Chiral effective field theory for dark matter direct detection

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Multi-Scale Problems Using Effective Field Theories

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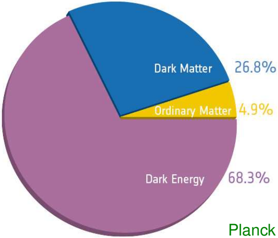
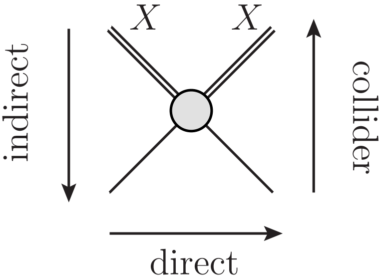
PLB 746 (2015) 410, PRD 94 (2016) 063505, PRL 119 (2017) 181803 with P. Klos, J. Menéndez, A. Schwenk

1802.04294, with A. Fieguth, P. Klos, J. Menéndez, A. Schwenk, C. Weinheimer

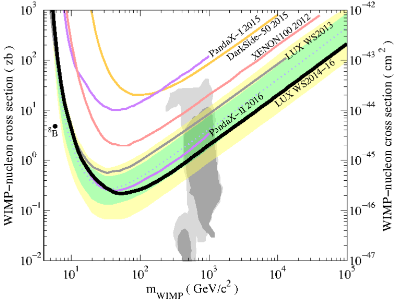
- Search strategies: direct, indirect, collider
- Assume DM particle is WIMP
- **Direct detection**: search for **WIMPs** scattering off nuclei in the large-scale detectors
- Ingredients for interpretation:
  - **DM halo**: velocity distribution
  - **Nucleon matrix elements**: WIMP–nucleon couplings
  - **Nuclear structure factors**: embedding into target nucleus

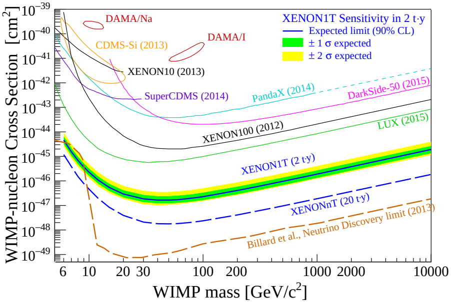


Planck 2013



Direct detection of dark matter: schematics

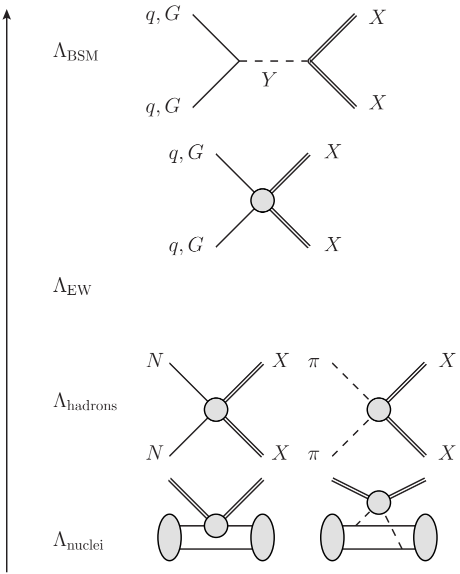




- **Nuclear recoil** in WIMP–nucleus scattering
  - **Flux factor** Φ: DM halo and velocity distribution
  - **WIMP–nucleus cross section**
- **Spin-independent**: coherent $\propto A^2$
- **Spin-dependent**: $\propto \langle \mathbf{S}_p \rangle$ or $\langle \mathbf{S}_n \rangle$
- Information on BSM physics encoded in normalization at $q = 0$

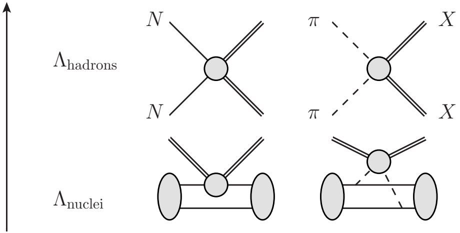
$\hookrightarrow$ for SI case: $\sigma_{\chi N}^{\mathrm{SI}}$

Direct detection of dark matter: scales



1. **BSM scale** Λ_{BSM}: $\mathcal{L}_{\text{BSM}}$
2. **Effective Operators**: $\mathcal{L}_{\text{SM}} + \sum_{i,k} \frac{1}{\Lambda^i_{\text{BSM}}} \mathcal{O}_{i,k}$
3. Integrate out **EW physics**
4. **Hadronic scale**: nucleons and pions
   $\hookrightarrow$ effective interaction Hamiltonian H_I
5. **Nuclear scale**: $\langle \mathcal{N} | H_I | \mathcal{N} \rangle$
   $\hookrightarrow$ nuclear wave function

Direct detection of dark matter: scales



4 **Hadronic scale**: nucleons and pions

$\hookrightarrow$ effective interaction Hamiltonian H_I

5 **Nuclear scale**: $\langle\mathcal{N}|H_I|\mathcal{N}\rangle$

$\hookrightarrow$ nuclear wave function

- Typical WIMP–nucleon **momentum transfer**

$$|\mathbf{q}_{\text{max}}| = 2\mu_{\mathcal{N}\chi}|\mathbf{v}_{\text{rel}}| \sim 200\,\text{MeV} \qquad |\mathbf{v}_{\text{rel}}| \sim 10^{-3} \qquad \mu_{\mathcal{N}\chi} \sim 100\,\text{GeV}$$

- QCD constraints: spontaneous breaking of chiral symmetry

  $\Rightarrow$ **Chiral effective field theory for WIMP–nucleon scattering**

  Prézeau et al. 2003, Cirigliano et al. 2012, 2013, Menéndez et al. 2012, Klos et al. 2013, MH et al. 2015, Bishara et al. 2017

- In NREFT Fan et al. 2010, Fitzpatrick et al. 2012, Anand et al. 2013 need to **match to QCD** to extract information on BSM physics $\Rightarrow$ "the" EFT approach not unique!

Outline

Chiral Perturbation Theory

- Effective theory of QCD based on **chiral symmetry**

$$\mathcal{L}_{\text{QCD}} = \bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R - \bar{q}_L \mathcal{M} q_R - \bar{q}_R \mathcal{M} q_L - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

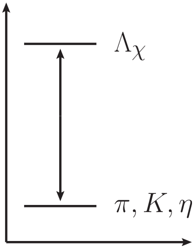
- Expansion in momenta p/Λ_χ and quark masses $m_q \sim p^2$

  $\hookrightarrow$ **scale separation**

- Two variants
  - ***SU*(2)**: u- and d-quark **dynamical**, m_s fixed at **physical value**

    $\hookrightarrow$ expansion in M_π/Λ_χ, usually nice convergence
  - ***SU*(3)**: u-, d-, and s-quark dynamical

    $\hookrightarrow$ expansion in M_K/Λ_χ, sometimes tricky



- Traditionally: meson-exchange potentials
- Chiral effective field theory
  - Based on **chiral symmetry** of QCD
  - **Power counting**
  - **Low-energy constants**
  - Hierarchy of multi-nucleon forces
  - Consistency of *NN* and 3*N*

