Transport phenomena in strong magnetic fields

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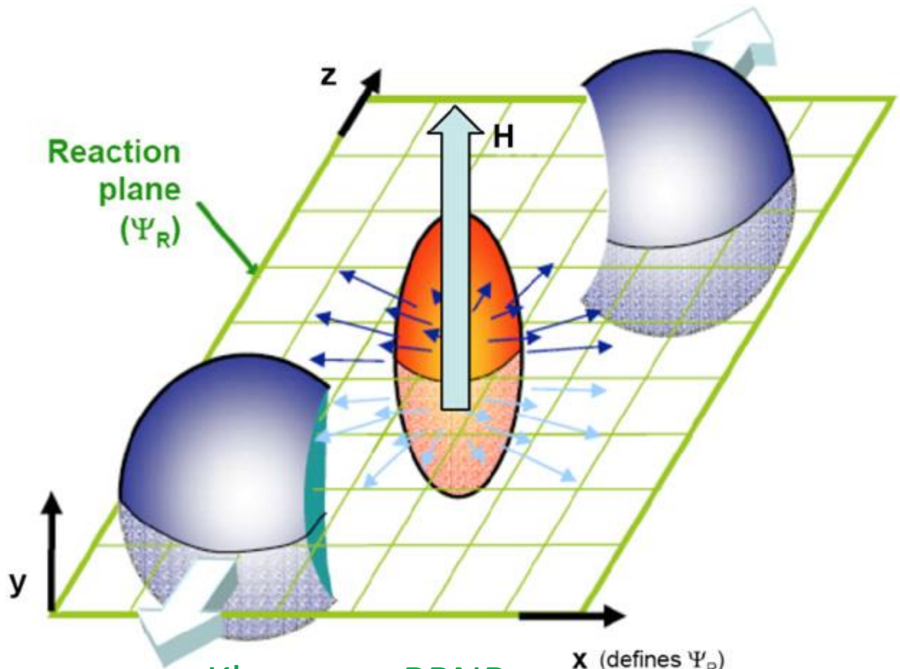
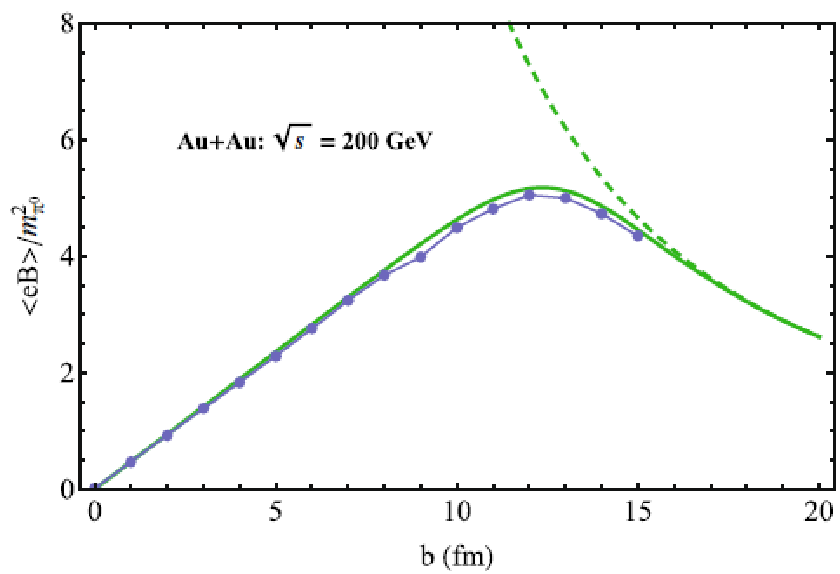
Seminar in INT Program INT-18-1b Week 1
Multi-Scale Problems Using Effective Field Theories

Strong magnetic fields
induced by relativistic heavy-ion collisions



Kharzeev, PPNP



W.-T. Deng & X.-G. Huang
KH and X.-G. Huang

Z ~ 80, v > 0.99999 c,
Length scale ~ $1/\Lambda_{QCD}$

$$eB \gtrsim m_\pi^2$$

One can study the interplay btw QCD and QED.

Besides,

- Weyl & Dirac semimetals
- Neutron stars/magnetars
- High intensity laser fields
- Strong B field by lattice QCD simulations
- Cosmology

Landau-level discretization

Diamagnetismus der Metalle.

Von **L. Landau**, zurzeit in Cambridge (England).

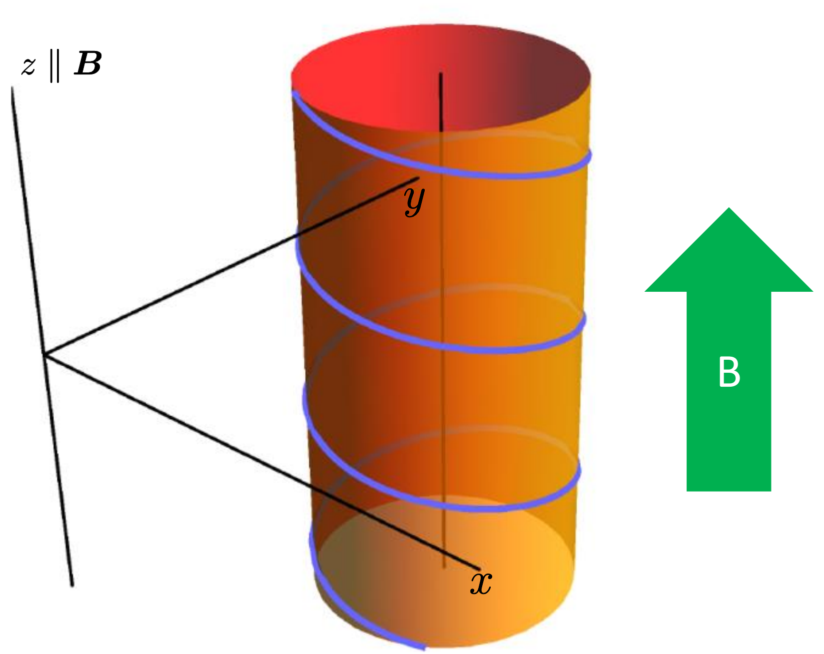
(Eingegangen am 25. Juli 1930.)

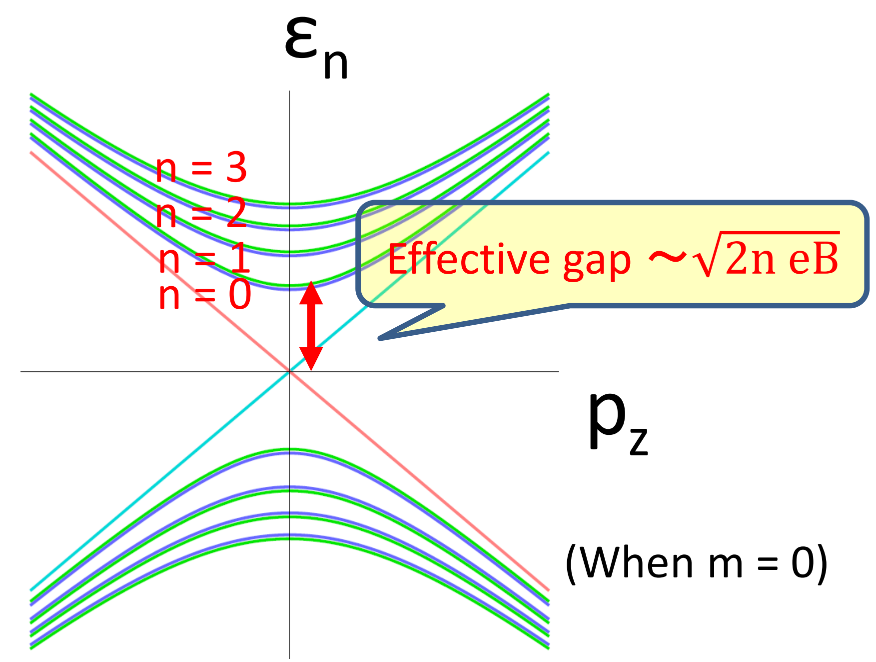
“Harmonic oscillator” in the transverse plane

Relativistic: $\epsilon_n = \sqrt{p_z^2 + (2n+1)eB + m^2}$

Nonrelativistic: $\epsilon_n = \dfrac{p_z^2}{2m^2} + (n + \dfrac{1}{2})\boxed{\dfrac{eB}{m^2}}$ Cyclotron frequency

The Zeeman effect for spin-1/2 cancels the “zero-point energy”.

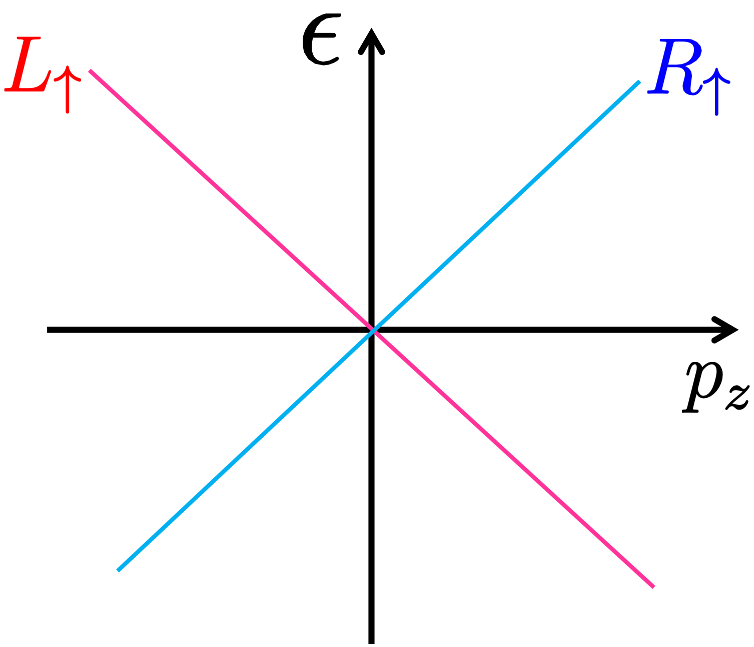




Dynamics in the lowest Landau level (LLL)

(1+1)-D dispersion relations
"Effective dimensional reduction"

$$\varepsilon = \pm p_z$$



Macroscopic consequences

Anomalous transport phenomena
→ Chiral magnetohydrodynamics (MHD)
KH, Y.Hirono, H.-U.Yee, Y.Yin

Any consequence in usual (dissipative) transport phenomena?

SSB and Kondo effect in analogy to dense systems

Chiral magnetohydrodynamics

KH, Yuji Hirono, Ho-Ung Yee, and Yi Yin, **arXiv:1711.08450** **[hep-th]**

Anomaly-induced transports in a magnetic and vortex field

$$\mu_V = (\mu_R + \mu_L)/2$$
$$\mu_A = (\mu_R - \mu_L)/2$$

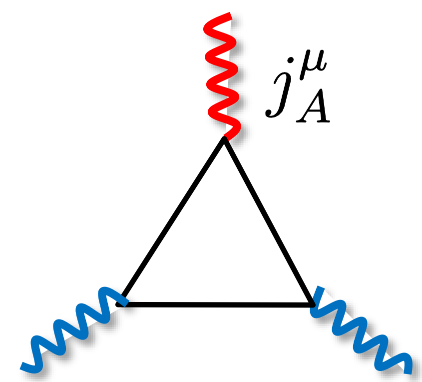
$$\begin{pmatrix} j_V^\mu \\ j_A^\mu \end{pmatrix} = C_A \begin{pmatrix} q_f \mu_A & \mu_V \mu_A \\ q_f \mu_V & (\mu_V^2 + \mu_A^2)/2 + C_A^{-1} T^2/12 \end{pmatrix} \begin{pmatrix} B^\mu \\ \omega^\mu \end{pmatrix}$$

$$B^\mu = \tilde{F}^{\mu\nu} u_\nu ,\ \omega^\mu = \frac{1}{2}\epsilon^{\mu\alpha\beta\gamma} u_\alpha \partial_\beta u_\gamma$$

Non-dissipative transport phenomena with time-reversal even and nonrenormalizable coefficients.

Anomaly relation: $\partial_\mu j_A^\mu = q_f^2 C_A \boldsymbol{E} \cdot \boldsymbol{B}$

$$C_A = \frac{1}{2\pi^2}$$



Cf., An interplay between the B and ω leads to a new nonrenormalizable transport coefficient for the magneto-vorticity coupling.

KH and Y.Yin, Phys.Rev.Lett. 117 (2016) 152002 [**1607.01513** **[hep-th]**]

Anomalous hydrodynamics in STRONG & DYNAMICAL magnetic fields

-- Anomalous hydrodynamics $\mu_A \neq 0,\ B \sim \mathcal{O}(\partial A)$ and external

Son & Surowka

-- Anomalous magnetohydrodynamics (MHD) $\mu_A \neq 0,\ B \sim \mathcal{O}(1)$ and dynamical

This work.

Slow variables in chiral MHD: $\{\epsilon,\ u^\mu,\ B^\mu,\ \text{and}\ n_A\}$

n_A: # density of axial charge
Neutral plasma (n_V = 0)
No E-field in the global equilibrium

EoM: $\partial_\mu T^{\mu\nu}_{\text{fluid+EM}} = 0,\ \partial_\mu \tilde{F}^{\mu\nu} = 0,\ \partial_\mu j^\mu_A = -C_A E^\mu B_\mu.$

Constitutive eqs. in the ideal order determined by the entropy conservation

$$
\begin{aligned}
T^{\mu\nu}_{(0)} &= \epsilon u^\mu u^\nu - X\Delta^{\mu\nu} - Y B^\mu B^\nu \\
\tilde{F}^{\mu\nu}_{(0)} &= B^\mu u^\nu - B^\nu u^\mu \\
j^\mu_{A(0)} &= n_A u^\mu
\end{aligned}
$$

$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu \ (u_\mu \Delta^{\mu\nu} = 0)$

E-field is first order.

$B^{(\mu}u^{\nu)}$ is absent in $T^{\mu\nu}$ when $n_V = 0$.

From EoM + thermodynamic relation $ds = \frac{1}{T}(d\epsilon - \mu_A dn_A - H_\mu dB^\mu)$

$$
\begin{aligned}
\partial_\mu (s u^\mu) &= u \cdot \partial s + s \partial \cdot u \\
&= (p - X)\partial \cdot u + (H^\mu - Y B^\mu) B \cdot \partial u_\mu \\
&= 0 \quad \text{for the ideal part.}
\end{aligned}
$$

Therefore, $T^{\mu\nu}_{(0)} = \epsilon u^\mu u^\nu - p\Delta^{\mu\nu} - \mu^{-1} B^\mu B^\nu$

ϵ and p are the total (fluid+magnetic) energy and pressure.

Constitutive eqs. and entropy current in the first order

$$
\begin{aligned}
T^{\mu\nu} &= T^{\mu\nu}_{(0)} + T^{\mu\nu}_{(1)} \\
\tilde{F}^{\mu\nu} &= \tilde{F}^{\mu\nu}_{(0)} - \epsilon^{\mu\nu\alpha\beta} u_\alpha E_{(1)\beta} \\
j^\mu_A &= j^\mu_{A(0)} + j^\mu_{A(1)}
\end{aligned}
$$

Note that $\partial_\mu j^\mu_A = -C_A E^\mu_{(1)} B_\mu$.

How much can we constrain $T^{\mu\nu}_{(1)}$, $E^\mu_{(1)}$, $j^\mu_{A(1)} \sim \mathcal{O}(\partial^1)$ from $\partial_\mu s^\mu \geq 0$?

Again, computing the entropy current,

$$
\begin{aligned}
\partial_\mu (s u^\mu) &= \partial_\mu [\cdots] + T^{\mu\nu}_{(1)} \partial_\mu (\beta u_\nu) - j^\mu_{A(1)} \partial_\mu (\beta \mu_A) \\
&\quad + E^\mu_{(1)} \{\mu_A C_A B_\mu - \epsilon_{\mu\nu\alpha\beta} u^\nu \partial^\alpha (\beta H^\beta)\}
\end{aligned}
$$

The total derivative term $\partial_\mu [\cdots]$ is identified as the first order correction to the entropy current, $\partial_\mu S^\mu_{(1)}$.

Insuring the semi-positivity with bilinear forms

The second law of the thermodynamics $\partial_\mu(su^\mu) \geq 0$ constrains the tensor structures of the first order corrections.

Each term should be semi-positive definite (see previous slide).
For example,

$$E^\mu_{(1)}\{\mu_A C_A B_\mu - \epsilon_{\mu\nu\alpha\beta}u^\nu\partial^\alpha(\beta H^\beta)\} \geq 0$$

Should have a bilinear form $E^\mu_{(1)} X_{\mu\nu} E^\nu_{(1)} \geq 0$

$$X_{\mu\nu} = \sigma_\parallel b_\mu b_\nu - \sigma_\perp(\Delta_{\mu\nu} + b_\mu b_\nu) - \sigma_{\mathrm{Hall}}\,\epsilon_{\mu\nu\alpha\beta}u^\alpha b^\beta$$

$$\sigma_{\parallel,\perp} \geq 0, \text{ but } \sigma_{\mathrm{Hall}} \propto \mu_V. \qquad b^\mu = B^\mu/(\sqrt{-B^\mu B_\mu})$$

Therefore, we get a "constitutive eq." of the E-field:

$$E^\mu_{(1)} = X^{-1\mu\rho}\{\mu_A C_A B_\rho - \epsilon_{\rho\nu\alpha\beta}u^\nu\partial^\alpha(\beta H^\beta)\}$$

Transport coefficients
--- CME and other dissipative terms

$$X_{\mu\nu}E^{\nu}_{(1)} = \mu_A C_A B_\mu - \epsilon_{\mu\nu\alpha\beta}u^\nu \partial^\alpha (\beta H^\beta)$$

with X given in the previous slide.

Eliminating $\partial_\alpha H_\beta$ by the Ampère's law $J^\nu = \partial_\mu F^{\mu\nu}$,

$$J^\mu_V = C_A \mu_A B^\mu + \left[\sigma_\parallel E^\mu_\parallel + \sigma_\perp E^\mu_\perp + \sigma_{\rm Hall}\epsilon^{\mu\nu\alpha\beta}u_\nu b_\alpha E_\beta\right] + \cdots$$

The CME current is given by C_A without any other unknown coefficient, and is necessary for insuring the semi-positive entropy production.

There appear 3 conductivities (see later slides).

Similarly,

$T^{\mu\nu}_{(1)}\partial_\mu(\beta u_\nu) \geq 0$ provides 5 shear and 2 bulk viscous coefficients (see later slides)

Landau & Lifshitz; Huang, Sedrakian, & Rischke; Tuchin; Hernandez & Kovtun; …

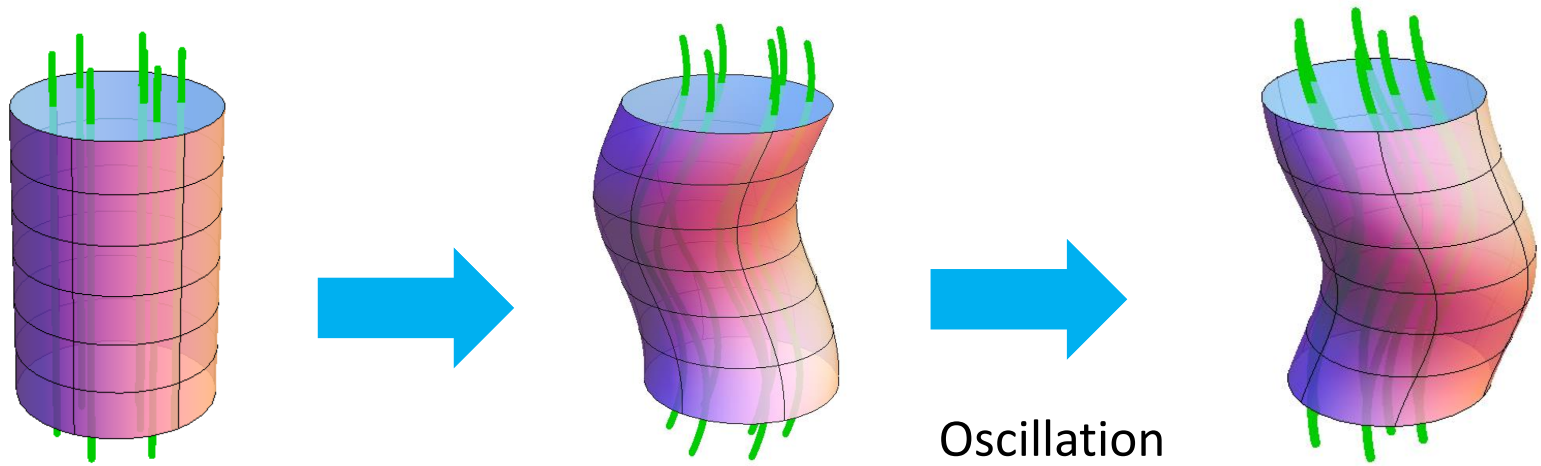
Phases of collective excitations and new instabilities from the CME

Collective excitations in MHD without anomaly

2 transverse waves (Alfven waves)

4 longitudinal waves (fast and slow magneto-sonic waves)

* Magnetic lines move together with the fluid volume.



