Production in NRQCD and pNRQCD

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NRQCD

- The effective field theory NRQCD is obtained by integrating out scales of heavy quark mass m and above
- Expansion of the QCD Lagrangian in powers of v yields the bilinear part of the NRQCD Lagrangian

$$\mathcal{L}_{\text{heavy}} = \psi^\dagger \left(iD_t + \frac{\boldsymbol{D}^2}{2m} \right) \psi + \chi^\dagger \left(iD_t - \frac{\boldsymbol{D}^2}{2m} \right) \chi,$$

$$\begin{aligned}
\delta\mathcal{L}_{\text{bilinear}} \quad = \quad & \frac{c_1}{8m^3} \left[\psi^\dagger (\boldsymbol{D}^2)^2 \psi - \chi^\dagger (\boldsymbol{D}^2)^2 \chi \right] \\
& + \frac{c_2}{8m^2} \left[\psi^\dagger (\boldsymbol{D} \cdot g_s \boldsymbol{E} - g_s \boldsymbol{E} \cdot \boldsymbol{D}) \psi + \chi^\dagger (\boldsymbol{D} \cdot g_s \boldsymbol{E} - g_s \boldsymbol{E} \cdot \boldsymbol{D}) \chi \right] \\
& + \frac{c_3}{8m^2} \left[\psi^\dagger (i\boldsymbol{D} \times g_s \boldsymbol{E} - g_s \boldsymbol{E} \times i\boldsymbol{D}) \cdot \boldsymbol{\sigma} \psi \right. \\
& \qquad \left. + \chi^\dagger (i\boldsymbol{D} \times g_s \boldsymbol{E} - g_s \boldsymbol{E} \times i\boldsymbol{D}) \cdot \boldsymbol{\sigma} \chi \right] \\
& + \frac{c_4}{2m} \left[\psi^\dagger (g_s \boldsymbol{B} \cdot \boldsymbol{\sigma}) \psi - \chi^\dagger (g_s \boldsymbol{B} \cdot \boldsymbol{\sigma}) \chi \right],
\end{aligned}$$

Quarkonium Decays

- Creation / annihilation of quarkonium states necessarily involves momentum scales of order m. Decays of quarkonium states can occur only through higher dimensional operators.
- Example : dimension-6 operators

$$
\begin{aligned}
\mathcal{O}_1({}^1S_0) &= \psi^\dagger \chi \chi^\dagger \psi, \\
\mathcal{O}_1({}^3S_1) &= \psi^\dagger \boldsymbol{\sigma} \chi \cdot \chi^\dagger \boldsymbol{\sigma} \psi, \\
\mathcal{O}_8({}^1S_0) &= \psi^\dagger T^a \chi \chi^\dagger T^a \psi, \\
\mathcal{O}_8({}^3S_1) &= \psi^\dagger \boldsymbol{\sigma} T^a \chi \cdot \chi^\dagger \boldsymbol{\sigma} T^a \psi,
\end{aligned}
$$

$$\Gamma_H = \sum_n c_n \langle H | \mathcal{O}(n) | H \rangle$$

Quarkonium Decays

$$\Gamma_H = \sum_n c_n \langle H|\mathcal{O}(n)|H\rangle$$

- This is a factorization formula, where all long-distance physics is encoded in the NRQCD matrix elements, and the Wilson coefficients c_n are only sensitive to short-distance physics.
- The perturbative proof is carried out by showing that all infrared (collinear and soft) divergences are absorbed into long-distance matrix elements. The proof is completed by the decoupling of the color and spin indices between the short-distance and long-distance quantities.

Quarkonium Decays

- Electromagnetic decays require the final state to be the QCD vacuum. Electromagnetic operators are obtained by inserting $|0\rangle\langle 0|$:

$$\begin{aligned} \mathcal{O}_1^{\mathrm{EM}}(^1S_0) &= \psi^\dagger \chi |0\rangle\langle 0| \chi^\dagger \psi \\ \mathcal{O}_1^{\mathrm{EM}}(^3S_1) &= \psi^\dagger \sigma^i \chi |0\rangle\langle 0| \chi^\dagger \sigma^i \psi \end{aligned}$$

$$\Gamma_H^{\mathrm{EM}} = \sum_n c_n \langle H|\mathcal{O}^{\mathrm{EM}}(n)|H\rangle$$

- Electromagnetic matrix elements are squares of meson-to-vacuum matrix elements. Hence, we can work in the amplitude level : $\mathcal{M}_H^{\mathrm{EM}} = \sum_n \mathcal{A}_n \langle 0|\mathcal{C}^{\mathrm{EM}}(n)|H\rangle$

Quarkonium Decays

- Operator matrix elements scale with v.
- Covariant derivatives and gauge fields are suppressed by powers of v. In general, higher-dimensional 4-quark operators are suppressed compared to dimension-6 operators.
- Quarkonium states have largest overlap with the $|Q\bar{Q}\rangle$ Fock state with same color and J^{PC} as the quarkonium. Higher Fock states such as $|Q\bar{Q}g\rangle$ are suppressed by powers of v.

