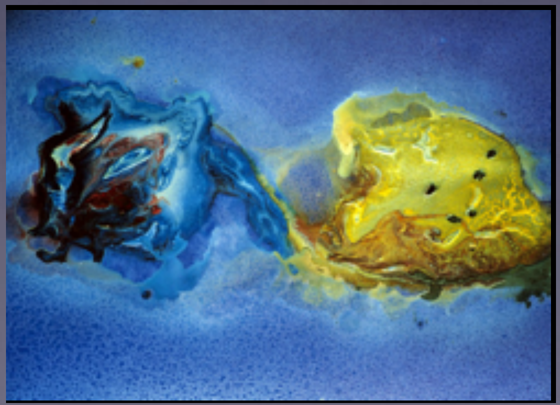
Spin in heavy hybrids
with Nonrelativistic Effective Field Theories



Physik Department TUM
T30f

Nora Brambilla

- Non relativistic Effective Field Theories (plus lattice) give a QCD description of quarkonia below the strong decay threshold

- the hierarchy of NREFT is based on the hierarchy of scales in quarkonium

- in this framework quarkonium becomes a golden system for the extraction of SM parameters (quark masses, alphas) and the study of confinement

- Can we use Non relativistic Effective Field Theories (plus lattice) to give a QCD description of exotic quarkonia (X,Y,Z) at or above the strong decay threshold?

- In this talk QQbar and glue: Hybrids multiplets with Lambda doubling effect and spin structure

- This EFT picture may be extended to a comprehensive description of X, Y, Z

Tetra quarks

van der Waals bottomonia interaction : bound states?

Material for discussion/references

- **Heavy quarkonium: progress, puzzles, and opportunities**
  N. Brambilla (Munich, Tech. U.) *et al.*. Oct 2010. 181 pp.
  Published in **Eur.Phys.J. C71 (2011) 1534**
  e-Print: **arXiv:1010.5827 [hep-ph]**-
- **QCD and Strongly Coupled Gauge Theories: Challenges and Perspectives**
  N. Brambilla (Munich, Tech. U.) *et al.*. Apr 2014. 241 pp.
  Published in **Eur.Phys.J. C74 (2014) no.10, 2981**
  e-Print: **arXiv:1404.3723**

chapter on exotics

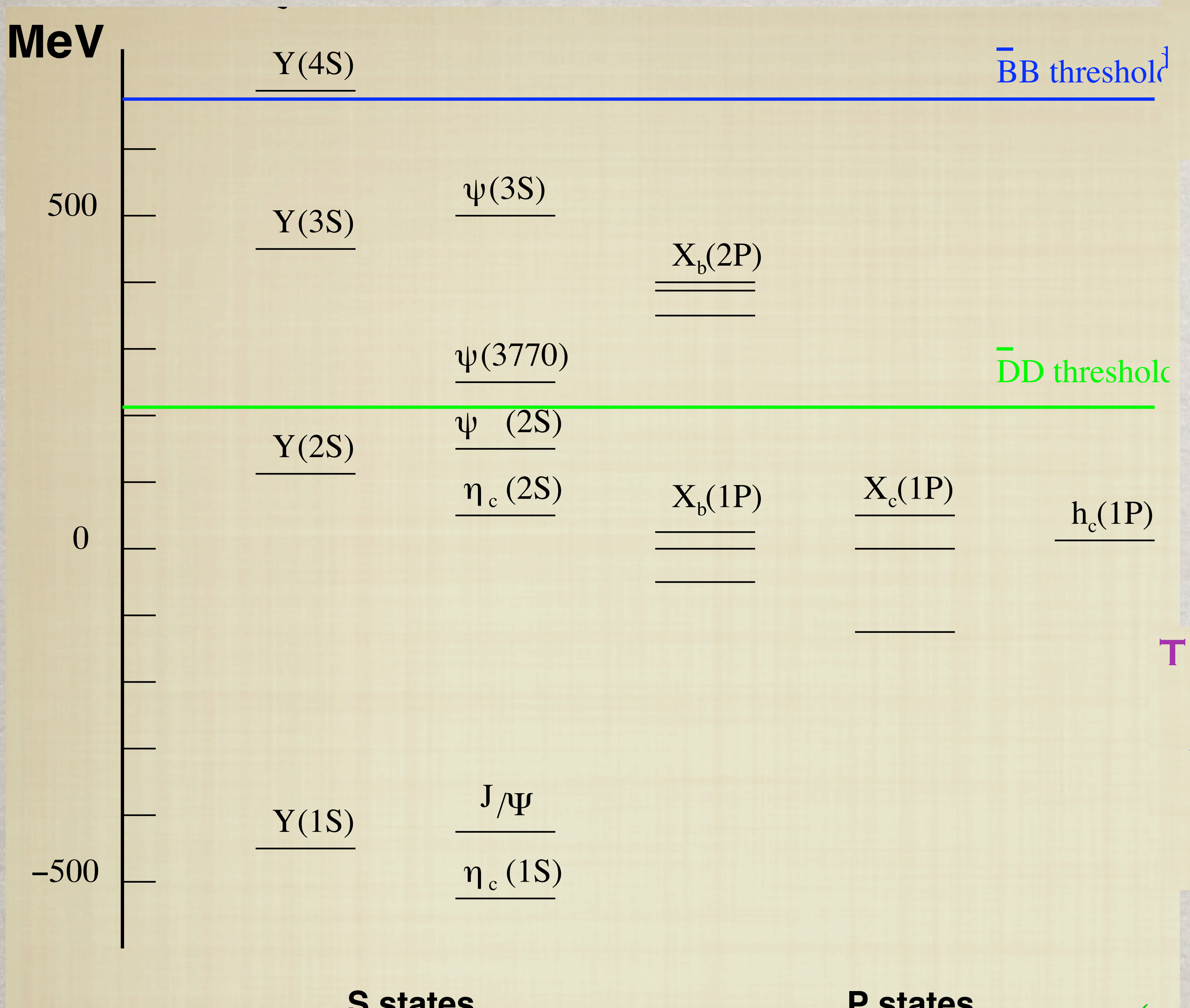
- **Effective field theories for heavy quarkonium**
  Nora Brambilla, Antonio Pineda, Joan Soto, Antonio Vairo
  Rev.Mod.Phys. 77 (2005) 1423
  e-Print: **hep-ph/0410047**

- **Quarkonium Hybrids with Nonrelativistic Effective Field Theories**
  Matthias Berwein , Nora Brambilla, Jaume Tarrús Castellà, Antonio Vairo
  Phys.Rev. D92 (2015) no.11, 114019
  e-Print: **arXiv:1510.04299**

- **Born-Oppenheimer approximation in an effective field theory language**
  Nora Brambilla , Gastão Krein, Jaume Tarrús Castellà, Antonio Vairo
  Phys.Rev. D97 (2018) no.1, 016016
  e-Print: **arXiv:1707.09647**

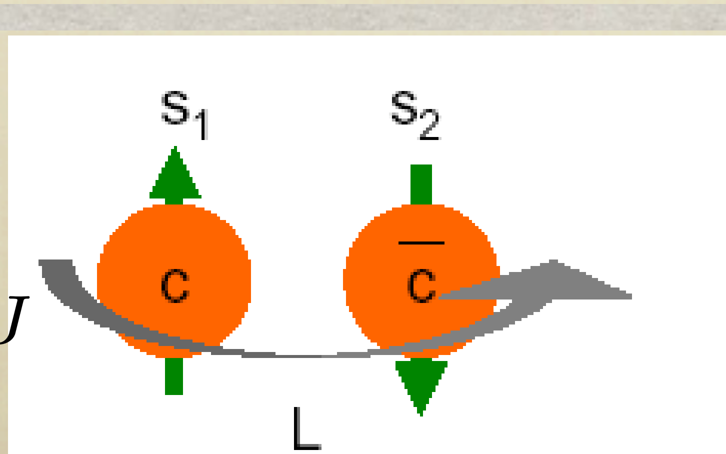
Spin structure of heavy-quark hybrids
Nora Brambilla, Wai Kin Lai, Jorge Segovia, Jaume Tarrús Castellà, Antonio Vairo. May 20, 2018.
e-Print: **arXiv:1805.07713**

Quarkonium scales



Normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$

$^{2S+1}L_J$



NR BOUND STATES HAVE AT LEAST 3 SCALES

$$m \gg mv \gg mv^2 \quad v \ll 1$$

$$mv \sim r^{-1}$$

and Λ_{QCD}

THE SYSTEM IS NONRELATIVISTIC(NR)

$$\Delta E \sim mv^2, \Delta_{fs} E \sim mv^4$$

$$v_b^2 \sim 0.1, v_c^2 \sim 0.3$$

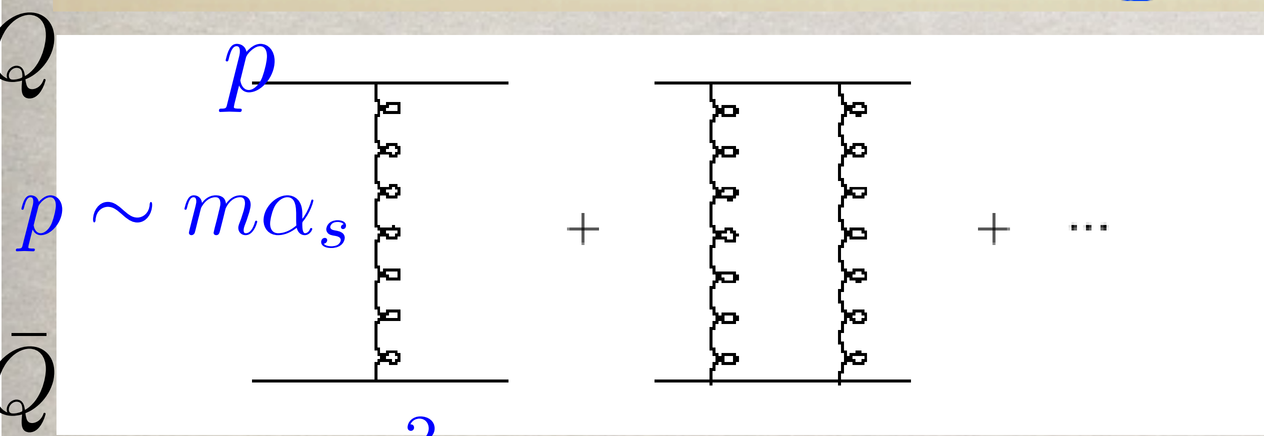
THE MASS SCALE IS PERTURBATIVE

$$m_Q \gg \Lambda_{\mathrm{QCD}}$$

$$m_b \simeq 5\,\mathrm{GeV}; m_c \simeq 1.5\,\mathrm{GeV}$$

QCD theory of Quarkonium: a very hard problem

Close to the bound state $\alpha_s \sim v$

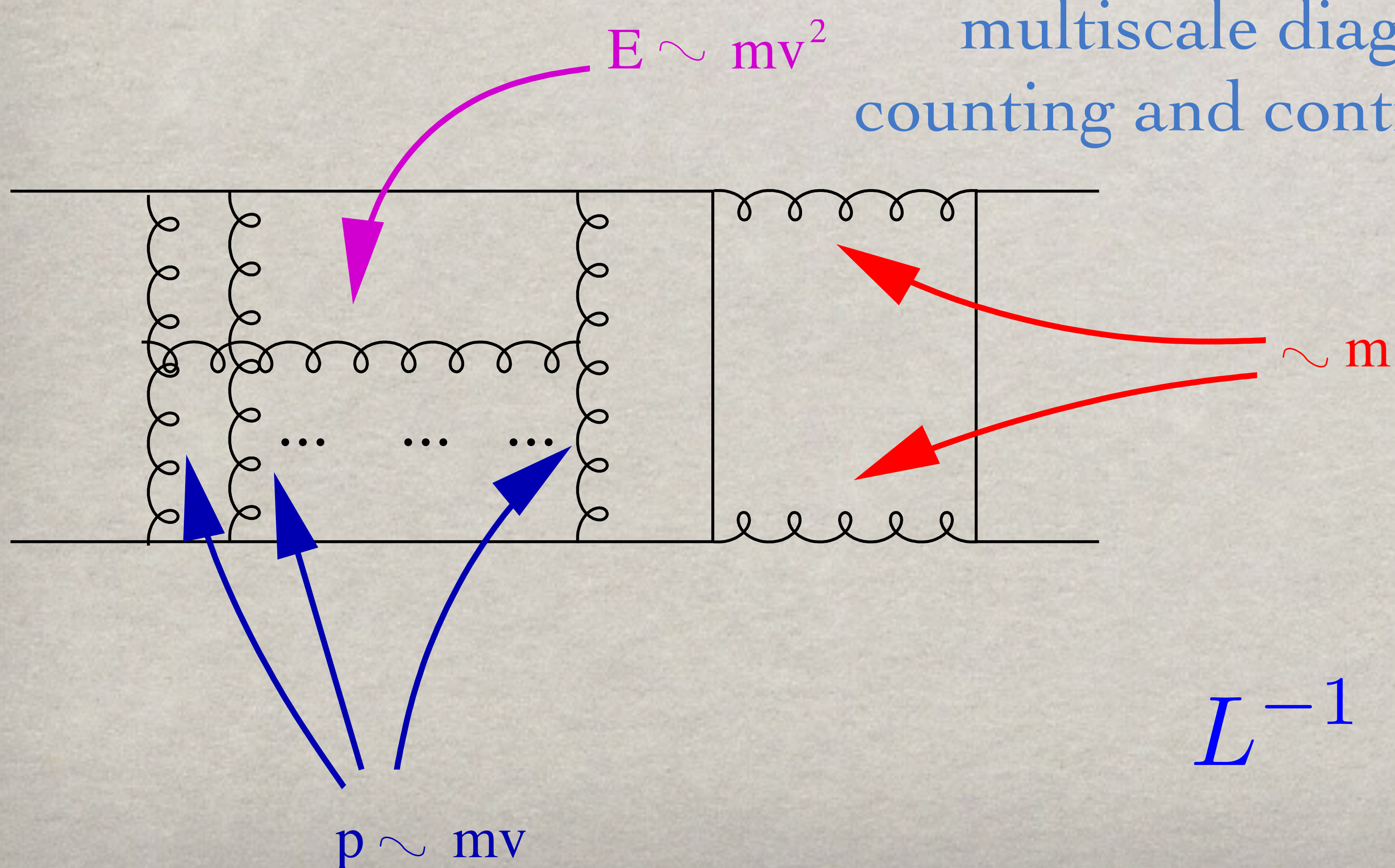


$$\frac{g^2}{p^2}\left(1 + \frac{m\alpha_s}{p}\right)$$

$$\sim \frac{1}{E - \left(\frac{p^2}{m} + V\right)}$$

- From $(\frac{p^2}{m} + V)\phi = E\phi \rightarrow p \sim mv$ and $E = \frac{p^2}{m} + V \sim mv^2$.

multiscale diagrams have a complicate power counting and contribute to all orders in the coupling

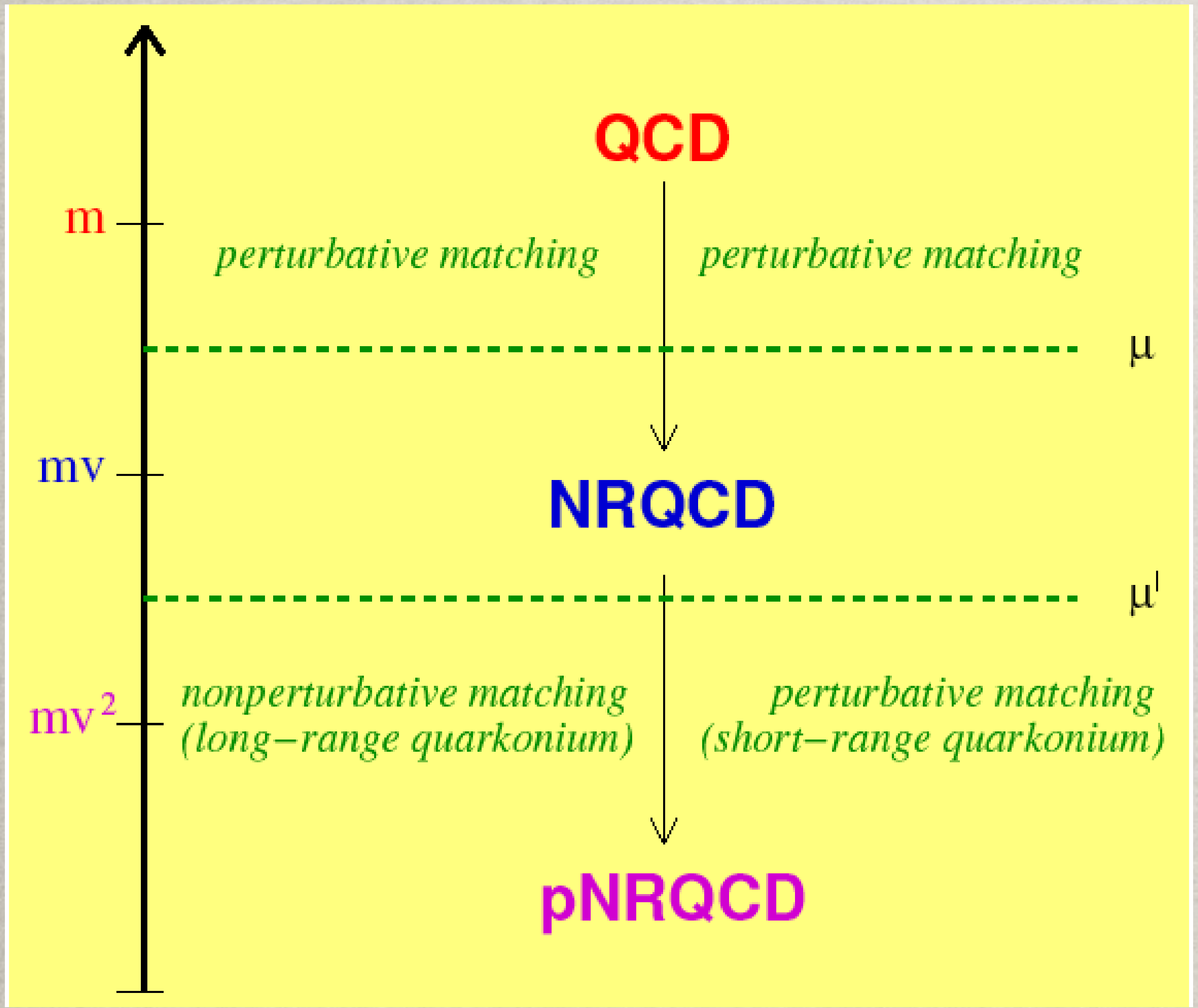


Difficult also for the lattice!

$$L^{-1} \ll \lambda \ll \Lambda \ll a^{-1}$$

Quarkonium with Non relativistic Effective Field Theories

Color degrees of freedom
3X3=1+8
singlet and octet QQbar



Hard

$$\frac{E_\lambda}{E_\Lambda} = \frac{mv}{m}$$

Soft (relative momentum)

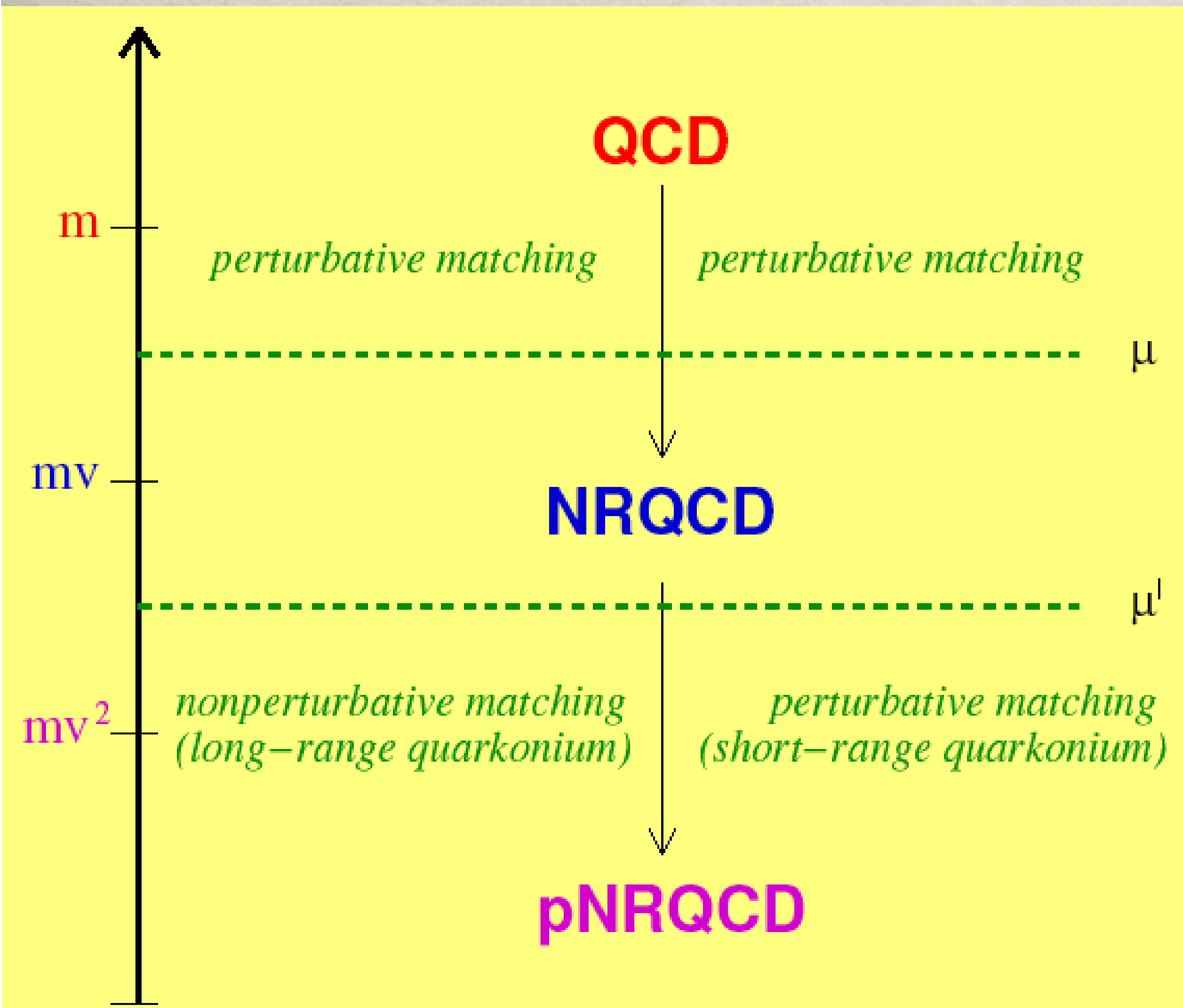
$$\frac{E_\lambda}{E_\Lambda} = \frac{mv^2}{mv}$$

