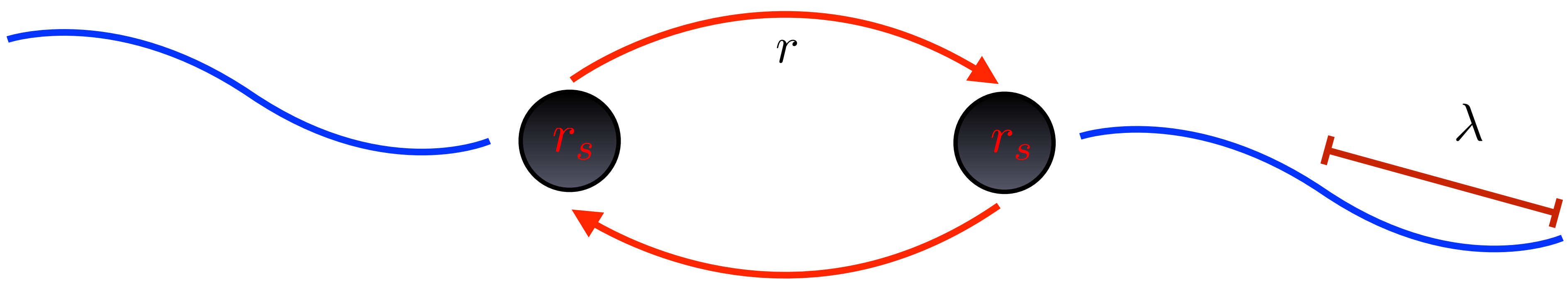
Dissipative effects in compact binary inspirals from EFT

Walter Goldberger
Yale U.

Based on work w/ Rothstein , past
+ in progress

Motivation:

Gravitational dynamics of radiating classical BH (or NS) binary systems in the non-relativistic limit is experimentally relevant (LIGO/VIRGO,…)



Even for $v \ll 1$, the non-linear nature of GR makes this a difficult problem, involving a hierarchy of length scales

Gravitational radius: $r_g = 2G_N M$

Physical radius: $r_s (= r_g \text{ for BH})$

Orbital scale: r

Radiation wavelength λ

$$r_g \sim r_s \gg r \gg \lambda$$

Experiments will be sensitive to at least v^6 corrections beyond Newtonian gravity (Thorne et al 1994). Numerical GR results also motivate computing higher order corrections.

In the NR limit $v/c \ll 1$ these scales are correlated:

$$r \sim r_g/v^2 \qquad\qquad \lambda \sim r/v \sim r_g/v^3$$

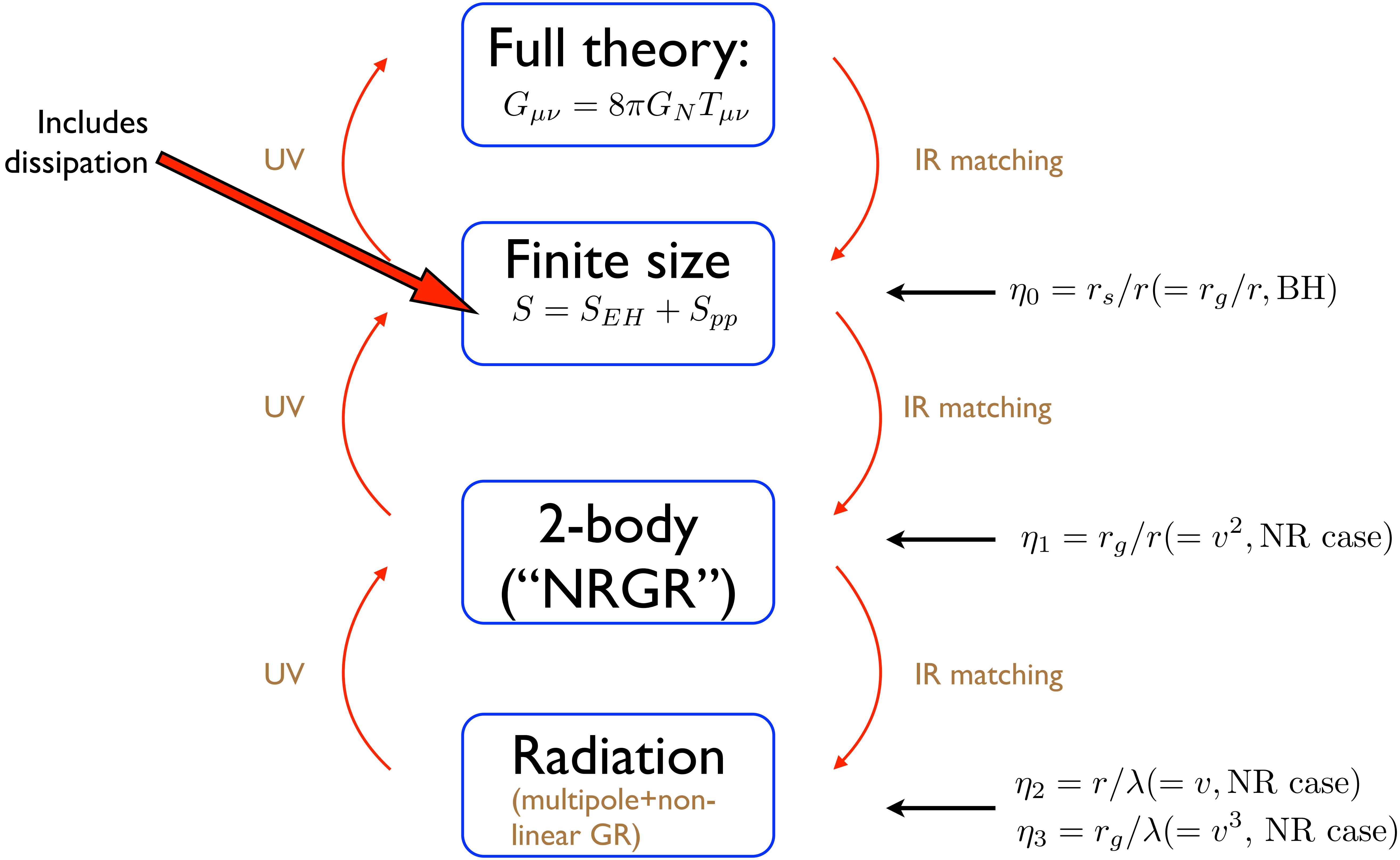
Thus at a fixed order in velocity ("Post-Newtonian expansion"), physics effects from all these scales may appear.

$\Longrightarrow$ Treat each scale separately, by constructing a tower of gravity Effective Field Theories

(WG+I. Rothstein, 2004)

The correct set of EFTs for the binary system has properties in common w/ its gauge theory counterparts (HQET, NRQED/NRQCD,…)

Tower of gravity EFTs:



Independent EFTs with distinct expansion parameter coincide in PN limit.
UV divergence in EFT_{i+1} corresponds to IR effect in EFT_i

EFT II: 2-body bound state

This is a theory of 2 pt non-relativistic particles, interacting gravitationally and emitting radiation:

$(\hbar = c = 1)$

$S_{EH} = -2m_{Pl}^2 \int d^4x \sqrt{g} R(x)$ $\quad S = S_{EH} + S_{pp}$ $\quad (m_{Pl}^2 = 1/(32\pi G_N))$

The most general (mod. e.o.m's) point particle Lagrangian consistent with symmetries:

$$S_{pp} = -m \int d\tau + c_E \int d\tau E_{\mu\nu} E^{\mu\nu} + c_B \int d\tau B_{\mu\nu} B^{\mu\nu} + \cdots$$

$E_{\mu\nu} = R_{\mu\alpha\nu\beta} v^\alpha v^\beta$ $\quad B_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\rho\sigma\lambda} v^\rho v^\alpha R_{\nu\alpha}{}^{\sigma\lambda}$

Systematically encodes finite size = tidal effects. Eg. "Love numbers"

(Flanagan+Hinderer, 2007) $c^{NS} \sim m R^4$ $\quad c_{BH,d=4} = 0$ (Damour et al; Poisson et al;Kol+Smolkin 2010)