$\hookrightarrow$ modern theory of nuclear forces

- Long-range part related to **pion–nucleon scattering**

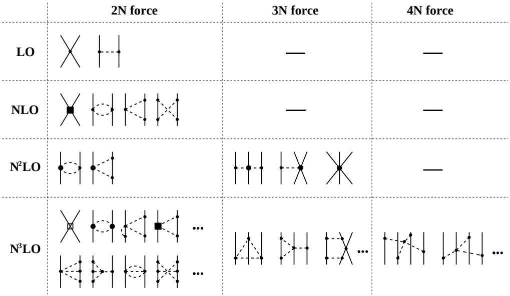
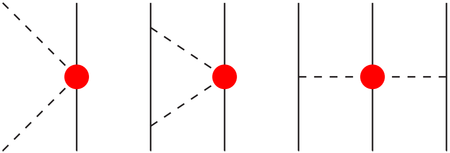
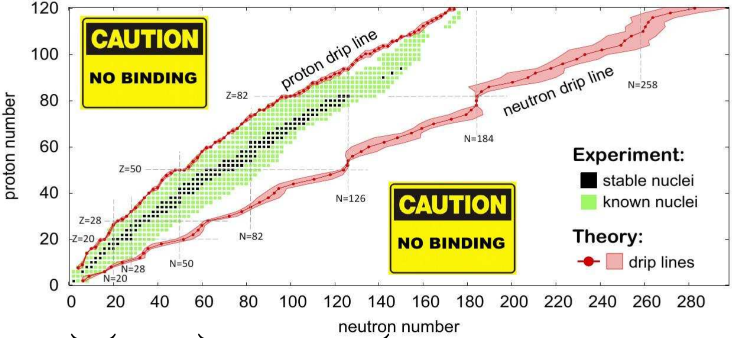


Figure taken from 1011.1343





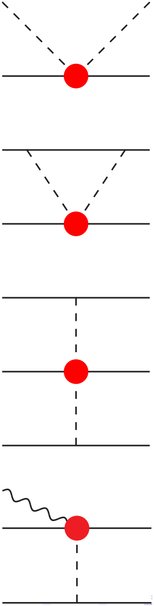
long-range plan 2015

QCD shell model, coupled cluster, in-medium SRG, ...

exact many-body: lattice EFT, NCSM, coupled-cluster, . . .

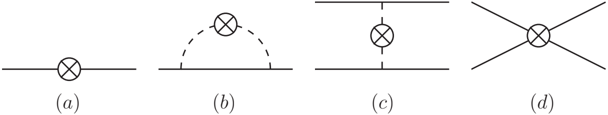
density functional, mean-field theory, ...

- Piecewise overlap of **ab-initio** and various **many-body** methods $\Rightarrow$ match to QCD
- Consistent *NN* interactions key at various stages
- Ab-initio not yet up to xenon, but impressive progress

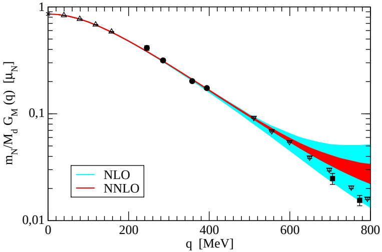


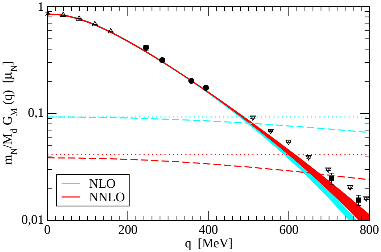
- Coupling to **external sources** $\mathcal{L}(v_\mu, a_\mu, s, p)$
- Same LECs appear in **axial current**

  $\hookrightarrow$ β decay, neutrino interactions, dark matter
- Vast literature for v_μ and a_μ, up to one-loop level
  - With unitary transformations: Kölling et al. 2009, 2011, Krebs et al. 2016
  - Without unitary transformations: Pastore et al. 2008, Park et al. 2003, Baroni et al. 2015
- For **dark matter** further currents: s, p, tensor, spin-2, θ^μ_μ

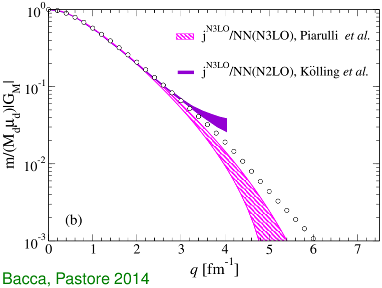


Vector current in chiral EFT: deuteron form factors, magnetic moments

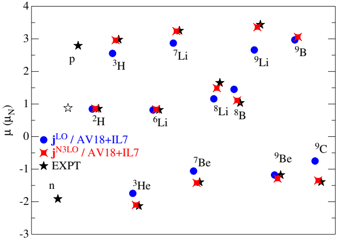




Kölling, Epelbaum, Phillips 2012

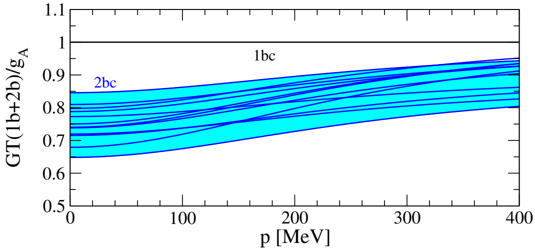


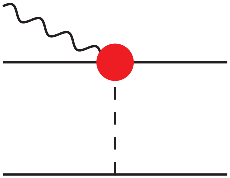
Bacca, Pastore 2014



Pastore et al. 2013

Axial-vector current in chiral EFT: ν-less double β decay

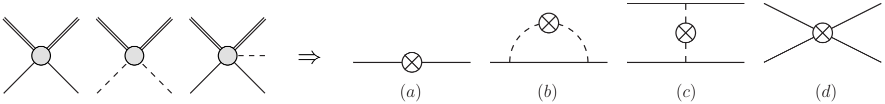




Menéndez, Gazit, Schwenk 2011

- Normal ordering over Fermi sea $\Rightarrow$ effective one-body currents
- **Two-body currents** contribute to **quenching of g_A** in Gamov–Teller operator

  $g_A \boldsymbol{\sigma} \tau^-$



- Expansion around **chiral limit** of QCD

  $\hookrightarrow$ simultaneous expansion in momenta and quark masses
- Three classes of corrections:
  - **Subleading one-body responses** (a)
  - **Radius corrections** (b)
  - **Two-body currents** (c), (d)
- NREFT covers (a), but misses (b)–(d)
  - (b): modifies coefficient of $\mathcal{O}_i$ by momentum-dependent factor
  - (c), (d): do not match directly onto NREFT, need **normal ordering**

$$\langle N^\dagger N\rangle N^\dagger N \to \mathcal{O}_i^{\text{eff}}$$

- (a)+(b) just **ChPT for nucleon form factors**, but (c)+(d) genuinely new effects

