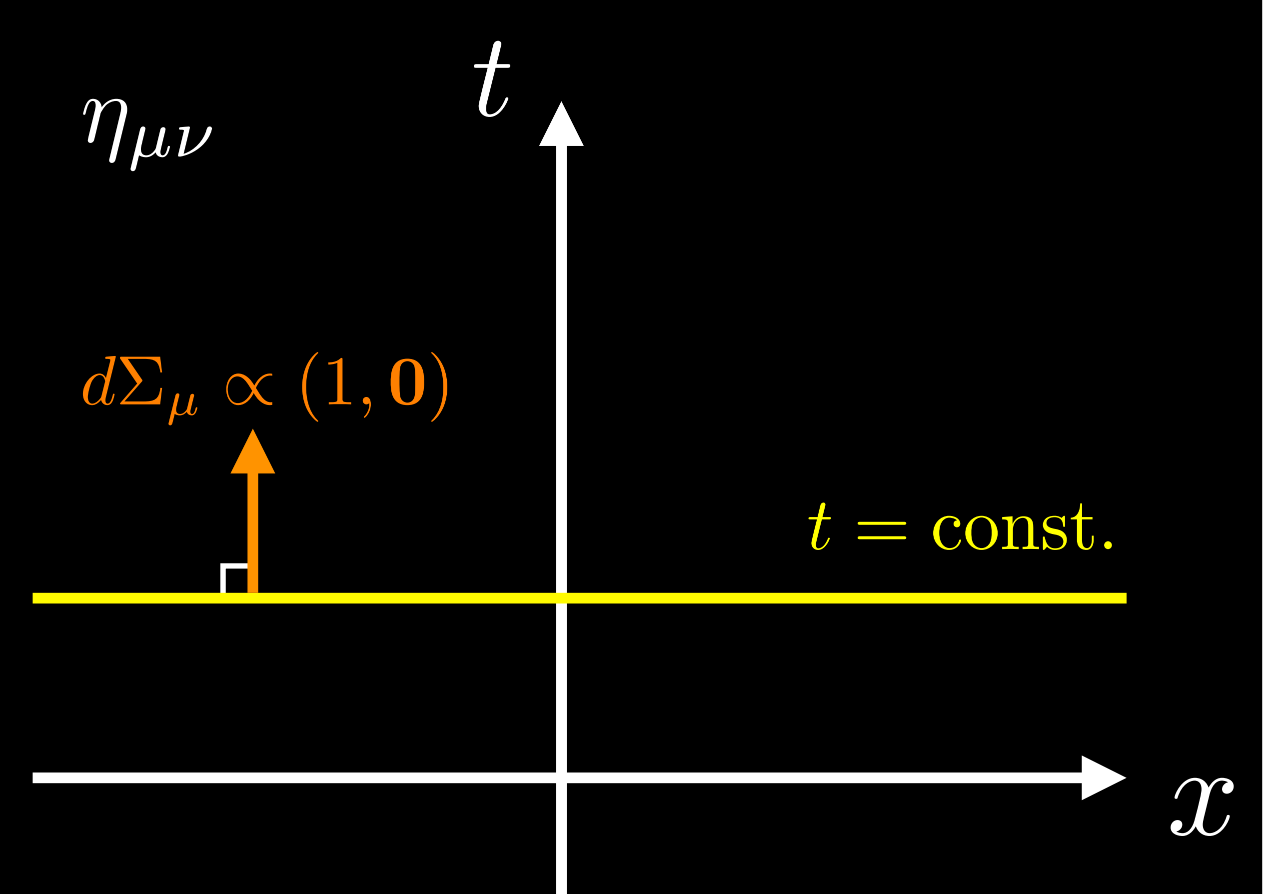
Answer:

$$\hat{\rho}_{\mathrm{LG}} = e^{-\int d^{d-1}x(\beta^\mu \hat{T}^0{}_\mu + \nu \hat{J}^0) - \Psi[\beta^\mu,\nu]}$$

Lagrange multipliers: $\lambda^a(x) = \{\beta^\mu(x), \nu(x)\}$

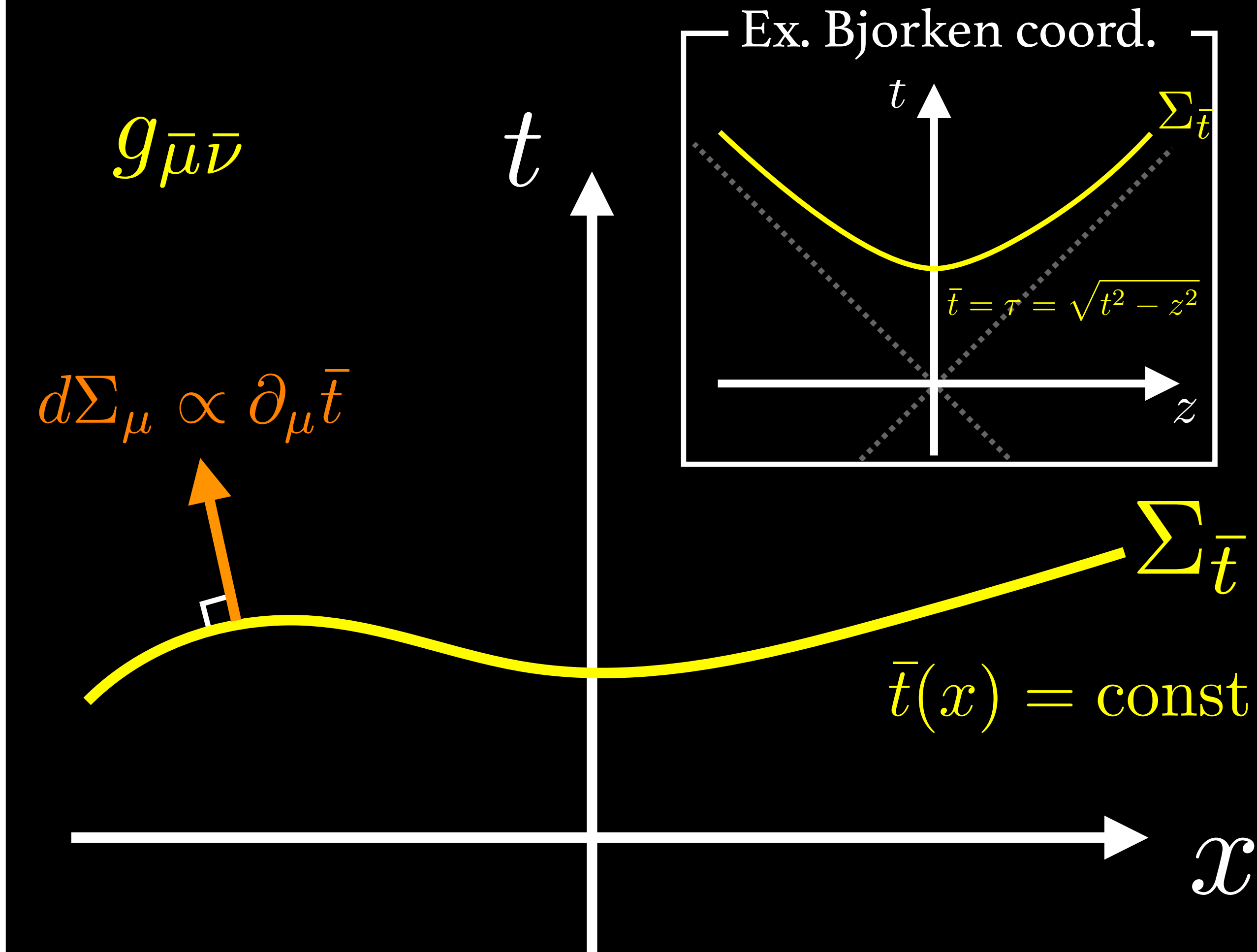
Introducing background metric

Flat spacetime



$$\hat{K} = -\int d^3x \left(\beta^\mu(\boldsymbol{x}) \hat{T}^0{}_\mu(\boldsymbol{x}) + \nu(\boldsymbol{x}) \hat{J}^0(\boldsymbol{x}) \right)$$

Curved spacetime



$$\hat{K} = -\int d\Sigma_{\bar{t}\nu} \left(\beta^\mu(x) \hat{T}^\nu{}_\mu(x) + \nu(x) \hat{J}^\nu(x) \right)$$

→ ① Formulation becomes manifestly covariant

② Background metric plays a role as external field coupled to $T^{\mu\nu}$

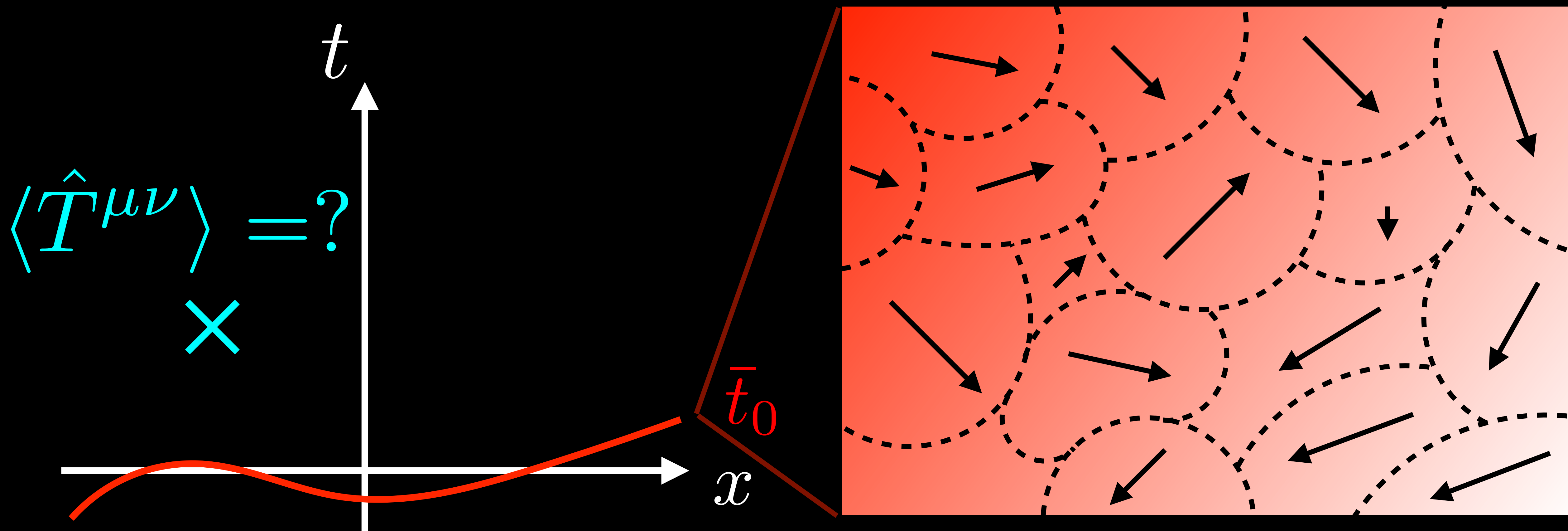
Dissipative part

Difficulty of problem

[Sasa PRL (2014), Hayata, Hidaka, MH, Noumi PRD (2015)]

Initial density operator: $\hat{\rho}(\bar{t}_0) = \hat{\rho}_{LG}[\bar{t}_0; \lambda] \equiv \exp\left[-\hat{S}[\bar{t}_0; \lambda]\right]$

Taking Heisenberg picture: $\langle \hat{\mathcal{O}}(x) \rangle = \mathrm{Tr}\hat{\rho}(\bar{t}_0)\hat{\mathcal{O}}(x)$



LG dist. with $\beta^\mu(x)$

→ **Naive perturbation breaks down due to time evolution!!**

Renormalized/optimized perturbation

When we cannot solve problem exactly

& naive perturbation breaks down

⇨ Choosing an appropriate "Ground State"

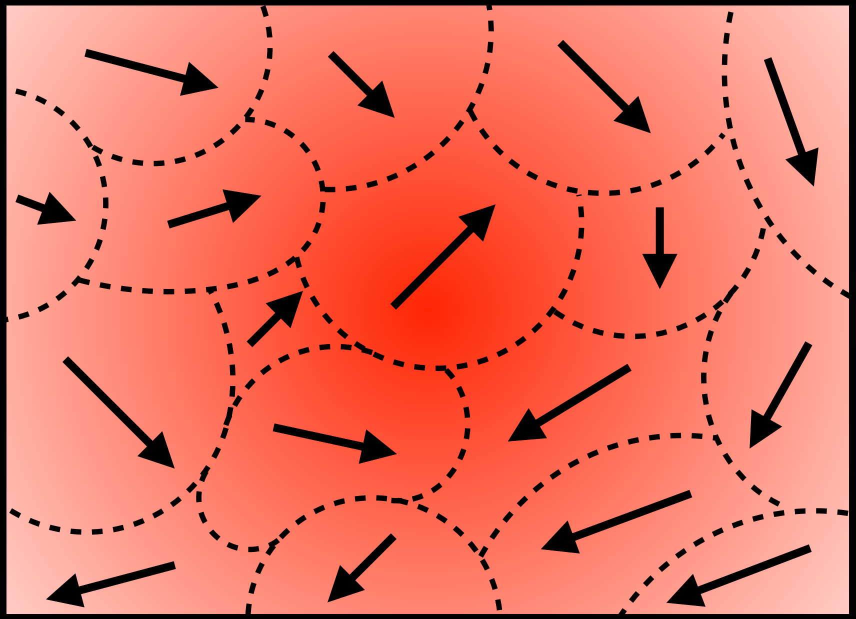
& reorganizing perturbation often works!!

