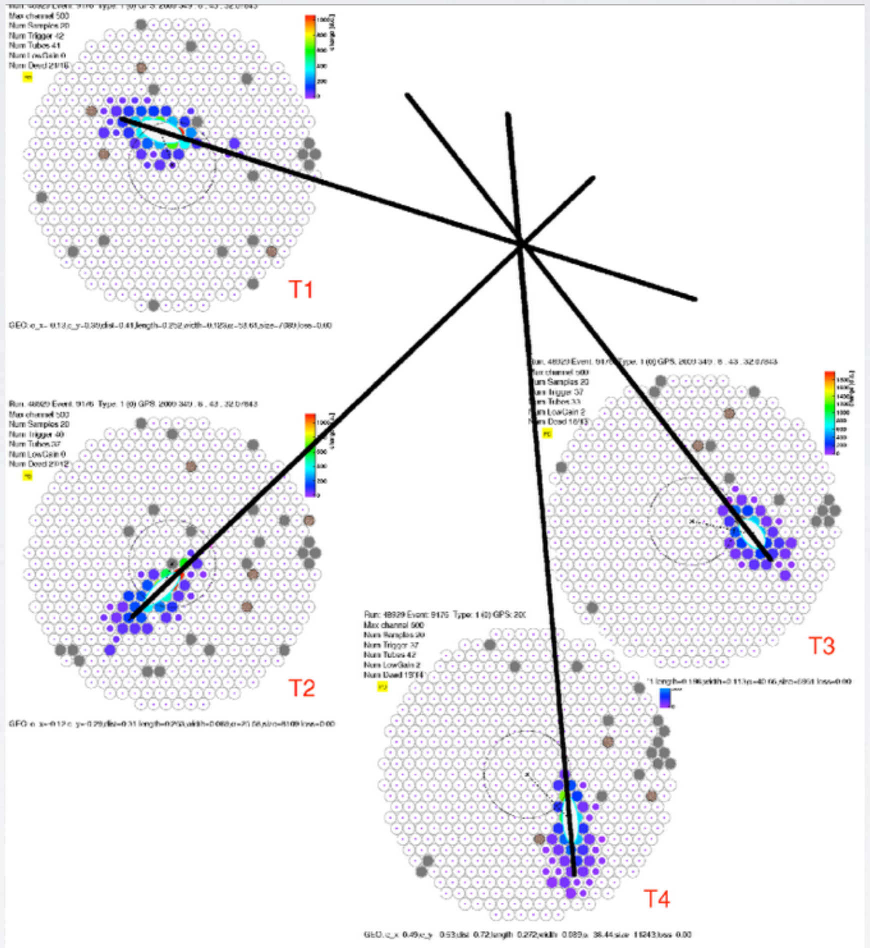
RESUMMED PHOTON SPECTRA FOR WIMP ANNIHILATION

Matthew Baumgart (Arizona State University)



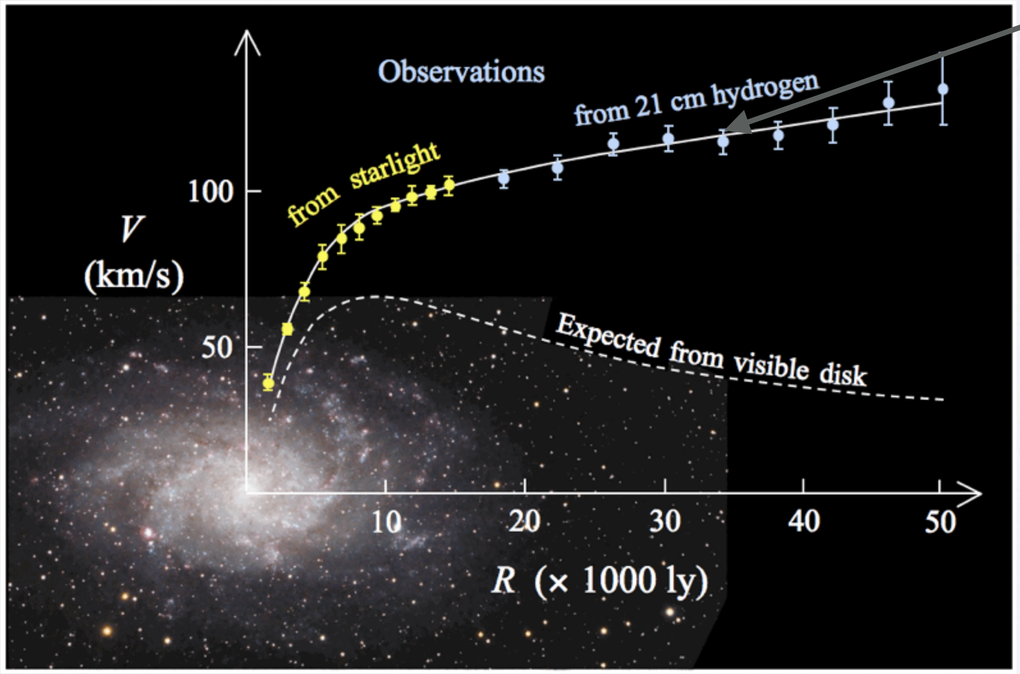
Multiscale Problems Using Effective Field Theories
University of Washington
5/17/2018

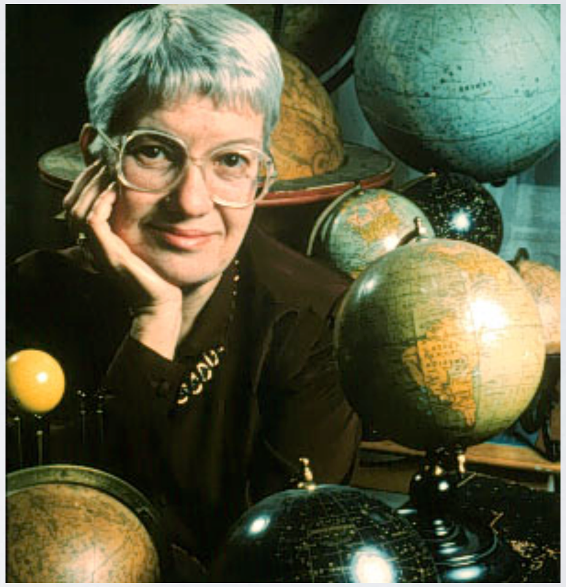
MB, I.Z. Rothstein, V. Vaidya: 1409.4415, PRL 114 (2015) 211301;
MB, I.Z. Rothstein, V. Vaidya: 1412.8698, JHEP 1504 (2015) 106;
MB and V. Vaidya: 1510.02470, JHEP 1603 (2016) 213;
MB, T. Cohen, I. Moult, N. Rodd, T. Slatyer, M. Solon, I, Stewart, V. Vaidya 1712.07656

WHY DARK MATTER?

Anomalies on 3 different astrophysical scales!

Galactic Rotation curves:
Stars move faster
than expected

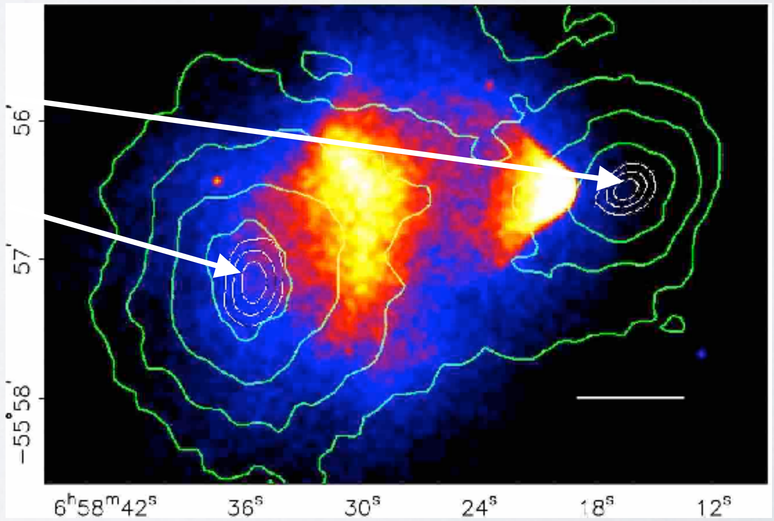




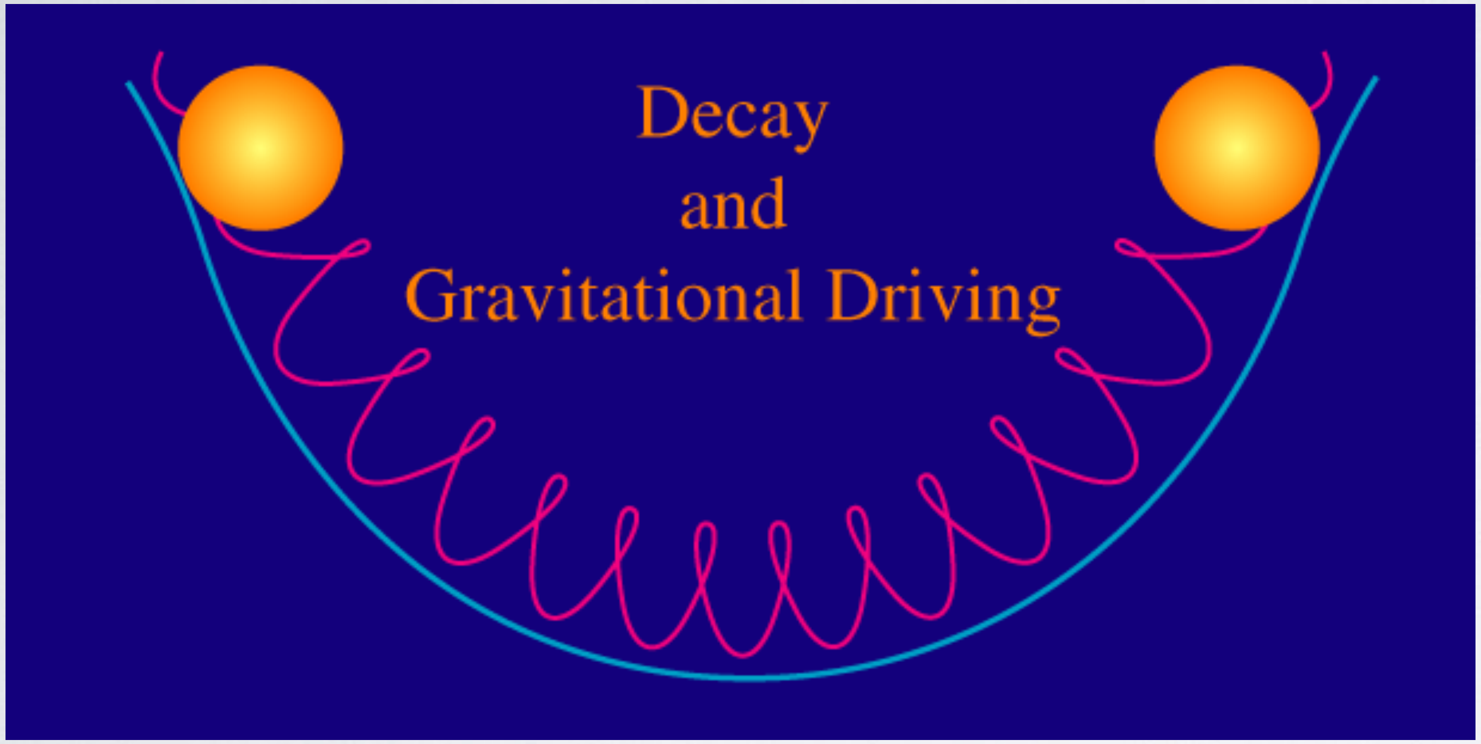
Vera Rubin 1928-2016
Established Rotation Curve anomaly

Colliding Clusters:
Gravitational wells
nowhere near visible peaks

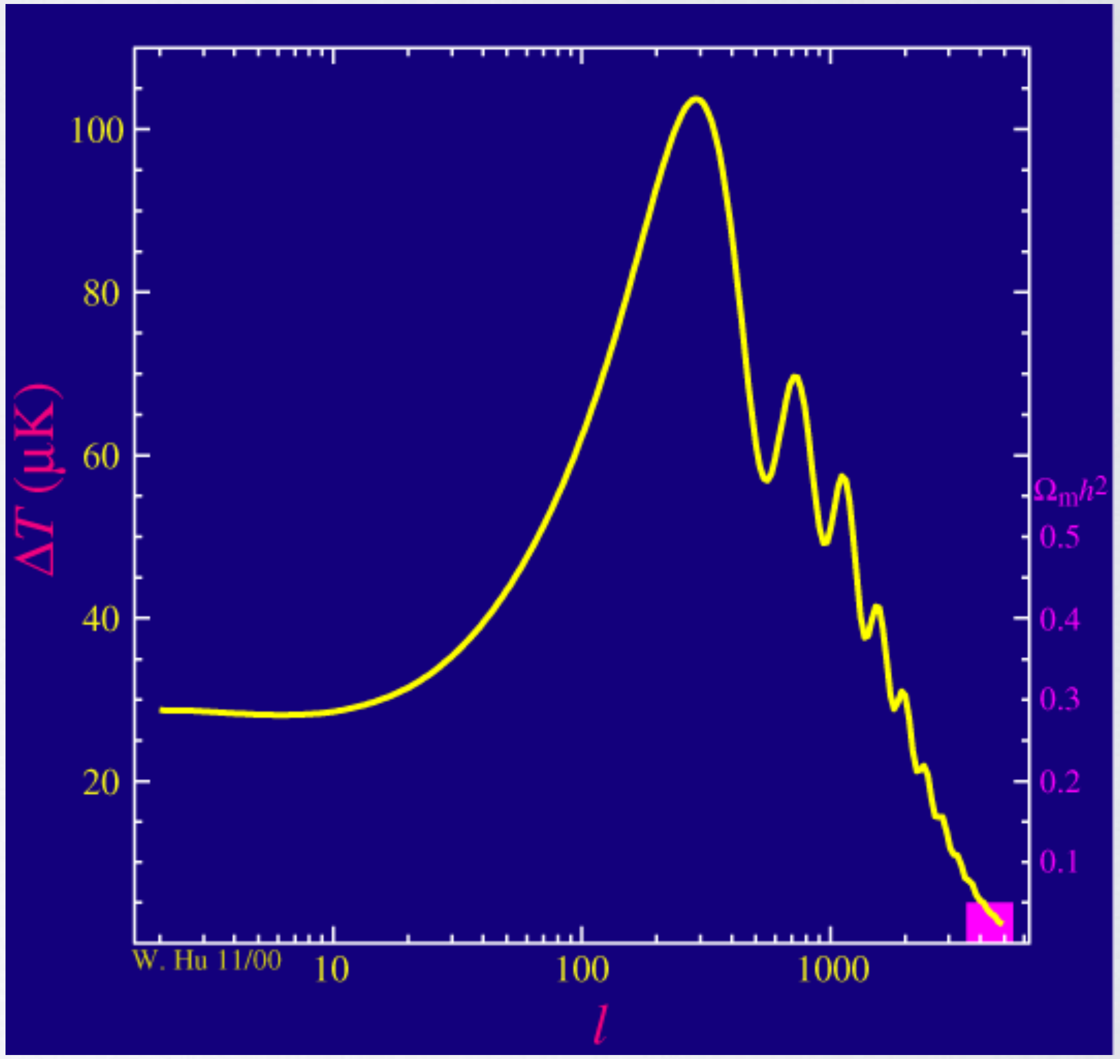
"Not modified gravity"



DARK MATTER ABUNDANCE



Cosmic Microwave Background:
Fluctuations measure Dark Matter as 27% of Universe's energy (Planck)



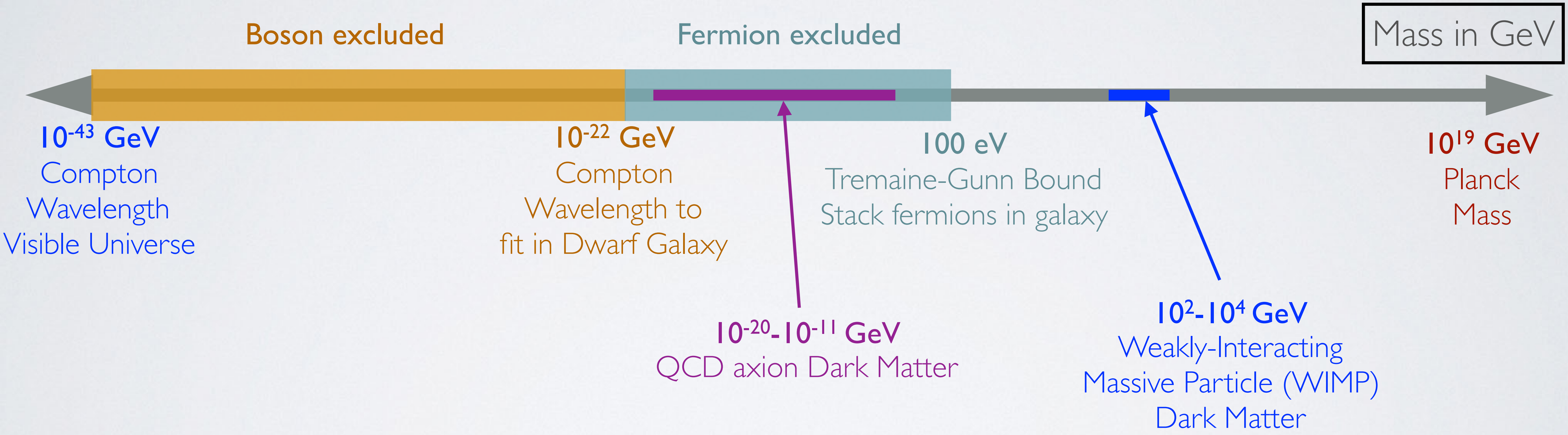
Animations from W. Hu

IT'S IN THERE SOMEWHERE

We know some "dark" particles! Neutrinos!