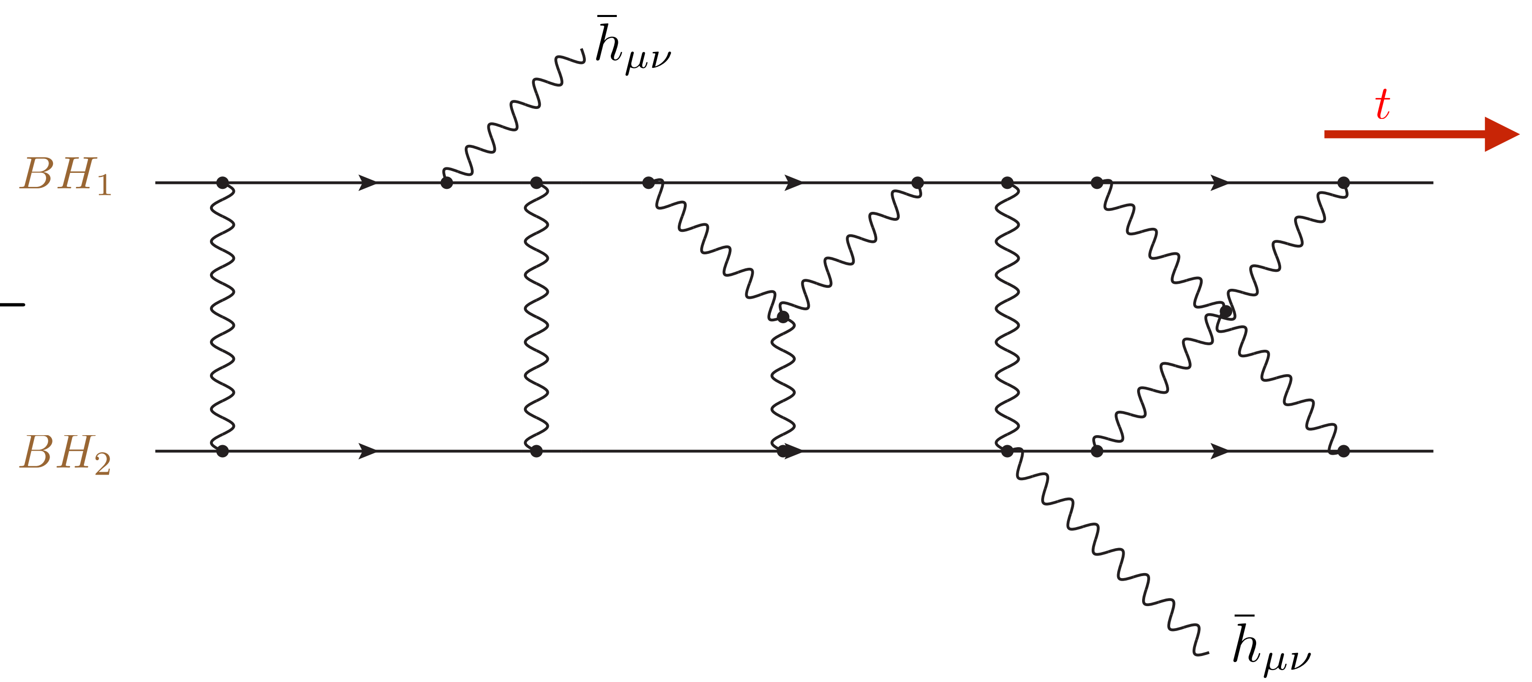
Ignoring spin and (until later in the talk) dissipation at the BH horizon.

The gravitational "Wilson line"

$$W = \exp i\Gamma[\bar{h}, x_a] = \int [\mathcal{D}h_{\mu\nu}]_{\text{b.c's}} e^{iS[h,\bar{h},x_a]}$$

generates all the observables of the (classical) binary system.
Diagrammatically:

$$W = \cdots +$$



$$+ \cdots$$

$$= e^{\sum(\text{BH irreducible diagrams})}$$

where we split up the metric into a background field and a "fluctuating part":

$$g_{\mu\nu} = \eta_{\mu\nu} + \bar{h}_{\mu\nu} + h_{\mu\nu}$$

background

fluctuation

and integrate out fluctuations. (work in background field gauge)

For example,

$$\Gamma[\bar{h}=0,x_a] = \int dt L(\mathbf{x}_a(t),\dot{\mathbf{x}}_a(t)) =$$ two-body Lagrangian

generates the equations of motion for the BH trajectories

The linear term in the background defines an effective energy-momentum tensor:

$$\Gamma[\bar{h}=,x_a] = \cdots + \frac{1}{2m_{Pl}}\int d^4x T^{\mu\nu}(x)\bar{h}_{\mu\nu} + \cdots$$

$$\partial_\mu T^{\mu\nu}(x) = 0$$

(Ward id. for diff invariance)

which can be used to compute radiation at infinity

In particular, with standard in/out (Feynman) b.c.'s, graviton emission amplitude is

$$\mathcal{A}_{h=\pm 2}(k) = \int d^4x e^{ik\cdot x} \epsilon^*_{\mu\nu}(h,k) T^{\mu\nu}(x)$$

and the graviton emission rate over $T \to \infty$

$$d\Gamma_h(\mathbf{k}) = \frac{1}{T} \frac{d^3\mathbf{k}}{(2\pi)^3 2|\mathbf{k}|} |\mathcal{A}_h(\mathbf{k})|^2,$$

yield time-averaged energy and momentum emission rates:

$$\langle \dot{P}^\mu \rangle_{h=\pm 2} = \int k^\mu d\Gamma_h(\mathbf{k}),$$

$$\langle \dot{\mathbf{J}} \rangle = 2\int \mathbf{n} d\Gamma_{h=2}(\mathbf{k}) - 2\int \mathbf{n} d\Gamma_{h=-2}(\mathbf{k}),$$

(Equivalently, the radiated power spectrum follows directly from the effective action:

$$\frac{1}{T}\mathrm{Im} S_{eff}[x_a] = \frac{1}{2}\int dE d\Omega \frac{d^2\Gamma}{dE d\Omega}, \longrightarrow \quad dP = E d\Gamma, \text{)}$$

Using in/in boundary conditions (as in cosmology) gives instantaneous observables, e.g. radiation field at infinity:

$$h_{\mu\nu}(\mathbf{x} \to \infty, t) = \int d^4y D^{\mathrm{ret}}_{\mu\nu;\alpha\beta}(x-y) T^{\alpha\beta}(y)$$

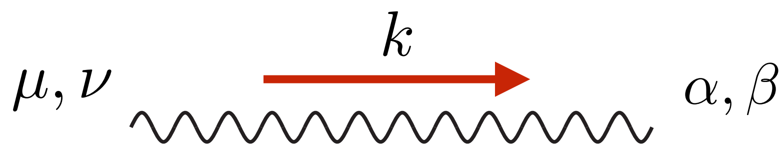
which yields the time-dep. waveform seen in the detector.

(C. Galley)

To compute the generating function W one could use standard covariant Feynman rules obtained by expanding $S_{EH} = -2m_{Pl}^2 \int d^4x \sqrt{g} R$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/m_{Pl}$$

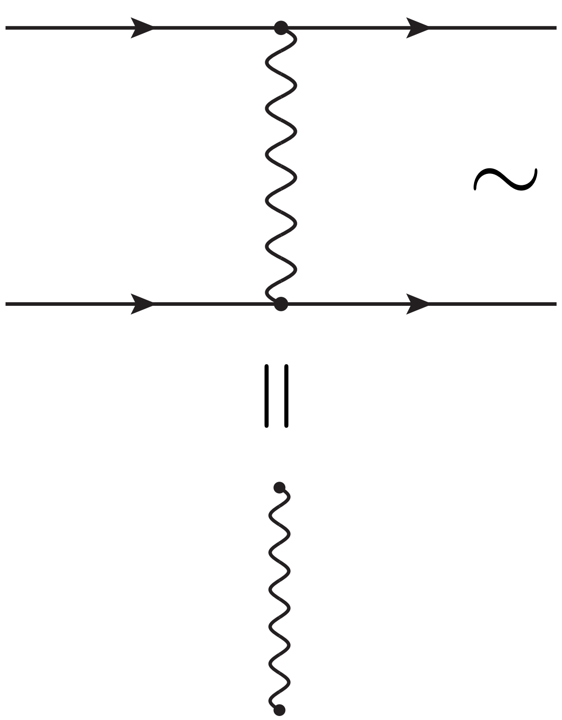
w/ e.g



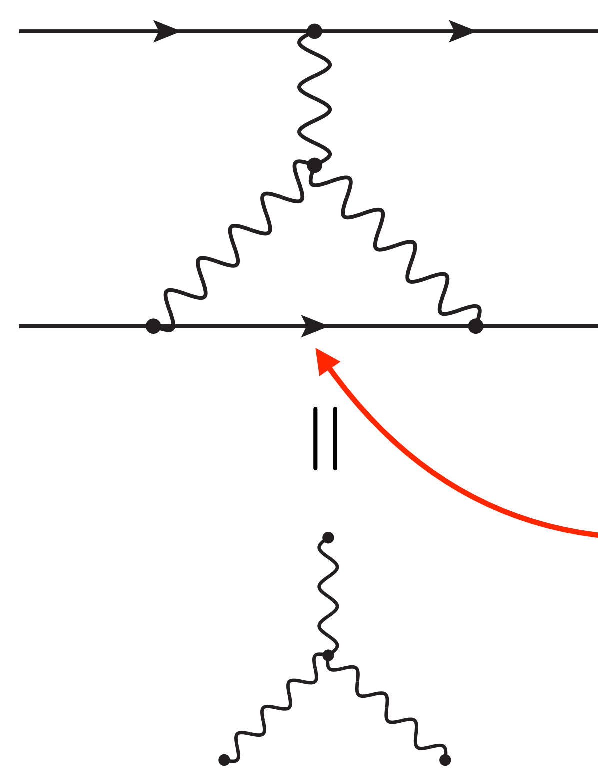
$$= \frac{i}{k^2} P_{\mu\nu;\alpha,\beta}$$

(Feynman gauge)

However, these Feynman rules are not optimal optimal for the NR limit $v \ll 1$
The diagrams don't have manifest power counting in the exp. parameter:



$$\sim v^2 + v^4 + \cdots$$



$$\sim v^?$$

(NOT A PROPAGATOR!)