Quarkonium Decays

- J/ψ has J^{PC} =1^{--}. $|Q\bar{Q}\rangle$ in color singlet S-wave spin triplet state has same J^{PC}, and the operator

$$\mathcal{O}_1(^3S_1) \quad = \quad \psi^\dagger \boldsymbol{\sigma} \chi \cdot \chi^\dagger \boldsymbol{\sigma} \psi$$

  destroys the $|Q\bar{Q}\rangle$ in the same state. Therefore at leading order in v, J/ψ decay rate is given by a single color-singlet operator matrix element

$$\Gamma_{J/\psi} = c_1 \langle J/\psi | \mathcal{O}_1(^3S_1) | J/\psi \rangle$$

Quarkonium Decays

- η_c has J^{PC}=0^{-+}. $|Q\bar{Q}\rangle$ in color singlet S-wave spin singlet state has same J^{PC}, and the operator

  $$\mathcal{O}_1(^1S_0) \quad = \quad \psi^\dagger \chi \chi^\dagger \psi,$$

  destroys the $|Q\bar{Q}\rangle$ in the same state. Therefore at leading order in v, η_c decay rate is given by a single color-singlet operator matrix element

  $$\Gamma_{\eta_c} = c_1 \langle \eta_c | \mathcal{O}_1(^1S_0) | \eta_c \rangle$$

Quarkonium Decays

- h_c has J^{PC} =1^{+-}. $|Q\bar{Q}\rangle$ in color singlet P-wave spin singlet state has same J^{PC}, and the operator

$$\mathcal{O}_1({}^1P_1) \quad = \quad \psi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}})\chi \cdot \chi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}})\psi$$

  destroys the $|Q\bar{Q}\rangle$ in the same state. This operator is suppressed by v^2 due to the covariant derivatives.

- On the other hand, the operator

$$\mathcal{O}_8({}^1S_0) \quad = \quad \psi^\dagger T^a \chi \chi^\dagger T^a \psi$$

  destroys the $|Q\bar{Q}g\rangle$ state with same J^{PC}. This Fock state is suppressed by v compared to $|Q\bar{Q}\rangle$.

- Hence, both matrix elements contribute at leading order:

$$\Gamma_{h_c} = c_1\langle h_c|\mathcal{O}_1({}^1P_1)|h_c\rangle + c_8\langle h_c|\mathcal{O}_8({}^1S_0)|h_c\rangle$$

Vacuum-Saturation Approximation

- The suppression of higher Fock states like $|Q\bar{Q}g\rangle$ compared to $|Q\bar{Q}\rangle$ leads to the vacuum-staturation approximation : a color-singlet operator can be replaced by its electromagnetic counterpart, making errors of v^4.

$$\mathcal{O}_n = \psi^\dagger \mathcal{K}'_n \chi \chi^\dagger \mathcal{K}_n \psi$$

$$\langle H|\mathcal{O}_n|H\rangle = \sum_X \langle H|\psi^\dagger \mathcal{K}'_n \chi|X\rangle\langle X|\chi^\dagger \mathcal{K}_n \psi|H\rangle = |\langle 0|\chi^\dagger \mathcal{K}_n \psi|H\rangle|^2 \left[1 + O(v^4)\right]$$

Quarkonium Production

- Inverse processes of decays are trivial. For example, $\eta_c \to \gamma\gamma$ is often measured by the production process $\gamma\gamma \to \eta_c$ with $\sqrt{s} = m_{\eta_c}$, followed by known decay modes of η_c .

- Exclusive production is related to electromagnetic decays if the initial state does not have strongly interacting particles. For example, exclusive production in lepton colliders of in decays of heavy particles.

- Inclusive production is nontrivial but most interesting.

Exclusive production

- In principle, can be described by analytical continuation from a decay process. For example, Higgs→Quarkonium+photon can be obtained from a calculation of the decay Quarkonium→Higgs+photon.
- The resulting formula is similar to that of the electromagnetic decay :

$$\mathcal{M}_H = \sum_n \mathcal{A}_n \langle H | \mathcal{C}^{\mathrm{EM}}(n) | 0 \rangle$$

Inclusive Production

- Inclusive production is described in a similar way. "Production" 4-quark operators :

$$
\begin{aligned}
\mathcal{O}_1^H({}^1S_0) &= \chi^\dagger \psi \left(a_H^\dagger a_H\right) \psi^\dagger \chi, \\
\mathcal{O}_1^H({}^3S_1) &= \chi^\dagger \sigma^i \psi \left(a_H^\dagger a_H\right) \psi^\dagger \sigma^i \chi, \\
\mathcal{O}_8^H({}^1S_0) &= \chi^\dagger T^a \psi \left(a_H^\dagger a_H\right) \psi^\dagger T^a \chi, \\
\mathcal{O}_8^H({}^3S_1) &= \chi^\dagger \sigma^i T^a \psi \left(a_H^\dagger a_H\right) \psi^\dagger \sigma^i T^a \chi.
\end{aligned}
$$

$$\sigma_H = \sum_n c_n \langle 0 | \mathcal{O}_n^H | 0 \rangle$$

Vacuum-Saturation Approximation

- Color-singlet production operators are also related to electromagnetic decay counterparts.

$$\mathcal{O}_n^H = \psi^\dagger \mathcal{K}'_n \chi \left(a_H^\dagger a_H \right) \chi^\dagger \mathcal{K}_n \psi$$

$$\begin{aligned}
\langle 0 | \mathcal{O}_n^H | 0 \rangle &\approx \langle 0 | \chi^\dagger \mathcal{K}_n \psi \left(\sum_{m_J} |H\rangle\langle H| \right) \psi^\dagger \mathcal{K}'_n \chi | 0 \rangle \\
&= (2J+1) \langle H | \psi^\dagger \mathcal{K}'_n \chi | 0 \rangle \langle 0 | \chi^\dagger \mathcal{K}_n \psi | H \rangle \\
&\approx (2J+1) \langle H | \mathcal{O}_n | H \rangle,
\end{aligned}$$

Production and decay of S-wave Quarkonia

- For S-wave quarkonia, production and decay rates at leading order in v are given by the a process-dependent Wilson coefficient times the leading order color singlet matrix element. This reproduces the color-singlet model.