But they aren't dark matter

The Great Ruler of Particle Physics



10^{-43} GeV
Compton
Wavelength
Visible Universe

10^{-22} GeV
Compton
Wavelength to
fit in Dwarf Galaxy

100 eV
Tremaine-Gunn Bound
Stack fermions in galaxy

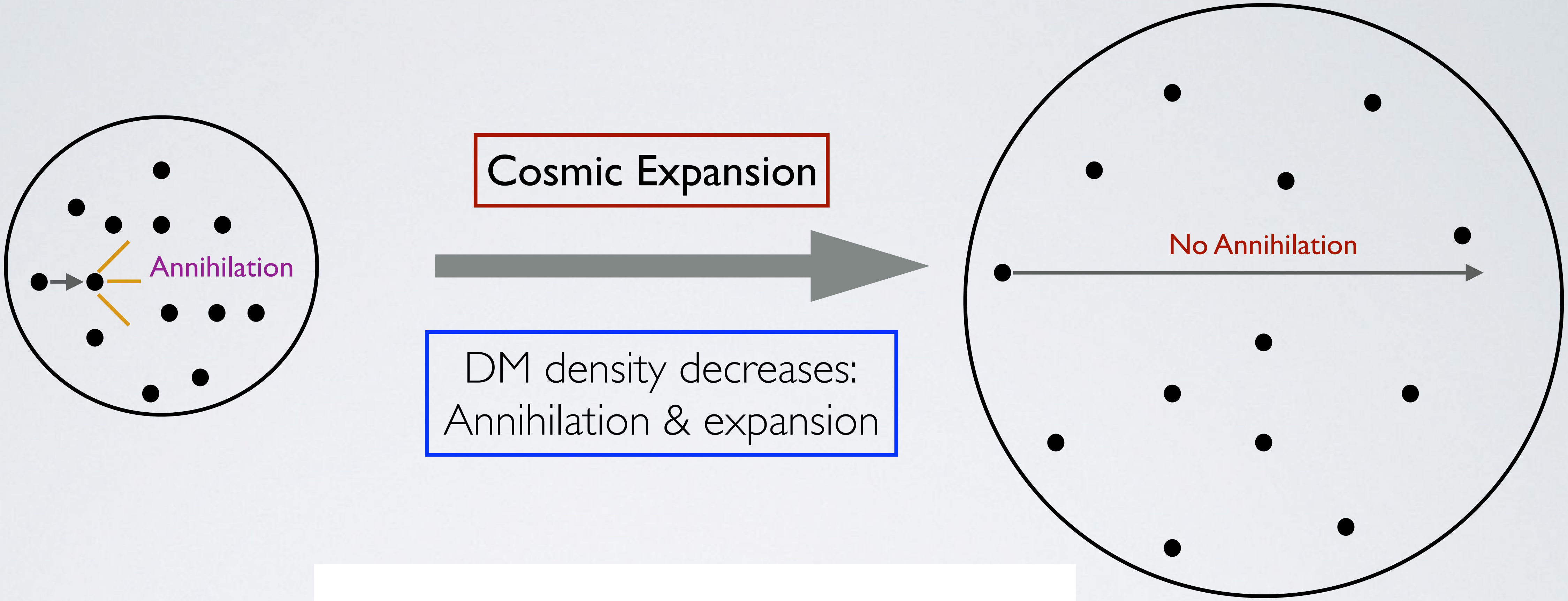
10^{19} GeV
Planck
Mass

10^{-20}-10^{-11} GeV
QCD axion Dark Matter

10^{2}-10^{4} GeV
Weakly-Interacting
Massive Particle (WIMP)
Dark Matter

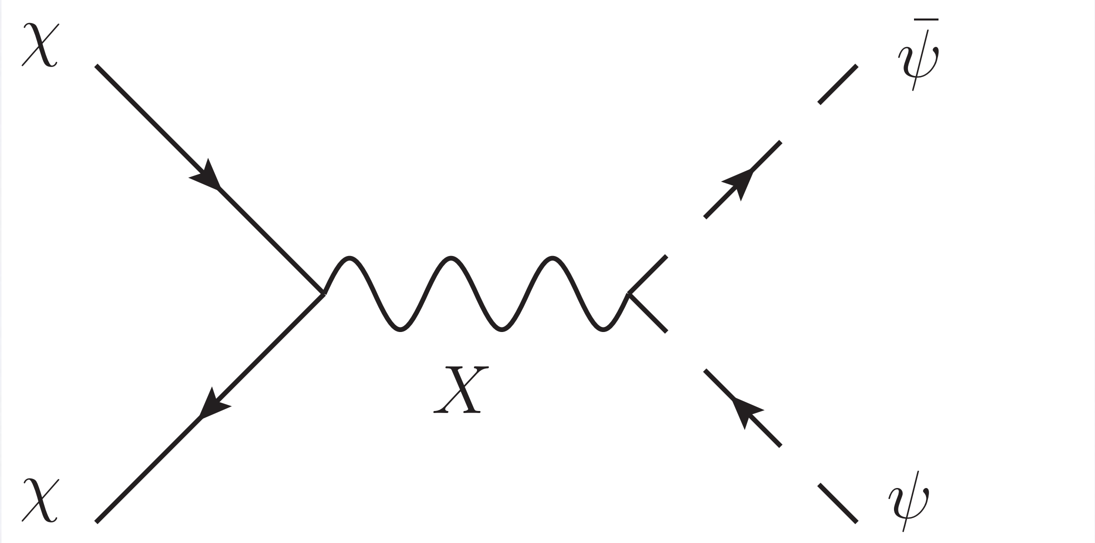
(After S. Rajendran)

WHY WIMPS?



Cosmic Expansion

DM density decreases:
Annihilation & expansion

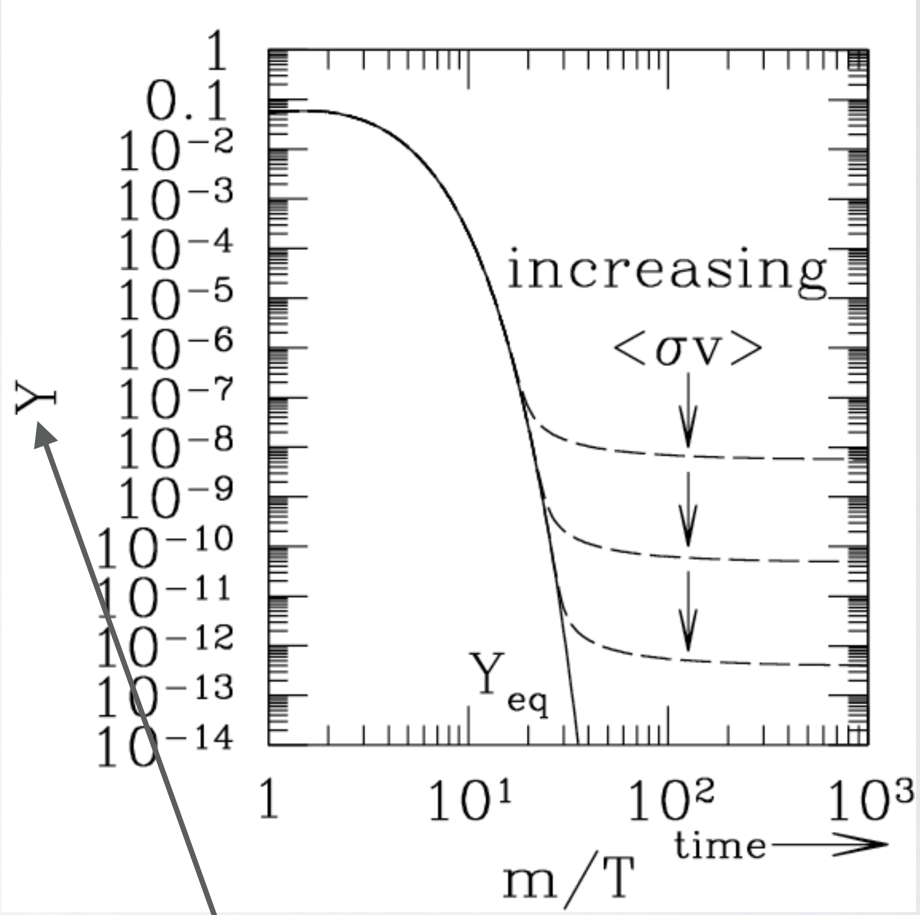


$$\langle \sigma v \rangle_{\text{annihilation}} \sim C\,\alpha^2/M_\chi^2$$

WIMP MIRACLE

$$\Omega_{\rm DM} \sim \frac{1}{10^3\langle\sigma v\rangle}\,\frac{1}{T_{\rm CMB}M_{Planck}} \sim \frac{1}{10^3\langle\sigma v\rangle}\,\frac{1}{\rm TeV^2}$$

$$\boxed{M_\chi \sim {\rm TeV}\left(10\sqrt{C}\alpha\right)\sqrt{\frac{\Omega_{\rm DM}}{0.27}}}$$

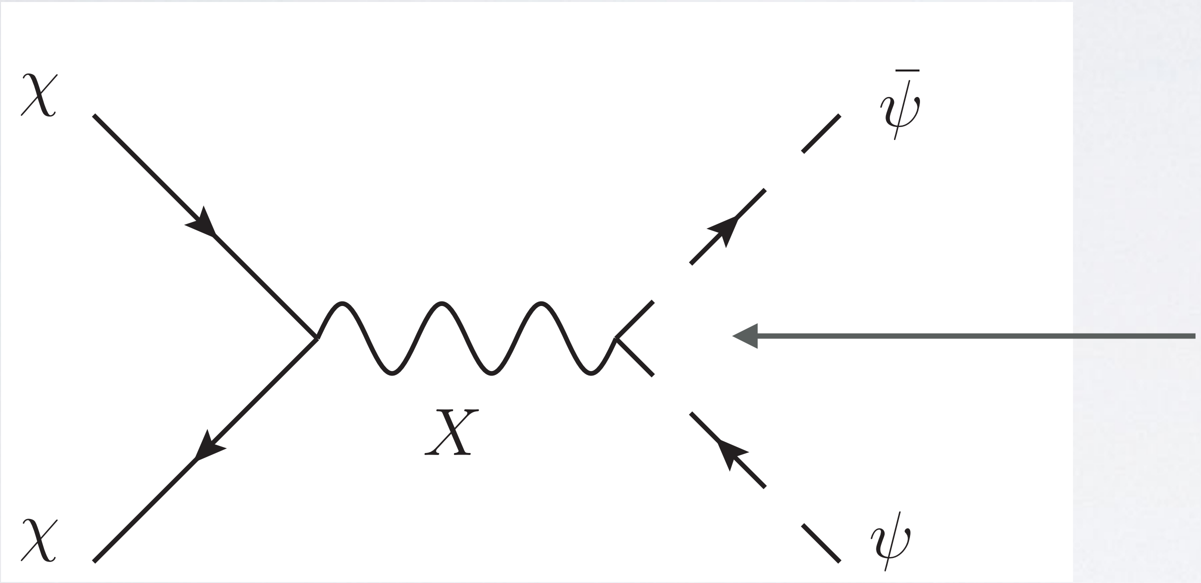


WIMP can be simple addition
to known particles & forces.
WHY?

DM density decreases:
Ω: Annihilation & expansion
Y: Annihilation

See Dimopoulos PLB 246(1990):347-52

STARTING SIMPLE W/ WIMPS



$\langle \sigma v \rangle_{\text{annihilation}} \sim C\, \alpha^2/M_X^2$

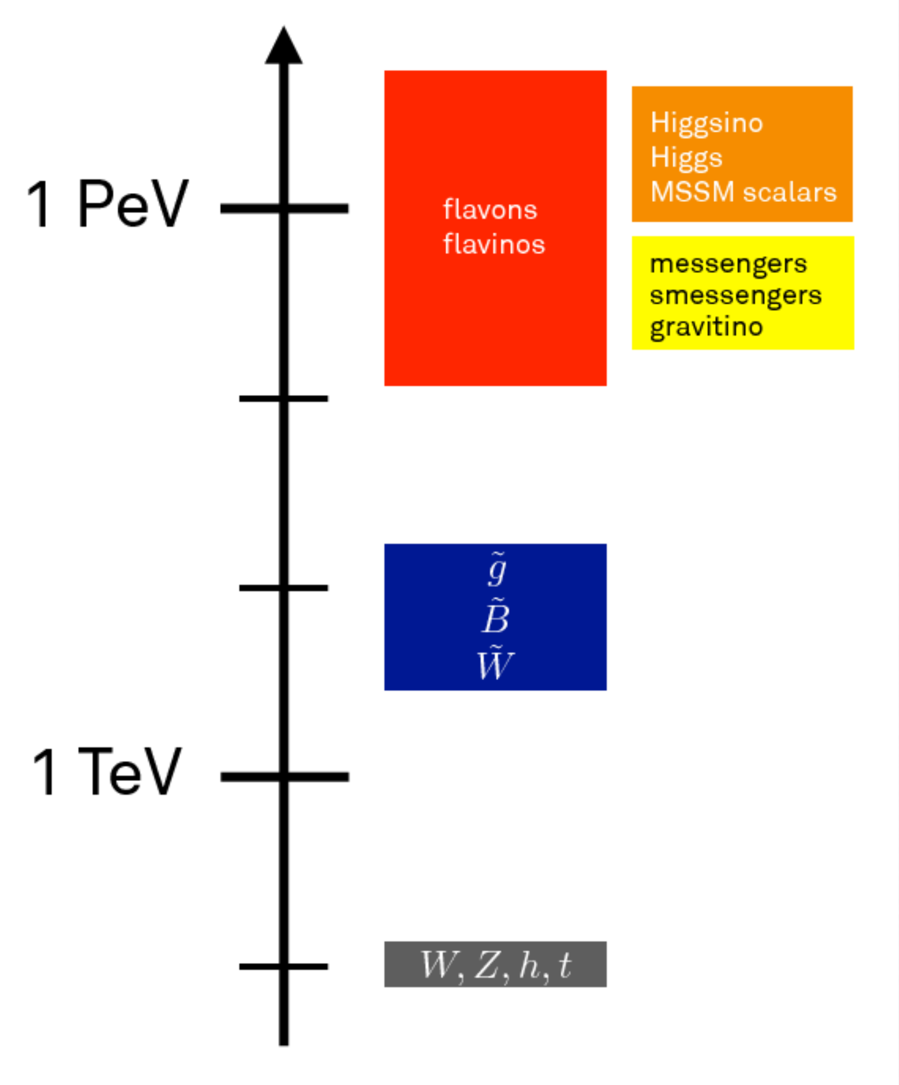
Maybe we already know
everything here **except χ?**
X: Z-boson, Higgs?
ψ: Elementary Fermion, Higgs?
α: α_{weak}?