The problem is that the diagrams involve momentum integrals over all momentum regions. However, for NR kinematics, two momentum space configurations dominate:

"potential": $(E \sim 0, \vec{p} \sim 1/r)$ (off-shell)

"radiation": $(E \sim v/r, \vec{p} \sim v/r)$

The solution to this problem is well known (see HQET, NRQED/NRQCD, SCET). Decompose graviton into distinct momentum modes and "pull out" short scales:

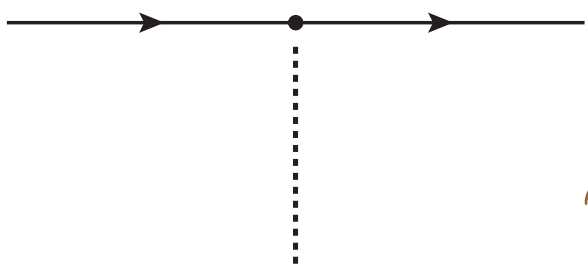
$$\partial_\mu H_{\mathbf{k}} \sim \frac{v}{r} H_{\mathbf{k}}$$

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \bar{h}_{\mu\nu}(x) + \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} H_{\mathbf{k}\,\mu\nu}(x^0)$$

$$\partial_\mu \bar{h} \sim \frac{v}{r}\bar{h} \qquad \mathbf{k} \sim \frac{1}{r}$$

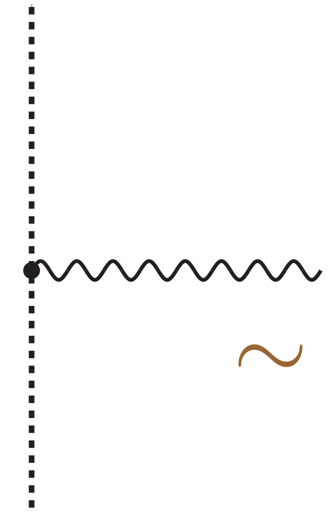
The radiation mode can be regarded as long wavelength background field in which potential gravitons propagate

In addition, need to multipole expand the couplings of the radiation mode to the particles and to the potentials. This yields an effective Lagrangian with manifest power counting in velocity:



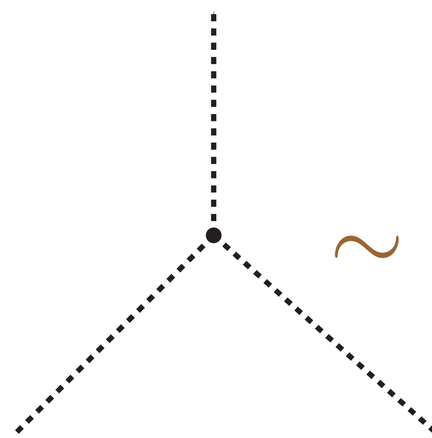
$$\sim \sqrt{L}$$

Pt. particle-Newton potential interaction:



$$\sim v^{5/2}/\sqrt{L}$$

Radiation-potential interaction

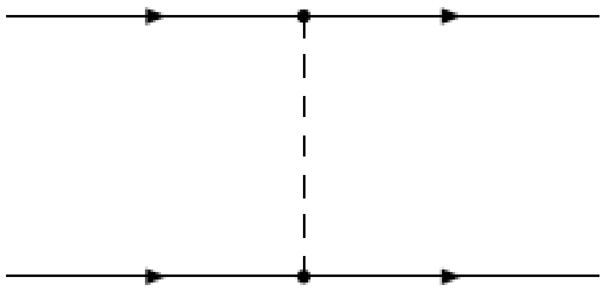


$$\sim v^2/\sqrt{L}$$

Potential graviton cubic self-interaction

By connecting vertices together, generate the 2-body potentials and the interactions of matter with radiation. Drop quantum corrections $\sim \hbar/L \ll 1$

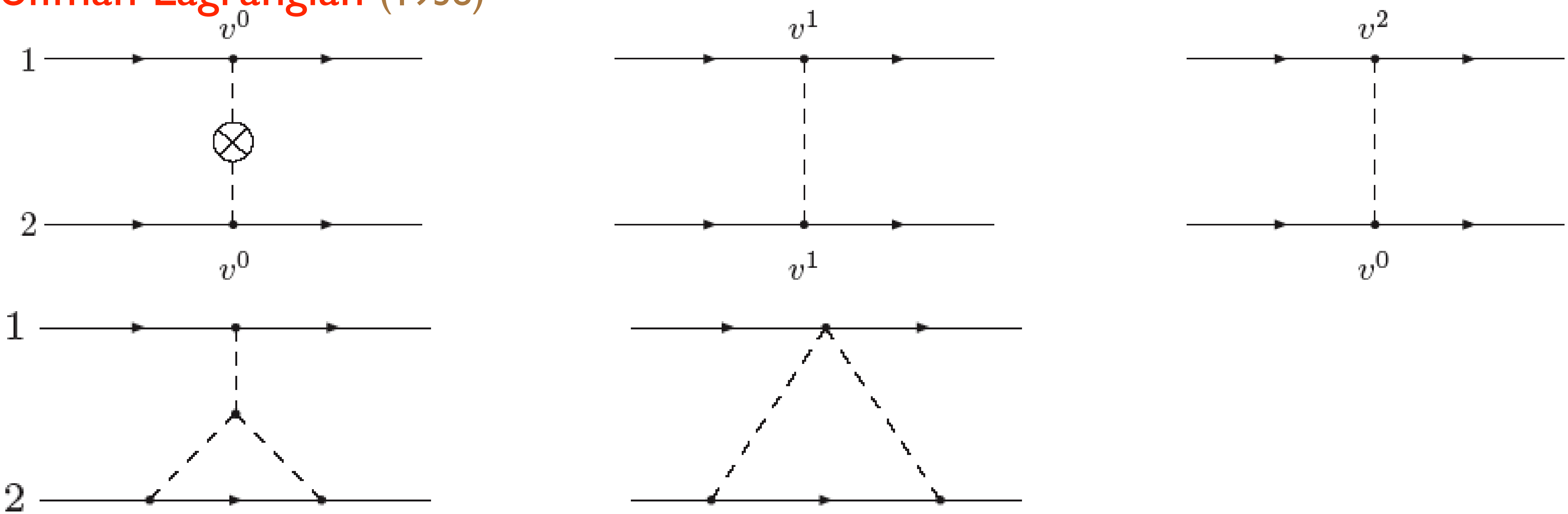
Leading order: Newton (1687)



(a)

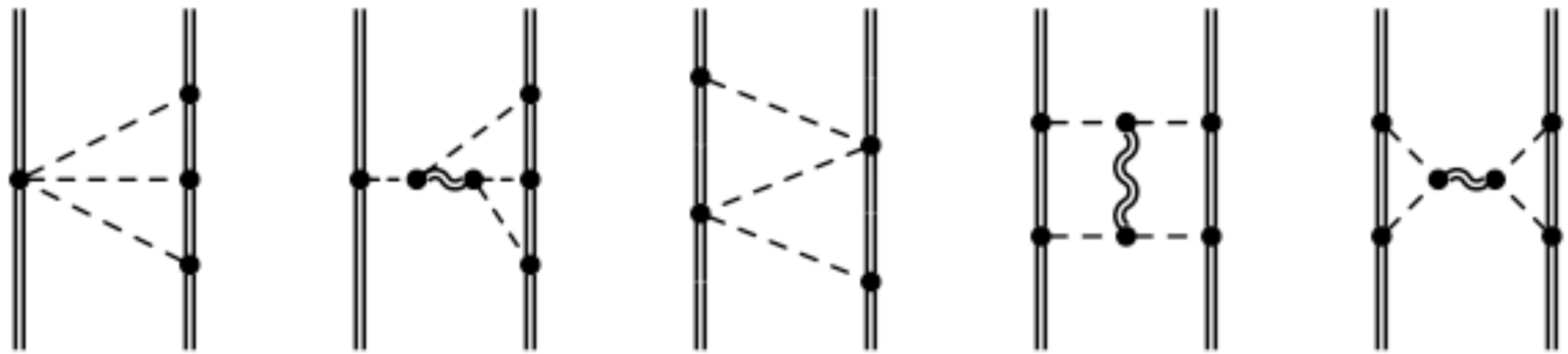
$$L = \frac{1}{2}\sum_a m_a \vec{v}_a^2 + \frac{G_N m_1 m_2}{r}$$

Next-to-leading (1PN): Einstein-Infeld Hoffman Lagrangian (1938)



$$L_{EIH} = \frac{1}{8}\sum_a m_a \vec{v}_a^4 + \frac{G_N m_1 m_2}{2r}\left[3(\vec{v}_1^2 + \vec{v}_2^2) - 7\vec{v}_1 \cdot \vec{v}_2 - (\vec{v}_1 \cdot n)(\vec{v}_1 \cdot n)\right]$$
$$- \frac{G_N^2 m_1 m_2}{2r^2}$$

2PN (1981-2002): Some of the diagrams are (Gilmore+Ross, PRD 2008)