Decay rate	$\Gamma_H = c_H \lvert \langle 0 \lvert \mathcal{O}_1 \rvert H \rangle \rvert^2$
Electromagnetic decay / exclusive production amplitudes	$\mathcal{M}_H = \mathcal{A} \langle 0 \lvert \mathcal{O}_1 \rvert H \rangle$
Inclusive production rate	$\sigma_H = \hat{\sigma} \lvert \langle 0 \lvert \mathcal{O}_1 \rvert H \rangle \rvert^2$

- The S-wave color-singlet matrix element is therefore one of the most important quantities in quarkonium phenomenology, and needs to be determined reliably.

Production and decay of S-wave Quarkonia

- S-wave color-singlet matrix element is certainly important to make reliable predictions to decay rates and exclusive production rates.

- Even for inclusive production rates, where large color-octet contributions are expected, color-singlet contribution should be well constrained so that the amount of color-octet contributions necessary to describe data is well determined.

S-wave color-singlet matrix element

- The color-singlet matrix element can be computed from lattice NRQCD, computed from potential models, or extracted from phenomenology.
- One of the most widely used result is obtained by using a potential-model description of the electromagnetic decay rates of J/ψ and η_c.

PHYSICAL REVIEW D 77, 094017 (2008)

Improved determination of color-singlet nonrelativistic QCD matrix elements for S-wave charmonium

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We present a new computation of S-wave color-singlet nonrelativistic QCD matrix elements for the

$$\langle \mathcal{O}_1 \rangle_{J/\psi} = 0.440^{+0.067}_{-0.055}\ \mathrm{GeV}^3$$

Bodwin, HSC, Kang, Lee, Yu, PRD77, 094017 (2008)

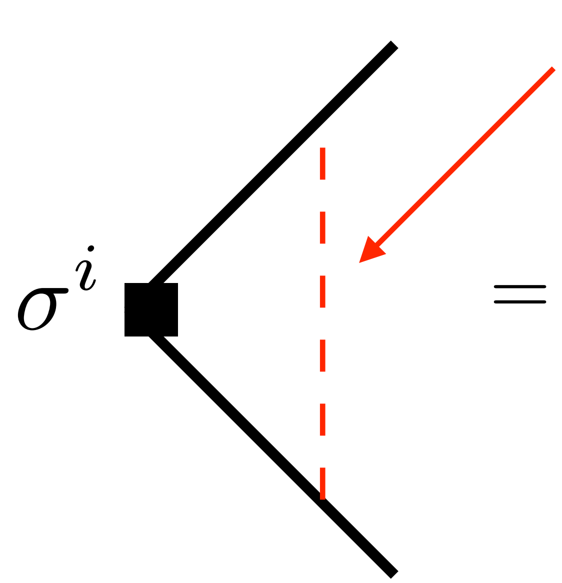
- In a potential model, the color-singlet matrix element at LO in v is given by the wavefunction at the origin. The order-v^2 matrix element is given by the derivative of the wavefunction. If we take the potential to be the Cornell potential, the linear potential can be fixed by the lattice calculation of the string tension.
- The short-distance Coulomb strength and the quark mass parameter is fixed by using the $1S$-$2S$ splitting of the charmonium states and the electromagnetic decay rate.
- Expressions for the electromagnetic decay rate includes corrections of order α_s and v^2.

Going to higher orders

- Predictions of decay / production rates can be systematically improved by computing corrections to Wilson coefficients at higher orders in α_s and including higher-dimensional operators that contribute at higher orders in v.
- While this sounds straightforward, the long-distance matrix elements are UV-divergent and require renormalization. Changing renormalization schemes and scales induce mixing between operators of different orders in v. Hence, there is tradeoff between adjusting long-distance matrix elements and Wilson coefficients.
- As is usual in effective field theories, radiative corrections involve power divergences as well as logarithmic ones.

1-loop example

- 1-loop correction to leading order spin-triplet matrix element at leading order in v in perturbative NRQCD



$$|\boldsymbol{q}| = mv$$

$$= -\frac{\pi\alpha_s C_F}{m}\sigma^i \int_{\boldsymbol{k}} \frac{1}{\boldsymbol{k}^2 - 2\boldsymbol{q}\cdot\boldsymbol{k} - i\varepsilon} + \frac{2\pi\alpha_s C_F}{m^2}\left[-m \int_{\boldsymbol{k}} \frac{k^i(\boldsymbol{k}\cdot\boldsymbol{\sigma})}{\boldsymbol{k}^2(\boldsymbol{k}^2 - 2\boldsymbol{q}\cdot\boldsymbol{k} - i\varepsilon)} + \frac{1}{2}\int_{\boldsymbol{k}} \frac{k^i(\boldsymbol{k}\cdot\boldsymbol{\sigma})}{|\boldsymbol{k}|^3} \right]$$

- The integrals are linearly and quadratically power divergent. A hard UV cutoff Λ yields contributions like $(\Lambda/m)^2$ and Λ/m.