= Renormalized/optimized perturbation

In the case of hydrodynamics,

an appropriate "Ground State" is

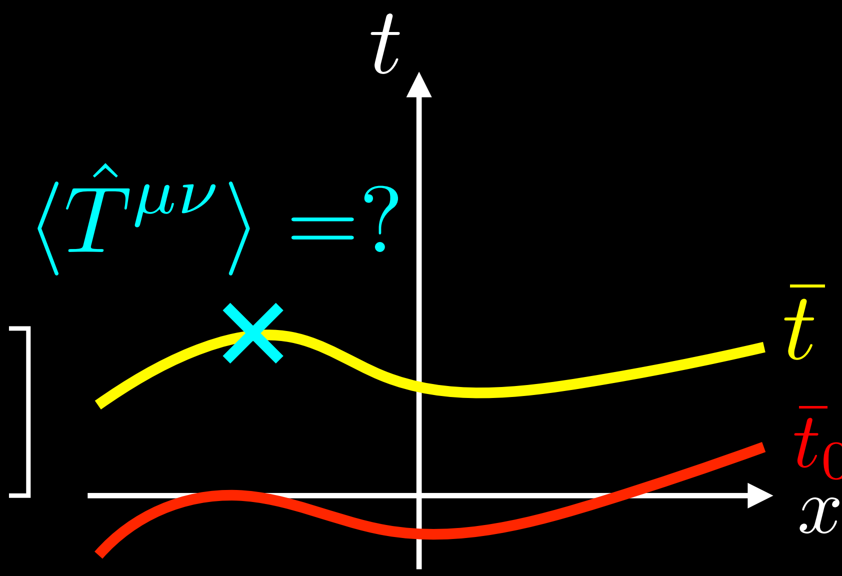
Local thermal equilibrium!!



RPT/OPT for Time evolution

$$\hat{\rho}(\bar{t}_0) = \hat{\rho}_{\rm LG}[\bar{t}_0;\lambda] \equiv \exp\left[-\hat{S}[\bar{t}_0;\lambda]\right]$$

$$= \exp\left[-\hat{S}[\bar{t};\lambda] + (\hat{S}[\bar{t};\lambda] - \hat{S}[\bar{t}_0;\lambda])\right]$$



LG dist. with $\lambda(\bar{t})$ (“Ground state”)

$\hat{\Sigma}[\bar{t},\bar{t}_0]$: Deviation from LG (“Perturbation”)

$$= \hat{\rho}_{\rm LG}[\bar{t};\lambda]\hat{U}[\bar{t},\bar{t}_0], \quad \hat{U}[\bar{t},\bar{t}_0] \equiv T_\tau \exp\left(\int_0^1 d\tau \hat{\Sigma}_\tau[\bar{t},\bar{t}_0]\right)$$

$$\langle \hat{\mathcal{O}}(x) \rangle = \langle \hat{U}\hat{\mathcal{O}}(x) \rangle^{\rm LG}_{\bar{t}}$$

Condition to determine parameters

$$\langle \hat{\mathcal{O}}(x) \rangle = \langle \hat{U} \hat{\mathcal{O}}(x) \rangle_{\bar{t}}^{\mathrm{LG}}$$ ←**Parameter ?**

Fastest Apparent Convergence (FAC)

Corrections for conserved charges should be minimized!

$$\langle \hat{U} \delta \hat{\mathcal{J}}^{\bar{0}}_{a}(x) \rangle_{\bar{t}}^{\mathrm{LG}} = 0 \;\Leftrightarrow\; \langle \hat{\mathcal{J}}^{\bar{0}}_{a}(x) \rangle = \langle \hat{\mathcal{J}}^{\bar{0}}_{a}(x) \rangle_{\bar{t}}^{\mathrm{LG}}$$

→ Determine new parameters $\beta^{\mu}(x),\ \nu(x)$ on $\Sigma_{\bar{t}}$

→ **Avg. of current is calculable !! (in principle)**

Entropy production and 2nd law

Entropy production operator

$$\hat{\Sigma}[\bar{t},\bar{t}_0] \equiv \hat{S}[\bar{t};\lambda] - \hat{S}[\bar{t}_0;\lambda] \qquad (\delta\hat{\mathcal{O}} \equiv \hat{\mathcal{O}} - \langle\hat{\mathcal{O}}\rangle_{\bar{t}}^{\mathrm{LG}})$$

$$= -\int_{\bar{t}_0}^{\bar{t}} d^4x \left[(\nabla_\mu \beta^\nu)\delta\hat{T}^\mu_{\ \nu} + (\nabla_\mu \nu)\delta\hat{J}^\mu\right]$$

- Quantum fluctuation theorem & 2nd law of thermodynamics

$$\langle\hat{\Sigma}[\bar{t},\bar{t}_0]\rangle \geq 0 \ \Leftrightarrow \ \langle\hat{S}[\bar{t},\lambda]\rangle - \langle\hat{S}[\bar{t}_0,\lambda]\rangle \geq 0$$

- $\hat{\Sigma}$ is proportional to derivatives: $\langle\hat{\mathcal{O}}(x)\rangle = \langle\hat{U}\hat{\mathcal{O}}(x)\rangle_{\bar{t}}^{\mathrm{LG}}$

"Expansion of $\hat{U} = T_\tau e^{\int_0^1 d\tau \hat{\Sigma}_\tau[\bar{t},\bar{t}_0]}$" = Derivative expansion

Dissipative derivative expansion

$$\langle \hat{T}^{\mu\nu}(x) \rangle = \langle \hat{U} \hat{T}^{\mu\nu}(x) \rangle_{\bar{t}}^{\mathrm{LG}}$$

$$= \langle \hat{T}^{\mu\nu}(x) \rangle_{\bar{t}}^{\mathrm{LG}} + \boxed{\int_0^1 d\tau \langle T_\tau \hat{\Sigma}_\tau [\bar{t}, \bar{t}_0] \hat{T}^{\mu\nu}(x) \rangle_{\bar{t}}^{\mathrm{LG}}} + \cdots$$

$$\equiv \langle \delta \hat{T}^{\mu\nu}(x) \rangle_{(0,1)}$$

$$\langle \hat{J}^{\mu}(x) \rangle = \langle \hat{U} \hat{J}^{\mu}(x) \rangle_{\bar{t}}^{\mathrm{LG}}$$

$$= \langle \hat{J}^{\mu}(x) \rangle_{\bar{t}}^{\mathrm{LG}} + \boxed{\int_0^1 d\tau \langle T_\tau \hat{\Sigma}_\tau [\bar{t}, \bar{t}_0] \hat{J}^{\mu}(x) \rangle_{\bar{t}}^{\mathrm{LG}}} + \cdots$$

$$\equiv \langle \delta \hat{J}^{\mu}(x) \rangle_{(0,1)}$$

Constitutive relation & Kubo formula

Constitutive relations

$$\langle \delta \hat{T}^{\mu\nu}(x) \rangle_{(0,1)} = -\frac{\zeta}{\beta} h^{\mu\nu} h^{\rho\sigma} \nabla_\rho \beta_\sigma - 2\frac{\eta}{\beta} h^{\mu\rho} h^{\nu\sigma} \nabla_{\langle\rho} \beta_{\sigma\rangle}$$

$$\langle \delta \hat{J}^{\mu}(x) \rangle_{(0,1)} = -\frac{\kappa}{\beta} h^{\mu\nu} \nabla_\nu \nu \qquad (h^{\mu\nu} n_\nu = 0)$$

Kubo formulae

$$\zeta = \beta(x) \int_{-\infty}^{\bar{t}} d^4x' \int_0^1 d\tau \langle e^{\hat{K}\tau} \tilde{\delta}\hat{p}(x') e^{-\hat{K}\tau} \tilde{\delta}\hat{p}(x) \rangle_{\bar{t}}^{\mathrm{LG}}$$

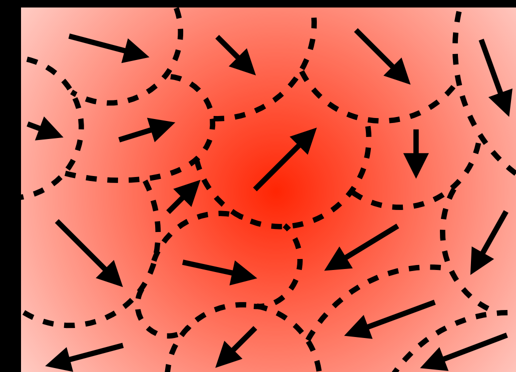
$$\eta = \frac{\beta(x)}{(d+1)(d-2)} \int_{-\infty}^{\bar{t}} d^4x' \int_0^1 d\tau \langle e^{\hat{K}\tau} \tilde{\delta}\hat{\pi}^{\mu\nu}(x') e^{-\hat{K}\tau} \tilde{\delta}\hat{\pi}^{\rho\sigma}(x) \rangle_{\bar{t}}^{\mathrm{LG}} h_{\mu\rho}(x) h_{\nu\sigma}(x)$$

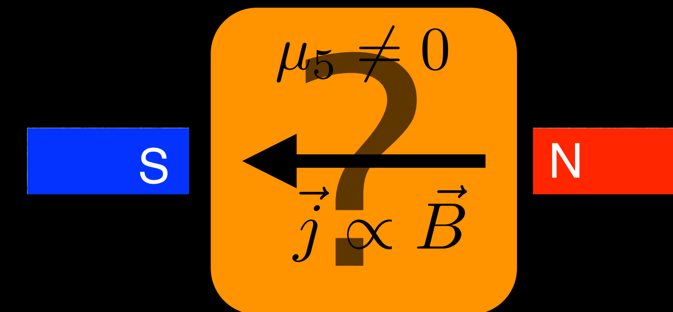
$$\kappa = \frac{\beta(x)}{d-1} \int_{-\infty}^{\bar{t}} d^4x' \int_0^1 d\tau \langle e^{\hat{K}\tau} \tilde{\delta}\hat{J}^{\mu}(x') e^{-\hat{K}\tau} \tilde{\delta}\hat{J}^{\nu}(x) \rangle_{\bar{t}}^{\mathrm{LG}} h_{\mu\nu}(x)$$

Outline

MOTIVATION:

Relativistic hydrodynamics
from **quantum field theory?**





APPROACH:

QFT for **initial local Gibbs distribution**

① Renormalized/optimized perturbation for dissipative part

②

RESULTS:

Derivation of Navier-Stokes eq.
& **anomaly-induced transports**

Non-dissipative part

Non-dissipative part

$$\langle \hat{T}^{\mu\nu}(x) \rangle = \langle \hat{U} \hat{T}^{\mu\nu}(x) \rangle_{\bar{t}}^{\mathrm{LG}}$$

$$= \langle \hat{T}^{\mu\nu}(x) \rangle_{\bar{t}}^{\mathrm{LG}} + \int_0^1 d\tau \langle T_\tau \hat{\Sigma}_\tau [\bar{t}, \bar{t}_0] \hat{T}^{\mu\nu}(x) \rangle_{\bar{t}}^{\mathrm{LG}} + \cdots$$

??

$$\equiv \langle \delta \hat{T}^{\mu\nu}_{(1)}(x) \rangle$$

$$\langle \hat{J}^{\mu}(x) \rangle = \langle \hat{U} \hat{J}^{\mu}(x) \rangle_{\bar{t}}^{\mathrm{LG}}$$