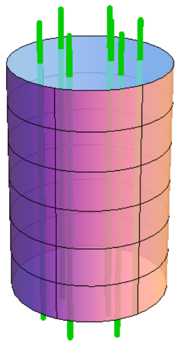
Tension of B-field = Restoring force

Fluid energy density = Inertia

Transverse Alfven wave

$$v_{\mathrm{Alf}}^2 = \frac{B_0^2}{\epsilon + p + B_0^2}$$

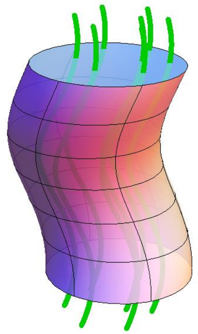
Alfven wave from a linear analysis



$$\boldsymbol{B}_0 \neq 0\,,\ T > 0\,,\ \mu_V = 0$$

0. Stationary solutions

$$u^\mu = (1, \mathbf{0}),\ \ B^\mu = (0, \boldsymbol{B}_0),\ \ j^\mu = (0, \mathbf{0})$$

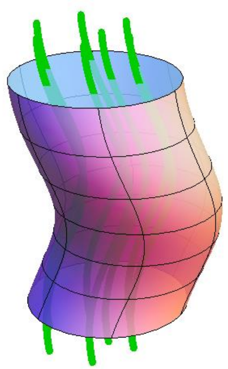


1. Transverse perturbations

$$\boldsymbol{v} \to \boldsymbol{v} + \delta \boldsymbol{v}$$

$$\boldsymbol{B}_0 \to \boldsymbol{B}_0 + \delta \boldsymbol{B}$$

Linearlize the set of hydrodynamic eqs. with respect to the perturbation.



2. Wave equation

$$\partial_t^2 \delta \boldsymbol{B}(t,z) = \boxed{\frac{B_0^2}{\epsilon + p}} \partial_z^2 \delta \boldsymbol{B}(t,z)$$

Alfven wave velocity

Transverse wave propagating along background B_0

$$\boldsymbol{B}_0 \parallel \boldsymbol{k}$$

Same wave equation for δv

→ Fluctuations of B and v propagate together.

How does the CME change the hydrodynamic waves in chiral fluid?

--- Drastic changes by only one term in the current

$$j^\mu = \sigma_{\mathrm{CME}} B^\mu$$

Excitations in anomalous MHD

$\boldsymbol{v}$ (3 d.o.f.)
$\boldsymbol{B}$ $(\nabla \cdot \boldsymbol{B} = 0)$ (2 d.o.f.)
ϵ (1 d.o.f.)

Linearlized EoM $(\boldsymbol{v} \to \boldsymbol{v} + \delta \boldsymbol{v},\ \boldsymbol{B} \to \boldsymbol{B} + \delta \boldsymbol{B})$

→ Secular eq. as a cubic eq. of ω^2

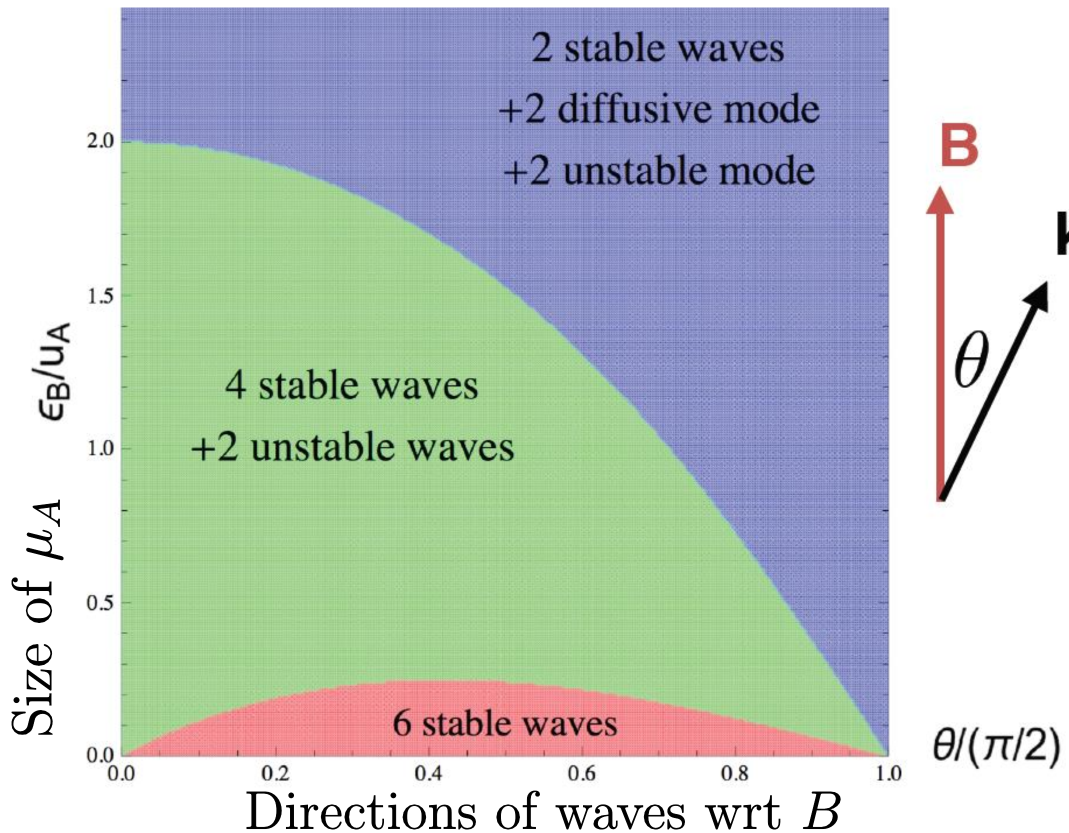
→ 3 modes propagating in the opposite directions (6 solutions in total)

$$(\omega^2 - x_1)(\omega^4 + b\omega^2 + c) = 0$$

x_1: Real solution

$$\mu_A \neq 0,\ \ \mu_V = 0$$

Stability of the waves from classification of solutions



New hydrodynamic instability in a chiral fluid

Signs of the imaginary parts
(Damping/growing modes in the hydrodynamic time evolution)

Positive
(Damping)

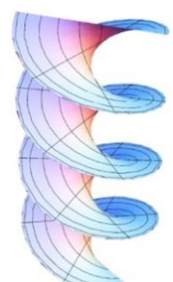
Negative
(Growing)

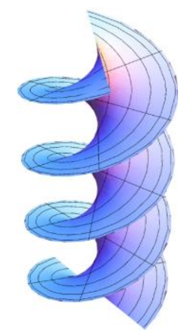
When $\mu_A > 0$

When $\mu_A < 0$

LH mode

RH mode





Helicity decomposition
(Circular R/L polarizations)

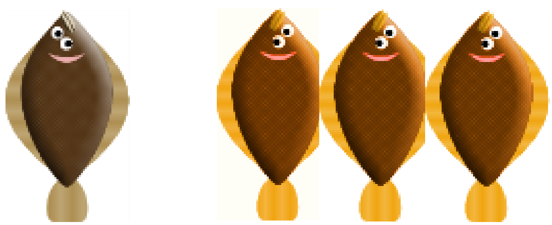
$$\nabla \times \boldsymbol{e}_{R/L} = \pm \boldsymbol{e}_{R/L}$$

A helicity selection, depending on the sign of μ_A .

Helicity conversions
as the topological origin of the instability

Chiral imbalance btw
R and L fermions

$$\mu_A$$



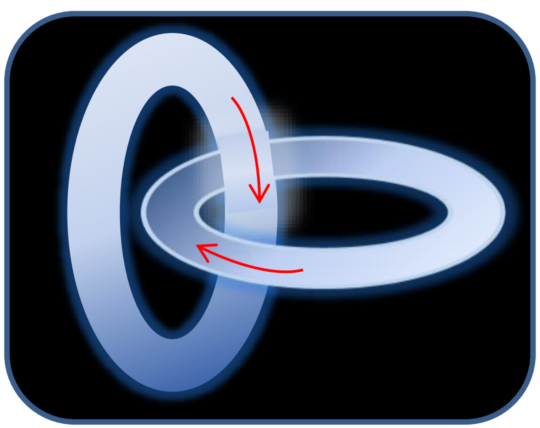
Chiral Plasma Instability (CPI)

Magnetic helicity

$$\int_V d^3x\, \boldsymbol{B} \cdot \boldsymbol{A}$$

Hirono

$$\int_V d^3x\, \boldsymbol{\omega} \cdot \boldsymbol{v}_{\text{fluid}}$$



Fluid helicity (structures of vortex strings)

Dynamical & beyond-linear analysis demanded.

Short summary 1:
Formulation of Chiral MHD

The second law of thermodynamics determines the zeroth order derivative expansion, and constrains the tensor structures in the first order.

In the MHD regime, the CME current is completely fixed by the anomaly coefficient without any ambiguity.

The other dissipative parts are characterized by
3 conductivities, and 5 shear and 2 bulk viscous coefficients.
(Will be discussed shortly.)

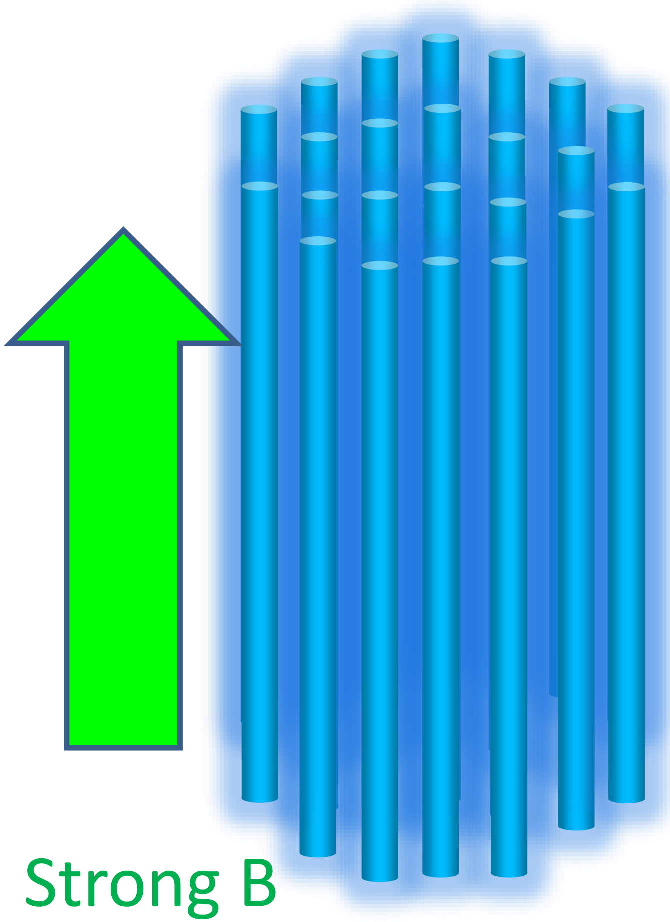
Short summary 2:
Collective excitations and instabilities of Chiral MHD

The CME drastically changes the time evolution of the fluid with the axial charge and B-field.

Not stable against a small perturbation on v and B.
→ New hydrodynamic instability!

Helicities of the unstable modes are selected by μ_A.

Dissipative transport phenomena in the lowest Landau Levels



Longitudinal, transverse, and Hall currents;
5 shear and 2 bulk viscous coefficients.

Landau & Lifshitz; Huang, Sedrakian, & Rischke;
Tuchin; Hernandez & Kovtun

In the LLL for the strong B limit,
charged fermions transport the charge
and momentum only along the B.

Longitudinal conductivity
Longitudinal bulk viscosity

KH, S.Li, D.Satow, H.-U. Yee
KH, X.-G.Huang, D.Satow, D.Rischke

1. Electrical conductivity in strong magnetic field

KH, Shiyong Li, Daisuke Satow, and Ho-Ung Yee, arXiv:1610.06839 [hep-ph].
KH and Daisuke Satow, arXiv:1610.06818 [hep-ph].

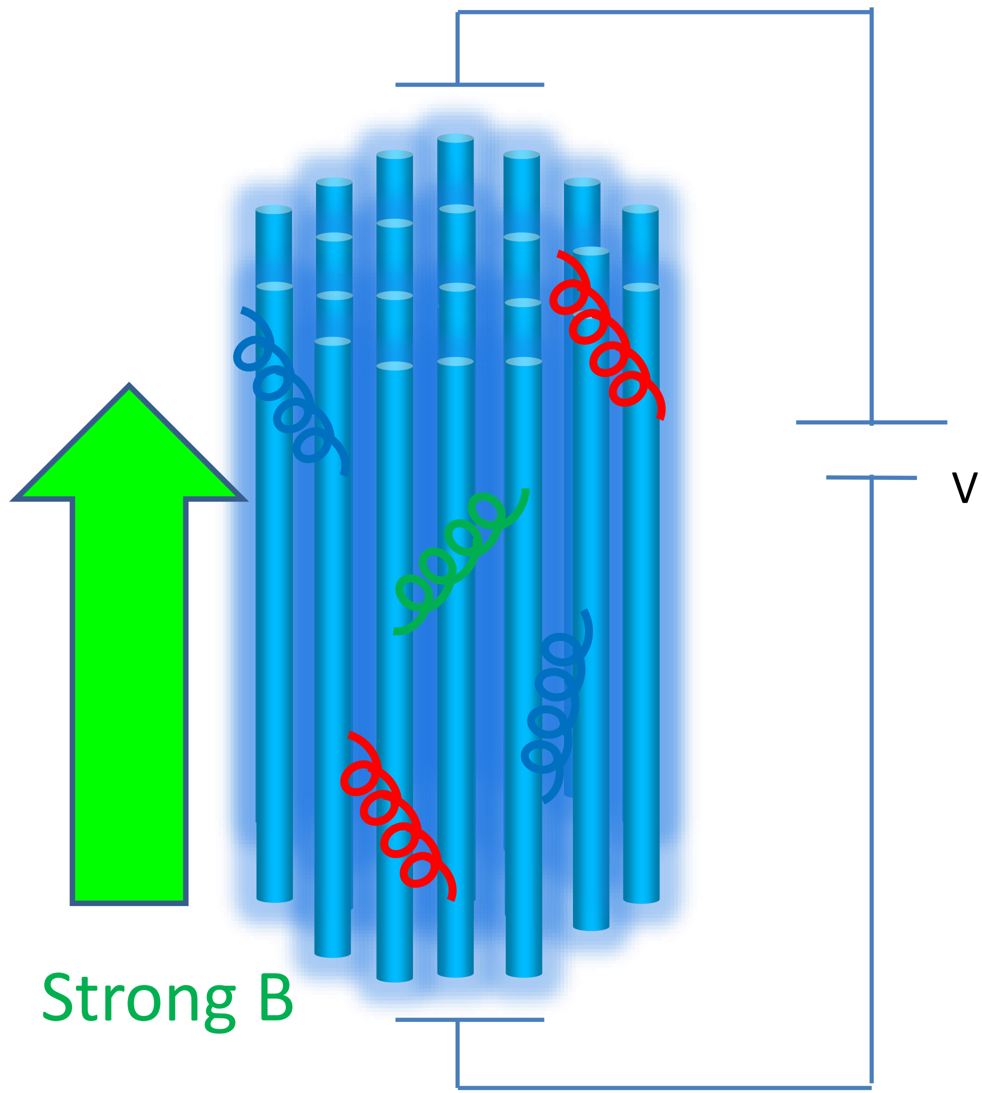
$$\boldsymbol{j} = \sigma_{\mathrm{Ohm}} \boldsymbol{E}$$

$$T \neq 0, \ \ \mu = 0$$

"Mismatched dimensions"

Quarks live in (1+1) D

Gluons live in (3+1) D

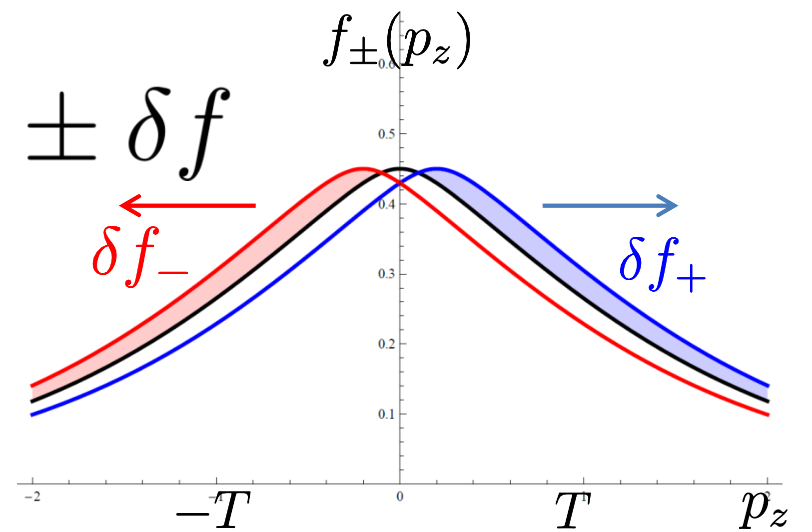


Linear response in kinetic theory

$$j_z = \sigma_{zz} E_z \qquad \sigma_{ij} = 0 \text{ except for } \sigma_{zz}.$$

Acceleration by the electric field

$$\dot{p}_z = \pm q_f E_z \qquad f_\pm \to f^{\mathrm{eq}} \pm \delta f$$



Total current integrated over p_z from the off-equilibrium components

$$j_z = \frac{|q_f B|}{2\pi} \cdot q_f \int \frac{dp_z}{2\pi} v_z (f_+ - f_-)$$

$2\delta f$

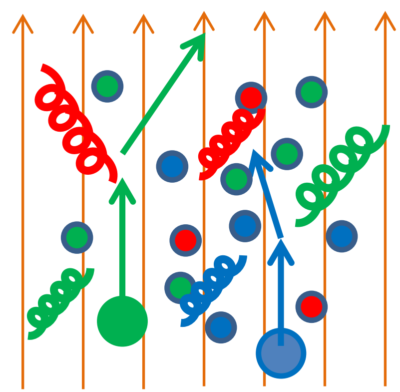
Density of states
"Landau degeneracy factor"

(1+1) Dim.

Boltzmann eq. in stationary and homogeneous limit

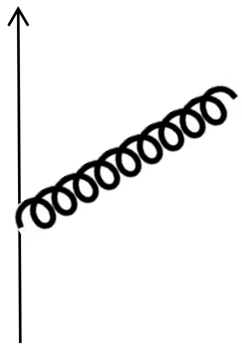
$$\cancel{\frac{\partial f_\pm}{\partial t}} + \dot{z}\cancel{\frac{\partial f_\pm}{\partial z}} + \dot{p}_z \frac{\partial f_\pm}{\partial p_z} = C[f_\pm]$$

External driving force v.s. Relaxation



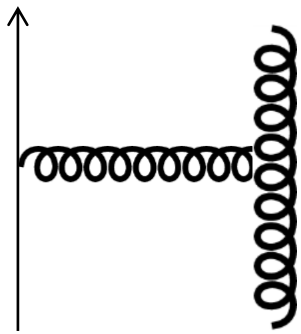
E field

Quark-damping mechanism in strong magnetic fields



$|\mathcal{M}|^2 \sim \mathcal{O}(\alpha_s)$

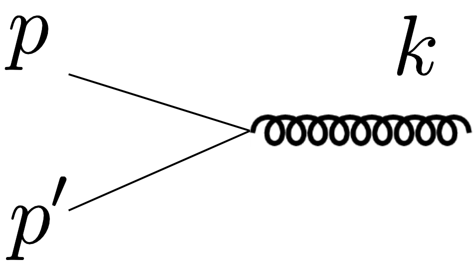
This work with B
(Cf., Cyclotron radiation)



$|\mathcal{M}|^2 \sim \mathcal{O}(\alpha_s^2)$

AMY without B

Finite B opens 1-2 processes



$$\epsilon_{\text{quark}}^2 = p_z^2 + m_f^2$$
$$\epsilon_{\text{gluon}}^2 = k_z^2 + |\boldsymbol{k}_\perp|^2$$

$|\boldsymbol{k}_\perp|$ works as a gluon mass for 2D kinematics.

Analogue of a massive weak boson production from $q\bar{q}$ annihilation in 4D.

Chirality selection in the massless limit

Chirality conservation at the vertex

$\epsilon_p = \pm p_z$

The scattering in the (1+1) D is prohibited by the chirality conservation in the massless limit (cf., CME).

Therefore, the scattering rate → 0 as m → 0.

$$\gamma \propto g^2 m_f^2$$

unless $|\boldsymbol{k}_\perp|$ is finite.