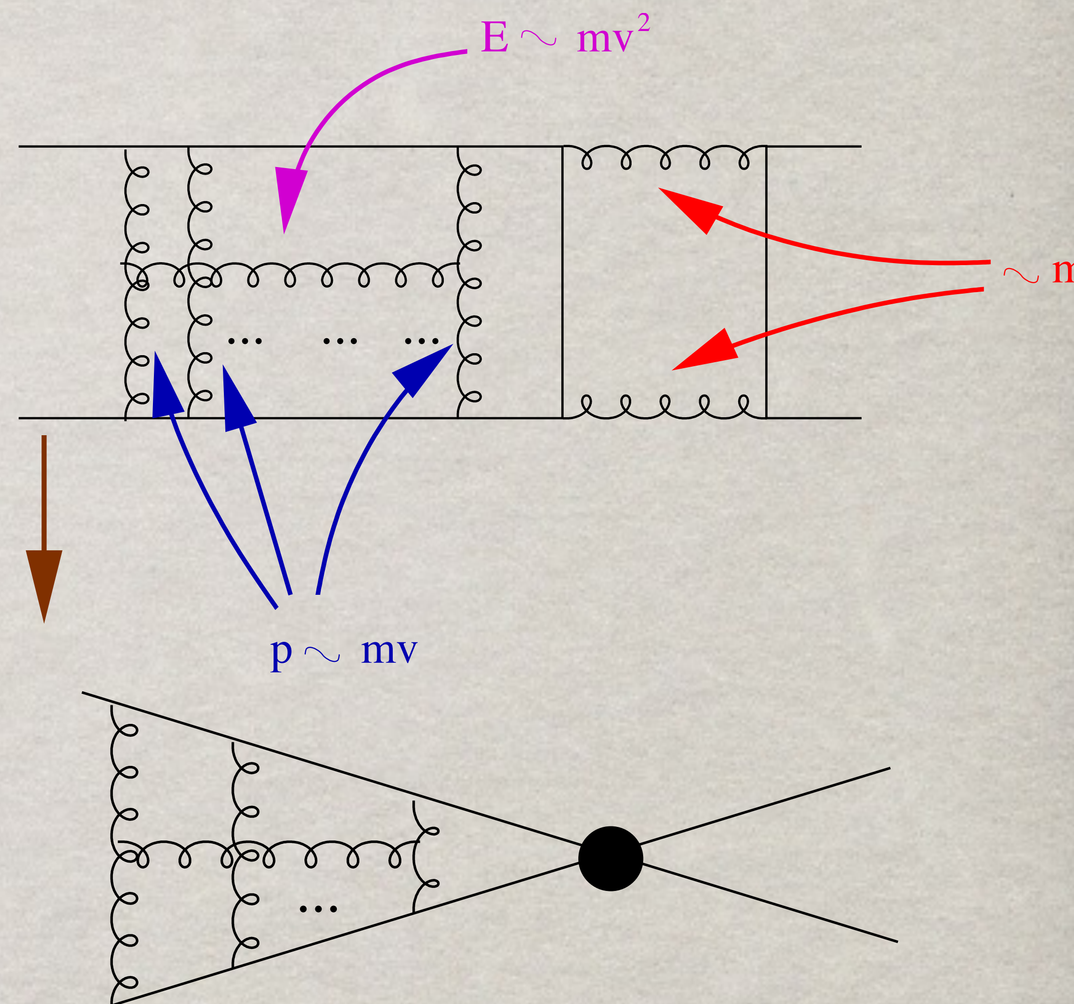
Ultrasoft (binding energy)

$$\mathcal{L}_{\rm EFT} = \sum_n c_n(E_\Lambda/\mu)\frac{O_n(\mu,\lambda)}{E_\Lambda}$$

$$\langle O_n \rangle \sim E_\lambda^n$$

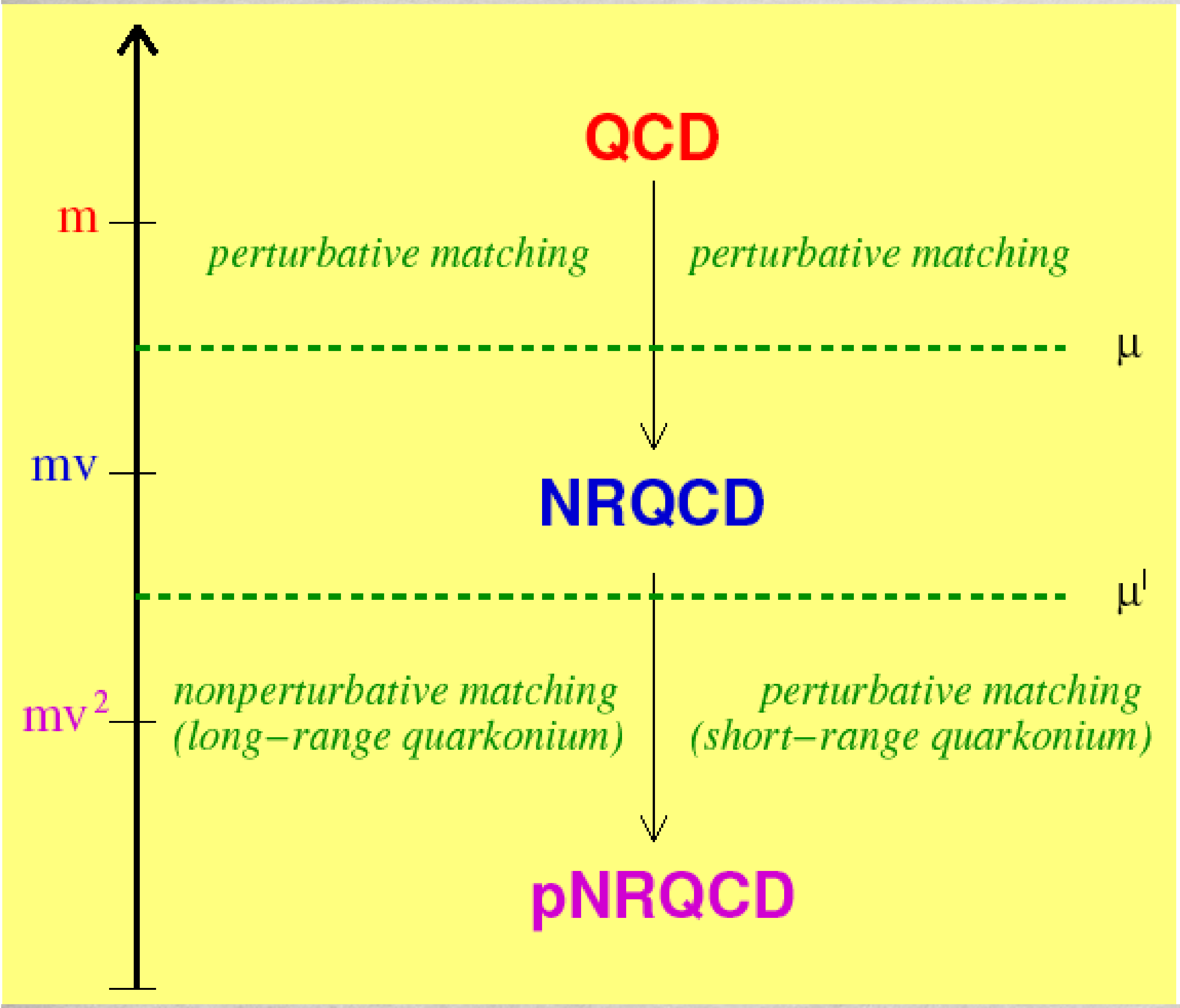
Quarkonium with NR EFT: Non Relativistic QCD (NRQCD)

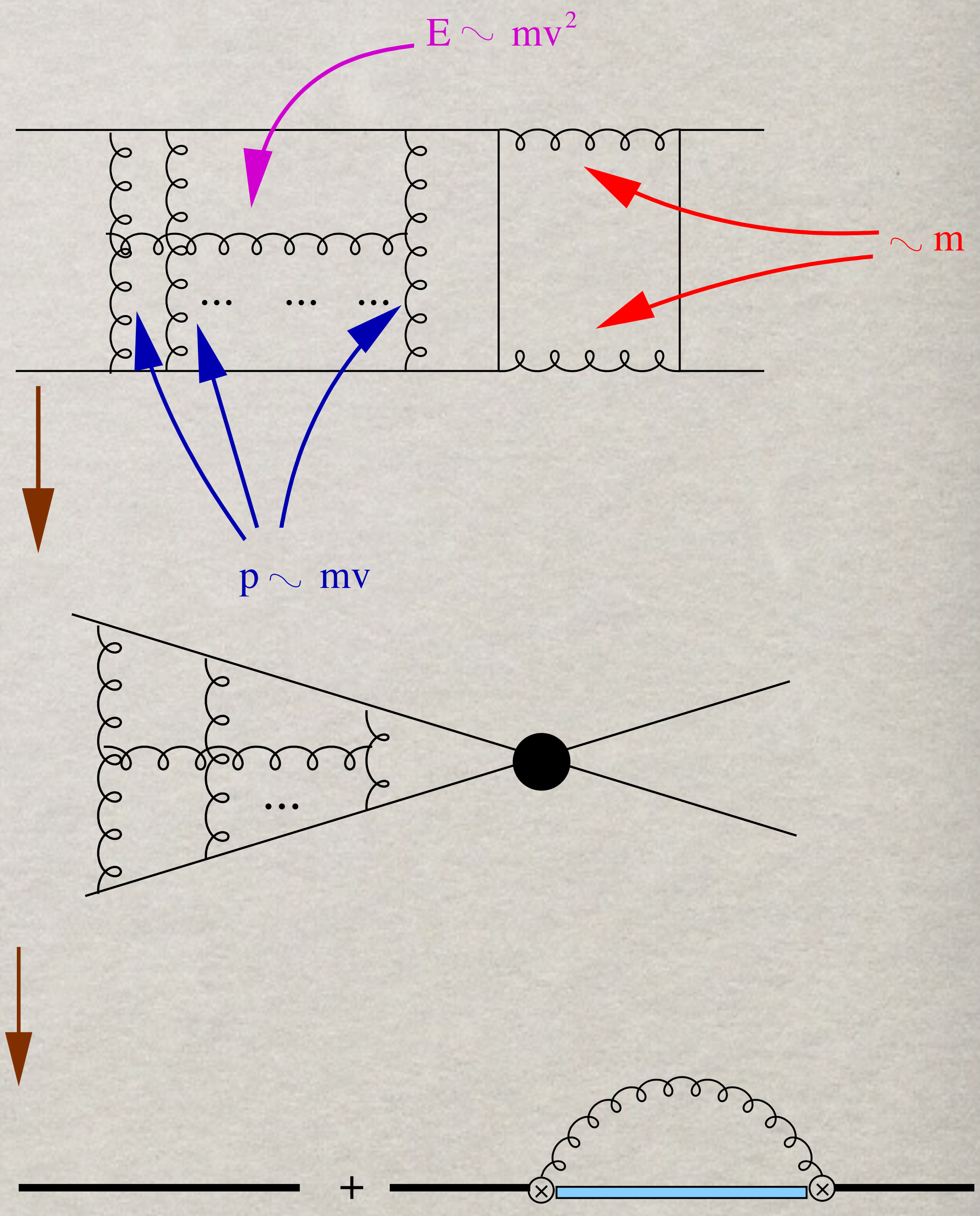




$$\mathcal{L}_{\rm NRQCD} = \sum_n c(\alpha_{\rm s}(m/\mu)) \times \frac{O_n(\mu,\lambda)}{m^n}$$

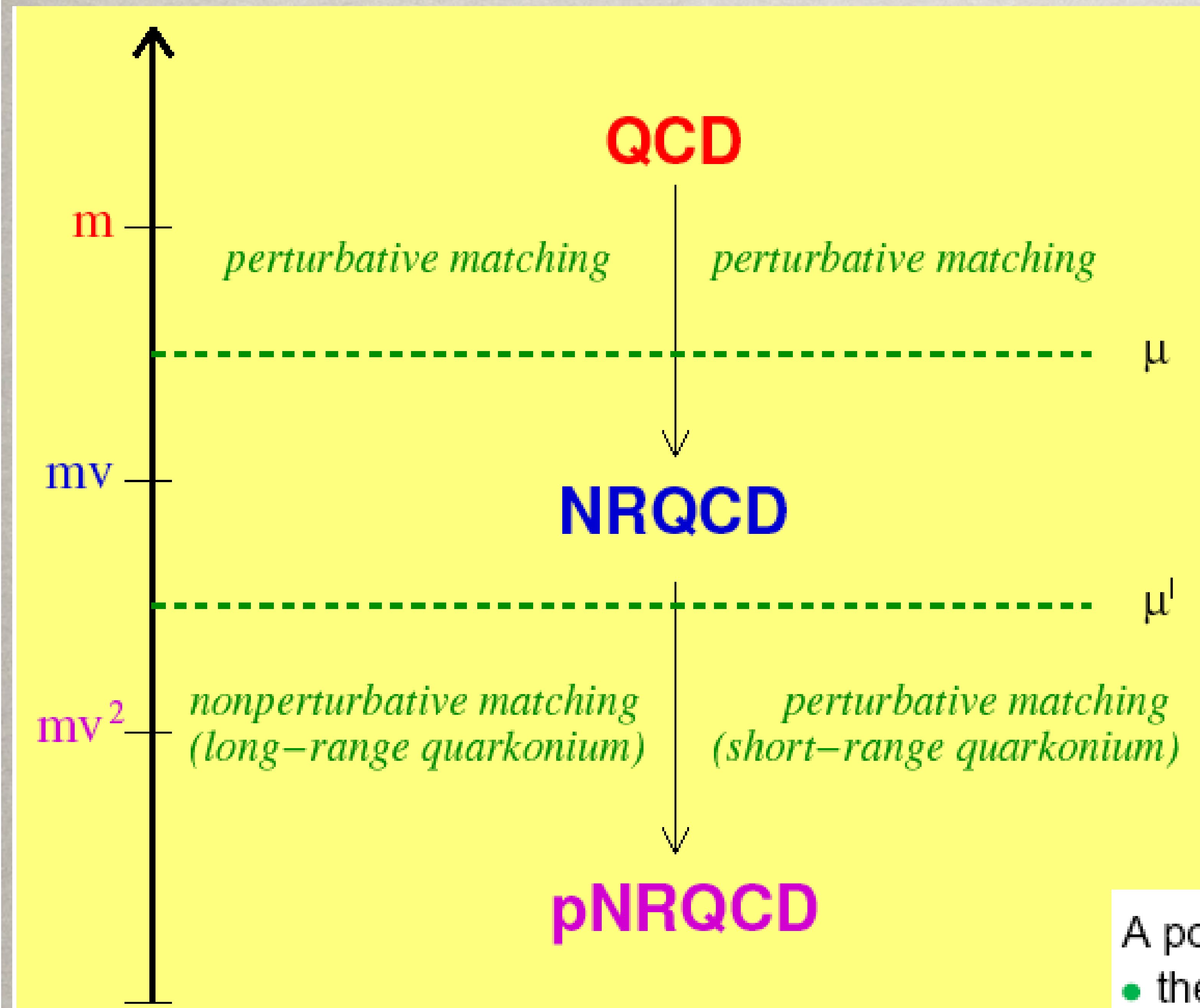
Quarkonium with NR EFT: potential NonRelativistic QCD (pNRQCD)





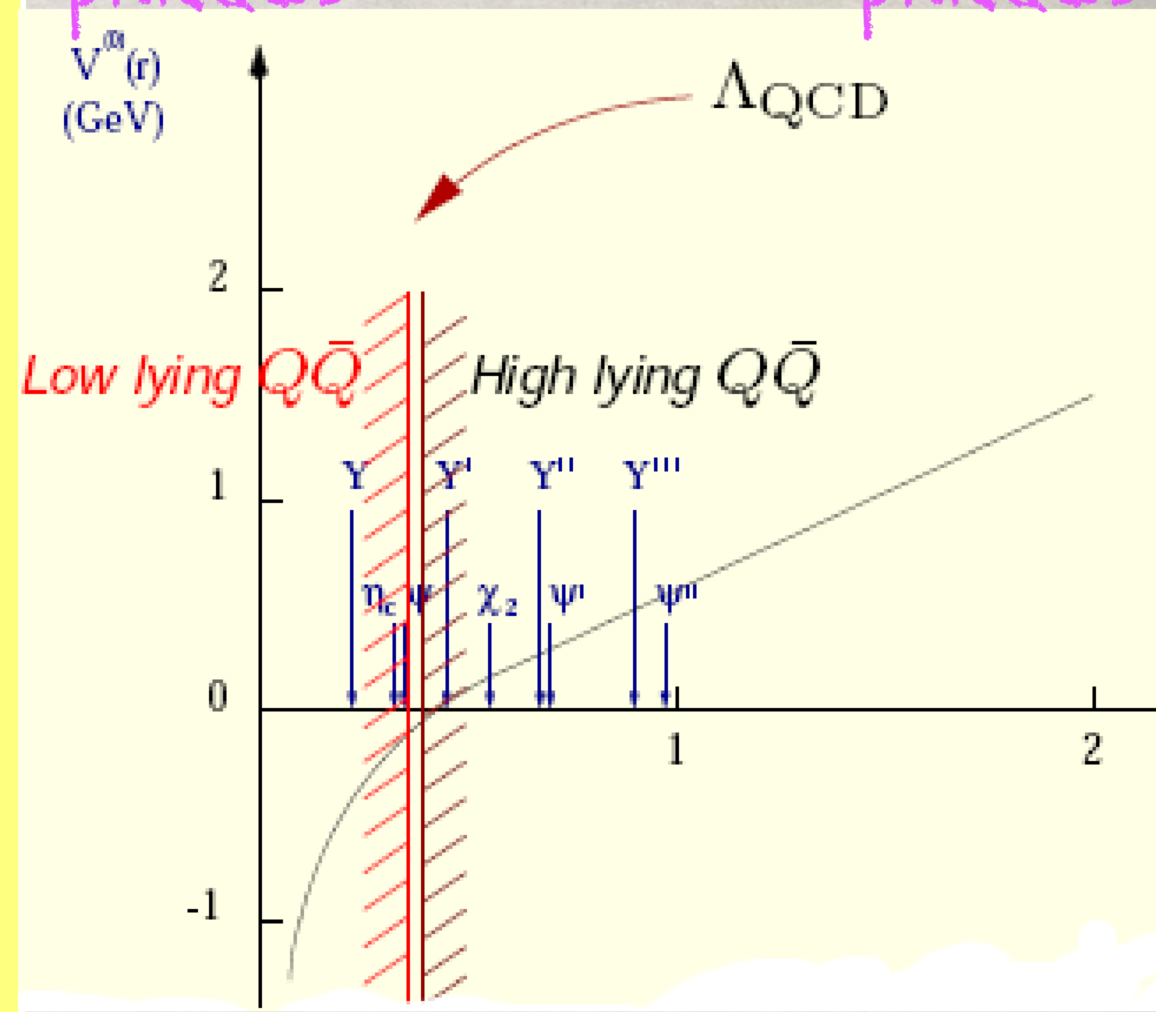
$$\mathcal{L}_{\mathrm{pNRQCD}} = \sum_k \sum_n \frac{1}{m^k} c_k(\alpha_s(m/\mu)) \times V(r\mu', r\mu) \times O_n(\mu', \lambda)\, r^n$$

Quarkonium with NR EFT: pNRQCD



weakly coupled pNRQCD

strongly coupled pNRQCD



A potential picture arises at the level of pNRQCD:

- the potential is perturbative if $mv \gg \Lambda_{QCD}$
- the potential is non-perturbative if $mv \sim \Lambda_{QCD}$

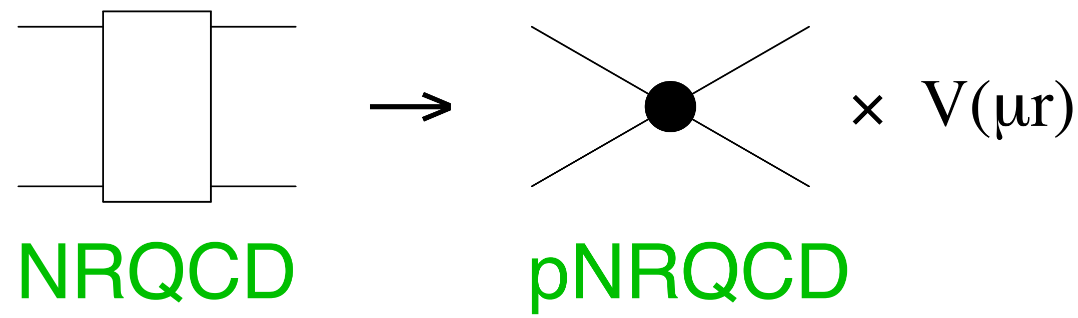
In QCD another scale is relevant Λ_{QCD}

Pineda, Soto 97, N.B., Pineda, Soto, Vairo 99
N.B. Vairo, Pineda, Soto 00--014
N.B., Pineda, Soto, Vairo Review of Modern Physis 77(2005) 1423

Quarkonium systems with small radius $r \ll \Lambda_{\mathrm{QCD}}^{-1}$

pNRQCD for quarkonia with small radius $r \ll \Lambda_{\rm QCD}^{-1}$

Degrees of freedom that scale like mv are integrated out:



- If $mv \gg \Lambda_{\rm QCD}$, the matching is perturbative

- Degrees of freedom: quarks and gluons

  Q-$\bar{Q}$ states, with energy $\sim \Lambda_{\rm QCD}, mv^2$ and momentum $\lesssim mv$
  $\Rightarrow$ (i) singlet $\rm S$ (ii) octet $\rm O$

  Gluons with energy and momentum $\sim \Lambda_{\rm QCD}, mv^2$

- Definite power counting: $r \sim \dfrac{1}{mv}$ and $t, R \sim \dfrac{1}{mv^2}, \dfrac{1}{\Lambda_{\rm QCD}}$

  The gauge fields are multipole expanded:
  $A(R,r,t) = A(R,t) + \mathbf{r} \cdot \boldsymbol{\nabla} A(R,t) + \ldots$

  Non-analytic behaviour in $r \to$ matching coefficients V

weak pNRQCD $\quad r \ll \Lambda_{\rm QCD}^{-1}$

Singlet static potential

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} + \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} + \mathrm{O}^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o \right) \mathrm{O} \right\}$$

LO in r

Octet static potential

- At leading order in r, the singlet S satisfies the QCD Schrödinger equation.
  - The (weak coupling) static potential is the Coulomb potential:

$$V_s(r) = -C_F \frac{\alpha_s}{r} + \ldots, \qquad V_o(r) = \frac{1}{2N} \frac{\alpha_s}{r} + \ldots, \qquad N = 3,\ C_F = \frac{4}{3}$$

S singlet field — singlet propagator

O octet field — octet propagator

Pineda, Soto 97; Brambilla, Pineda, Soto, Vairo 99-

weak pNRQCD $\quad r \ll \Lambda_{\rm QCD}^{-1}$

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\,a} + {\rm Tr}\left\{ {\rm S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) {\rm S} + {\rm O}^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o \right) {\rm O} \right\}$$