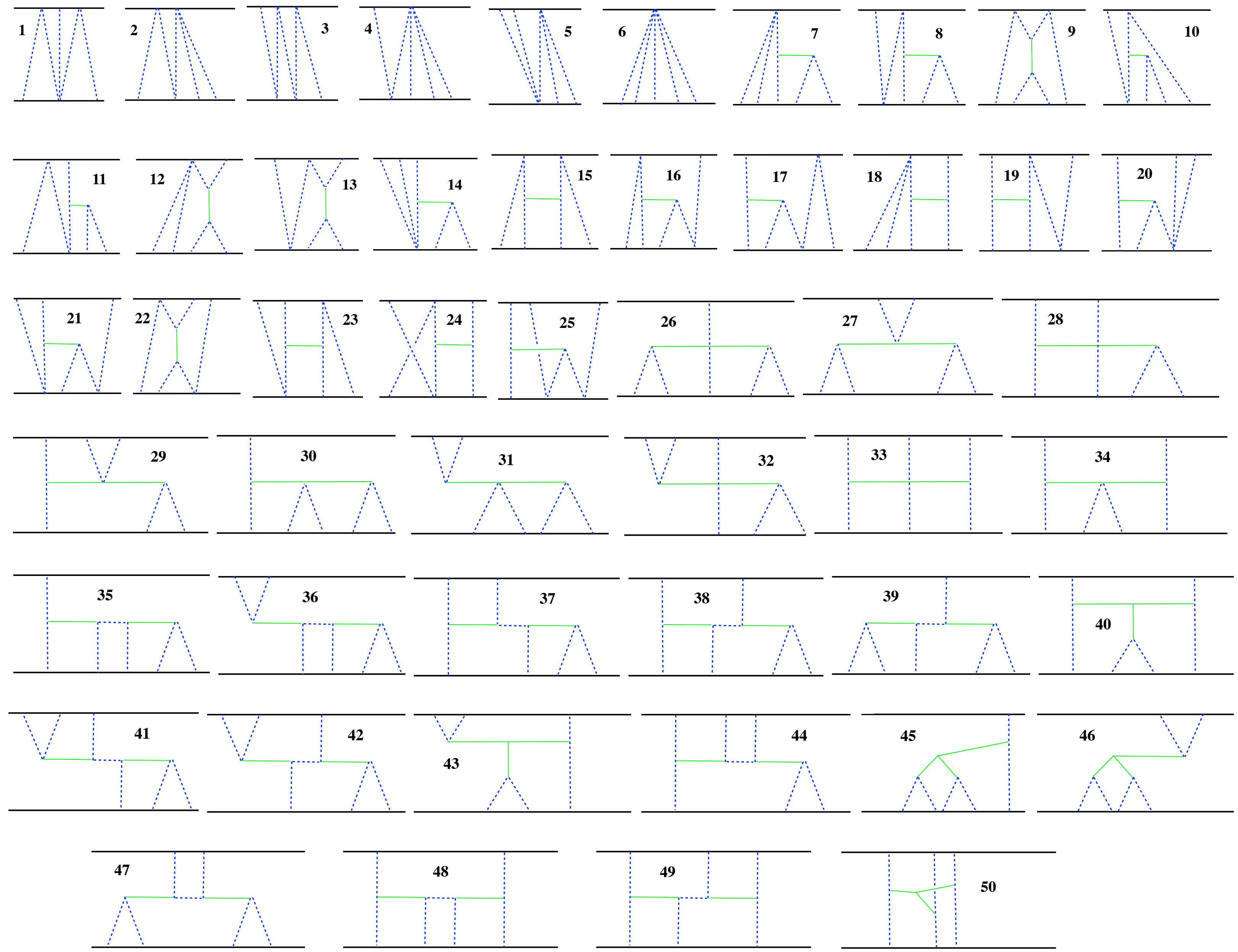
$$
\begin{aligned}
L_{2PN} =& \frac{m_1 \mathbf{v}_1^6}{16} \\
&+ \frac{G m_1 m_2}{r}\left(\frac{7}{8}\mathbf{v}_1^4 - \frac{5}{4}\mathbf{v}_1^2 \mathbf{v}_1\cdot\mathbf{v}_2 - \frac{3}{4}\mathbf{v}_1^2 \mathbf{n}\cdot\mathbf{v}_1 \mathbf{n}\cdot\mathbf{v}_2 + \frac{3}{16}\mathbf{v}_1^2\mathbf{v}_2^2 + \frac{1}{8}(\mathbf{v}_1\cdot\mathbf{v}_2)^2 \right.\\
&\qquad \left. - \frac{1}{8}\mathbf{v}_1^2(\mathbf{n}\cdot\mathbf{v}_2)^2 + \frac{3}{4}\mathbf{n}\cdot\mathbf{v}_1\mathbf{n}\cdot\mathbf{v}_2\mathbf{v}_1\cdot\mathbf{v}_2 + \frac{3}{16}(\mathbf{n}\cdot\mathbf{v}_1)^2(\mathbf{n}\cdot\mathbf{v}_2)^2\right) \\
&+ G m_1 m_2 \left(\frac{1}{8}\mathbf{a}_1\cdot\mathbf{n}\mathbf{v}_2^2 + \frac{3}{2}\mathbf{a}_1\cdot\mathbf{v}_1\mathbf{n}\cdot\mathbf{v}_2 - \frac{7}{4}\mathbf{a}_1\cdot\mathbf{v}_2\mathbf{n}\cdot\mathbf{v}_2 - \frac{1}{8}\mathbf{a}_1\cdot\mathbf{n}(\mathbf{n}\cdot\mathbf{v}_2)^2\right) \\
&+ G m_1 m_2 r\left(\frac{15}{16}\mathbf{a}_1\cdot\mathbf{a}_2 - \frac{1}{16}\mathbf{a}_1\cdot\mathbf{n}\mathbf{a}_2\cdot\mathbf{n}\right) \\
&+ \frac{G^2 m_1 m_2^2}{r^2}\left(\frac{7}{4}\mathbf{v}_1^2 + 2\mathbf{v}_2^2 - \frac{7}{2}\mathbf{v}_1\cdot\mathbf{v}_2 + \frac{1}{2}(\mathbf{n}\cdot\mathbf{v}_1)^2\right) \\
&+ \frac{G^3 m_1 m_2^3}{2r^3} + \frac{3G^3 m_1^2 m_2^2}{2r^3} + (1\leftrightarrow 2),
\end{aligned}
$$

reducible to one-loop integrals via IBP:

$$\int \frac{d^{d-1}\mathbf{k}}{(2\pi)^{d-1}} \frac{1}{[(\mathbf{k}+\mathbf{p})^2]^\alpha [\mathbf{k}^2]^\beta}$$

(simplification of PT via field redefs: B. Kol+M. Smolkin, 2007-2008.)

State of the art: Potentials at 4PN (Foffa, Sturani, Mastrolia, Sturm, PRD 2017). All diagram topologies

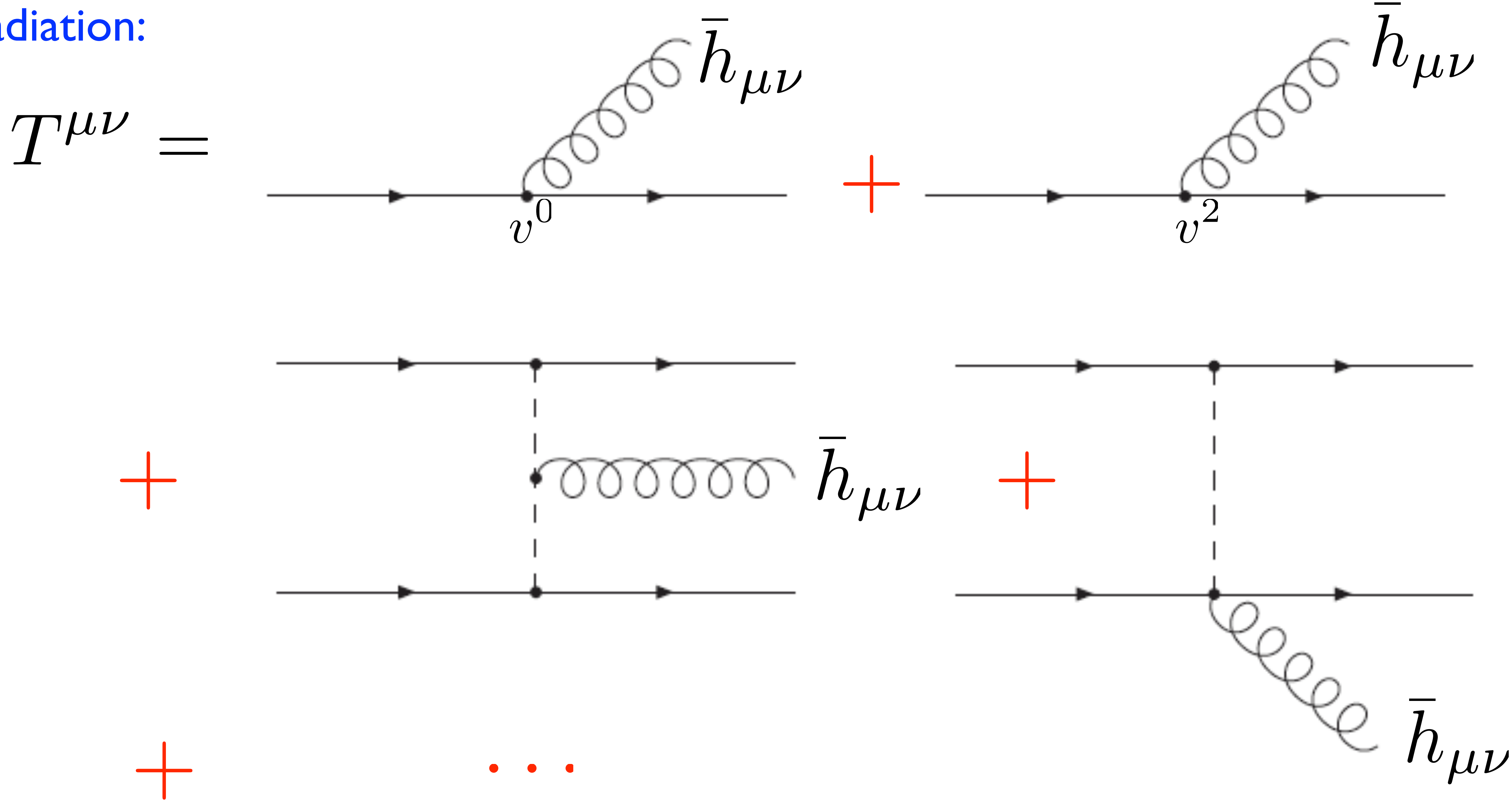


static part of the 2-body potentials:

$$\sum_{a=1}^{50} \mathcal{L}_a = \frac{3}{8}\frac{G_N^5 m_1^5 m_2}{r^5} + \frac{31}{3}\frac{G_N^5 m_1^4 m_2^2}{r^5} + \frac{141}{8}\frac{G_N^5 m_1^3 m_2^3}{r^5}\,.$$