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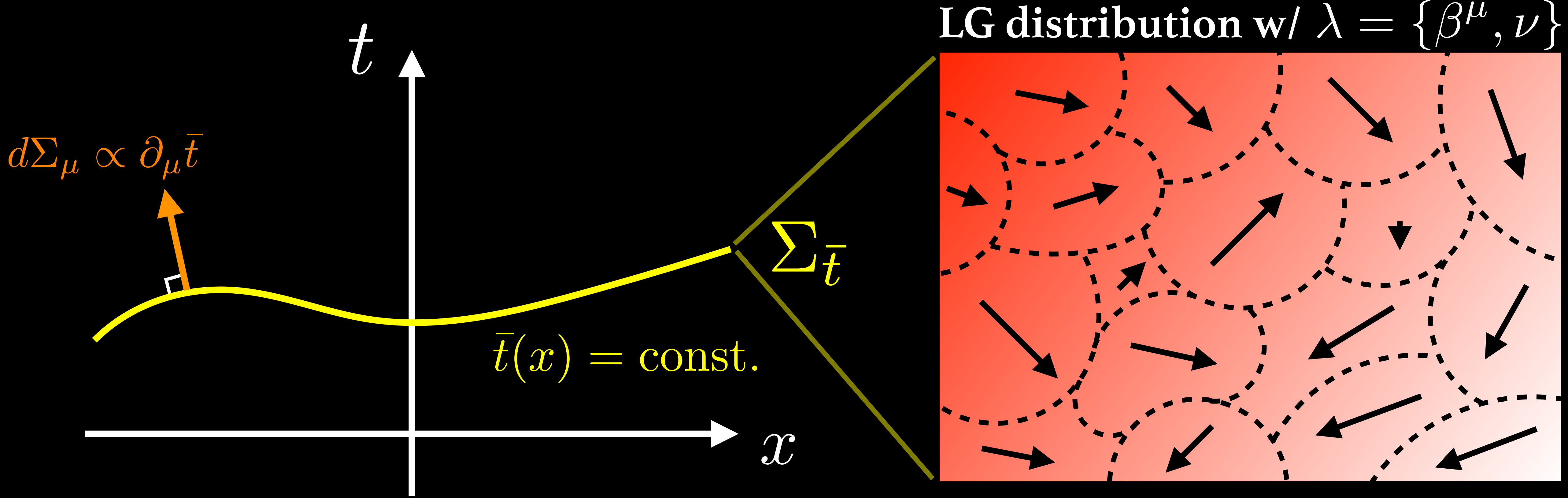
??

$$\equiv \langle \delta \hat{J}^{\mu}_{(1)}(x) \rangle$$

(Local) Thermodynamic Potential



Masseiu-Planck functional

$$\Psi[\bar{t};\lambda] \equiv \log \mathrm{Tr} \exp \left[\int d\Sigma_{\bar{t}\nu} \left(\beta^\mu(x) \hat{T}^\nu{}_\mu(x) + \nu(x) \hat{J}^\nu(x) \right) \right]$$

$$= \log \mathrm{Tr} \exp \left[- \int d^3\bar{x} \sqrt{-g} \left(\beta^{\bar{\mu}}(\bar{x}) \hat{T}^{\bar{0}}{}_{\bar{\mu}}(\bar{x}) + \nu(\bar{x}) \hat{J}^{\bar{0}}(\bar{x}) \right) \right]$$

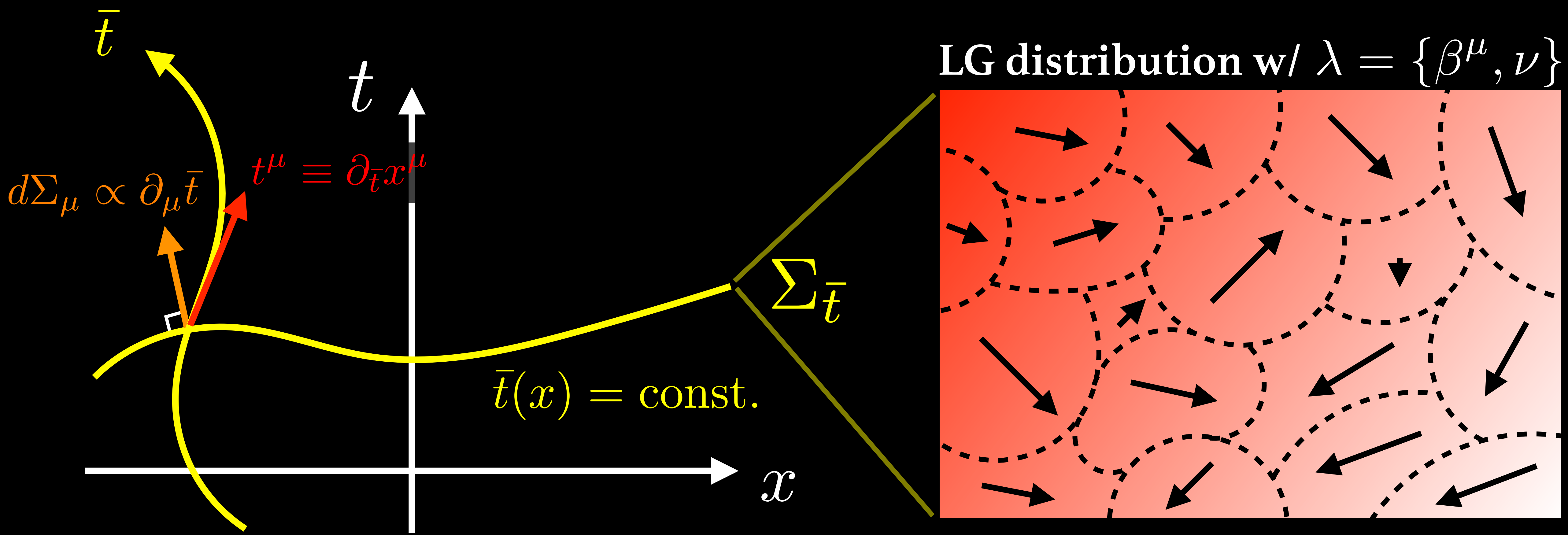
Variation formula for local equil.

[Banerjee et al. (2012), Jensen et al. (2012), Haehl et al. (2015), **MH** (2017)]

Variation formula in "hydrostatic gauge"

$$\langle \hat{T}^{\mu\nu}(x) \rangle_{\bar{t}}^{\mathrm{LG}} = \frac{2}{\sqrt{-g}} \frac{\delta}{\delta g_{\mu\nu}(x)} \Psi[\bar{t};\lambda], \quad \langle \hat{J}^{\mu}(x) \rangle_{\bar{t}}^{\mathrm{LG}} = \frac{1}{\sqrt{-g}} \frac{\delta}{\delta A_{\mu}(x)} \Psi[\bar{t};\lambda]$$

(Local) Thermodynamic Potential



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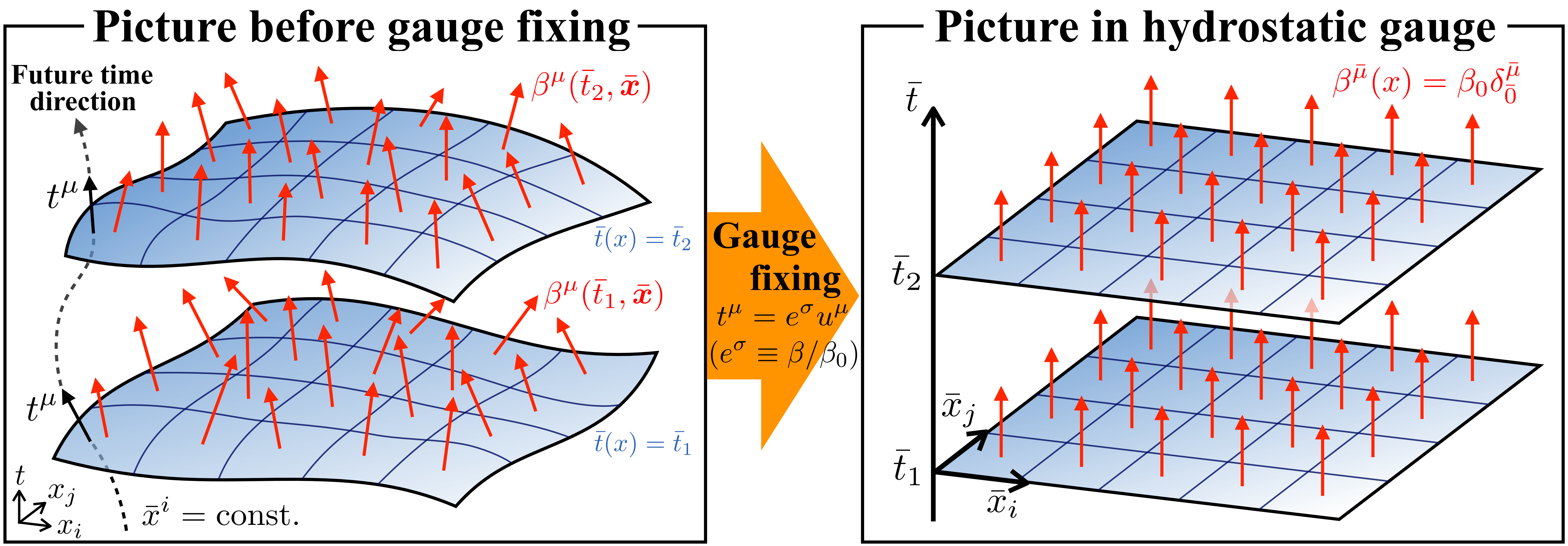
$$= \log \mathrm{Tr} \exp \left[- \int d^3\bar{x} \sqrt{-g} \left(\beta^{\bar{\mu}}(\bar{x}) \hat{T}^{\bar{0}}{}_{\bar{\mu}}(\bar{x}) + \nu(\bar{x}) \hat{J}^{\bar{0}}(\bar{x}) \right) \right]$$

Hydrostatic gauge fixing



We can choose the time direction vector $t^\mu(x) \equiv \partial_{\bar{t}} x^\mu$

Hydrostatic gauge fixing

Let us choose $t^\mu(x) = \beta^\mu(x)/\beta_0, \ A_{\bar{0}}(x) = \nu(x)$

Variation formula for local equil.

[Banerjee et al. (2012), Jensen et al. (2012), Haehl et al. (2015), MH (2017)]

Variation formula in "hydrostatic gauge"

$$\langle \hat{T}^{\mu\nu}(x)\rangle_{\bar{t}}^{\mathrm{LG}} = \frac{2}{\sqrt{-g}}\frac{\delta}{\delta g_{\mu\nu}(x)}\Psi[\bar{t};\lambda], \quad \langle \hat{J}^{\mu}(x)\rangle_{\bar{t}}^{\mathrm{LG}} = \frac{1}{\sqrt{-g}}\frac{\delta}{\delta A_{\mu}(x)}\Psi[\bar{t};\lambda]$$

Proof. Consider time derivative of $\Psi[\lambda]$

$$\partial_{\bar{t}}\Psi[\bar{t};\lambda] = \int d^{d-1}\bar{x}\sqrt{-g}\left(\nabla_\mu\beta_\nu\langle\hat{T}^{\mu\nu}\rangle_{\bar{t}}^{\mathrm{LG}} + (\nabla_\mu\nu + F_{\nu\mu}\beta^\nu)\langle\hat{J}^\mu\rangle_{\bar{t}}\right)$$

$$= \int d^{d-1}\bar{x}\sqrt{-g}\left(\frac{1}{2}(\nabla_\mu\beta_\nu + \nabla_\nu\beta_\mu)\langle\hat{T}^{\mu\nu}\rangle_{\bar{t}}^{\mathrm{LG}} + (\beta^\nu\nabla_\nu A_\mu + A_\nu\nabla_\mu\beta^\nu)\langle\hat{J}^\mu\rangle_{\bar{t}}\right)$$

$$= \int d^{d-1}\bar{x}\sqrt{-g}\left(\frac{1}{2}\pounds_\beta g_{\mu\nu}\langle\hat{T}^{\mu\nu}\rangle_{\bar{t}}^{\mathrm{LG}} + \pounds_\beta A_\mu\langle\hat{J}^\mu\rangle_{\bar{t}}\right)$$

On the other hand, since $t^\mu = \beta^\mu$, we can express the LHS as

$$\partial_{\bar{t}}\Psi[\bar{t};\lambda] = \int d^{d-1}\bar{x}\left(\pounds_\beta g_{\mu\nu}\frac{\delta\Psi}{\delta g_{\mu\nu}} + \pounds_\beta A_\mu\frac{\delta\Psi}{\delta A_\mu}\right)$$