Chiral counting

- Starting point: **effective WIMP Lagrangian** Goodman et al. 2010

$$\mathcal{L}_\chi = \frac{1}{\Lambda^3}\sum_q \Big[C_q^{SS}\bar\chi\chi\, m_q\bar q q + C_q^{PS}\bar\chi i\gamma_5\chi\, m_q\bar q q + C_q^{SP}\bar\chi\chi\, m_q\bar q i\gamma_5 q + C_q^{PP}\bar\chi i\gamma_5\chi\, m_q\bar q i\gamma_5 q\Big]$$
$$+\frac{1}{\Lambda^2}\sum_q \Big[C_q^{VV}\bar\chi\gamma^\mu\chi\,\bar q\gamma_\mu q + C_q^{AV}\bar\chi\gamma^\mu\gamma_5\chi\,\bar q\gamma_\mu q + C_q^{VA}\bar\chi\gamma^\mu\chi\,\bar q\gamma_\mu\gamma_5 q + C_q^{AA}\bar\chi\gamma^\mu\gamma_5\chi\,\bar q\gamma_\mu\gamma_5 q\Big]$$
$$+\frac{1}{\Lambda^3}\Big[C_g^S\bar\chi\chi\,\alpha_s G^a_{\mu\nu}G_a^{\mu\nu}\Big]$$

- **Chiral power counting**

$$\partial = \mathcal{O}(p), \qquad m_q = \mathcal{O}(p^2) = \mathcal{O}(M_\pi^2), \qquad a_\mu, v_\mu = \mathcal{O}(p), \qquad \frac{\partial}{m_N} = \mathcal{O}(p^2)$$

$\hookrightarrow$ construction of effective Lagrangian for nucleon and pion fields

$\hookrightarrow$ organize in terms of **chiral order ν**, $\mathcal{M} = \mathcal{O}(p^\nu)$

Chiral counting: summary

Nucleon		V		A	
WIMP		t	$\mathbf{x}$	t	$\mathbf{x}$
V	1b	0	$1+2$	2	$0+2$
	2b	4	$2+2$	2	$4+2$
	2b NLO	—	—	5	$3+2$
A	1b	$0+2$	1	$2+2$	0
	2b	$4+2$	2	$2+2$	4
	2b NLO	—	—	$5+2$	3

Nucleon		S	P
WIMP			
S	1b	2	1
	2b	3	5
	2b NLO	—	4
P	1b	$2+2$	$1+2$
	2b	$3+2$	$5+2$
	2b NLO	—	$4+2$

- $+2$ from NR expansion of WIMP spinors, terms can be dropped if $m_\chi \gg m_N$
- Red: all terms up to $\nu = 3$
- Two-body currents: *AA* Menéndez et al. 2012, Klos et al. 2013, *SS* Prézeau et al. 2003, Cirigliano et al. 2012, but **new currents in *AV* and *VA* channel** 1503.04811
- Worked out the matching to NREFT and BSM Wilson coefficients for spin-$1/2$
  $\hookrightarrow$ **hierarchy** predicted from chiral expansion

Matching to nonrelativistic EFT

- Operator basis in NREFT Fan et al. 2010, Fitzpatrick et al. 2012, Anand et al. 2013

$$\mathcal{O}_1 = \mathbb{1} \qquad \mathcal{O}_2 = (\mathbf{v}^\perp)^2 \qquad \mathcal{O}_3 = i\mathbf{S}_N \cdot (\mathbf{q} \times \mathbf{v}^\perp) \qquad \mathcal{O}_4 = \mathbf{S}_\chi \cdot \mathbf{S}_N$$

$$\mathcal{O}_5 = i\mathbf{S}_\chi \cdot (\mathbf{q} \times \mathbf{v}^\perp) \qquad \mathcal{O}_6 = \mathbf{S}_\chi \cdot \mathbf{q}\, \mathbf{S}_N \cdot \mathbf{q} \qquad \mathcal{O}_7 = \mathbf{S}_N \cdot \mathbf{v}^\perp \qquad \mathcal{O}_8 = \mathbf{S}_\chi \cdot \mathbf{v}^\perp$$

$$\mathcal{O}_9 = i\mathbf{S}_\chi \cdot (\mathbf{S}_N \times \mathbf{q}) \qquad \mathcal{O}_{10} = i\mathbf{S}_N \cdot \mathbf{q} \qquad \mathcal{O}_{11} = i\mathbf{S}_\chi \cdot \mathbf{q}$$

- Matching to chiral EFT ($f_N, \ldots$: Wilson coefficients + nucleon form factors)

$$\mathcal{M}^{SS}_{1,\mathrm{NR}} = \mathcal{O}_1 f_N(t) \qquad \mathcal{M}^{SP}_{1,\mathrm{NR}} = \mathcal{O}_{10} g_5^N(t) \qquad \mathcal{M}^{PP}_{1,\mathrm{NR}} = \frac{1}{m_\chi}\mathcal{O}_6 h_5^N(t)$$

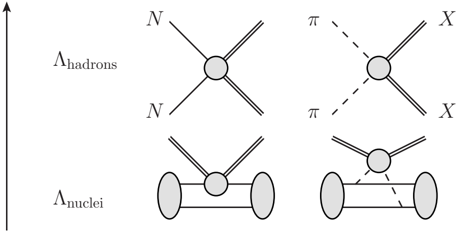
$$\mathcal{M}^{VV}_{1,\mathrm{NR}} = \mathcal{O}_1\Big(f_1^{V,N}(t) + \frac{t}{4m_N^2} f_2^{V,N}(t)\Big) + \frac{1}{m_N}\mathcal{O}_3 f_2^{V,N}(t) + \frac{1}{m_N m_\chi}\Big(t\mathcal{O}_4 + \mathcal{O}_6\Big) f_2^{V,N}(t)$$

$$\mathcal{M}^{AV}_{1,\mathrm{NR}} = 2\mathcal{O}_8 f_1^{V,N}(t) + \frac{2}{m_N}\mathcal{O}_9\Big(f_1^{V,N}(t) + f_2^{V,N}(t)\Big)$$

$$\mathcal{M}^{AA}_{1,\mathrm{NR}} = -4\mathcal{O}_4 g_A^N(t) + \frac{1}{m_N^2}\mathcal{O}_6 g_P^N(t) \qquad \mathcal{M}^{VA}_{1,\mathrm{NR}} = \Big\{ -2\mathcal{O}_7 + \frac{2}{m_\chi}\mathcal{O}_9 \Big\} h_A^N(t)$$

- Conclusions
  - $\mathcal{O}_2$, $\mathcal{O}_5$, and $\mathcal{O}_{11}$ do not appear at $\nu = 3$, not all $\mathcal{O}_i$ independent
  - 2b operators of similar or even greater importance than some of the 1b operators

Direct detection of dark matter: scales



4. **Hadronic scale**: nucleons and pions

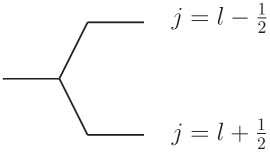
   $\hookrightarrow$ effective interaction Hamiltonian H_I

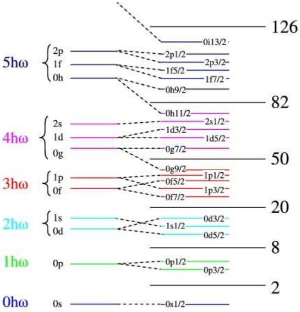
5. **Nuclear scale**: $\langle \mathcal{N} | H_I | \mathcal{N} \rangle$

