Rederiving overclosure bounds for wimp-like dark matter models

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Multi-Scale Problems using Effective Field Theories

University of Washington - Institute for Nuclear Theory
Wednesday, May 16

in collaboration with Mikko Laine: JHEP **1708** (2017) 047 and JHEP **1804** (2018) 072

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Outline

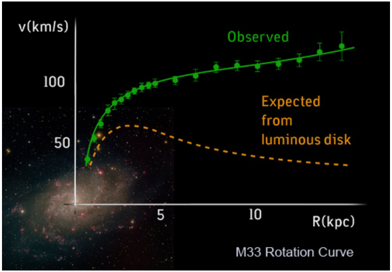
1. Motivation and Introduction
2. Non-relativistic WIMPs in a thermal bath
3. The inert doublet model
4. A look at strongly interacting mediators
5. Conclusions and Outlook

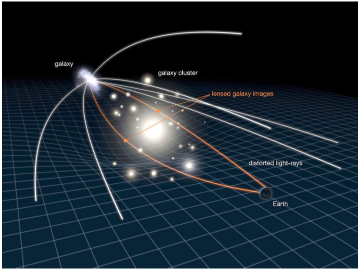
Evidence for Dark Matter I

- We can infer the existence of dark matter from its gravitational effects

At different scales

1. Star-velocity distribution in a galaxy V. Rubin and W. Ford (1970)
2. Galaxy-velocity distribution in a cluster of galaxies F. Zwicky (1937)
3. Strong and weak gravitational lensing J. K. Adelman-McCarthy et al. (2005)

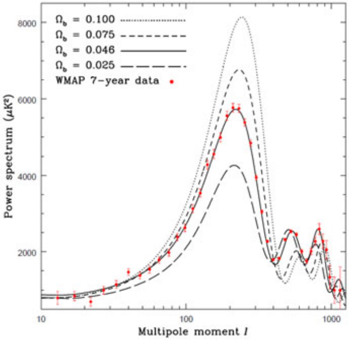




Evidence for Dark Matter II

Even at cosmological scales

- clear evidence from the Cosmic Microwave Background P.A.R. Ade et al. 1502.01589
- early universe before *recombination*: baryon-photon fluid oscillations



- Ω_m, Ω_b and photons
- dynamics of the fluid: gravitational collapse vs expansion due to pressure

$$\Omega_{dm} h^2 = 0.1186 \pm 0.0020$$

$$\Omega_b h^2 = 0.02226 \pm 0.00023$$

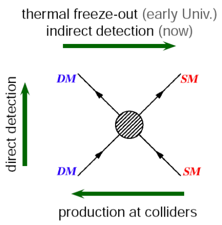
- Ω_b consistent with BBN predictions!
- only with dark matter structure formation could occur

Weakly interacting massive particles

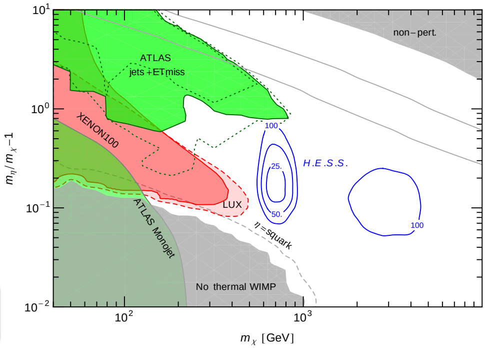
- Many candidates: axions, sterile neutrinos, composite dark matter ... G. Gelmini 1502.01320

WIMPs are attractive for some reasons

- arise to solve problems within particle physics realm (SUSY, extra dimensions...)
- relic abundance from freeze-out ($\Omega_{\rm dm} h^2$ today)
- testable experimentally with direct, indirect and collider searches



How reliable is the curve obtained from the cosmological relic abundance?



1403.4634

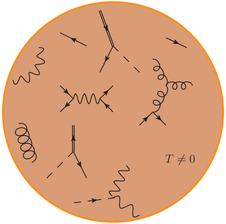
Wimp relic density

The thermal history in brief... P. Gondolo and G. Gelmini (1991)

- χ participates in weak interactions: equilibirum abundance in the early universe

$$\chi\chi \leftrightarrow f\,\bar{f}$$

- Massive particle, introduce a scale besides the **temperature T**
- Recombination $f\,\bar{f} \rightarrow \chi\chi$ is Boltzmann suppressed at $T < M$ ($n_{\rm F,B} \sim e^{-M/T}$)
- Eventually the DM pairs do not annihilate any more: freeze-out abundance



Boltzmann Equation

- n_χ total number density of DM particles
- annihilation and creation processes
- expanding background

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma v \rangle (n_\chi^2 - n_{\chi,\rm eq}^2)$$

- Kinetic equilibrium is assumed

Annihilation cross section

- Early universe thermodynamics: particles in a hot plasma

$$f_B^{\rm eq}(E)=\frac{1}{e^{E/T}-1}\,,\quad f_F^{\rm eq}(E)=\frac{1}{e^{E/T}+1}$$

- particle number density $n_\chi^{\rm eq}=g_\chi\int_{\mathbf{p}}f_F^{\rm eq}(E)\to g_\chi\left(\frac{MT}{2\pi}\right)^{\frac{3}{2}}e^{-\frac{M}{T}}$ $\left(n_i^{\rm eq}\approx g_i\frac{T^3}{\pi^2}\right)$
- **kinetic equilibrium**: momenta distribution, e.g. $\chi f\to\chi f$

$$p\sim T\;,\quad p\sim\sqrt{MT}\approx M\sqrt{\frac{T}{M}}\equiv Mv \qquad f_i(E)=f_i^{\rm eq}(E)\frac{n_i}{n_i^{\rm eq}}$$

- **chemical equilibrium**: detailed balance of a reaction, e. g. $\chi\chi\leftrightarrow f\bar{f}$

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- **chemical equilibrium**: detailed balance of a reaction, e. g. $\chi\chi \leftrightarrow f\bar{f}$

- thermally averaged cross section

$$\langle \sigma v \rangle = \frac{\int_{\mathbf{p}_1}\int_{\mathbf{p}_2} \sigma v \, e^{-E_1/T} e^{-E_2/T}}{\int_{\mathbf{p}_1}\int_{\mathbf{p}_2} e^{-E_1/T} e^{-E_2/T}}\,, \quad v = |\mathbf{v}_1 - \mathbf{v}_2|\,, \quad \frac{d\sigma}{d\Omega} = \frac{1}{4M^2 v}|\mathcal{M}|^2 \frac{1}{32\pi}$$

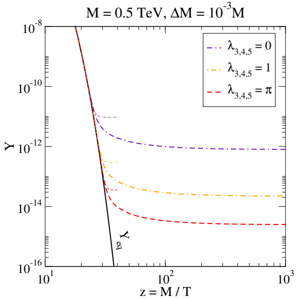
The overclosure bound from relic density

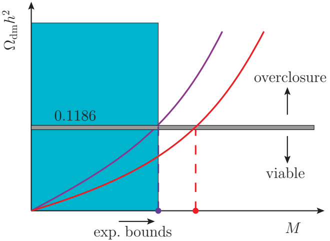
- $\langle \sigma v \rangle$: input from particle physics and thermal averaged, with $v \sim \sqrt{T/M} < 1$

$$\langle \sigma v \rangle \approx \langle a + bv^2 + \dots \rangle = a + \frac{3}{2} b \frac{T}{M} + \dots \Rightarrow \boxed{\langle \sigma v \rangle \approx \frac{\alpha^2}{M^2}}$$