RELIC DENSITY

$$\Omega_{DM} = 0.27$$

Measured Dark Matter Density

Top Down Candidates:
Supersymmetric Dark Matter

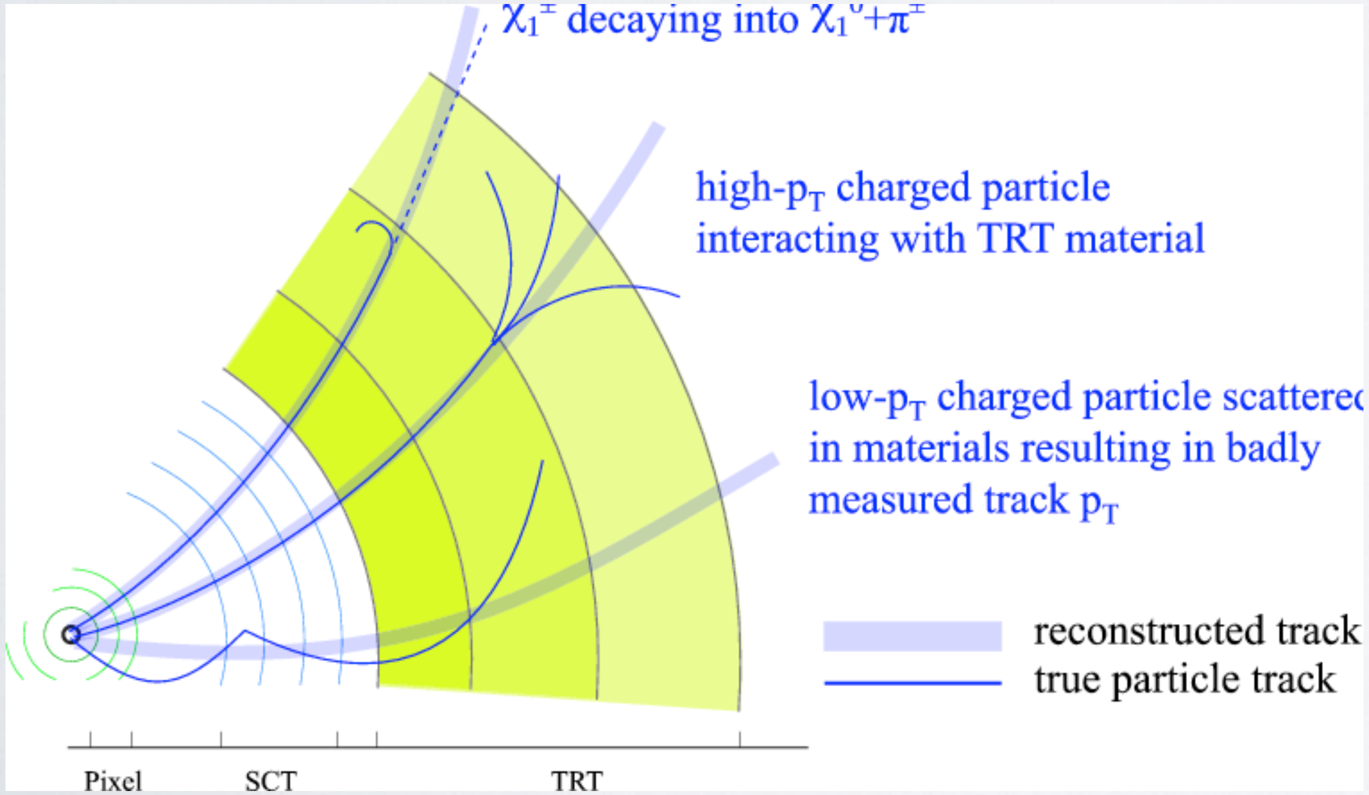


Simple Candidates!
Dark Matter ↔ Weak Scale:
Weak Triplet: "Wino"
Weak Doublet: "Higgsino"

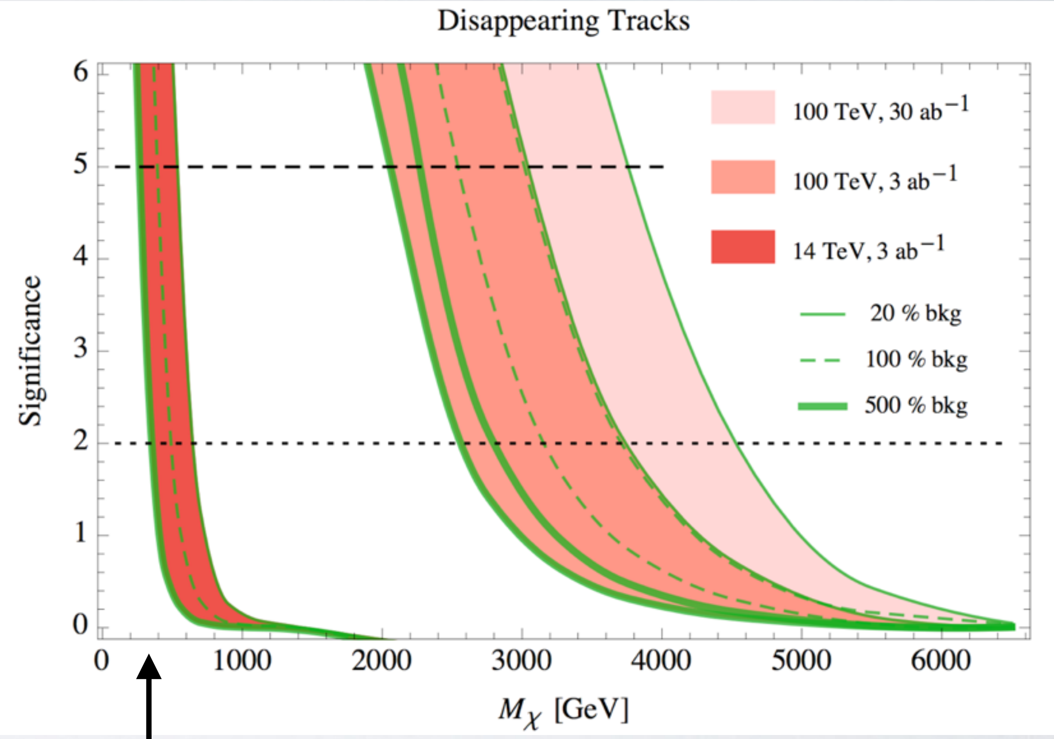
Correct Dark Matter Density fixes M_X:
Wino: 3 TeV
Higgsino: 1 TeV

DISCOVERY AT THE LHC? NO

Find Winos
by their charged partner's
disappearing track
$M_\chi > 270$ GeV



1202.4847 (ATLAS)



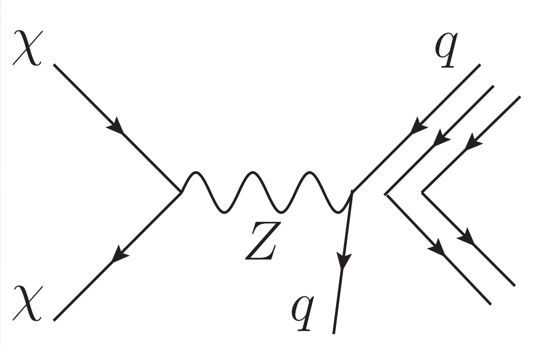
From Cirelli et al. 1407.7058

500 GeV,
LHC Wino reach

Higgsino reach
may not improve
over LEP: 110 GeV

Han et al.: 1401.1235

MINING FOR WINOS? NO



M. Goodman & E. Witten: PRD31 (1985)

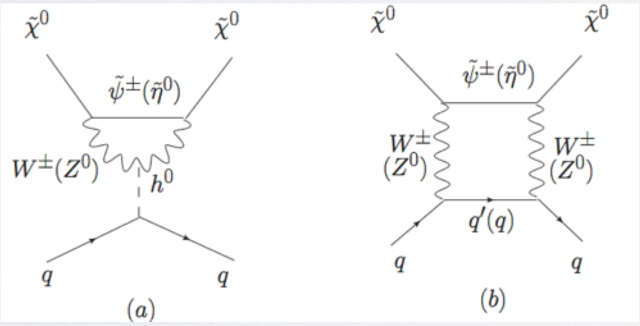
10^{-35}cm^2

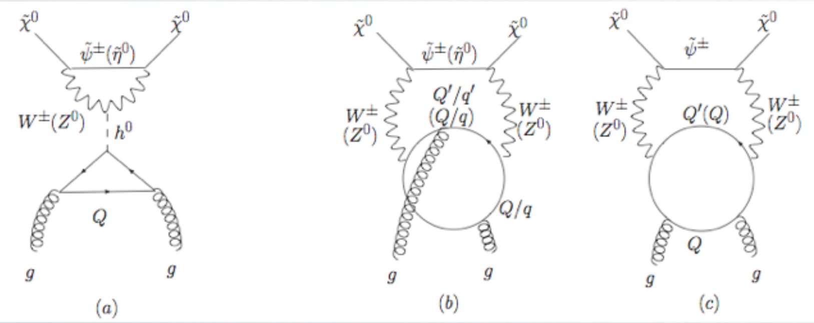
Z portal excluded

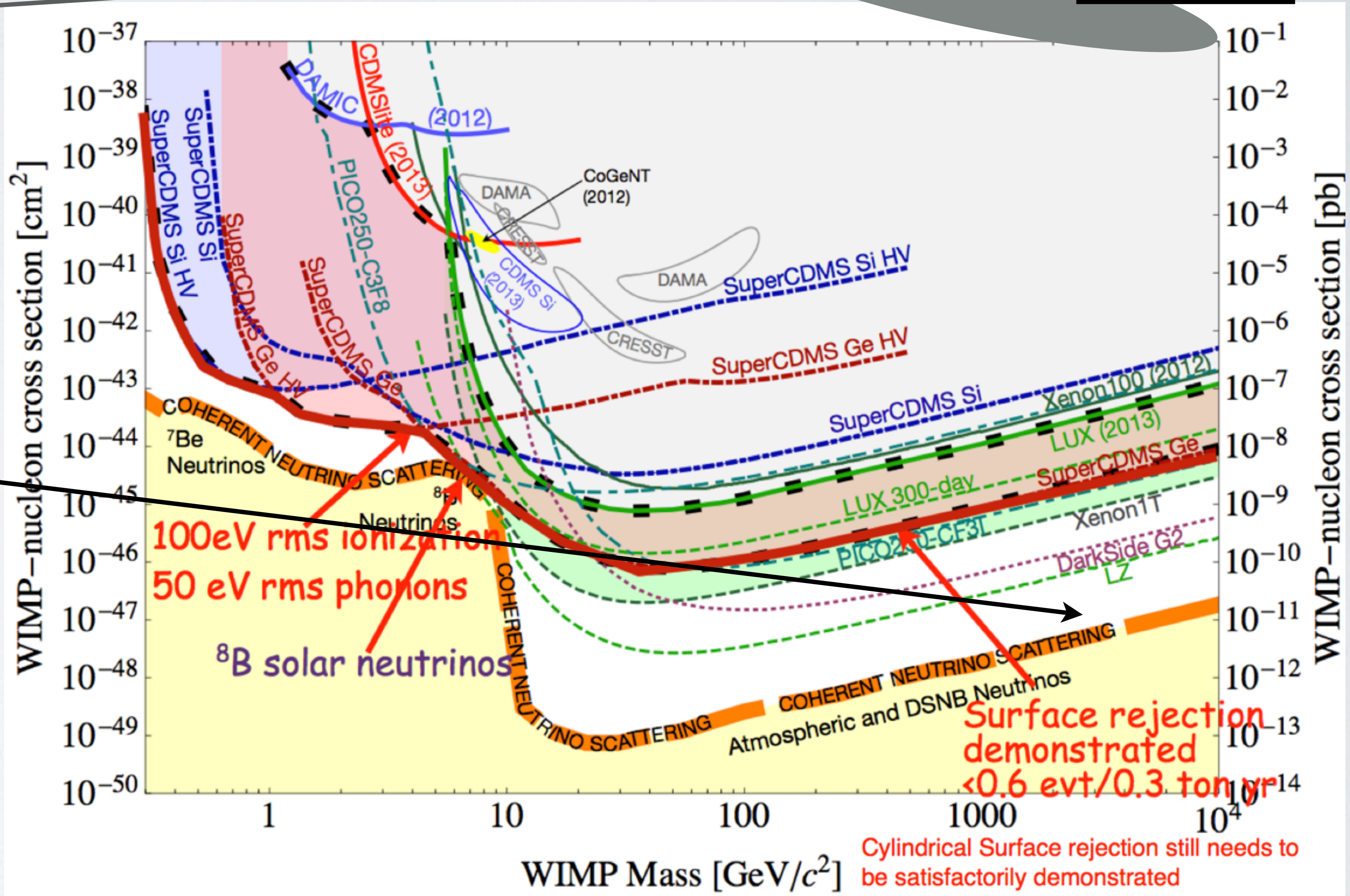
Example: Coupling Dark Matter to Nucleons through Z
Ruled Out

$\sigma_{\text{wino}} \sim 10^{-47}$ cm^2

Wino: Simple Model, but not Simple Calculation





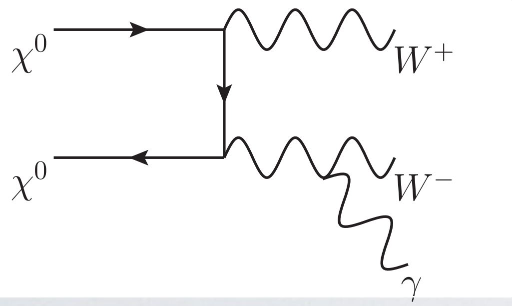


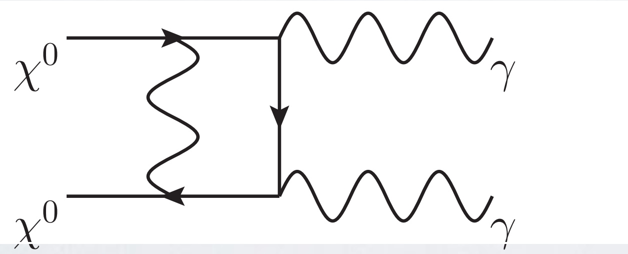
From Rick Gaitskell (LUX) talk

J. Hisano et al.: 1104.0228
R. Hill & M. Solon: 1309.4092

Higgsino limits even weaker

ECHO OF THE WIMP MIRACLE



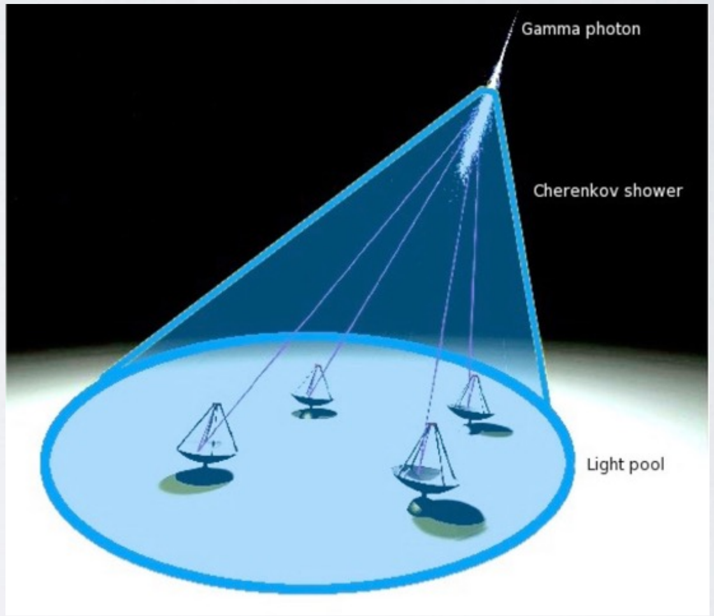


Indirect Detection:

Photons from Dark Matter Annihilation

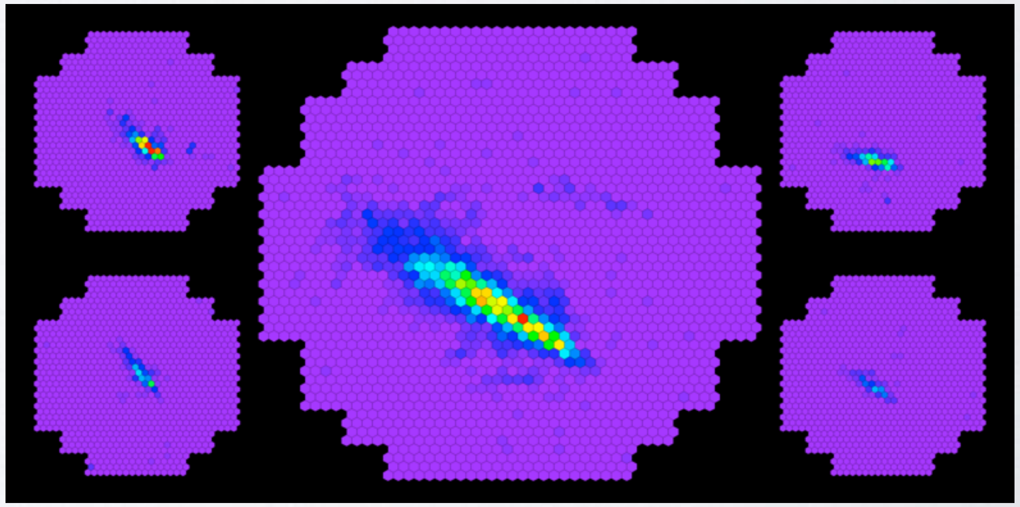
HESS/VERITAS can probe
Dark Matter Masses
up to 20 TeV

Successor **CTA will start in 2023**,
will test **up to 100 TeV**



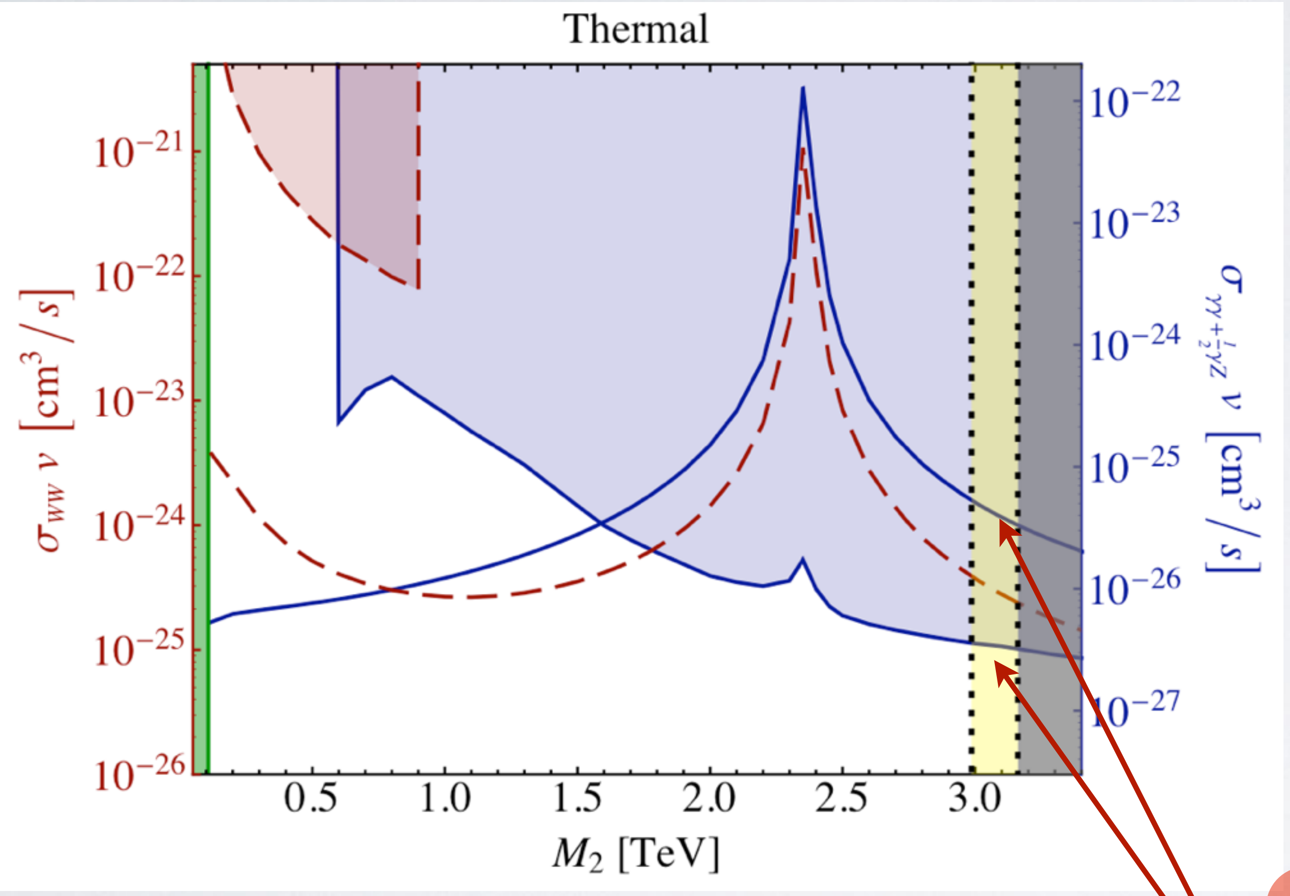
O(TeV) γ leads to $O(10^4 m)$ light pool on ground

Schematic of air shower observed by Cherenkov Telescope (spie.org)



HESS event

WINO SHOT DEAD BY HESS?



Ruled out by 16x?

From 1307.4082:

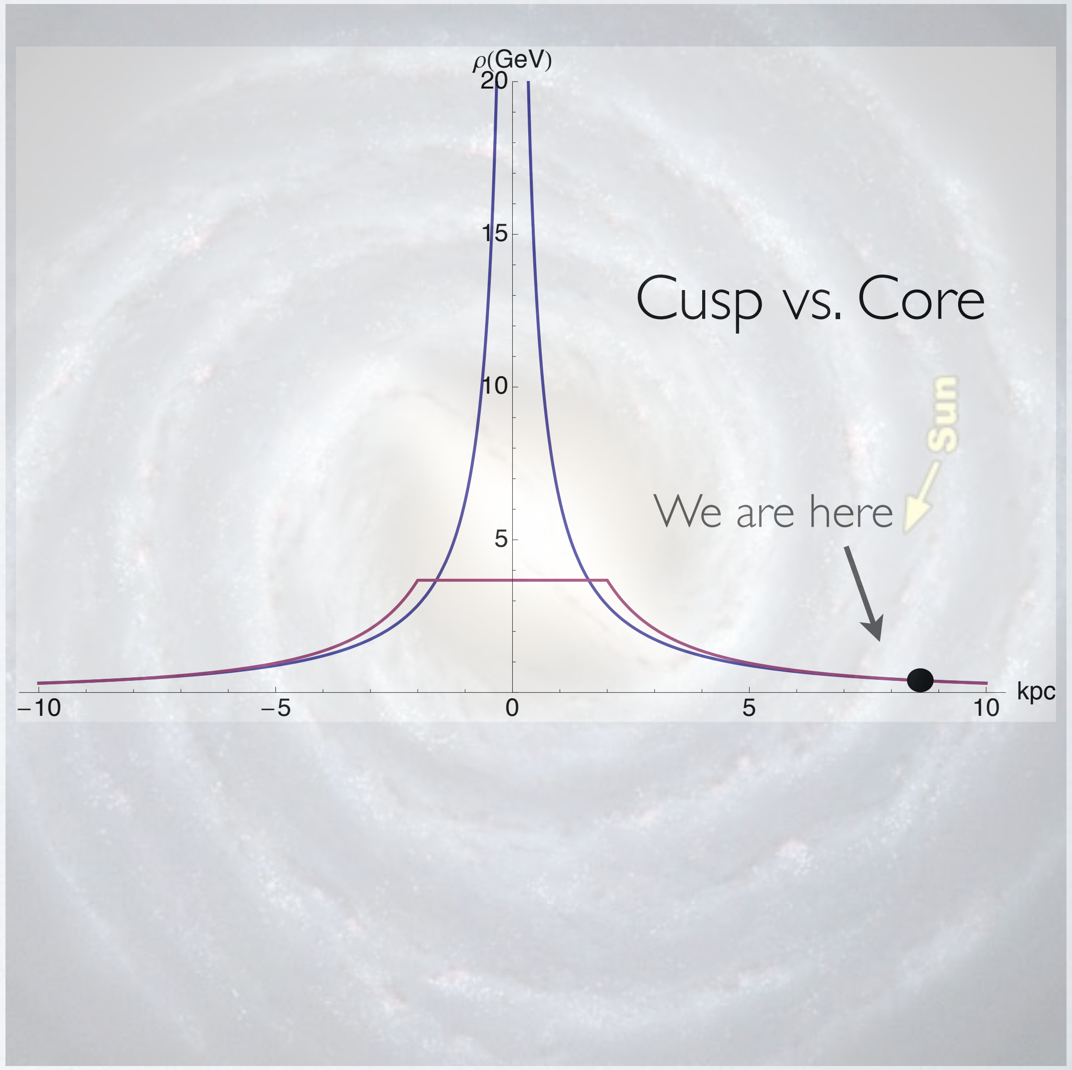
Blue line: Wino annihilation rate

Blue shade: Exclusion from HESS

Yellow shade: All DM is thermal wino

The Loophole

Navarro-Frank-White
(cusped) halo profile assumed



Cusp vs. Core

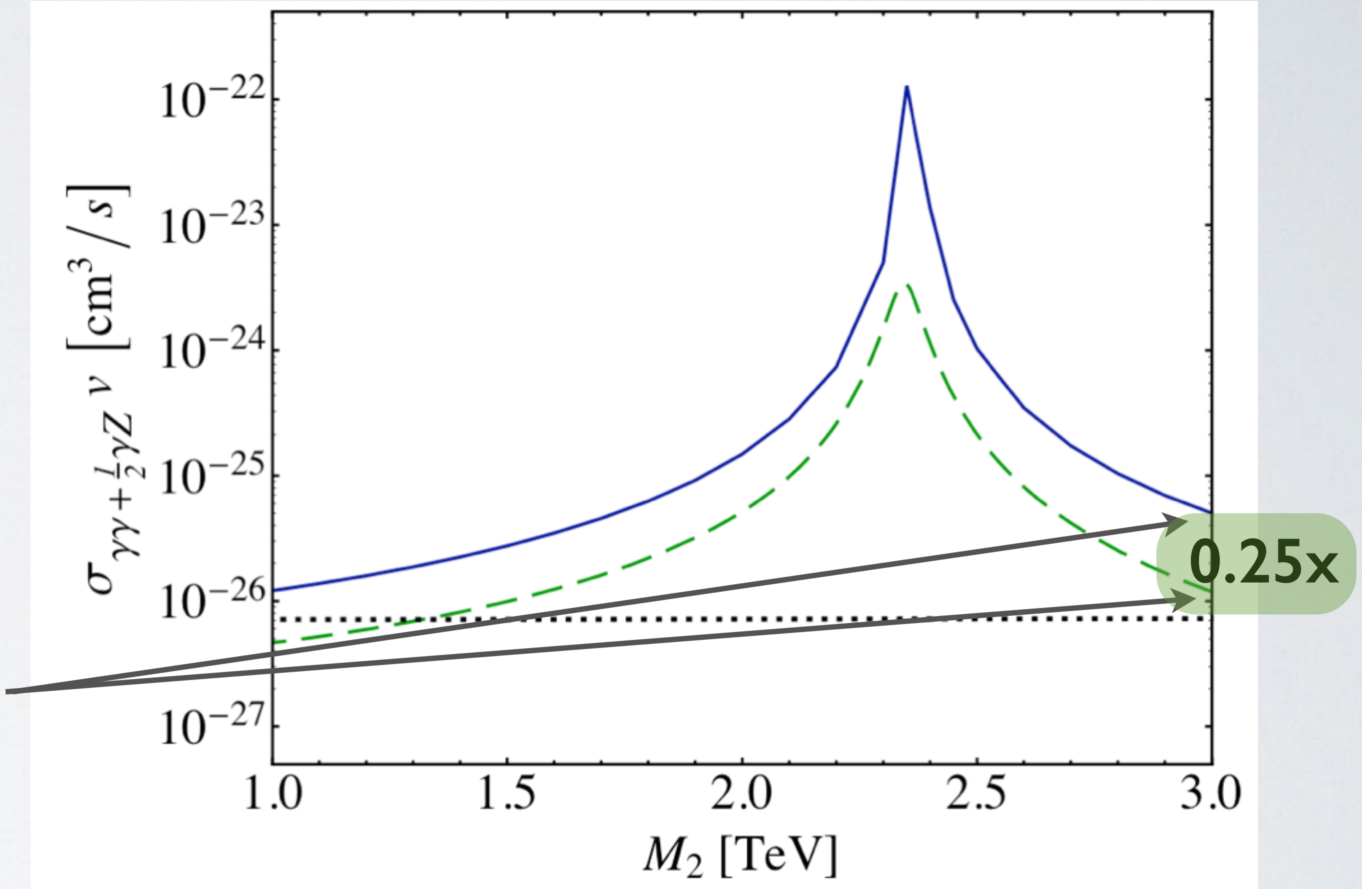
We are here

*1307.4082: Cohen, Lisanti, Pierce, and Slatyer;
see also "In Wino Veritas", 1307.4400: Fan & Reece

WHAT COULD SAVE THE WINO?