   $\hookrightarrow$ nuclear wave function

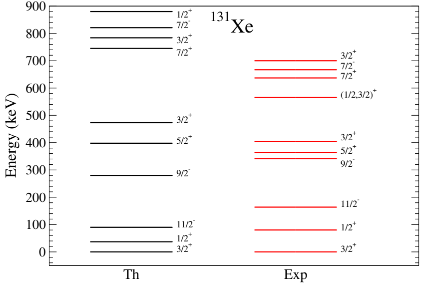
Coherence effects

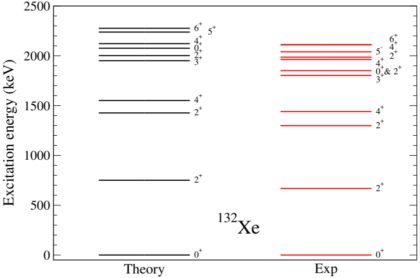
- Six distinct nuclear responses
  Fitzpatrick et al. 2012, Anand et al. 2013
  - $M \leftrightarrow \mathcal{O}_1 \leftrightarrow$ SI
  - $\Sigma', \Sigma'' \leftrightarrow \mathcal{O}_4, \mathcal{O}_6 \leftrightarrow$ SD
  - $\Phi'' \leftrightarrow \mathcal{O}_3 \leftrightarrow$ quasi-coherent, spin-orbit operator
  - $\Delta, \tilde{\Phi}'$: not coherent
- **Quasi-coherence** of Φ''
  - Spin-orbit splitting
  - Coherence until mid-shell
  - About 20 coherent nucleons in Xe
  - Interference M–$\Phi'' \leftrightarrow \mathcal{O}_1$–$\mathcal{O}_3$
- Further coherent M-responses from $\mathcal{O}_5$, $\mathcal{O}_8$, $\mathcal{O}_{11}$, but no interference with $\mathcal{O}_1$ due to sum over $\mathbf{S}_\chi$





Spectra and shell-model calculation



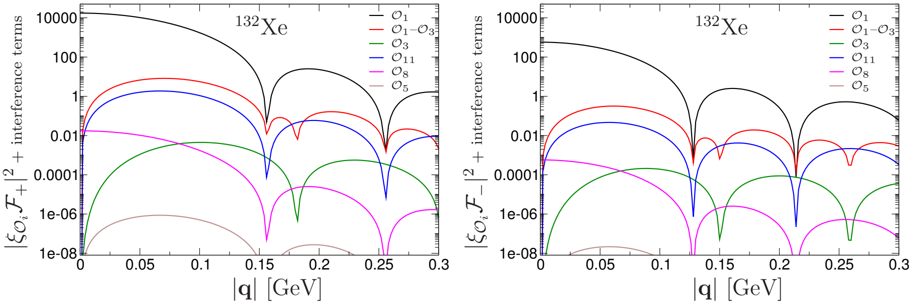


- **Shell-model diagonalization** for Xe isotopes with ^{100}Sn core
- **Uncertainty estimates**: currently phenomenological shell-model interaction

  $\hookrightarrow$ chiral-EFT-based interactions in the future?

  $\hookrightarrow$ **ab-initio calculations for light nuclei?**

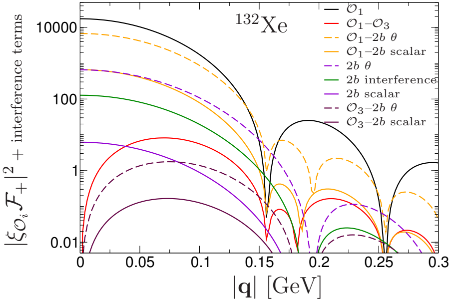
Consequences for the structure factors

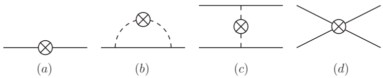


- $\xi_{\mathcal{O}_i}$ kinematic factors for $\mathcal{O}_i$, e.g. $\xi_{\mathcal{O}_1} = 1$, $\xi_{\mathcal{O}_3} = \frac{\mathbf{q}^2}{2m_N^2}$
- $\mathcal{O}_{11}$ assumes $m_\chi = 2\,\text{GeV}$

  $\hookrightarrow$ much stronger suppressed for heavy WIMPs
- Structure factors imply **hierarchy** as long as coefficients do not differ strongly

Two-body currents



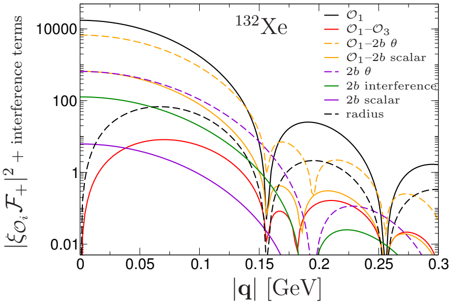


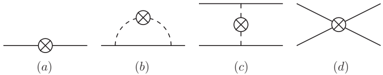
- Finite at $|\mathbf{q}| = 0$
- Most important next to IS and IV $\mathcal{O}_1$
- Sensitive to **new combination of Wilson coefficients**, e.g. for scalar channel

$$f_N = \frac{m_N}{\Lambda^3}\bigg(\sum_{q=u,d,s} C_q^{SS} f_q^N - 12\pi f_Q^N C_g^{\prime S}\bigg) \qquad f_\pi = \frac{M_\pi}{\Lambda^3}\sum_{q=u,d}\bigg(C_q^{SS} + \frac{8\pi}{9}C_g^{\prime S}\bigg) f_q^\pi \qquad f_\pi^\theta = -\frac{M_\pi}{\Lambda^3}\frac{8\pi}{9}C_g^{\prime S}$$

- Typically (5–10)% effect, enhanced whenever cancellations occur: **blind spots**, **heavy WIMP limit**

Radius corrections

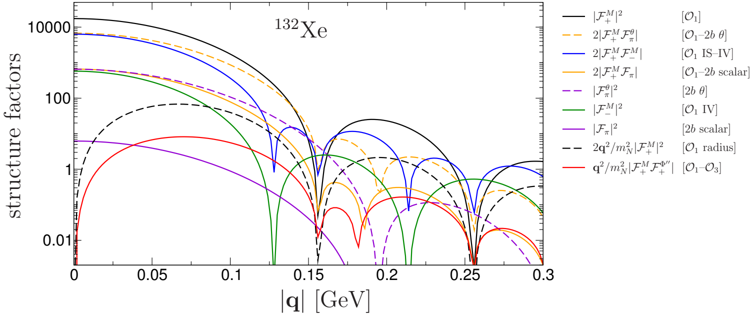




- Set scale as $\mathbf{q}^2/m_N^2$
- Strong suppression at small $|\mathbf{q}|$, but potentially relevant later
- Yet another new combination

$$\dot{f}_N = \frac{m_N}{\Lambda^3}\left(\sum_{q=u,d,s} C_q^{SS}\dot{f}_q^N - 12\pi \dot{f}_Q^N C_g^{\prime S}\right)$$