$A_\mu \propto (0, 1, \pm i, 0)$

Results

$$\sigma^{zz} = q_f^2 \frac{|q_f B|}{2\pi} \frac{4T}{g^2 m_f^2 \ln(T/M)}$$

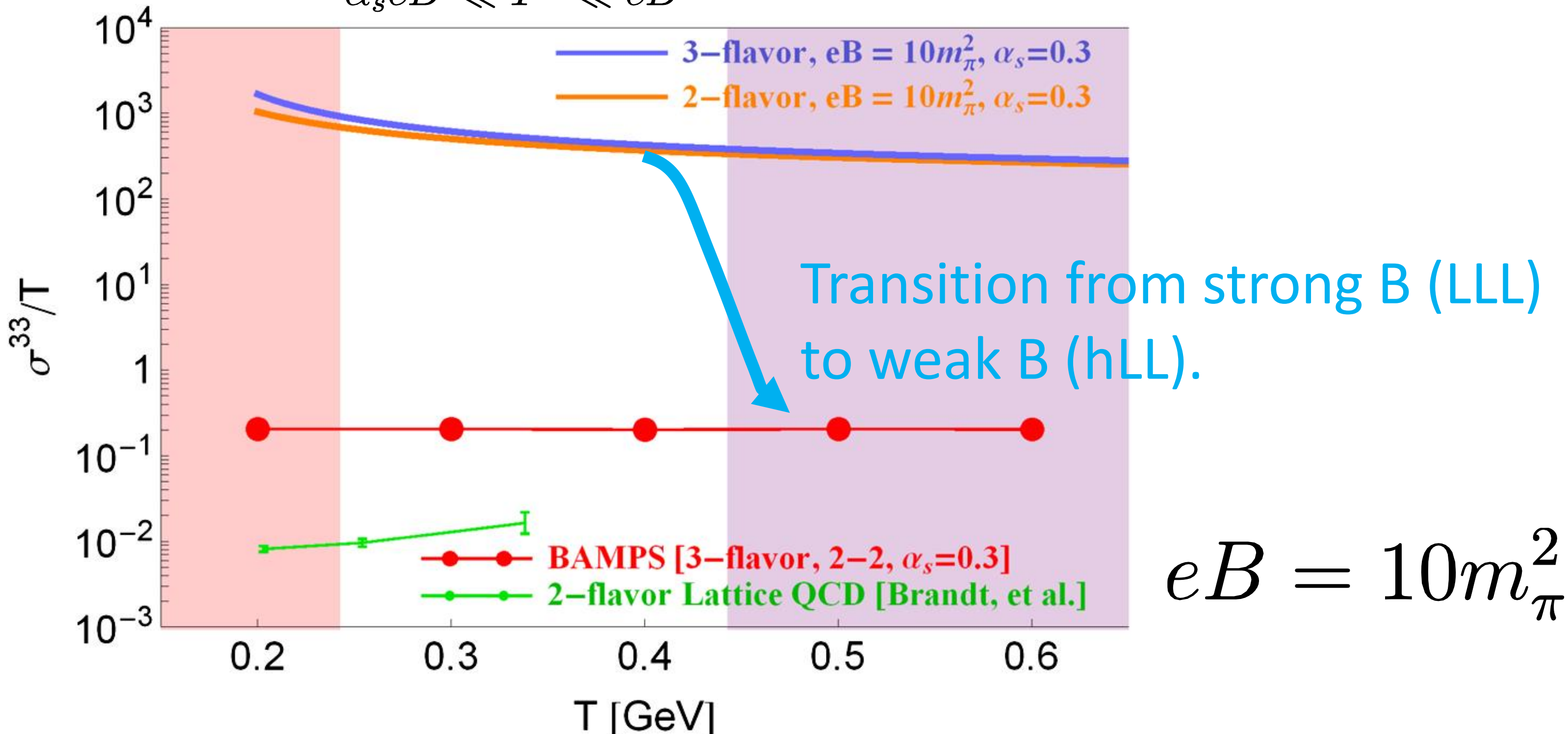
See KH and Satow for a consistent result from diagrammatic calculation, and Fukushima and Hidaka for effects of the higher Landau levels.

When eB = 0,

Arnold, Moore, and Yaffe

$$\sigma_{B=0} = q_f^2 \frac{T}{g^4 \ln[T/(gT)]}$$

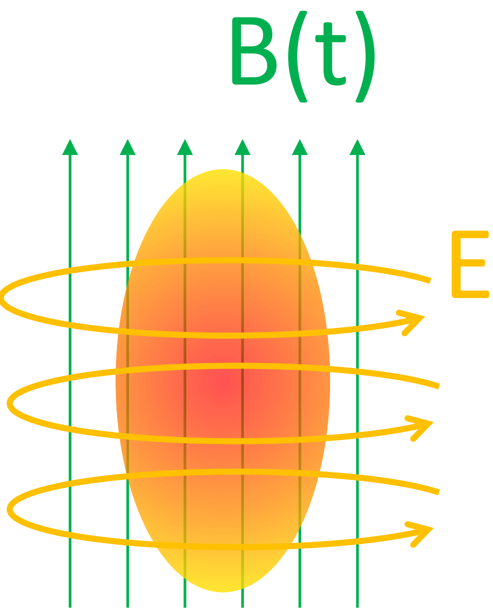
$\alpha_s eB \ll T^2 \ll eB$



$$eB = 10 m_\pi^2$$

Lifetime of the B-field in HIC

A longer lifetime due to the Lenz's law?   Tuchin



$$\partial_t B(t) < 0$$

Time dependent B induces E.

E induces J if QGP is conducting.

→ Induced J sustains B.

Important to know the conductivity of QGP.

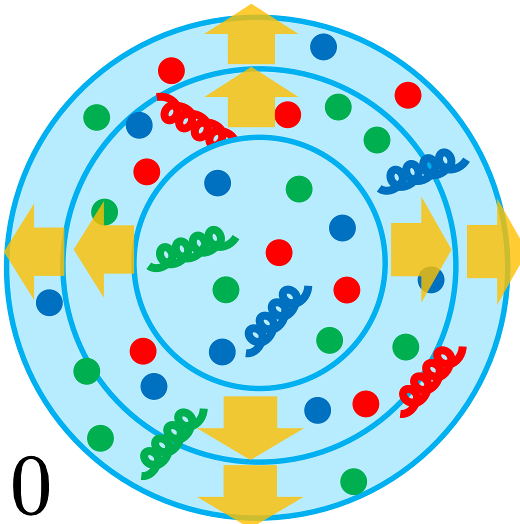
There is no transverse current in the LLL, because the quarks are confined in the longitudinal direction.

→No backreaction effect in the "very" strong magnetic field.

2. Bulk viscosity in a strong magnetic field

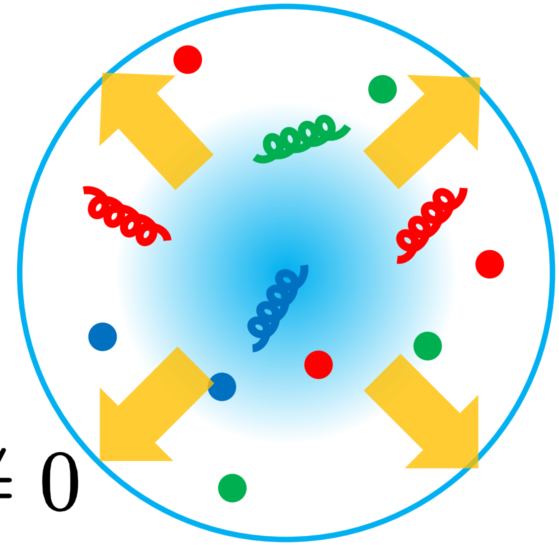
KH, X.-G. Huang, D. Rischke, D. Satow, Phys. Rev. D96 (2017) 094009
arXiv:1708.00515 **[hep-ph]** .

Bulk viscosity of the QGP



$\zeta = 0$

Adiabatic expansion
in an equilibrium



$\zeta \neq 0$

Rapid expansion
in (slightly) off equilibrium

Scale inv. in the massless & classical limits: $\mathcal{L}_{\rm QCD} = \bar{\psi} i \not{D} \psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$

A finite bulk viscosity demands conformal-symmetry breakings.

$$T^\mu_\mu = m^2 \bar{\psi}\psi + \frac{\beta(g)}{g^3} F^{\mu\nu} F_{\mu\nu}$$

$$\zeta \sim m^4 \#_1 + \beta^2 \#_2$$

Arnold, Dogan, Moore (2006)

Pressure evolution in response to an expansion

$$\delta P_{\parallel} = \frac{|eB|}{2\pi} \int \frac{dk_z}{2\pi} k_z v_z \delta f(k_z) \qquad v_z = k_z/\epsilon_k$$

In the linear response regime, $\delta f \propto \partial_z u_z$.

$$\zeta_{\parallel} = -\frac{1}{3} \frac{\delta P_{\parallel}}{(\partial_z u_z)}$$

Boltzmann eq. in an expanding system

$$\frac{\partial f}{\partial t} + v_z \frac{\partial f}{\partial z} = C[f]$$

Perturbation by the expansion of the system

$$f(t, z; k_z) = \frac{1}{\exp[\beta(t)(\epsilon - k_z u_z(z))] + 1}$$

Solve the linearlized Boltzmann eq. for $\delta f = f - f_{\mathrm{eq}}$

Competition between the conformal symmetry
and the chirality conservation in the massless limit.

$$\zeta_{\|} = \frac{|eB|}{2\pi} \cdot \frac{1}{T} \int \frac{dk_z}{2\pi \epsilon_k^2} \frac{[\epsilon^2 - k_z^2]^2}{\gamma_{\mathrm{damp}}} [\cdots]$$

Conformal symmetry
~ [m^2]^2

Chirality selection rule
~ m^2

Remember $\gamma_{\mathrm{damp}} \propto m^2 g^2$ in B !!

Results

$$M^2 = \alpha_s eB$$

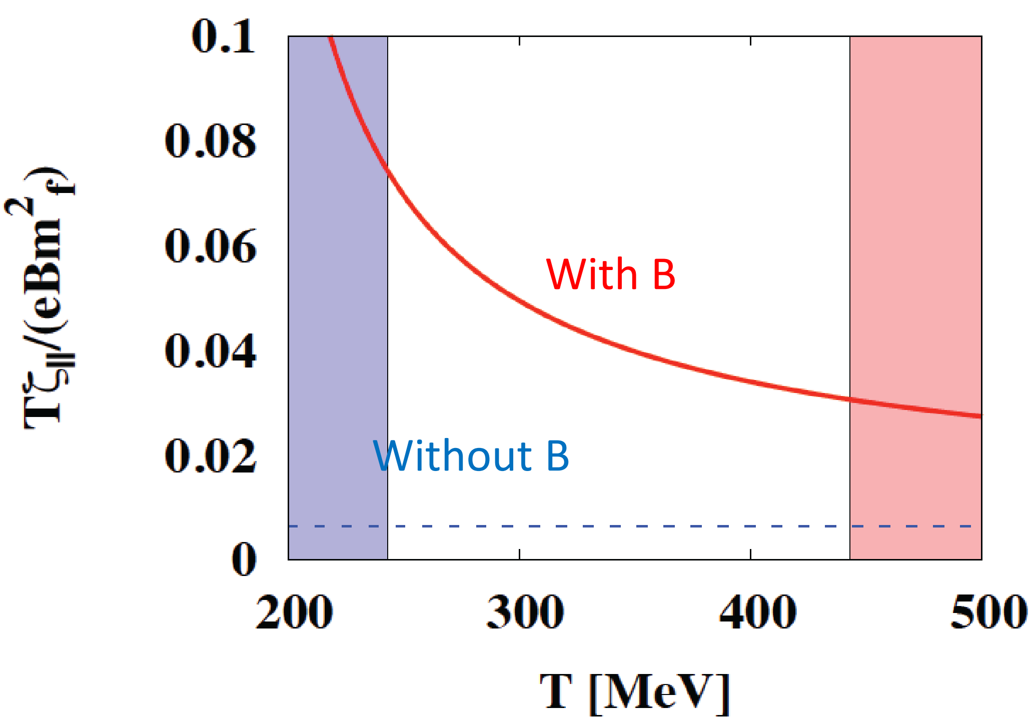
$$\zeta_\parallel \sim \frac{|eB|}{2\pi} \frac{m^2}{g^2 T \ln(T/M)}$$

Consistent result can be obtained from diagrammatic calculation with Kubo formula.

When eB = 0,

Arnold, Dogan, Moore (2006)

$$\zeta_{B=0} \sim (\text{typical momentum})^4 \frac{(\text{conformal breaking factor})^2}{(\text{mean free path})^{-1}} \sim T^4 \left(\frac{m_f^2}{T^2}\right)^2 \frac{1}{g^4 T \ln(1/g)}.$$



Short summary

The chirality selection plays crucial roles in the non-anomalous transports.

Consequences of the chirality selection rule and the competition with the conformal symmetry.

Electrical conductivity: $\sigma_{zz} \propto eB \dfrac{1}{g^2 m_f^2}$

Bulk viscosity: $\zeta_{\parallel} \propto eB \dfrac{m_f^2}{g^2}$

Analogy between the systems at high density and in strong B --- Consequences of dimensional reductions

Condensed matter in finite density		Quark matter in strong B-fields
BCS instability	⟺	Magnetic catalysis of χSB
Kondo effect	⟺	Kondo effect in B-field

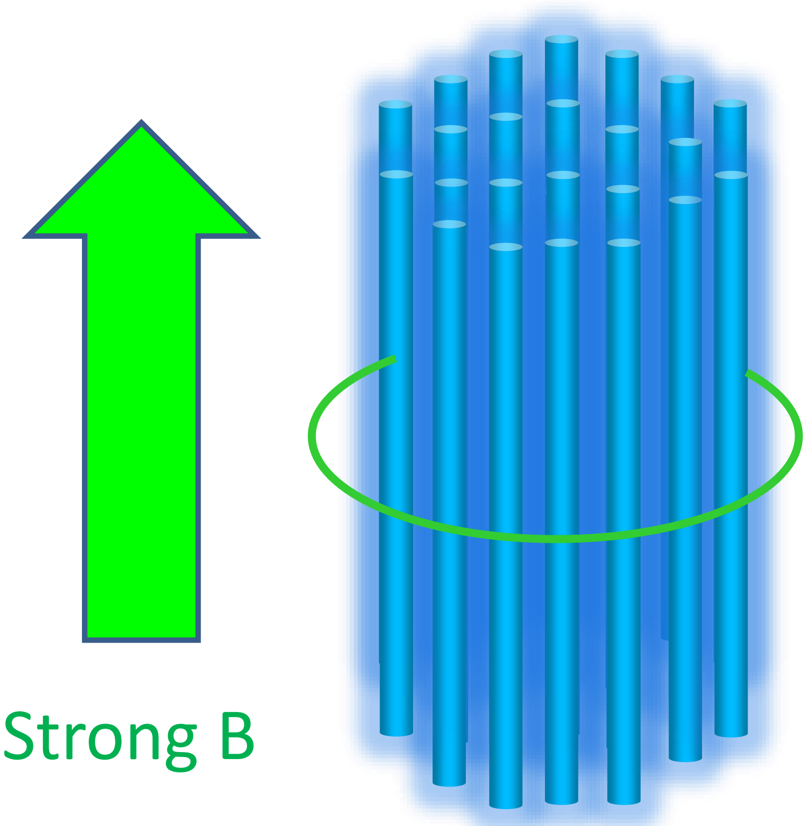
They are all understood from the dimensional reduction.

KH, K.Itakura, S.Ozaki, A review paper to appear in PPNP

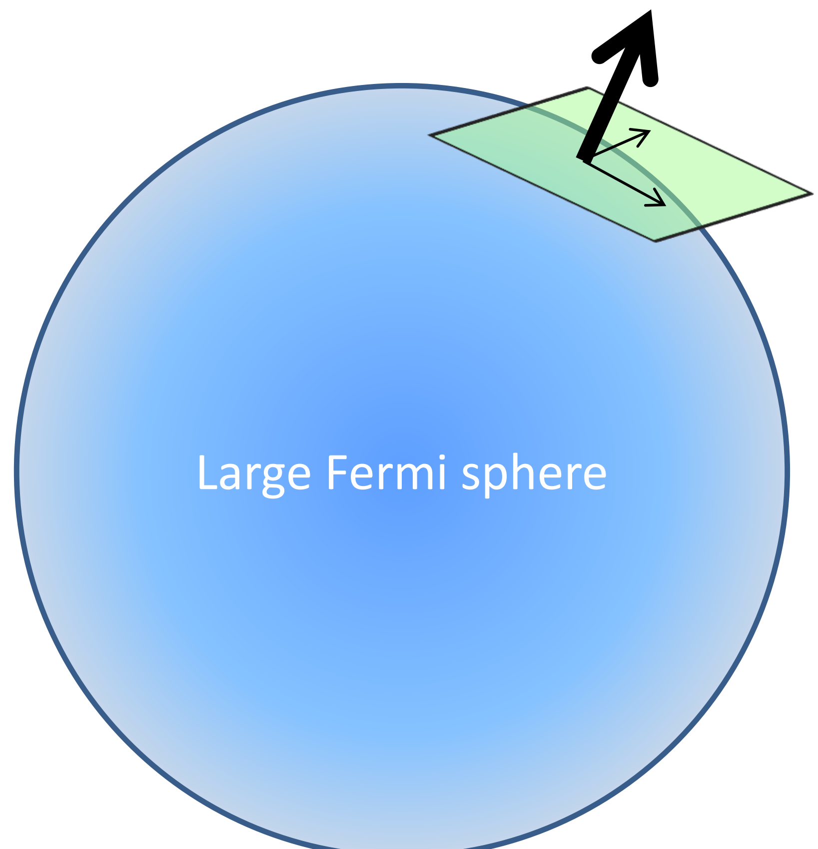
Analogy btw the dimensional reduction in a large B and μ

(1+1)-D dispersion relations

$$\varepsilon = \pm p_z \quad \Leftrightarrow \quad \epsilon = \pm \ell_{\parallel} \quad (\ell_{\parallel} \ll \mu)$$



Strong B



2-dimensional density of states

$$\rho = \frac{N_{\rm state}}{S} = \frac{eB}{2\pi} \sim 1/r^2_{cyclotron} \quad \Leftrightarrow \quad \rho_F \sim \mu^2$$

IR scaling dimensions

$$\text{When } \epsilon \to s\epsilon,\ \ell_\parallel \to s\ell_\parallel. \quad (s<1)$$

Kinetic term

$$\mathcal{S}^{\rm kin} = \int dt \sum_{\boldsymbol{v}_F} \int \frac{d^2\boldsymbol{\ell}_\perp d\ell_\parallel}{(2\pi)^3} \bar{\psi}_+ (i\partial_t - \ell_\parallel)\gamma^0 \psi_+ + \mathcal{O}(1/\mu)$$

$$0 = \underbrace{2d_\psi}_{\bar{\psi}\cdot\psi} + \underbrace{(-1)}_{dt} + \underbrace{1}_{d\ell_\parallel} + \underbrace{1}_{\partial_t}$$

$$\boxed{d_\psi = -\frac{1}{2}}$$

Four-Fermi operators for superconductivity

Polchinski (1992)

$$\mathcal{S}^{\rm int} = \int dt \left[\int \frac{d^2\boldsymbol{\ell}_\perp d\ell_\parallel}{(2\pi)^3} \right]^4 G[\bar{\psi}_+^{(4)} \hat{\gamma}_\parallel^\mu \psi_+^{(2)}][\bar{\psi}_+^{(3)} \hat{\gamma}_\mu^\parallel \psi_+^{(1)}] \delta^{(3)}(\boldsymbol{p}^{(1)} + \boldsymbol{p}^{(2)} - \boldsymbol{p}^{(3)} - \boldsymbol{p}^{(4)})$$

In general momentum config.

$$p^{(1)} + p^{(2)} \sim \mu$$

$$d_{4-\text{Fermi}} = \underbrace{(-1)}_{dt} + \underbrace{4(1-\frac{1}{2})}_{4(d\ell_\parallel + d_\psi)} = +1$$

In the BCS config.

$$p^{(1)} + p^{(2)} \sim \ell_\parallel \ll \mu$$

$$d_{4-\text{Fermi}} = (-1) + 4(1-\frac{1}{2}) \boxed{-1} = 0$$

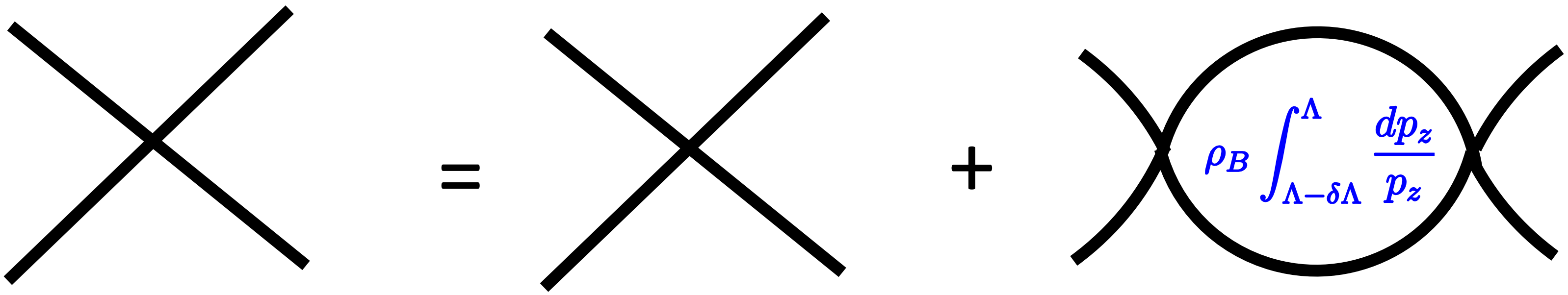
SSB by the dimensional reduction

KH, K.Itakura, S.Ozaki, hep-ph/1706.04913
and a review paper to appear in PPNP.

(1+1)-D dispersion relation → d_ψ = - 1/2

Again, the 4-Fermi interaction is a marginal operator in the (1+1) dimensions in a strong B!

Wilsonian RG flow driven by the logarithmic quantum corrections



Coupling at Λ - δΛ

Coupling at Λ

Logarithmic quantum correction
2-dim. integral results in ρ.

$$\Lambda \frac{dG(\Lambda)}{d\Lambda} = -G^2(\Lambda)\rho_B$$

Negative beta function!!

→ The solution diverges at $\Lambda_{\rm dyn} = \Lambda_{\rm UV} \exp\left(-\frac{1}{\rho_B G(\Lambda_{\rm UV})}\right)$

The IR regime of the LLL is strong coupling.