LO in r

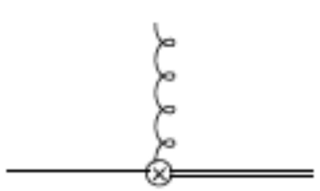
$$+V_A {\rm Tr}\left\{ {\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}\,{\rm S} + {\rm S}^\dagger \mathbf{r}\cdot g\mathbf{E}\,{\rm O} \right\}$$
$$+\frac{V_B}{2} {\rm Tr}\left\{ {\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}\,{\rm O} + {\rm O}^\dagger {\rm O}\mathbf{r}\cdot g\mathbf{E} \right\}$$
$$+\cdots$$

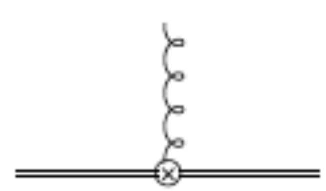
NLO in r

- Feynman rules:

 $= \theta(t)\, e^{-itH_s}$

 $= \theta(t)\, e^{-itH_o} \left(e^{-i\int dt\, A^{\rm adj}} \right)$

 $= {\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}\,{\rm S}$

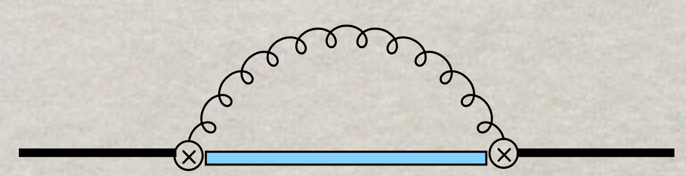
 $= {\rm O}^\dagger \{\mathbf{r}\cdot g\mathbf{E},\, {\rm O}\}$

The QQbar potential is a matching coefficient of pNRQCD and can be calculated in perturbation theory

Quarkonium singlet potential

$$V = V_0 + \frac{1}{m} V_1 + \frac{1}{m^2}(V_{SD} + V_{VD})$$

Small systems: QQ energies at $m\alpha_{\mathrm{s}}^5$

$$E_n = 2m + \langle n | \frac{p^2}{m} + V_s | n \rangle + \langle n |$$  $$| n \rangle$$

$$E_n = \langle n | H_s(\mu) | n \rangle - i \frac{g^2}{3N_c} \int_0^\infty dt \, \langle n | \mathbf{r} e^{it(E_n^{(0)} - H_o)} \mathbf{r} | n \rangle \, \langle \mathbf{E}(t) \, \mathbf{E}(0) \rangle (\mu)$$

Applications to Quarkonium physics: systems with small radius

for references see the QWG doc arXiv:1010.5827

- c and b masses at NNLO, N^{3}LO*, NNLL*;
- B_c mass at NNLO; Penin et al 04
- B_c^*, η_c, η_b masses at NLL; Kniehl et al 04
- Quarkonium 1P fine splittings at NLO;
- $\Upsilon(1S)$, η_b electromagnetic decays at NNLL;
- $\Upsilon(1S)$ and J/ψ radiative decays at NLO;
- $\Upsilon(1S) \to \gamma\eta_b$, $J/\psi \to \gamma\eta_c$ at NNLO;
- $t\bar{t}$ cross section at NNLL;
- QQq and QQQ baryons: potentials at NNLO, masses, hyperfine splitting, ... ; N. B. et al 010
- Thermal effects on quarkonium in medium: potential, masses (at $m\alpha_s^5$), widths, ...;

$$\mathcal{B}(J/\psi \to \gamma\eta_c(1S)) = (1.6 \pm 1.1)\%$$
$$\mathcal{B}(\Upsilon(1S) \to \gamma\eta_b(1S)) = (2.85 \pm 0.30) \times 10^{-4}$$

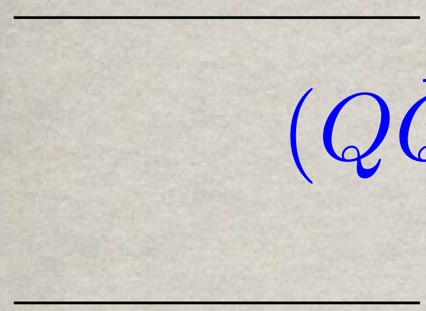
N. B. Yu Jia A. Vairo 2005

$$\Gamma(\eta_b(1S) \to \gamma\gamma) = 0.54 \pm 0.15 \text{ keV}\,.$$
$$\Gamma(\eta_b(1S) \to \text{LH}) = 7\text{-}16 \text{ MeV}$$

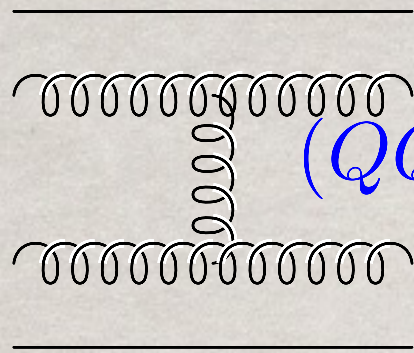
Y. Kiyo, A. Pineda, A. Signer 2010

Quarkonium systems with large radius $r \sim \Lambda_{QCD}^{-1}$

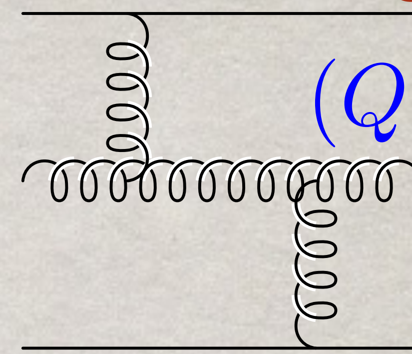
Hitting the scale $\Lambda_{\rm QCD}$ $\quad r \sim \Lambda_{QCD}^{-1}$



$(Q\bar{Q})_1$



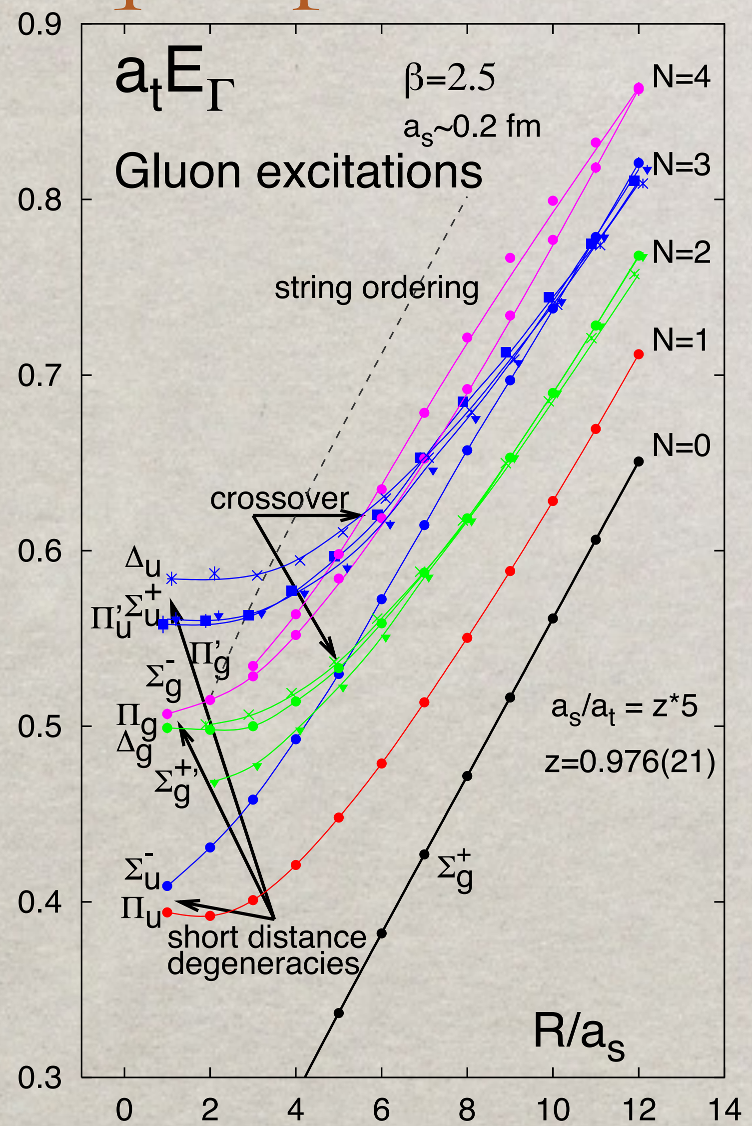
$(Q\bar{Q})_1 +$ Glueball



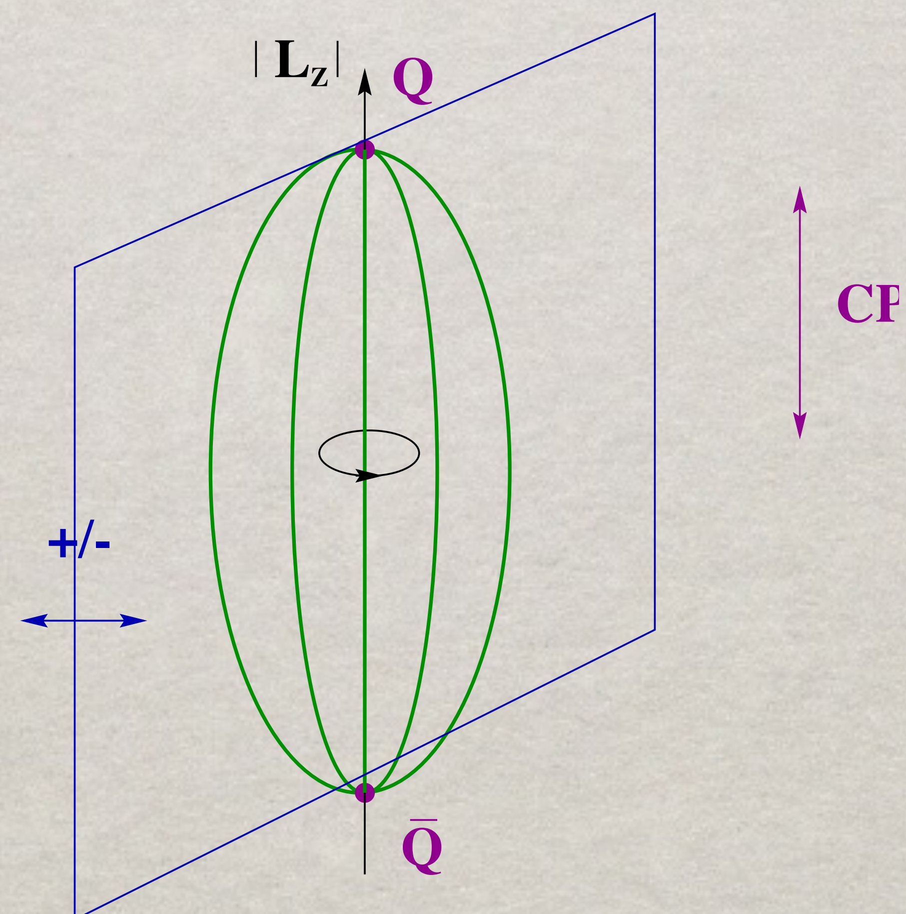
$(Q\bar{Q})_8 G$ hybrid

Static qcd spectrum

L a t t i c e



Juge Kuti Mornigstar 98-06 Static NRQCD



Symmetries of a diatomic molecule + C.C.

a) $|L_z| = 0, 1, 2, \ldots$
$= \Sigma, \Pi, \Delta \ldots$

b) CP (u/g)

c) Reflection $(+/-)$
(for Σ only)

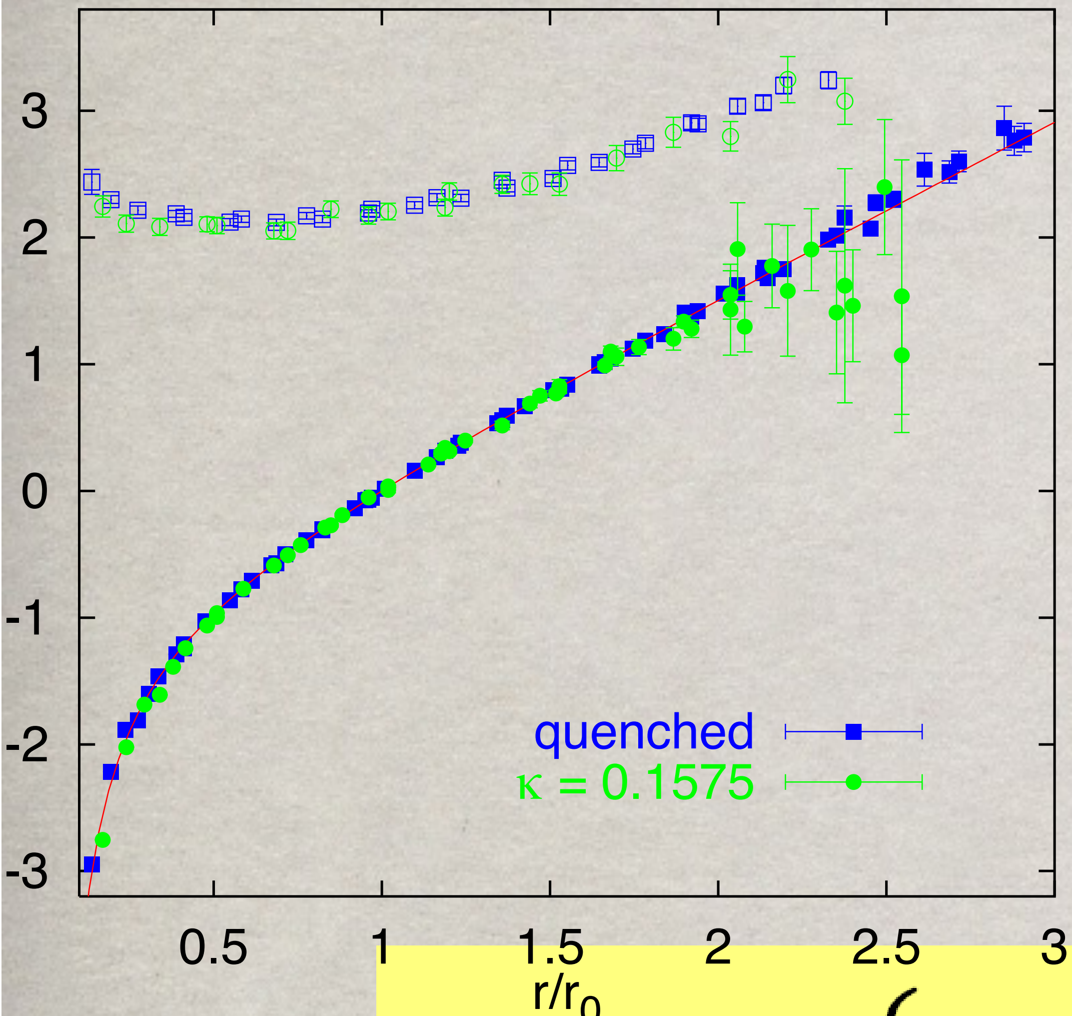
$$\mathcal{H}^{(0)}|\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)} = E_n^{(0)}(\mathbf{x}_1, \mathbf{x}_2)|\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)}$$

$$|\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)} = \psi^\dagger(\mathbf{x}_1)\chi(\mathbf{x}_2)|n; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)}$$

$|\underline{0}\rangle^{(0)} = |(Q\bar{Q})_1\rangle \rightarrow$ Quarkonium Singlet

$|\underline{n > 0}\rangle^{(0)} = |(Q\bar{Q})g^{(n)}\rangle \rightarrow$ Higher Gluonic Excitations

strongly coupled pNRQCD $r \sim \Lambda_{QCD}^{-1}$ $mv \sim \Lambda_{QCD}$



- integrate out all scales above mv^2
- gluonic excitations develop a gap $\Lambda_{\rm QCD}$ and are integrated out

$\Rightarrow$ The singlet quarkonium field S of energy mv^2 is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).

$$\mathcal{L} = \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} \right\}$$

- A potential description emerges from the EFT Brambilla Pineda Soto Vairo 00
- The potentials $V = \mathrm{Re}V + ImV$ from QCD in the matching: get spectra and decays
- V to be calculated on the lattice or in QCD vacuum models

The matching condition is: $\langle H|\mathcal{H}|H\rangle = \langle nljs|\frac{\mathbf{p}^2}{m} + \sum_n \frac{V_s^{(n)}}{m^n}|nljs\rangle$

$$H_{\rm NRQCD} = H^{(0)} + \frac{1}{m_Q}H^{(1,0)} + \frac{1}{m_{\bar{Q}}}H^{(0,1)} + \dots ,$$

$$H^{(0)} = \int d^3x \frac{1}{2}(\boldsymbol{E}^a\cdot\boldsymbol{E}^a + \boldsymbol{B}^a\cdot\boldsymbol{B}^a) - \sum_{j=1}^{n_f}\int d^3\boldsymbol{x}\,\bar{q}_j\, i\boldsymbol{D}\cdot\boldsymbol{\gamma}\, q_j ,$$

$$H^{(1,0)} = -\frac{1}{2}\int d^3x\, \psi^\dagger \left(\boldsymbol{D}^2 + gc_F\,\boldsymbol{\sigma}\cdot\boldsymbol{B}\right)\psi ,$$

$$H^{(0,1)} = \frac{1}{2}\int d^3x\, \chi^\dagger \left(\boldsymbol{D}^2 + gc_F\,\boldsymbol{\sigma}\cdot\boldsymbol{B}\right)\chi ,$$

$$\mathcal{H}^{(0)}|\underline{n};\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)} = E_n^{(0)}(\mathbf{x}_1,\mathbf{x}_2)|\underline{n};\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)}$$

$$|\underline{n};\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)} = \psi^\dagger(\mathbf{x}_1)\chi(\mathbf{x}_2)|n;\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)}$$

The matching condition is: $\langle H|\,\mathcal{H}\,|H\rangle = \langle nljs|\,\frac{\mathbf{p}^2}{m} + \sum_n \frac{V_s^{(n)}}{m^n}\,|nljs\rangle$

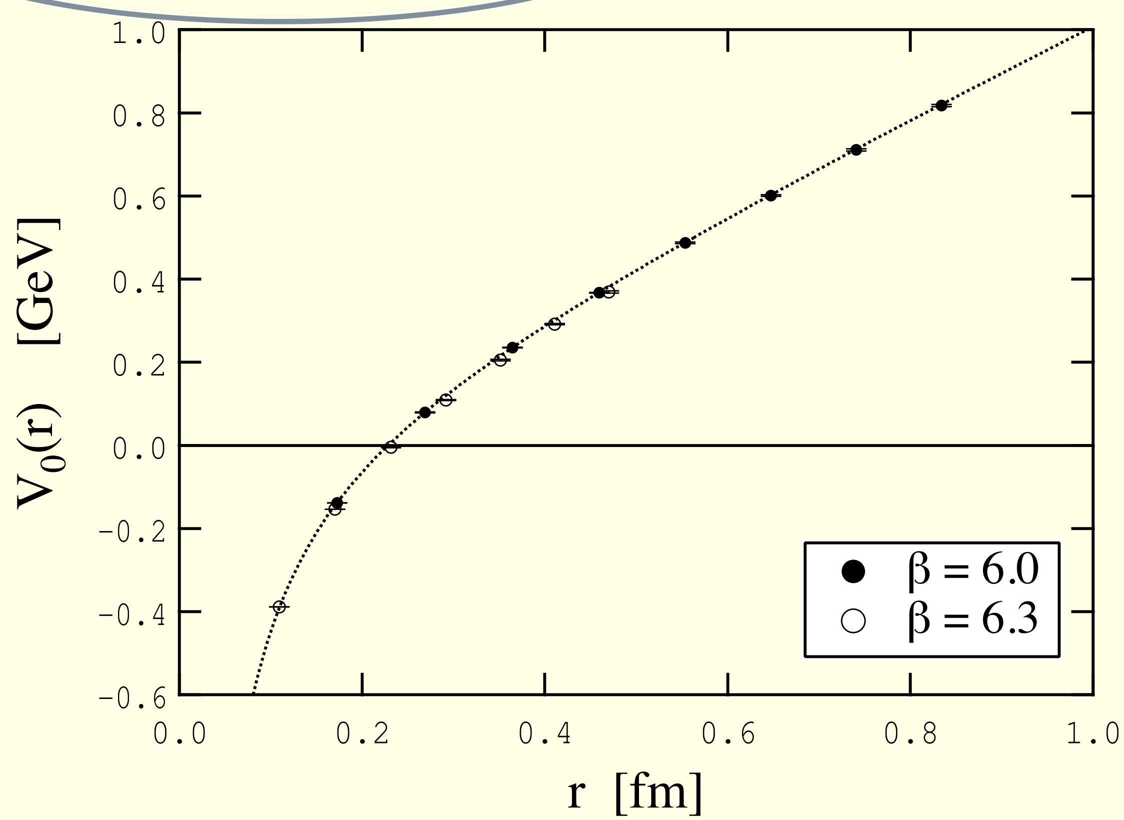
and from this we obtain the

Quarkonium singlet potential

$$V = V_0 + \frac{1}{m}V_1 + \frac{1}{m^2}(V_{SD} + V_{VD})$$

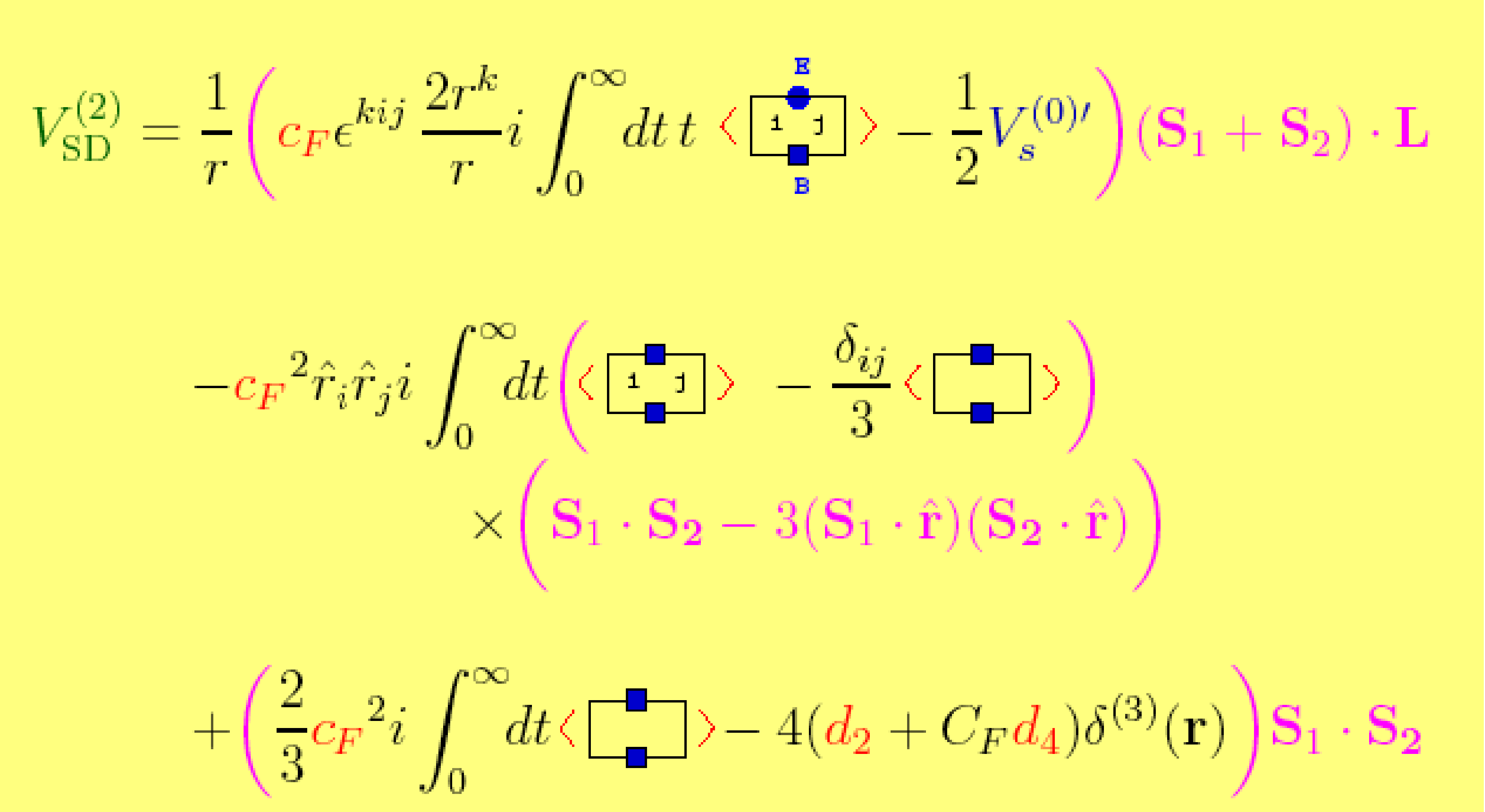
$$V_s^{(0)} = \lim_{T\to\infty} \frac{i}{T} \ln\langle W(r \times T)\rangle = \lim_{T\to\infty} \frac{i}{T} \ln\langle \square \rangle$$

$$W = \langle \exp\{ig \oint A^\mu dx_\mu\}\rangle$$



○ Koma Koma NPB 769(07)79

QCD Spin dependent potentials



Eichten Feinberg 81, Gromes 84, Chen et al. 95 Brambilla Vairo 99 Pineda, Vairo 00