One graviton sector: radiation couplings (WG+A. Ross, PRD 2010)

Integrating out potential modes gives the couplings of 2-body system to radiation:



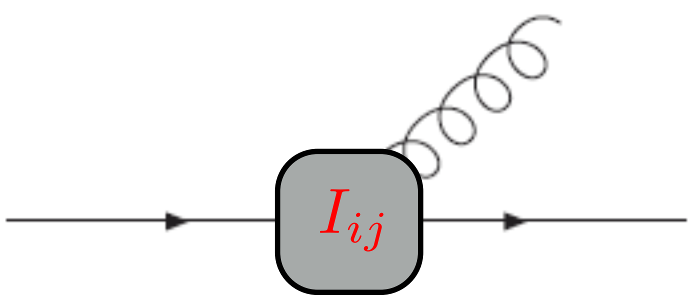
(1st graph=LO. Last three graphs are NLO).

The resulting action consists of a set of multipole moments coupled to the worldline of composite object. In the CM frame,

$\ell = 2_E$ $\quad$ $\ell = 2_B$ $\quad$ $\ell = 3_E$

$$\Gamma[\bar{h}] = \frac{1}{2m_{Pl}} \int dx^0 \left[I_E^{ij}(x^0) R_{0i0j} + \frac{4}{3} I_B^{i,jk}(x^0) R_{0jik} + \frac{1}{3} I_E^{ijk}(x^0) \nabla_k R_{0i0j} + \cdots \right]$$

For example, the quadrupole moment to NLO (Will+Wagoner, 1970's)



$$= \int d^3\mathbf{x} \left[T^{00} + T^{aa} + \frac{11}{42} \mathbf{x^2} \ddot{T}^{00} - \frac{4}{3} \dot{T}^{0k} x^k \right] \left[x^i x^j \right]^{TF} + \mathcal{O}(v^4)$$

$$= \sum_a m_a \mathbf{x}_a^i \mathbf{x}_a^j \left[1 + \frac{3}{2} \mathbf{v}_a^2 - \sum_b \frac{G_N m_b}{|\mathbf{x}_a - \mathbf{x}_b|} \right] + \frac{11}{42} \sum_a m_a \frac{d^2}{dt^2} (\mathbf{x_a}^2 \mathbf{x}_a^i \mathbf{x}_a^j)$$

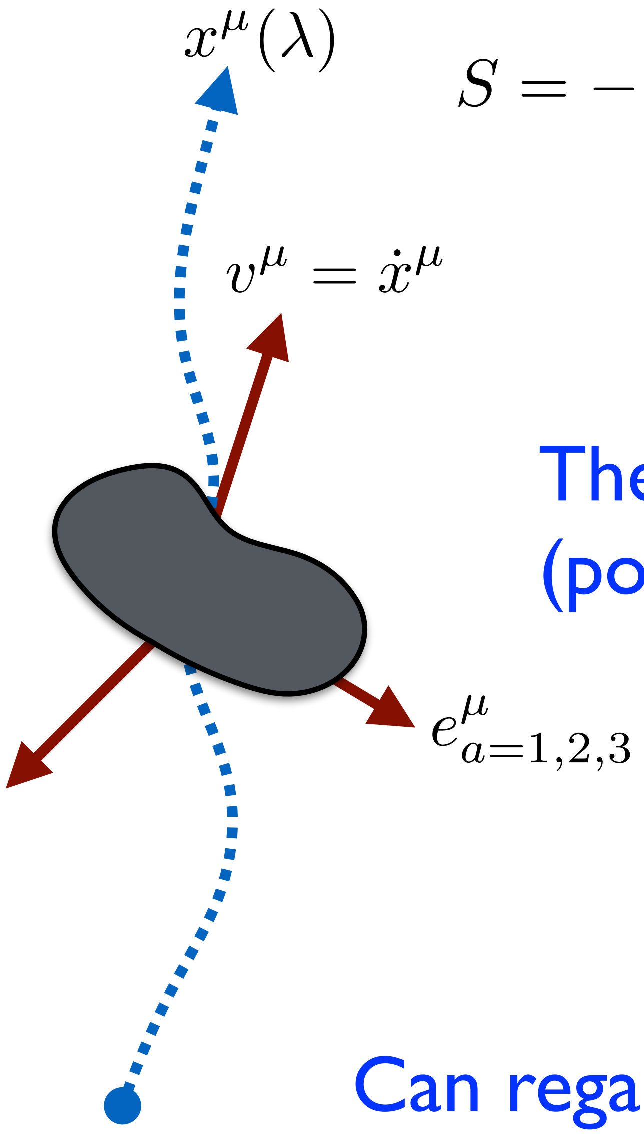
$$- \frac{4}{3} \sum_a m_a \frac{d}{dt} (\mathbf{x_a} \cdot \mathbf{v}_a \mathbf{x}_a^i \mathbf{x}_a^j) - \text{traces} + \mathcal{O}(v^4)$$

EFTIII: Radiation

(Double expansion: $\eta_2 = r/\lambda \sim v$
$\eta_3 = r/r_g \sim v^3$)

(WG+Ross, PRD 2010)

This is a field theory of radiation coupled to a point object with multipole moments. Most general diff. invariant action:



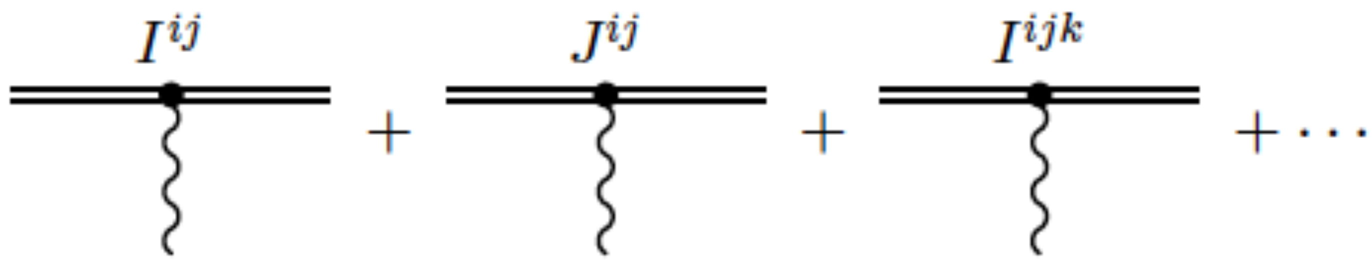
$$S = -\int d\tau(\lambda) m(\lambda) - \int dx^\mu L_{ab}(\lambda)\, \omega_\mu^{ab}(x(\tau)) + \frac{1}{2}\int d\tau(\lambda) I_{ab}(\lambda) E^{ab}(x(\tau))$$
$$-\frac{2}{3}\int d\tau J_{ab}(\lambda) B^{ab}(x) + \frac{1}{6}\int d\tau I_{abc}(\lambda) \nabla^c E^{ab}(x) + \cdots$$

The time evolution of the moments arises from short dist. (potentials) as well as radiative corrections (radiation reaction).

Can regard the moments as time-dependent Wilson coefficients (coupling constants). Radiative corrections in the EFT will generate RG flows for them.

Can use this theory to compute observables at infinity, even if the short distance time evolution of the moments is not known. For example, the graviton emission amplitude involving the 1st three moments:

$$i\mathcal{A}_h(\mathbf{k}) =$$



$$= \frac{i}{4m_{Pl}}\epsilon^*_{ij}(\mathbf{k},h)\left[\mathbf{k}^2 I^{ij}(k) + \frac{4}{3}|\mathbf{k}|\,\mathbf{k}^l\epsilon^{ikl}J^{jk}(k) - \frac{i}{3}\mathbf{k}^2\mathbf{k}^l I^{ijl}(k) + \cdots\right]$$