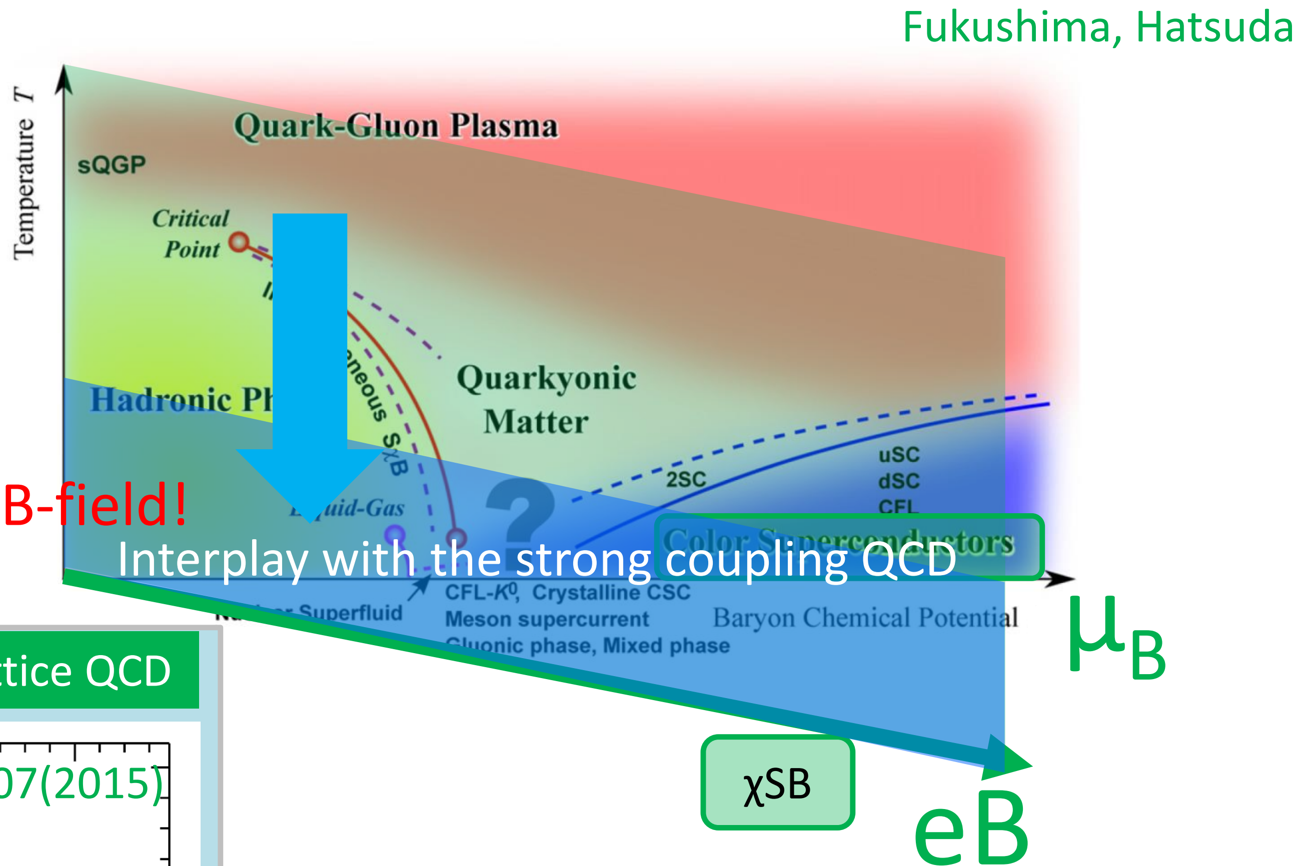
→ χSB occurs even in QED!!!
(Cf., SC occurs with any weak attraction.)

Emergent scale in the IR !

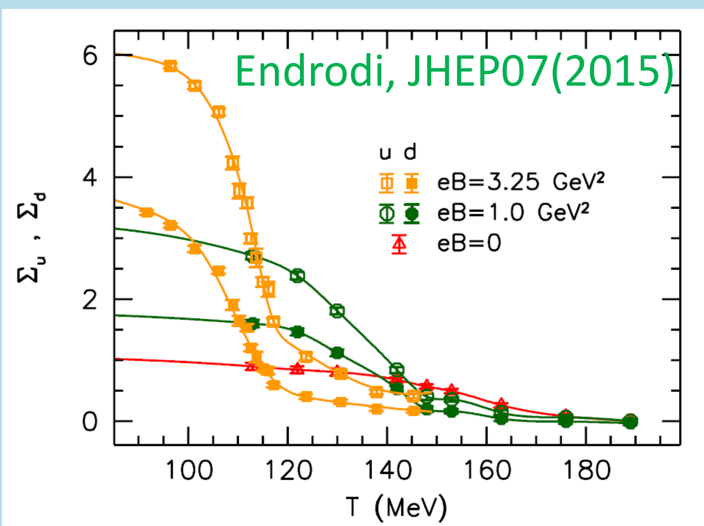
Cf., Similarity to $\Lambda_{\rm QCD}$

Chiral symmetry and its spontaneous breaking

No sign problem in B-field!



Thermodynamics from lattice QCD



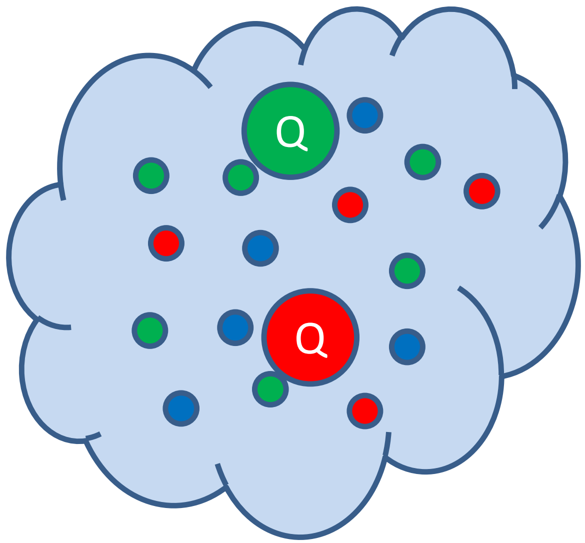
Lattice QCD results inspired so many authors,
Including KH, T.Kojo, N.Su; KH, K.Itakura, S.Ozaki.

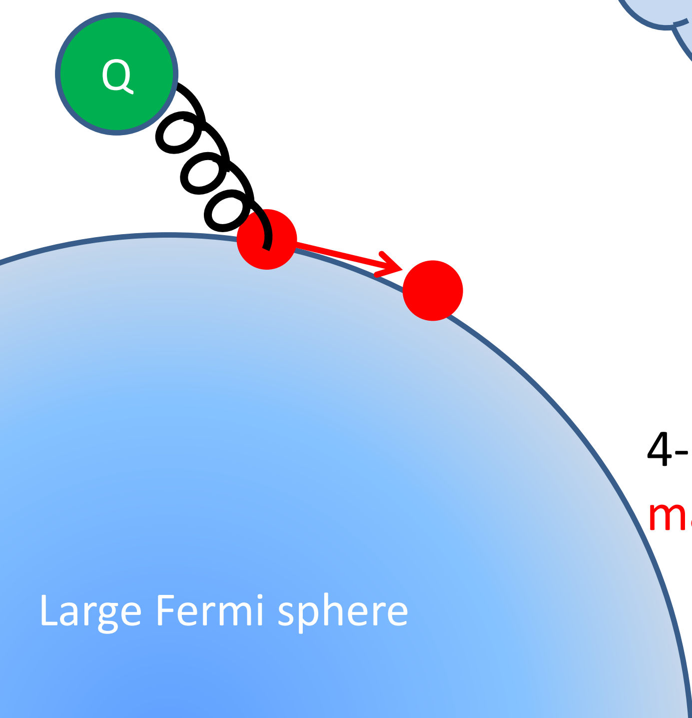
Kondo effect

--- Another consequence of the dimensional reduction

Impurity scatterings near a Fermi surface

+ Electron-impurity scattering in conde. matt.
+ Light-Heavy quark scattering in quark matter

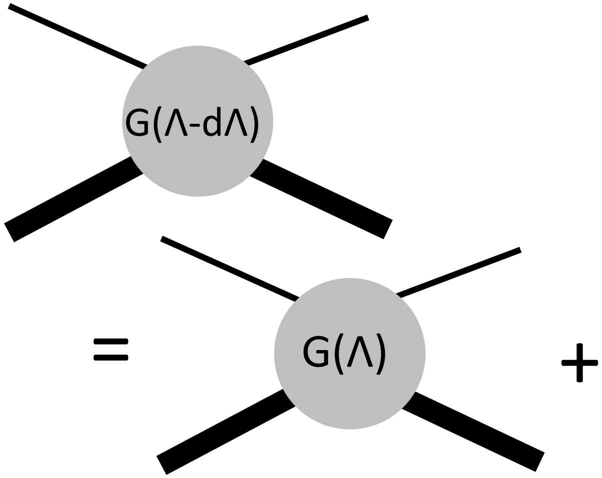




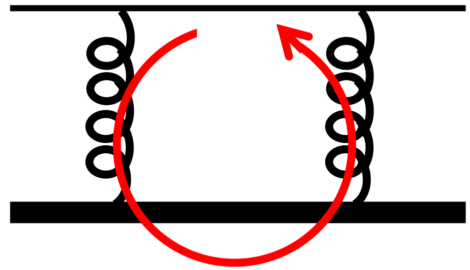
$$G(\Lambda)(\bar{\psi}\psi)(\bar{\Psi}\Psi)$$

4-Fermi operator for the heavy-light scattering is marginal when the light particle lives in (1+1) dimensions.

RG analysis for "QCD Kondo effect"



Particle contribution

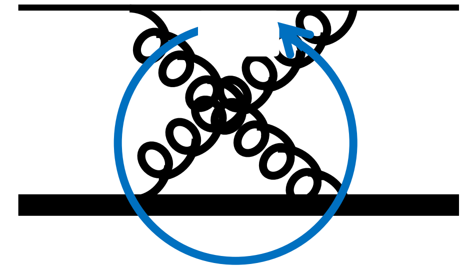


$$\rho_F \int_{\epsilon_F} \frac{d\epsilon}{\epsilon} \sim \log \frac{\Lambda}{\Lambda - d\Lambda}$$

From above to ε_F

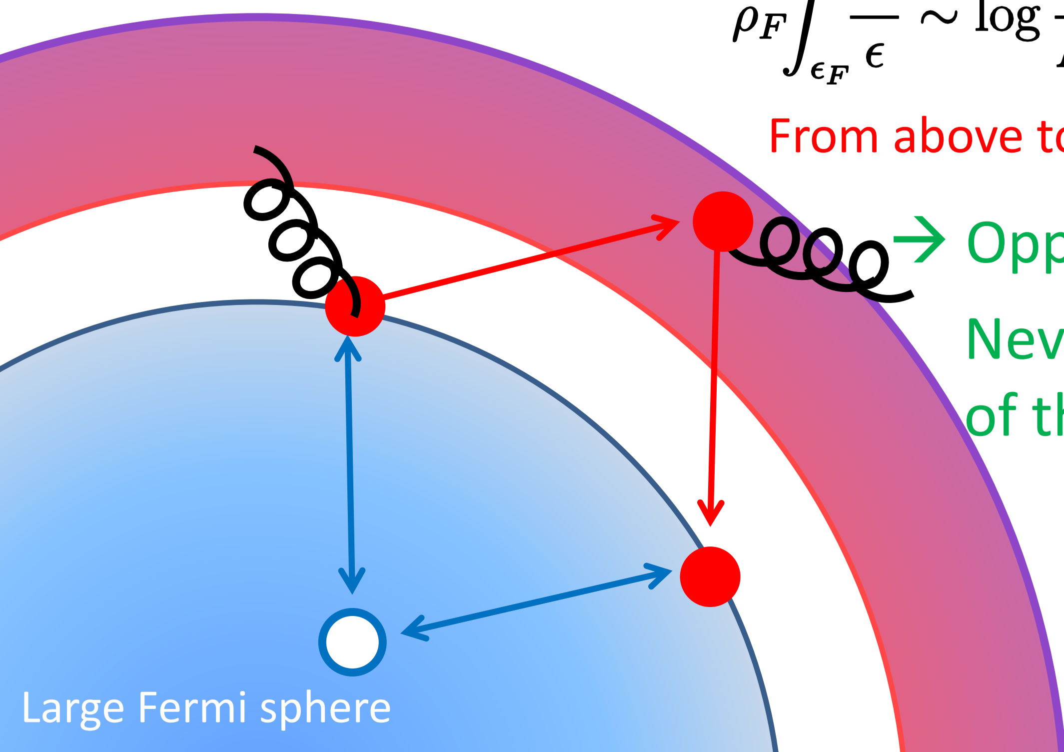
+

Hole contribution



$$\rho_F \int^{\epsilon_F} \frac{d\epsilon}{\epsilon} \sim -\log \frac{\Lambda}{\Lambda - d\Lambda}$$

From below to ε_F



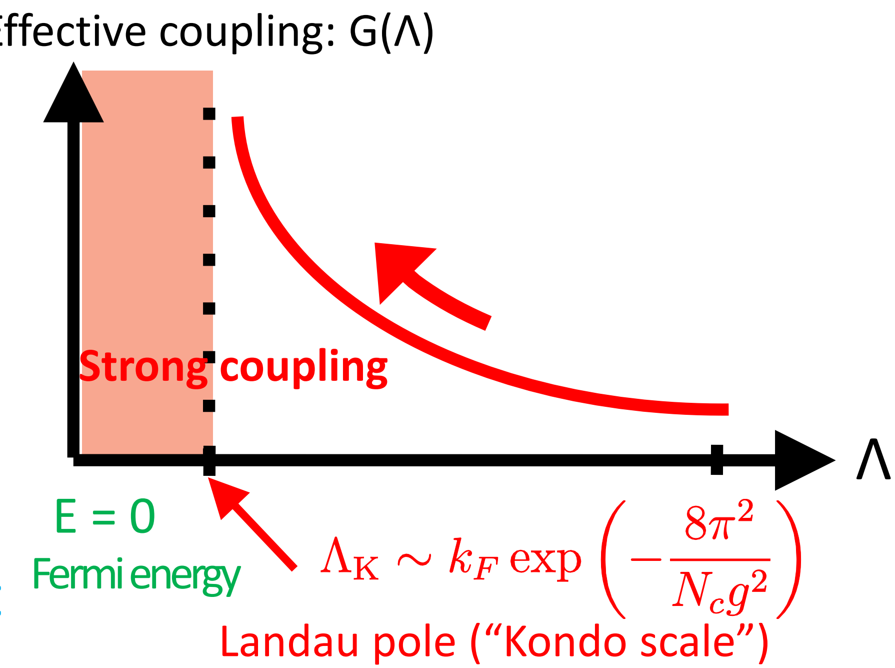
→ Opposite signs in the log corrections.

Nevertheless, do not cancel because of the non-Abelian matrices.

RG equation

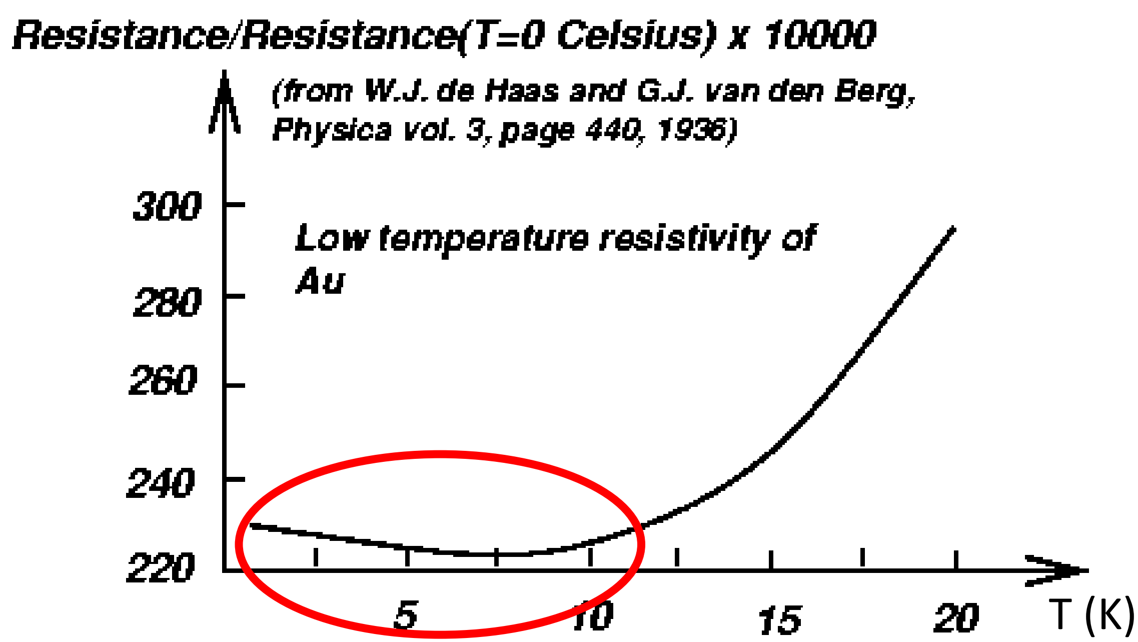
$$\Lambda \frac{dG(\Lambda)}{d\Lambda} = -\frac{k_F^2}{8\pi^2} g^2 N_c G^2(\Lambda)$$

Negative beta function
→ Asymptotic-free solution



Consequence of the Kondo effect

Anomalous increase of the resistance



Kondo effect at high density and in a strong B --- Analogy from the dimensional reduction

Kondo Effect at high density

$$\Lambda_{\rm K} \sim k_F \exp\left(-\frac{8\pi^2}{N_c g^2}\right)$$

Kondo Effect in B-field

$$\Lambda_{\rm K} \sim \sqrt{q_{\rm em} B}\, \exp\left(-\frac{8\pi^2}{N_c g^2}\right)$$

$$k_F \leftrightarrow \sqrt{eB}$$

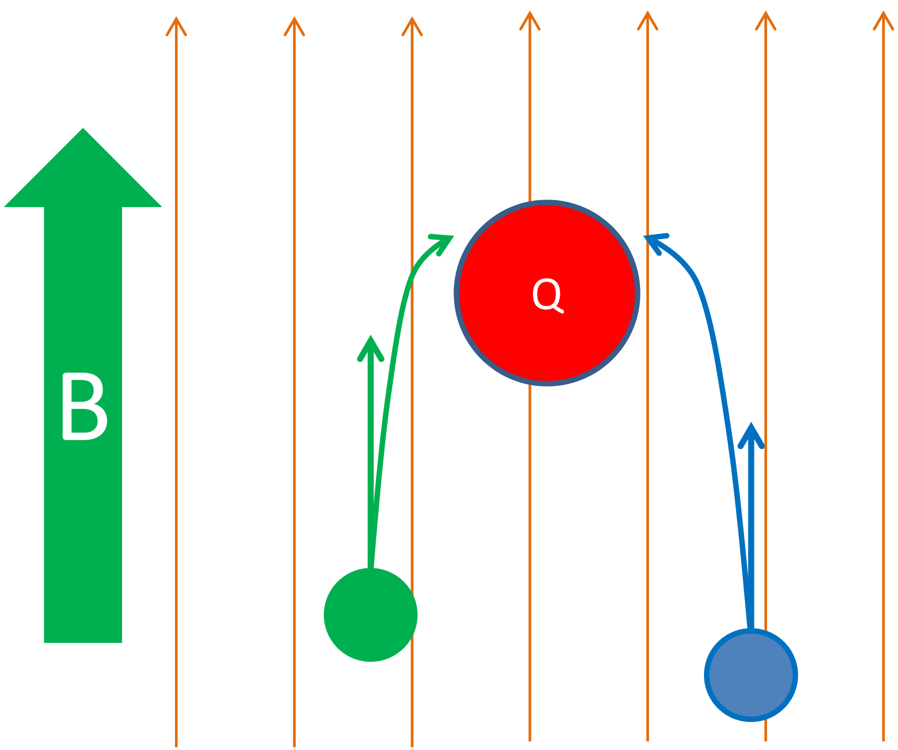
Correspondence btw the density of states

KH, K. Itakura, S. Ozaki, S. Yasui

S. Ozaki, K. Itakura, Y. Kuramoto

Possible application to the heavy-quark diffusion dynamics in QGP

The drag force on the heavy quarks may be enhanced by the Kondo effect in the strong magnetic field.

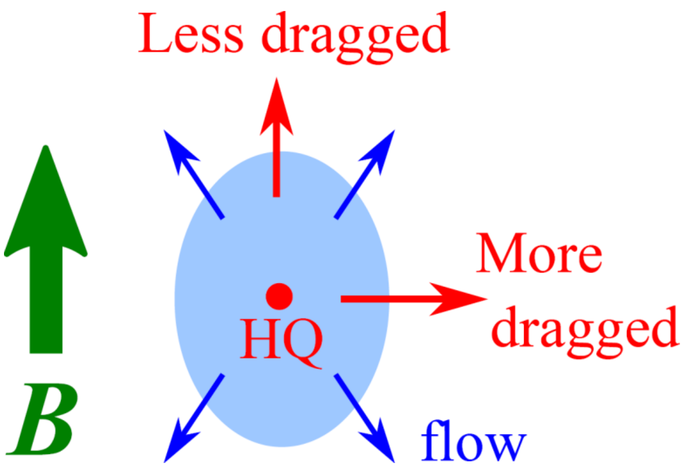


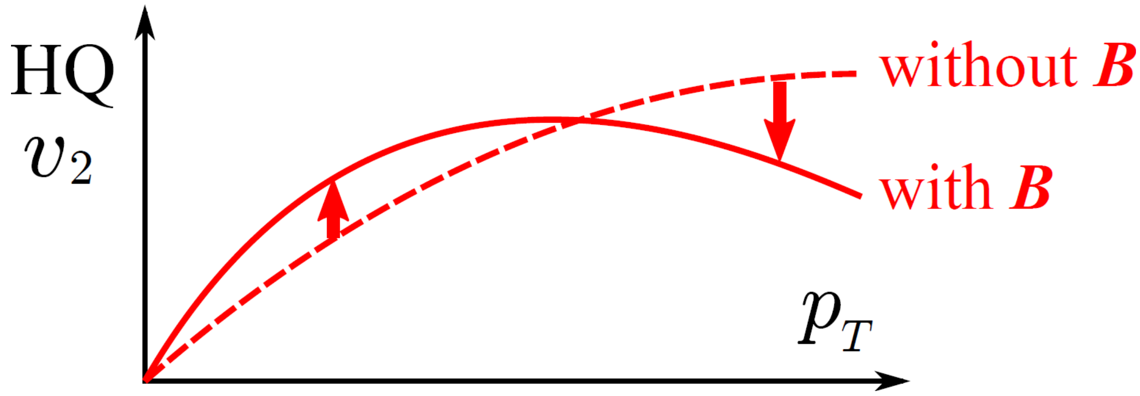
Interactions btw the heavy and light particles are strongly coupled in the low-energy.