Matching them gives the above variation formula! □

Q. How can we calculate $\Psi \equiv \log Z$?

Thermal QFT in a Nutshell

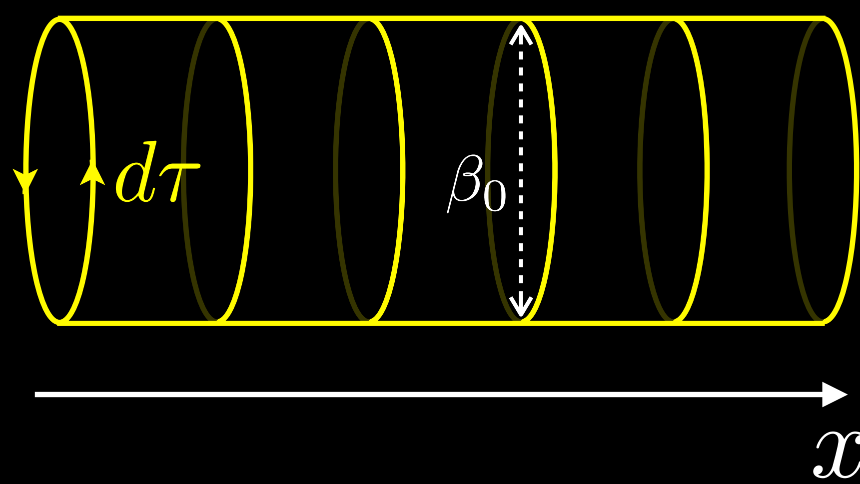
Global equil. β_0

$$T = \text{const.}$$

Path int. →

Thermal QFT (Matsubara formalism)

[Matsubara, 1955]



QFT in the flat spacetime with size β_0

Gibbs dist.: $$\hat{\rho}_G = \frac{e^{-\beta(\hat{H}-\mu\hat{N})}}{Z} = e^{-\beta(\hat{H}-\mu\hat{N})-\Psi[\beta,\nu]}$$

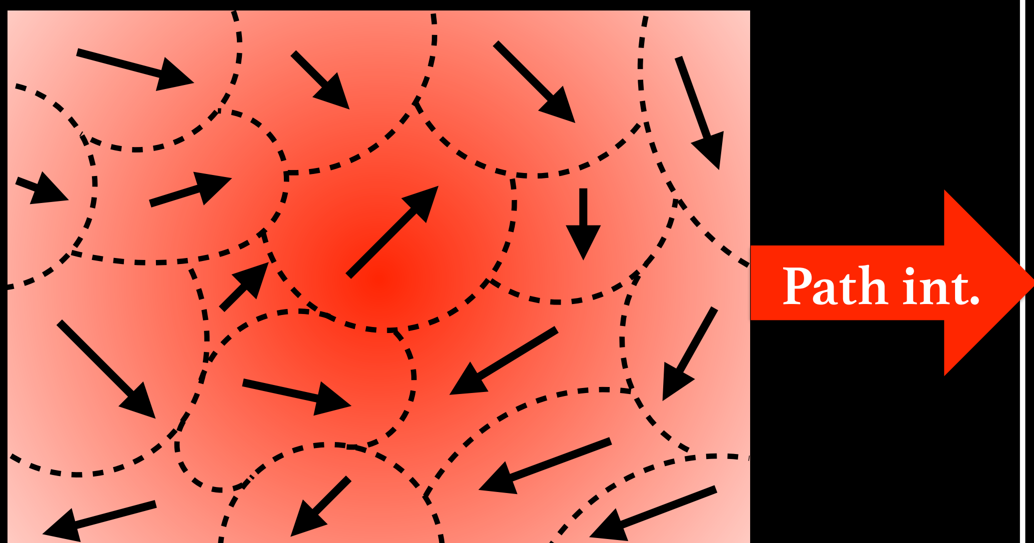
Thermodynamic potential with Euclidean action

$$\Psi[\beta,\nu] = \log \mathrm{Tr}\, e^{-\beta(\hat{H}-\mu\hat{N})} = \log \int d\varphi \langle \pm\varphi | e^{-\beta(\hat{H}-\mu\hat{N})} | \varphi \rangle$$

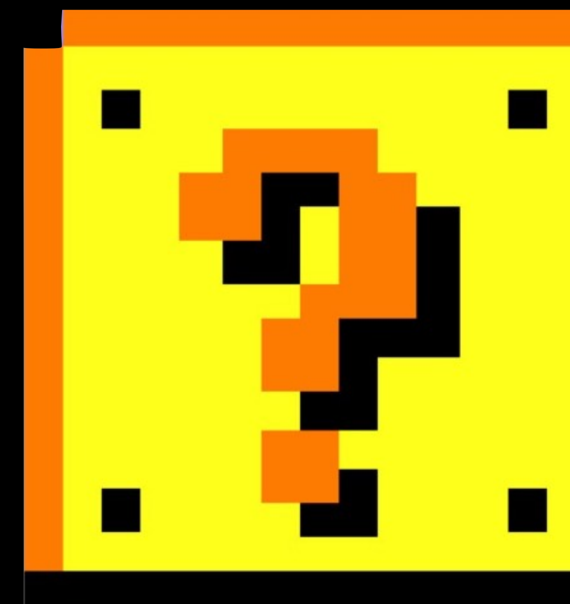
$$= \log \int_{\varphi(\beta)=\pm\varphi(0)} \mathcal{D}\varphi\, e^{+S_E[\varphi]}, \quad S_E[\varphi] = \int_0^\beta d\tau \int d^3x\, \mathcal{L}_E(\varphi, \partial_\mu\varphi)$$

QFT for local thermal equilibrium?

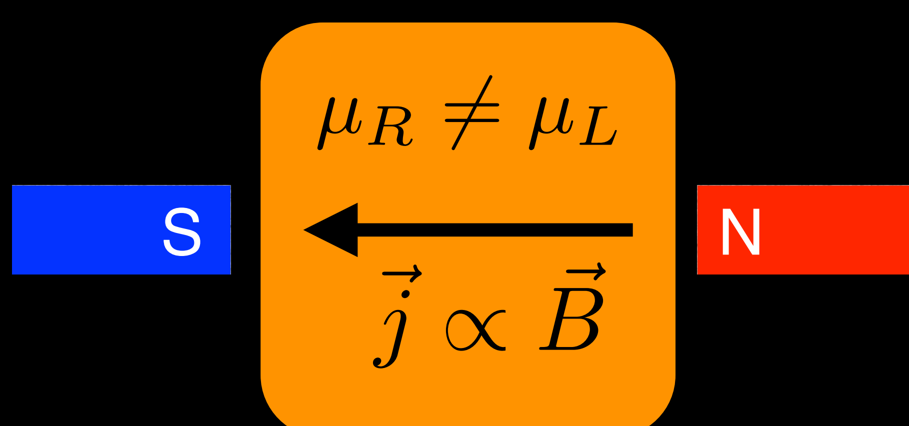
Local equil. $\{\beta(x),\ \vec{v}(x)\}$



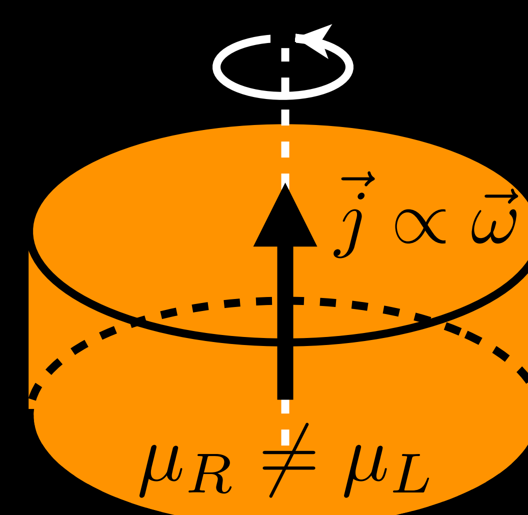
Local Thermal QFT



Local thermal QFT can describe anomaly-induced transport



Chiral Magnetic Effect



Chiral Vortical Effect

Case study 1: Scalar field

$$\mathcal{L} = -\frac{g^{\bar{\mu}\bar{\nu}}}{2}\partial_{\bar{\mu}}\phi\partial_{\bar{\nu}}\phi - V(\phi)$$

$$\Rightarrow \quad \hat{T}^{\mu\nu} \equiv \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g_{\mu\nu}} = \partial^{\mu}\hat{\phi}\partial^{\nu}\hat{\phi} + g^{\mu\nu}\mathcal{L}(\hat{\phi}, \partial_{\rho}\hat{\phi})$$

$$\begin{aligned}\Psi[\bar{t};\lambda] &= \log \mathrm{Tr}\exp\left[-\int d^{d-1}\bar{x}\sqrt{-g}\beta^{\mu}(x)\hat{T}^{\bar{0}}{}_{\mu}(x)\right] \\ &= \log\int \mathcal{D}\phi \exp\left(S_E[\phi,\beta^{\mu}]\right) = \log\int \mathcal{D}\phi \exp\left(S_E[\phi,\tilde{g}]\right)\end{aligned}$$

$$\begin{aligned}S[\phi,\beta^{\mu}] &= \int_0^{\beta_0} d\tau \int d^3\bar{x}\sqrt{-g}e^{\sigma}u^{\bar{0}}\left[-\frac{e^{-2\sigma}}{2u^{\bar{0}}u_{\bar{0}}}(i\dot{\phi})^2 - \frac{-e^{-\sigma}u^{\bar{i}}}{u^{\bar{0}}u_{\bar{0}}}(i\dot{\phi})\partial_{\bar{i}}\phi - \frac{1}{2}\left(\gamma^{\bar{i}\bar{j}} + \frac{u^{\bar{i}}u^{\bar{j}}}{u^{\bar{0}}u_{\bar{0}}}\right)\partial_{\bar{i}}\phi\partial_{\bar{j}}\phi - V(\phi)\right] \\ &= \int_0^{\beta_0} d\tau \int d^3\bar{x}\sqrt{-\tilde{g}}\left[-\frac{\tilde{g}^{\bar{\mu}\bar{\nu}}}{2}\partial_{\bar{\mu}}\phi\partial_{\bar{\nu}}\phi - V(\phi)\right] \qquad \left(e^{\sigma(\bar{x})} \equiv \beta(\bar{x})/\beta_0\right)\end{aligned}$$

ψ in terms of thermal metric

$$\Psi[\bar{t};\lambda] = \log \int \mathcal{D}\phi \exp\left(S_E[\phi,;\tilde{g}]\right)$$

Thermal metric

$$\tilde{g}_{\bar{\mu}\bar{\nu}} = \begin{pmatrix} -e^{2\sigma} & e^{\sigma}u_{\bar{j}} \\ e^{\sigma}u_{\bar{i}} & \gamma_{\bar{i}\bar{j}} \end{pmatrix}$$

$$(e^{\sigma(\bar{x})} \equiv \beta(\bar{x})/\beta_0)$$

Inverse thermal metric

$$\tilde{g}^{\bar{\mu}\bar{\nu}} = \begin{pmatrix} \dfrac{e^{-2\sigma}}{u^{\bar{0}}u_{\bar{0}}} & -\dfrac{e^{-\sigma}u^{\bar{j}}}{u^{\bar{0}}u_{\bar{0}}} \\ -\dfrac{e^{-\sigma}u^{\bar{i}}}{u^{\bar{0}}u_{\bar{0}}} & \gamma^{\bar{i}\bar{j}} + \dfrac{u^{\bar{i}}u^{\bar{j}}}{u^{\bar{0}}u_{\bar{0}}} \end{pmatrix}$$

◆ Interpretation of above result

$\Psi[\bar{t};\lambda]$ **is described by QFT in "curved spacetime" s. t.**

$$d\tilde{s}^2 = -e^{2\sigma}(d\tilde{t} + a_{\bar{i}}dx^{\bar{i}})^2 + \gamma'_{\bar{i}\bar{j}}dx^{\bar{i}}dx^{\bar{j}}$$

$$(a_{\bar{i}} \equiv e^{-\sigma}u_{\bar{i}}, \quad \gamma'_{\bar{i}\bar{j}} \equiv \gamma_{\bar{i}\bar{j}} + u_{\bar{i}}u_{\bar{j}}, \quad d\tilde{t} = -id\tau)$$

Case study 2: Dirac field

$$\mathcal{L} = -\frac{1}{2}\bar{\psi}\left(\gamma^a e_a{}^{\bar{\mu}}\overrightarrow{D}_{\bar{\mu}} - \overleftarrow{D}_{\bar{\mu}}\gamma^a e_a{}^{\bar{\mu}}\right)\psi - m\bar{\psi}\psi$$

Symmetric energy-momentum tensor

$$T^{\bar{\mu}}{}_{\bar{\nu}} = -\delta^{\bar{\mu}}_{\bar{\nu}}\mathcal{L} - \frac{1}{4}\bar{\psi}(\gamma^{\bar{\mu}}\overrightarrow{D}_{\bar{\nu}} + \gamma_{\bar{\nu}}\overrightarrow{D}^{\bar{\mu}} - \overleftarrow{D}_{\bar{\nu}}\gamma^{\bar{\mu}} - \overleftarrow{D}^{\bar{\mu}}\gamma_{\bar{\nu}})\psi$$