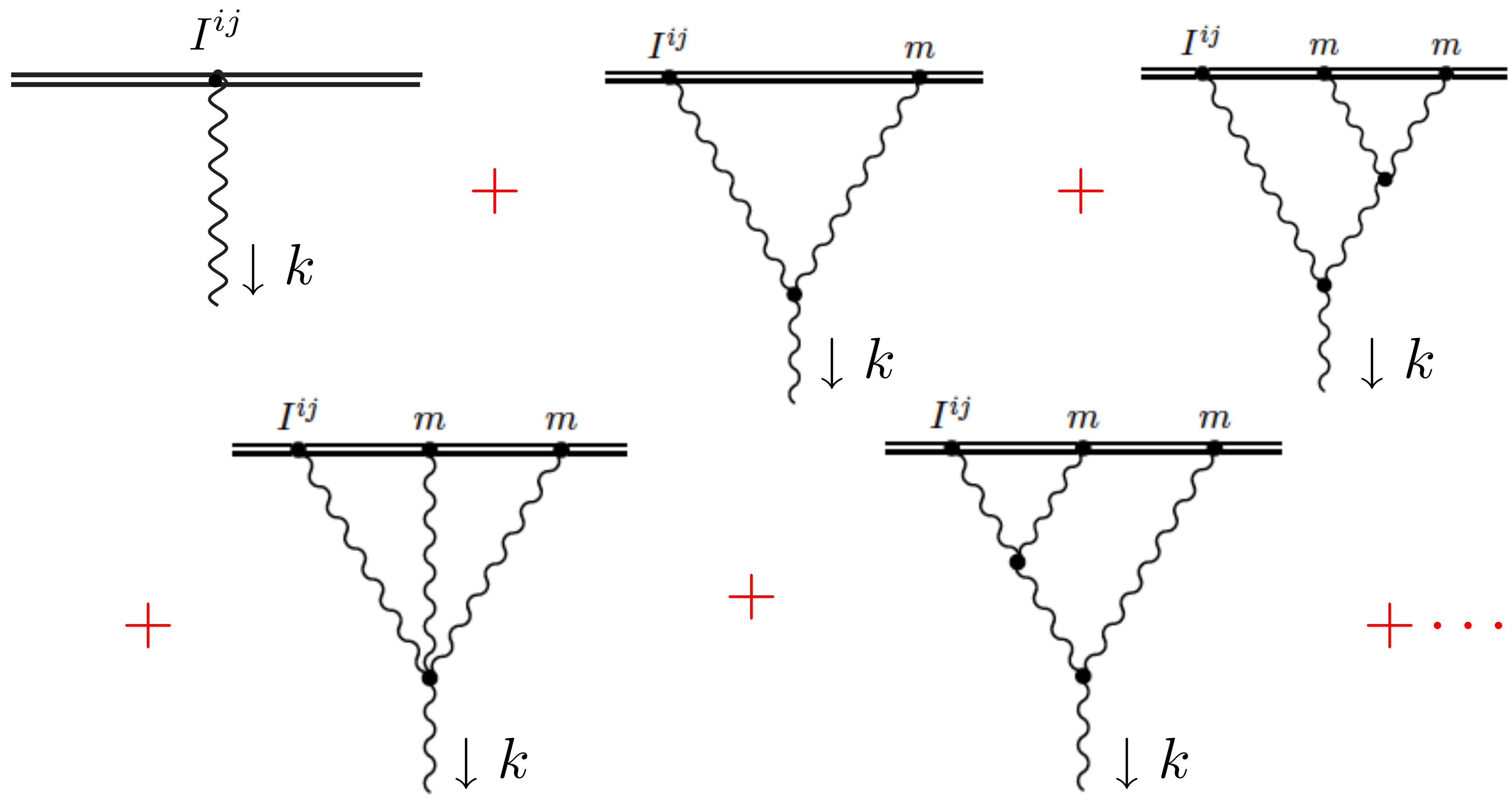
Determines the time averaged energy loss rate of the composite system:

$$\dot{P}^0 = \frac{G_N}{5}\left\langle\left(\frac{d^3}{dt^3}I^{ij}(t)\right)^2\right\rangle + \frac{16G_N}{45}\left\langle\left(\frac{d^3}{dt^3}J^{ij}(t)\right)^2\right\rangle + \frac{G_N}{189}\left\langle\left(\frac{d^4}{dt^4}I^{ijk}(t)\right)^2\right\rangle + \cdots$$

UV and IR divergences in radiation

Focus on the $\ell = 2$ channel. The amplitude to 3PN order is

$i\mathcal{A}(k) =$



Non-linear interaction of emitted gravitons with multipole moments introduces both UV and IR divergences.

Dissipative Effects in Compact Binaries

The formalism outlined so far neglects dissipation, ie absorption of GWs by the compact object.

On general grounds, dissipation implies the existence of low frequency modes with $\omega \sim \omega_{GW}$ (eg NS: hydro modes,.... BH: horizon absorption) not captured by the point particle EFT

$$S_{pp} = -m\int d\tau + c_E \int d\tau E_{\mu\nu}E^{\mu\nu} + c_B \int d\tau B_{\mu\nu}B^{\mu\nu} + \cdots$$

eg, for a Schwarschild black hole, the spectrum contains an infinite tower of modes labeled by $SO(3)$. In this case there are some zero modes:

Mode	Freq.	J^P
$m(\lambda)$	0	0
$x^\mu(\lambda)$	0	1^+
$\omega_{ij}(\lambda)$ (spin)	0	1^-

there are also an infinite tower of quasinormal modes

n	$\ell = 2$		$\ell = 3$		$\ell = 4$	
0	0.37367	-0.08896 i	0.59944	-0.09270 i	0.80918	-0.09416 i
1	0.34671	-0.27391 i	0.58264	-0.28130 i	0.79663	-0.28443 i
2	0.30105	-0.47828 i	0.55168	-0.47909 i	0.77271	-0.47991 i
3	0.25150	-0.70514 i	0.51196	-0.69034 i	0.73984	-0.68392 i

(from Kokkotas and Schmidt, gr-qc/9909058).

Table 1: *The first four QNM frequencies (ωM) of the Schwarzschild black hole for $\ell = 2, 3,$ and 4 [135]. The frequencies are given in geometrical units and for conversion into kHz one should multiply by $2\pi(5142Hz) \times (M_\odot/M)$.*

which are increasingly "broad resonances"

Even though the form of the internal spectrum depends on the details of the internal structure, can incorporate the effects of dissipation in a model independent way using open system EFT methods.

The idea is to treat the compact object as $R \to 0$ as an "atom", i.e a worldline with local operators coupled to gravitons. For a spherical symmetric object, the leading interactions with gravitons take the form

$$S_{pp} = \cdots + \frac{1}{2}\int d\tau(\lambda) Q_E^{ab}(\lambda) E_{ab}(x(\lambda)) + \frac{1}{2}\int d\tau(\lambda) Q_B^{ab}(\lambda) B_{ab}(x(\lambda)) + \cdots$$

With operators $Q_E^{ab}(\lambda), \; Q_B^{ab}(\lambda) \quad \ldots$ acting on the Hilbert space of internal states.

Microscopic properties are then encoded in the correlation functions

$$\langle Q^{E,B} \cdots Q^{E,B} \rangle$$

which can be related to observable quantities of the compact object.

Example: Graviton absorption and power dissipation

Consider an compact object of mass M . Graviton absorption amplitude in the object's rest frame:

$$i\mathcal{A}(g_h(k) + M \to X) = \langle X|Te^{-i\int dt H_{int}}|k, h; M\rangle$$

$$\approx \frac{i\omega^2}{2m_{Pl}}\epsilon_{ij}(k)\int dt e^{-i\omega t}\langle X|Q^E_{ij}(t)|M\rangle + \text{magnetic}$$

absorption cross section is

$$\sigma_{abs}(\omega) = \lim_{T\to\infty}\frac{1}{T}\cdot\frac{1}{2\omega}\sum_X |\mathcal{A}(g(k) + M \to X)|^2$$

then, assuming unitarity (even for BHs!):

$$\sum_X |X\rangle\langle X| = \mathbb{I} \quad \Longrightarrow \quad \sigma_{abs}(\omega) = \frac{\omega}{8m_{Pl}^2}\int dx^0 e^{-i\omega x^0}[\omega^2\epsilon^*_{ab}\epsilon_{cd}\langle Q^E_{ab}(0)Q^E_{cd}(x^0)\rangle$$

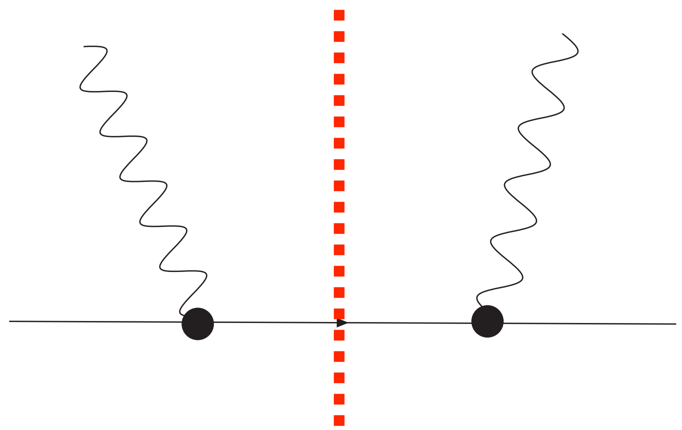
$$+(\mathbf{k}\times\epsilon^*)_{ab}(\mathbf{k}\times\epsilon)_{cd}\langle Q^B_{ab}(0)Q^B_{cd}(x^0)\rangle]$$

where the 2-pt. correlators are in the initial state of the compact object

$$\langle Q^E(0)Q^E(x^0)\rangle = \langle M|Q^E(0)Q^E(x^0)|M\rangle$$

(alternatively, initial state could be mixed/thermal)