The conductivity is enhanced because the carriers are “trapped” near the impurity.

There are reactions on the heavy particles, which may enhance the drag force.

Anisotropic diffusion constant at LO
---Generation of additional v2 of heavy flavors





K. Fukushima, KH, H.-U. Yee, Yi Yin, [arXiv:1512.03689 [hep-ph]]

Magnetic anisotropy gives rise to v2 of HQs
even without the v2 of medium.
→ Possible to generate v2 of HQs in the early QGP stage.

Kondo effect in B-field may occur in the NLO!

Summary

The effective dimensional reduction in the LLL gives rise to rich macroscopic consequences.

The low energy dynamics of the chiral fluid in a dynamical magnetic field is captured by the chiral MHD and contains novel collective excitations and instabilities.

Analogy between the systems at high density and in strong magnetic fields can be understood in terms of the dimensional reduction.

Backup slides

Kubo formulas

Huang, Rischke, Sedrakian (2011)
Hernandez and Kovtun (2017)

$$\kappa_{\parallel} = \frac{\partial}{\partial \omega} \mathrm{Im} G^R_{j^3 j^3}|_{\mathbf{p}=\mathbf{0},\omega\to 0},$$

$$\zeta_{\parallel} = \frac{1}{3}\frac{\partial}{\partial \omega}\left[2\mathrm{Im}G^R_{\tilde{P}_\perp \tilde{P}_\parallel} + \mathrm{Im}G^R_{\tilde{P}_\parallel \tilde{P}_\parallel}\right]_{\mathbf{p}=\mathbf{0},\omega\to 0},$$

$$\zeta_{\perp} = \frac{1}{3}\frac{\partial}{\partial \omega}\left[2\mathrm{Im}G^R_{\tilde{P}_\perp \tilde{P}_\perp} + \mathrm{Im}G^R_{\tilde{P}_\parallel \tilde{P}_\perp}\right]_{\mathbf{p}=\mathbf{0},\omega\to 0},$$

$$\eta_0 = \frac{\partial}{\partial \omega} \mathrm{Im} G^R_{T^{12} T^{12}}|_{\mathbf{p}=\mathbf{0},\omega\to 0},$$

$$\eta_1 = -\frac{4}{3}\eta_0 - 2\frac{\partial}{\partial \omega} \mathrm{Im} G^R_{\tilde{P}_\parallel \tilde{P}_\perp}|_{\mathbf{p}=\mathbf{0},\omega\to 0},$$

$$\eta_2 = -\eta_0 + \frac{\partial}{\partial \omega} \mathrm{Im} G^R_{T^{13} T^{13}}|_{\mathbf{p}=\mathbf{0},\omega\to 0},$$

$$\eta_3 = \frac{1}{2}\frac{\partial}{\partial \omega} \mathrm{Im} G^R_{\tilde{P}_\perp T^{12}}|_{\mathbf{p}=\mathbf{0},\omega\to 0},$$

$$\eta_4 = \frac{\partial}{\partial \omega} \mathrm{Im} G^R_{T^{13} T^{23}}|_{\mathbf{p}=\mathbf{0},\omega\to 0},$$

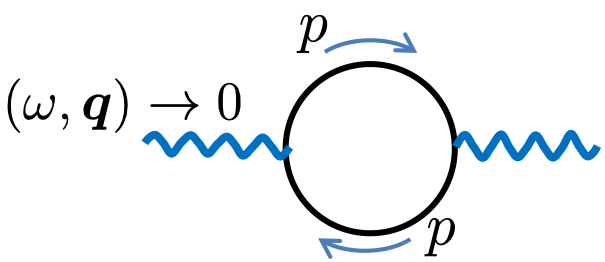
$$\tilde{P}_\parallel \equiv P_\parallel - \Theta_\beta \epsilon,$$
$$\tilde{P}_\perp \equiv P_\perp - (\Theta_\beta + \Phi_\beta)\epsilon,$$
with $\Theta_\beta \equiv (\partial P_\parallel / \partial \epsilon)_B$
and $\Phi_\beta \equiv -B(\partial M/\partial \epsilon)_B$.

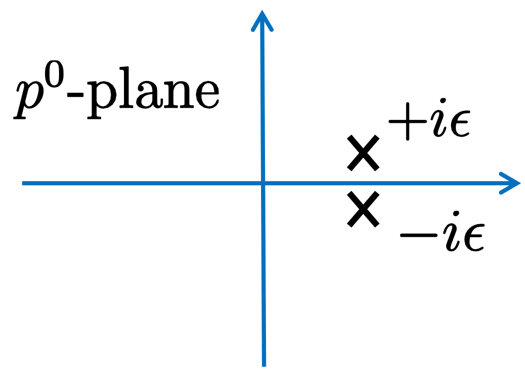
Consistent result from diagrammatic calculation

KH, D. Satow

Response function $\longrightarrow$ Kubo formula

$$j^\mu = \Pi_R^{\mu\nu} A_\nu(q) \qquad \boldsymbol{E} = i\omega \boldsymbol{A} \qquad j^i = \frac{\Pi_R^{ij}}{i\omega} \boldsymbol{E}^j$$

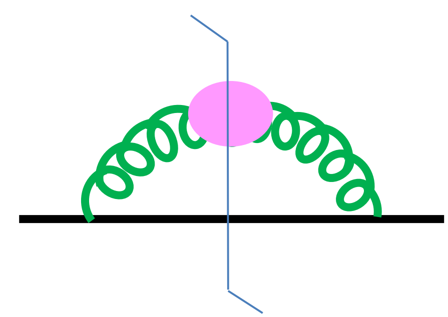




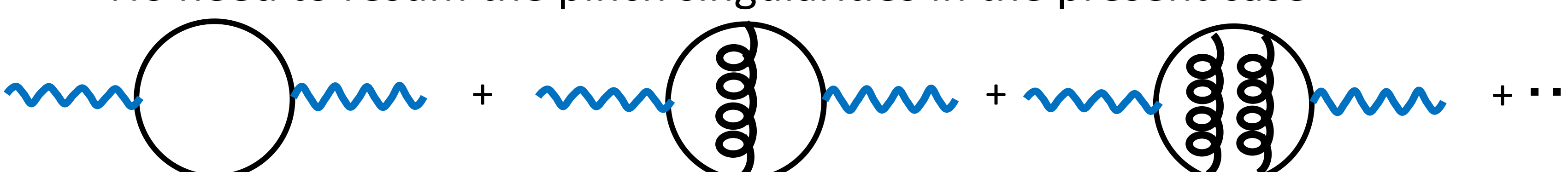
Divergence in free theory $\quad \Pi_R^{33} \sim \int \frac{dp_z}{2\pi} \frac{1}{i\epsilon}$

$$i\epsilon \to i\gamma_{\mathrm{damp}} \propto g^2 m_f^2 \ln(T/m_f)$$

Regularized by quark damping rate $\quad \Pi_R^{33} \sim \int \frac{dp_z}{2\pi} \frac{1}{i\gamma}$



No need to resum the pinch singularities in the present case



Energy-momentum tensor in the LLL

$$T^{\mu\nu}(x) = \frac{i}{2}\mathcal{S}\sum_{f}\left[\overline{\psi}\overleftarrow{D}^{\mu}\gamma^{\nu}\psi + \overline{\psi}D^{\mu}\gamma^{\nu}\psi\right]$$

In the Landau gauge,

$$\psi(x) = \int_{p_L, p^2} e^{-i(p_L \cdot x_L - p^2 x^2)}\mathcal{H}(x^1 - \frac{p^2}{eB})\mathcal{P}_{+}\,\chi(p_L) + \text{ (Contributions from n} \geq 1)$$

$$p_L^{\mu} \equiv (p^0, 0, 0, p^3),\ \bar{p}^{\mu} \equiv (p^0, 0, p^2, p^3)$$

$$\mathcal{P}_{\pm} = (1 + \mathrm{sgn}(eB) i\gamma^1\gamma^2)/2$$

When q → 0,

$$T^{\mu\nu}(q_L) = \int_{\bar{p}} \bar{\chi}(p_L + q_L)\gamma_L^{\mu} p_L^{\nu}\mathcal{P}_{+}\chi(p_L)$$

Subtraction of the equilibrium component

$$\delta P_{\|} \to \delta \tilde{P}_{\|} = \delta P_{\|} - \frac{\partial P_{\|}}{\partial e} e \quad e\text{: Energy density}$$

$$\delta \tilde{P}_{\|} = \frac{|eB|}{2\pi} \int \frac{dk_z}{2\pi} \frac{k_z^2 - \Theta \epsilon_k^2}{\epsilon_k} \delta f(k_z) \qquad \Theta = \frac{\partial P_{\|}}{\partial e}$$

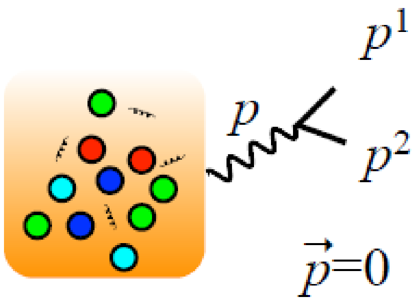
Note that $\Theta = \frac{1}{d_{\mathrm{space}}}$ in the massless limit.

$$\epsilon_k^2 = k_z^2 + m^2$$

$\delta \tilde{P}_{\|} \propto m^2$ due to a small deviation from the conformal limit.

Possible Phenomenological Implications

2. Soft Dilepton Production



G. D. Moore and J. -M. Robert, hep-ph/0607172.

$$\frac{d\Gamma}{d^4p} = \frac{\alpha}{12\pi^4\omega^2} T\sigma^{33}$$

$\because$(virtual photon emission rate)~$n_B(\omega)\mathrm{Im}\Pi^{\mu}{}_{\mu}$~$T\sigma^{33}$

(photon interaction energy w leptons) (quark mean free path)$^{-1}$

$$e\sqrt{eB} \ll \omega \ll \frac{g^2m^2}{T}\ln\left(\frac{T}{M}\right)$$

σ^{33} is large

Soft dilepton production is enhanced by B?

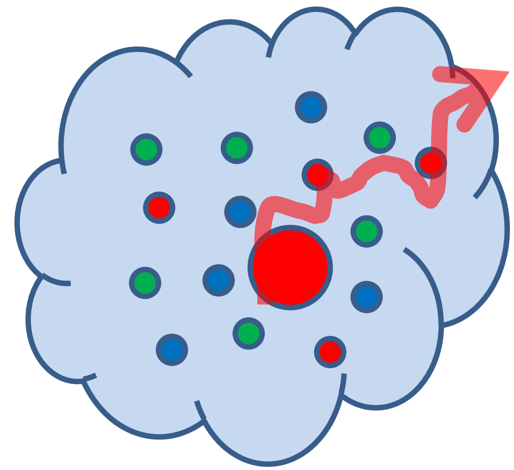
Heavy quark (HQ) dynamics in the QPG
-- In soft regime

Langevin equation

$$\frac{d\boldsymbol{P}}{dt} = \boldsymbol{\xi}(t) - \eta_D \boldsymbol{P}$$

Random kick (white noise)

$$\langle \xi_i(t)\xi_j(t')\rangle = \kappa\delta_{ij}\delta(t-t')$$



Drag force coefficient: η_D
Diffusion constant: κ

Einstein relation

$$\eta_D = \frac{\kappa}{2MT}$$

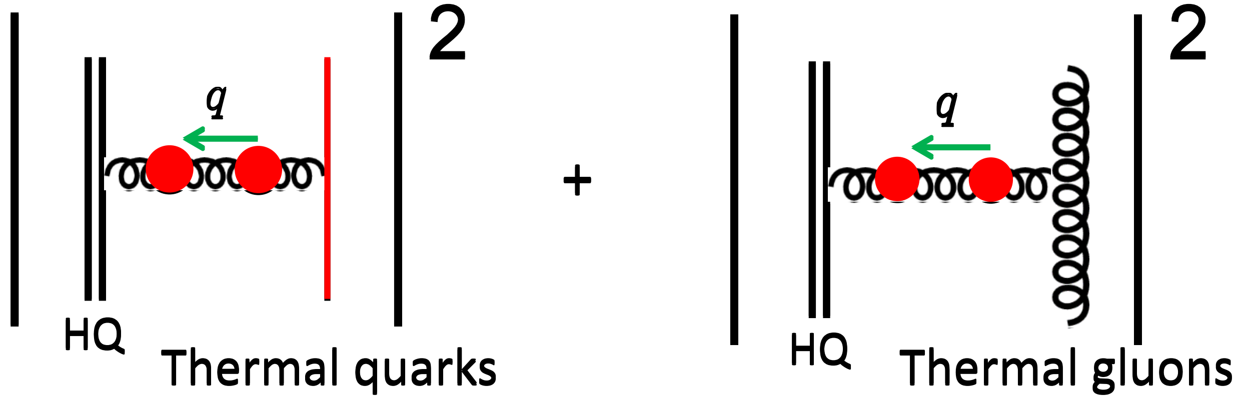
Perturbative calculation by finite-T field theory (Hard Thermal Loop resummation)
LO and NLO without B are known (Moore & Teaney, Caron-Huot & Moore).

Perturbative computation of momentum diffusion constant

$$\kappa_i = \int d^3\boldsymbol{q}\, q_i^2 \frac{d\Gamma}{d^3\boldsymbol{q}}$$

Momentum transfer rate in the LO Coulomb scatterings

$$\frac{d\Gamma}{d^3\boldsymbol{q}} =$$

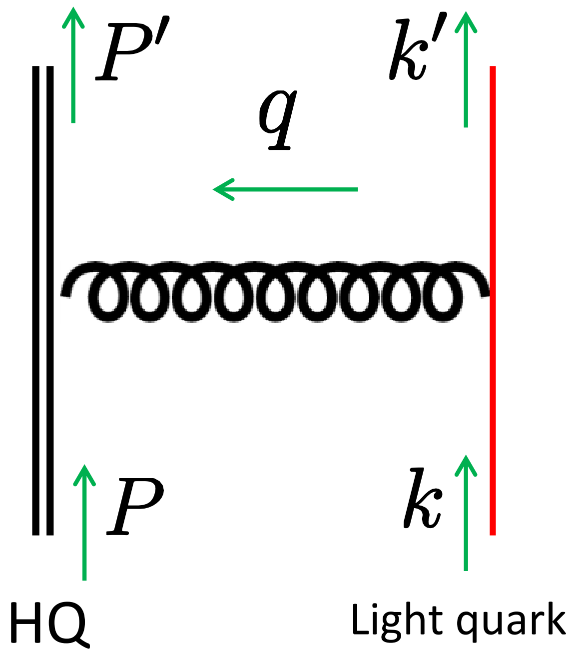


c.f.) LO and NLO without B (Moore & Teaney, Caron-Huot & Moore)

Effects of a strong magnetic field (eB >> T^2)

1. Modification of the dispersion relation of thermal quarks
2. Modification of the Debye screening mass

1. Prohibition of the longitudinal momentum transfer



Linear dispersion relation $k^0 = \pm k_z$

From the chirality conservation

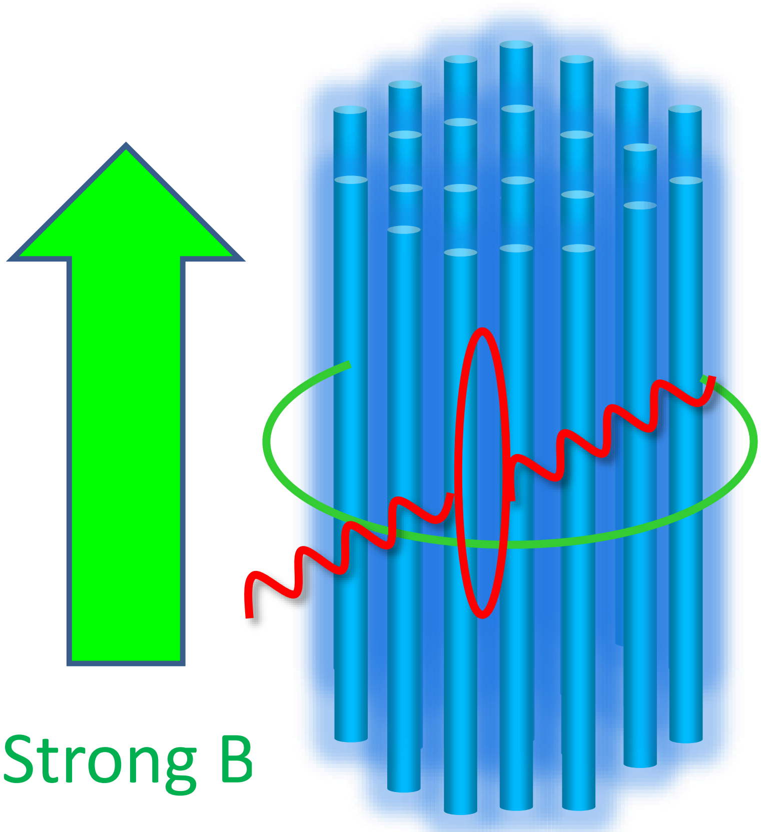
$$q^0 \;=\; \pm(k'_z - k_z) = \pm q_z$$

In the static limit (or HQ limit) $q^0 \to 0$

$$q_z \to 0.$$

$$\kappa_\parallel = 0 \text{ in massless limit, while } \kappa_\perp \neq 0.$$

2. Screening effect in a strong B



$$\rho = \frac{N_{\text{state}}}{S} = \frac{eB}{2\pi}$$

Gluon self-energy $\Pi^{\mu\nu}(q) = \frac{eB}{2\pi}\Pi^{\mu\nu}_{1+1}$

Schwinger model

$$\Pi^{\mu\nu}_{1+1} = \mathrm{tr}[t^a t^a]\frac{g^2}{\pi}(q_\parallel^2 g_\parallel^{\mu\nu} - q_\parallel^\mu q_\parallel^\nu)$$

$$m_D^2 \sim \frac{eB}{2\pi}\cdot\frac{g^2}{\pi} \gg (gT)^2$$

Anisotropic momentum diffusion constant

	Longitudinal	Perpendicular
Quarks	$\kappa_{\parallel}^{\text{quark}} = 0$	$\kappa_{\perp}^{\text{quark}} \sim \alpha_s^2 T \times eB \times \log \frac{T^2}{\alpha_s eB}$
Gluons	$\kappa_{\parallel}^{\text{gluon}} \sim \alpha_s^2 T \times T^2 \times \log \frac{T^2}{\alpha_s eB}$	$\kappa_{\perp}^{\text{gluon}} \sim \alpha_s^2 T \times T^2 \times \log \frac{T^2}{\alpha_s eB}$

Remember the density of states in B-field, $\rho = \frac{N_{\text{state}}}{S} = \frac{eB}{2\pi}$

In the strong B limit,

$$\frac{\kappa_{\parallel}}{\kappa_{\perp}} \sim \frac{\kappa_{\text{gluon}}}{\kappa_{\text{quark}} + \kappa_{\text{gluon}}} \sim \frac{T^2}{eB} < 1$$

Transverse diffusion constant in massless limit

$$\kappa_\perp = \alpha_s \lim_{q^0 \to 0} \frac{T}{q^0} \int d^3\boldsymbol{q}\, q_\perp^2 \frac{\mathrm{Im}\Pi(\boldsymbol{q})}{[\,\boldsymbol{q}^2 + m_D^2\,]^2}$$

Distribution function $n(q^0) \sim \dfrac{T}{q^0}$

Screened Coulomb scattering amplitude (squared)

$$m_D^2 \sim \alpha_s eB$$

Spectral density

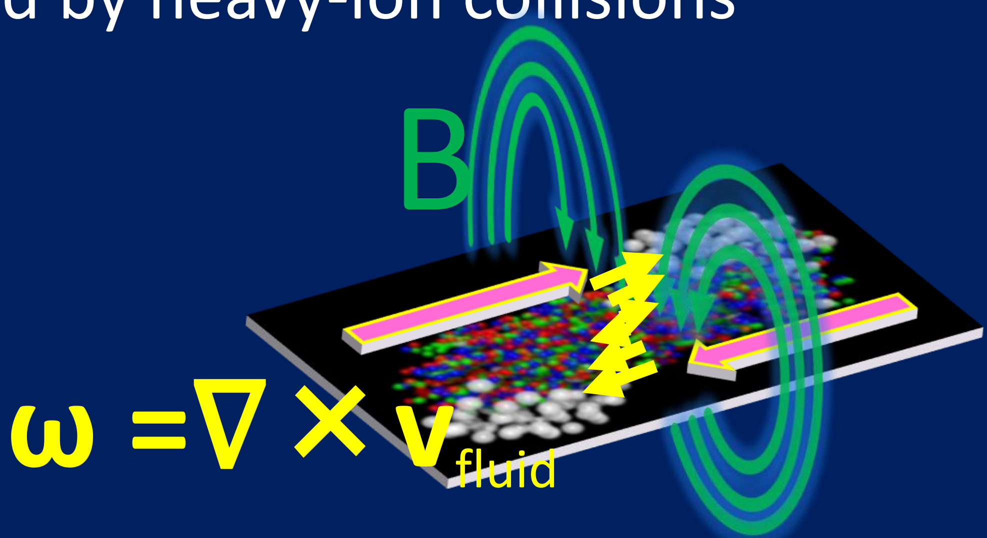
$$2\mathrm{Im}\Pi(\boldsymbol{q}) = \rho(\boldsymbol{q}) \sim m_D^2\, q^0 \delta(q_z)$$

$$\kappa_\perp \sim \alpha_s T \int d^2\boldsymbol{q}_\perp q_\perp^2 \frac{m_D^2}{[\,\boldsymbol{q}_\perp^2 + m_D^2\,]^2} \sim \alpha_s T m_D^2 \log 1/\alpha_s$$