Full set of coherent contributions

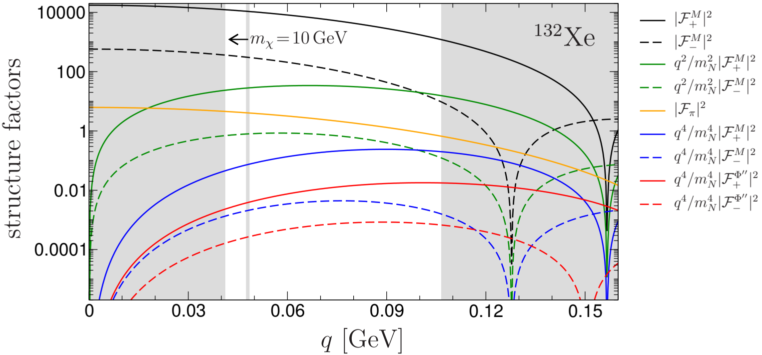


- Parameterize cross section as

$$\frac{d\sigma^{\mathrm{SI}}_{\chi\mathcal{N}}}{d\mathbf{q}^2}=\frac{1}{4\pi\mathbf{v}^2}\bigg|\bigg(c_+^M-\frac{\mathbf{q}^2}{m_N^2}\,\dot{c}_+^M\bigg)\mathcal{F}_+^M(\mathbf{q}^2)+\bigg(c_-^M-\frac{\mathbf{q}^2}{m_N^2}\,\dot{c}_-^M\bigg)\mathcal{F}_-^M(\mathbf{q}^2)$$
$$+c_\pi\mathcal{F}_\pi(\mathbf{q}^2)+c_\pi^\theta\mathcal{F}_\pi^\theta(\mathbf{q}^2)+\frac{\mathbf{q}^2}{2m_N^2}\Big[c_+^{\Phi''}\mathcal{F}_+^{\Phi''}(\mathbf{q}^2)+c_-^{\Phi''}\mathcal{F}_-^{\Phi''}(\mathbf{q}^2)\Big]\bigg|^2$$

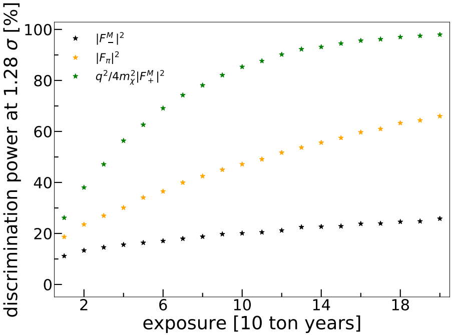
- Single-nucleon cross section: $\sigma^{\mathrm{SI}}_{\chi N}=\mu_N^2|c_+^M|^2/\pi$
- c related to Wilson coefficients and nucleon form factors

Discriminating different response functions



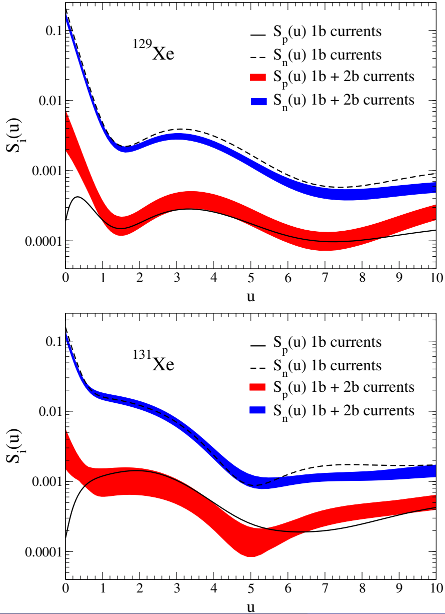
- White region accessible to XENON-type experiment
- Can one tell these curves apart in a realistic experimental setting?
- Consider XENON1T-like, XENONnT-like, DARWIN-like settings

Discriminating different response functions



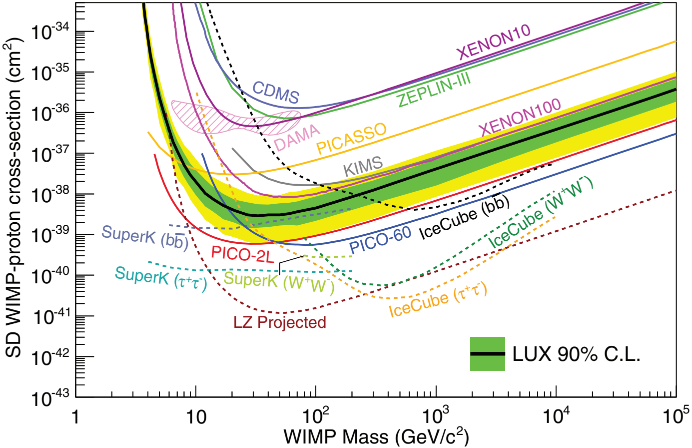
- DARWIN-like setting, $m_\chi = 100\,\text{GeV}$
- q-dependent responses more easily distinguishable
- If interaction not much weaker than current limits, DARWIN could discriminate most responses from standard SI structure factor

Two-body currents: SD case



- Nuclear structure factors for **spin-dependent interactions** Klos et al. 2013
  - Based on chiral EFT currents (1b+2b)
  - Shell model
  - $u = q^2 b^2/2$ related to momentum transfer
- 2b currents absorbed into redefinition of 1b current

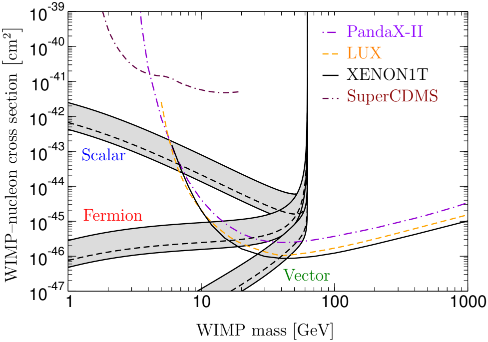
Two-body currents: SD case



Xenon becomes competitive for σ_p thanks to two-body currents!

Higgs Portal dark matter

- **Higgs Portal**: WIMP interacts with SM via the Higgs
  - **Scalar**: $H^\dagger H S^2$
  - **Vector**: $H^\dagger H V_\mu V^\mu$
  - **Fermion**: $H^\dagger H \bar{f} f$
- If $m_h > 2m_\chi$, should happen at the LHC
  $\hookrightarrow$ limits on **invisible Higgs decays**



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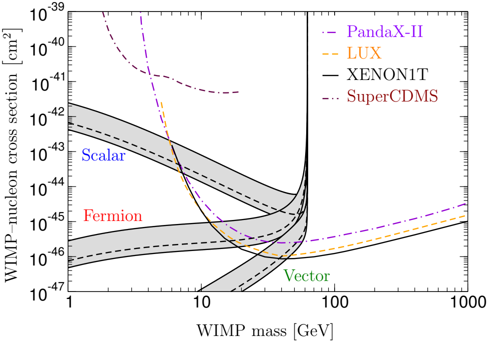
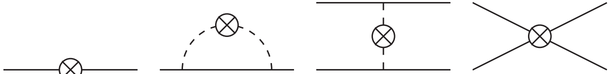


- Translation requires input for **Higgs–nucleon coupling**

$$f_N = \sum_{q=u,d,s,c,b,t} f_q^N = \frac{2}{9} + \frac{7}{9} \sum_{q=u,d,s} f_q^N + \mathcal{O}(\alpha_s) \qquad m_N f_q^N = \langle N | m_q \bar{q} q | N \rangle$$

- Issues: input for $f_N = 0.260 \ldots 0.629$ outdated, two-body currents missing