- new variables $Y_\chi = n_\chi / s$ and $z = M/T \Rightarrow$ connect to the observed abundance

$$Y_{\rm phys} = Y(z_{\rm final}) : \quad \Omega_{\rm dm} = \frac{M Y_{\rm phys} s(T_0)}{\rho_{\rm cr}(T_0)} \Rightarrow \Omega_{\rm dm} h^2 = \frac{M}{\rm GeV} \frac{Y_{\rm phys}}{3.645 \times 10^{-9}}$$

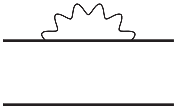




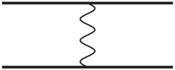
Wimp in a thermal bath

- at $T > 160$ GeV the electroweak symmetry is restored $\langle \phi \rangle = 0$
- χ are **non-relativistic**: have time to undergo several interactions

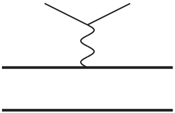
A) Mass correction



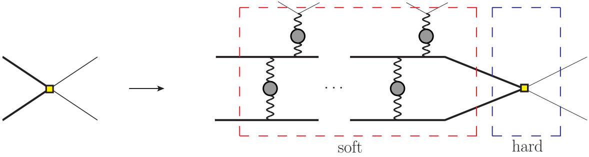
B) Sommerfeld effect and bound states



C) Interaction rate

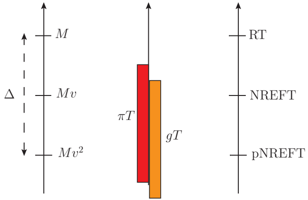


- How does all this reflect into the $\chi\chi$ annihilation?



Non-relativistic and thermal scales I

- Non-relativistic scales: $M \gg Mv \gg Mv^2$ (Coulomb potential $v \sim \alpha$)
- Thermal scales: πT and $m_D \approx \alpha^{1/2} T$, if weakly-coupled plasma $\pi T \gg m_D$



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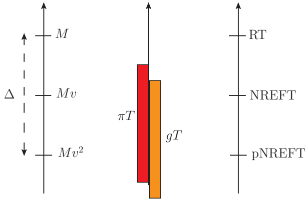
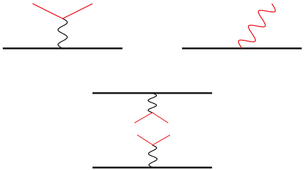




1. **Thermal widths**: the heavy particle is constantly kicked by plasma constituents

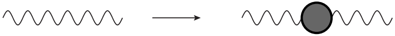
M. Laine, O. Philipsen, P. Romatschke and M. Tassler hep-ph/0611300; N. Brambilla, J. Ghiglieri, A. Vairo and P. Petreczky 0804.0993;

N. Brambilla, M. A. Escobedo, J. Ghiglieri and A. Vairo 1109.5826 and 1303.6097

$$\Gamma_{\rm GD} \sim \alpha^3 T\,, \quad \Gamma_{\rm LD} \sim \alpha T\,, \quad \Gamma^{\rm pair}_{\rm LD} \sim \alpha^2 T^3 r^2 \sim \begin{cases} \Gamma \sim \frac{\alpha^2 T^3}{M^2 v^2} \sim \alpha^2 \frac{T^2}{M}\,, & v \sim \sqrt{T/M} \\ \Gamma \sim \frac{\alpha^2 T^3}{M^2 v^2} \sim \frac{T^3}{M^2}\,, & v \sim \alpha \end{cases}$$

Non-relativistic and thermal scales II

2. **Thermal masses**: gauge-boson exchange $m_D \sim \alpha^{1/2} T$



$m \qquad\qquad m + m_D$

- the heavy dark matter particles experience thermal mass shifts
  if $T/M < \alpha^{1/2}$ the resummed one is larger P.M. Chesler, A. Gynther and A. Vuorinen 0906.3052

$\delta M_{\rm th} \sim \alpha T^2/M \qquad \delta M_{\rm th} \sim -\alpha m_D/2 \sim -\alpha^{3/2} T$

- Salpeter correction in nuclear theory: annihilation rate is enhanced

$$\gamma \sim e^{-2M/T} \rightarrow \gamma \sim e^{-2M/T} e^{\alpha m_D/T}$$

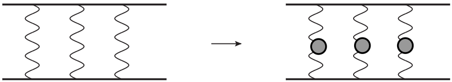
Non-relativistic and thermal scales III

3. **Sommerfeld effect**: distortion of the wave function of the annihilating pair

J. Hisano, S. Matsumoto, M. Nagai, O. Saito and M. Senami hep-ph/0610249; J.L. Feng, M. Kaplinghat and H.-B. Yu 1005.4678, M. Cirelli and A. Strumia 0903.3381, M. Beneke, A. Bharucha, F. Dighera, C. Hellmann, A. Hryczuk, S. Recksiegel and P. Ruiz-Femenia 1601.04718 ...

$$S_{\text{att.}} = \left(\frac{\pi\alpha}{v}\right)\frac{1}{1-\exp(-\frac{\pi\alpha}{v})}\,, \quad S_{\text{rep.}} = \left(\frac{\pi\alpha}{v}\right)\frac{1}{\exp(\frac{\pi\alpha}{v})-1}$$

$\rightarrow$ how do thermal effects change this?



4. **Bound state**: if they exist, they have binding energies $|\Delta E| \sim \alpha^2 M$

B. von Harling and K. Petraki 1407.7874; S.P. Liew and F. Luo 1611.08133; A. Mitridate, M. Redi, J. Smirnov and A. Strumia 1702.01141

$$\gamma \sim e^{-2M/T} \rightarrow \gamma \sim e^{-2M/T} e^{\alpha^2 M/T}$$

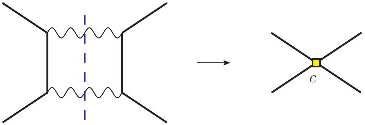
$\rightarrow$ of $\mathcal{O}(1)$ for $T \sim \alpha^2 M$: really important if bound states exist at freeze-out!

Annihilation rate

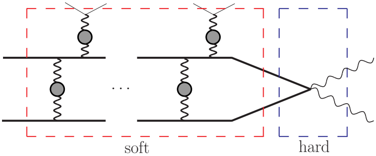
- Annihilation of a heavy pair: DM-DM, with energies $\sim 2M$ (forget about T)

$$\mathcal{O} = i\frac{c}{M^2}\phi^\dagger\phi^\dagger\phi\phi\,,\ c \approx \alpha^2 \quad \text{(inclusive s-wave annihilation)}$$

G. T. Bodwin, E. Braaten and G. P. Lepage hep-ph/9407339



- $M \gg T \Rightarrow \Delta x \sim \frac{1}{k} \sim \frac{1}{M} \ll \frac{1}{T}$ local and insensitive to the thermal scales



- we want to "thermal-average"

$$\langle \phi^\dagger\phi^\dagger\phi\phi \rangle_T$$

Beyond the free case: the spectral function

Compare Boltzmann equation with linear response theory

$$(\partial_t + 3H)n = -\langle \sigma v \rangle (n^2 - n_{\rm eq}^2) \quad \text{and} \quad (\partial_t + 3H)n = -\Gamma_{\rm chem}(n - n_{\rm eq})$$

$$\langle \sigma v \rangle \equiv \frac{\Gamma_{\rm chem}}{2 n_{\rm eq}} \Rightarrow \langle \sigma v \rangle = \frac{4}{n_{\rm eq}^2} \frac{c}{M^2} \gamma \quad \text{where } \gamma = \langle \phi^\dagger \phi^\dagger \phi \phi \rangle_T$$