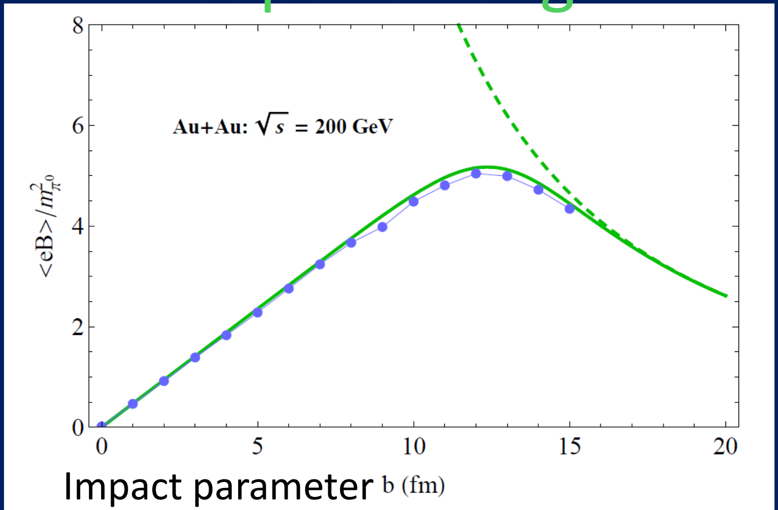
3. Anomalous transports from magneto-vorticity coupling

KH and Yi Yin, Phys. Rev. Lett. 117 (2016) 15. [arXiv:1607.01513 [hep-th]]

Strong magnetic field & vorticity/angular momentum induced by heavy-ion collisions

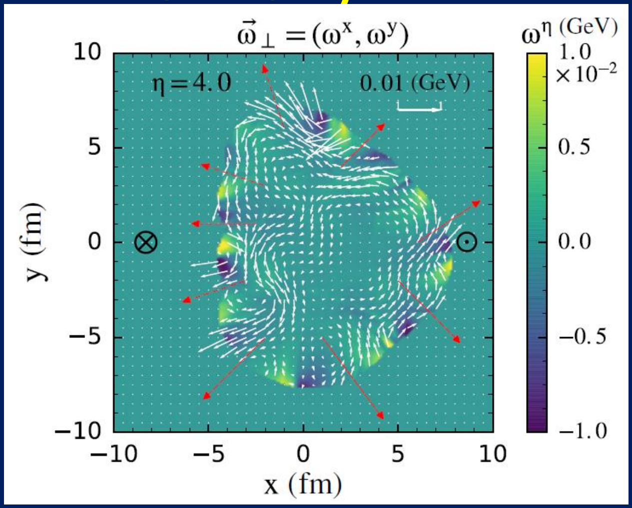


Super-strong B



Deng & Huang (2012), KH & Huang (2016)
Skokov et al. (2009), Voronyuk et al. (2011), Bzdak, Skokov (2012) McLerran, Skokov (2014)

Vorticity in HIC

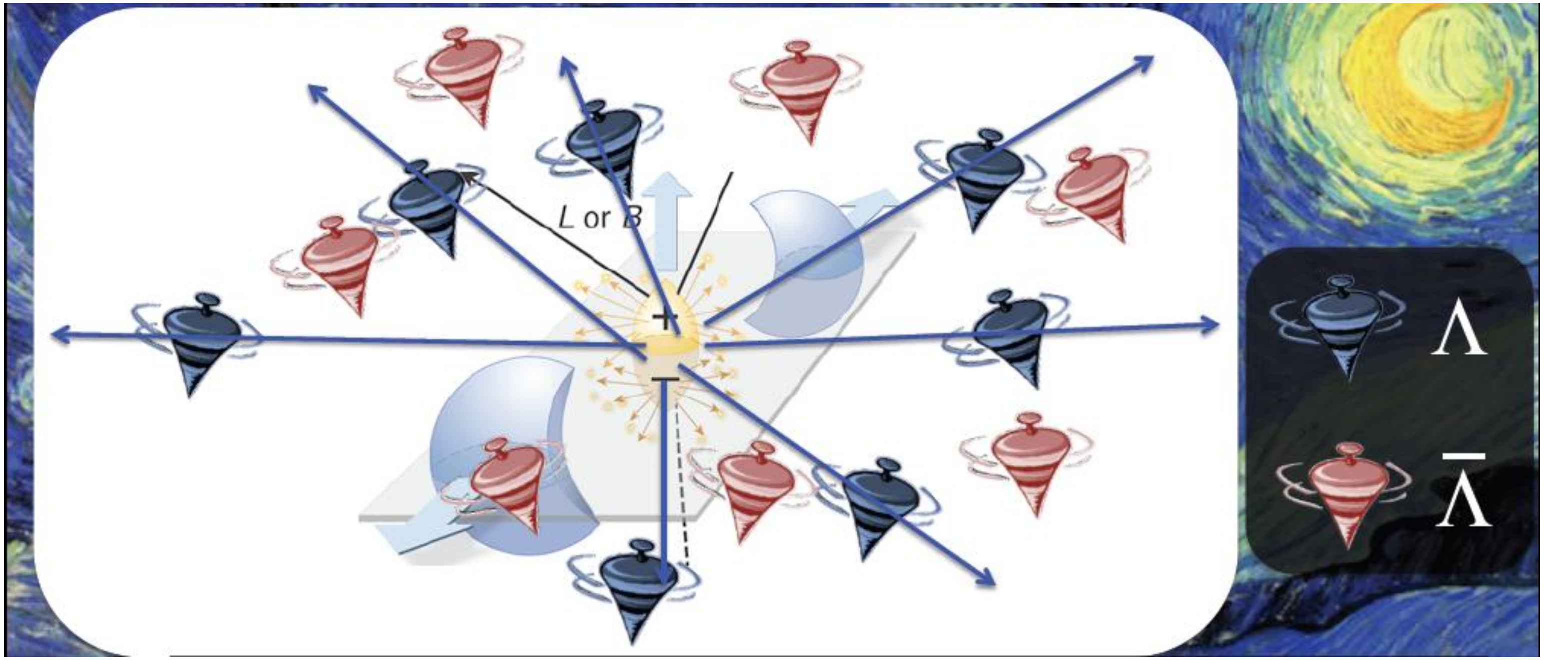


Pang, Petersen, Wang, Wang (2016)
Becattini et al., Csernai et al., Huang, Huovinen, Wang
Jiang, Lin, Liao(2016) Deng, Huang (2016)

Spin polarizations from spin-rotation coupling

$$f^{\pm}(\epsilon, \omega) = f_0^{\pm}(\epsilon - \boldsymbol{S} \cdot \boldsymbol{\omega}) \qquad f_0^{\pm}(\epsilon) = \frac{1}{e^{\pm\beta(\epsilon-\mu)} + 1}$$

Λ polarization



Slide by M. Lisa

See talks by Becattini, Niida, Konyushikhin, Li

Becattini et al., Glastad & Csernai, Gyulassy & Torrieri, Xie,,,,

An interplay B $\otimes$ ω

For dimensional reason, one would get

$$j \sim \# \; \boldsymbol{B} \cdot \boldsymbol{\omega}$$

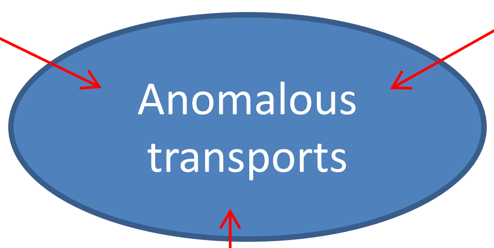
Cf., In CVE, it was

$$j^{\mu}_{R/L} = C_A \mu^2_{R/L} \omega^{\mu}$$

Could the magneto-vorticity coupling be important ??

QFT with Kubo formula

Anomalous hydrodynamics due to Son and Surowka



Chiral kinetic theory

$$j^{\mu} = \sigma_{\mathrm{Ohm}} E^{\mu} + \xi_B B^{\mu} + \xi_\omega \omega^{\mu}$$

The first-order derivative expansion $[A^{\mu} \sim \mathcal{O}(\partial^0),\ v^{\mu} \sim \mathcal{O}(\partial^0)]$

$$E^{\mu} \sim B^{\mu} \sim \omega^{\mu} \sim \mathcal{O}(\partial^1)$$

→ Yes, it is important when B is so strong that B >> O(∂^{1}).

Q1. Is the coefficient related to any quantum anomaly?
Q2. How is T and/or μ dependence?

Consequences of a magneto-vorticity coupling

Shift of thermal distribution functions by the spin-vorticity coupling

Spin-vorticity coupling

$$f^{\pm}(\epsilon, \omega) = f_0^{\pm}(\epsilon - \boldsymbol{S} \cdot \boldsymbol{\omega}) \qquad f_0^{\pm}(\epsilon) = \frac{1}{e^{\pm\beta(\epsilon-\mu)} + 1}$$

Landau & Lifshitz, Becattini et al.

In the LLL, the spin direction is aligned along the magnetic field .

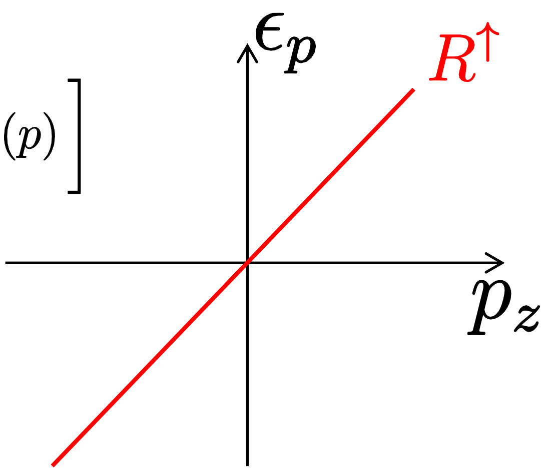
$$\Delta\epsilon^{\pm} \equiv -\boldsymbol{S} \cdot \boldsymbol{\omega} = \mp \mathrm{sgn}(q_f) \frac{1}{2} \hat{\boldsymbol{B}} \cdot \boldsymbol{\omega}$$

- for particle

+ for antiparticle

Number density

$$n_R = \frac{|q_f B|}{2\pi} \left[\int_0^{\infty} \frac{dp_z}{2\pi} f^+(p) + \int_{-\infty}^{0} \frac{dp_z}{2\pi} f^-(p) \right]$$



At the LO in the energy shift Δε

$$\Delta n_R = \frac{|q_f B|}{2\pi} \left[\Delta\epsilon^+ \int_0^{\infty} \frac{dp_z}{2\pi} \frac{\partial f_0^+(p_z)}{\partial p_z} + \Delta\epsilon^- \int_{-\infty}^{0} \frac{dp_z}{2\pi} \frac{\partial f_0^-(p_z)}{\partial p_z} \right]$$

$$\Delta n_R = q_f \frac{C_A}{4} \boldsymbol{B} \cdot \boldsymbol{\omega} [f_0^+(0) + f_0^-(0)]$$
$$= q_f \frac{C_A}{4} \boldsymbol{B} \cdot \boldsymbol{\omega}$$

$f_0^+(0) + f_0^-(0) = 1$ identically for any T and μ.

The shift is independent of the chirality, and depends only on the spin direction.

$$\Delta n_L = \Delta n_R$$

In the V-A basis,

$$\Delta n_V = \Delta n_R + \Delta n_L = q_f \frac{C_A}{2} \boldsymbol{B} \cdot \boldsymbol{\omega}$$
$$\Delta n_A = \Delta n_R - \Delta n_L = 0$$

½ from the size of the spin

Spatial components of the current

$$\Delta j_R^3 = v_R \Delta n_R \qquad j^1 = j^2 = 0 \text{ for the LLL}$$

Velocity: $v_{R/L} = \pm \mathrm{sgn}(q_f B)$

The shift depends on the chirality through the velocity.

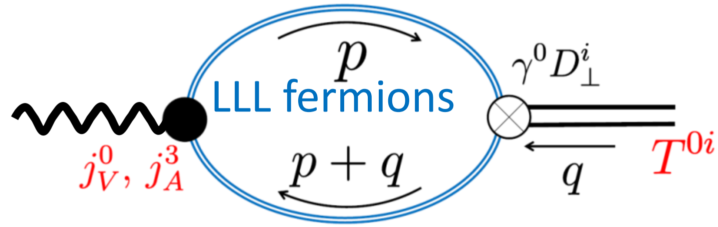
$$\Delta j_R^3 = -\Delta j_L^3$$

In the V-A basis,

$$\Delta j_V^3 = 0$$
$$\Delta j_A^3 = |q_f| \mathrm{sgn}(B) \frac{C_A}{2} \boldsymbol{B} \cdot \omega$$

Field-theoretical computation by Kubo formula

Perturbative ω in a strong B

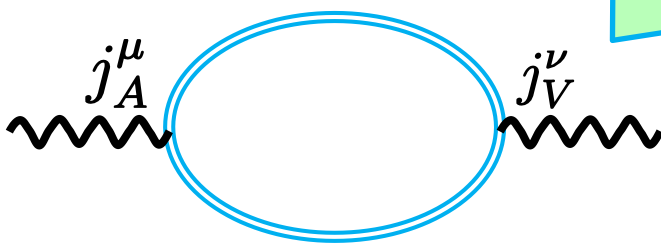


$$\lambda = -2i \lim_{q_x \to 0} \frac{\partial}{\partial q_x} \langle n_V(x) T^{02}(x') \rangle \theta(t-t')$$

Similar to the Kubo formula used to get the T^2 term in CVE (Landsteiner, Megias, Pena-Benitez)

We confirm

1. the previous results obtained from the shift of distributions.
2. a relation of $\langle n_V T^{02} \rangle$ to the chiral anomaly diagram in the (1+1) dim.

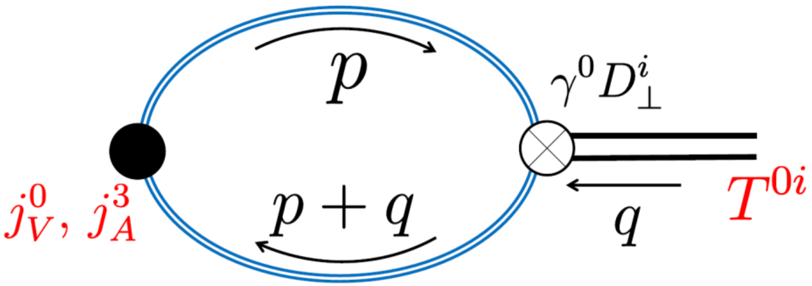
$\Pi_{AV}^{\mu\nu} =$ 

$$q_\mu \Pi_{AV}^{\mu\nu} \neq 0 \ !!!$$

There is no T or μ correction in the massless limit, since it is related to the chiral anomaly!

Field-theoretical computation by Kubo formula

$$\lambda = -2i \lim_{q_x \to 0} \frac{\partial}{\partial q_x} \langle n_V T^{02} \rangle$$



$$n_V(x) = \bar{\psi}(x)\gamma^0\psi(x)$$

$$T^{0i}(x) = \tfrac{i}{2}\bar{\psi}(x)(\gamma^0 D^i + \gamma^i D^0)\psi(x)$$

$$S_{LLL} = 2e^{-\frac{|\boldsymbol{p}_\perp|^2}{q_f B}} \frac{i}{\not{p}_\parallel + m_f} \mathcal{P}_+ \qquad \mathcal{P}_+ = (1 + i\mathrm{sgn}(q_f B)\gamma^1\gamma^2)/2$$

$$\langle n_V T^{02} \rangle \propto \frac{|q_f B|}{2\pi} q_x \Pi^{00}_{1+1}$$

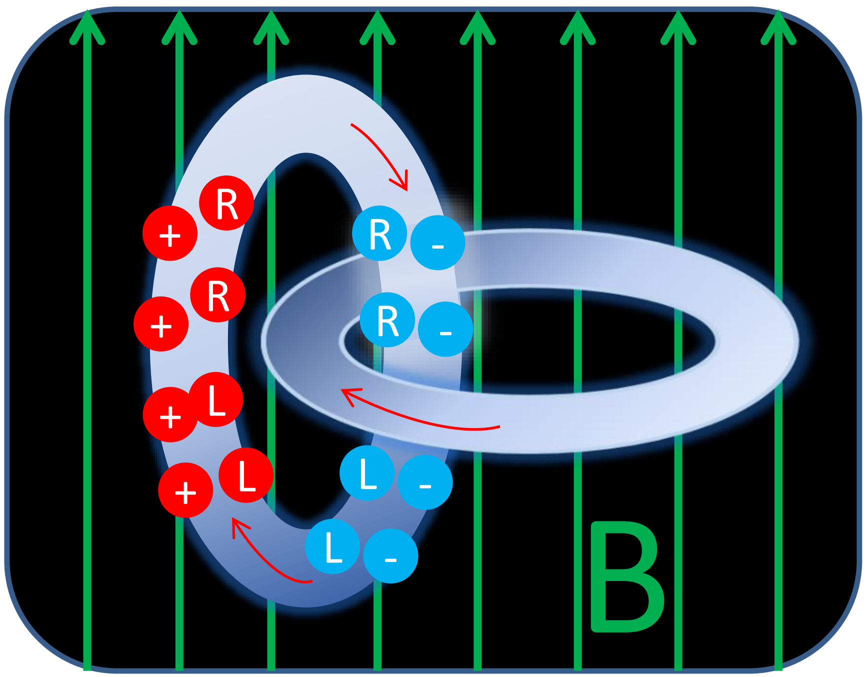
$$\Pi^{\mu\nu}_{1+1} = \int \frac{d^2 p_\parallel}{(2\pi)^2} \mathrm{tr}[\gamma^\mu_\parallel S_{1+1}(p_\parallel + q_\parallel)\gamma^\nu_\parallel S_{1+1}(p_\parallel)] = \frac{1}{\pi}\frac{1}{q^2_\parallel}(q^2_\parallel g^{\mu\nu}_\parallel - q^\mu_\parallel q^\nu_\parallel)$$

There is no T or μ correction in the massless limit!
→ Consistent with the previous observation from the shift of distributions.

Summary 2

A magneto-vorticity coupling B ⊗ ω induces charge redistributions without μ_A.

- Related to the chiral anomaly in the (1+1) dimensions.
- No T or μ correction.



When $\boldsymbol{B}\cdot\boldsymbol{\omega}\neq 0$,

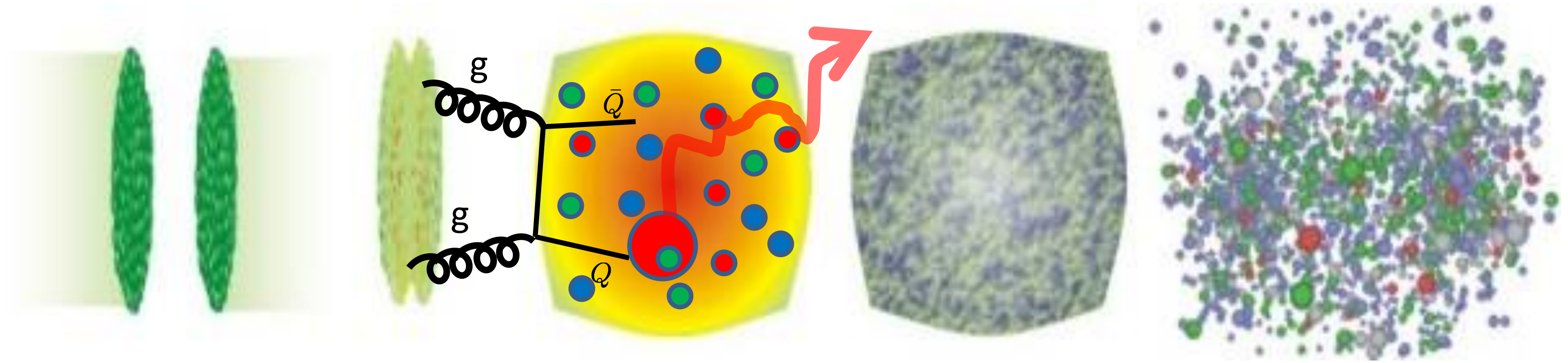
$$j^0_{EM,V} = q_f^2\frac{C_A}{2}(\boldsymbol{B}\cdot\boldsymbol{\omega})$$

$$j^3_{EM,A} = \mathrm{sgn}(q_f)q_f^2\frac{C_A}{2}(\boldsymbol{B}\cdot\omega)\hat{\boldsymbol{B}}$$

Emerges even without μ_A.