-factorization: the NRQCD matching coefficients encode the physics at the large scale m, the potentials are given in terms of low energy nonperturbative Wilson loops

-the spin dependent potential has the usual structure with spin-orbit, tensor and spin-spin terms. The spin-orbit term has a confining contribution

-the spin dependent potentials in the Schroedinger eq. give the multiplet spin structure

EFTs (plus lattice) give a QCD description of quarkonium below threshold

For states close or above the strong decay threshold the situation is much more complicated.

there is no mass gap between quarkonium and the creation of a heavy-light mesons couple

$$m_{Q\bar{q}} + m_{\bar{Q}q} = 2m + 2\Lambda_{QCD}$$

Near theshold heavy-light mesons and gluons excitations have to be included and many additional states built using the light quark quantum numbers may appear

No systematic treatment is yet available; also lattice calculations are challenging

Many phenomenological models exist

States made of two heavy and light quarks

- Pairs of heavy-light mesons: $D\bar{D}$, $B\bar{B}$, ...
- Molecular states, i.e. states built on the pair of heavy-light mesons.
  - Tornqvist PRL 67(91)556
- The usual quarkonium states, built on the static potential, may also give rise to molecular states through the interaction with light hadrons (hadro-quarkonium).
  - Dubynskiy Voloshin PLB 666 (2008) 344
- Tetraquark states. MAIANI, PICCININI, POLOSA ET AL. 2005--
  - Jaffe PRD 15(77)267
  - Ebert Faustov Galkin PLB 634(06)214

Vijande, Valcarce, Richard

Having the spectrum of tetraquark potentials, like we have for the gluonic excitations, would allow us to build a plethora of states on each of the tetraquark potentials, many of them developing a width due to decays through pion (or other light hadrons) emission. Diquarks have been recently investigated on the lattice.

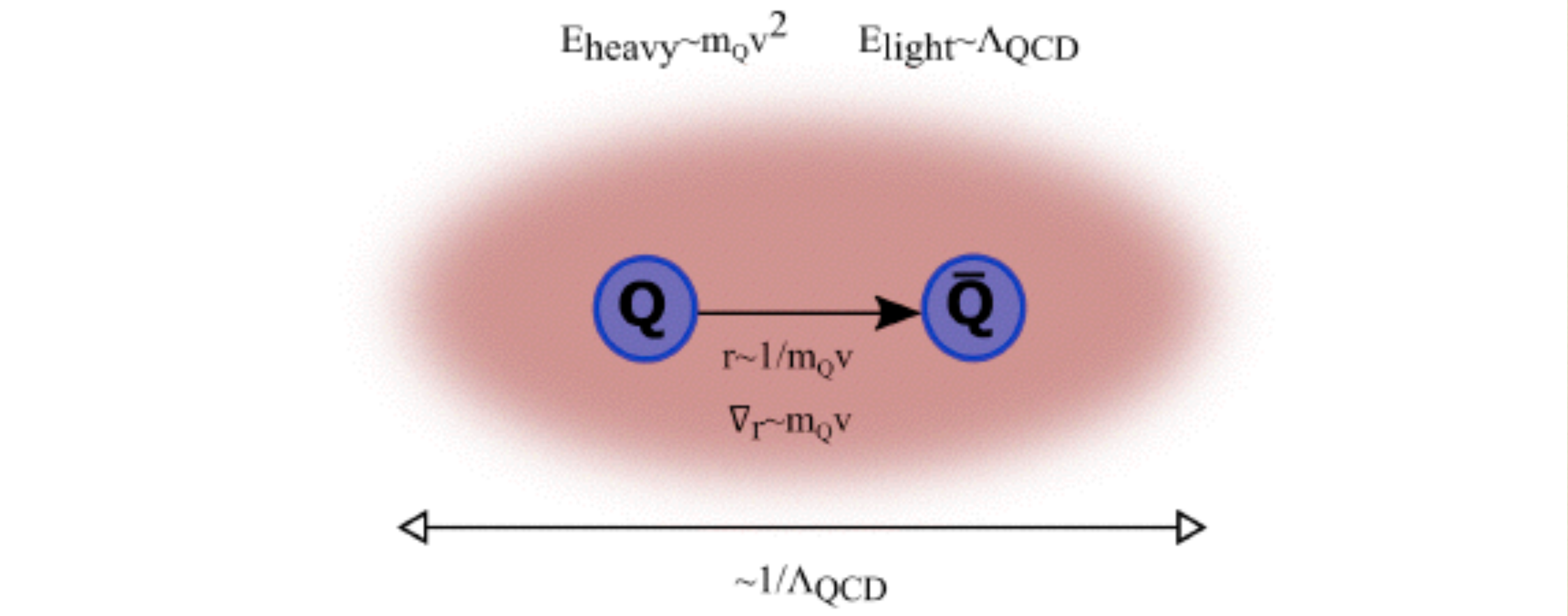
- Alexandrou et al. PRL 97(06)222002
- Fodor et al. PoS LAT2005(06)310

- Pairs of heavy-light baryons.
  - Qiao PLB 639 (2006) 263

choosing one of these degrees of freedom and an interaction originates a model for exotics.

It is particularly difficult to insert spin in these models, and when it is done the spin interaction is taken from standard quarkonium

Start considering the simplified case of heavy quark, heavy antiquark plus glue



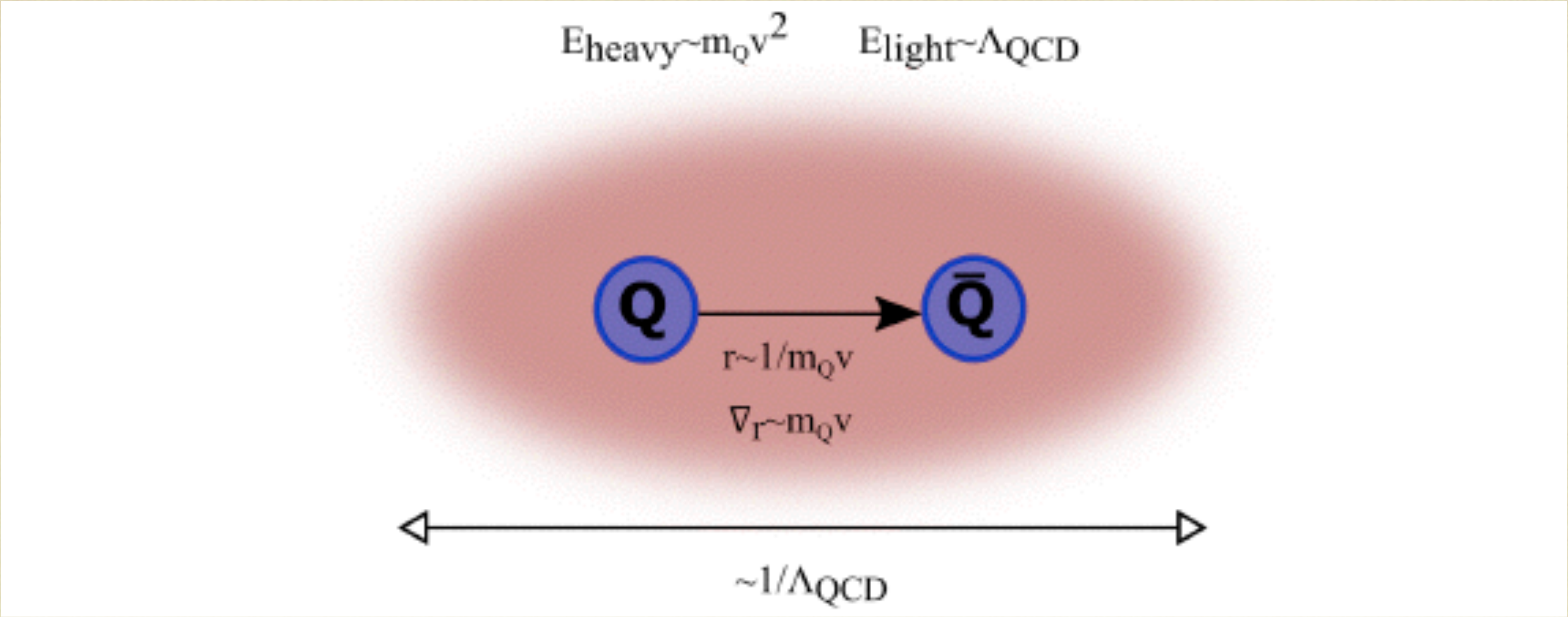
Characteristic Scales

- Heavy-quarks are **non-relativistic** $m_Q \gg \Lambda_{QCD}$.
- Two components with very different dynamical time scales $\Lambda_{QCD} \gg m_Q v^2$.
  - Light d.o.f state $E_{light} \sim \Lambda_{QCD}$.
  - Heavy-quark binding $E_Q \sim m_Q v^2$ ($v \ll 1$ relative velocity).
  - Adiabatic expansion, **Born-Oppenheimer** approximation in atomic physics. L. Griffiths, C. Michael, P. Rakow Phys.Lett.129B (1983); K.. Juge, J. Kuti, C. Morningstar Nucl.Phys.Proc.Suppl.63 (1998); E. Braaten, C. Langmack, D. Smith Phys.Rev.D90 (2014); C. Meyer, E. Swanson Prog.Part.Nucl.Phys.82 (2015)...

Quarkonium hybrids are a similar system to diatomic molecules

- Heavy d.o.f: Nuclei $\rightarrow$ Heavy Quark
- Light d.o.f: Electrons $\rightarrow$ Gluons&Light-quarks

Start considering the simplified case of heavy quark, heavy antiquark plus glue



EFT approach: Exploit the hierarchy of scales at the Lagrangian level

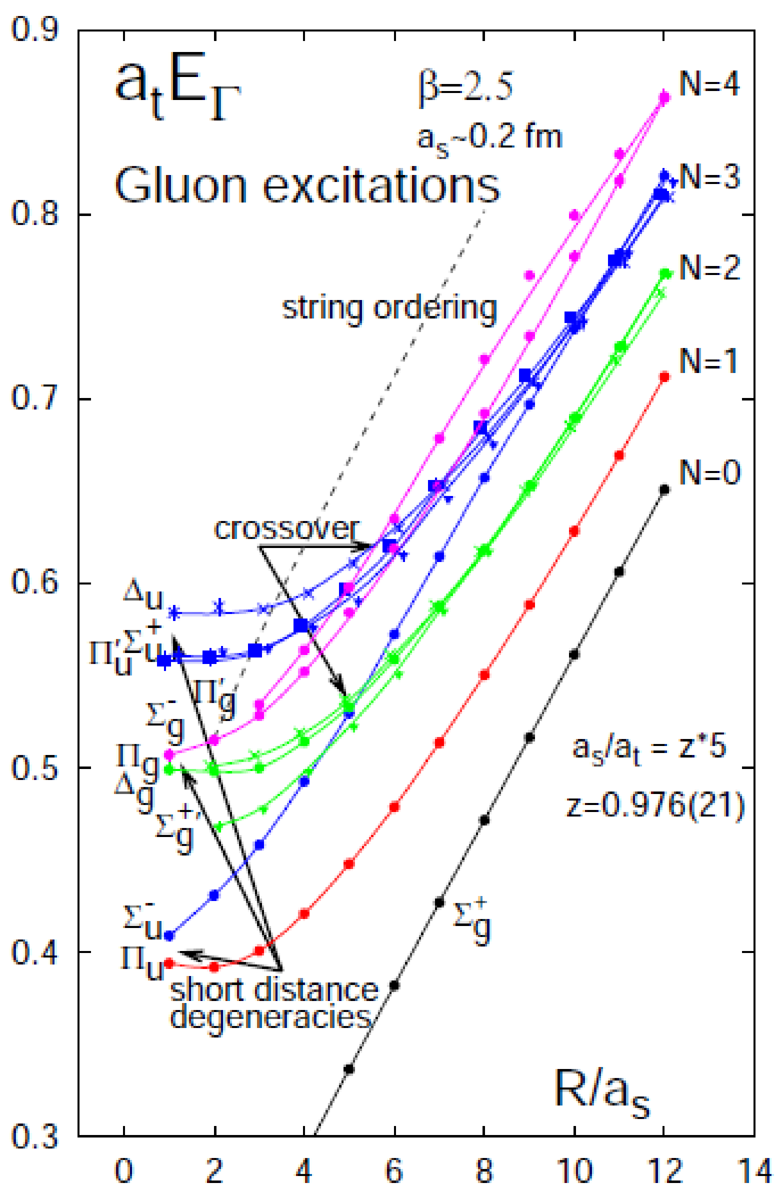
* Integrate out m_Q modes: NRQCD W. Caswell, G. Lepage Phys.Lett.167B (1986); G. Bodwin, E. Braaten, G. Lepage Phys.Rev.D51 (1995)
* In the short distance regime $r \lesssim 1/\Lambda_{QCD}$: integrate out $m_Q v \sim 1/r$ modes: (weakly-coupled) pNRQCD A. Pineda, J. Soto Nucl.Phys.Proc.Suppl.64 (1998); N. Brambilla, A. Pineda, J. Soto, A. Vairo Nucl.Phys.B566 (2000)
* Integrate out Λ_{QCD}: Hybrid and tetraquarks EFT (BOEFT) at $E \sim mv^2$. M. Berwein, N. Brambilla, JTC, A. Vairo. Phys.Rev.D92 (2015); N. Brambilla, G. Krein, JTC, A. Vairo Phys.Rev.D97 (2018); R. Oncala, J. Soto Phys.Rev.D96 (2017)

Heavy-quark heavy antiquark plus glue

Define the symmetries of the system and the system static energies in NRQCD

Static
Lattice energies

Juge Kuti Morningstar 2003

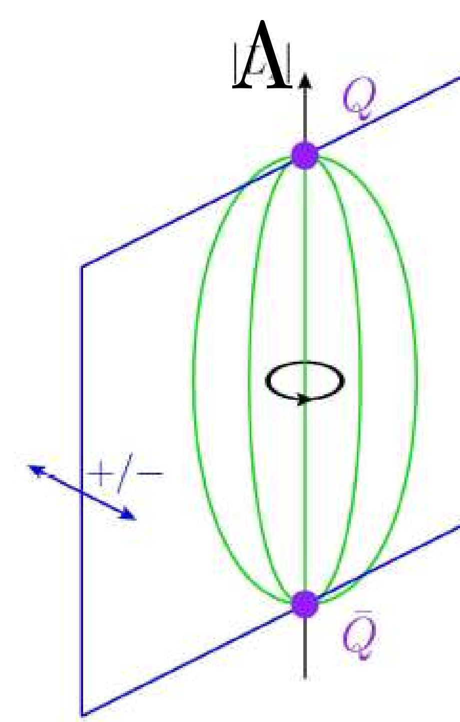


Symmetries

Static states classified by symmetry group $D_{\infty h}$
Representations labeled Λ^σ_η

Representations of $D_{\infty h}$

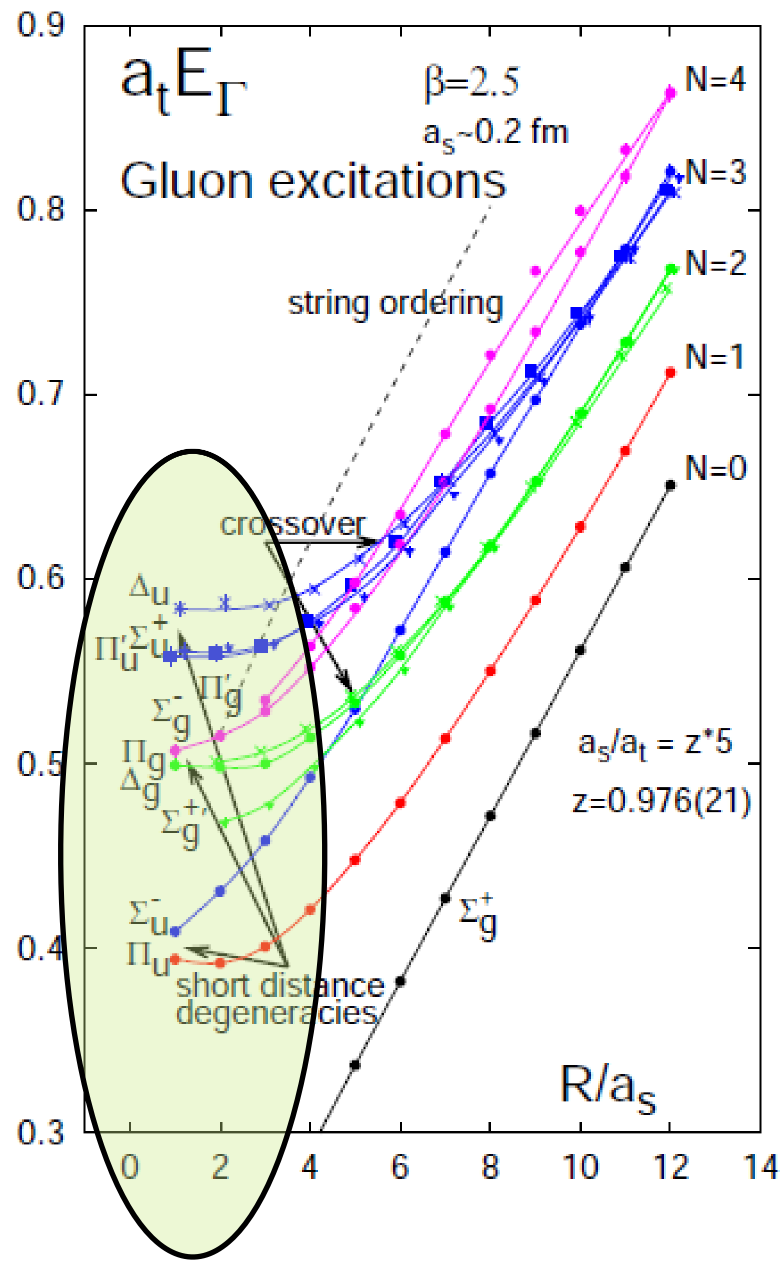
- $\Lambda = |\lambda|$ rotational quantum number
  $\lambda = \hat{r} \cdot \boldsymbol{K} = 0, \pm 1, \pm 2 \ldots$ corresponds to
  $\Lambda = \Sigma, \Pi, \Delta \ldots$
- η eigenvalue of CP:
  $g \hat{=} +1$, $u \hat{=} -1$
- σ eigenvalue of reflections (only Σ states)



- The static energies correspond to the irreducible representations of $D_{\infty h}$
- In general it can be more than one state for each irreducible representation of $D_{\infty h}$, usually denoted by primes, e.g. Π_u, Π'_u, Π''_u...

Heavy-quark heavy antiquark plus glue

static Lattice energies



- Σ_g^+ is the ground state potential that generates the standard quarkonium states.
- The rest of the static energies correspond to excited gluonic states that generate hybrids.
- The two lowest hybrid static energies are Π_u and Σ_u^-, they are nearly degenerate at short distances.
- The static energies have been computed in quenched lattice QCD, the most recent data by Juge, Kuti, Morningstar, 2002 and Bali and Pineda 2003.
- Quenched and unquenched calculations for Σ_g^+ and Π_u were compared in Bali et al 2000 and good agreement was found below string breaking distance.