D. Bodeker and M. Laine 1205.4987; S. Kim and M. Laine 1602.08105; S. Kim and M. Laine 1609.00474

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- thermal expectation value of the operators that annihilate/create a DM-DM pair

$$\gamma = \frac{1}{Z} \sum_{m,n} e^{-E_m/T} \langle m | \phi^\dagger \phi^\dagger | n \rangle \langle n | \phi \phi | m \rangle$$

- any correlator in equilibrium can be expressed in term of the *spectral function*

$$\rho(\omega, \mathbf{k}) = \int_{-\infty}^{\infty} dt \int_{\mathbf{r}} e^{i\omega t - i\mathbf{k}\cdot\mathbf{r}} \langle \frac{1}{2} \left[(\phi\phi)(t, \mathbf{r}), (\phi^\dagger \phi^\dagger)(0, \mathbf{0}) \right] \rangle_T$$

$$\gamma = \int_{2M-\Lambda}^{\infty} \frac{d\omega}{\pi} e^{-\frac{\omega}{T}} \int_{\mathbf{k}} \rho(\omega, \mathbf{k}) + \mathcal{O}(e^{-4M/T}) \,, \; \alpha^2 M \ll \Lambda \sim M$$

FROM ρ TO A SCHRÖDINGER EQUATION

- ρ can be extracted from a the imaginary part of a Green's function'
- non-relativistic dynamics

Y. Burnier, M. Laine and M. Vepsalainen, (2007)

$$E_m \equiv \omega = E' + 2M + \frac{k^2}{4M} \text{ and } H = -\frac{\nabla^2}{M} + V(r)$$

$$\left[H - i\Gamma - E'\right] G(E';\mathbf{r},\mathbf{r'}) = N\delta^3(\mathbf{r}-\mathbf{r'}) \qquad \lim_{\mathbf{r},\mathbf{r'}\to 0} \mathrm{Im}\, G(E';\mathbf{r},\mathbf{r'}) = \rho(E')$$

From ρ to a Schrödinger equation

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$$\Gamma \to 0^+ : \quad \lim_{T\to 0} \rho(E') = N \sum_m |\psi_m(\mathbf{0})|^2 \pi \delta(E_m - E')$$

- E_m are the s-wave energy eigenvalues of $H = -\frac{\nabla^2}{M} + V(r)$

- In the free case $V(r) \to 0$ and $\Gamma \to 0^+$

$$\rho_{\text{free}}(E') = N \frac{M^{\frac{3}{2}} \theta(E') \sqrt{E'}}{4\pi}$$

From ρ to a Schrödinger equation

- from the inhomogeneous to homogeneous equation M.J. Strassler and M.E. Peskin (1991)

$$\varrho = \alpha M r \quad V = \alpha^2 M \tilde{V}, \quad \Gamma = \alpha^2 M \tilde{\Gamma} \quad E' = \alpha^2 M \tilde{E}'$$

$$\left[-\frac{d^2}{d\varrho^2} + \frac{\ell(\ell+1)}{\varrho^2} + \tilde{V} - i\tilde{\Gamma} - \tilde{E}' \right] u_\ell(\varrho) = 0 \quad \Rightarrow \rho(E') = \frac{\alpha M^2 N}{4\pi} \int_0^\infty d\varrho \, \mathrm{Im} \left[\frac{1}{u_0(\varrho)} \right]$$

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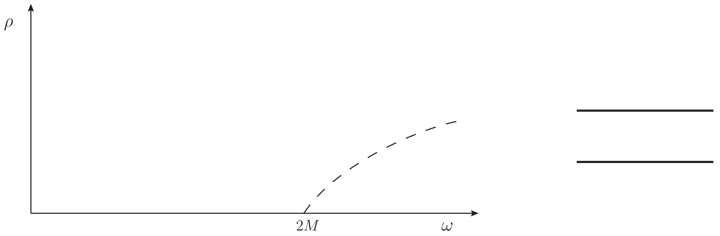

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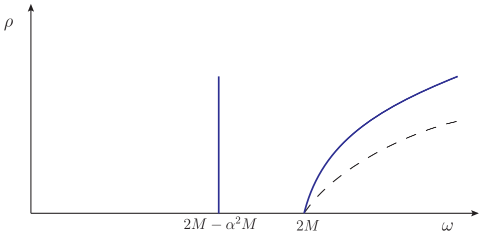

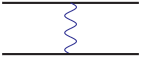

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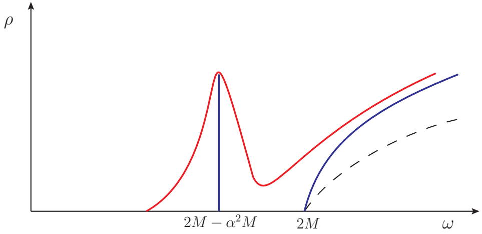
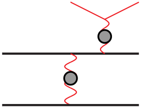
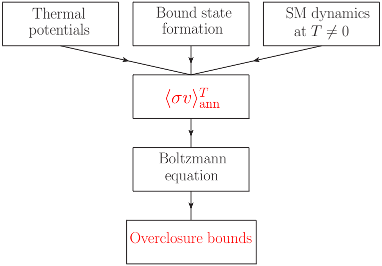


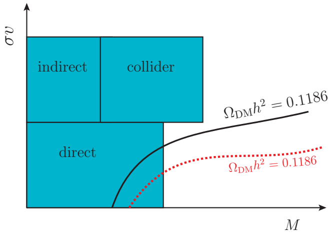


Summary of the theoretical framework

Relic density can be factorized in some steps — M. Laine and S.'Kim 1609.00474

- Calculate the matching coefficients from the hard annihilation process, $E \sim 2M$
- Compute the static potentials and thermal widths induced by the particle exchanged by the heavy ones
- Extract the spectral function $\Rightarrow$ annihilation rate
- Solve the Boltzmann equation with the **thermal** cross section





The inert doublet model

- Supplement SM with χ SU(2) doublet, no coupling with fermions, unbroken vacuum
- We focus on the high-mass regime of the model: $M \gtrsim 530$ GeV
- Degenerate case: 4 states, $H_0, H_{\bar{0}}, H_{\pm}$ with the same mass

T. Hambye, F.-S. Ling, L. Lopez Honorez and J. Rocher, 0903.4010

$$\begin{aligned}\mathcal{L}_\chi &= (D^\mu \chi)^\dagger (D_\mu \chi) - M^2 \chi^\dagger \chi \\ &- \left\{ \lambda_2 (\chi^\dagger \chi)^2 + \lambda_3 \, \phi^\dagger \phi \, \chi^\dagger \chi + \lambda_4 \, \phi^\dagger \chi \, \chi^\dagger \phi + \left[\frac{\lambda_5}{2} (\phi^\dagger \chi)^2 + h.c. \right] \right\}\end{aligned}$$

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- Annihilations happening at $T \sim M/20 ... M/10^3 \Rightarrow v \sim \sqrt{T/M} \ll 1$