Halo Model	Quantum Corrections
Flatten distribution in galactic center (core)	Claim in literature of **75% reduction** from first quantum corrections
Core needed: **1.5 kpc**	Core needed: **0.5 kpc**

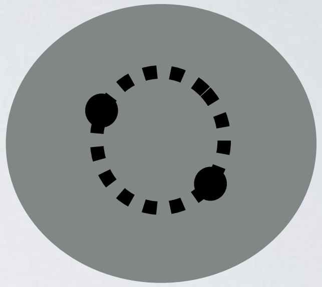
Simulations with baryons show **cusped profiles down to 1 kpc** (1208.4844, 1305.5360, 1306.0898)

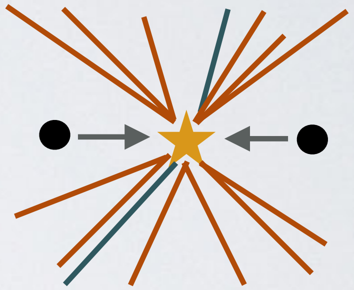


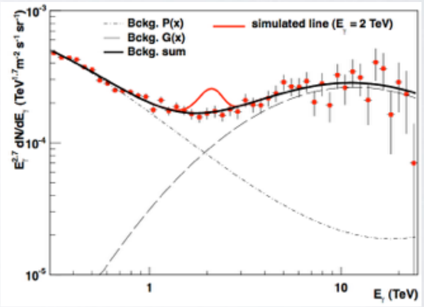
From Cohen et al.1307.4082

O(1-10) Factor at stake, need state of the art calculation to determine

- **3 separate threats to perturbation theory!**
  - $M_\chi/m_W >> 1$ → Long range force
  - $M_\chi/m_W >> 1$ → Electroweak shower
  - Log(1-zcut) → Phase space restriction
- Proliferation of scales → **Effective Field Theory**







LONG-RANGE FORCES

- Sommerfeld Enhancement, **generic for WIMP Dark Matter** (v~10^{-3-6})

- Quantum-Mechanically **Potential drags wavefunction** to peak at origin

$$\langle \sigma v \rangle = |\psi(0)|^2\, \Gamma_{\text{pert.}}$$

Wavefunction at the origin

Classical Analogy

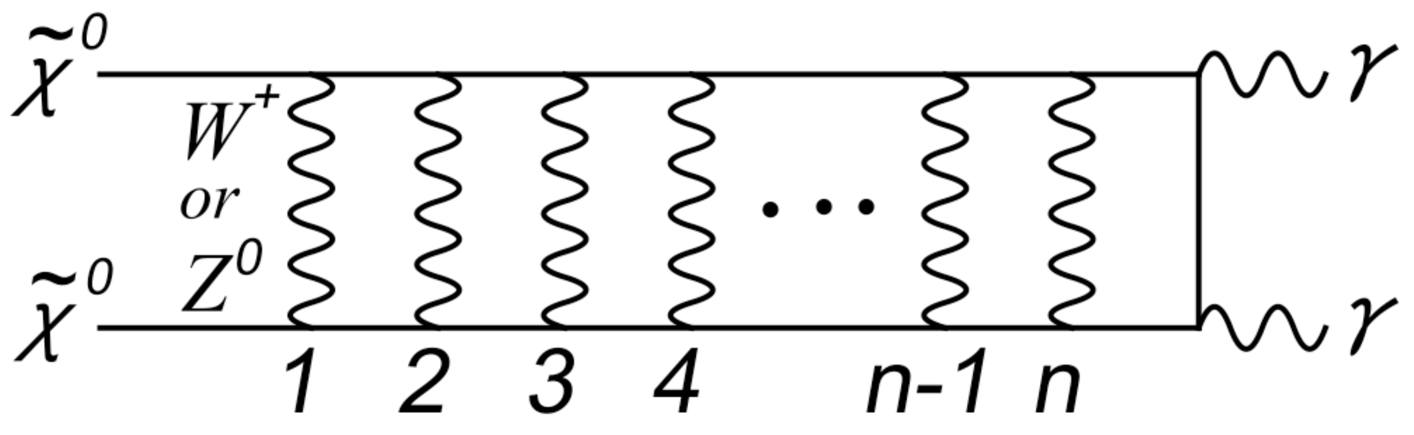
In absence of gravitation, capture radius is geometric, R

Turning on gravity, cross section grows for slower projectile:

$$b_{\text{capture}} = R\sqrt{1 + \frac{2GM}{v^2 R}}$$

$$b_{\text{capture}} \sim \frac{1}{v}$$

SOMMERFELD ENHANCEMENT



$$\mathcal{A}_n \simeq \alpha\left(\frac{\alpha_W M_\chi}{m_W}\right)^n$$

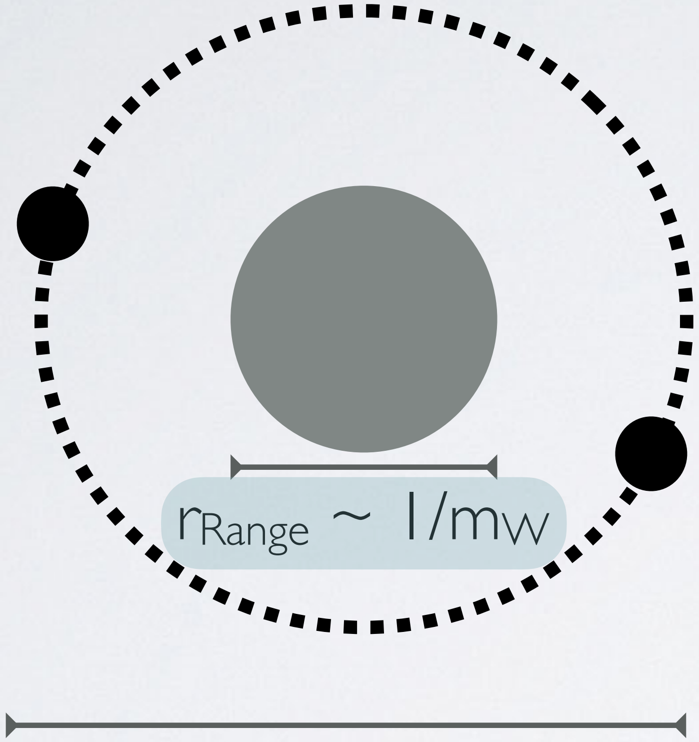
$\alpha_W M_\chi/m_W$ or $\alpha_W/v>1$
→ Sum to all orders!

$$|\psi(0)|^2 \propto \min\left[\frac{\alpha_W}{v}, \frac{\alpha_W M_\chi}{m_W}\right]$$

Parametrically enhanced wavefunction

Reduce Relativistic WIMP QFT to Quantum Mechanics + Potential
[MB, Rothstein, I., Vaidya, V.: 1412.8698]

SOMMERFELD RESONANCES

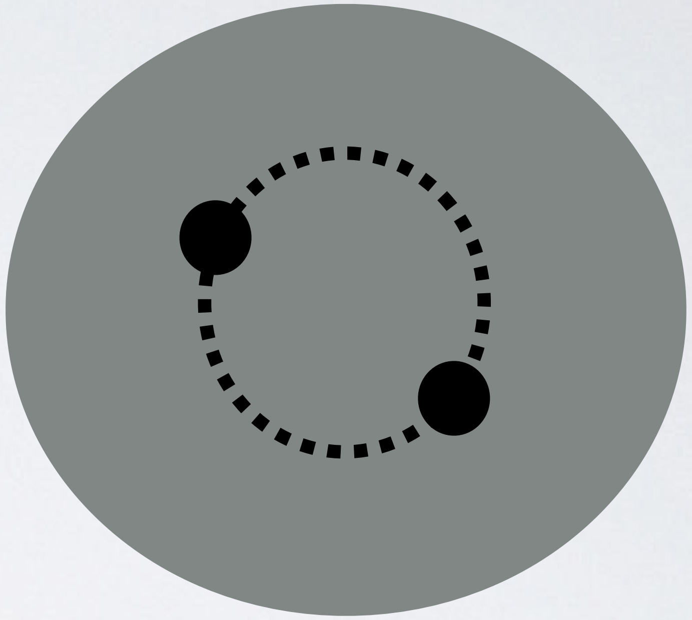


$r_{\text{Range}} \sim 1/m_W$

$r_{\text{Bohr}} \sim 1/\alpha M_\chi$

$r_{\text{Bohr}} >> r_{\text{Range}}$

No bound state



$r_{\text{Range}} >> r_{\text{Bohr}}$

Bound state forms

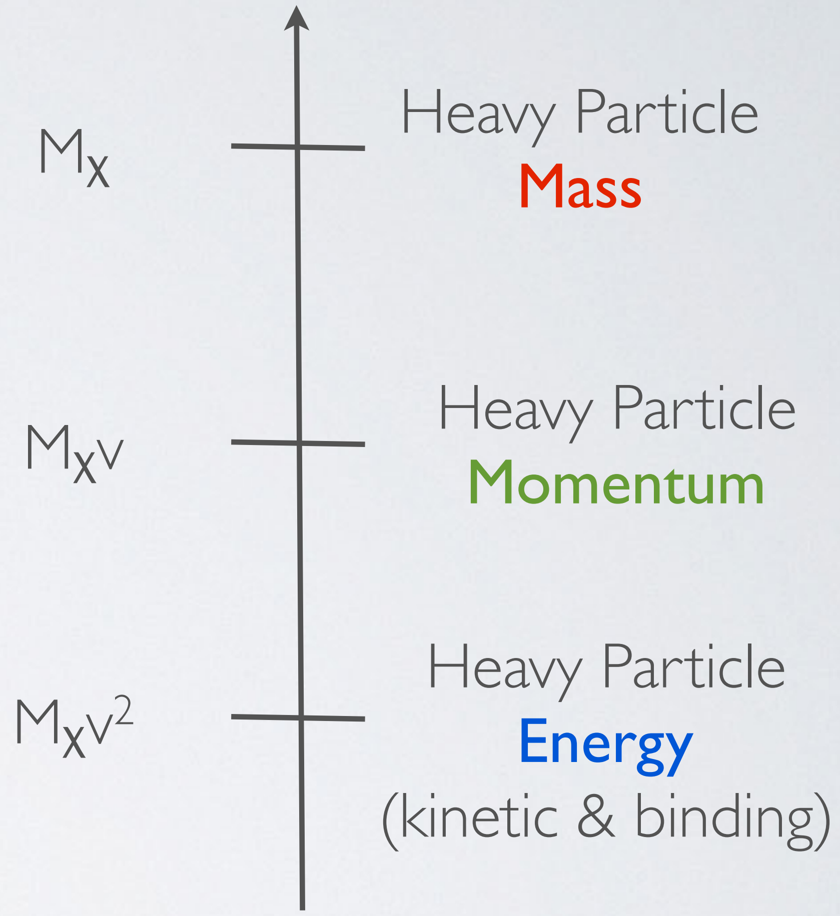
$m_W = \alpha_W M_\chi$ @ $M_\chi = 2.4$ TeV

Transition from short to long-range force leads to resonance

NR"QCD" FOR WIMPS

Multimodal Effective Field Theory

WIMP χ	$(E,p)\sim(M_\chi v^2, M_\chi v)$
Potential $A^\mu \to V(r)$	$(E,p)\sim(M_\chi v^2, M_\chi v)$
Ultrasoft $S\sim \mathrm{Exp}[i\int v.A_{us}]$	$(E,p)\sim(M_\chi v^2, M_\chi v^2)$
Soft A_s^μ	$(E,p)\sim(M_\chi v, M_\chi v)$



M_χ Heavy Particle **Mass**

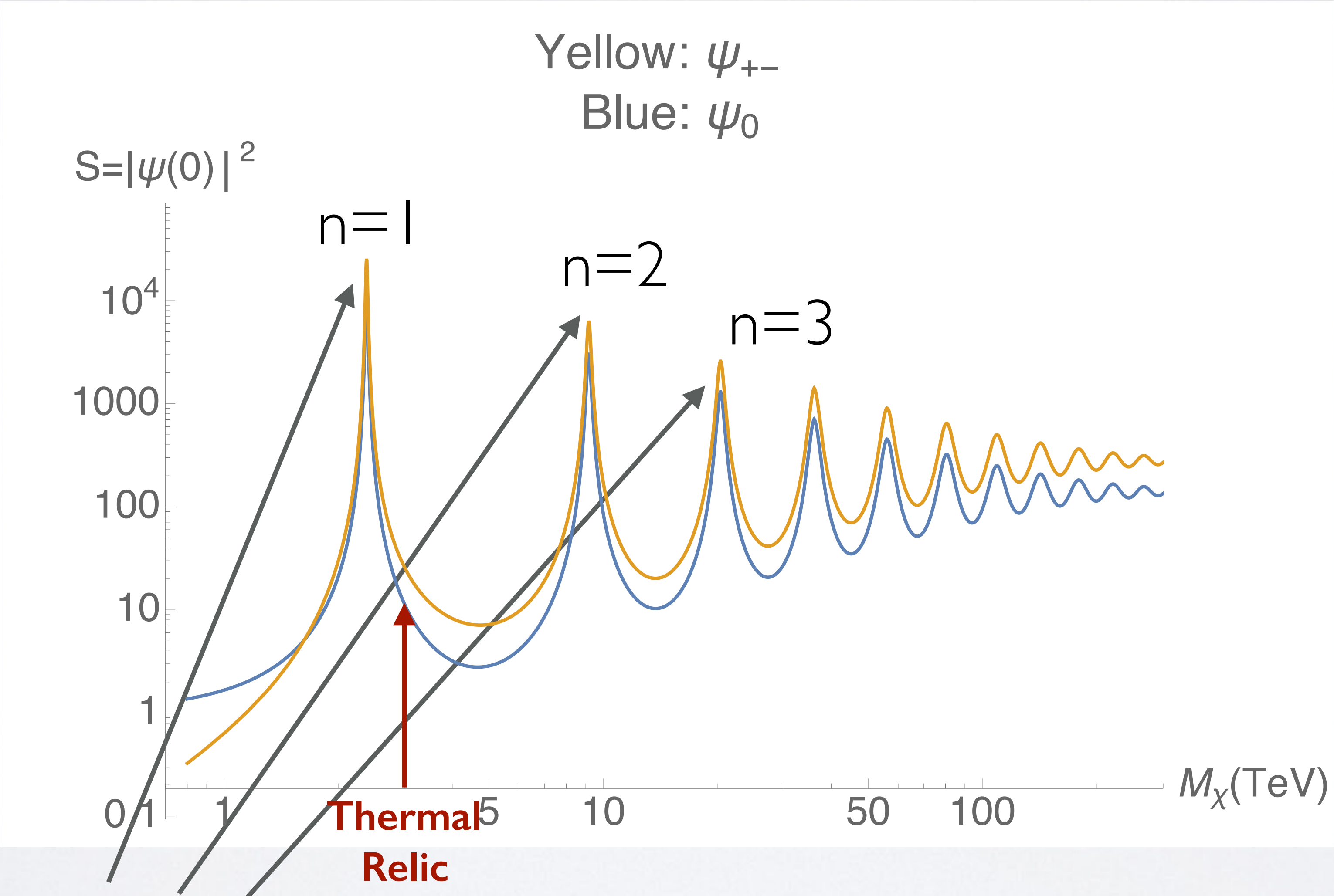
$M_\chi v$ Heavy Particle **Momentum**

$M_\chi v^2$ Heavy Particle **Energy** (kinetic & binding)

Hierarchy of scales for nonrelativistic particles

*Classic treatment of NRQCD in Bodwin, Braaten, LePage hep-ph/9407339

NR COMPUTATION



Zero-energy bound states → Peaks

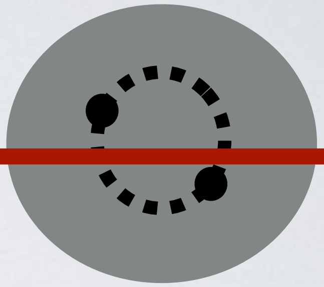
$\alpha_W M_\chi = n^2 m_W$

$$\left\langle 0 \right| \chi_v^{3\,T}\, i\sigma_2\, \chi_v^3 \left| (\chi^0\chi^0)_S \right\rangle = 4\sqrt{2}\, M_\chi\, s_{00}\,;$$

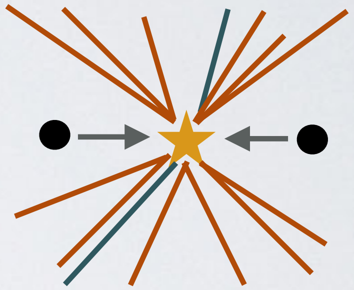
$$\left\langle 0 \right| \chi_v^{+\,T}\, i\sigma_2\, \chi_v^- \left| (\chi^0\chi^0)_S \right\rangle = 4\, M_\chi\, s_{0\pm}$$

Wavefunction at the origin

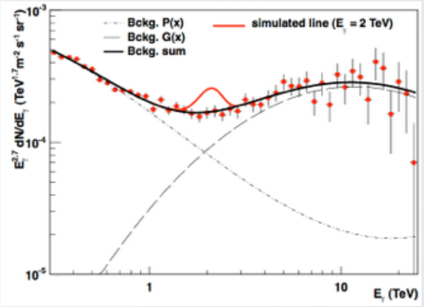
- **3 separate threats to perturbation theory!**
  - ~~$M_\chi/m_W >> 1$ → Long range force~~



  - $M_\chi/m_W >> 1$ → Electroweak shower

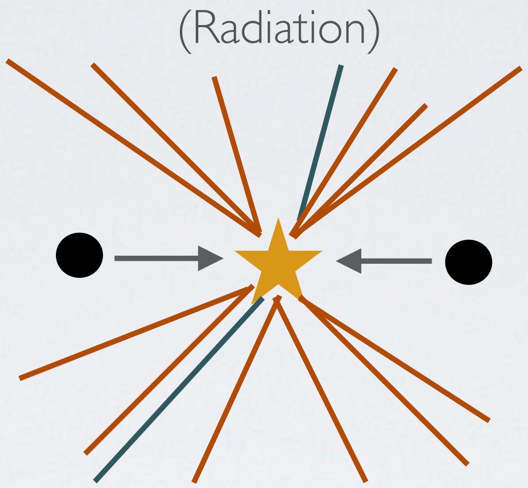


  - Log(1-zcut) → Phase space restriction



- Proliferation of scales → **Effective Field Theory**

HUGE ACCELERATION → CLASSICAL RADIATION



Charged particles in annihilation process radiate (γ, W, Z) from acceleration

Perturbative factor picks up kinematic enhancements "Sudakov double log"

$$\sigma v = \sigma v_0 \left| \exp\left[-\frac{\alpha}{2\pi} \log(E_{\mathrm{high}}/E_{\mathrm{low}}) \log(E_{\mathrm{high}}/E_{\mathrm{collinear}}) \right] \right|^2$$

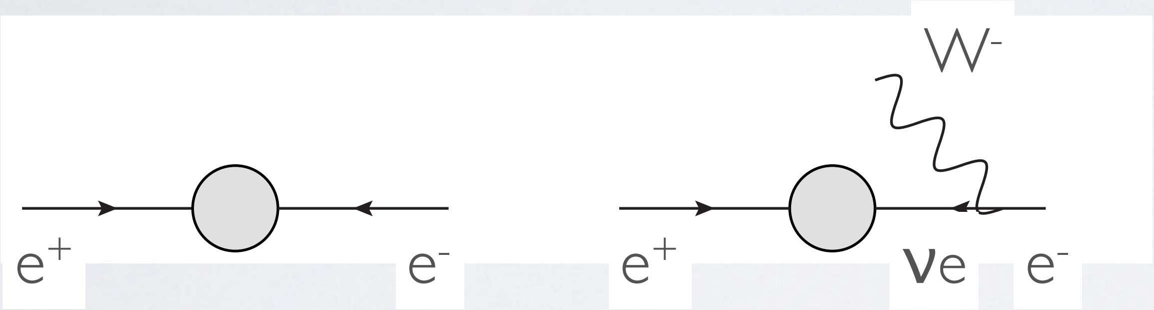
Above rate **produces classical spectrum**, but **hard to see in quantum perturbation theory**

$$\boxed{\frac{\alpha_W}{\pi} \log(M_{\mathrm{wino}}^2/m_W^2)^2 \approx 0.6}$$

Double log
Large correction!