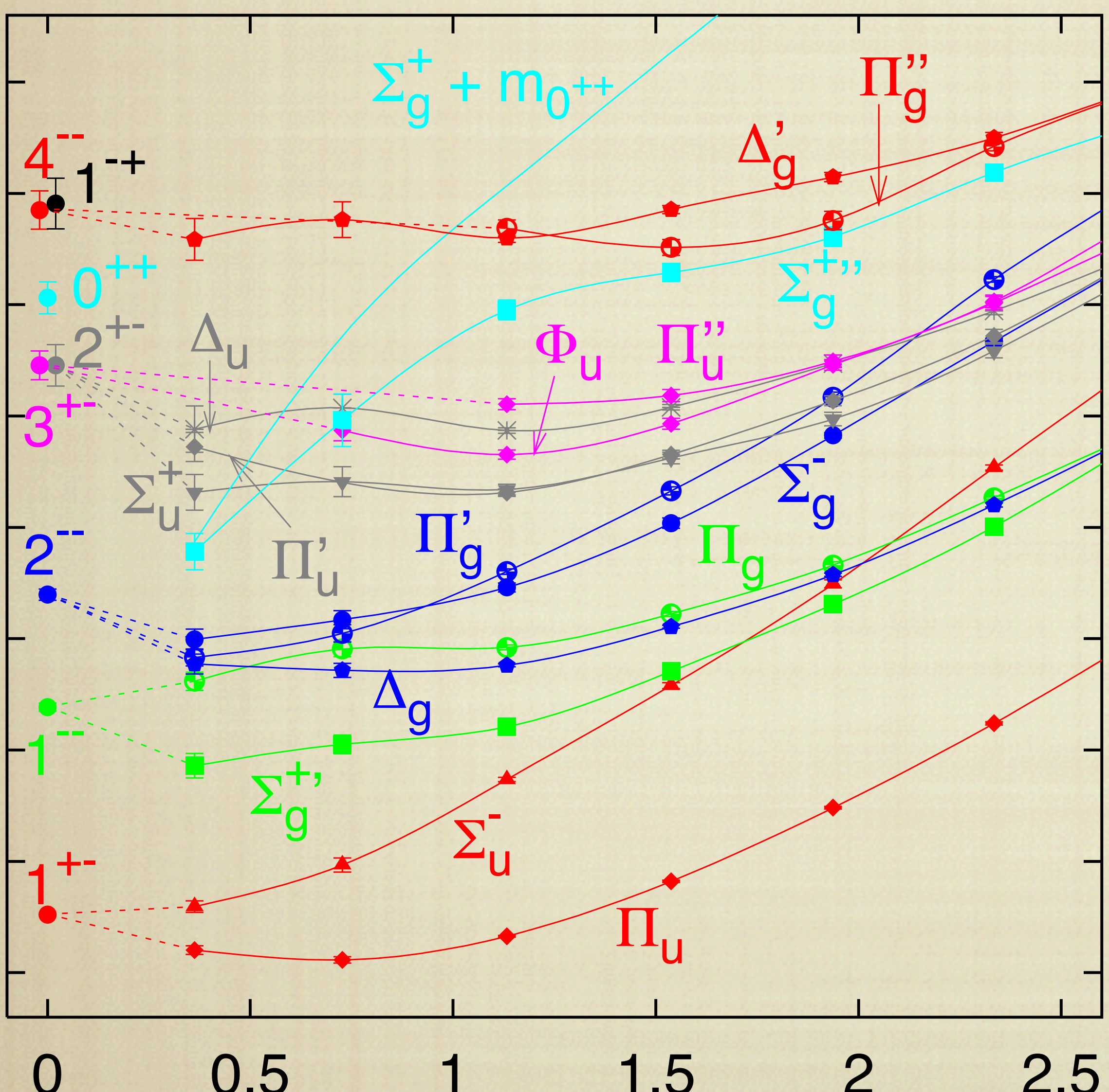
- Juge Kuti Morningstar PRL 90 (2003) 161601

pNRQCD gives the multiplets at short distance:gluelumps

In the short-range hybrids become gluelumps, i.e., quark-antiquark octets, O^a, in the presence of a gluonic field, H^a: $H(R,r,t) = H^a(R,t)O^a(R,r,t)$.

In the limit $r \to 0$ more symmetry: $D_{\infty h} \to O(3) \times C$

- Several Λ_η^σ representations contained in one J^{PC} representation:
- Static energies in these multiplets have same $r \to 0$ limit.



Gluonic excitation operators up to dim 3

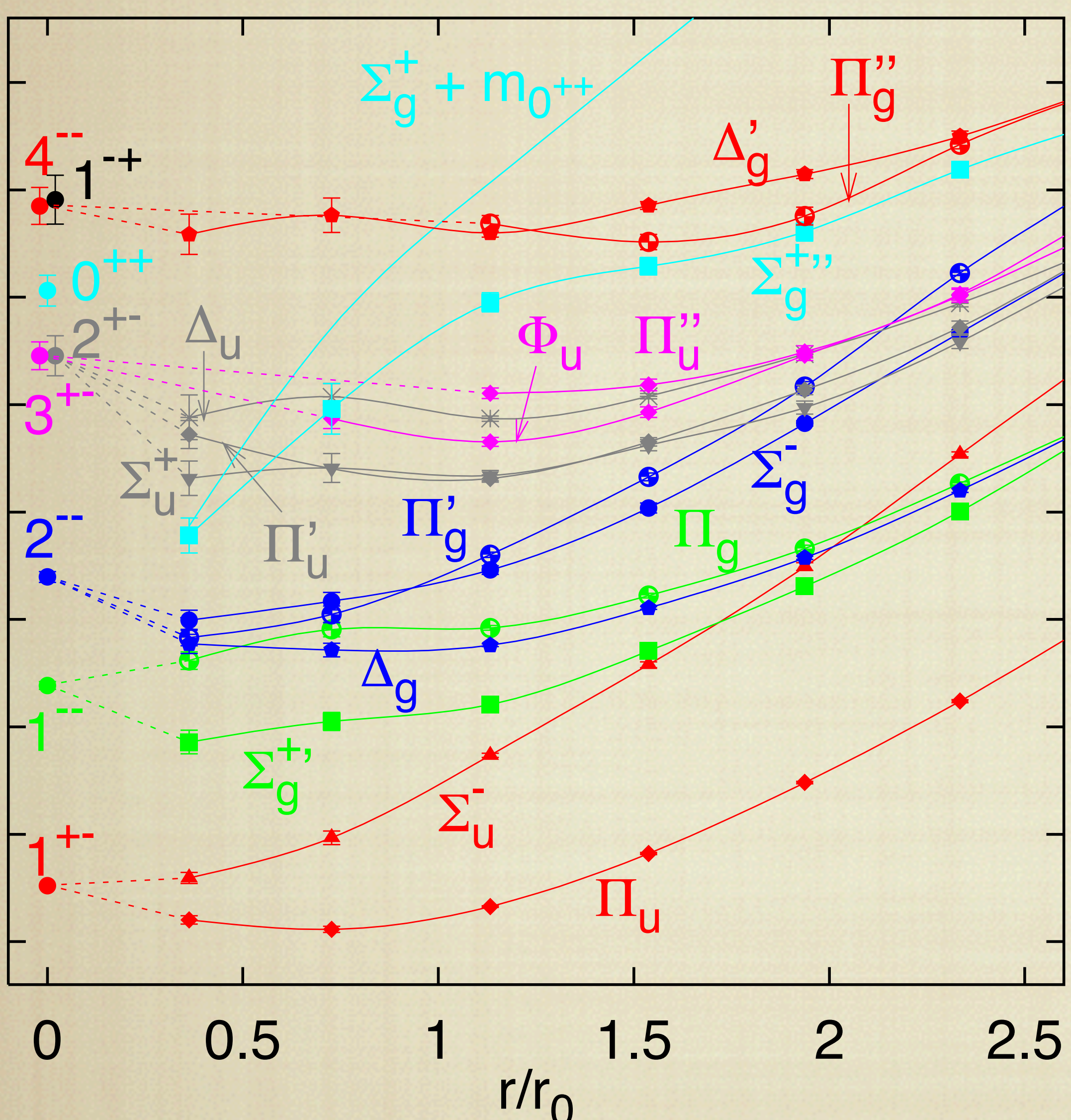
Λ_η^σ	K^{PC}	H^a
Σ_u^-	1^{+-}	$\mathbf{r}\cdot\mathbf{B}\,,\mathbf{r}\cdot(\mathbf{D}\times\mathbf{E})$
Π_u	1^{+-}	$\mathbf{r}\times\mathbf{B}\,,\mathbf{r}\times(\mathbf{D}\times\mathbf{E})$
$\Sigma_g^{+\prime}$	1^{--}	$\mathbf{r}\cdot\mathbf{E}\,,\mathbf{r}\cdot(\mathbf{D}\times\mathbf{B})$
Π_g	1^{--}	$\mathbf{r}\times\mathbf{E}\,,\mathbf{r}\times(\mathbf{D}\times\mathbf{B})$
Σ_g^-	2^{--}	$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{B})$
Π_g'	2^{--}	$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{B}+\mathbf{D}(\mathbf{r}\cdot\mathbf{B}))$
Δ_g	2^{--}	$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{B})^j+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{B})^i$
Σ_u^+	2^{+-}	$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{E})$
Π_u'	2^{+-}	$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{E}+\mathbf{D}(\mathbf{r}\cdot\mathbf{E}))$
Δ_u	2^{+-}	$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{E})^j+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{E})^i$

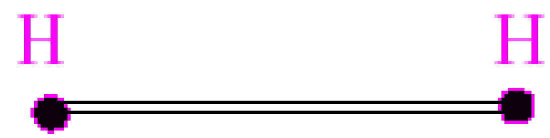
Brambilla Pineda Soto Vairo 00

The gluelump multiplets Σ_u^-, Π_u; $\Sigma_g^{+\prime}, \Pi_g$; $\Sigma_g^-, \Pi_g', \Delta_g$; $\Sigma_u^+, \Pi_u', \Delta_u$ are degenerate.

Match to pNRQCD: one can determine the form of the potential

In the short-range hybrids become gluelumps, i.e., quark-antiquark octets, O^a, in the presence of a gluonic field, H^a: $H(R,r,t) = H^a(R,t)O^a(R,r,t)$.





$$= e^{-iTE_H}$$

$$E_H = V_o + \frac{i}{T}\ln\langle H^a(\frac{T}{2})\phi^{\mathrm{adj}}_{ab} H^b(-\frac{T}{2})\rangle$$

$$\langle H^a(\frac{T}{2})\phi^{\mathrm{adj}}_{ab} H^b(-\frac{T}{2})\rangle^{\mathrm{np}} \sim h\, e^{-iT\Lambda_H}$$

$$E_H(r) = V_o(r) + \Lambda_H + b_{\Lambda_H} r^2$$

octet potential

gluelump mass

correction softly breaking the symm

Match to pNRQCD: one can determine the form of the potential

$$E_H(r) = V_O(r) + \Lambda_H + b_H r^2$$

Λ_H

- It is a non-perturbative quantity.
- It depends on the particular operator H^a, however it is the same for operators corresponding to different projections of the same gluonic operators.
- The gluelump masses have been determined in the lattice. Foster *et all* 1999; Bali, Pineda 2004; Marsh Lewis 2014
- At the subtraction scale $\nu_f = 1$ GeV: $\Lambda^{RS}_{1^{+-}} = 0.87(15)$ GeV.

b_H

- It is a non-perturbative quantity.

- Proportional to r^2 due to rotational invariance and the multipole expansion.
- We are going to fix it through a fit to the static energies lattice data.
- Breaks the degeneracy of the potentials.

Octet potential at two loops; renormalon subtraction realised among pole mass, octet potential and gluelump mass, use RS scheme

State multiplets

Hybrid spectrum for $\kappa = 1^{+-} \rightarrow \Lambda_\eta^\sigma = \Sigma_u^-, \Pi_u$

We consider hybrids that are excitations of the lowest lying static energies Π_u and Σ_u^-. In the $r \rightarrow 0$ limit Π_u and Σ_u^- are degenerate and correspond to a gluonic operator with quantum numbers 1^{+-}.

States are organized in spin multiplets.

	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

Braaten PRL 111 (2013) 162003
Braaten Langmack Smith PRD 90 (2014) 014044

Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019

$$E_H(r) = V_O(r) + \Lambda_H + b_H r^2$$

The Lambda -doubling effect breaks the degeneracy between opposite parity spin-symmetry
multiplets and lowers the mass of the multiplets that get mixed contributions of different static
energies.

1st solution

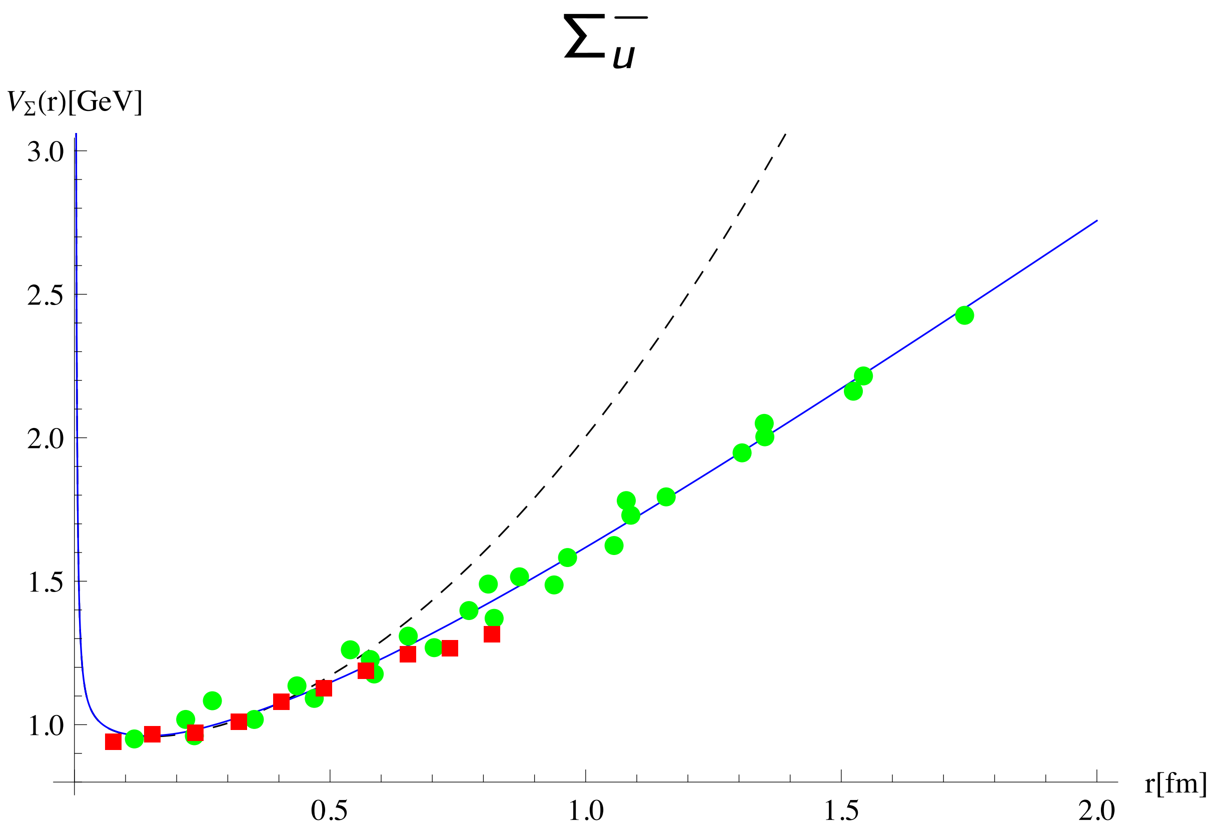
$$\left[-\frac{1}{2\mu r^2}\partial_r r^2 \partial_r + \frac{1}{2\mu r^2}\begin{pmatrix} l(l+1)+2 & 2\sqrt{l(l+1)} \\ 2\sqrt{l(l+1)} & l(l+1) \end{pmatrix} + \begin{pmatrix} E_\Sigma^{(0)} & 0 \\ 0 & E_\Pi^{(0)} \end{pmatrix}\right]\begin{pmatrix} \psi_\Sigma \\ \psi_\Pi \end{pmatrix} = \mathcal{E}\begin{pmatrix} \psi_\Sigma \\ \psi_\Pi \end{pmatrix}$$

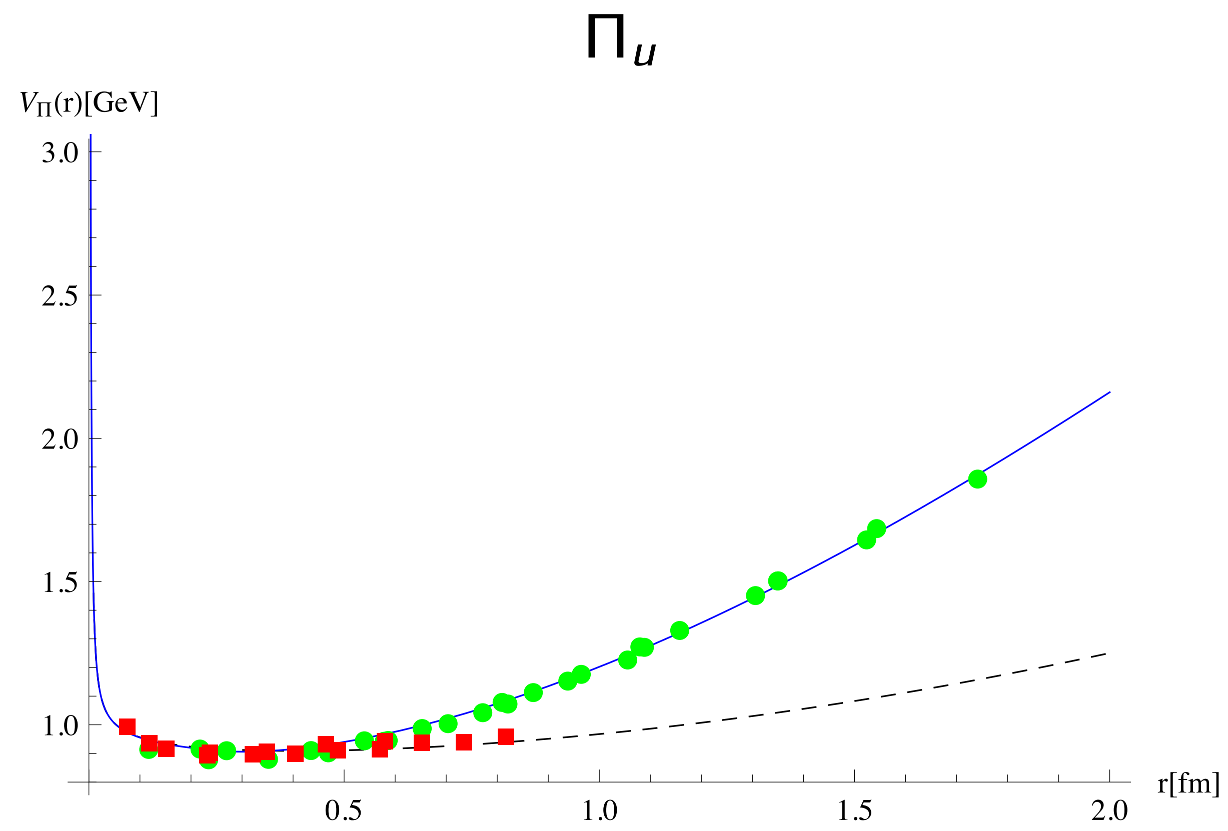
2nd solution

$$\left[-\frac{1}{2\mu r^2}\partial_r r^2 \partial_r + \frac{l(l+1)}{2\mu r^2} + E_\Pi^{(0)}\right]\psi_\Pi = \mathcal{E}\,\psi_\Pi$$

- energy eigenvalue $\mathcal{E}$ gives hybrid mass: $m_H = m_Q + m_{\bar{Q}} + \mathcal{E}$
- $l(l+1)$ is the eigenvalue of angular momentum $\boldsymbol{L}^2 = (\boldsymbol{L}_{Q\bar{Q}} + \boldsymbol{L}_g)^2$
- the two solutions correspond to **opposite parity** states: $(-1)^l$ and $(-1)^{l+1}$
- corresponding eigenvalues under charge conjugation: $(-1)^{l+s}$ and $(-1)^{l+s+1}$
- Schrödinger equations can be solved numerically

For $l = 0$ the off-diagonal terms vanish, so the equations for $\psi_\Sigma^{(N)}$ and $\psi_{-\Pi}^{(N)}$ decouple. There exists only one parity state, and its radial wave function is given by a Schrödinger equation with the $E_\Sigma^{(0)}$ potential and an angular part $2/mr^2$.





Lattice data: Bali, Pineda 2004; Juge, Kuti, Morningstar 2003, dashed line $V^{(0.5)}$, solid line $V^{(0.25)}$

$V^{(0.25)}$

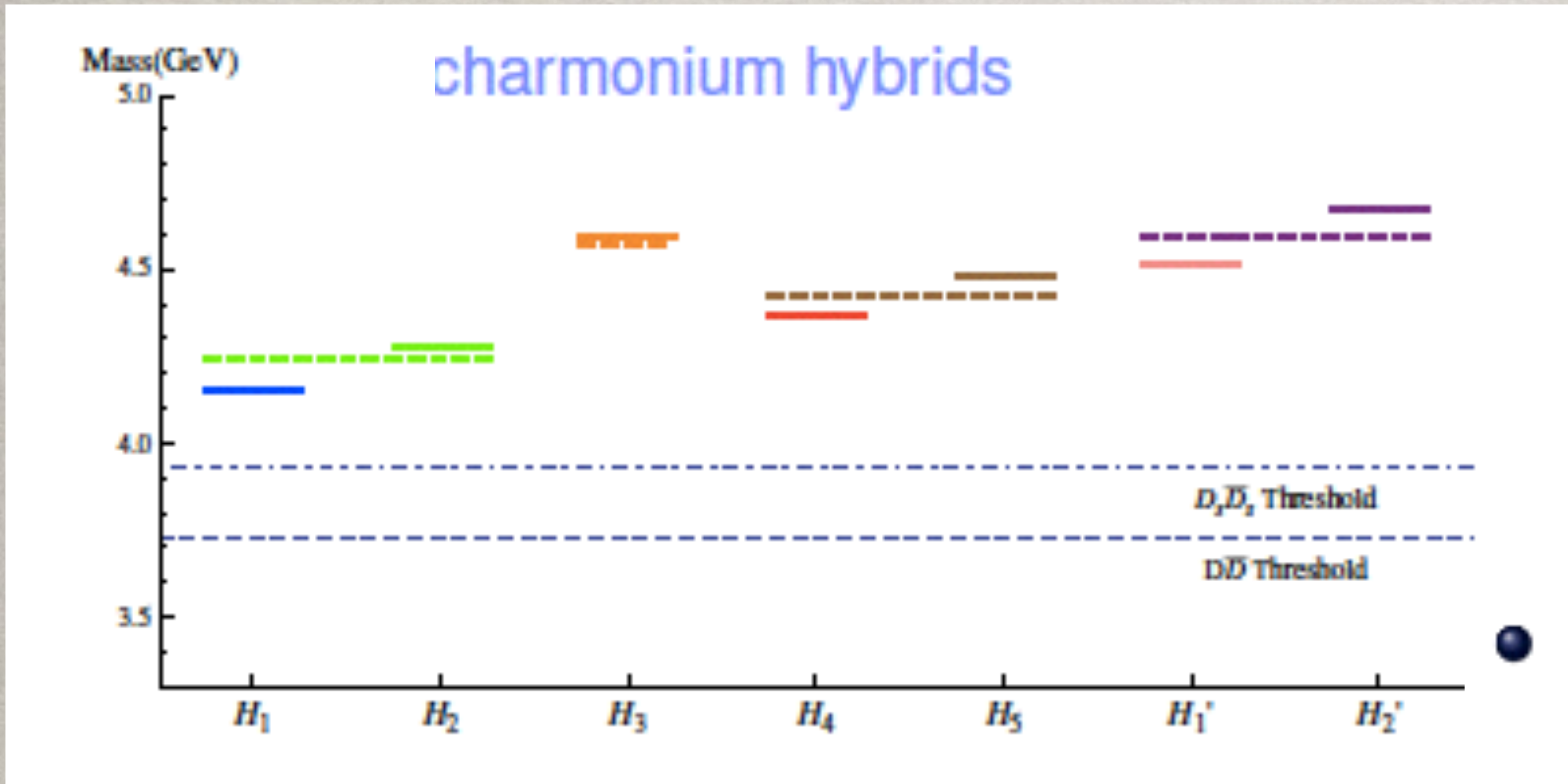
- $r \leq 0.25$ fm: pNRQCD potential.
  - Lattice data fitted for the $r = 0 - 0.25$ fm range with the same energy offsets as in $V^{(0.5)}$.