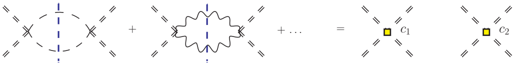
$$\chi = \frac{1}{\sqrt{2M}} \left(C e^{-iMt} + D^\dagger e^{iMt} \right) , \quad \chi^\dagger = \frac{1}{\sqrt{2M}} \left(D e^{-iMt} + C^\dagger e^{iMt} \right)$$

$$\delta\mathcal{L}_{\mathrm{NREFT}} = i \left(\frac{c_1}{M^2} \underbrace{C_p^\dagger D_p^\dagger D_q C_q}_{\equiv O_1} + \frac{c_2}{M^2} \underbrace{C_p^\dagger T^a_{pq} D_q^\dagger D_r T^a_{rs} C_s}_{\equiv O_2} + \frac{c_3}{M^2} \underbrace{D_p^\dagger D_q^\dagger D_p D_q}_{\equiv O_3} + \frac{c_4}{M^2} \underbrace{C_p^\dagger C_q^\dagger C_p C_q}_{\equiv O_4} \right)$$

Matching the hard process

- Matching matrix elements of four-particle states: imaginary part of c_i

$$c_1 = \frac{g_1^4 + 3g_2^4 + 8\lambda_3^2 + 8\lambda_3\lambda_4 + 2\lambda_4^2}{256\pi}$$
$$c_2 = \frac{g_1^2 g_2^2 + \lambda_4^2}{32\pi}$$
$$c_3 = c_4 = \frac{\lambda_5^2}{128\pi}$$



Degenerate case: cross section with free-heavy scalar

- $\langle \sigma v \rangle = \frac{4}{n_{\rm eq}^2} \sum_i^4 c_i \gamma_i$ and with $N_1 = 2$, $N_2 = \frac{3}{2}$, $N_3 = N_4 = 6$ we obtain

$$\langle \sigma_{\rm eff}\, v \rangle^{(0)} = \frac{c_1}{2} + \frac{3c_2}{8} + \frac{3(c_3 + c_4)}{2}$$

- we redefined the $c_i \to c_i / M^2$

Including the potentials

- the quasi-static heavy scalars interact with gauge bosons, $W_0^{\pm}, B_0, A_0$

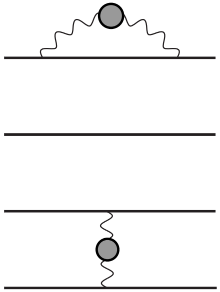
$$\left[-\frac{\nabla_r^2}{M} + \mathcal{V}_i(r) - E'\right] G_i(E';\mathbf{r},\mathbf{r}') = N_i\,\delta^{(3)}(\mathbf{r}-\mathbf{r}')\,, \quad \lim_{\mathbf{r}\to\vec{0}} \mathrm{Im}\,G_i(E';\mathbf{r},\mathbf{r}') = \rho_i(E')$$

Electroweak thermal potentials

$$\mathcal{V}_{\mathrm{W}}(r) \equiv \frac{g_2^2}{4}\int_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}\, i\langle W_0^+ W_0^-\rangle_{\mathrm{T}}(0,k)\,,$$

$$\mathcal{V}_{\mathrm{A}}(r) \equiv \frac{g_2^2}{4}\int_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}\, i\langle A_0^3 A_0^3\rangle_{\mathrm{T}}(0,k)$$

$$\mathcal{V}_{\mathrm{B}}(r) \equiv \frac{g_1^2}{4}\int_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}\, i\langle B_0 B_0\rangle_{\mathrm{T}}(0,k)$$



HTL propagators for gauge bosons

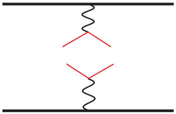
- $m \ll \pi T$, capture thermal effects with Hard-Thermal Loop (i.e. $T \gg 30$ GeV for Z, W)

J. Frenkel and J.C. Taylor (1990), E. Braaten and R.D. Pisarski (1990), J.C. Taylor and S.M.H. Wong (1990)

$$i\langle W_0^+ W_0^- \rangle_T = \frac{1}{\mathbf{k}^2 + m_{\widetilde{W}}^2} - \frac{i\pi T}{k} \frac{m_{\mathrm{E2}}^2}{(\mathbf{k}^2 + m_{\widetilde{W}})^2} \qquad \text{(static limit)}$$

- $m_{\widetilde{W}}^2 = m_W^2 + m_{\mathrm{E2}}^2$ and $m_W = g_2 v_T/2$

$$m_{\mathrm{E1}}^2 = \left(\frac{n_S}{6} + \frac{5n_G}{9}\right) g_1^2 T^2, \quad m_{\mathrm{E2}}^2 = \left(\frac{2}{3} + \frac{n_S}{6} + \frac{n_G}{3}\right) g_2^2 T^2$$



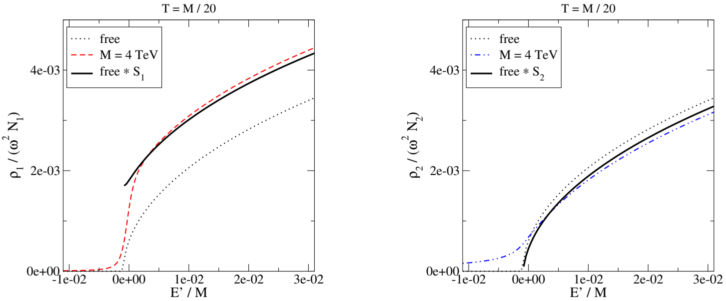
$$\mathcal{V}_{\mathrm{W}}(r) = \frac{g_2^2}{16\pi}\left[\frac{\exp(-m_{\widetilde{W}} r)}{r} - i\,\frac{T m_{\mathrm{E2}}^2 \phi(m_{\widetilde{W}} r)}{m_{\widetilde{W}}^2}\right]$$

$$\mathcal{V}_{\mathrm{W}}(0) = -\frac{g_2^2}{16\pi}\left(m_{\widetilde{W}} + i\,\frac{T m_{\mathrm{E2}}^2}{m_{\widetilde{W}}^2}\right) + \left.\frac{g_2^2 m_W}{16\pi}\right|_{T=0}$$

Results for the spectral functions

- the potential for the attractive channel reads

$$\mathcal{V}_1 = 2\mathcal{V}_{\rm W}(0) + \mathcal{V}_{\rm A}(0) + \mathcal{V}_{\rm B}(0) - 2\mathcal{V}_{\rm W}(r) - \mathcal{V}_{\rm A}(r) - \mathcal{V}_{\rm B}(r)$$

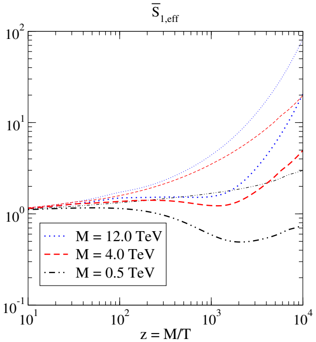


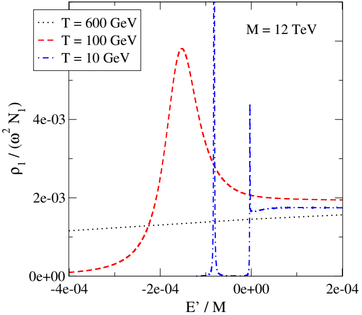
- there is no large deviation with respect to a $T = 0$ Sommerfeld factor
- no bound states around the freeze-out, non-zero tail in the repulsive channel