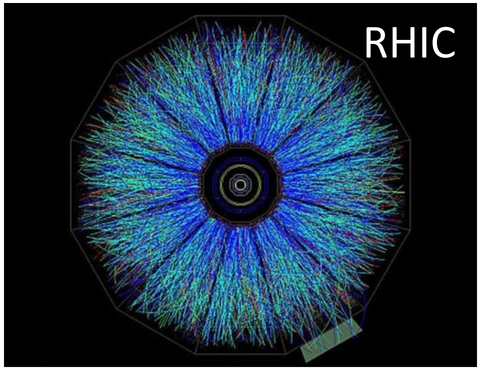
Coupling between the CME and fluid velocity induces a new instability in MHD. Take by Y. Hiron. KH, Hirono, Yee, Yin, In preparation.

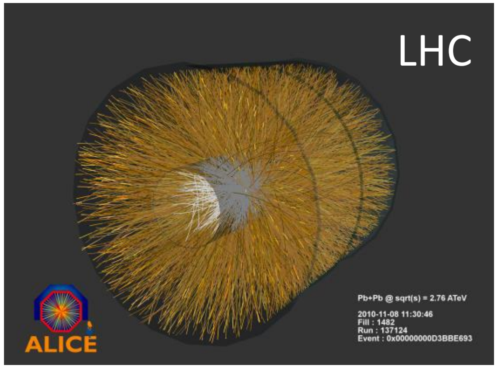
Heavy quarks as a probe of QGP



Thermal Quark-Gluon Plasma (QGP)

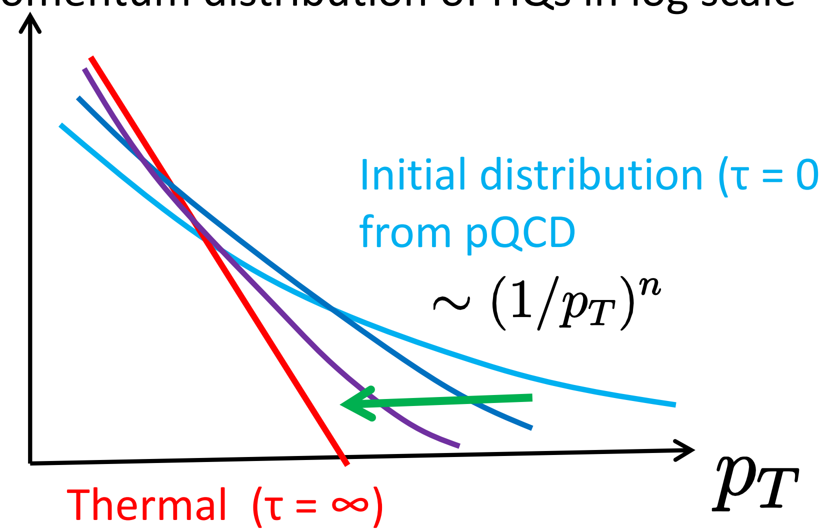
Hadrons





Non-thermal heavy-quark production in hard scatterings

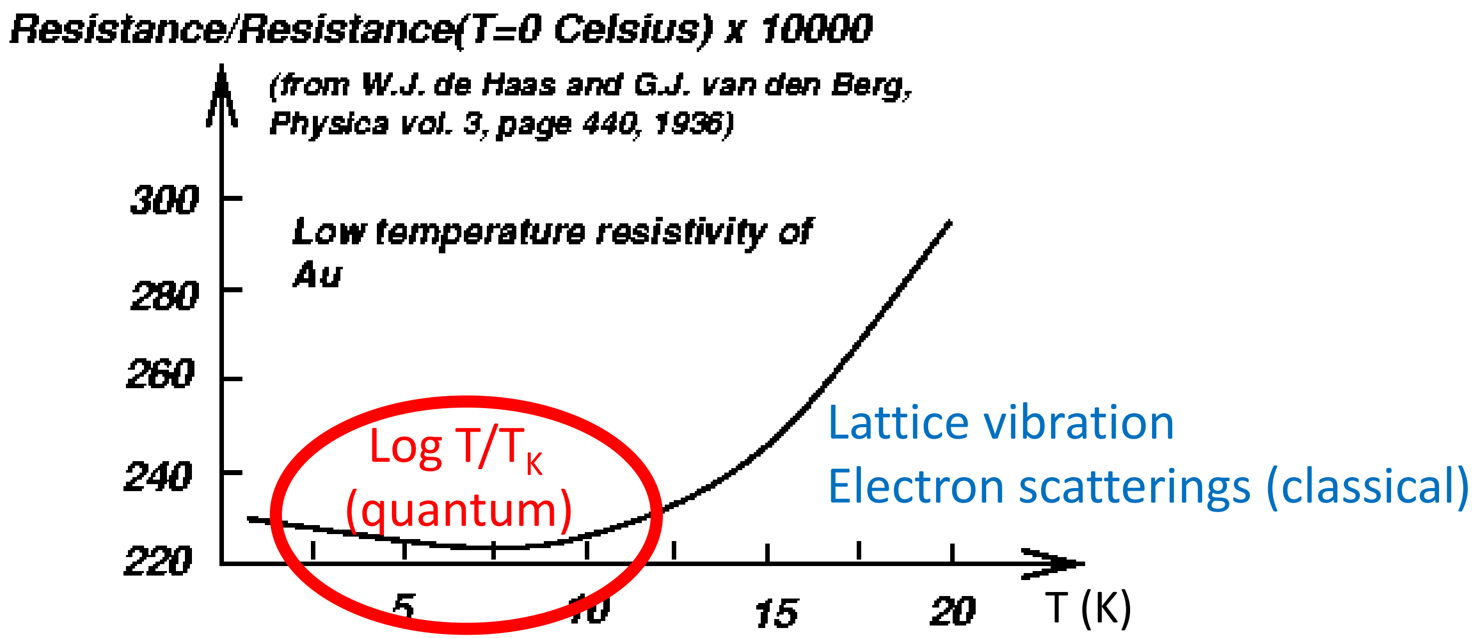
Momentum distribution of HQs in log scale



Relaxation time is controlled by transport coefficients (Drag force, diffusion constant)

Brief Introduction to Kondo effect in cond. matt.

Measurement of the resistance of alloy (with impurities)



T_K: Kondo Temp. (Location of the minima)

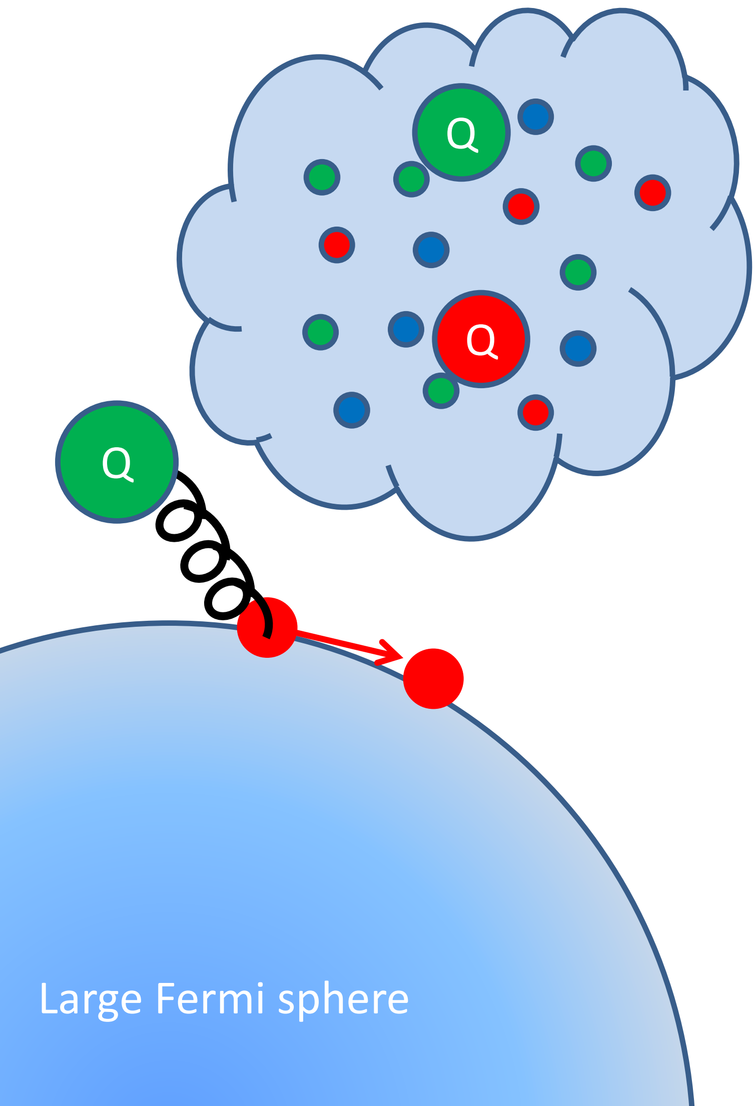
Progress of Theoretical Physics, Vol. 32, No. 1, July 1964

Resistance Minimum in Dilute Magnetic Alloys

Jun KONDO

Impurity scatterings near a Fermi surface

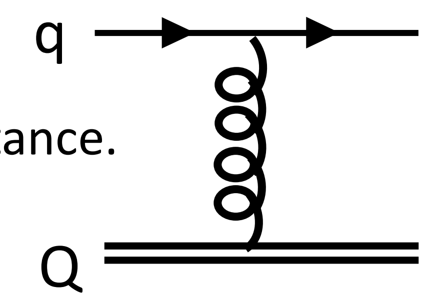
+ Electron-impurity scattering in conde. Matt.
+ Light-Heavy quark scattering in quark matter



$$G(\Lambda)(\bar{\psi}\psi)(\bar{\Psi}\Psi)$$

How does the coupling evolve with the energy scale, Λ --> 0, on the basis of Wilsonian RG?

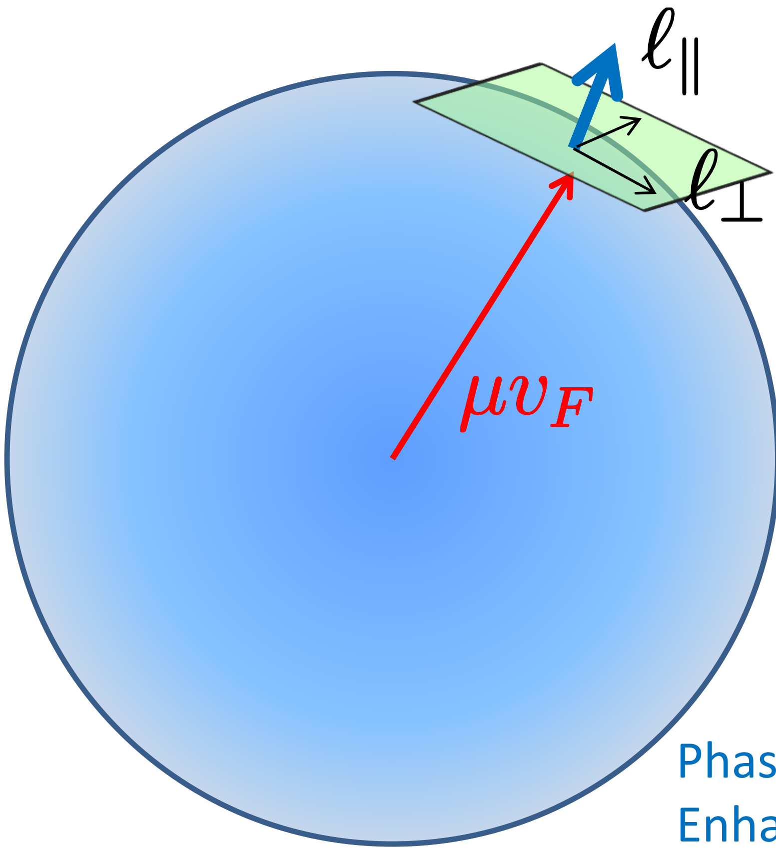
The LO does not explain the minimum of the resistance.



Logarithmic quantum corrections arise in special kinematics and circumstances.

→ BCS, Kondo effect, etc.

"Dimensional reduction" in dense systems
-- (1+1)-dimensional low-energy effective theory



+ Low energy excitation along radius [(1+1) D]

$$\epsilon = \pm \ell_{\parallel} \quad (\ell_{\parallel} \ll \mu)$$

+ Degenerated states in the tangential plane [2D]

Phase space volume ~ p^{D-1} dp.
Enhanced IR dynamics induces nonperturbative physics, such as superconductivity and Kondo effect.

Cf., Superconductivity occurs no matter how weak the attraction is.

IR scaling dimensions

$$\text{When } \epsilon \to s\epsilon,\ \ell_\parallel \to s\ell_\parallel. \quad (s<1)$$

Kinetic term

$$\mathcal{S}^{\text{kin}} = \int dt \sum_{\boldsymbol{v}_F} \int \frac{d^2\boldsymbol{\ell}_\perp d\ell_\parallel}{(2\pi)^3} \bar{\psi}_+ (i\partial_t - \ell_\parallel)\gamma^0 \psi_+ + \mathcal{O}(1/\mu)$$

$$0 = \underset{\bar{\psi}\cdot\psi}{2d_\psi} + \underset{dt}{(-1)} + \underset{d\ell_\parallel}{1} + \underset{\partial_t}{1} \qquad \boxed{d_\psi = -\frac{1}{2}}$$

Four-Fermi operators for superconductivity

Polchinski (1992)

$$\mathcal{S}^{\text{int}} = \int dt \left[\int \frac{d^2\boldsymbol{\ell}_\perp d\ell_\parallel}{(2\pi)^3} \right]^4 G[\bar{\psi}_+^{(4)} \hat{\gamma}_\parallel^\mu \psi_+^{(2)}][\bar{\psi}_+^{(3)} \hat{\gamma}_\mu^\parallel \psi_+^{(1)}] \delta^{(3)}(\boldsymbol{p}^{(1)} + \boldsymbol{p}^{(2)} - \boldsymbol{p}^{(3)} - \boldsymbol{p}^{(4)})$$

In general momentum config.

$$p^{(1)} + p^{(2)} \sim \mu \qquad d_{4-\text{Fermi}} = \underset{dt}{(-1)} + \underset{4(d\ell_\parallel + d_\psi)}{4(1-\frac{1}{2})} = +1$$

In the BCS config.

$$p^{(1)} + p^{(2)} \sim \ell_\parallel \ll \mu \qquad d_{4-\text{Fermi}} = (-1) + 4(1-\frac{1}{2}) \boxed{-1} = 0$$

Scaling dimensions in the LLL

KH, K.Itakura, S.Ozaki, hep-ph/1706.04913; a review paper to appear in PPNP.

When $\epsilon_{\mathrm{LLL}} \to s\epsilon_{\mathrm{LLL}}$, $p_z \to s p_z$. ($s < 1$; $\boldsymbol{p}_\perp$ does not scale.)

Kinetic term

$$S^{\mathrm{kin}}_{LLL} = \int dt \int dp_z \bar{\psi}_{LLL}(p_z)(i\partial_t \gamma^0 - p_z \gamma^3 - m_f)\psi_{LLL}(p_z)$$

$$0 = \underbrace{2d_\psi}_{\bar{\psi}\cdot\psi} + \underbrace{(-1)}_{dt} + \underbrace{1}_{dp_z} + \underbrace{1}_{\partial_t} \quad \Rightarrow \quad d_\psi = -\frac{1}{2}$$

A four-Fermi operator for the LLL

$$\mathcal{S}^{\mathrm{int}} = \int dt \left[\int \frac{dp_z}{2\pi}\right]^4 G[\bar{\psi}^{(4)}_{\mathrm{LLL}} \hat{\gamma}^\mu_\parallel \psi^{(2)}_{\mathrm{LLL}}][\bar{\psi}^{(3)}_{\mathrm{LLL}} \hat{\gamma}^\parallel_\mu \psi^{(1)}_{\mathrm{LLL}}]\delta(p_z^{(1)} + p_z^{(2)} - p_z^{(3)} - p_z^{(4)})$$

$$d_4 = -1 + 4 \times (1 - 1/2) - 1 = 0$$

Always marginal irrespective of the interaction type and the coupling constant thanks to the “dimensional reduction” in the LLL.

IR scaling dimension for Kondo effect

Heavy-quark Kinetic term

$$S_H^{\rm kin} = \int dt \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\, \Psi_+^\dagger(\boldsymbol{k})\, i\partial_t \Psi_+(\boldsymbol{k}) + \mathcal{O}(1/m_H)$$

$$\boxed{d_\Psi = (-1) + 1 = 0}$$

Heavy-light four-Fermi operator

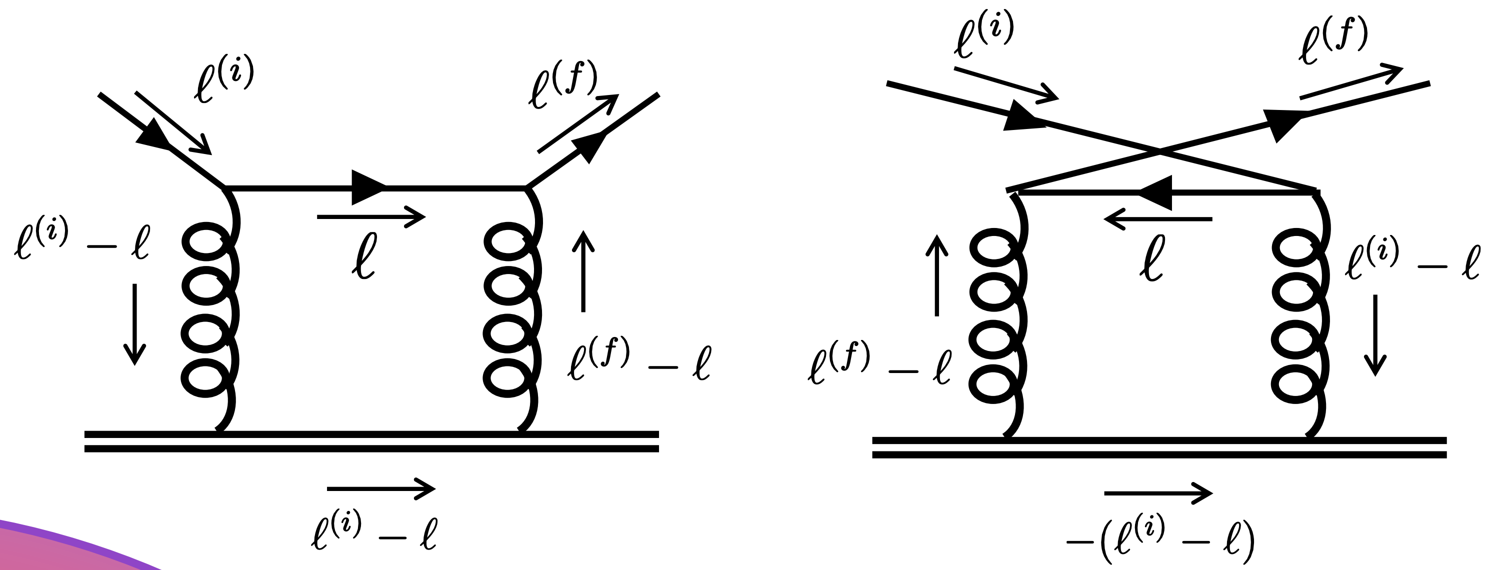
$$S_{\rm H-L}^{\rm int} = \int dt \left[\int \frac{d^2\boldsymbol{\ell}_\perp d\ell_\parallel}{(2\pi)^3}\right]^2 \left[\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\right]^2 G[\bar{\psi}_+^{(3)} t^a \psi_+^{(1)}][\bar{\Psi}_+^{(4)} t^a \Psi_+^{(2)}]$$

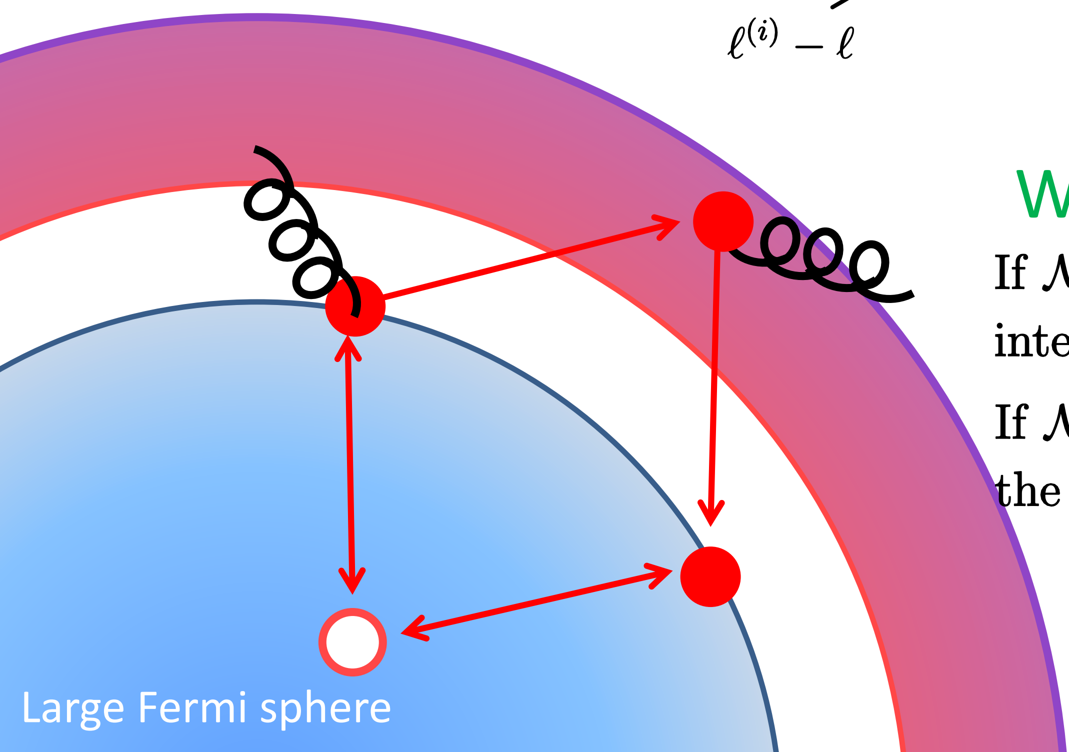
$$d_{\rm H-L} = (-1) + 2(1 + d_\psi) + 2d_\Psi = 0$$

Marginal !! Let us proceed to diagrams.