- In DR, power divergences are discarded, and the loop integral do not have logarithmic divergences. Thus, the 1-loop correction ***appears*** finite in DR. ***This does not mean that power divergences do not play a role in DR.***

1-loop example

- Consider the Wilson coefficient for the electromagnetic current $\langle 0|\bar{\psi}\gamma^\mu\psi|H\rangle$. This is a conserved current in QCD, and hence, it is free of UV divergences to all orders in perturbation theory.

- The factorization formula in NRQCD at LO in v :

$$\langle 0|\bar{\psi}\gamma^i\psi|H\rangle = c\langle 0|\chi^\dagger\boldsymbol{\sigma}\psi|H\rangle$$

  Since the full-QCD quantity is scale invariant, the scale dependence of the matrix element is cancelled by the scale dependence of the Wilson coefficient c.

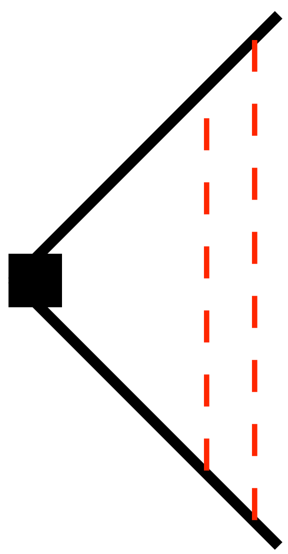
- In DR, the 1-loop correction to the matrix element vanishes, and so, the Wilson coefficient c is finite at NLO in α_s.

- However, in cutoff regularization, the 1-loop correction to c involves power divergences of the form $(\Lambda/m)^n$.

2-loop example

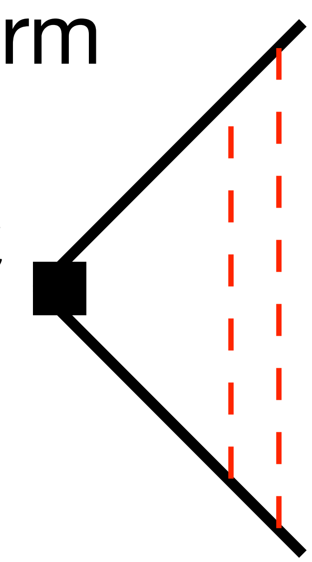
- 2-loop correction to leading order spin-triplet matrix element at leading order in v in perturbative NRQCD.

- In cutoff regularization,

σ^i 

$$+ \ldots = \alpha_s^2 C_F \left(-\frac{C_F}{3} - \frac{C_A}{2} \right) \log \Lambda + \text{UV-finite terms}$$

- In DR, the loop correction is scaleless, and hence vanishes in the form

σ^i 

$$+ \ldots = \alpha_s^2 C_F \left(-\frac{C_F}{6} - \frac{C_A}{4} \right) \left(\frac{1}{\epsilon_{\rm UV}} - \frac{1}{\epsilon_{\rm IR}} \right)$$

2-loop example

- Returning to the factorization formula in NRQCD at LO in v :

$$\langle 0|\bar{\psi}\gamma^i\psi|H\rangle = c\langle 0|\chi^\dagger \boldsymbol{\sigma}\psi|H\rangle$$

- In DR, the UV pole in the NNLO correction to the matrix element is subtracted by renormalizing the matrix element. The 2-loop corrections to the matrix element gives

$$\left[1+\alpha_s^2 C_F\left(-\frac{C_F}{6}-\frac{C_A}{4}\right)\left(\frac{1}{\epsilon_{\text{UV}}}-\frac{1}{\epsilon_{\text{IR}}}\right)\right]\langle 0|\chi^\dagger \boldsymbol{\sigma}\psi|H\rangle^{\text{Bare}}$$
$$=\left[1+\alpha_s^2 C_F\left(-\frac{C_F}{6}-\frac{C_A}{4}\right)\left(-\log\mu^2-\frac{1}{\epsilon_{\text{IR}}}\right)\right]\langle 0|\chi^\dagger \boldsymbol{\sigma}\psi|H\rangle^{\text{MS}}$$

- The IR pole cancels the infrared divergence of the 2-loop perturbative correction to the QCD current $\langle 0|\bar{\psi}\gamma^\mu\psi|H\rangle$.

- Hence, in DR, scale dependence in the Wilson coefficient first appears at two loops.

EM Current in DR

- The quarkonium EM current has been calculated up to 3 loops in DR. 2-loop result in DR has been known since 1997.
- The NNLO correction factor in $\overline{\text{MS}}$ scheme is given by

$$1-\frac{8\alpha_s(m)}{3\pi}-\left[\frac{35\pi^2}{27}\log\left(\frac{\mu^2}{m^2}\right)-\frac{11n_f}{27}+\frac{125\zeta(3)}{9}+\frac{511\pi^2}{324}+\frac{89}{54}+\frac{14}{9}\pi^2\log 2\right]\left(\frac{\alpha_s(m)}{\pi}\right)^2+O(\alpha_s^3)$$

$$=1-\frac{8\alpha_s(m)}{3\pi}-\left[44.55+12.8\log\left(\frac{\mu^2}{m^2}\right)-0.41n_f\right]\left(\frac{\alpha_s(m)}{\pi}\right)^2+O(\alpha_s^3)$$

Czarnecki and Melnikov, PRL80, 2531 (1998)
Beneke, Signer, Smirnov, PRL80, 2535 (1998)

- Scale dependence agrees with what is expected from the 2-loop correction to the matrix element.
- Not only the finite piece is large, but also the scale dependence is strong.