◆ **Result of path integral**

$$\Psi[\bar{t};\lambda] \equiv \log \mathrm{Tr}\exp\left[\int d\Sigma_{\bar{t}\nu}\left(\beta^\mu(x)\hat{T}^\nu{}_\mu(x) + \nu(x)\hat{J}^\nu(x)\right)\right]$$

$$= \log\int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left(S_E[\psi,\bar{\psi};\tilde{e}]\right)$$

ψ in terms of thermal vielbein

$$\Psi[\bar{t};\lambda] = \log \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left(S_E[\psi,\bar{\psi};\tilde{e}]\right)$$

◆ **Euclidean action with thermal vielbein**

$$S_E[\psi,\bar{\psi};\tilde{e}] = \int_0^{\beta_0} d\tau \int d^3\bar{x}\tilde{e}\left[-\frac{1}{2}\bar{\psi}\left(\gamma^a \tilde{e}_a{}^{\bar{\mu}}\overrightarrow{D}_{\bar{\mu}} - \overleftarrow{D}_{\bar{\mu}}\gamma^a\tilde{e}_a{}^{\bar{\mu}}\right)\psi - m\bar{\psi}\psi\right]$$

Thermal vielbein : $\tilde{e}_{\bar{0}}{}^a = e^\sigma u^a,\ \tilde{e}_{\bar{i}}{}^a = e_{\bar{i}}{}^a \quad (e^\sigma \equiv \beta(x)/\beta_0)$

◆ **Interpretation of above result**

$\Psi[\bar{t};\lambda]$ **is described by QFT in "curved spacetime" s. t.**

$$d\tilde{s}^2 = \tilde{e}_{\bar{\mu}}{}^a\tilde{e}_{\bar{\nu}}{}^b\eta_{ab}dx^{\bar{\mu}}dx^{\bar{\nu}} = -e^{2\sigma}(d\tilde{t} + a_{\bar{i}}dx^{\bar{i}})^2 + \gamma'_{\bar{i}\bar{j}}dx^{\bar{i}}dx^{\bar{j}}$$

$$(a_{\bar{i}} \equiv e^{-\sigma}u_{\bar{i}}, \quad \gamma'_{\bar{i}\bar{j}} \equiv \gamma_{\bar{i}\bar{j}} + u_{\bar{i}}u_{\bar{j}}, \quad d\tilde{t} = -id\tau)$$

Local Thermal QFT

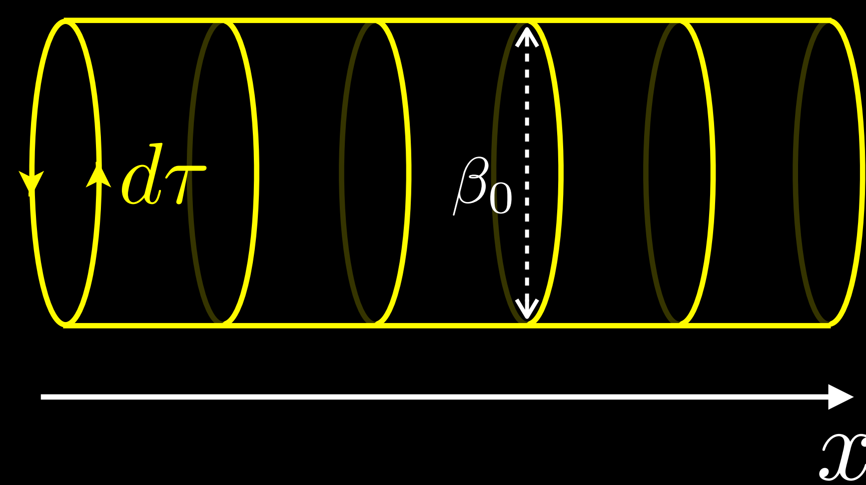
Global equil. β_0

$$T = \text{const.}$$

Path int.

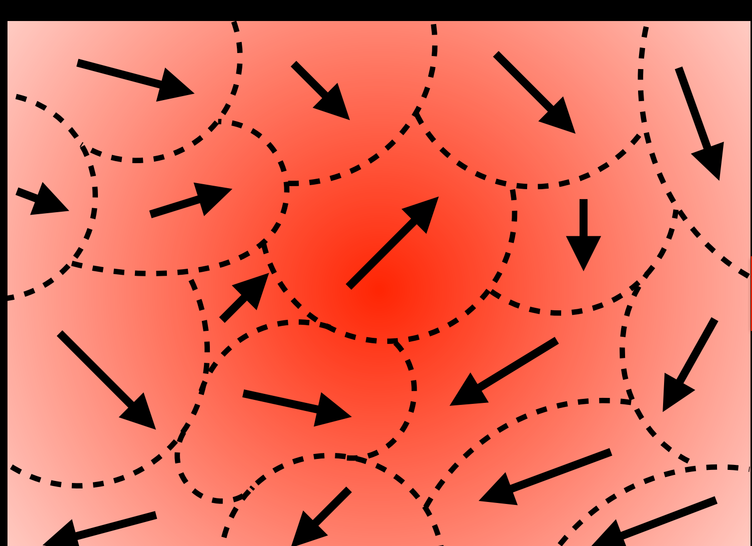
Thermal QFT (Matsubara formalism)

[Matsubara, 1955]



QFT in the flat spacetime with size β_0

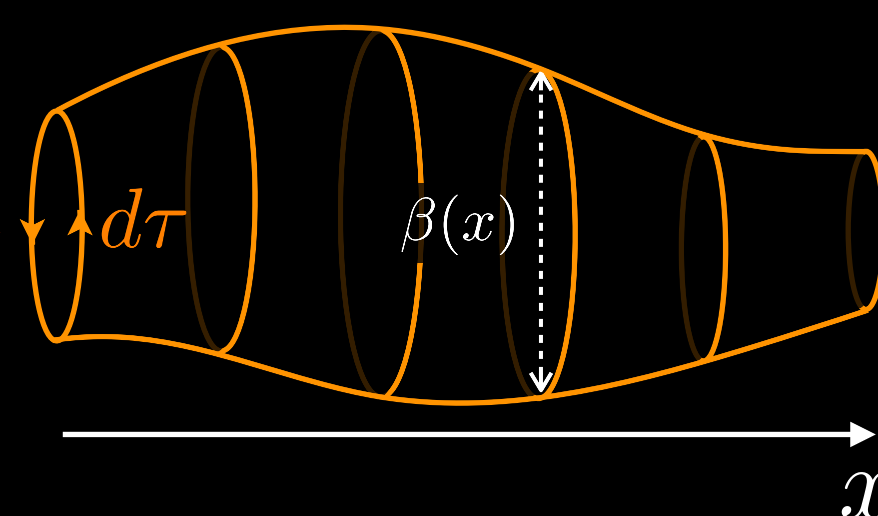
Local equil. $\{\beta(x),\ \vec{v}(x)\}$



Path int.

Local Thermal QFT

[Hayata-Hidaka-MH-Noumi PRD(2015)]
[MH (2017)]



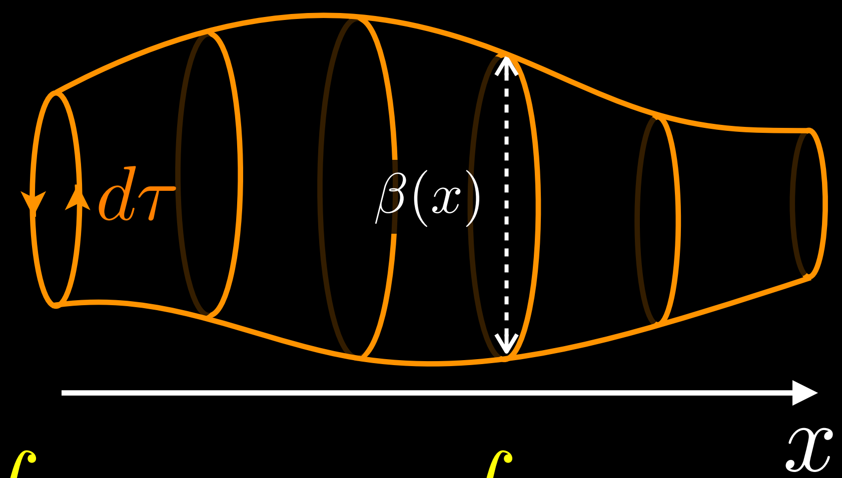
QFT in the "curved spacetime" with "line element"

$$d\tilde{s}^2 = d\tilde{s}^2(\beta, \vec{v})$$

Two ways to construct $\Psi \equiv \log Z$

1. Use diffeo & gauge invariance!

- Ψ is expressed by $\{\tilde{g}_{\mu\nu}, \tilde{A}_\mu\}$
- Ψ is diffeo & gauge invariant!



➡ Ψ is expressed in terms of $\beta = \oint d\tilde{s}, \ \beta\mu = \oint \tilde{A}, \ \tilde{R}, \ \tilde{F}_{\mu\nu}$

2. Use symmetry from imaginary-time nature!

- Ψ is spatial diffeomorphism invariant
- Ψ is Kaluza-Klein gauge invariant!

➡ $\Psi \equiv \log Z$ should respect these two symmetries!!

[cf. Hydrostatic partition function method Banerjee et al.(2012), Jensen et al.(2012)]

Two ways to construct $\Psi \equiv \log Z$

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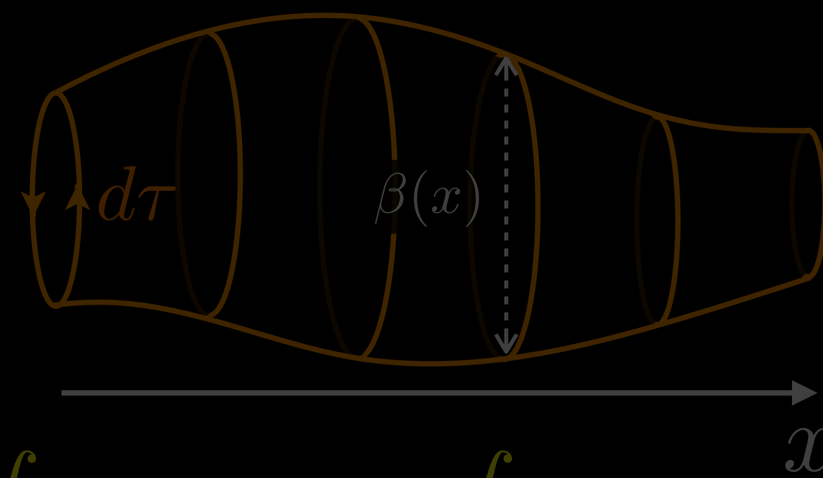

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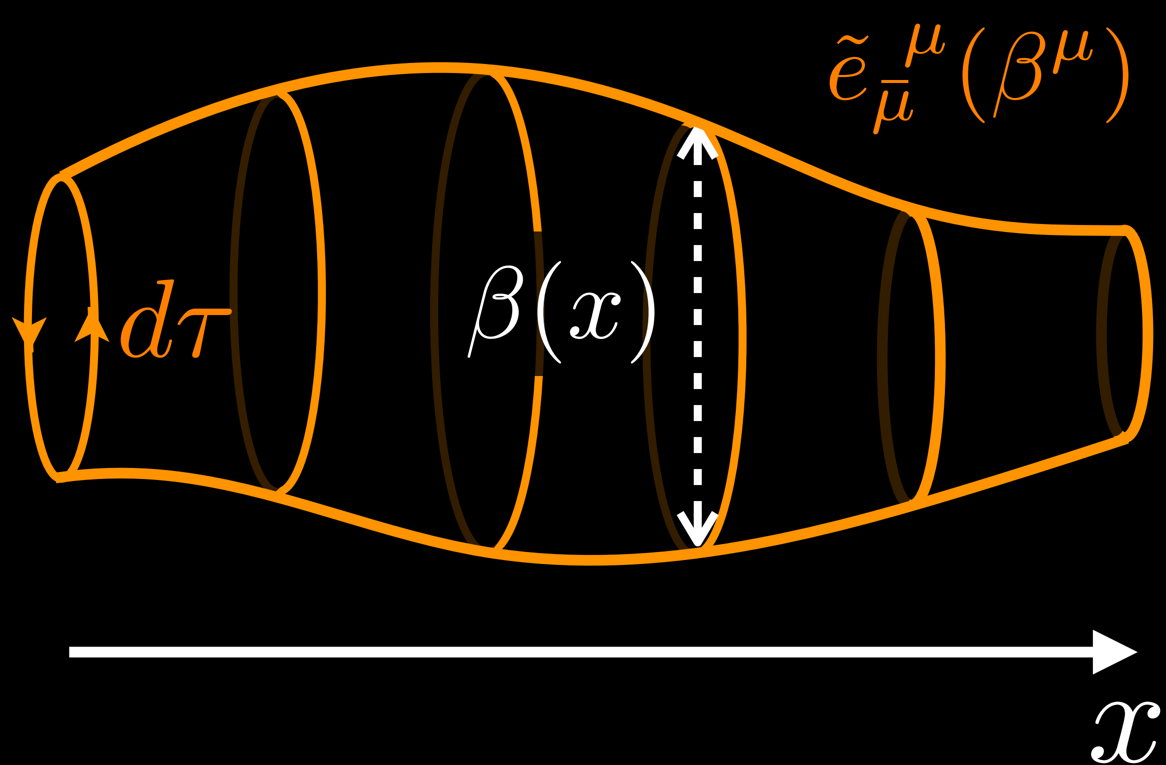
Kaluza-Klein gauge symmetry

$$d\tilde{s}^2 = -e^{2\sigma}(d\tilde{t} + a_{\bar{i}} dx^{\bar{i}})^2 + \gamma'_{\bar{i}\bar{j}} dx^{\bar{i}} dx^{\bar{j}} \quad (d\tilde{t} = -i d\tau)$$



Parameters λ don't depend on imaginary time τ.