Higgs–nucleon coupling



- **One-body contribution**

$$f_N^{\text{1b}} = 0.307(9)_{ud}(15)_s(5)_{\text{pert}} = 0.307(18)$$

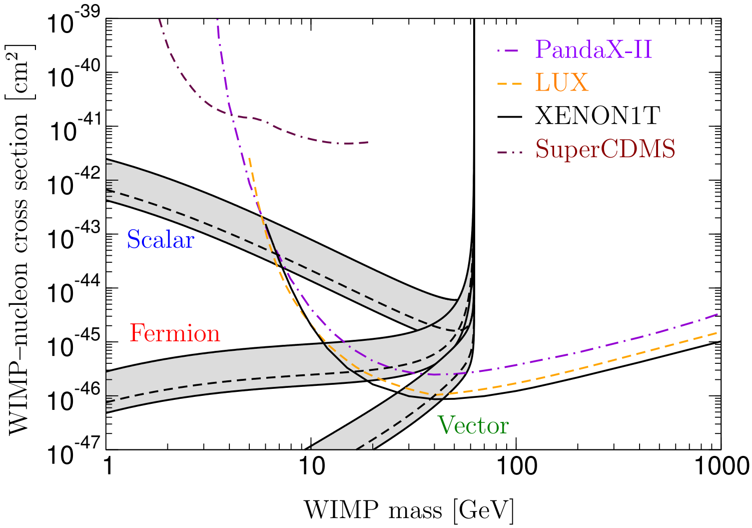
- Limits on WIMP–nucleon cross section subsume **two-body effects**

  $\hookrightarrow$ have to be included for meaningful comparison

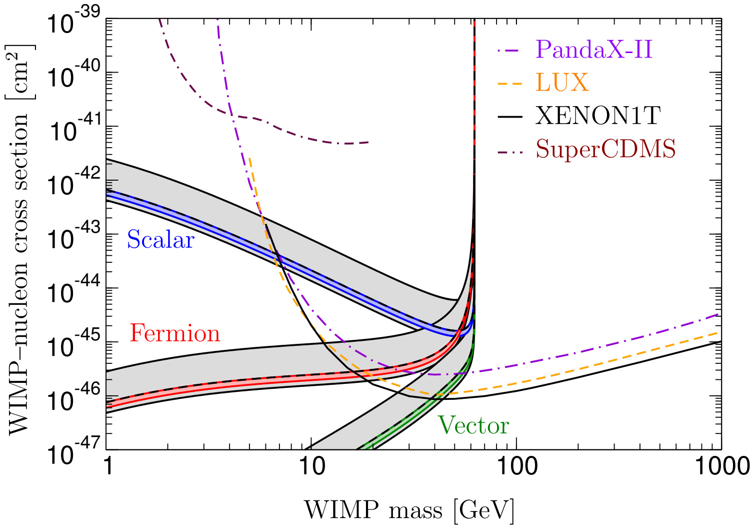
- **Two-body contribution**
  - Need s and θ^μ_μ currents
  - Treatment of θ^μ_μ tricky: several ill-defined terms combine to $\langle\Psi|T + V_{NN}|\Psi\rangle = E_{\text{b}}$
  - A cancellation makes the final result anomalously small

$$f_N^{\text{2b}} = \big[-3.2(0.2)_A(2.1)_{\text{ChEFT}} + 5.0(0.4)_A\big] \times 10^{-3} = 1.8(2.1) \times 10^{-3}$$

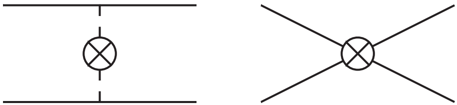
Improved limits for Higgs Portal dark matter



Improved limits for Higgs Portal dark matter



Contact terms



- Scalar source suppressed for $(N^\dagger N)^2$

  $\hookrightarrow$ **long-range contribution dominant** (in Weinberg counting)

- Typical size **(5–10)%**

  $\hookrightarrow$ reflected by results for structure factors

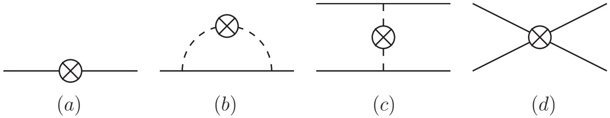
  $\hookrightarrow$ more important in case of cancellations

- Contact terms do appear for other sources, e.g. θ^μ_μ

  $\hookrightarrow$ related to **nuclear binding energy** E_b

- Same structure factor in spin-2 two-body currents MH, Klos, Menéndez, Schwenk, in preparation

Conclusions



- **Chiral EFT** for WIMP–nucleon scattering
- Predicts **hierarchy** for corrections to leading coupling
- Connects nuclear and hadronic scales
- Ingredients: **nuclear matrix elements** and **structure factors**
- Applications:
  - discriminating nuclear responses
  - σ_p^{SD} limits from xenon via two-body currents
  - improved limits on Higgs Portal dark matter from LHC searches

Rate and structure factors

- **Rate**

$$\frac{dR}{d\mathbf{q}^2} = \frac{\rho M}{m_A m_\chi} \int_{v_{\text{min}}}^{v_{\text{esc}}} d^3v \, |\mathbf{v}| f(|\mathbf{v}|) \, \frac{d\sigma_{\chi\mathcal{N}}}{d\mathbf{q}^2}$$

- **Halo-independent methods** Drees, Shan 2008, Fox, Liu, Weiner 2010, ...
- **Nuclear structure factors** Engel, Pittel, Vogel 1992

$$\frac{d\sigma_{\chi\mathcal{N}}}{d\mathbf{q}^2} = \frac{8G_F^2}{(2J+1)v^2} \Big[S_A(q) + S_S(q) \Big]$$

- Normalization at $|\mathbf{q}| = 0$:

$$S_S(0) = \frac{2J+1}{4\pi} \Big| c_0 A + c_1(Z-N) \Big|^2$$

$$S_A(0) = \frac{(2J+1)(J+1)}{4\pi J} \Big| (a_0 + a_1)\langle \mathbf{S}_p \rangle + (a_0 - a_1)\langle \mathbf{S}_n \rangle \Big|^2$$

- Assume $c_1 = 0$ and SI scattering

$$\frac{d\sigma^{\text{SI}}_{\chi\mathcal{N}}}{d\mathbf{q}^2} = \frac{\sigma^{\text{SI}}_{\chi N}}{4\mathbf{v}^2 \mu_N^2} \mathcal{F}^2_{\text{SI}}(\mathbf{q}^2)$$

$\hookrightarrow$ phenomenological **Helm form factor** $\mathcal{F}^2_{\text{SI}}(\mathbf{q}^2)$

Gell-Mann–Oakes–Renner relation

- Leading order in $SU(2)$ **meson ChPT**

$$\mathcal{L}_{\text{ChPT}} = \frac{F_\pi^2}{4}\text{Tr}\Big(d^\mu U^\dagger d_\mu U + 2B\mathcal{M}\big(U+U^\dagger\big)\Big) + \cdots$$
$$= (m_u+m_d)BF_\pi^2 - \frac{1}{2}(m_u+m_d)B\,(\pi^0)^2 - (m_u+m_d)B\,\pi^+\pi^- + \cdots$$

- Comparison with **QCD** Lagrangian

$$\big\langle\mathcal{L}_{\text{QCD}}\big\rangle = -m_u\langle\bar{u}u\rangle - m_d\langle\bar{d}d\rangle + \cdots \quad\Rightarrow\quad BF_\pi^2 = -\langle\bar{q}q\rangle \qquad \langle\bar{q}q\rangle = \langle\bar{u}u\rangle = \langle\bar{d}d\rangle$$