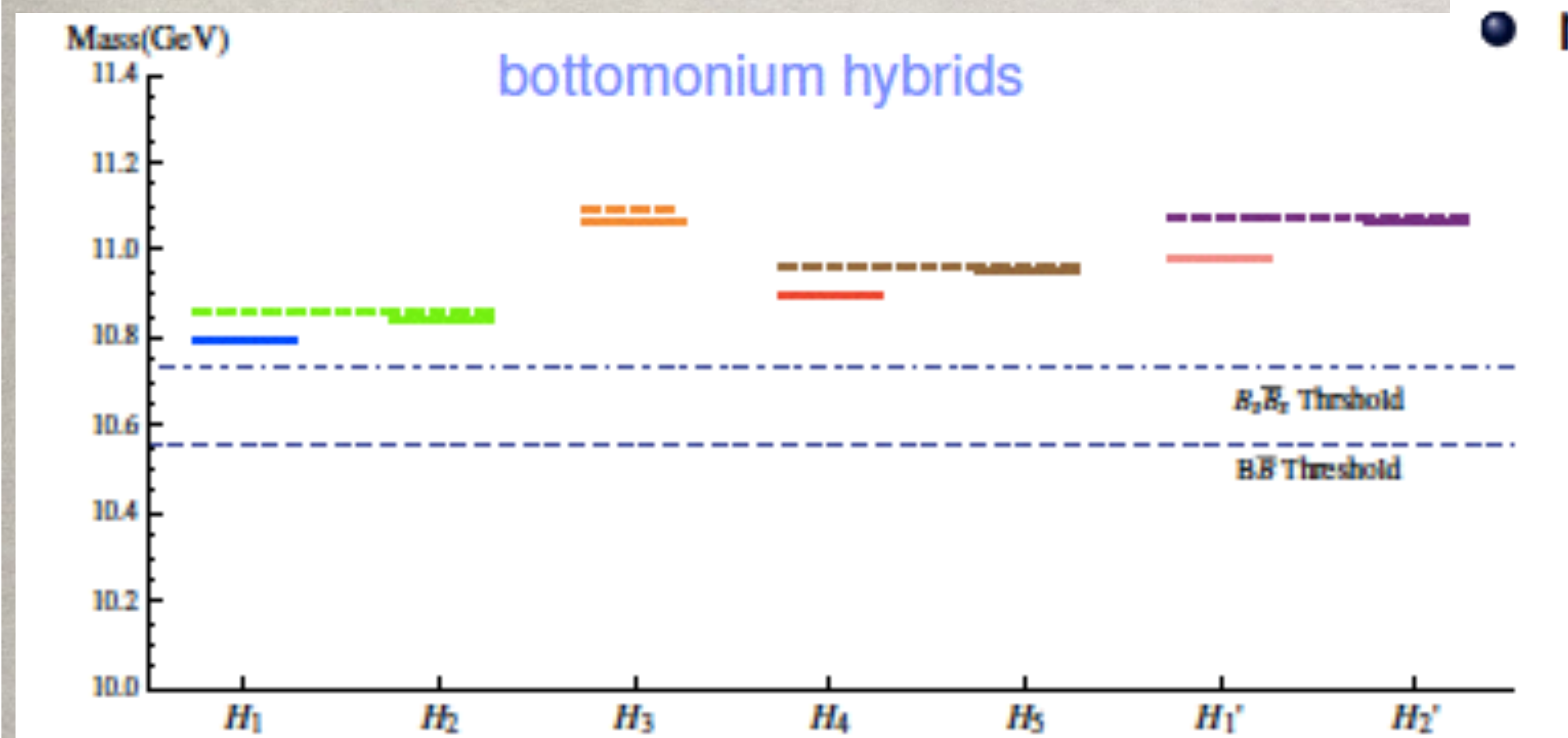
$$b_\Sigma^{(0.25)} = 1.246\,\mathrm{GeV/fm^2}, \qquad b_\Pi^{(0.25)} = 0.000\,\mathrm{GeV/fm^2}\,.$$

- $r > 0.25$ fm: phenomenological potential.
  - $\mathcal{V}'(r) = \frac{a_1}{r} + \sqrt{a_2 r^2 + a_3} + a_4$.
  - Same energy offsets as in $V^{(0.25)}$.
  - *Constraint:* Continuity up to first derivatives.

Berwein, N.B. , Tarrus, Vairo arXiv:1510.04299

Λ doubling in quarkonium hybrid states





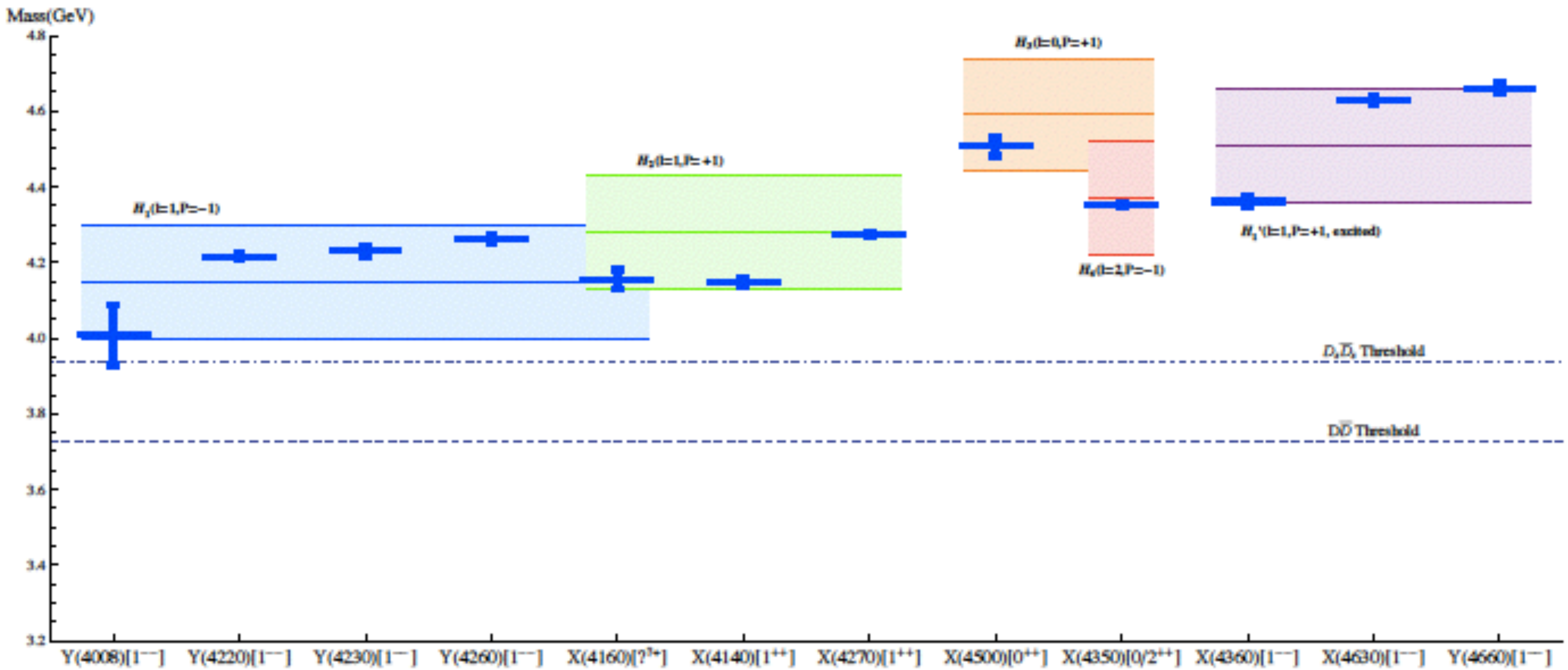
- no distinction between opposite parity states in BO
- mixed states lie lower than pure

- Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019

  data without mixing (dashed) from Braaten et al PRD 90 (2014) 114044

Charmonium Hybrid spectrum for $\kappa = 1^{+-}$

M. Berwein, N. Brambilla, JTC, A. Vairo. Phys.Rev.D92 (2015)



1. Solid blue bars: Neutral exotic charmonium states (Belle, CDF, BESIII, Babar, LHCb).
2. Bands: Predicted masses for hybrid spin-symmetry multiplets $\pm$ uncertainty of Λ_{1+-}.

- Spin-symmetry multiplets

	l	J^{PC}	
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u

Introducing the spin of the quark

Up to now we have worked at the leading order, now we want to include the correction coming from the quark spin

We calculate the spin dependent potentials matching NRQCD and pNRQCD: we get a purely perturbative contribution in the form of spin dependent octet potential integrating out mv and then we get nonperturbative correlators depending only on glue when integrating out Lambda_QCD.

the nonperturbative correlators should be calculated on the lattice or in QCD vacuum models

we fix them on lattice data of charmonium—> we can then predict hybrids spin multiplet for bottomonium

Born-Oppenheimer EFT for hybrids

Gluelumps operators G

In the short distance limit the static energies are characterized by $O(3) \times C$ instead of $D_{\infty h}$.

- At the pNRQCD level a basis of hybrid states is defined as

$$|\kappa, \lambda\rangle = P^i_{\kappa\lambda} O^{a\,\dagger}(\boldsymbol{r}, \boldsymbol{R})\, G^{i\,a}_{\kappa}(\boldsymbol{R})|0\rangle$$

- The hybrid EFT is formulated for the subspace spanned by

$$\int d^3r d^3R \sum_{\kappa} |\kappa, \lambda\rangle\, \Psi_{\kappa\lambda}(t, \boldsymbol{r}, \boldsymbol{R})$$

- $G^{i\,a}_{\kappa}$ create a basis of color-octet eigenstates of $h_0(\boldsymbol{R})$ in the presence of a static, local, color-octet source O^a.

$$h_0(\boldsymbol{R}) G^{i\,a}_{\kappa}(\boldsymbol{R})|0\rangle = \Lambda_{\kappa} G^{i\,a}_{\kappa}(\boldsymbol{R})|0\rangle$$

- The light d.o.f Hamiltonian density leading order in the multipole expansion.

$$h_0 = \frac{1}{2}\left(\boldsymbol{E}^2 - \boldsymbol{B}^2\right) - \left(\sum_{j=1}^{n_f} \bar{q}_j i\boldsymbol{D}\cdot\boldsymbol{\gamma} q_j\right)$$

- States are constrained to satisfy the Gauss law.

- $\Psi_{\kappa\lambda}$ is the basic degree of freedom upon which we build the EFT.
- $P^i_{\kappa\lambda}$ projects $G^{i\,a}_{\kappa}$ into a representation of $D_{\infty h}$.

- After projecting and integrating out Λ_{QCD}:

$$\mathcal{L}_{BOEFT} = \int d^3r \sum_{\kappa} \sum_{\lambda\lambda'} \Psi^{\dagger}_{\kappa\lambda}(t, \boldsymbol{r}, \boldsymbol{R}) \left\{ \delta_{\lambda\lambda'} i\partial_t - V_{\kappa\lambda\lambda'}(r) - P^{i\,\dagger}_{\kappa\lambda} \frac{\boldsymbol{\nabla}^2_r}{m_Q} P^i_{\kappa\lambda'} \right\} \Psi_{\kappa\lambda'}(t, \boldsymbol{r}, \boldsymbol{R}) + \dots$$

The potential $V_{\kappa\lambda\lambda'}$ can be organized into an expansion in $1/m_Q$

$$V_{\kappa\lambda\lambda'}(r) = V^{(0)}_{\kappa\lambda}(r)\delta_{\lambda\lambda'} + \frac{V^{(1)}_{\kappa\lambda\lambda'}(r)}{m_Q} + \frac{V^{(2)}_{\kappa\lambda\lambda'}(r)}{m_Q^2} + \dots$$

Born-Oppenheimer EFT for hybrids

- After projecting and integrating out Λ_{QCD}:

$$\mathcal{L}_{BOEFT} = \int d^3r \sum_{\kappa} \sum_{\lambda\lambda'} \Psi^{\dagger}_{\kappa\lambda}(t,\, \boldsymbol{r},\, \boldsymbol{R}) \left\{ \delta_{\lambda\lambda'} i\partial_t - V_{\kappa\lambda\lambda'}(r) - P^{i\,\dagger}_{\kappa\lambda} \frac{\boldsymbol{\nabla}^2_r}{m_Q} P^{i}_{\kappa\lambda'} \right\} \Psi_{\kappa\lambda'}(t,\, \boldsymbol{r},\, \boldsymbol{R}) + \dots$$

The potential $V_{\kappa\lambda\lambda'}$ can be organized into an expansion in $1/m_Q$

$$V_{\kappa\lambda\lambda'}(r) = V^{(0)}_{\kappa\lambda}(r)\delta_{\lambda\lambda'} + \frac{V^{(1)}_{\kappa\lambda\lambda'}(r)}{m_Q} + \frac{V^{(2)}_{\kappa\lambda\lambda'}(r)}{m_Q^2} + \dots$$

The static potential, $V^{(0)}_{\kappa\lambda}$, can be matched to the lattice NRQCD static energies and a short distance weak-couplig pNRQCD description:

$$E^{(0)}_{|\lambda|^{\sigma}_{\eta}}(r) = V_o(r) + \Lambda_\kappa + b_{\kappa\lambda} r^2 + \dots = V^{(0)}_{\kappa\lambda}(r)$$

The nonadiabatic coupling mixes states which are different projections of the same light d.o.f operator.

$$P^{i\,\dagger}_{\kappa\lambda} \left[\frac{\boldsymbol{\nabla}^2_r}{m_Q}, P^{i}_{\kappa\lambda'} \right] = P^{i\,\dagger}_{\kappa\lambda} \frac{\boldsymbol{\nabla}^2_r}{m_Q} P^{i}_{\kappa\lambda'} - \frac{\boldsymbol{\nabla}^2_r}{m_Q}$$

Hybrids spin dependent potentials

$\kappa = 1^{+-} \to \Lambda_\eta^\sigma = \Sigma_u^-, \Pi_u$

$$V^{(1)}_{1\lambda\lambda'\,SD}(r) = V_{1\,SK}(r)\left(P^{i\dagger}_{1\lambda} K^{ij}_1 P^j_{1\lambda'}\right)\cdot S + \dots,$$

$$V^{(2)}_{1\lambda\lambda'\,SD}(r) = V_{1\,LSa}(r)\left(P^{i\dagger}_{1\lambda} L_{Q\bar{Q}} P^i_{1\lambda'}\right)\cdot S + V_{1\,LSb} P^{i\dagger}_{1\lambda}(r)\left(L^i_{Q\bar{Q}} S^j + S^i L^j_{Q\bar{Q}}\right) P^j_{1\lambda'}$$
$$+ V_{1\,S^2}(r) S^2 \delta_{\lambda\lambda'} + V_{1\,S_{12}a}(r) S_{12} \delta_{\lambda\lambda'} + V_{1\,S_{12}b}(r) P^{i\dagger}_{1\lambda} P^j_{1\lambda'}\left(S^i_1 S^j_2 + S^i_2 S^j_1\right) + \dots$$

—>Unlike standard quarkonium spin appear at 1/m

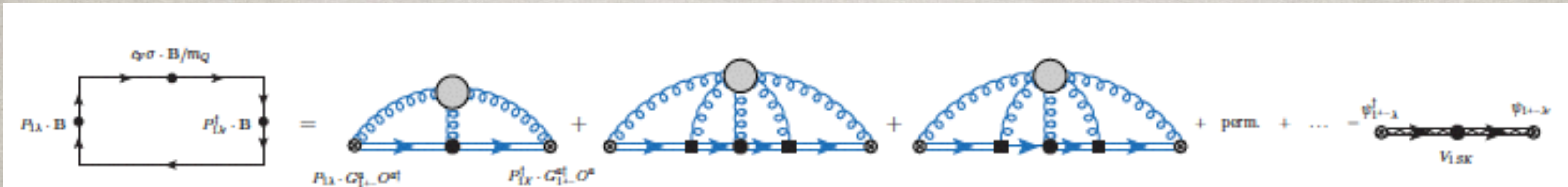
—>These are new operators not present in standard quarkonium

where $\boldsymbol{L}_{Q\bar{Q}}$ is the orbital angular momentum of the heavy-quark-antiquark pair, $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$ are the spin vectors of the heavy quark and heavy antiquark respectively, $\boldsymbol{S} = \boldsymbol{S}_1 + \boldsymbol{S}_2$ and $S_{12} = 12(\boldsymbol{S}_1\cdot\hat{\boldsymbol{r}})(\boldsymbol{S}_2\cdot\hat{\boldsymbol{r}}) - 4\boldsymbol{S}_1\cdot\boldsymbol{S}_2$.

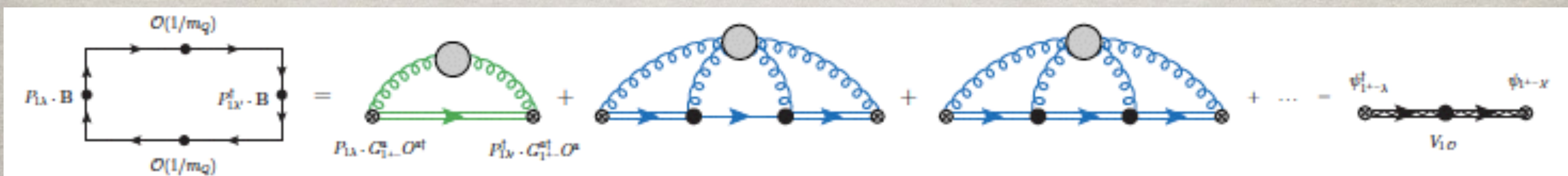
$(K^{ij})^k = i\epsilon^{ijk}$ is the angular momentum operator for the spin-1 gluons.

Hybrids spin dependent potentials

$\kappa = 1^{+-} \rightarrow \Lambda_\eta^\sigma = \Sigma_u^-, \Pi_u$



$$V_{1\,SK} = V_{SK}^{np\,(0)} + V_{SK}^{np\,(1)} r^2 + \dots ,$$



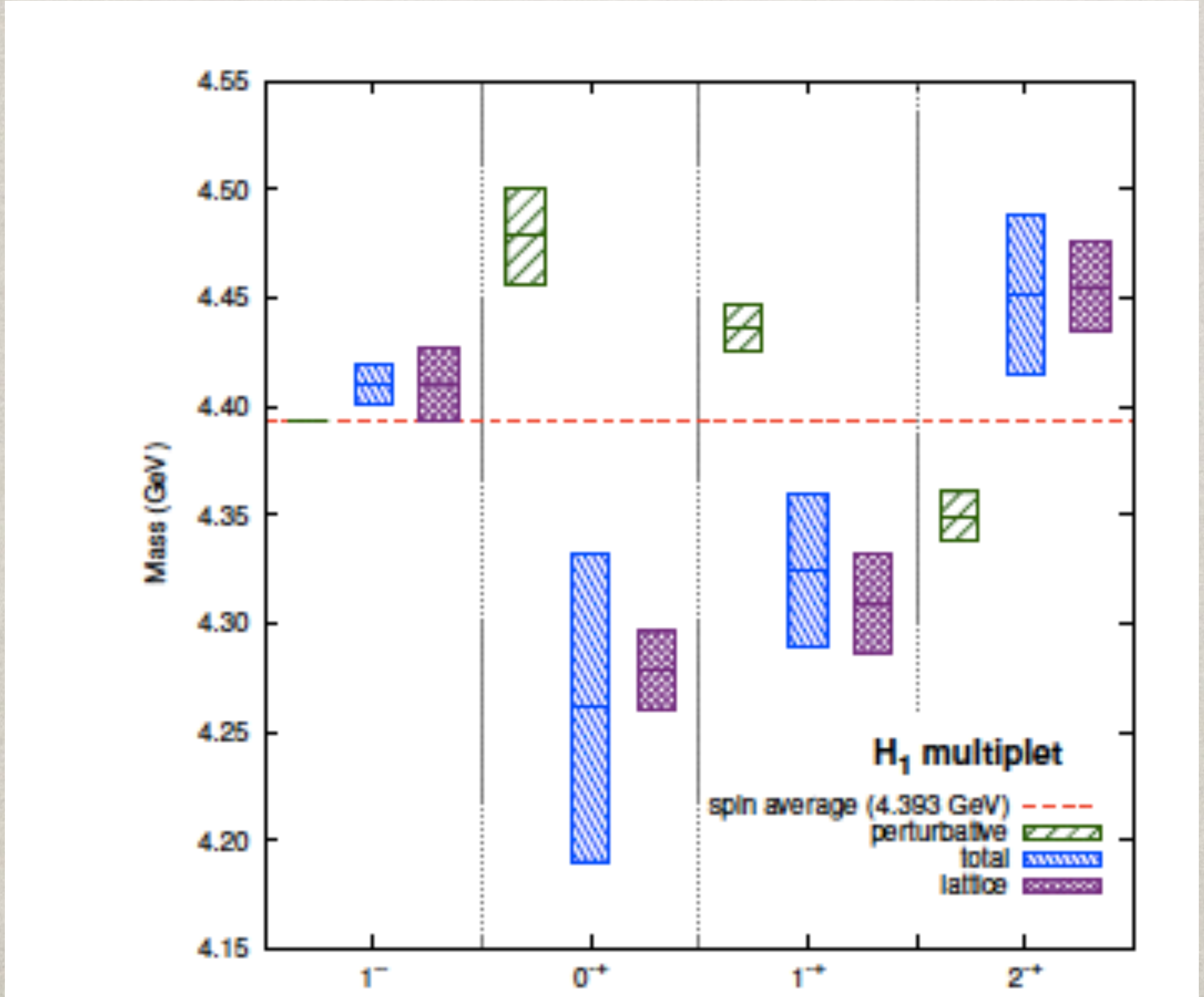
$$V_{1\,\mathcal{O}} = V_{o\,\mathcal{O}}(r) + V_{\mathcal{O}}^{np\,(0)} + \dots , \quad \mathcal{O} = SLa,\ SLb,\ S^2,\ S_{12},\ S_{12}b$$

-In the short distance we can use weakly coupled pNRQCD to calculate V_0, it is given by the QQbar octet potential