T-Averaged Sommerfeld factors

- $\bar{S}_i$: **distortion of the wave function, thermal widths, Salpeter correction, bound states**

$$\bar{S}_i \equiv \frac{e^{2\Delta M_T/T}}{N_i}\left(\frac{4\pi}{MT}\right)^{\frac{3}{2}}\int_{-\Lambda}^{\infty}\frac{\mathrm{d}E'}{\pi}e^{-E'/T}\rho_i(E')\,, \quad 2\Delta M_T \equiv \mathrm{Re}\left[2\mathcal{V}_W(0)+\mathcal{V}_A(0)+\mathcal{V}_B(0)\right]$$

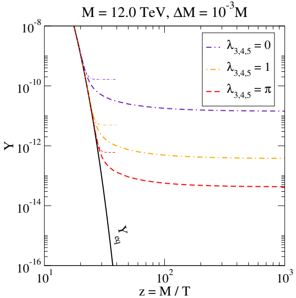


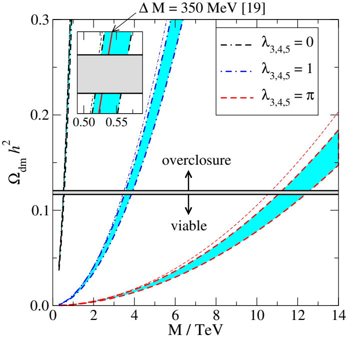


$$\langle \sigma v \rangle^{(0)} \to \langle \sigma v \rangle = \frac{c_1 \bar{S}_1}{2} + \frac{3\, c_2 \bar{S}_2}{8} + \frac{3\,(c_3+c_4)\bar{S}_3}{2}$$

Overclosure bound for IDM

$$Y'(z) = -\langle\sigma v\rangle M m_{\rm Pl} \frac{c(T)}{\sqrt{24\pi e(T)}} \left.\frac{Y^2(z)-Y^2_{\rm eq}(z)}{z^2}\right|_{T=M/z}$$





- $\lambda_i = 0$: $M < 519 \pm 4\text{GeV}$ $\rightarrow$ $M < 523 \pm 4\text{GeV}$ or $M < 562 \pm 4\text{GeV}$
- $\lambda_i = \pi$: $M < 10.6 \pm 0.1\text{TeV}$ $\rightarrow$ $M < 11.1 \pm 0.1\text{TeV}$ or $M < 12.1 \pm 0.1\text{TeV}$

Simplified models

To link effectively a BSM theory and dark matter

- example: SUSY has a rather large parameter space
- Constraints are set on a simple model that captures the most relevant physics

A. De Simone and T. Jacques 1603.08002

Majorana fermion DM + Coloured mediator

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_\chi + \mathcal{L}_\eta + \mathcal{L}_{\rm int}$$

$$\mathcal{L}_\chi^{\rm M} = \frac{1}{2}\bar{\chi} i \not{\partial} \chi - \frac{M}{2}\bar{\chi}\chi \,, \quad \mathcal{L}_\eta = (D^\mu \eta)^\dagger (D_\mu \eta) - M_\eta^2 \eta^\dagger \eta - \lambda_2 \left(\eta^\dagger \eta\right)^2$$

$$\mathcal{L}_{\rm int} = -{\rm y}\, \eta^\dagger \bar{\chi} P_R q - {\rm y}^* \, \bar{q} P_L \chi \eta - \lambda_3 \eta^\dagger \eta H^\dagger H$$

M. Garny, A. Ibarra and S. Vogl 1503.01500

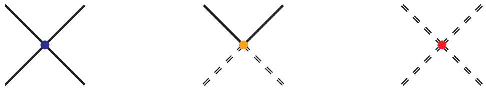
- the annihilation of $\chi\chi$ pairs is p-wave suppressed — J. Edsjö and P. Gondolo hep/ph-9704361

$\Rightarrow$ the role of the (co)annihilating η is important and driven by QCD

$$\langle \sigma v \rangle \approx \langle \sigma v \rangle_{\chi\chi} + e^{-\frac{\Delta M}{M}} \langle \sigma v \rangle_{\eta\chi} + e^{-2\frac{\Delta M}{M}} \langle \sigma v \rangle_{\eta\eta}$$

Strong interactions enter...

- Again $\eta = \frac{1}{\sqrt{2M}}\left(\phi e^{-iMt} + \varphi^\dagger e^{iMt}\right)$ and $\chi = (\psi e^{-iMt}, -i\sigma_2\psi^* e^{iMt})$



$$\begin{aligned}\mathcal{L}_{\rm abs} &= i\left\{c_1\, \psi_p^\dagger\psi_q^\dagger\psi_q\psi_p + c_2\left(\psi_p^\dagger\phi_\alpha^\dagger\psi_p\phi_\alpha + \psi_p^\dagger\varphi_\alpha^\dagger\psi_p\varphi_\alpha\right)\right.\\ &+ \left. c_3\,\phi_\alpha^\dagger\varphi_\alpha^\dagger\varphi_\beta\phi_\beta + c_4\,\phi_\alpha^\dagger\varphi_\beta^\dagger\,\varphi_\gamma\phi_\delta\,T^a_{\alpha\beta}T^a_{\gamma\delta} + c_5\left(\phi_\alpha^\dagger\phi_\beta^\dagger\phi_\beta\phi_\alpha + \varphi_\alpha^\dagger\varphi_\beta^\dagger\varphi_\beta\varphi_\alpha\right)\right\}\end{aligned}$$

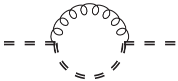
- the matching coefficients are

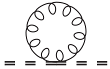
$$c_1 = 0\,, \quad c_2 = \frac{|y|^2(|h|^2 + g_s^2 C_F)}{128\pi M^2}\,,$$

$$c_3 = \frac{1}{32\pi M^2}\left(\lambda_3^2 + \frac{g_s^4 C_F}{N_c}\right), \quad c_4 = \frac{g_s^4(N_c^2-4)}{64\pi M^2 N_c}\,, \quad c_5 = \frac{|y|^4}{128\pi M^2}\,.$$

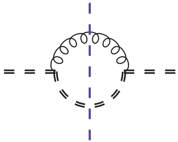
Thermal masses and interaction rates I

- the gluonic contribution are IR sensitive $\rightarrow$ need to be resummed for a correct result

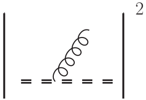




$$\frac{\mathrm{Re}\Pi_R}{2M_\eta} = \frac{g_s^2 C_F T^2}{12 M_\eta}$$



$\sim$



$$\frac{\mathrm{Im}\Pi_R}{2M_\eta} = 0$$

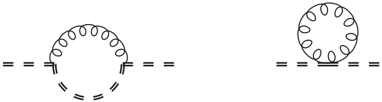
- the real part is analogous to that for a heavy fermion

  J.F. Donoghue, B.R. Holstein and R.W. Robinett (1986)

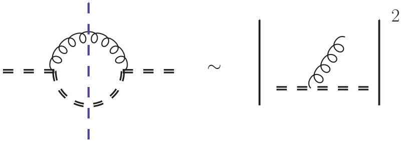
- the imaginary part vanishes because there is no phase space for the $1 \leftrightarrow 2$ process

Thermal masses and interaction rates II

- at high temperatures these naive results are misleading



$$\frac{\mathrm{Re}\Pi_R}{2M_\eta} =: \frac{g_s^2 C_F T^2}{12 M_\eta}$$

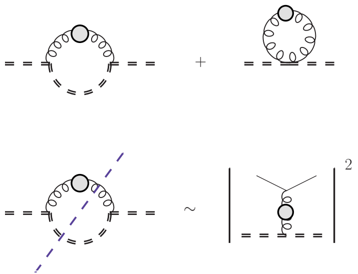


$$\frac{\mathrm{Im}\Pi_R}{2M_\eta} =: 0$$

- $M_\eta + \Delta M$ and $\Delta M \ll \pi T \ll M_\eta$
- real part $\sim g_s^2 C_F \Delta M$ and imaginary part $\sim g_s^2 C_F |\Delta M| n_B(|\Delta M|) \sim g_s^2 C_F T$
- Bose enhancement of the soft contribution compensates against the phase-space suppression