Gell-Mann–Oakes–Renner relation

$$M_\pi^2 = (m_u+m_d)B + \mathcal{O}\big(m_q^2\big) \qquad B = -\frac{\langle\bar{q}q\rangle}{F_\pi^2}$$

Gell-Mann–Oakes–Renner relation

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- Mass difference entirely due to **electromagnetism**

$$M_{\pi^\pm}^2 = M_{\pi^0}^2 + 2e^2F_\pi^2 Z + \mathcal{O}\big(m_d-m_u\big)^2$$

Gell-Mann–Oakes–Renner relation

$$M_{\pi^0}^2 = 2\hat{m}B + \mathcal{O}\big(m_q^2\big) \qquad \hat{m} = \frac{m_\text{u}+m_\text{d}}{2} \qquad B = -\frac{\langle\bar{q}q\rangle}{F_\pi^2}$$

Example: chiral counting in scalar channel

- Leading pion–nucleon Lagrangian

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}\left[i\gamma_\mu\left(\partial^\mu - i{\color{red}v^\mu}\right) - m_N + \frac{g_A}{2}\gamma_\mu\gamma_5\left(2{\color{red}a^\mu} - \frac{\partial^\mu \boldsymbol{\pi}}{F_\pi}\right) + \cdots\right]\Psi$$

$\hookrightarrow$ **no scalar source**!

WIMP	Nucleon	S
	1b	2
S	2b	3

Example: chiral counting in scalar channel

- Leading pion–nucleon Lagrangian

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}\Big[i\gamma_\mu\big(\partial^\mu - i v^\mu\big) - m_N + \frac{g_A}{2}\gamma_\mu\gamma_5\Big(2a^\mu - \frac{\partial^\mu \boldsymbol{\pi}}{F_\pi}\Big) + \cdots\Big]\Psi$$

$\hookrightarrow$ **no scalar source**!

- Scalar coupling

$$f_N = \frac{m_N}{\Lambda^3}\sum_{q=u,d,s} C_q^{SS} f_q^N + \cdots \qquad \langle N|m_q\bar{q}q|N\rangle = f_q^N m_N$$

$\hookrightarrow$ for $q=u,d$ related to **pion–nucleon σ-term** $\sigma_{\pi N}$

	Nucleon	S
WIMP		
	1b	2
S	2b	3

- Chiral expansion

$$\sigma_{\pi N} = -4c_1 M_\pi^2 - \frac{9g_A^2 M_\pi^3}{64\pi F_\pi^2} + \mathcal{O}(M_\pi^4) \qquad \dot{\sigma} = \frac{5g_A^2 M_\pi}{256\pi F_\pi^2} + \mathcal{O}(M_\pi^2)$$

$\hookrightarrow$ slow convergence due to strong $\pi\pi$ rescattering

$\hookrightarrow$ use phenomenology for the full scalar form factor!

σ-term from Roy–Steiner analysis of pion–nucleon scattering

Error analysis

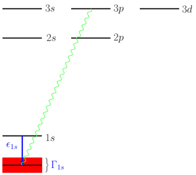
$$\sigma_{\pi N} = 59.1 \pm \underbrace{0.7}_{\text{flat directions}} \pm \underbrace{0.3}_{\text{matching}} \pm \underbrace{0.5}_{\text{systematics}} \pm \underbrace{1.7}_{\text{scattering lengths}} \pm \underbrace{3.0}_{\text{low-energy theorem}} \text{ MeV}$$

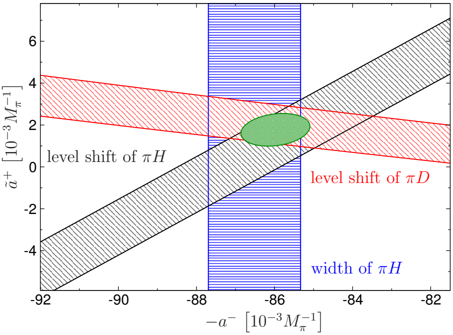
$$= 59.1 \pm 3.5 \text{ MeV}$$

- Crucial result: relation between $\sigma_{\pi N}$ and **πN scattering lengths**

$$\sigma_{\pi N} = 59.1 \text{ MeV} + \sum_{I_s} c_{I_s} \Delta a_{0+}^{I_s}$$

- **Pionic atoms**: $\pi^- p/d$ bound states





A new σ-term puzzle

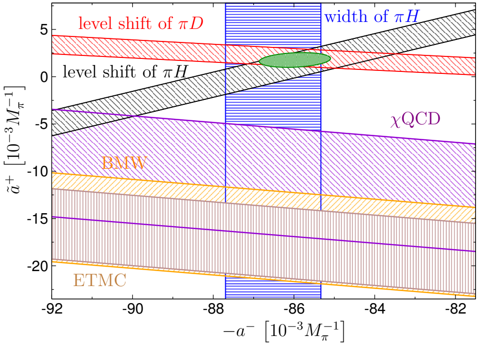
- Recent lattice calculations of $\sigma_{\pi N}$
  - BMW 1510.08013:

    $\sigma_{\pi N} = 38(3)(3)$ MeV
  - χQCD 1511.09089:

    $\sigma_{\pi N} = 45.9(7.4)(2.8)$ MeV
  - ETMC 1601.01624:

    $\sigma_{\pi N} = 37.2(2.6)\left({}^{+4.7}_{-2.9}\right)$ MeV
  - RQCD 1603.00827:

    $\sigma_{\pi N} = 35(6)$ MeV



- Similar puzzle in lattice calculation of

  $K \to \pi\pi$ RBC/UKQCD 1505.07863, also 3σ level

- Both puzzles with profound implications for BSM searches:

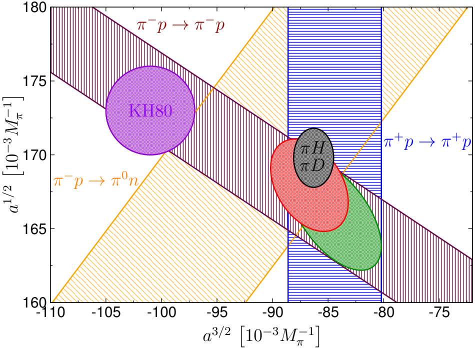
  scalar nucleon couplings, *CP* violation in K_0–$\bar{K}_0$ mixing

A new σ-term puzzle: issues with the pionic-atom scattering lengths?

- Something wrong with pionic-atom data?
- **Direct fit to pion–nucleon data base**

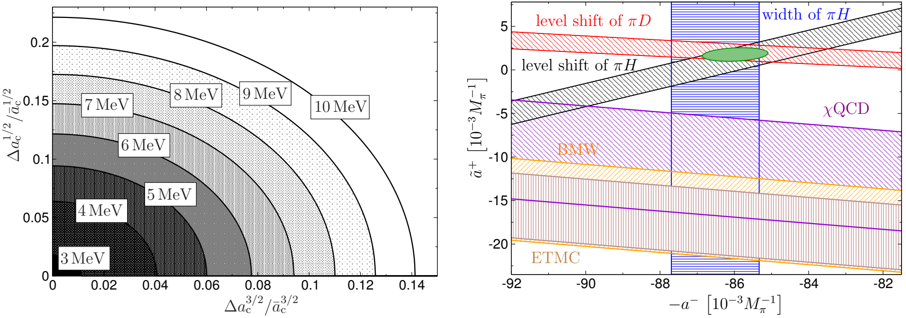
  1706.01465, requires careful treatment of
  - Radiative corrections
  - Experimental normalization uncertainties
- Bottom line:

$$\sigma_{\pi N} = 58(5)\,\text{MeV}$$



$\hookrightarrow$ independent confirmation of pionic-atom data

A new σ-term puzzle: what could lattice do?