-The V^{np} depend on non perturbative gluon correlators not yet calculated on the lattice: 6 unknown

-The only flavour dependence is carried by the NRQCD matching coefficients

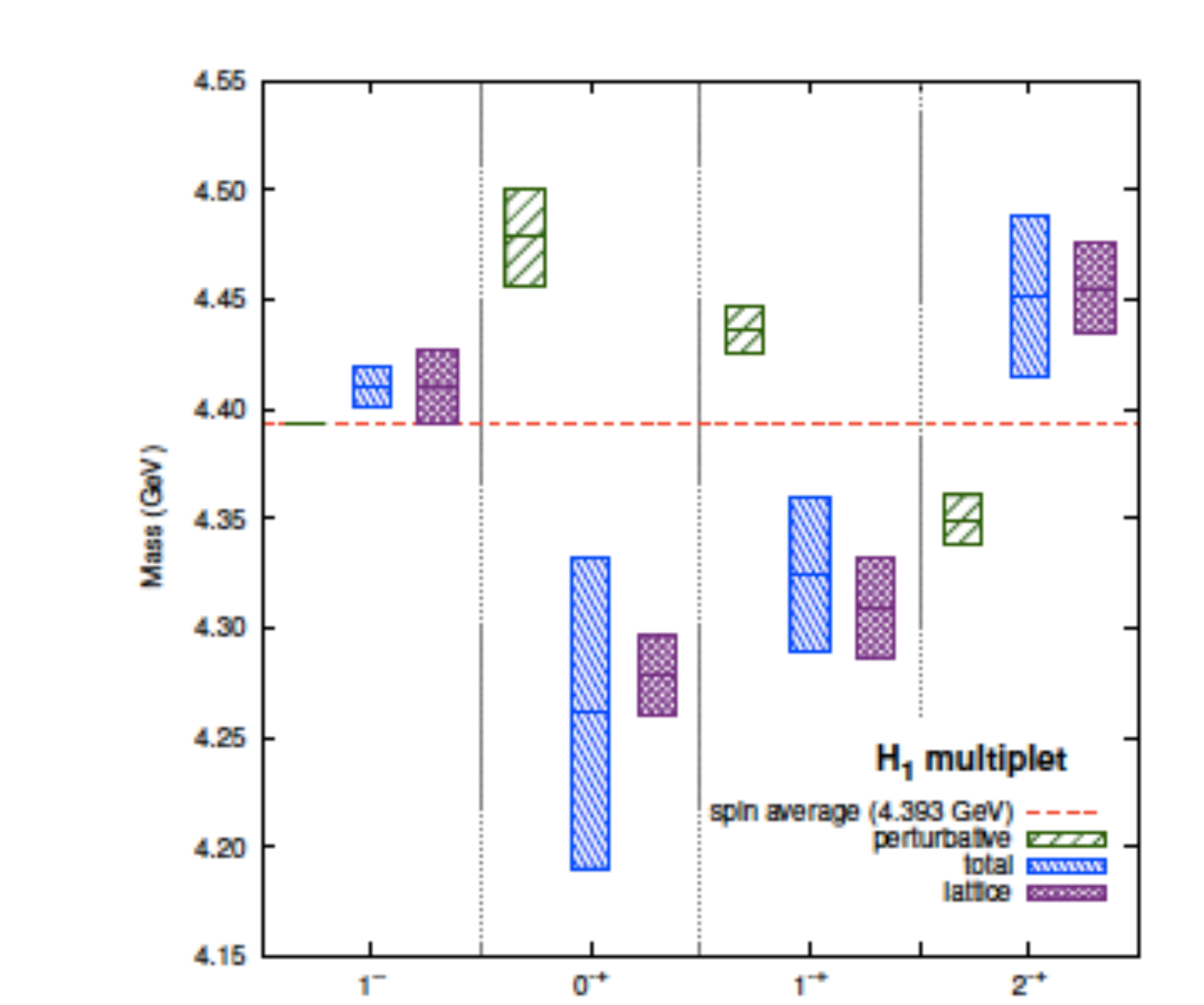
Charmonium Hybrids Multiplets H_1



lattice data from G. K. C. Cheung, C. O'Hara, G. Moir, M. Peardon, S. M. Ryan, C. E. Thomas, and D. Tims (Hadron Spectrum), JHEP **12**, 089 (2016), arXiv:1610.01073 [hep-lat].

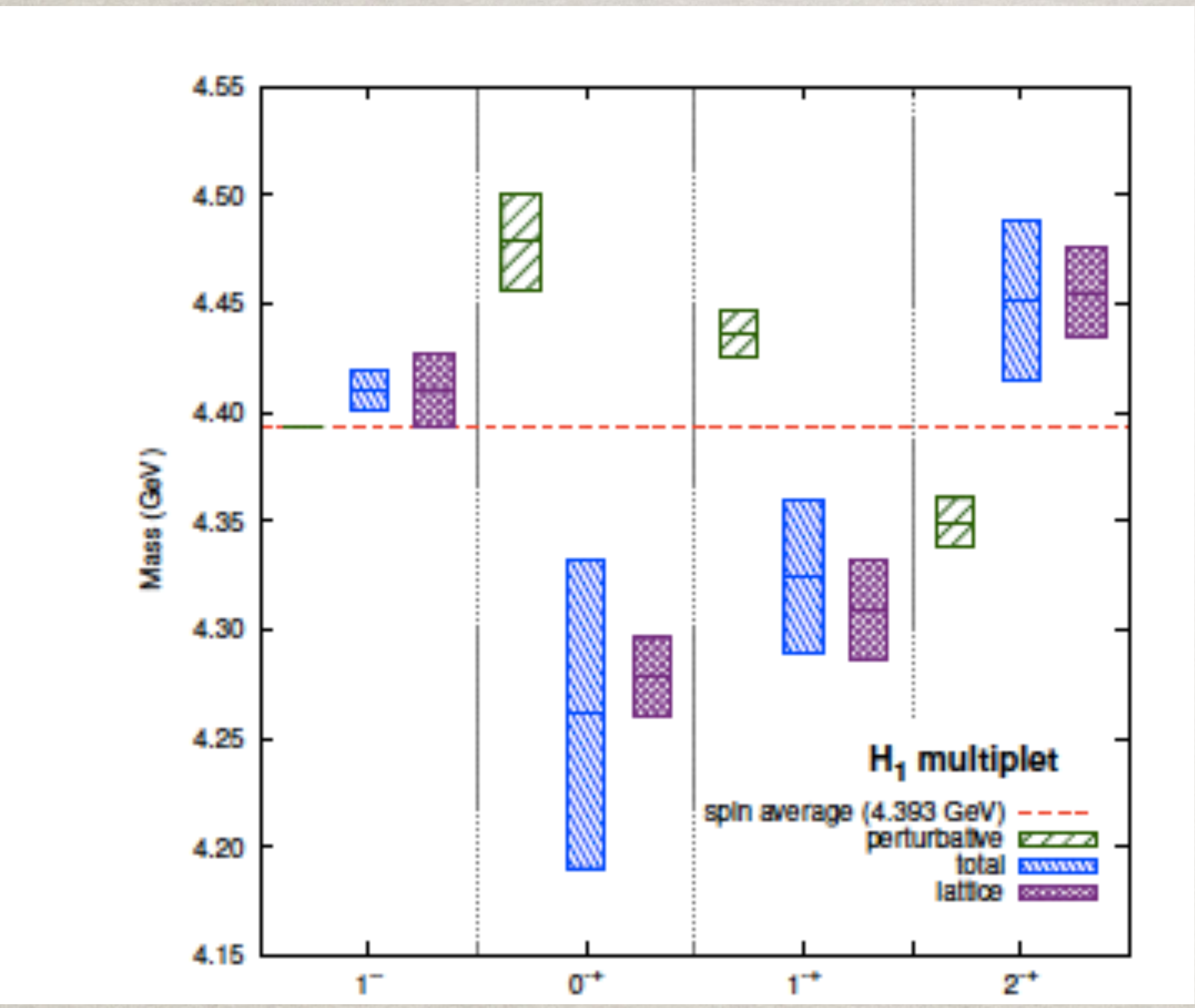
with a pion of about 240 MeV

Charmonium Hybrids Multiplets H_1



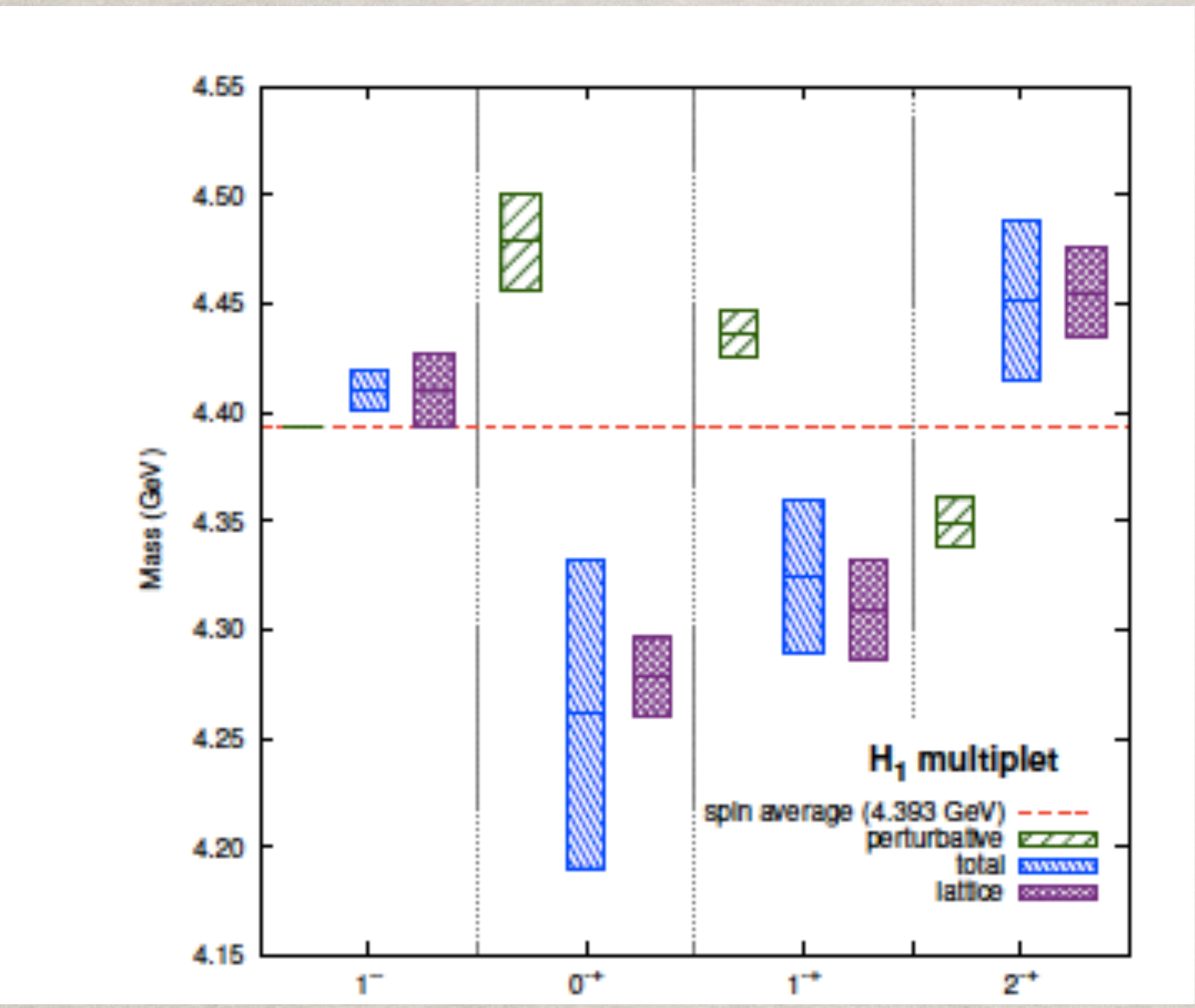
Power counting: we include terms up to order Lambda^3/m^2 and m v^4 to the spin splittings

Charmonium Hybrids Multiplets H_1



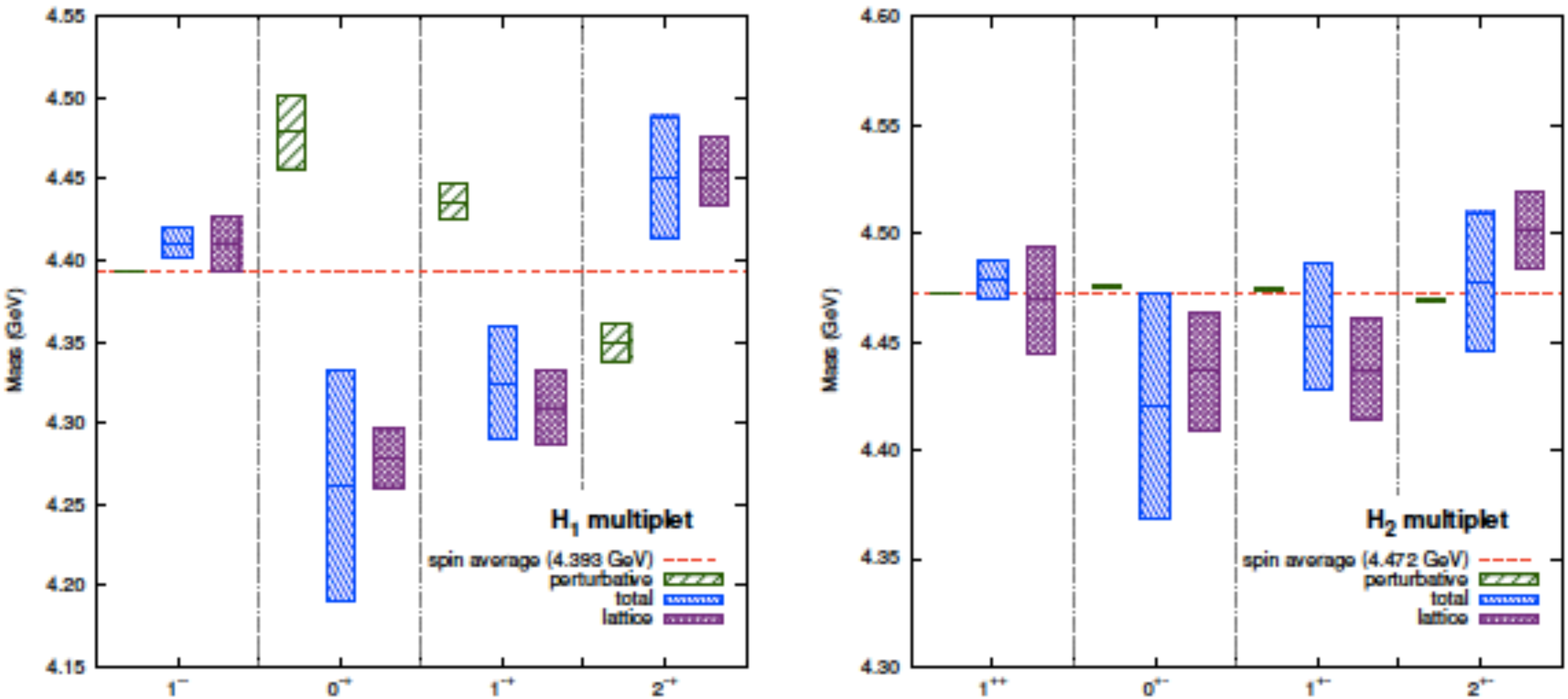
height of the boxes is an estimate of the uncertainty: estimated by the parametric size of higher order corrections, m alpha_s^5 for the perturbative part, powers of Lambda_qcd/m for the nonperturbative part, plus the statistical error on the fit

Charmonium Hybrids Multiplets H_1



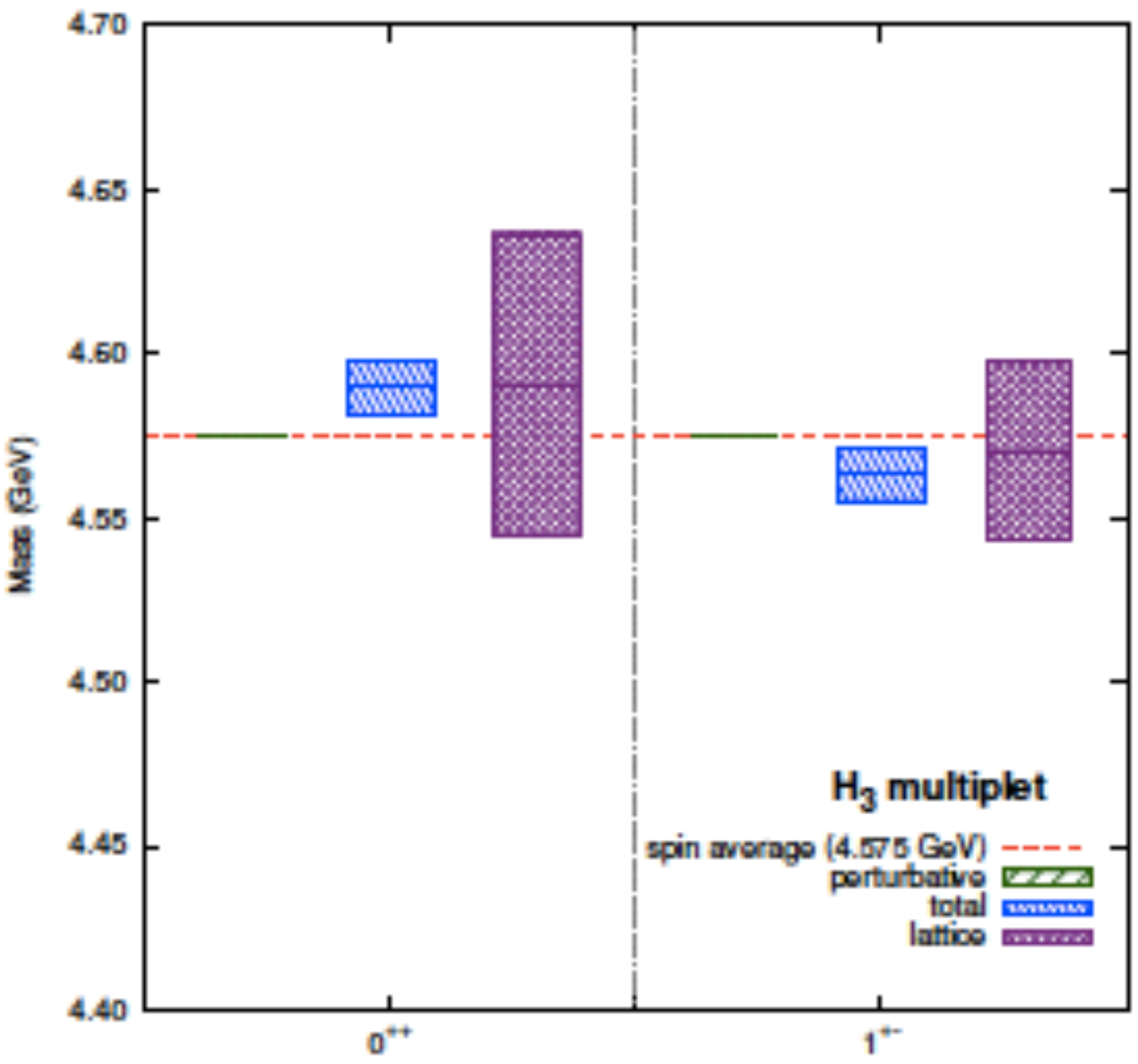
the   perturbative part produces a pattern opposite to the lattice and to ordinary quarkonia —> discrepancy can be reconciled thanks to the nonperturbative parts, especially the one at order 1/m which goes like Lambda^2/m and is parametrically larger than the perturbative contribution that i border m v^4

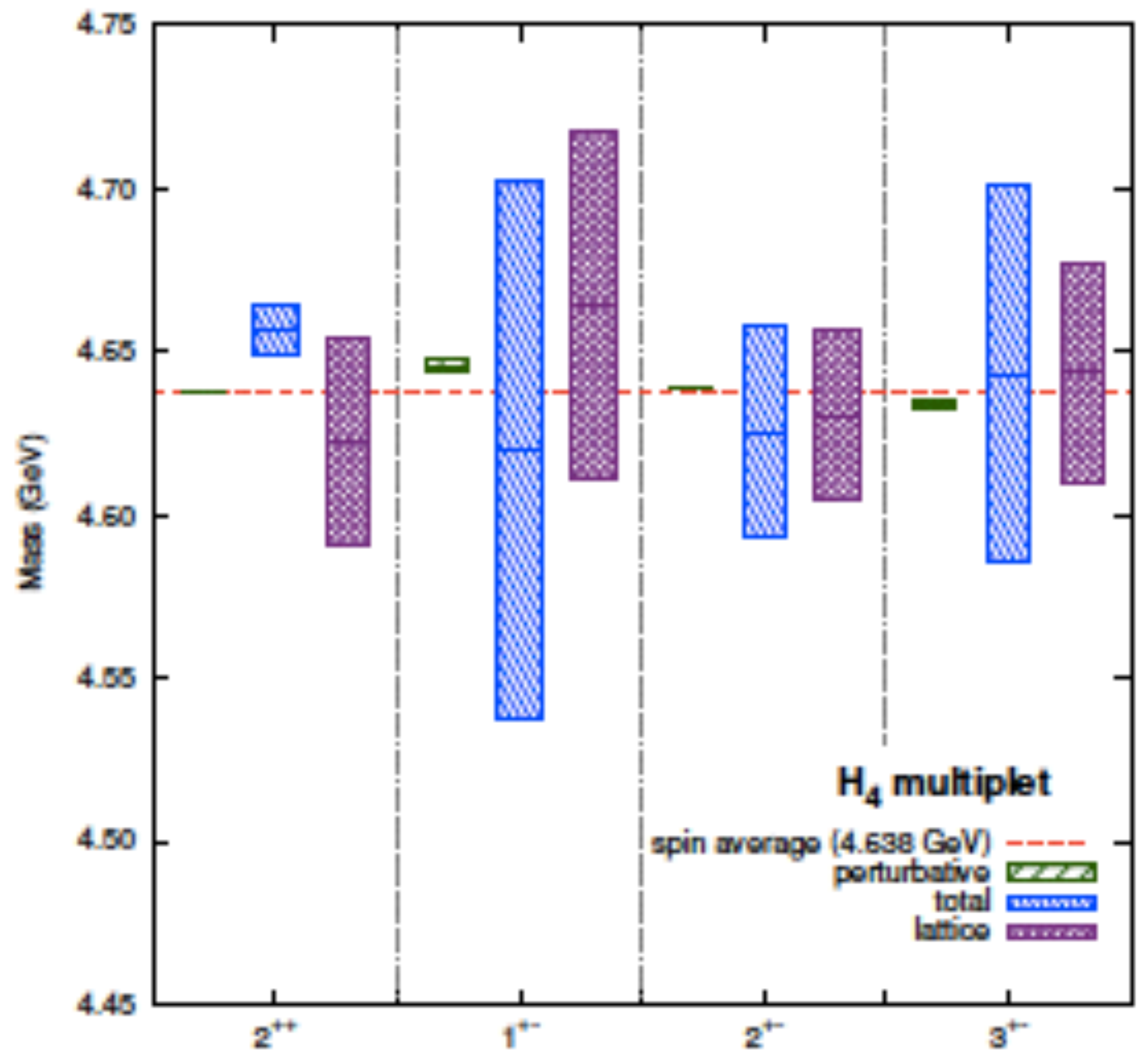
Charmonium Hybrids Multiplets H_1 and H_2