Thermal masses and interaction rates III

- η quasi static and interact with A_0^a, in a plasma Debye screened $m_D \sim g_s T$



$$\frac{\mathrm{Re}\Pi_R}{2M_\eta} = \frac{g_s^2 C_F T^2}{12 M_\eta} + \frac{g_s^2 C_F}{2} \int_{\mathbf{p}} \frac{1}{p^2 + m_D^2} = \frac{g_s^2 C_F T^2}{12 M_\eta} - \frac{g_s^2 C_F m_D}{8\pi}$$

$$\frac{\mathrm{Im}\Pi_R}{2M_\eta} = -\frac{g_s^2 C_F}{2} \int_{\mathbf{p}} \frac{\pi T m_D^2}{p(p^2 + m_D^2)^2} = -\frac{g_s^2 C_F T}{8\pi}$$

- Real part: Debye-screened Coulomb self-energy

$$g_s^2 \frac{T^2}{M} \lesssim g_s^2 (g_s T) \Rightarrow \frac{T}{M} \lesssim g_s$$

- imaginary part: reflects fast colour and phase-changing $2 \to 2$ scatterings off light medium particles (first derived for heavy quarks)

Rates and annihilation cross section

- $\Delta M_T \equiv \Delta M + \frac{(g_s^2 C_F + \lambda_3) T^2}{12M} - \frac{g_s^2 C_F m_D}{8\pi}$

Rates and annihilation cross section

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- the potential that plays a role involves QCD gluons

$$V(r) \equiv \frac{g_s^2}{2} \int_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} \left[\frac{1}{\mathbf{k}^2 + m_D^2} - i \frac{\pi T}{k} \frac{m_D^2}{(\mathbf{k}^2 + m_D^2)^2} \right] , \quad m_D = g_s \sqrt{\frac{N_c}{3} + \frac{N_f}{6}}$$

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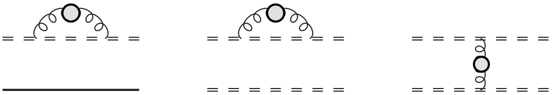
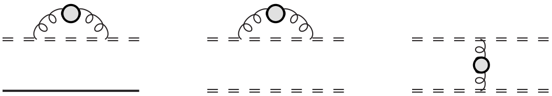
- after the Fourier transform

$$V(r) = \frac{g_s^2}{2} \begin{cases} \frac{\exp(-m_D r)}{4\pi r} - \frac{iT}{2\pi m_D r} \int_0^\infty \frac{dz \sin(z m_D r)}{(1+z^2)^2} , & r > 0 \\ -\frac{m_D}{4\pi} - \frac{iT}{4\pi} , & r = 0 \end{cases}$$

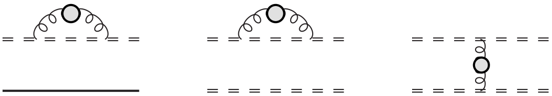




$$\mathcal{V}_1 = 0\,, \quad \mathcal{V}_2 = C_F V(0)\,, \quad \mathcal{V}_3 = 2C_F[V(0) - V(r)]$$
$$\mathcal{V}_4 = 2C_F V(0) + \frac{V(r)}{N_c}\,, \quad \mathcal{V}_5 = 2C_F V(0) + \frac{(N_c - 1)V(r)}{N_c}$$

- the thermally modified Sommerfeld factors are

$$\bar{S}_i = \left(\frac{4\pi}{MT}\right)^{\frac{3}{2}} \int_{-\Lambda}^{\infty} \frac{\mathrm{d}E'}{\pi}\, e^{[\mathrm{Re}\mathcal{V}_i(\infty) - E']/T}\, \frac{\rho_i(E')}{N_i}$$



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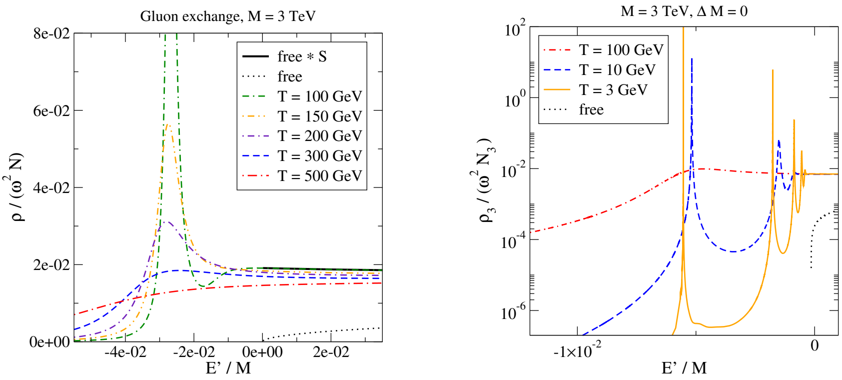
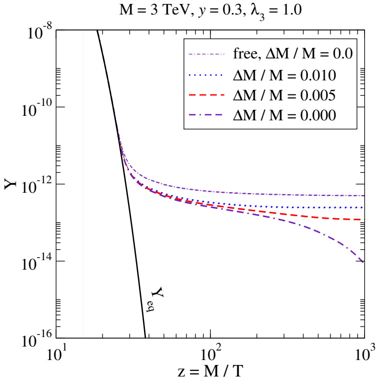
$$\left\langle \sigma_{\rm eff}\, v \right\rangle = \frac{2c_1 + 4c_2 N_c\, e^{-\Delta M_T/T} + [c_3 \bar{S}_3 + c_4 \bar{S}_4 C_F + 2c_5 \bar{S}_5 (N_c + 1)] N_c\, e^{-2\Delta M_T/T}}{\left(1 + N_c\, e^{-\Delta M_T/T}\right)^2}$$

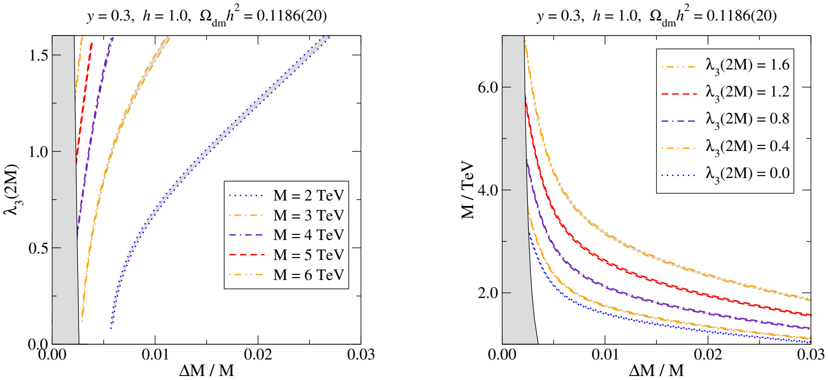
Bound states and thermal widths



- bound states already start to form at $z \sim 15$ and visible at $z \sim 20$
- at high temperatures: reduced Sommerfeld effect with respect to a massless gluon