EM Current in DR

- Relative sizes of NLO and NNLO corrections

	NLO	NNLO ($\mu=m$)	NNLO ($\mu=mv$)
J/ψ	-30%	-54%	-35%
Υ	-18%	-19%	-6%

EM Current in DR

- It is possible that large radiative corrections are associated with renormalons, which, in turn, is related to power divergences in loop corrections to matrix elements.
- Renormalons imply ambiguities in dimensionally regulated matrix elements and corresponding Wilson coefficients.
- Renormalon ambiguities cancel in observables, and hence, in factorization formulae. However, in NRQCD, cancellation of renormalon ambiguities can occur among matrix elements of different orders in v. For example, renormalon ambiguities cancel in the vector current in the following combination :

$$(\Delta c) \times \langle 0|\chi^\dagger \boldsymbol{\sigma} \psi|H\rangle + \Delta\langle 0|\chi^\dagger \boldsymbol{\sigma} \psi|H\rangle + \frac{1}{6}\Delta\langle 0|\chi^\dagger \boldsymbol{D}^2 \boldsymbol{\sigma} \psi|H\rangle = 0$$

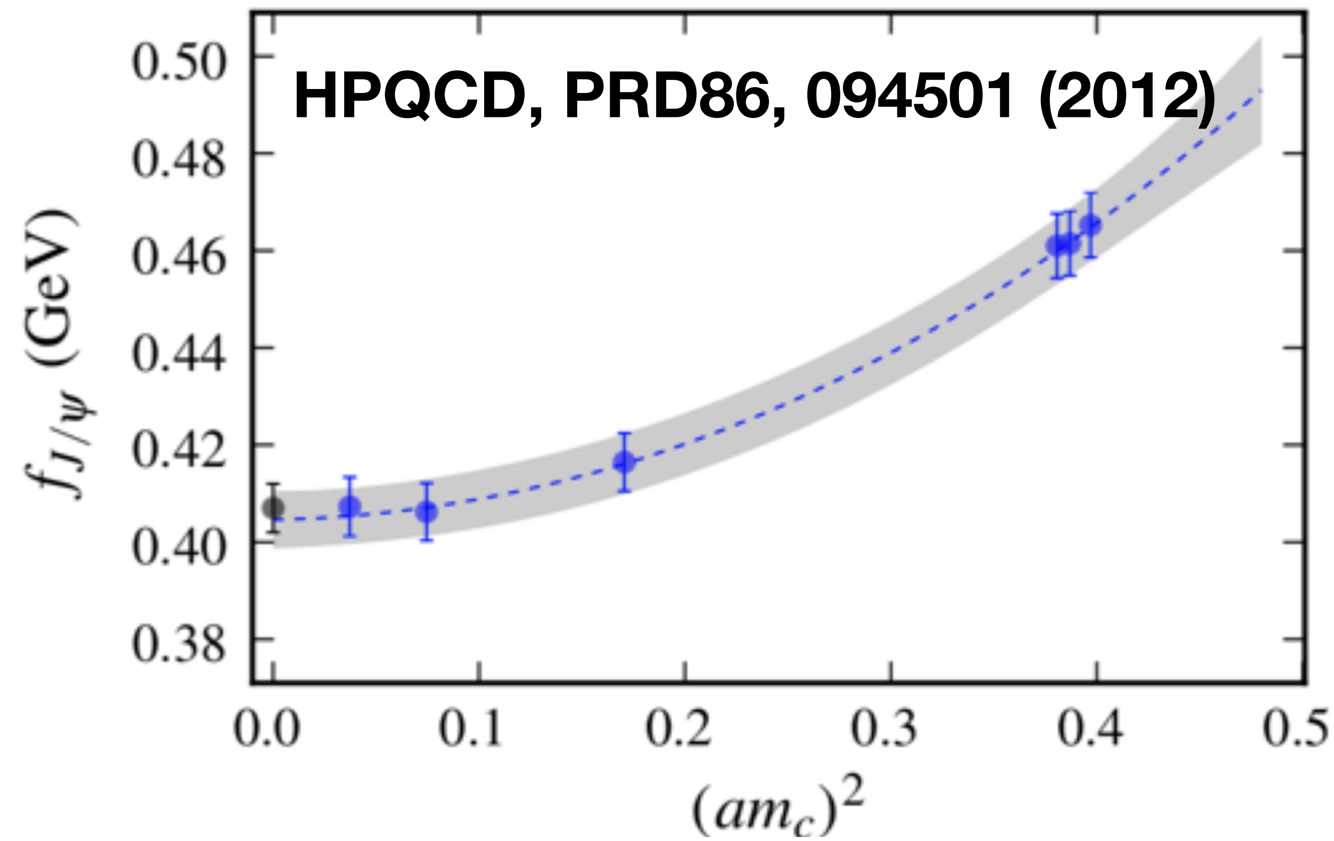
Braaten and Chen, PRD57, 4236 (1998)