Equivalently, using the optical theorem

Im 

$$\sigma_{abs,p}(\omega) = \frac{\omega^3}{4m_{Pl}^2}\mathrm{Im}\left[F_E(\omega) + F_B(\omega)\right]$$

$$(\omega > 0)$$

where the time-ordered two-point function is, using rotational invariance

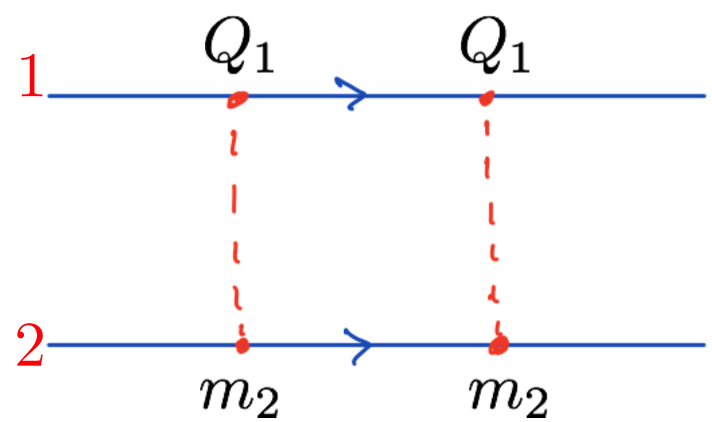
$$\int dx^0 e^{-i\omega x^0} \langle T Q^E_{ab}(0) Q^E_{cd}(x^0)\rangle = -\frac{i}{2}\left[\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc} - \frac{2}{3}\delta_{ab}\delta_{cd}\right] F_E(\omega),$$

and similarly for the magnetic contribution

(see also Klebanov's computation of greybody factors in AdS/CFT…)

The same two-point correlators show up in the two-body sector and control dissipative effects in the evolution of non-relativistic binaries. Using the NRGR formalism, this is given by the box diagram with potential exchange:

$$i\Gamma_{eff}[x_1, x_2] =$$



$$+(1 \leftrightarrow 2) + \cdots$$

$$= \frac{m_2^2}{8m_{Pl}^4} \int dx_1^0 d\bar{x}_1^0 dx_2^0 d\bar{x}_2^0 \langle T H_{00}(x_2^0) E_{ij}(x_1^0)\rangle \langle T H_{00}(\bar{x}_2^0) E_{rs}(\bar{x}_1^0)\rangle \times \langle T Q_{ij}^E(x_1^0) Q_{rs}^E(\bar{x}_2^0)\rangle + (1 \leftrightarrow 2) + \cdots .$$

$$= \frac{1}{4} G_N^2 \sum_{a \neq b} \int \frac{d\omega}{2\pi} F_b^E(\omega) m_a^2 |q_{ij}^{(a)}(\omega)|^2 + \cdots .$$

$$q_{ij}^{(a)}(t) = \partial_i^a \partial_j^a |\mathbf{x}_{12}(t)|^{-1}$$

Absorption power spectrum:

$$\frac{2}{T}\mathrm{Im}\Gamma_{eff} = \int \frac{d\omega}{\omega}\frac{dP}{d\omega} \quad \Longrightarrow \quad \frac{dP_{abs}}{d\omega} = -\frac{1}{T}\frac{G_N}{64\pi^2}\sum_{a\neq b}\frac{\sigma_{abs}^{(b)}(\omega)}{\omega^2} m_a^2 |q_{ij}^{(a)}(\omega)|^2,$$

is related to the low frequency graviton absorption cross section by the compact binary.

For the case of black holes, the low frequency $\sigma_{abs}(\omega)$ can be calculated analytically, by finding the graviton wavefunctions in the BH background:

$$\Box_{BH} h_{\mu\nu} = 0$$

$$h_{\mu\nu}(x) = e^{-i\omega t}\frac{R_\ell(r)}{r}Y^{\ell m}_{\mu\nu}(\Omega)$$

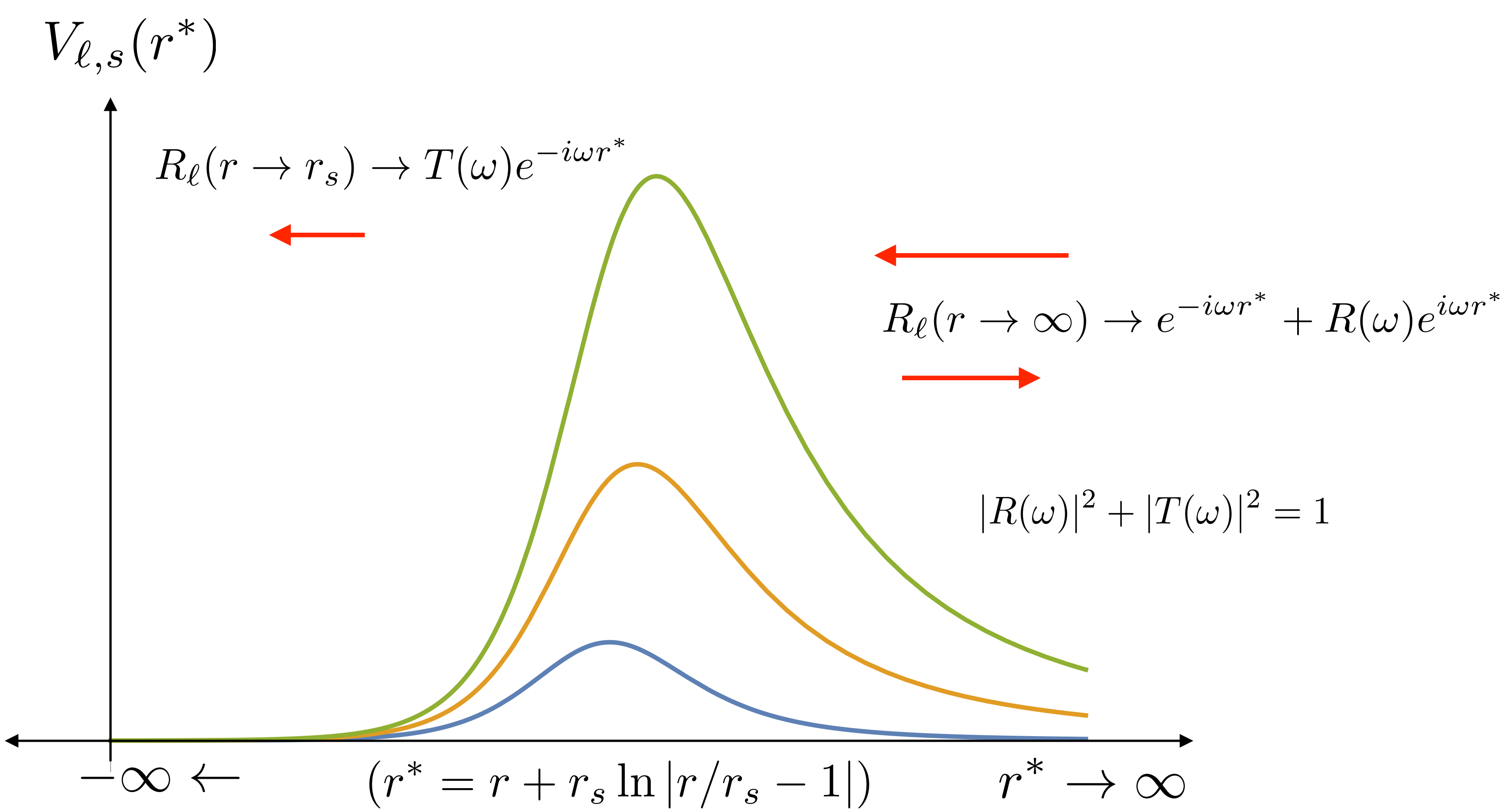
$$\longrightarrow$$

$$\left(-\frac{d^2}{dr^{*2}} + V_\ell(r)\right)R_\ell(r) = \omega^2 R_\ell(r)$$

$$V_\ell(r) = \left(1-\frac{r_s}{r}\right)\left(\frac{\ell(\ell+1)}{r^2} - \frac{3r_s}{r^3}\right)$$

Schrodinger eqn for radial modes
="Regge-Wheeler" eqn.

BCs for scattering:



$$\sigma_{abs}(\omega) \sim |T(\omega)|^2$$

(QNMs: Same eqn. but purely outgoing bc's at the horizon and infinity)

These absorption coefficients were computed by Page (1975) for massless particles of arbitrary spin in the case of Kerr black holes:

$$\sigma_s(\omega) = \pi\omega^{-2}\sum_{l,m}\Gamma_{s\omega lmp} \underset{\omega\to 0}{\sim} \begin{cases} A, & s=0 \\ 2\pi M^2, & s=\frac{1}{2} \\ \frac{4}{9}A(3M^2-a^2)\omega^2, & s=1 \\ \frac{16}{225}A(5M^2+\frac{5}{2}M^2a^2+a^4)\omega^4, & s=2. \end{cases}$$

Using his result we can match the two-point functions in the case $s=2$

$$\mathrm{Im}F_E(\omega) = \mathrm{Im}F_B(\omega) = 16G_N^5 m^6|\omega|/45$$

yielding predictions for horizon power absorption in NR binary evolution:

$$P_{abs} = \frac{16}{45}G_N^7 m_1^2 m_2^2\left\langle \frac{1}{2}\sum_{a\neq b} m_a^4 \dot{q}_{ij}^{(b)}\dot{q}_{ij}^{(b)}\right\rangle \qquad = \frac{32}{5}G_N^7 m^6\mu^2\left\langle \frac{\mathbf{v}^2}{|\mathbf{x}|^8} + 32\frac{(\mathbf{x}\cdot\mathbf{v})^2}{|\mathbf{x}|^{10}}\right\rangle.$$

(in agreement with Poisson (1995) in the extreme mass limit $m_1 \gg m_2$)

Note: $P_{abs}/P_{quad} \sim v^8$ is a (small) 4PN effect. Absorption enhanced to v^5 for rotating black holes (see Porto; Endlich+Pencco for EFT description)

Hawking emission and EFT (w/ Rothstein, in progress)

In principle, same methods can be applied to capture long distance effects of emission by black hole horizon. (phenomenological applications: none whatsoever)

E.g neutral black hole electrodynamics:

$$S_{int} = -\int d\tau p_a(\tau) E^a(\tau) - \int d\tau m_a(\tau) B^a(\tau)$$

as in the graviton case, the correlators $\langle p_a p_b \rangle$ $\langle m_a m_b \rangle$ are related to absorptive processes.