H_1 and H_2 corresponds to l=1 and are negative and positive parity resp. The mass splitting between H_1 and H_2 is a result of lambda-doubling

Charmonium Hybrids Multiplets H_3 and H_4





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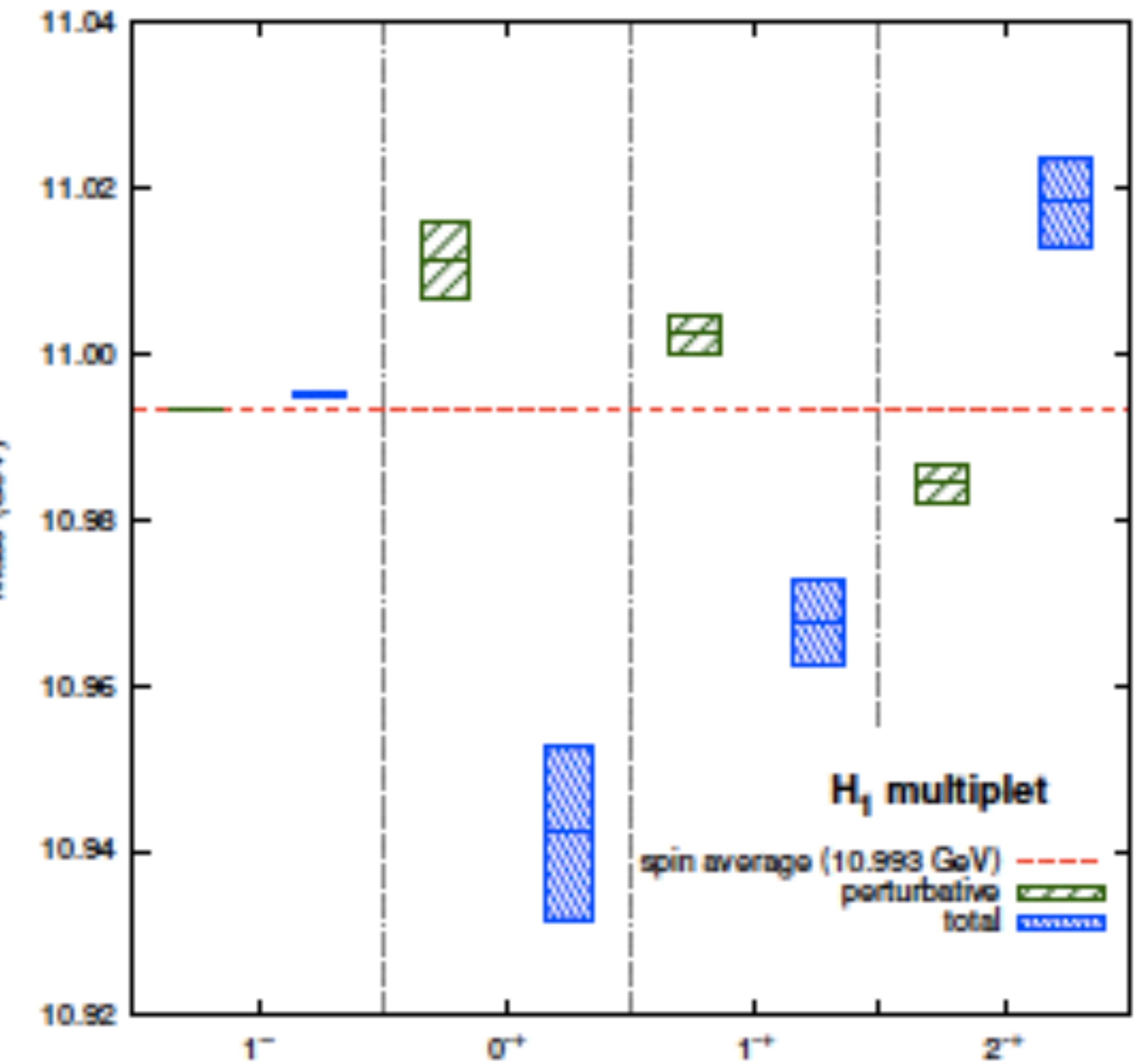
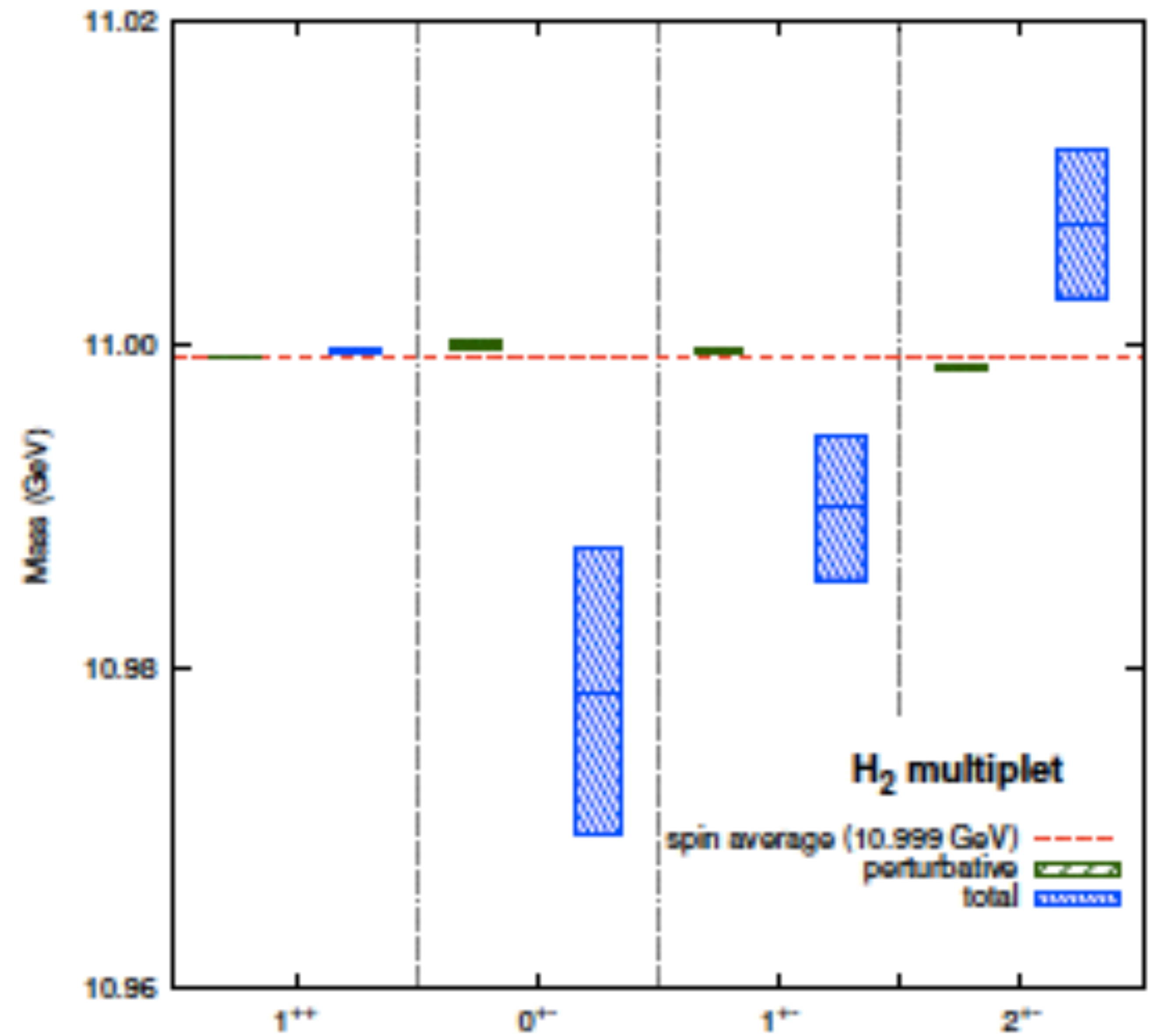
with a pion of about 240 MeV

Since the nonperturbative correlators
are not depending on flavour we can
use the result of the fits
on charmonium to predict
the hybrid bottomonium spin multiplets

Prediction for the Bottomonium Hybrids Multiplets H_1, H_2

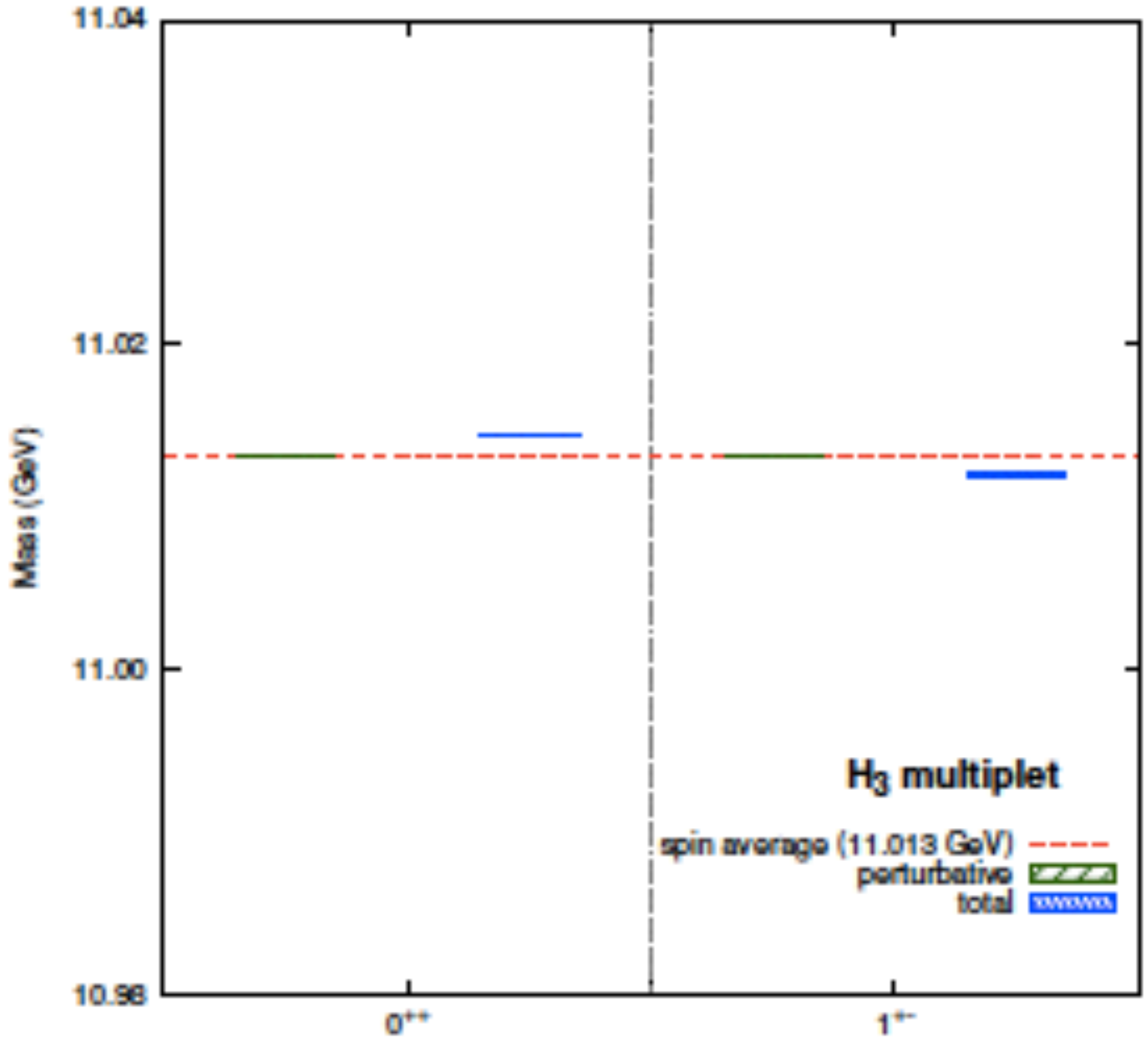


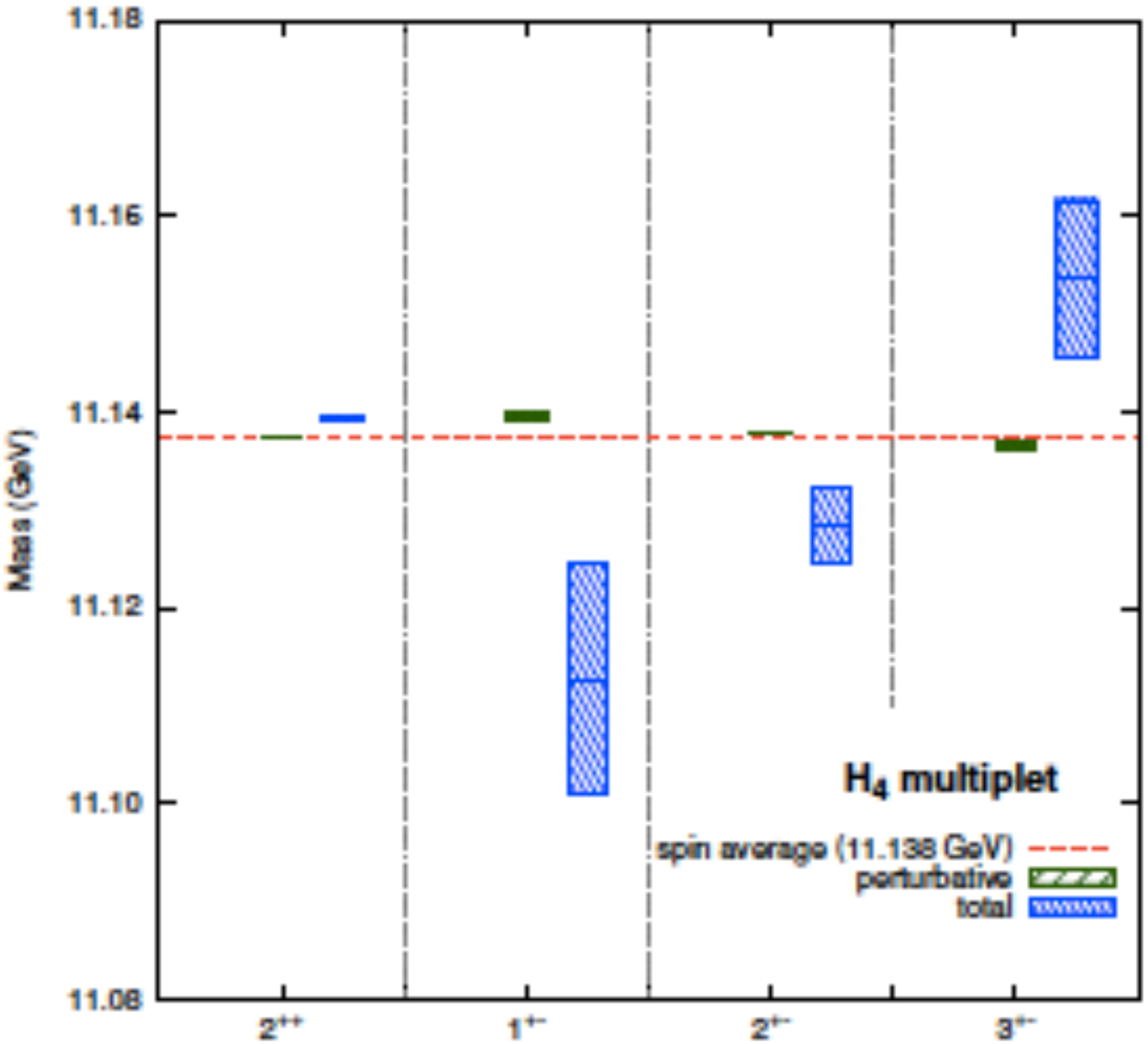


or bottomonium hybrids lattice data are scarce and not at the level of charmonium hybrids

- **Spin structure of heavy-quark hybrids**
  Nora Brambilla, Wai Kin Lai, Jorge Segovia, Jaume Tarrús Castellà, Antonio Vairo. May 20, 2018.
  e-Print: **arXiv:1805.07713**

Prediction for the Bottomonium Hybrids Multiplets H_3, H_4





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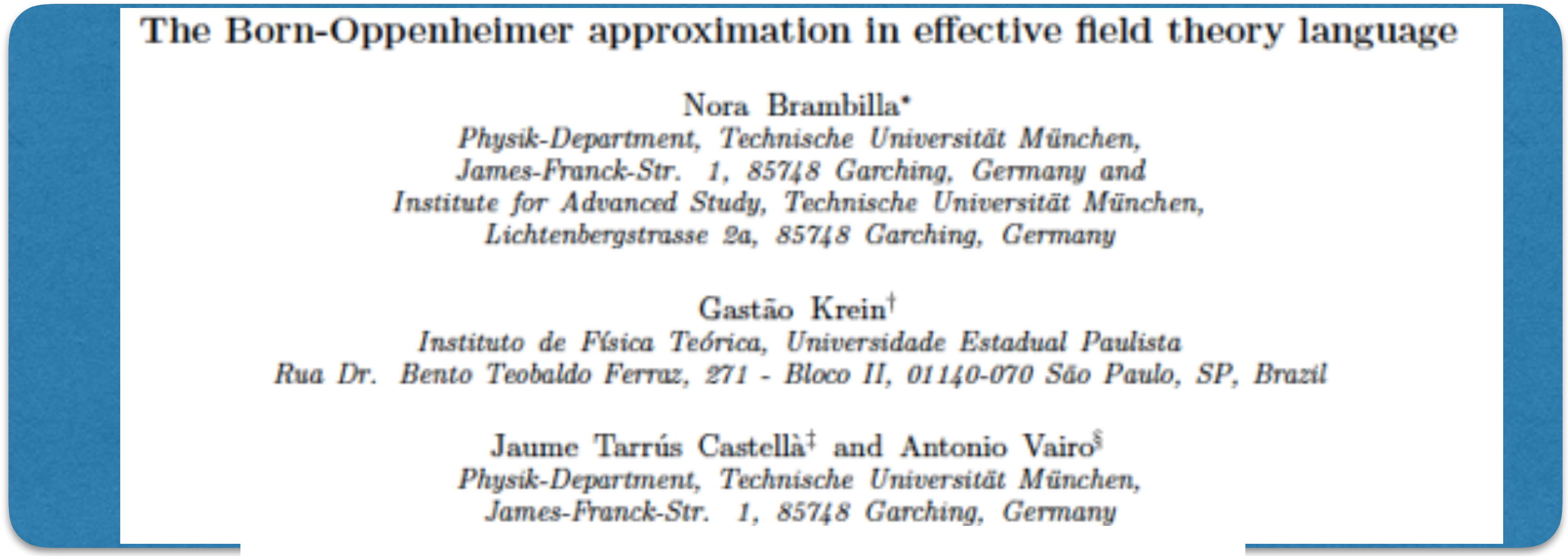
we can consider more general eigenstates of the octet sector the pNRQCD hamiltonian



The Born-Oppenheimer approximation in effective field theory language

Nora Brambilla*

Physik-Department, Technische Universität München, James-Franck-Str. 1, 85748 Garching, Germany and Institute for Advanced Study, Technische Universität München, Lichtenbergstrasse 2a, 85748 Garching, Germany

Gastão Krein†

Instituto de Física Teórica, Universidade Estadual Paulista Rua Dr. Bento Teobaldo Ferraz, 271 - Bloco II, 01140-070 São Paulo, SP, Brazil

Jaume Tarrús Castellà‡ and Antonio Vairo§

Physik-Department, Technische Universität München, James-Franck-Str. 1, 85748 Garching, Germany

$$|\kappa\rangle = O^{a\dagger}(\boldsymbol{r},\boldsymbol{R})\, G^a_{i\kappa}(\boldsymbol{R})|\mathrm{US}\rangle\,,$$

$$\kappa = \{J^{PC}, f\}\,,$$

light flavour

project on $\int d^3r d^3R \sum_{i\kappa} |\kappa\rangle\, \Psi_{i\kappa}(t,\boldsymbol{r},\boldsymbol{R}).$

obtain

$$L_{BO} = \int d^3R d^3r \sum_{\kappa} \Psi^\dagger_{i\kappa}(t,\boldsymbol{r},\boldsymbol{R})\left[(i\partial_t - h_o - \Lambda_\kappa)\,\delta^{ij} - \sum_\lambda P^i_{\kappa\lambda} b_{\kappa\lambda} r^2 P^j_{\kappa\lambda} + \cdots\right]\Psi_{j\kappa}(t,\boldsymbol{r},\boldsymbol{R})\,,$$

gives origin to a coupled Schroedinger equation

$$i\partial_t \Psi_{\kappa\lambda}(t,\boldsymbol{r},\boldsymbol{R}) = \left[\left(-\frac{\nabla^2_r}{M} + V_o(r) + \Lambda_\kappa + b_{\kappa\lambda} r^2\right)\delta_{\lambda\lambda'} - \sum_{\lambda'} C_{\kappa\lambda\lambda'}\right]\Psi_{\kappa\lambda'}(t,\boldsymbol{r},\boldsymbol{R})\,.$$

that can describe “**tetraquarks**” —> needs lattice calculations of tetraquarks static energies

Conclusions

Quarkonium is a golden system to study strong interactions

For states below threshold non relativistic EFTs provide a systematic tool to investigate a wide range of observables in the realm of QCD and quarkonium becomes a

NREFT Allow us to make calculations with unprecented precision, where high order perturbative calculations are possible
and to systematically factorize short from long range contributions where observables are sentitive to the nonperturbative dynamics of QCD

For states close or above the strong decay threshold the situation is much more complicated.

Many degrees of freedom show up and the absence of a clear systematic is an obstacle to a universal picture

We have presented results obtained for the hybrid masses in pNRQCD that show a very rich structure of multiplets.

Conclusions

- study of hybrids in EFT framework (NRQCD and pNRQCD)
- study of the spectrum of the static Hamiltonian with non-static corrections
- short distance degeneracy of static energies
- mixing of different static states
- breaking of degeneracy between opposite parity states

We have included spin in the hybrids multiplet structure:
—could interpret the lattice result
—make independent predictions for the bottomonium sector

Same approach can be used to include light quarks: "tetraquarks"
This approach holds the promise to be able to explain all exotics (including pentaquark) from QCD in the same framework

Input from the lattice is needed: more precise calculations of the gluelump masses, static energies for the hybrids and the tetra quarks, correlators of gluons fields..

Exotics may be generated also by QCD van der Waals forces: for example eta_b-eta_b bound states?