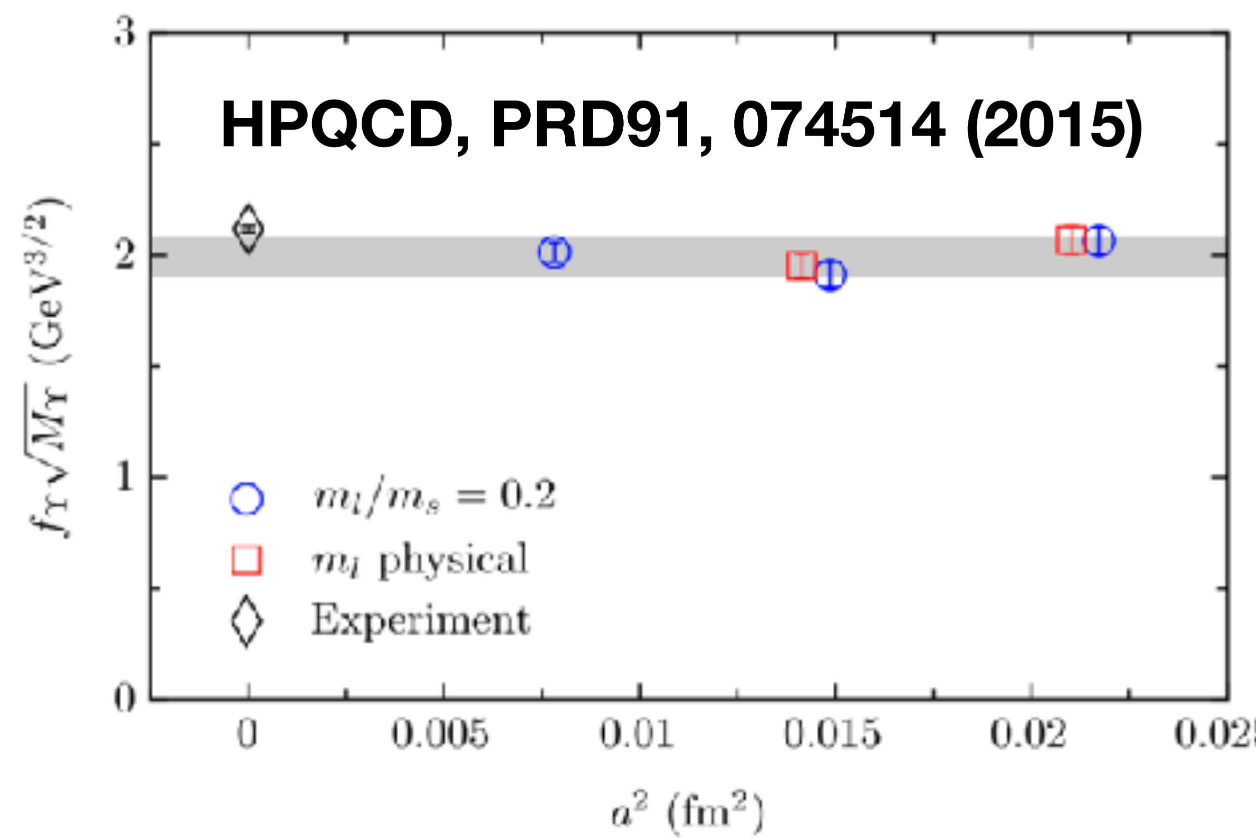
EM Current in DR

- So far, renormalons have generally been studied in the large-n_f limit. In quarkonium production and decay processes, the large-n_f limit does not seem to work very well. Hence, the usual treatment of renormalons has limited applicability.
- It might be worth investigating other regularization schemes that make power divergences explicit.
- Hard cutoff regularization does not work well beyond one loop due to the breakdown of Ward identities.

EM Current in Lattice

- Quarkonium EM current have been investigated in lattice QCD and lattice NRQCD.
- J/ψ EM current has been computed with relativistic charm quarks. Excellent agreement with experiment.
- Υ EM current has been computed with lattice NRQCD, also good agreement with experiment.