- πN: lattice calculation of $a^{1/2}$, $a^{3/2}$

  $\hookrightarrow$ test input for πN scattering lengths
- Possible issues of σ-term calculations:
  - Finite-volume corrections
  - Discretization effects
  - Excited-state contamination

Status of the phenomenological determination of $\sigma_{\pi N}$

- **Karlsruhe/Helsinki** partial-wave analysis KH80 Höhler et al. 1980s

  $\hookrightarrow$ comprehensive analyticity constraints, old data

- Formalism for the extraction of $\sigma_{\pi N}$ via the **Cheng–Dashen low-energy theorem**

  Gasser, Leutwyler, Locher, Sainio 1988, Gasser, Leutwyler, Sainio 1991

  $\hookrightarrow$ "canonical value" $\sigma_{\pi N} \sim 45\,\text{MeV}$, based on KH80 input

- **GWU/SAID** partial-wave analysis Pavan, Strakovsky, Workman, Arndt 2002

  $\hookrightarrow$ much larger value $\sigma_{\pi N} = (64 \pm 8)\,\text{MeV}$

- ChPT fits vary according to PWA input Fettes, Meißner 2000

  (same problem in different regularizations (w/ and w/o Δ) Alarcón et al. 2012)

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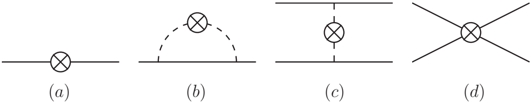
- Our work: two new sources of information on low-energy πN scattering
  - Precision extraction of **πN scattering lengths** from **hadronic atoms**
  - **Roy-equation constraints**: analyticity, unitarity, crossing symmetry

  1506.04142,1510.06039

QCD constraints for subleading nuclear corrections



- One-body operators: known **nuclear form factors**

  $\hookrightarrow$ determines **radius corrections** (b)

- **Axial Ward identity** relates $g^N_{A,P}(t)$ and

$$\mathcal{M}^{AA}_{1,\text{NR}} = -4\mathcal{O}_4 g^N_A(t) + \frac{1}{m_N^2}\mathcal{O}_6 g^N_P(t)$$

  $\hookrightarrow$ fixed combination of $\mathcal{O}_{4,6}$ in (a)

- $\mathcal{O}_{10}$ only appears in SP channel $\Rightarrow$ not coherent and vanishes at $\mathbf{q} = 0$

Comparison to NREFT

- For the **leading corrections** all $\mathcal{O}_i$ but $\mathcal{O}_3$ are small

  $\hookrightarrow$ not necessary to keep 2×14 parameters in first step

- But: some new parameters for two-body effects and radius corrections

  $\hookrightarrow$ cover **coherent responses** (+ SD), same order in chiral counting

- Nucleon operators: $\mathbb{1}$, $\mathbf{S}_N$, $\mathbf{v}^\perp$, $\mathbf{v}^\perp \times \mathbf{q}$, $\mathbf{v}^\perp \cdot \mathbf{q} = 0$

  $\hookrightarrow$ only $\mathbf{v}^\perp \to \boldsymbol{\nabla}$ can produce new coherent (nuclear) effect

- Similarly to SD searches: define subleading "cross sections"

  $\hookrightarrow$ pion–WIMP scattering

- NREFT only first step in chain of EFTs

  $\hookrightarrow$ need **matching to QCD** to make connection to BSM, ChEFT one crucial step

Analysis strategies

- Parameters ($\zeta = 1(2)$ for Dirac (Majorana)):

$$c_\pm^M = \frac{\zeta}{2}\Big[f_p \pm f_n + f_1^{V,p} \pm f_1^{V,n}\Big] \qquad c_\pi = \zeta f_\pi \qquad c_\pi^\theta = \zeta f_\pi^\theta \qquad c_\pm^{\Phi''} = \frac{\zeta}{2}\Big(f_2^{V,p} \pm f_2^{V,n}\Big)$$

$$\dot{c}_\pm^M = \frac{\zeta m_N^2}{2}\bigg[\dot{f}_p \pm \dot{f}_n + \dot{f}_1^{V,p} \pm \dot{f}_1^{V,n} + \frac{1}{4m_N^2}\Big(f_2^{V,p} \pm f_2^{V,n}\Big)\bigg]$$

- Couplings

$$f_N = \frac{m_N}{\Lambda^3}\bigg(\sum_{q=u,d,s} C_q^{SS} f_q^N - 12\pi f_Q^N C_g'^{S}\bigg) \qquad f_\pi = \frac{M_\pi}{\Lambda^3}\sum_{q=u,d}\bigg(C_q^{SS} + \frac{8\pi}{9}C_g'^{S}\bigg) f_q^\pi \qquad f_\pi^\theta = -\frac{M_\pi}{\Lambda^3}\frac{8\pi}{9}C_g'^{S}$$

- Conclusions
  - Different c probe **different linear combinations** of Wilson coefficients
  - Ideally: global analysis of different experiments
  - **One-operator-at-a-time strategy**: producing limits e.g. on c_-^M and c_π in addition to c_+^M would provide additional information on BSM parameter space
  - **QCD constraints**: when considering $\mathcal{O}_3$ should also keep radius corrections

Spin-2 and coupling to the energy-momentum tensor

- Effective Lagrangian truncated at dim-7, but if WIMP heavy $m_\chi/\Lambda = \mathcal{O}(1)$

  $\hookrightarrow$ heavy-WIMP EFT Hill, Solon 2012, 2014

$$\mathcal{L} = \frac{1}{\Lambda^4}\bigg\{\sum_q C_q^{(2)}\bar{\chi}\gamma_\mu i\partial_\nu\chi\frac{1}{2}\bar{q}\Big(\gamma^{\{\mu}iD_-^{\nu\}} - \frac{m_q}{2}g^{\mu\nu}\Big)q + C_g^{(2)}\bar{\chi}\gamma_\mu i\partial_\nu\chi\Big(\frac{g_{\mu\nu}}{4}G^a_{\lambda\sigma}G_a^{\lambda\sigma} - G_a^{\mu\lambda}G^\nu_{a\lambda}\Big)\bigg\}$$

  $\hookrightarrow$ leading order: **nucleon pdfs**

  $\hookrightarrow$ similar two-body current as in scalar case, pion pdfs, EMC effect

- Coupling of trace anomaly θ^μ_μ to $\pi\pi$

$$\theta^\mu_\mu = \sum_q m_q\bar{q}q + \frac{\beta_{\text{QCD}}}{2g_s}G^a_{\mu\nu}G_a^{\mu\nu} \quad\Leftrightarrow\quad \langle\pi(p')|\theta_{\mu\nu}|\pi(p)\rangle = p_\mu p'_\nu + p'_\mu p_\nu + g_{\mu\nu}\,(M_\pi^2 - p\cdot p')$$

  $\hookrightarrow$ probes gluon Wilson coefficient C_g^S