Using Page's result $\sigma_{abs}(\omega) = \frac{4\pi}{3} r_s^4 \omega^2$:

$$F(\omega > 0)\delta_{ab} = \int d\tau e^{i\omega\tau} \langle p_a(\tau) p_b(0) \rangle = \int d\tau e^{i\omega\tau} \langle m_a(\tau) m_b(0) \rangle = \frac{4\pi}{3} r_s^4 \omega \delta_{ab}$$

BH emission involves the same correlators, but at negative frequency:

$$d\Gamma(M \to \gamma(k) + X) = \frac{1}{T} \frac{d^3\vec{k}}{(2\pi)^3 2\omega} \sum_X |\mathcal{A}(M \to \gamma(k) + X)|^2$$

(assuming unitarity)

$$\frac{d^2\Gamma}{d\Omega d\omega} = \frac{\omega^3}{16\pi^3} \int d\tau e^{-i\omega\tau} [\epsilon_a \epsilon_b^* \langle p_a(\tau) p_b(0) \rangle + (\vec{n} \times \vec{\epsilon})_a (\vec{n} \times \vec{\epsilon}^*)_b \langle m_a(\tau) m_b(0) \rangle]$$

$$= \frac{\omega^3}{8\pi^2} F(-\omega)$$

In this case we match to Hawking's prediction for the photon emission rate:

$$\frac{d\Gamma_{s,\ell,m,h}}{d\omega} = \frac{\Gamma_{s,\ell,m,h}(\omega)}{e^{\hbar\omega/T_H} - 1} = \frac{\Gamma_{s,\ell,m,h}(\omega)}{e^{4\pi r_s \omega} - 1}$$

Greybody factors from Page:

$$\Gamma_{s\omega l m p} = \left[\frac{(l-s)!(l+s)!}{(2l)!(2l+1)!!}\right]^2 \prod_{n=1}^{l}\left[1+\left(\frac{\omega - m\Omega}{n\kappa}\right)^2\right] 2\left(\frac{\omega - m\Omega}{\kappa}\right)\left(\frac{A\kappa}{2\pi}\omega\right)^{2l+1}, \quad 2s \text{ even},$$

$$\Gamma_{s\omega l m p} = \left[\frac{(l-s)!(l+s)!}{(2l)!(2l+1)!!}\right]^2 \prod_{n=1}^{l+1/2}\left[1+\left(\frac{\omega - m\Omega}{n\kappa - \frac{1}{2}\kappa}\right)^2\right]\left(\frac{A\kappa}{2\pi}\omega\right)^{2l+1}, \quad 2s \text{ odd},$$

Our operators correspond to the case $s = \ell = 1$:

$$\frac{d\Gamma}{d\omega} = \frac{\omega^3}{2\pi} F(-\omega) \approx \frac{T_H}{\hbar}\frac{\Gamma_{s,\ell,m,h}(\omega)}{\omega} \xrightarrow{(r_s\omega \ll 1)} F(-\omega) = \frac{1}{3} r_s^3$$

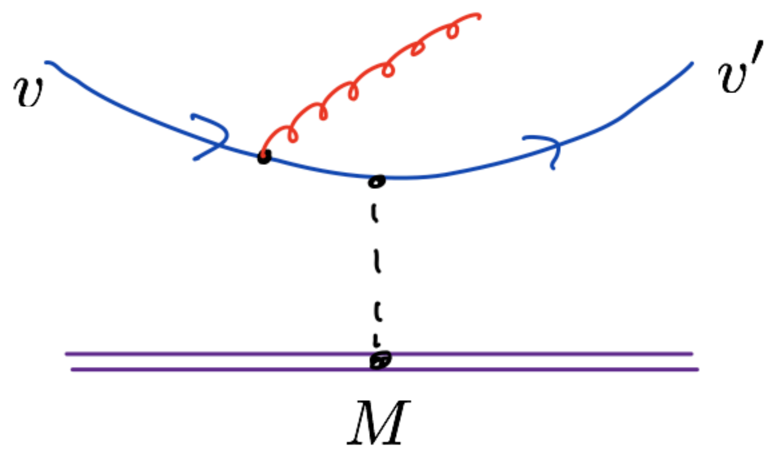
(Can also obtain same result by imposing KMS condition on absorptive part:

Giddings+Witten, unpublished)

Application: Soft Hawking photon emission in glancing BH collisions

$$\mathrm{BH}(v) + M \to \mathrm{BH}(v') + \gamma(k)$$

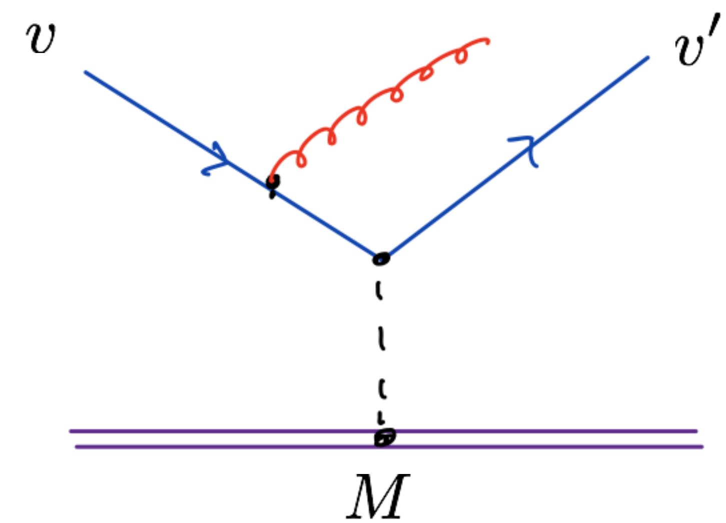
Small neutral BH scattering off the gravitational field of heavy point mass M.



$$\frac{d^2}{d\tau^2} v^\mu(\tau) = -\Gamma^\mu_{\alpha\beta} v^\alpha v^\beta$$

$$\approx$$

$(\omega = k^0 \to 0)$



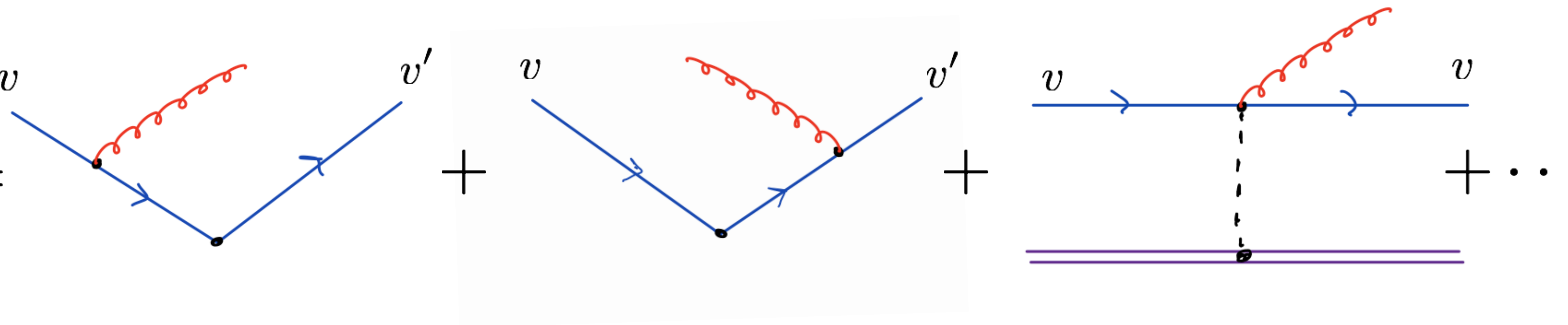
$$x^\mu(\tau) \approx b^\mu + v^\mu \tau \theta(-\tau) + v'^\mu \tau \theta(\tau)$$

Work in the weak gravity limit with small deflections/large impact parameter:

$$\Delta v^\mu = v'^\mu - v^\mu = \frac{4G_N M}{b^2} \frac{(u \cdot v)^2 - 1/2}{u \cdot v \sqrt{(u \cdot v)^2 - 1}} b^\mu \ll 1$$

Hawking emission amplitude in the point EFT limit:

$i\mathcal{A} =$



Emission rate to leading order in deflection:

$$\frac{d^2\Gamma}{d\omega d\Omega} = \frac{r_s^3\omega}{24\pi^3}(k\cdot v)^2\left[1+\frac{k\cdot\Delta v}{k\cdot v}\right] + \mathcal{O}(\Delta v^2) + \mathcal{O}(r_s\omega)$$

Radiated power:

$$dP = \hbar\omega d\Gamma$$

Conclusions

Proliferation of length scales in the binary merger problem motivates the construction of a tower of EFTs

Dissipation (eg black hole absorption) incorporated by including worldline degrees of freedom

Same approach can be used to account for particle emission:

EFT for ultralight axions emission/superradiance in binary dynamics.

(Baumann, Chia, Porto, 2018)

Hawking soft photon theorems in hard scattering processes?

(w/ Rothstein, in progress)