"Kaluza-Klein" gauge tr.

$$\begin{cases} \tilde{t} \to \tilde{t} + \chi(\bar{\boldsymbol{x}}) \\ \bar{\boldsymbol{x}} \to \bar{\boldsymbol{x}} \\ a_{\bar{i}}(\bar{\boldsymbol{x}}) \to a_{\bar{i}}(\bar{\boldsymbol{x}}) - \partial_{\bar{i}}\chi(\bar{\boldsymbol{x}}) \end{cases}$$

$$\Psi[\lambda] = \log \int \mathcal{D}\bar{\psi}\mathcal{D}\psi e^{S[\psi,\bar{\psi},\tilde{e}]} \ni$$

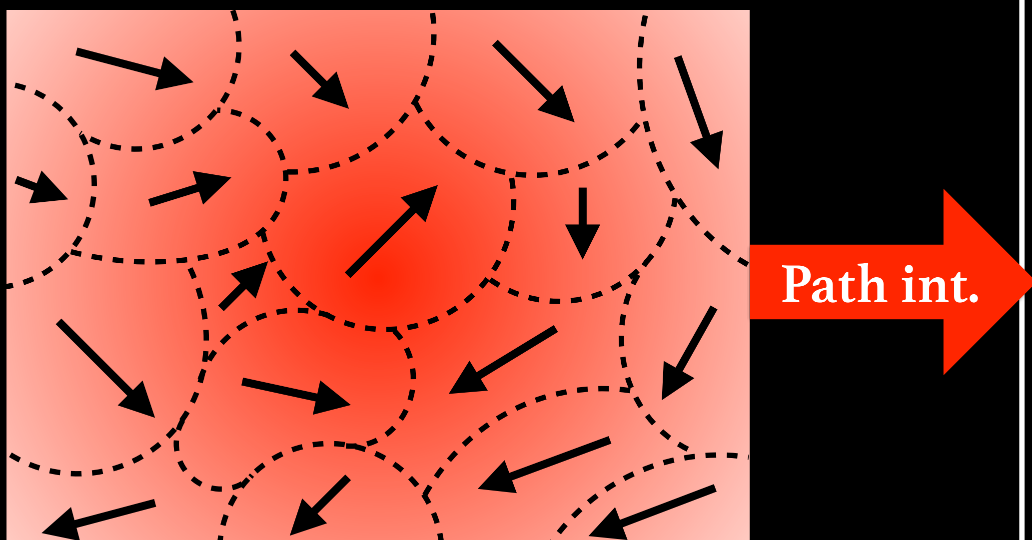
$$(f_{\bar{i}\bar{j}} \equiv \partial_{\bar{i}} a_{\bar{j}} - \partial_{\bar{j}} a_{\bar{i}})$$

$f^{\bar{i}\bar{j}} f_{\bar{i}\bar{j}}, \cdots$

$a_{\bar{i}},\ a_{\bar{i}} a^{\bar{i}}, \cdots$

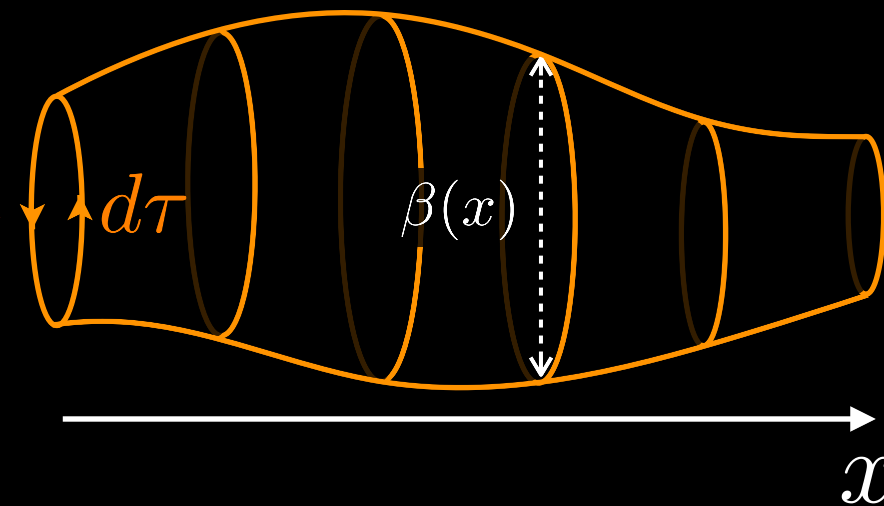
Short Summary: Local Thermal QFT

Local equil. $\{\beta(x),\ \vec{v}(x)\}$



Local Thermal QFT

[Hayata-Hidaka-MH-Noumi PRD(2015)]
[MH Ann. Phys. (2017)]



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$$d\tilde{s}^2 = d\tilde{s}^2(\beta, \vec{v})$$

$$\Psi[\bar{t};\lambda] \equiv \log \mathrm{Tr} \exp\left[\int d\Sigma_{\bar{t}\nu}\left(\beta^\mu(x)\hat{T}^\nu{}_\mu(x) + \nu(x)\hat{J}^\nu(x)\right)\right]$$

① $\Psi[\lambda]$ plays a role as the generating functional: $\langle \hat{T}^{\mu\nu}(x)\rangle^{\mathrm{LG}} = \dfrac{2}{\sqrt{-g}}\dfrac{\delta}{\delta g_{\mu\nu}(x)}\Psi[\lambda]$

② $\Psi[\lambda]$ is written in terms of QFT in curved spacetime

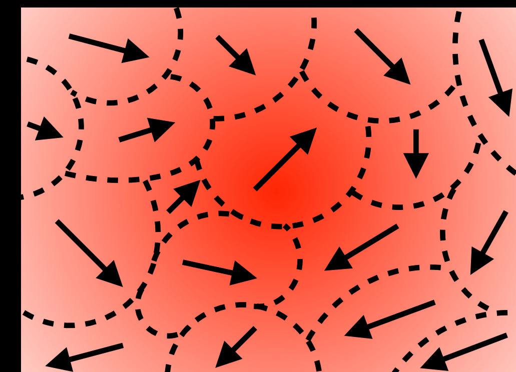
$$d\tilde{s}^2 = -e^{2\sigma}(d\tilde{t} + a_{\bar{i}}dx^{\bar{i}})^2 + \gamma'_{\bar{i}\bar{j}}dx^{\bar{i}}dx^{\bar{j}}$$

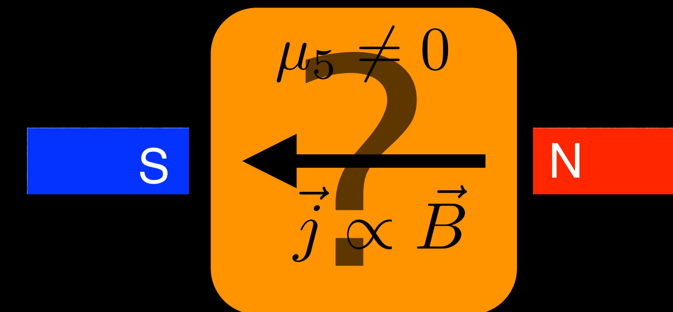
Symmetry = Spatial diffeomorphism + Kaluza-Klein gauge

Outline

MOTIVATION:

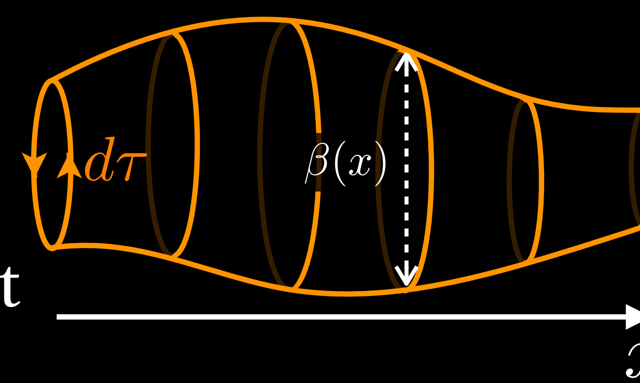
Relativistic hydrodynamics
from **quantum field theory?**





APPROACH:

QFT for **initial local Gibbs distribution**



① Renormalized/optimized perturbation for dissipative part

② Path-integral formula for non-dissipative part

$$\Psi[\lambda] \leftarrow \text{QFT in } d\tilde{s}^2 = -e^{2\sigma}(d\tilde{t} + a_{\bar{i}} dx^{\bar{i}})^2 + \gamma'_{\bar{i}\bar{j}} dx^{\bar{i}} dx^{\bar{j}}$$

Symmetry = Spatial diffeomorphism + Kaluza-Klein gauge

RESULTS:

Derivation of Navier-Stokes eq.
& **anomaly-induced transports**

Double derivative expansion

(1) Dissipative derivative expansion

$$\hat{\rho}_{\mathrm{LG}}(\bar{t}_0) = \hat{\rho}_{\mathrm{LG}}(\bar{t}) \left(1 + \mathcal{O}(\partial) + \mathcal{O}(\partial^2) + \cdots \right)$$

→ Dissipative correction (viscosity etc.)

(2) Non-dissipative derivative expansion

$$\Psi[\beta^\mu, \nu] = \Psi^{(0)}[\beta^\mu, \nu] + \Psi^{(1)}[\beta^\mu, \nu, \partial] + \mathcal{O}(\partial^2) + \cdots$$

Derivative expansion of ψ

(2) Derivative expansion of ψ

$$\Psi[\beta^\mu, \nu] = \Psi^{(0)}[\beta^\mu, \nu] + \Psi^{(1)}[\beta^\mu, \nu, \partial] + \mathcal{O}(\partial^2) + \cdots$$

$\simeq \beta p$ Symmetry property

$= 0$ **Parity-even system**

Non-dissipative constitutive relation

$$\langle \hat{T}^{\mu\nu}(x) \rangle_{\bar{t}}^{\mathrm{LG}} = \frac{2}{\sqrt{-g}} \frac{\delta}{\delta g_{\mu\nu}(x)} \Psi[\bar{t}; \lambda] = T^{\mu\nu}_{(0)}[\lambda(x)] + T^{\mu\nu}_{(1)}[\lambda(x), \nabla\lambda(x)] + \cdots$$

$$\langle \hat{J}^{\mu}(x) \rangle_{\bar{t}}^{\mathrm{LG}} = \frac{2}{\sqrt{-g}} \frac{\delta}{\delta A_{\mu}(x)} \Psi[\bar{t}; \lambda] = J^{\mu}_{(0)}[\lambda(x)] + J^{\mu}_{(1)}[\lambda(x), \nabla\lambda(x)] + \cdots$$

$= 0$

Parity-even case

$$\mu_R = \mu_L$$

Recipe for Masseiu-Planck fcn.