Scattering in the NLO
-- Renormalizaiton in the low energy dynamics

$$\mathcal{M} =$$





Wilsonian RG

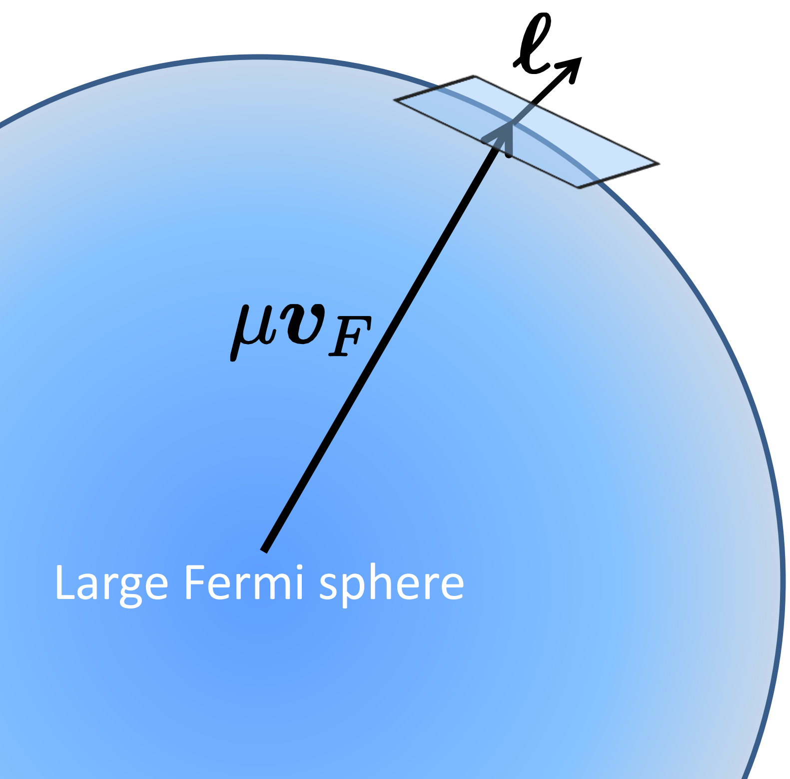
If $\mathcal{M} \sim \Lambda$,
interactions become less and less important.

If $\mathcal{M} \sim \log \Lambda$,
the fate depens on the sign of β func.

High-Density Effective Theory (LO)

Expansion around the large Fermi momentum

$$p^0 = \ell^0 , \quad \boldsymbol{p}^i = \mu \boldsymbol{v}_F^i + \boldsymbol{\ell}^i$$

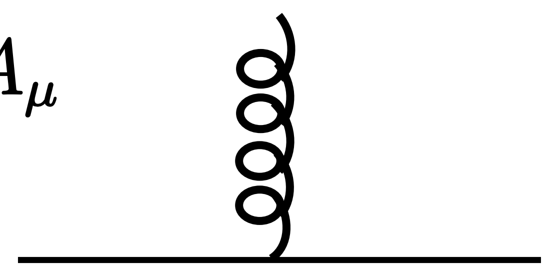


(1+1)-dimensional dispersion relation

$$\ell^0 = \boldsymbol{v}_F \cdot \boldsymbol{\ell} \equiv \ell_{\parallel}$$

Spin flip suppressed
when the mass is small m << μ.

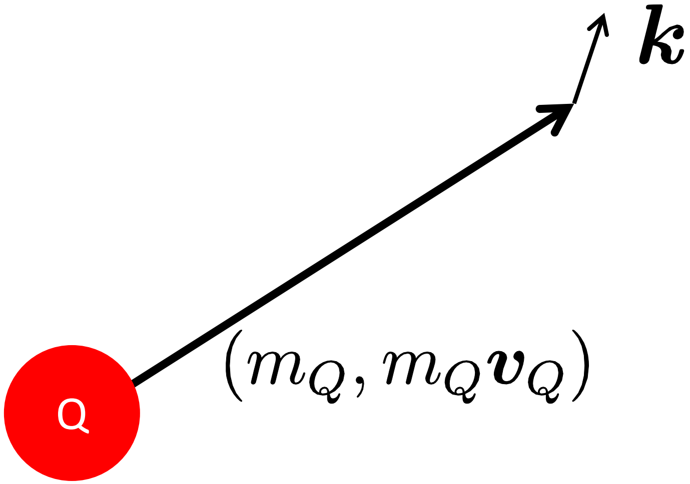
$$\gamma^\mu A_\mu \to \gamma^0 v_F^\mu A_\mu$$



Heavy-Quark Effective Theory (LO)

HQ-momentum decomposition

$$p^\mu = m_Q v_Q^\mu + k^\mu$$



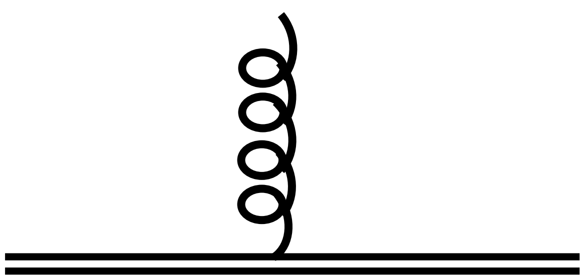
HQ velocity

$$v_Q^\mu = \frac{1}{m_Q} P^\mu \Big|_{P^2=m_Q^2}$$

Nonrelativistic magnetic moment suppressed by $1/m_Q$

$$\gamma^\mu A_\mu \to v_Q^\mu A_\mu$$

$$\gamma^\mu A_\mu = A^0 \text{ when } \vec{\boldsymbol{v}}_Q = 0.$$



Gluon propagator in dense matter

$$D^{\mu\nu}(k) = \frac{P_L^{\mu\nu}}{k^2 - \Pi_L} + \frac{P_T^{\mu\nu}}{k^2 - \Pi_T} - \xi \frac{k^\mu k^\nu}{k^4}$$

$$P_T^{\mu\nu} = \delta^{\mu i}\delta^{\nu j}\left(\delta^{ij} - \frac{k^i k^j}{|\boldsymbol{k}|^2}\right)$$

$$P_L^{\mu\nu} = -\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right) - P_T^{\mu\nu}$$

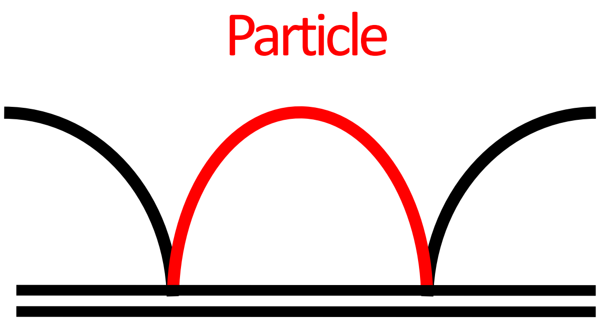
Screening of the <A^0A^0> from the HDL

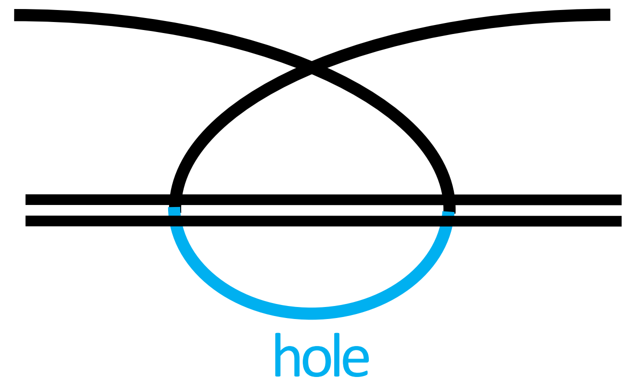
$$\Pi_L \sim m_{\mathrm{Debye}}^2 \sim (g\mu)^2$$

Cf., Son, Schaefer, Wilczek, Hsu, Schwetz, Pisarski, Rischke, ……,
showed that unscreened magnetic gluons play a role in the cooper paring.

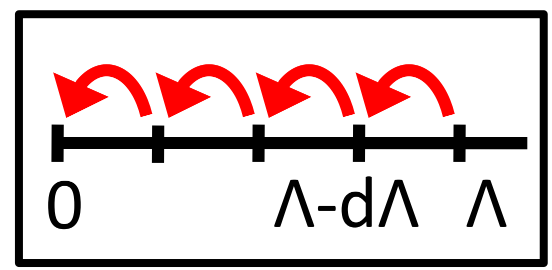
Important ingredients for Kondo effect

1. Quantum corrections



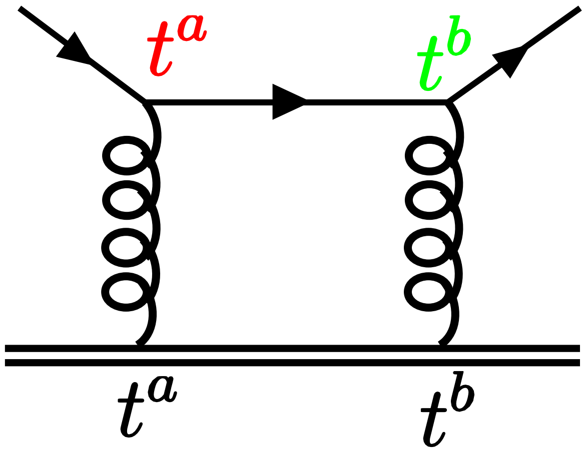


2. Log enhancements from the IR dynamics

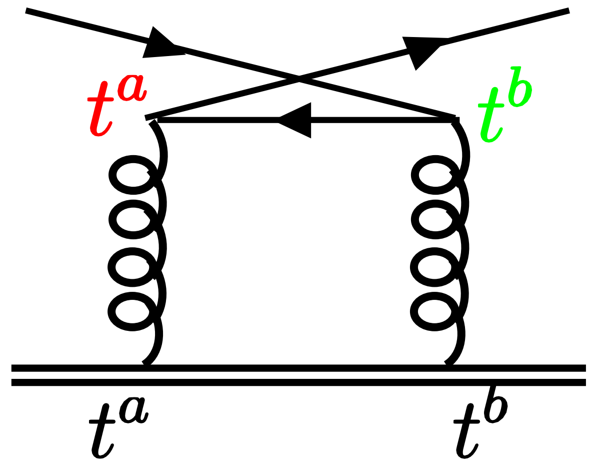


$$\rho_F \int_{\epsilon_F} \frac{d\epsilon}{\epsilon} \sim \log \frac{\Lambda}{\Lambda - d\Lambda} \qquad \rho_F \int^{\epsilon_F} \frac{d\epsilon}{\epsilon} \sim -\log \frac{\Lambda}{\Lambda - d\Lambda}$$

Color-matrix structures



$$\bar{u}^k (t^b)^{kj} (t^a)^{ji} u^i$$



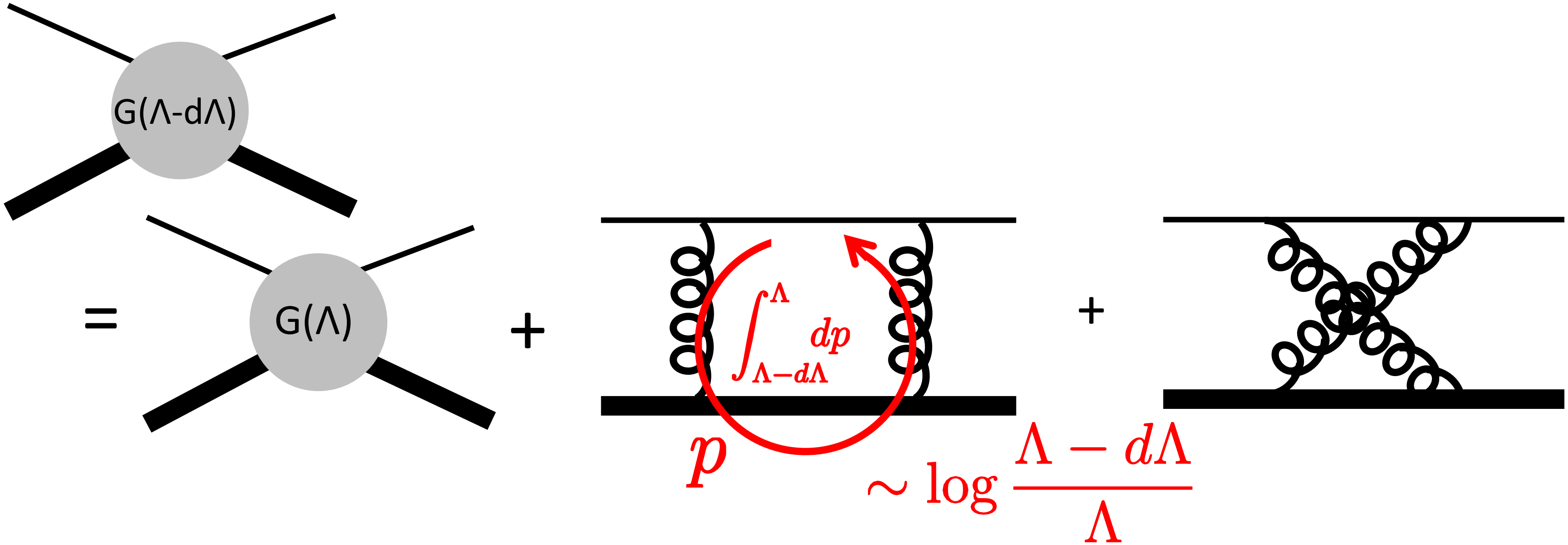
$$\bar{u}^k (t^a)^{kj} (t^b)^{ji} u^i$$

3. Incomplete cancellation due to non-Abelian interactions

Particle contribution $$[t^a t^b]_{ij} [t^a t^b]_{k\ell} = c\,\delta_{ij}\delta_{k\ell} - \frac{1}{n} t^a_{k\ell} t^a_{ij}$$

Hole contribution $$[t^a t^b]_{ij} [t^b t^a]_{k\ell} = c\,\delta_{ij}\delta_{k\ell} - \frac{1}{n} t^a_{k\ell} t^a_{ij} + \frac{n}{2} t^a_{k\ell} t^a_{ij}$$

RG analysis for “QCD Kondo effect”

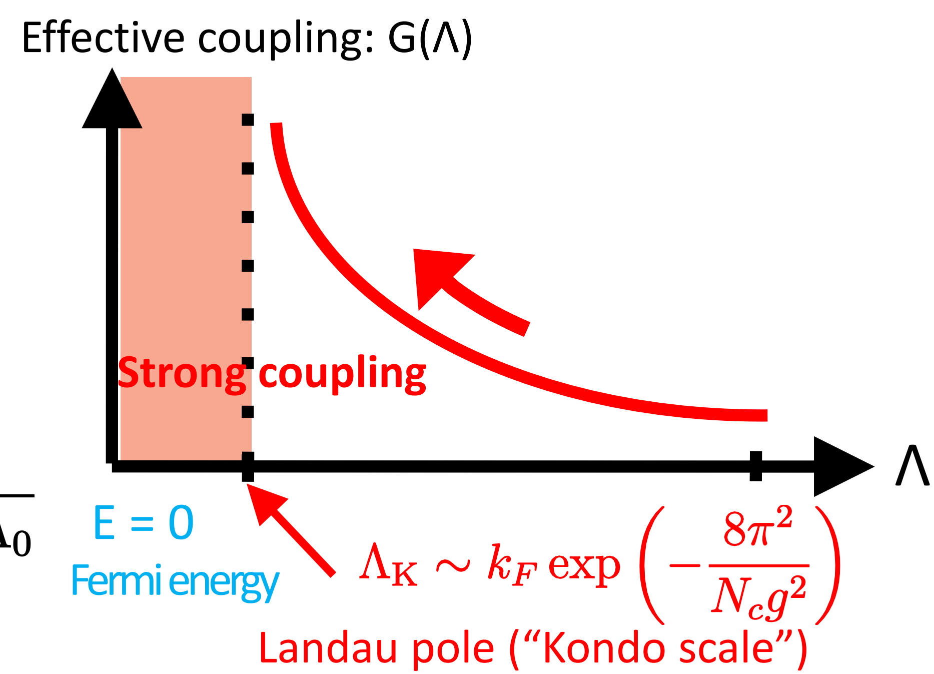


RG equation

$$\Lambda \frac{dG(\Lambda)}{d\Lambda} = -\frac{k_F^2}{8\pi^2} g^2 N_c G^2(\Lambda)$$

Asymptotic-free solution

$$G(\Lambda) = \frac{G(\Lambda_0)}{1 + g^2 N_c/(8\pi^2) G(\Lambda_0) \log \Lambda/\Lambda_0}$$

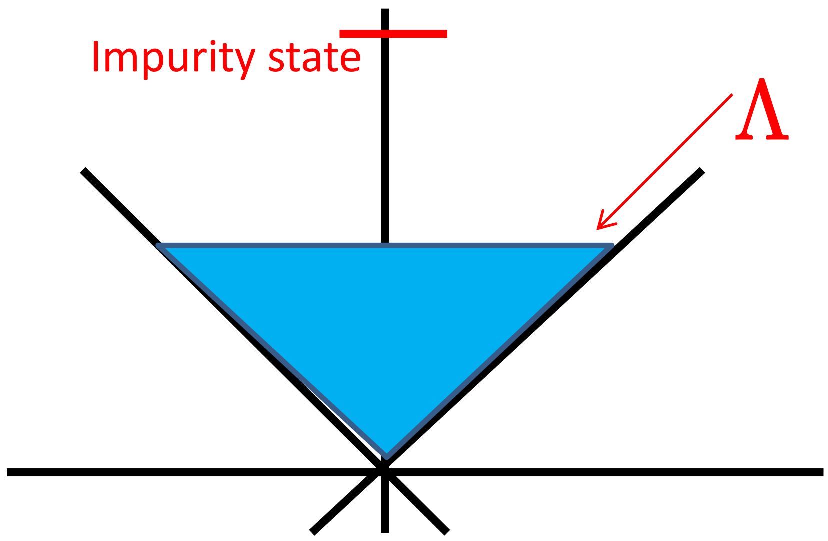


Short summary for Kondo effect in quark matter

1. Non-Ablelian interaction (QCD)

2. Dimensional reduction near the Fermi surface

3. Continuous spectra near the Fermi surface,
   and heavy impurities (gapped spectra).



An analogy between the dimensional reductions
in high-density matter and in strong magnetic field

Cf. S. Ozaki, K. Itakura, Y. Kuramoto, "Magnetically Induced QCD Kondo Effect ", **arXiv:1509.06966 [hep-ph]**

Cf., KH, K. Itakura, S. Ozaki, To appear in Prog. Part. Nucl. Phys.

Scaling dimensions in the LLL

When $\epsilon_{\rm LLL} \to s\epsilon_{\rm LLL}$, $p_z \to sp_z$. ($\boldsymbol{p}_\perp$ does not scale.)

(1+1)-D dispersion relation → $d_\psi = -1/2$

Four-light-Fermi operator

$$\mathcal{S}^{\rm int} = \int dt \left[\int \frac{dp_z}{2\pi}\right]^4 G[\bar\psi^{(4)}_{\rm LLL}\hat\gamma^\mu_\parallel \psi^{(2)}_{\rm LLL}][\bar\psi^{(3)}_{\rm LLL}\hat\gamma^\parallel_\mu \psi^{(1)}_{\rm LLL}]\delta(p_z^{(1)}+p_z^{(2)}-p_z^{(3)}-p_z^{(4)})$$

Always marginal thanks to the dimensional reduction in the LLL.

→ Magnetic catalysis of chiral condensate.

Chiral symmetry breaking occurs even in QED.

Gusynin, Miransky, and Shovkovy. Lattice QCD data also available (Bali et al.).

Heavy-light four-Fermi operator

$$S^{\rm int}_{\rm H-L} = \int dt \left[\int \frac{dp_z}{2\pi}\right]^2 \left[\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\right]^2 G[\bar\psi^{(3)}_{\rm LLL} t^a \psi^{(1)}_{\rm LLL}][\bar\Psi^{(4)}_{+} t^a \Psi^{(2)}_{+}]$$

Marginal !! Just the same as in dense matter.

Important ingredients of Kondo effect -- Revisited with strong B fields

1. Quantum corrections (loop effects)

2. Log enhancement from the IR dynamics due to the dimensional reduction in the strong B.

3. Incomplete cancellation due to non-Abelian color-exchange interactions

"QCD Kondo Effect"

$$\Lambda_{\mathrm{K}} \sim k_F \exp\left(-\frac{8\pi^2}{N_c g^2}\right)$$

KH, K. Itakura, S. Ozaki, S. Yasui, arXiv:1504.07619 [hep-ph]

"Magnetically Induced QCD Kondo Effect"

$$\Lambda_{\mathrm{K}} \sim \sqrt{q_{\mathrm{em}} B} \exp\left(-\frac{8\pi^2}{N_c g^2}\right)$$

S. Ozaki, K. Itakura, Y. Kuramoto, "Magnetically Induced QCD Kondo Effect ", **arXiv:1509.06966 [hep-ph]**