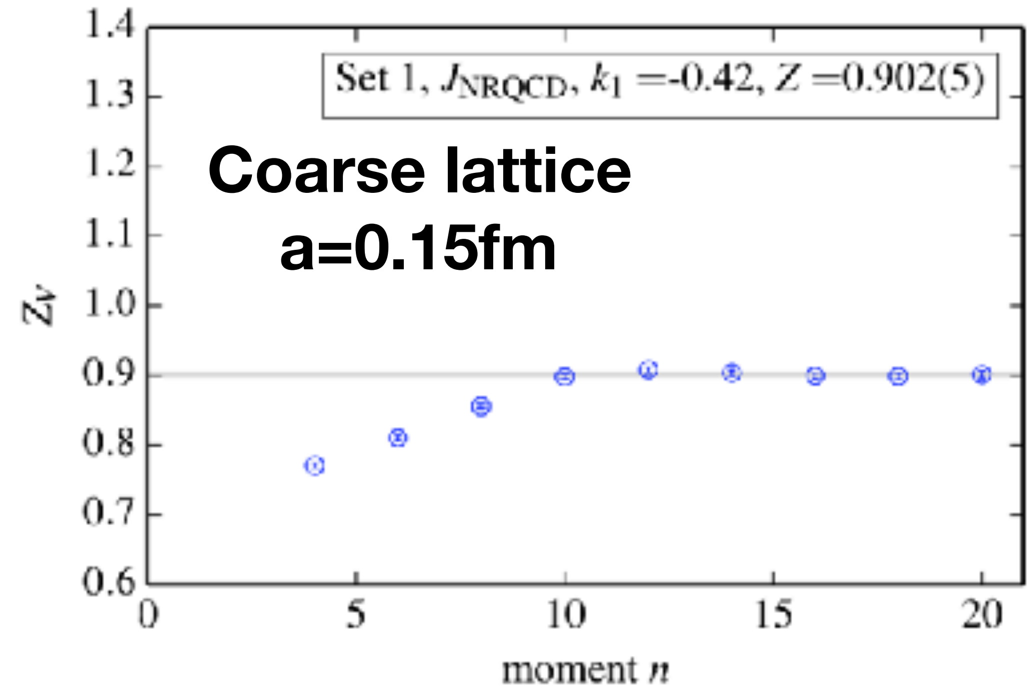


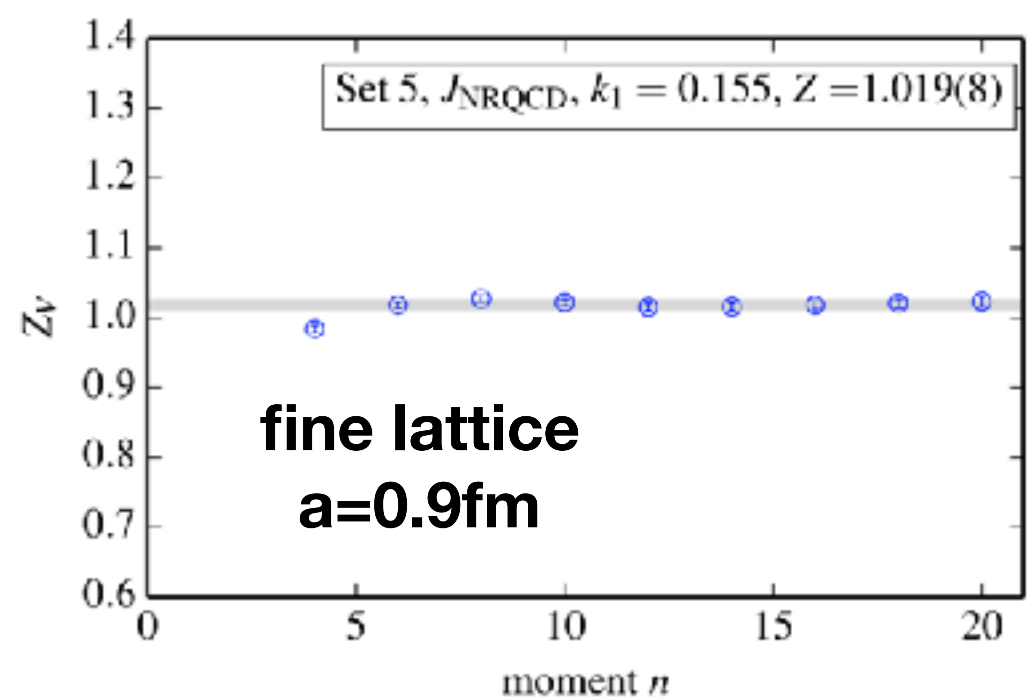
EM Current in Lattice

- In the Υ EM current, the matching coefficients were computed nonperturbatively by comparing 3-loop perturbative calculation of hadronic vacuum polarization.
- The matching coefficients depend on the lattice spacing nontrivially, but the coefficient at leading order in v is close to the tree-level value.
- One-loop lattice perturbation theory calculations also show nontrivial dependence on lattice spacing.

Hart, Hippel, Horgan, PRD75, 014008 (2007)

HPQCD, PRD91, 074514 (2015)





pNRQCD

- The pNRQCD effective field theory describes NRQCD matrix elements in terms of wavefunctions and gluonic correlators.

$$\langle V_Q(nS)|O_1(^3S_1)|V_Q(nS)\rangle = C_A \frac{|R^V_{n0}(0)|^2}{2\pi}\left(1 - \frac{E^{(0)}_{n0}}{m}\frac{2\mathcal{E}_3}{9} + \frac{2\mathcal{E}_3^{(2,t)}}{3m^2} + \frac{c_F^2\mathcal{B}_1}{3m^2}\right),$$

Brambilla, Eiras, Pineda, Soto, Vairo, PRD67, 034018 (2003)

- The weak-coupling regime is well suited for investigating the UV (short-distance) properties of matrix elements.
  For example, the 1- and 2-loop corrections to the NRQCD matrix elements can be related to the corrections to the wavefunction from potentials of higher orders in $1/m$.

pNRQCD example

- The correction from the perturbative spin-dependent potential corresponds to part of a divergent 2-loop correction to the matrix element.

$$V_{S=1} = \frac{\tilde{V}_S}{m^2}\delta(\boldsymbol{r})$$

$$\psi_n(0) = \psi_n^{(0)}(0) - \hat{G}(E_n^{(0)})\frac{\tilde{V}_S}{m^2}\psi_n^{(0)}(0)$$

$$\hat{G}(E_n^{(0)}) \equiv \sum_{m\neq n}\frac{|\psi_m^{(0)}(0)|^2}{E_m^{(0)} - E_n^{(0)}} = \lim_{E\to E_n^{(0)}}\left(G(E) - \frac{|\psi_n^{(0)}(0)|^2}{E_n^{(0)} - E}\right)$$