BLOCH-NORDSIECK THEOREM VIOLATION*

- Electroweak physics has **infrared divergences**, even in fully inclusive observables



$\sigma_{e+e-} \neq \sigma_{e+\nu}$, virtual corrections only cancel emission upon color averaging

$$S_{12}^{a'b'ab} = \left\langle 0 \middle| \left(Y_n^{3k} Y_{\bar{n}}^{dk}\right)^\dagger (x) \delta(\mathcal{M} - \hat{\mathcal{M}}_s) \left(Y_n^{3g} Y_{\bar{n}}^{df} Y_v^{ag} Y_v^{bf}\right) (0) \middle| 0 \right\rangle \delta^{a'b'} \,,$$

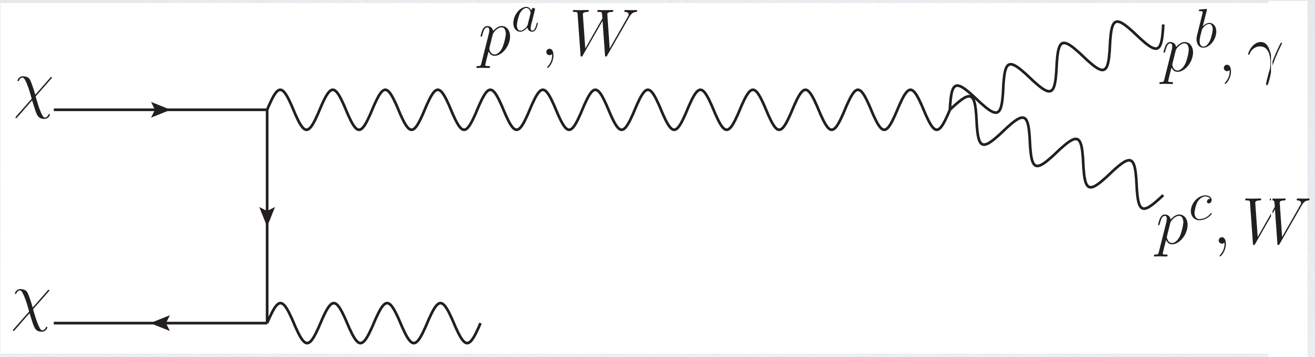
$$\to \delta^{a'b'} \delta(m_X^2) \langle 0| \left(Y_{\bar{n}}^\dagger\right)^{e3} Y_v^{ae} Y_v^{bf} Y_{\bar{n}}^{3f} |0\rangle$$

- Wilson lines **collapse from measurement inclusivity**, but identifying photon in final state **precludes sum over degenerate states.**

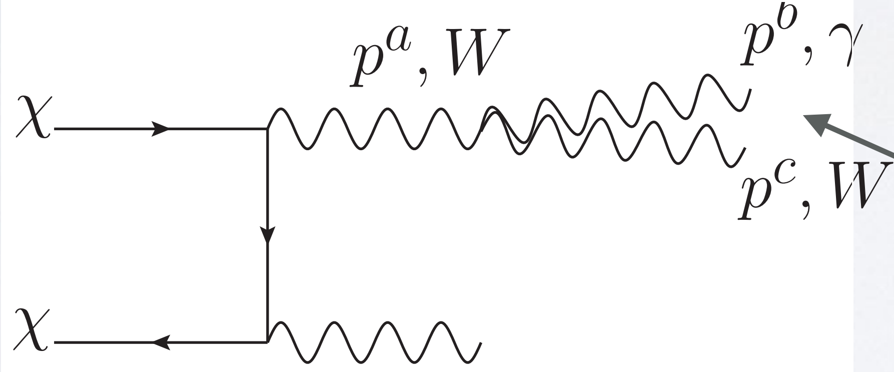
*hep-ph/0001142: Ciafaloni, Ciafaloni, & Comelli; hep-ph/0103315: Ciafaloni

SOFT/COLLINEAR ENHANCEMENT

Soft radiation: Time-scales much longer than annihilation



Collinear Radiation: Narrow splitting of one particle into 2



$$\propto \frac{1}{p_A^2} = \frac{1}{2E_bE_c(1-\cos\theta)}$$

θ

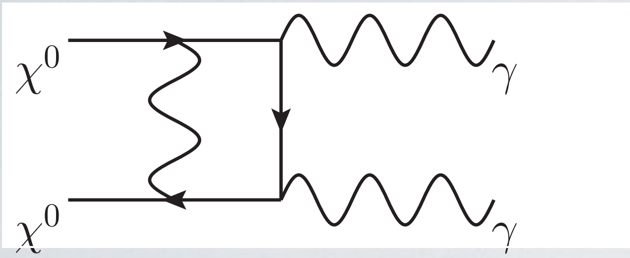
Keep modes with **kinematic enhancement** (soft, collinear)

SCET for
Dark Matter annihilation

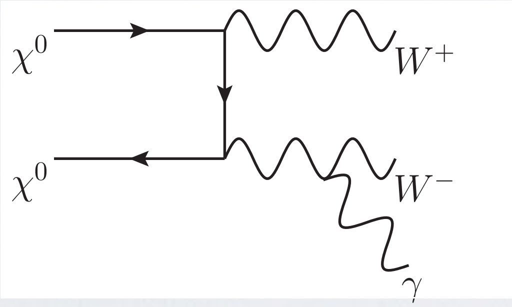
[MB, Rothstein, I., Vaidya, V.: 1409.4415]

*Originally developed for study of QCD
hep-ph/0005275: Bauer, Fleming, Luke
hep-ph/0011336: Bauer et al.

SEMI-INCLUSIVE ANNIHILATION

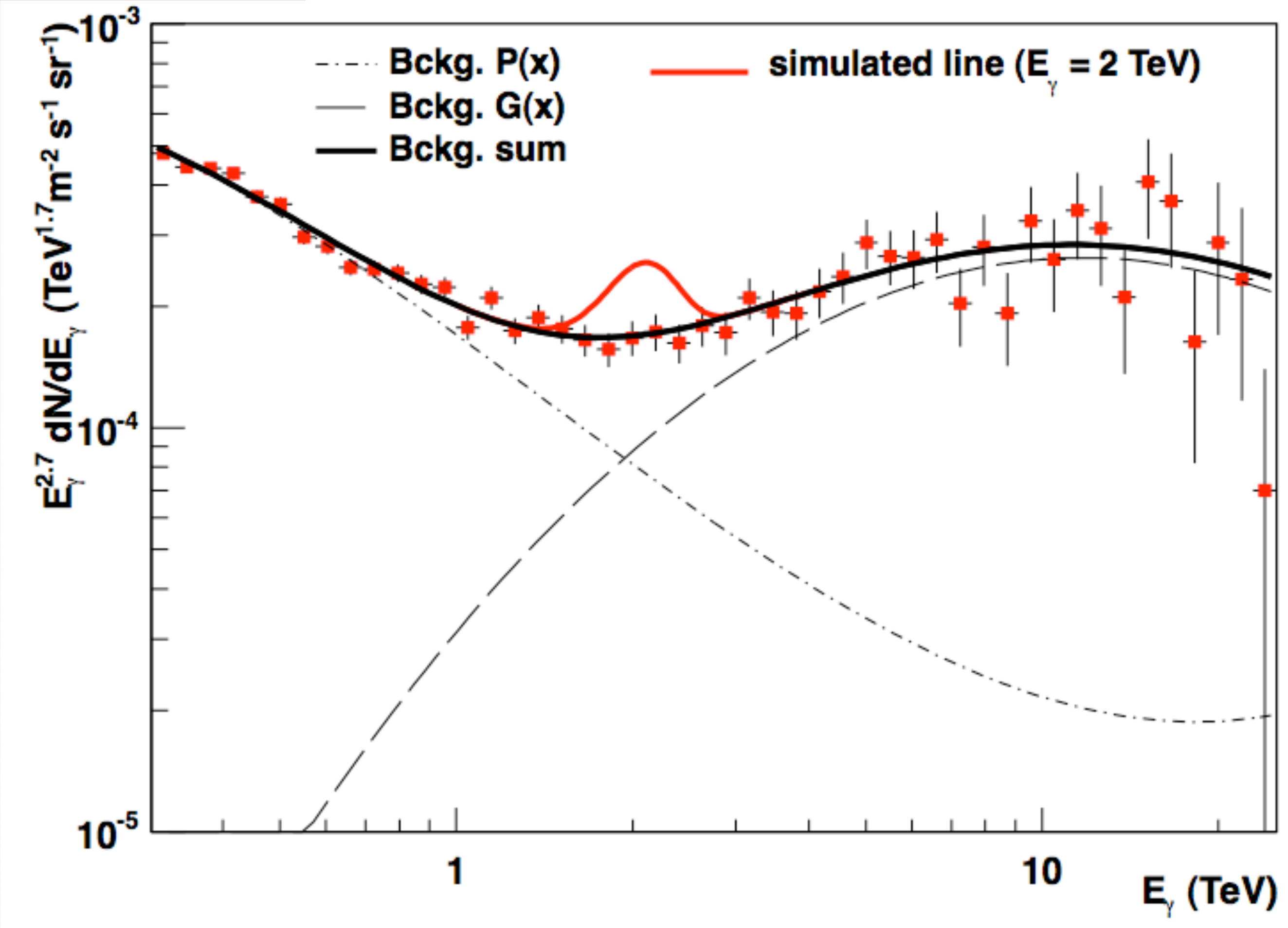


or



or … ?

- HESS/VERITAS observes photons colliding with the atmosphere
- Therefore, we compute $\chi\chi \rightarrow \gamma$+(Whatever else)
- But we have introduced a **new scale, $M_\chi(1\text{-}z_{cut})$~ Bin size**



From HESS collaboration 1301.1173

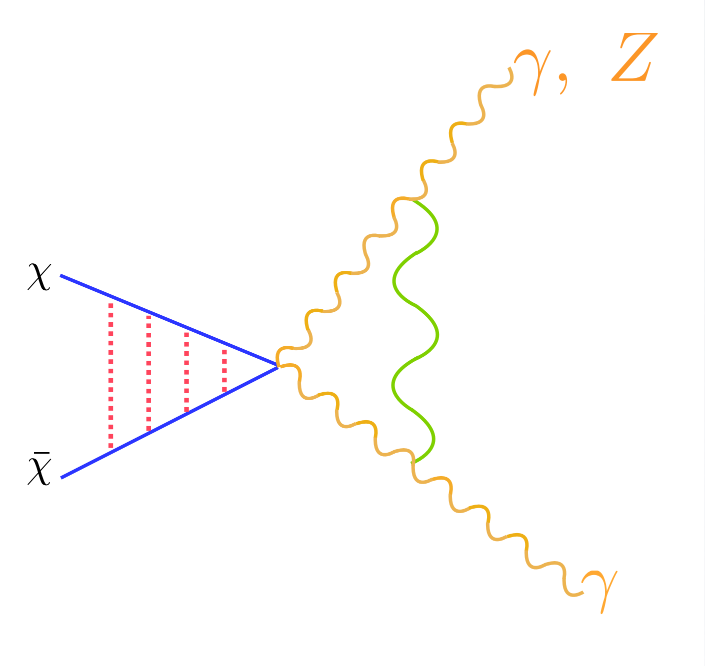
at 3 TeV, **energy resolution is ~400 GeV**

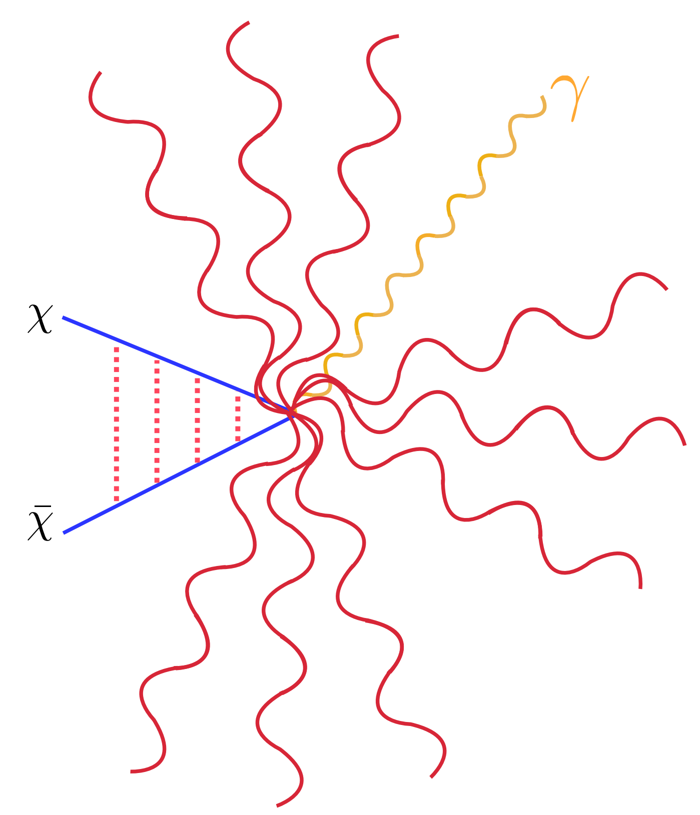
m_W = 80 GeV

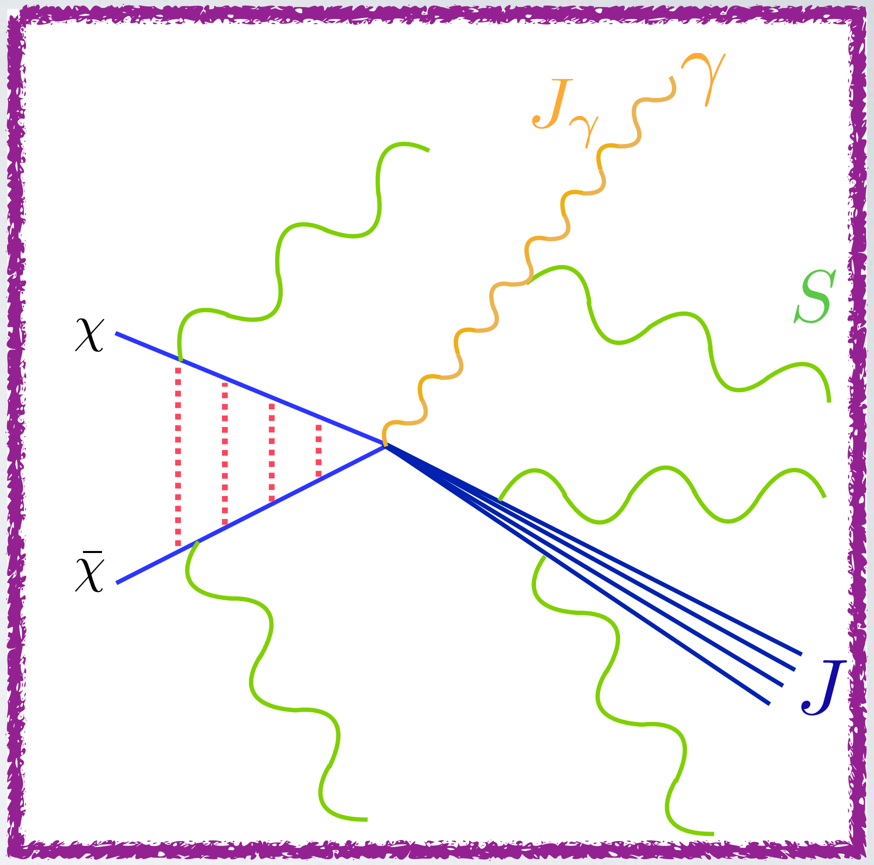
E_γ (soft W)=M_χ-m_W/2

E_γ (collinear Ws) = M_χ-m_W^2/M_χ

GOLDILOCKS & THE 3 RESEARCH GROUPS







Exclusive

2→2 annihilation:

γγ+γZ

Ovanesyan, Slatyer, and Stewart: 1409.8294; Cohen et al.: 1409.7392

Semi-inclusive

Integrate out recoil state with OPE: γ+X

MB, I.Z. Rothstein, V. Vaidya: 1409.4415, MB, I.Z. Rothstein, V. Vaidya: 1412.8698

Endpoint Region

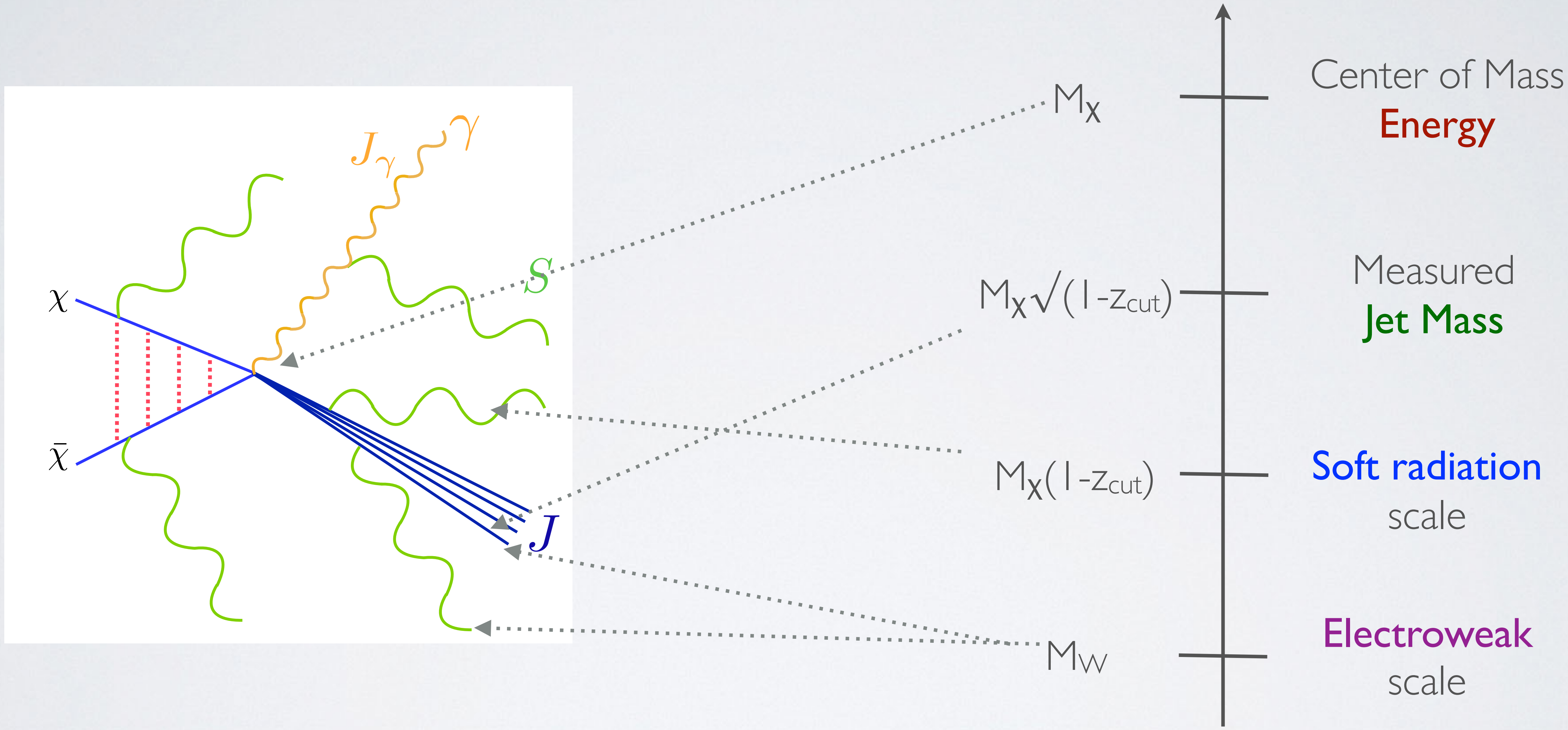
Measurement forces recoil into jet: γ+X

MB, Cohen, Moult et al.: 1712.07656

Semi-inclusive with cutoff

MB and V. Vaidya: 1510.02470

SCET WITH 2 EXPANSIONS



M_χ — Center of Mass **Energy**

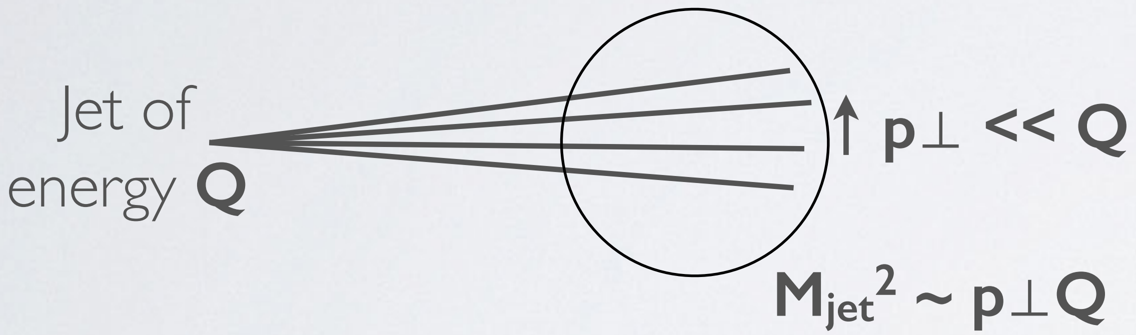
$M_\chi\sqrt{(1-z_{cut})}$ — Measured **Jet Mass**

$M_\chi(1-z_{cut})$ — **Soft radiation** scale

M_W — **Electroweak** scale

SOFT-COLLINEAR EFFECTIVE THEORY

- Large scale-hierarchies can arise within one field



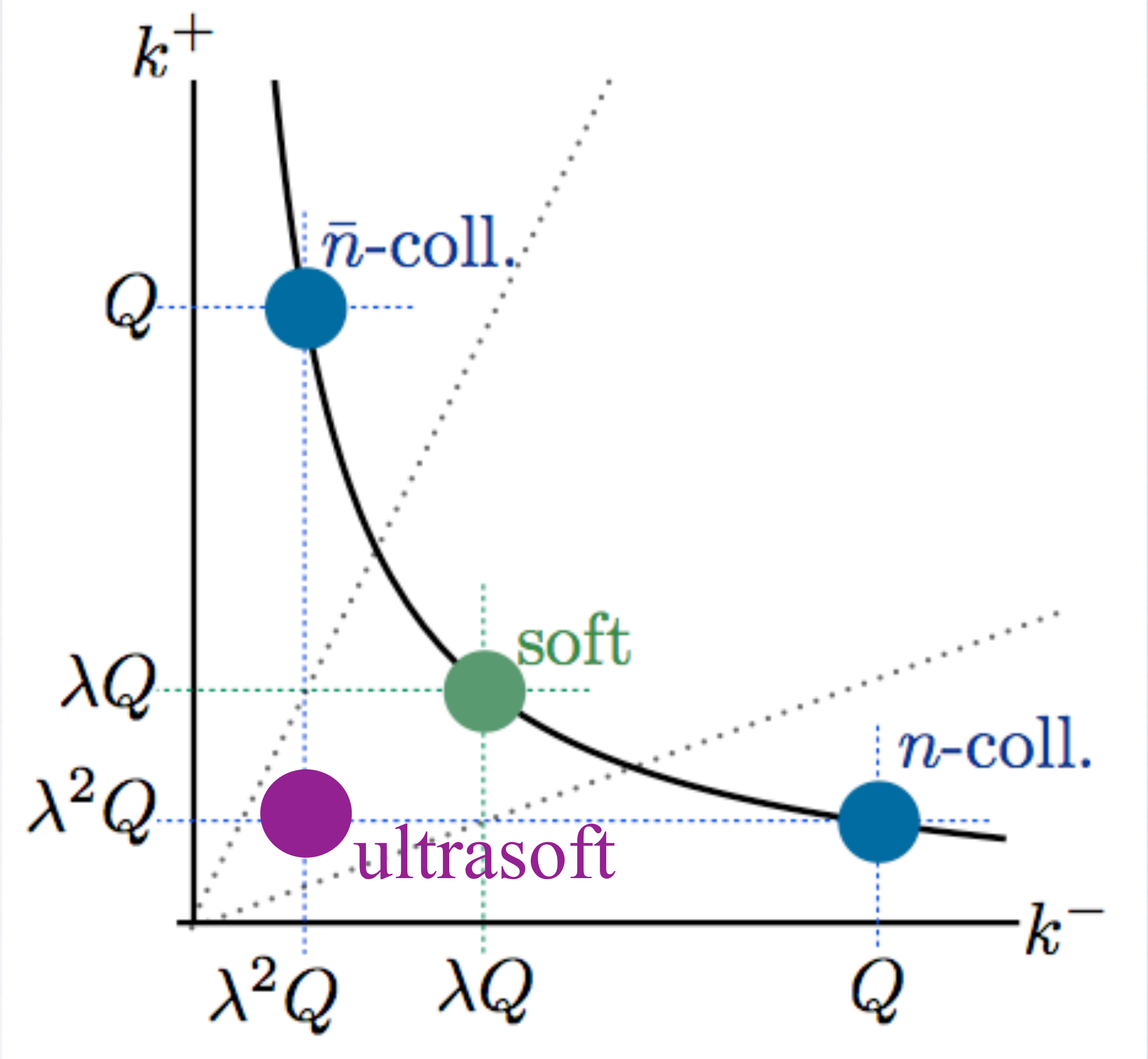
Jet of energy **Q**

↑ p⊥ << Q

M_{jet}^2 ~ p⊥Q

- We can use Renormalization Group to resum kinematic logs

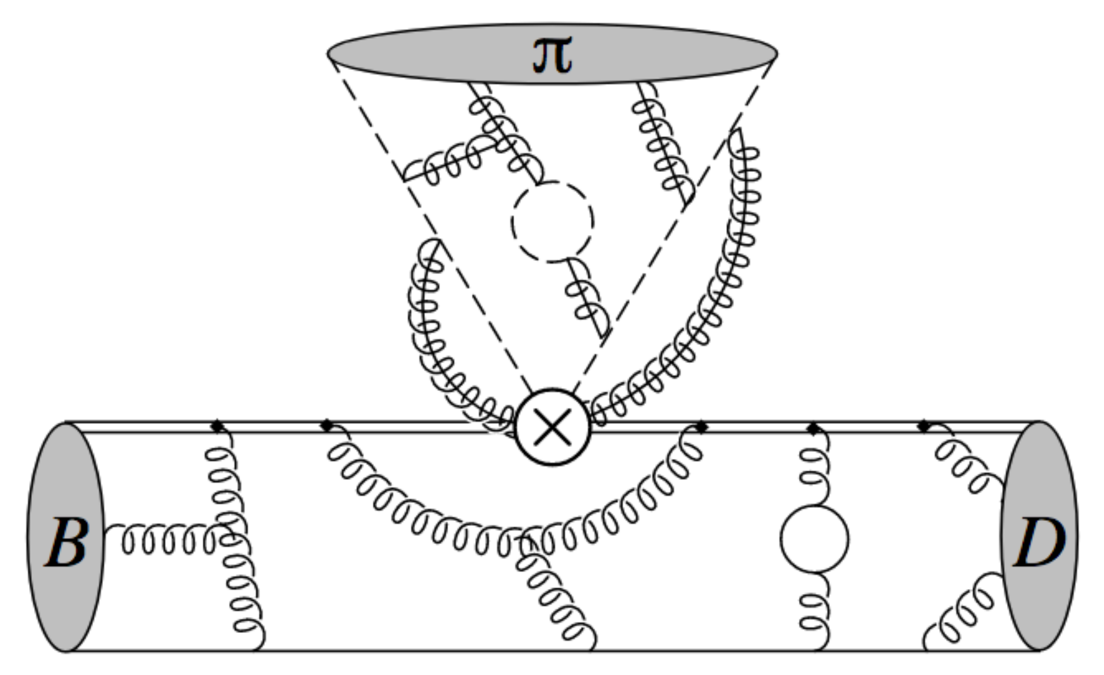
Lightcone momenta
$k^+ = k^0 + k^3$
$k^- = k^0 - k^3$



Integrate out hard modes, keep those collinear to null directions and soft fields

TRADITIONAL FACTORIZATION

- SCET modes' lightcone power counting
  - Collinear $(p_+, p_-, p_\perp) \sim Q(1, \lambda^2, \lambda)$
  - Soft $(p_+, p_-, p_\perp) \sim Q(\lambda, \lambda, \lambda)$
  - Ultrasoft $(p_+, p_-, p_\perp) \sim Q(\lambda^2, \lambda^2, \lambda^2)$



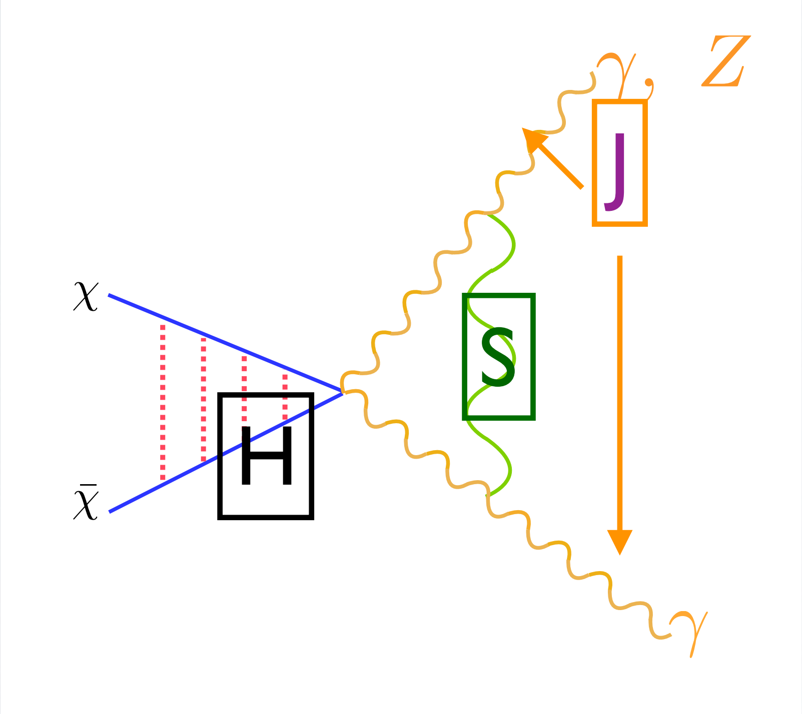
$\xi \rightarrow Y\xi$

Pull ultrasoft Wilson line out of collinear field →

Collinear/Soft Factorization!