[Banerjee et al.(2012), Jensen et al.(2012)]

Masseiu-Planck functional

$$\Psi[\lambda] = \log \int \mathcal{D}\phi e^{S[\phi,\tilde{g}]} = \Psi^{(0)}[\lambda] + \Psi^{(1)}[\lambda, \partial] + \mathcal{O}(\partial^2)$$

$$\mathcal{O}(p^0) \qquad \mathcal{O}(p^1)$$

- **Building blocks** : $\lambda = \{e^{\sigma},\ a_{\bar{i}},\ \mu,\ A_{\bar{i}}\}$

- **Symmetry** : Spatial diffeo, Kaluza-Klein, Gauge

$A_{\bar{i}}$: not Kaluza-Klein inv. ➡ $\bar{A}_{\bar{i}} \equiv A_{\bar{i}} - \mu a_{\bar{i}}$

- **Power counting scheme** : $\lambda = \mathcal{O}(p^0)$

$f_{\bar{i}\bar{j}} \equiv \partial_{\bar{i}} a_{\bar{j}} - \partial_{\bar{j}} a_{\bar{i}} = \mathcal{O}(p^1)$ ➡ $f f = \mathcal{O}(p^2)$

ψ⁽⁰⁾ : Order $\mathcal{O}(p^0)$

Masseiu-Planck functional

$$\Psi[\lambda] = \log \int \mathcal{D}\phi e^{S[\phi,\tilde{g}]} = \underbrace{\Psi^{(0)}[\lambda]}_{\mathcal{O}(p^0)} + \underbrace{\Psi^{(1)}[\lambda,\partial]}_{\mathcal{O}(p^1)} + \mathcal{O}(\partial^2)$$

- Building blocks : $\lambda = \{e^{\sigma},\ \cancel{a_{\bar{i}}},\ \mu,\ \cancel{\bar{A}_{\bar{i}}}\}$

$$\Psi^{(0)}[\lambda] = \int_0^{\beta_0} d\tau \int d^3\bar{x}\sqrt{\gamma'}e^{\sigma}p(\beta,\mu)$$

Perfect fluid

$$\langle \hat{T}^{\mu\nu}(x)\rangle_{\bar{t}}^{\mathrm{LG}} = (e+p)u^{\mu}u^{\nu} + p\eta^{\mu\nu}$$

$$\langle \hat{J}^{\mu}(x)\rangle_{\bar{t}}^{\mathrm{LG}} = nu^{\mu}$$

Derivation of Navier-Stokes eq.

Given

Conservation laws

$$\nabla_\mu \langle \hat{T}^{\mu\nu}(x) \rangle = 0, \quad \nabla_\mu \langle \hat{J}^\mu(x) \rangle = 0$$

✓ Constitutive relations (1st order)

$$\langle \hat{T}^{\mu\nu}(x) \rangle = (e+p) u^\mu u^\nu + p g^{\mu\nu} - \frac{\zeta}{\beta} h^{\mu\nu} h^{\rho\sigma} \nabla_\rho \beta_\sigma - 2\frac{\eta}{\beta} \nabla^{\langle \mu} \beta^{\nu \rangle}$$

$$\langle \hat{J}^\mu(x) \rangle = n u^\mu - \frac{\kappa}{\beta} h^{\mu\nu} \nabla_\nu \nu$$

✓ Physical properties

Static properties: $\Psi[\lambda] = \log \int \mathcal{D}\varphi_i e^{S_E[\varphi_i;\tilde{g}]} = \int d^3\bar{x} \sqrt{\gamma'} \beta p(\beta, \mu)$

Dynamic properties: $\zeta = \beta(x) \int_{-\infty}^{\bar{t}} d^4x' \int_0^1 d\tau \langle e^{\hat{K}\tau} \tilde{\delta}\hat{p}(x') e^{-\hat{K}\tau} \tilde{\delta}\hat{p}(x) \rangle_{\bar{t}}^{\mathrm{LG}}$ etc.

Parity-odd case

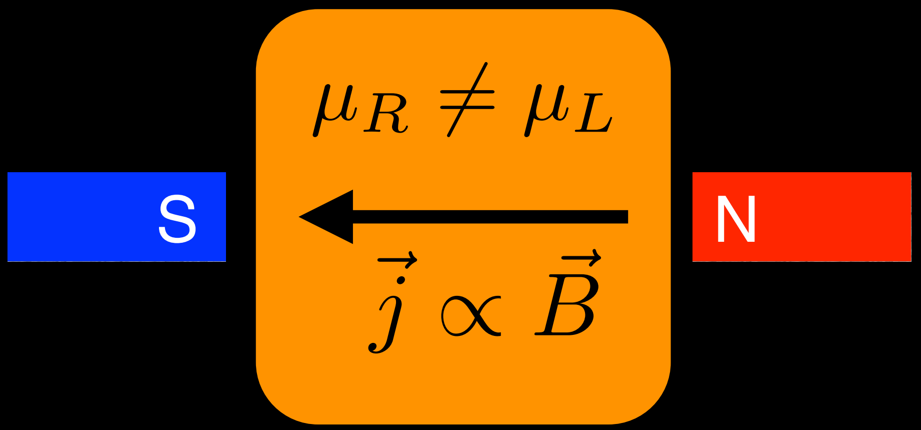
$$\mu_R \neq \mu_L$$

Anomaly-induced transport

[Fukushima et al.(2008), Vilenkin (1980)]

◆ **Chiral Magnetic Effect (CME)**

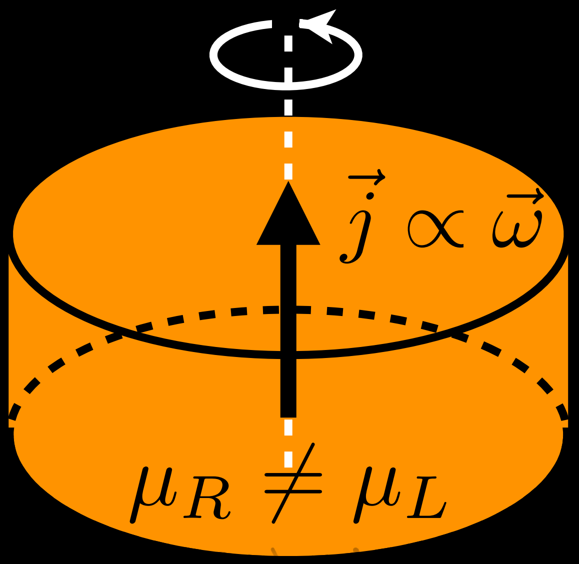
$$\vec{j} = \frac{e\mu_5}{2\pi^2}\vec{B}$$



[Erdmenger et al. (2008), Son-Surowka (2009)]

◆ **Chiral Vortical Effect (CVE)**

$$\vec{j} = \frac{\mu\mu_5}{2\pi^2}\vec{\omega}$$



Double derivative expansion

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➡ Dissipative correction (viscosity etc.)

(2) Non-dissipative derivative expansion

$$\Psi[\beta^\mu, \nu] = \Psi^{(0)}[\beta^\mu, \nu] + \Psi^{(1)}[\beta^\mu, \nu, \partial] + \mathcal{O}(\partial^2) + \cdots$$

Derivative expansion of ψ

(2) Derivative expansion of ψ

$$\Psi[\beta^\mu, \nu] = \Psi^{(0)}[\beta^\mu, \nu] + \Psi^{(1)}[\beta^\mu, \nu, \partial] + \mathcal{O}(\partial^2) + \cdots$$

$\Psi^{(0)}[\beta^\mu, \nu] \simeq \beta p$ — Symmetry property

$\Psi^{(1)}[\beta^\mu, \nu, \partial] = 0$ **Parity-even system**

$\Psi^{(1)}[\beta^\mu, \nu, \partial] \neq 0$ **Parity-odd system**

Non-dissipative constitutive relation

$$\langle \hat{T}^{\mu\nu}(x) \rangle_{\bar{t}}^{\mathrm{LG}} = \frac{2}{\sqrt{-g}} \frac{\delta}{\delta g_{\mu\nu}(x)} \Psi[\bar{t}; \lambda] = T^{\mu\nu}_{(0)}[\lambda(x)] + T^{\mu\nu}_{(1)}[\lambda(x), \nabla\lambda(x)] + \cdots$$

$$\langle \hat{J}^{\mu}(x) \rangle_{\bar{t}}^{\mathrm{LG}} = \frac{2}{\sqrt{-g}} \frac{\delta}{\delta A_{\mu}(x)} \Psi[\bar{t}; \lambda] = J^{\mu}_{(0)}[\lambda(x)] + J^{\mu}_{(1)}[\lambda(x), \nabla\lambda(x)] + \cdots$$

$T^{\mu\nu}_{(1)}, J^{\mu}_{(1)}$: $= 0$ (parity-even) $\neq 0$ (parity-odd)

Recipe for Massieu-Planck fcn.

Weyl fermion : $\mathcal{L} = \frac{i}{2}\xi^\dagger \left(e_m^{\ \mu} \sigma^m \overrightarrow{D}_\mu - \overleftarrow{D}_\mu \sigma^m e_m^{\ \mu} \right) \xi$

$$\Psi[\lambda] = \log \int \mathcal{D}\xi^\dagger \mathcal{D}\xi e^{S[\xi,\xi^\dagger,A,\tilde{e}]} = \underbrace{\Psi^{(0)}[\lambda]}_{\mathcal{O}(p^0)} + \underbrace{\Psi^{(1)}[\lambda,\partial]}_{\mathcal{O}(p^1)} + \mathcal{O}(\partial^2)$$

- **Building blocks** : $\lambda = \{e^\sigma,\ a_{\bar{i}},\ \mu_R,\ \bar{A}_{\bar{i}}\}$

- **Symmetry** : Spatial diffeo, Kaluza-Klein, Gauge

  $A_{\bar{i}}$: not Kaluza-Klein inv. ➡ $\bar{A}_{\bar{i}} \equiv A_{\bar{i}} - \mu_R a_{\bar{i}}$

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  $f_{\bar{i}\bar{j}} \equiv \partial_{\bar{i}} a_{\bar{j}} - \partial_{\bar{j}} a_{\bar{i}} = \mathcal{O}(p^1)$ ➡ $ff = \mathcal{O}(p^2)$

ψ⁽⁰⁾ : Order $\mathcal{O}(p^0)$

Weyl fermion : $\mathcal{L} = \frac{i}{2}\xi^\dagger \left(e_m^{\ \mu}\sigma^m \overrightarrow{D}_\mu - \overleftarrow{D}_\mu \sigma^m e_m^{\ \mu} \right) \xi$

$$\Psi[\lambda] = \log \int \mathcal{D}\xi^\dagger \mathcal{D}\xi e^{S[\xi,\xi^\dagger,A,\tilde{e}]} = \underbrace{\Psi^{(0)}[\lambda]}_{\mathcal{O}(p^0)} + \underbrace{\Psi^{(1)}[\lambda,\partial]}_{\mathcal{O}(p^1)} + \mathcal{O}(\partial^2)$$

- Building blocks : $\lambda = \{e^\sigma, \cancel{a_{\bar{i}}}, \mu_R, \cancel{\bar{A}_{\bar{i}}}\}$

$$\Psi^{(0)}[\lambda] = \int_0^{\beta_0} d\tau \int d^3\bar{x}\sqrt{\gamma'}e^\sigma p(\beta,\mu_R)$$

→ **Perfect fluid**

$$\langle \hat{T}^{\mu\nu}(x)\rangle_{\bar{t}}^{\mathrm{LG}} = (e+p)u^\mu u^\nu + p\eta^{\mu\nu}$$

$$\langle \hat{J}^{\mu}_R(x)\rangle_{\bar{t}}^{\mathrm{LG}} = n_R u^\mu$$

ψ^(I) : Order $\mathcal{O}(p)$

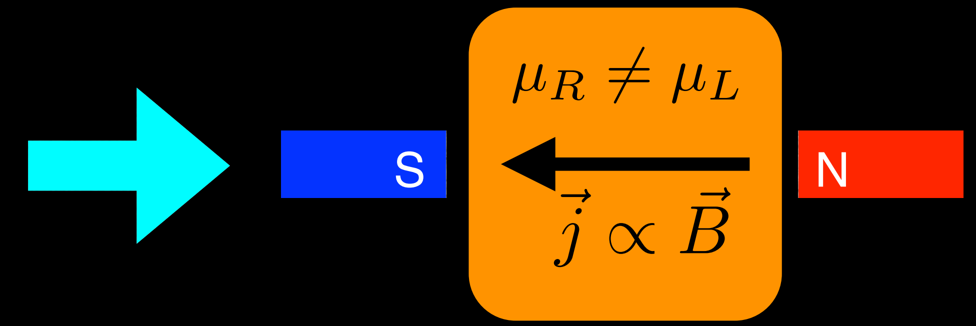
Weyl fermion : $\mathcal{L} = \frac{i}{2}\xi^\dagger\left(e_m^{\ \mu}\sigma^m\overrightarrow{D}_\mu - \overleftarrow{D}_\mu\sigma^m e_m^{\ \mu}\right)\xi$

$$\Psi[\lambda] = \log\int\mathcal{D}\xi^\dagger\mathcal{D}\xi e^{S[\xi,\xi^\dagger,A,\tilde{e}]} = \Psi^{(0)}[\lambda] + \Psi^{(1)}[\lambda,\partial] + \mathcal{O}(\partial^2)$$