- a blind $\Delta M = 0$ brings to very large masses M
- **however the splitting cannot be arbitrary small**!
  if $2\Delta M - |E_1| < 0$ the lightest two-particle states are $(\eta^\dagger \eta)$
  $\Rightarrow$ $(\chi\chi)$ rapidly convert into $(\eta^\dagger \eta)$ that are short lived and promptly annihilate



- gray bands implement the constraint $2\Delta M - |E_1| > 0$
- the model can be phenomenologically viable up to $M \sim 5...7$ TeV
- y and h have a small impact on $\Omega_{\rm dm}$, whereas λ_3 enters the very efficient singlet channel thorough $c_3 = (\lambda_3^2 + g_s^2 C_F/N_c)/(32\pi^2 M^2)$
- Note: a $\lambda_3 \neq 0$ is always generated at high scale (from RGEs)

Summary and Outlook

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- the freeze-out calculation is factorized into $\langle \sigma v \rangle \approx c_i \langle \mathcal{O}_i \rangle_T$
- $E \sim M$: matching coefficients at $T = 0 \rightarrow$ NREFTs
- $E \sim T, m_D$ solve a thermally modified Schrödinger equation for $\rho(E)$:
  distortion of the wave function, thermal widths, Salpeter correction, bound states

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- Attempt to refine the calculation of the thermal freeze-out for WIMPs

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- **Outlook**: Address other models, study the impact of the Higgs (scalar) exchange, assess the impact on experimental analysis (SB and Stefan Vogl in preparation)

IDM mass ranges

- Low-mass regime: $M \lesssim M_W$
- Intermediate regime: $M_W \lesssim M \lesssim 535$ GeV, ruled out by XENON

  XENON100 Collaboration, E. Aprile et al. (2012), 1207.5988
- High-mass regime: $M \gtrsim 535$ GeV, unitary bound $\lambda_i \sim 4\pi \Rightarrow M \sim 58$ TeV

  T. Hambye, F.-S. Ling, L. Lopez Honorez and J. Rocher, 0903.4010

Thermally average cross section and freeze-out

- thermally averaged cross section

$$\langle \sigma v \rangle = \frac{\int d^3p_1 d^3p_2 \; \sigma v \; e^{-E_1/T} e^{-E_1/T}}{\int d^3p_1 d^3p_2 e^{-E_1/T} e^{-E_1/T}}$$

- Freeze-out estimation

$$H \sim n\langle \sigma v \rangle \Rightarrow \frac{T^2}{m_{\rm Pl}} \sim \left(\frac{MT}{2\pi} \right)^{3/2} e^{-\frac{M}{T}} \frac{\alpha^2}{M^2}$$

- Thermal expectation value

$$\gamma = \frac{1}{\mathcal{Z}} e^{-E_m/T} \sum_m \langle m | \theta^\dagger \eta^\dagger \eta \theta | m \rangle$$

- kinetically equilibrated particle: $E_{kin} \approx Mv^2 \sim T$

Sommerfeld factors at $T = 0$

- electroweak potentials: short distance part $r \ll m_{\widetilde{W}}$

$$\mathcal{V}_1(r) \simeq \frac{3g^2 + g'^2}{16\pi r}\,, \quad \mathcal{V}_2(r) \simeq \frac{g^2 - g'^2}{16\pi r}\,, \quad \mathcal{V}_3(r) \simeq \frac{g^2 + g'^2}{16\pi r}$$

- then we can use the standard form of the Sommerfeld factors

$$S_1 = \frac{X_1}{1 - e^{-X_1}}\,, \quad S_{2,3} = \frac{X_{2,3}}{e^{-X_{2,3}} - 1}$$

- where $X_i = \pi\alpha_i/v$ and $E' = 2\Delta M_T + Mv^2$

HTL approximation

- HTL is justified when the particle with which the gauge fields interact are ultrarelativistic, i.e. $m \ll \pi T$
- top and bottom common mass m_f, $W^{\pm}, Z, h$ with a common mass m_g

$$m^2_{\mathrm{E1}} \simeq \frac{g'^2}{2}\left[\frac{49T^2}{18} + \frac{11\chi_F(m_f)}{3} + \chi_B(m_g)\right]$$

$$m^2_{\mathrm{E2}} \simeq \frac{g'^2}{2}\left[\frac{3T^2}{2} + 3\chi_F(m_f) + 5\chi_B(m_g)\right]$$

- this is however a pure phenomenological recipe $m_b < \pi T < m_t$
- temperature dependent Higgs expectation value (it vanishes for $T \approx 160 GeV$)

$$v_T^2 \equiv -\frac{m_\phi^2}{\lambda} \text{ for } m_\phi^2 < 0\,, \quad m_\phi^2 \equiv -\frac{m_h^2}{2} + \frac{(g'^2 + 3g^2 + 8\lambda + 4h_t^2)T^2}{16}$$

Low-temperature and mass splitting

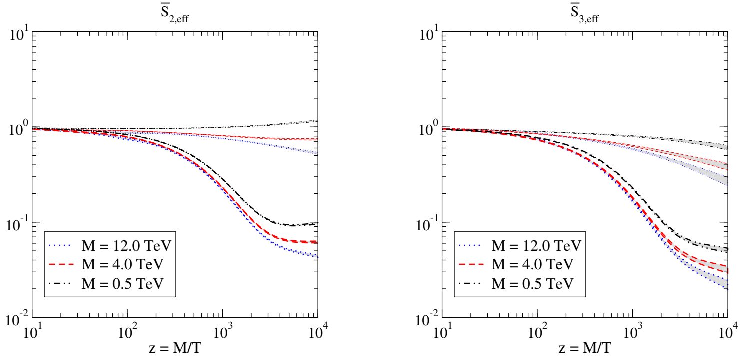
- the vacuum mass difference ΔM becomes important at very low temperature
- the effect is to reduce the importance of the coannihilating species
- it can be phenomenologically included via the substitution

$$\bar{S}_1 \;\to\; \bar{S}_{1,\mathrm{eff}} \equiv \bar{S}_1 \left[\frac{1}{4} + \frac{3e^{-2\Delta M/T}}{4}\right]$$

$$\bar{S}_{2,3,4} \;\to\; \bar{S}_{2,3,4,\mathrm{eff}} \equiv \bar{S}_{2,3,4} \left[\frac{1}{12} + \frac{e^{-\Delta M/T}}{3} + \frac{7e^{-2\Delta M/T}}{12}\right]$$

- the appearance of $2\Delta M_T$ in $\bar{S}_i$ is due to

$$n_{eq} \approx 4 \left(\frac{MT}{2\pi}\right)^{\frac{3}{2}} e^{-(M+\Delta M_T)/T} \qquad (1)$$


$\overline{S}_{2,\mathrm{eff}}$
$\overline{S}_{3,\mathrm{eff}}$
10^1
10^0
10^{-1}
10^{-2}
10^1
10^2
10^3
10^4
z = M/T
M = 12.0 TeV
M = 4.0 TeV
M = 0.5 TeV