Kiyo, Pineda, Signer, NPB841 (2010) 231

- The zero-distance Green's function is UV divergent.

pNRQCD example

- In DR, the perturbative Green's function is given by

$$G_{\rm DR}(E) = \frac{\alpha_s C_F m^2}{4\pi}\left(\frac{-mE}{\pi e^{-\gamma_{\rm E}}}\right)^{-2\epsilon}\left[\frac{1}{4\epsilon} - \frac{1}{2\lambda} + \frac{1}{2} - \gamma_{\rm E} - \psi(1-\lambda) + O(\epsilon)\right]$$

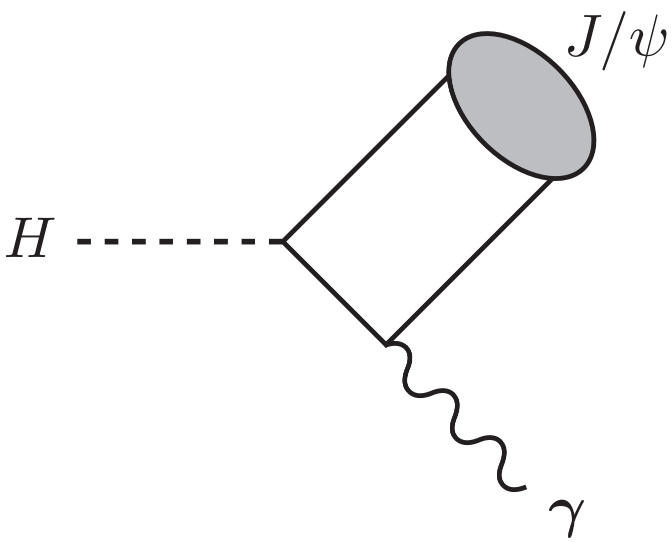
- An alternative regularization is to compute the wavefunction at a finite distance r from origin

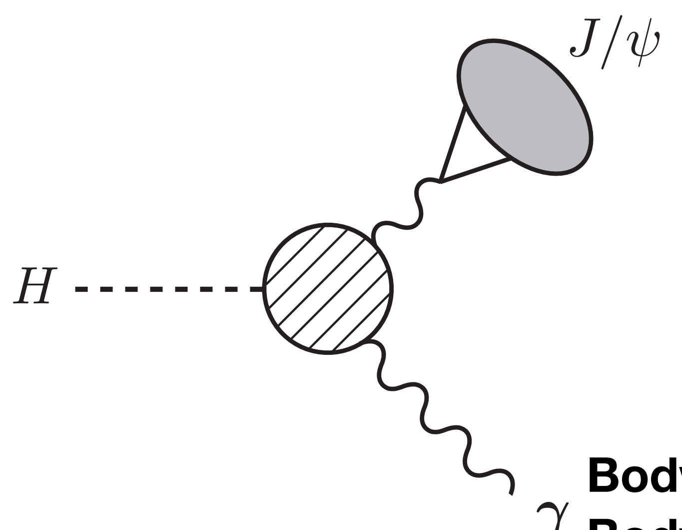
$$G_r(E) = \frac{\alpha_s C_F m^2}{4\pi}\left[\frac{1}{\alpha_s C_F m r} - \log(\mu e^{\gamma_{\rm E}} r) - \frac{1}{2\lambda} - \frac{1}{2}\log\left(\frac{-4mE}{\mu^2}\right) + 1 - \gamma_{\rm E} - \psi(1-\lambda)\right]$$

- The difference between the two Green's functions gives the 1-loop finite renormalization needed to convert finite-r regularization to DR.

Neat example

- Sometimes the scale and scheme dependences in the Wilson coefficients can be made cancel manifestly by using ratios.
- Example : Higgs decay into J/ψ + photon





$$\sim J_V = \langle 0|\bar{\psi}\gamma^\mu\psi|J/\psi\rangle$$

$$\sim J_T = \langle 0|\bar{\psi}[\gamma^\mu, \gamma^\nu]\psi|J/\psi\rangle$$

Bodwin, Petriello, Stoynev, Velasco, PRD88, 053003 (2013)
Bodwin, HSC, Ee, Lee, Petriello, PRD90, 113010 (2014)
Bodwin, HSC, Ee, Lee, PRD95, 054018 (2017),
PRD96, 116014 (2017)
König, Neubert, JHEP 1508 (2015) 012

- At LO in v, the ratio J_T/J_V is perturbative and independent on the renormalization scheme. Thus,

$$J_T = (J_T/J_V)_{\text{perturbative}} \times (J_V)_{\text{measured}}$$

Summary

- Quarkonium phenomenology requires well-determined long-distance matrix elements and Wilson coefficients with good perturbative control.
- Perturbative corrections are often quite large in quarkonium production and decay processes in dimensional regularization. Renormalization scheme dependence also seem to be large.
- This might have to do with the way power divergences are dealt with in DR, as power divergences are related to renormalons.
- Rewriting matrix elements in pNRQCD may give new insights to these issues.