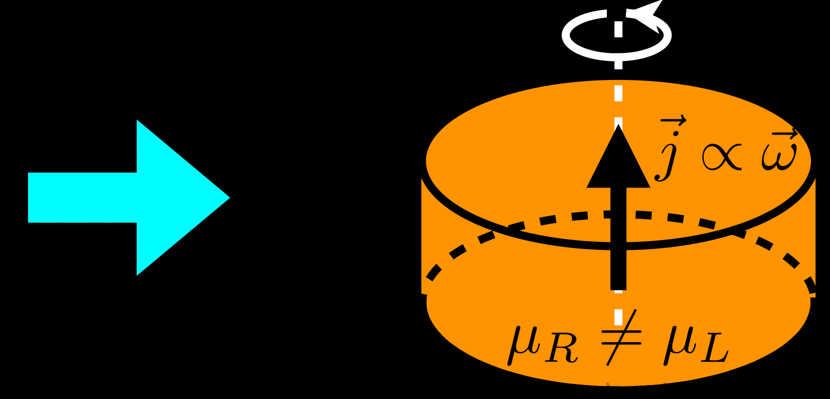
$\mathcal{O}(p^0)$ $\mathcal{O}(p^1)$

- Building blocks : $\lambda = \{e^\sigma,\ a_{\bar{i}},\ \mu_R,\ \bar{A}_{\bar{i}}\}$

$$\int d^3\bar{x}\sqrt{\gamma'}C_1(\beta,\mu_R)\epsilon^{\bar{i}\bar{j}\bar{k}}\bar{A}_{\bar{i}}\partial_{\bar{j}}\bar{A}_{\bar{k}}$$



$$\int d^3\bar{x}\sqrt{\gamma'}C_2(\beta,\mu_R)\epsilon^{\bar{i}\bar{j}\bar{k}}\bar{A}_{\bar{i}}\partial_{\bar{j}}a_{\bar{k}}$$



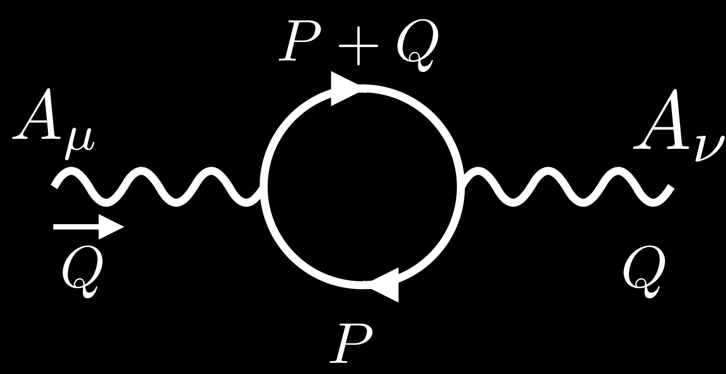
Anomalous transport coefficients

① **Non-perturbative way (WZ consistency condition ...)**

[Banerjee et al.(2012), Jensen et al.(2012) , Haehl et al. (2015)]

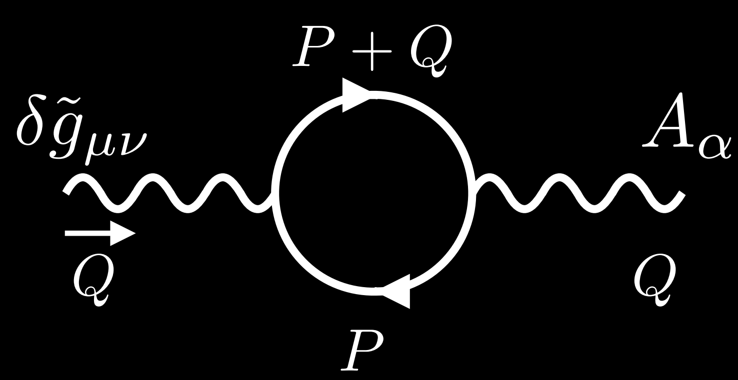
② Perturbative evaluation of ψ in external field

$$\frac{\delta^2\Psi}{\delta A_\mu \delta A_\nu} =$$



$$\simeq -i\varepsilon^{0\mu\rho\nu}\tilde{Q}_\rho \frac{\mu_R}{4\pi^2}$$

$$\frac{\delta^2\Psi}{\delta \tilde{g}_{\mu\nu} \delta A_\alpha} =$$



$$\simeq i\tilde{Q}_\rho \underbrace{C}_{=\frac{\mu_R^2}{8\pi^2}+\frac{T^2}{24}}(\eta^{\nu 0}\varepsilon^{\rho\mu 0\alpha} + \delta_{ij}\eta^{\nu i}\epsilon^{\rho\mu j\alpha})$$

$$\Rightarrow \Psi^{(1)}[\lambda] = \int d^3x \varepsilon^{0ijk}\left[\frac{\nu_R}{8\pi^2}A_i\partial_j A_k + \left(\frac{\nu_R\mu_R}{8\pi^2}+\frac{T}{24}\right)A_i\partial_j\tilde{g}_{0k}\right]$$

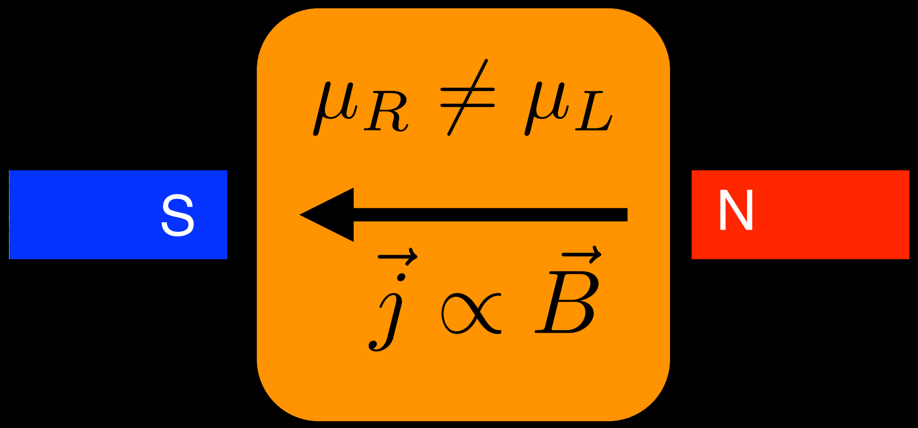
Derivation of CME/CVE

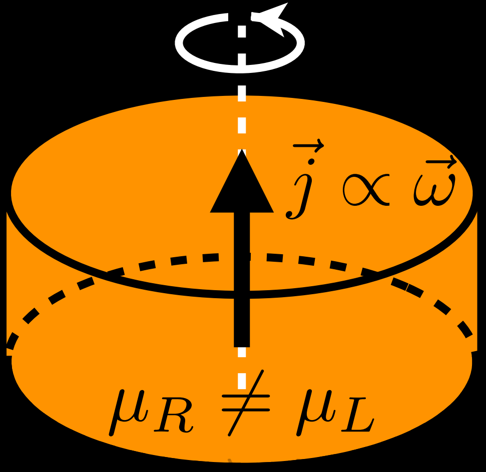
$$\Psi^{(1)}[\lambda] = \int d^3x \varepsilon^{0ijk} \left[\frac{\nu_R}{8\pi^2} A_i \partial_j A_k + \left(\frac{\nu_R \mu_R}{8\pi^2} + \frac{T}{24} \right) A_i \partial_j \tilde{g}_{0k} \right]$$

$$\Rightarrow \langle \hat{J}^i_R(x) \rangle^{\mathrm{LG}}_{(0,1)} = \frac{1}{\sqrt{-g}} \frac{\delta \Psi^{(1)}}{\delta A_i(x)} = \frac{\mu_R}{4\pi^2} B^i + \left(\frac{\mu_R^2}{8\pi^2} + \frac{T^2}{24} \right) \omega^i$$

$$\langle \hat{J}^i_V(x) \rangle^{\mathrm{LG}}_{(0,1)} = \frac{\mu_5}{2\pi^2} B^i + \frac{\mu \mu_5}{2\pi^2} \omega^i$$

$$\langle \hat{J}^i_A(x) \rangle^{\mathrm{LG}}_{(0,1)} = \frac{\mu}{2\pi^2} B^i + \left(\frac{\mu^2 + \mu_5^2}{4\pi^2} + \frac{T^2}{12} \right) \omega^i$$

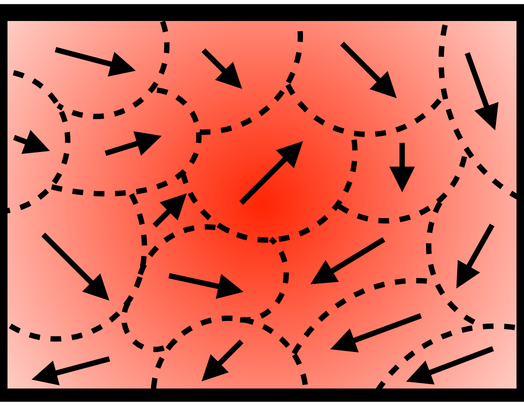


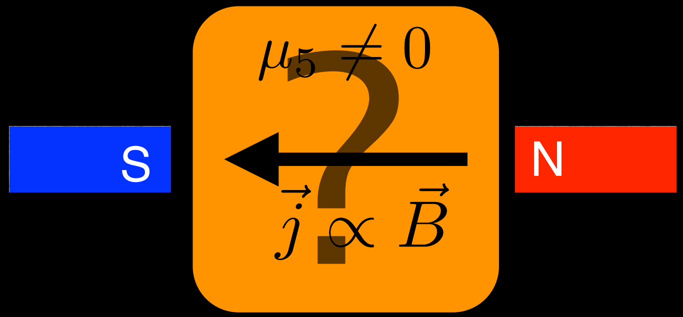


Summary

MOTIVATION:

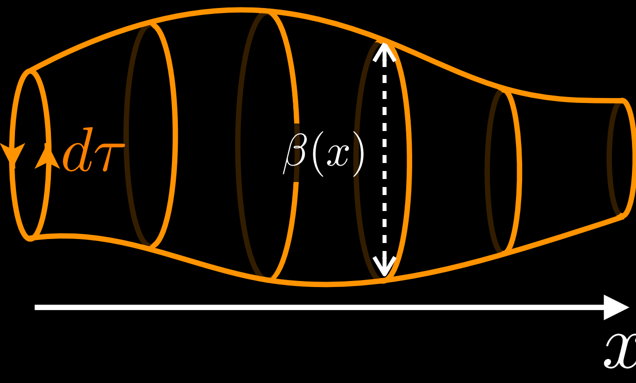
Relativistic hydrodynamics
from **quantum field theory?**





APPROACH:

QFT for **initial local Gibbs distribution**



① Renormalized/optimized perturbation for dissipative part

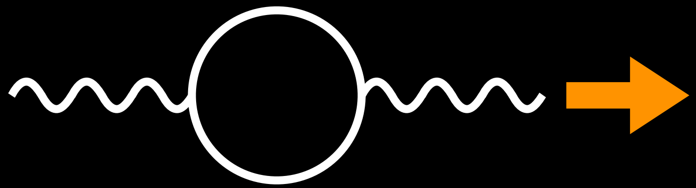
② Path-integral formula for non-dissipative part

$$\Psi[\lambda] \leftarrow \text{QFT in } d\tilde{s}^2 = -e^{2\sigma}(d\tilde{t} + a_{\bar{i}}dx^{\bar{i}})^2 + \gamma'_{\bar{i}\bar{j}}dx^{\bar{i}}dx^{\bar{j}}$$

Symmetry = Spatial diffeomorphism + Kaluza-Klein gauge

RESULTS:

Derivation of Navier-Stokes eq.
& **anomaly-induced transports**



$$\Psi^{(1)} \rightarrow \vec{j} = \frac{e\mu_5}{2\pi^2}\vec{B}$$

Outlook

DISSIPATION AND FLUCTUATION:

How to implement dissipation and fluctuation based on QFT?

- Zubarev et al. (1979)
- Becattini et al. (2015)
- Hayata, Hidaka, **MH**, Noumi (2015)

- Haehl, Loganayagam, Rangamani (2015-)
- Harder, Kovtun, Ritz (2015)
- Crossley, Giorioso, Liu (2015-)
- Jensen et al. (2017-)

NON-DISSIPATIVE TRANSPORT:

Evaluation of Masseiu-Planck fcn. in several situations

s.t. in the presence of magnetic field/vorticity ...

- Hattori, Yin(2016) **- Becattinil et al. (2015)**

SUPERFLUID / MAGNETO-HYDRODYNAMICS:

Extension to cases with other zero modes

s.t. Nambu-Goldstone-mode, Photon

- Grozdanov et al. (2017)