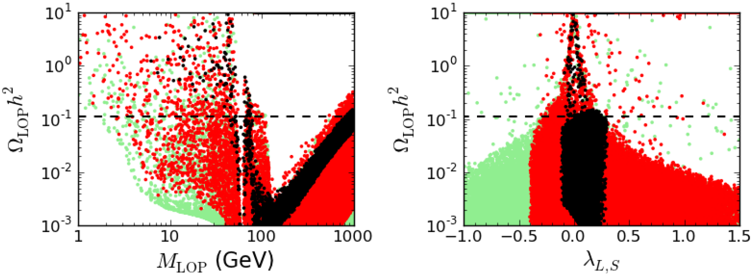
IDM scalar masses

- with $v \equiv \langle \phi \rangle$

$$
\begin{aligned}
M_{H_0} &= M^2 + \frac{1}{2}(\lambda_3 + \lambda_4 + \lambda_5) v^2 \,, \\
M_{H_{\bar{0}}} &= M^2 + \frac{1}{2}(\lambda_3 + \lambda_4 - \lambda_5) v^2 \,, \\
M_{H_0} &= M^2 + \frac{1}{2}\lambda_3 v^2 \,, \\
\Delta M_{\text{SM}} &= \frac{g^2}{4\pi} M_W \sin^2 \frac{\theta_W}{2}
\end{aligned}
$$

- the different components can be non degenerate in mass

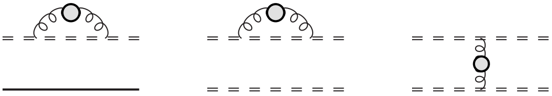
$$
C = \begin{pmatrix} H_+ \\ \frac{H_0 - iH_{\bar{0}}}{\sqrt{2}} \end{pmatrix} \,, \quad D = \begin{pmatrix} H_- \\ \frac{H_0 + iH_{\bar{0}}}{\sqrt{2}} \end{pmatrix}
$$



A. Goudelis, B. Herrmann and O. Stal 1303.3010

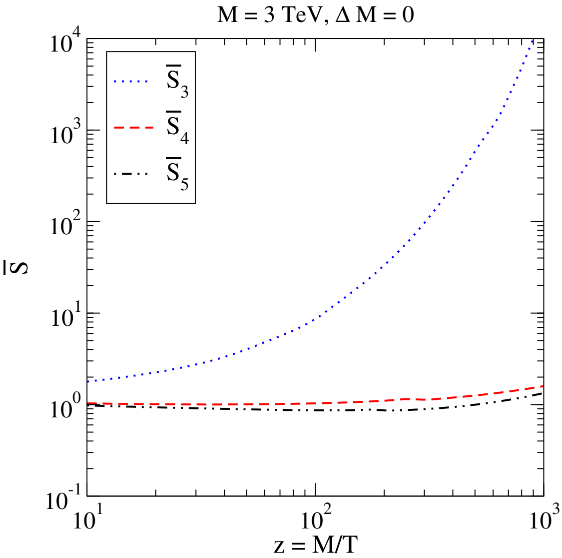
Scalar QCD potential

$$V(r) = \frac{g_s^2}{2}\begin{cases} \frac{\exp(-m_D r)}{4\pi r} - \frac{iT}{2\pi m_D r}\int_0^\infty \frac{dz \sin(z m_D r)}{(1+z^2)^2}\,, & r > 0 \\ -\frac{m_D}{4\pi} - \frac{iT}{4\pi}\,, & r = 0 \end{cases}$$

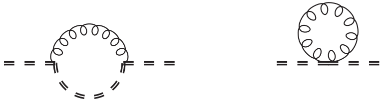


$$\mathcal{V}_1 = 0\,, \quad \mathcal{V}_2 = C_F V(0)\,, \quad \mathcal{V}_3 = 2C_F[V(0) - V(r)]$$
$$\mathcal{V}_4 = 2C_F V(0) + \frac{V(r)}{N_c}\,, \quad \mathcal{V}_4 = 2C_F V(0) + \frac{(N_c-1)V(r)}{N_c}$$

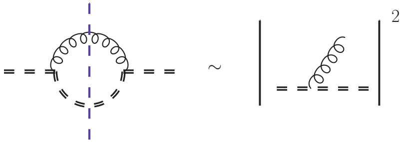
Sommerfeld for scalar QCD



Rates I



$$\frac{\mathrm{Re}\Pi_R}{2M_\eta} = \frac{g_s^2 C_F T^2}{12 M_\eta}$$

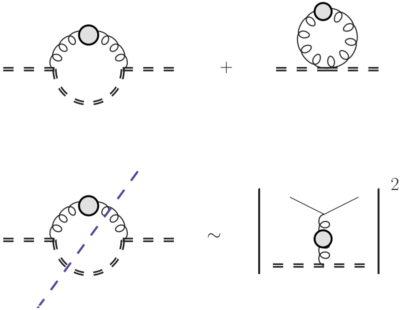


$$\frac{\mathrm{Im}\Pi_R}{2M_\eta} = 0$$

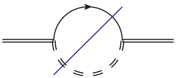
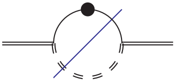
- $M_\eta + \Delta M$ and $\Delta M \ll \pi T \ll M_\eta$
- real part $\sim g_s^2 C_F \Delta M$ and imaginary part $\sim g_s^2 C_F |\Delta M| n_B(|\Delta M|) \sim g_s^2 C_F T$
- Resummed mass correction dominates over the unresummed when

$$g_s^2 \frac{T^2}{M} \lesssim g_s^2 \underbrace{g_s T}_{m_D} \Rightarrow \frac{T}{M} \lesssim g_s$$

Rates II



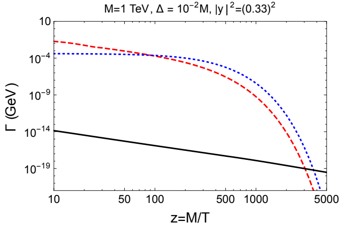
Rates III: equilibrium in the dark sector

- $1 \to 2$ and $2 \to 2$ scattering

$$\Gamma_{1\to 2} = \frac{|y|^2 N_c M}{4\pi} \left(\frac{\Delta}{M}\right)^2 n_F(\Delta)$$

$$\Gamma_{2\to 2} = \frac{N_c |y|^2}{8M} \int \frac{d^3p}{(2\pi)^3} \frac{\pi m_q^2}{p(p^2+m_q^2)} n_F\left(\Delta + \frac{p^2}{2M}\right)$$



Gluodissociation in quarkonium

- $M \gg 1/r \gg T \gg \Delta V$, start with pNRQCD
- difference between the octet and singlet potential

$$\Delta V = \frac{1}{r}\left(\frac{\alpha_s}{2N_c} + C_F\alpha_s\right) = \frac{N_c\alpha_s}{2r} \sim M\alpha_s^2$$

- the thermal width is

$$\Gamma = \frac{4}{3}C_F\alpha_s r^2(\Delta V)^3 n_B(\Delta V) \approx \frac{1}{3}N_c^2 C_F\alpha_s^3 T$$

- at small distances the two contributions are

$$\Gamma_{\text{LD}} \sim g_s^2 C_F T m_D^2 r^2 \,, \quad \Gamma_{\text{GD}} \sim g_s^2 C_F T(\Delta E)^2 r^2$$

RGEs for the models

IDM

- for $m_Z < \mu < M$, the couplings are evolved like in the Standard Model
- for $\mu > M$ (in the annihilation process, c_i) we use IDM P.M. Ferreira and D.R.T. Jones 0903.2856

Simplified model

- The only coupling that we need at a scale $\mu \ll M$ is the strong coupling
- we evaluate it at 2-loop level for $\mu \lesssim M$
- For $\mu > M$, the contribution of the coloured scalar is added and we switch over to 1-loop running
- in the thermal potential we have small and large distance scales:
  1. short distances: $\mu = e^{-\gamma_E}/r$, and no scalar in the running
  2. large distances: thermal couplings from EQCD at finite T