hep-ph/0109045:
Bauer, Pirjol, & Stewart

SCET OBSERVABLES



Factorized Hilbert Space:

$$\big|X\big\rangle = \big|X_{\text{collinear}}\big\rangle\big|X_{\text{soft}}\big\rangle$$

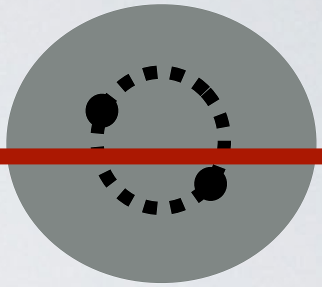
$$d\sigma = H(Q)\, J(Q, z_{\text{collinear}}) \otimes S(z_{\text{soft}})$$

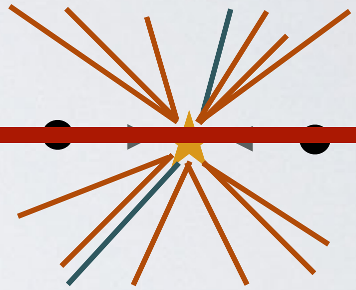
Squared Wilson coefficient

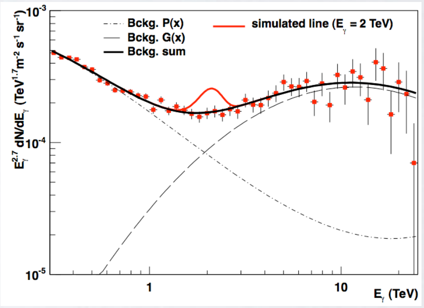
$$S = \langle 0\big|(YY)^\dagger\,\delta\big[f(z_{\text{soft}})\big](YY)\big|0\rangle$$

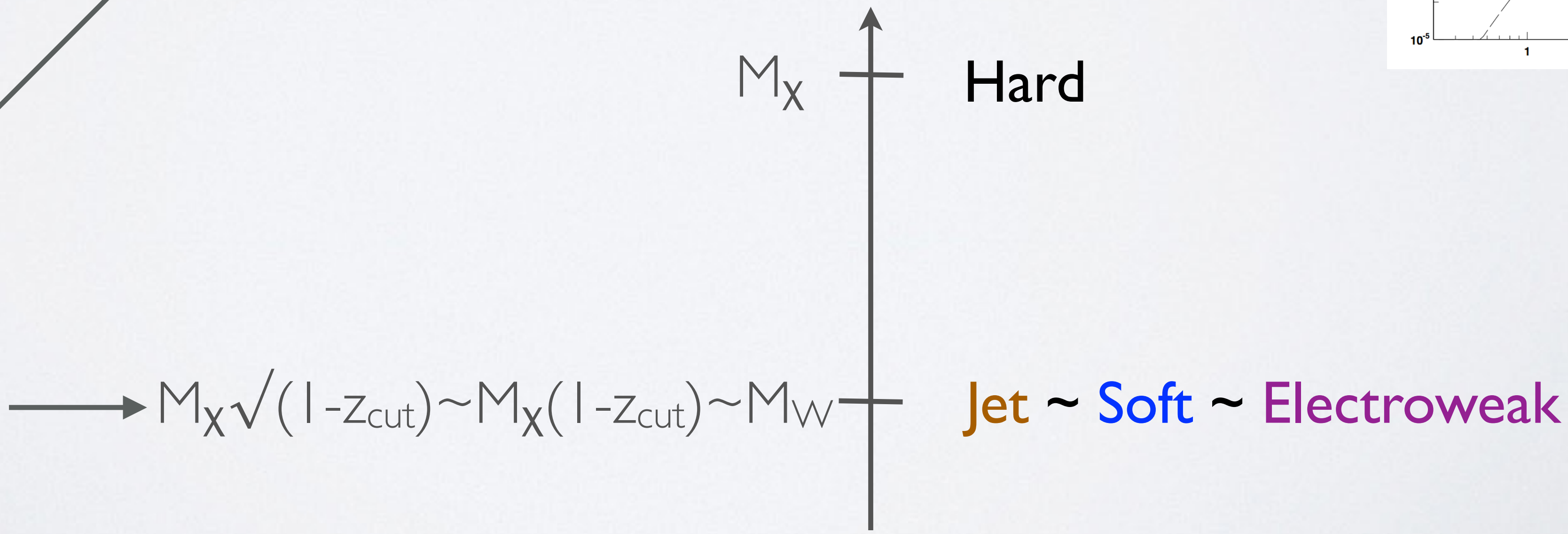
$$J_n = \langle 0\big|B_{n\perp}\delta\big[f(Q, z_{\text{collinear}})\big]\big|X_n\rangle\langle X_n\big|B_{n\perp}\big|0\rangle$$

- 3 separate threats to perturbation theory!
  - ~~$M_\chi/m_W >> 1$ → Long range force~~
  - ~~$M_\chi/m_W >> 1$ → Electroweak shower~~
  - $\mathrm{Log}(1\text{-}z_{cut})$ → Phase space restriction









If $J(M_\chi,(1\text{-}z_{coll}),m_W)$

$M_\chi\sqrt{(1\text{-}z_{cut})} \sim M_\chi(1\text{-}z_{cut}) \sim M_W$ — Jet ~ Soft ~ Electroweak

M_χ — **Hard**

COLLINEAR REFACTORIZATION

Original SCET **doesn't distinguish soft scales**: **$(1\text{-}z_{coll})$**, m_W

J_n: Perform matching

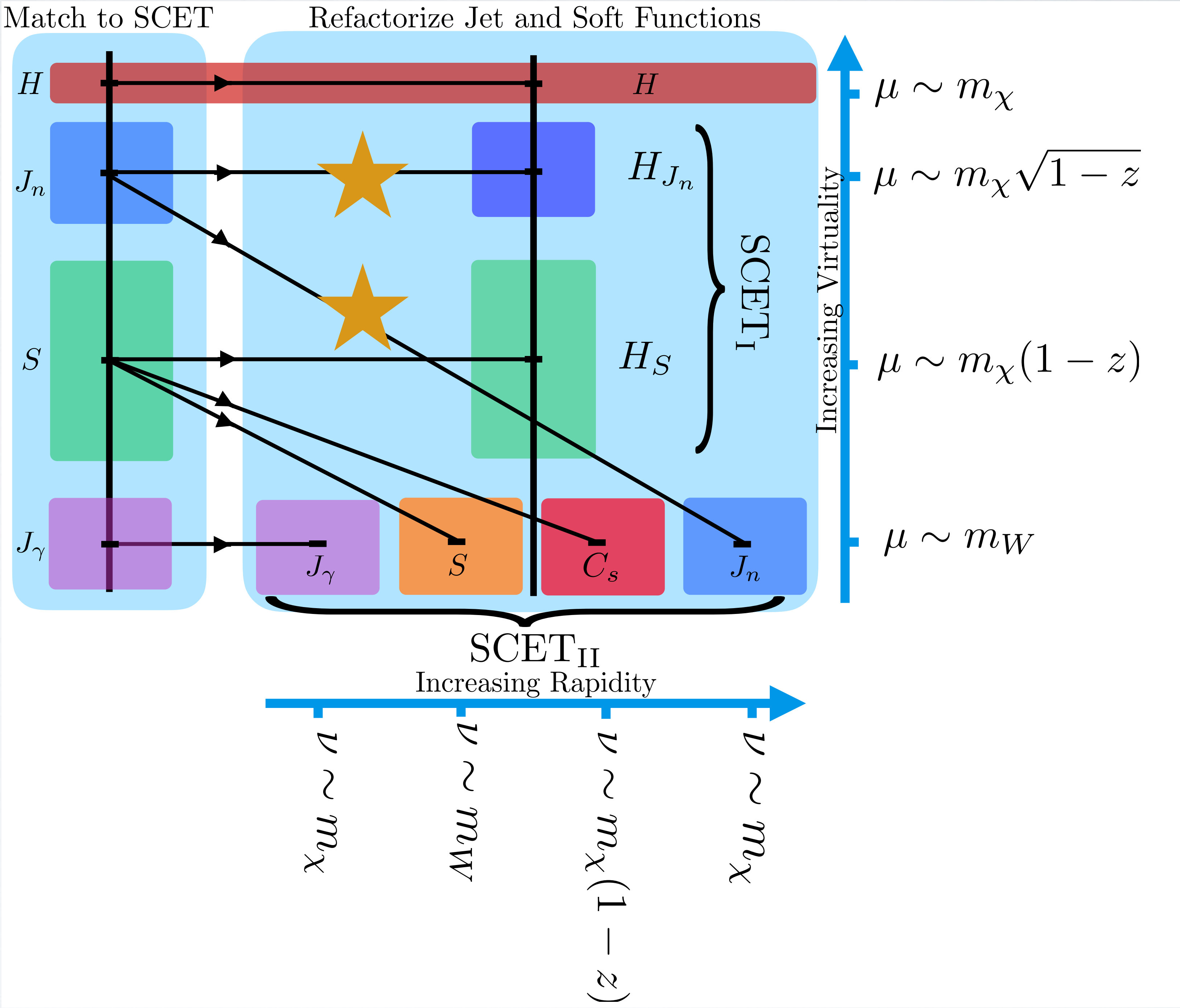
@ $M_\chi\sqrt{(1\text{-}z_{cut})}$

$J_n \rightarrow H_{J_n}(M_\chi\sqrt{(1\text{-}z_{cut})})J_n(m_W)$

Remaining **collinear**:

$(p_+,p_-,p_\perp) \sim M(1,\lambda^2,\lambda)$

$\lambda = m_W/M_\chi$



SOFT REFACTORIZATION

S: Perform matching

@ $M_\chi\sqrt{(1\text{-}z_{cut})}$

$S \to H_S(M_\chi\sqrt{(1\text{-}z_{cut})})S(m_W)$

???

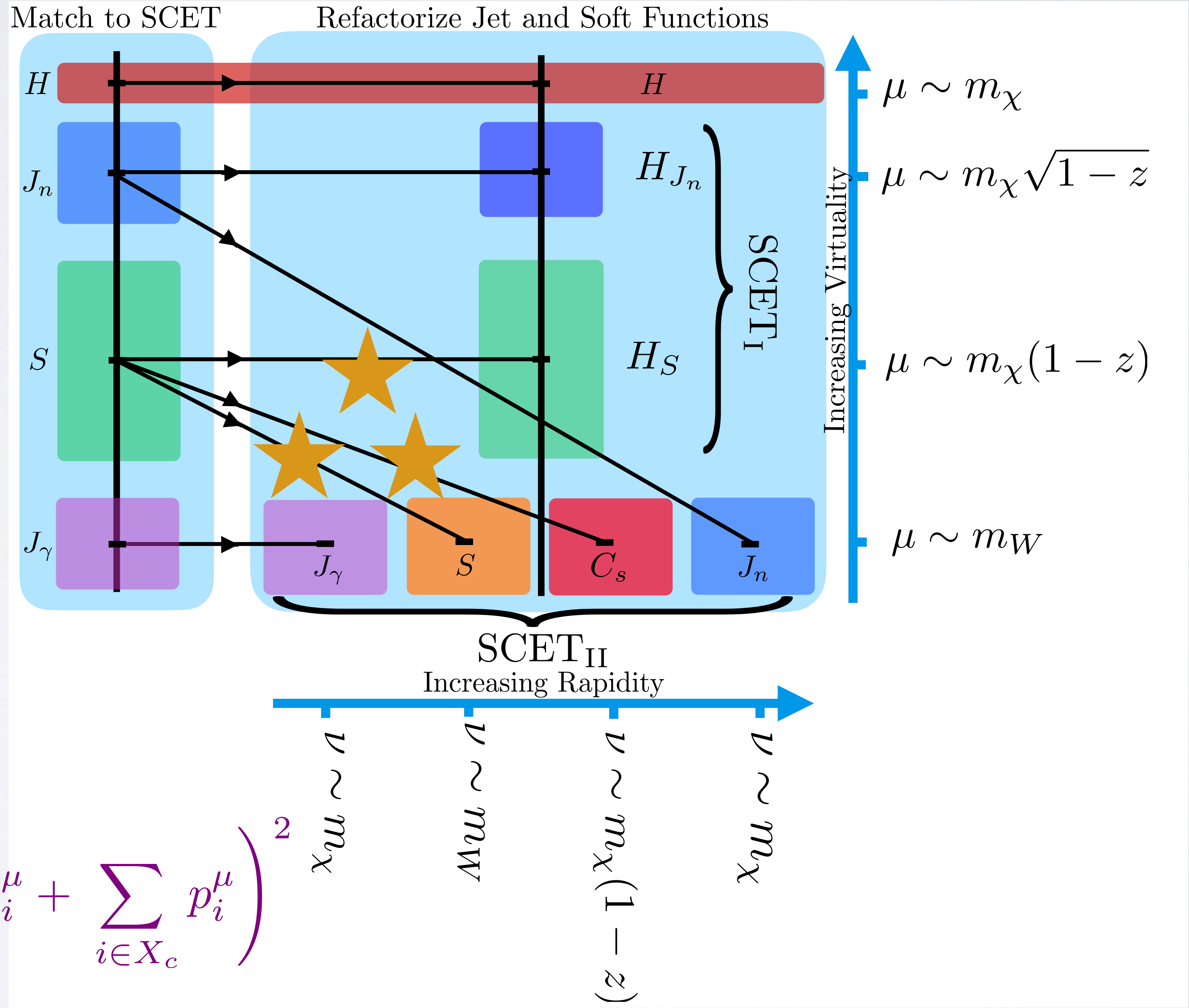
Remaining **soft:**

$(p_+,p_-,p_\perp) \sim M(\lambda,\lambda,\lambda)$

$\lambda = m_W/M_\chi$

BUT...

what about measurement function?



$$(1-z) = \frac{1}{4\,M_\chi^2}\,m_X^2 = \frac{1}{4\,M_\chi^2}\left(\sum_{i\in X_s} p_i^\mu + \sum_{i\in X_c} p_i^\mu\right)^2$$

$$\equiv (1-z_s) + (1-z_c) + \mathcal{O}(\lambda^2)$$

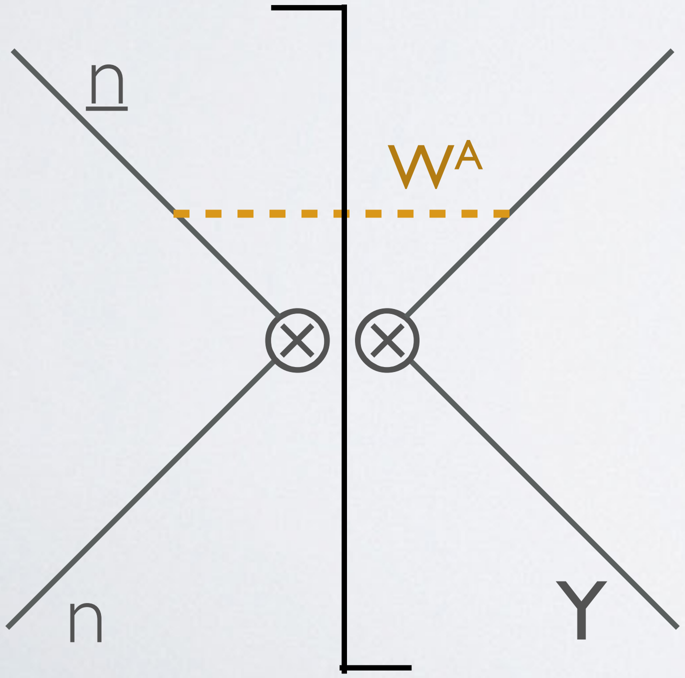
WHITHER SOFT DIVERGENCE?

$$S_{11}^{a'b'ab}(x) = \Big\langle 0 \Big| \left(Y_n^{3k} Y_{\bar{n}}^{dk}\right)^\dagger \delta\left[M_\chi^2(1 - z_{\text{soft}}) - M_\chi^2(1 - \hat{z}_{\text{soft}})\right]$$

$$\times\, |X_s\rangle\langle X_s| \left(Y_n^{3j} Y_{\bar{n}}^{dj}\right) \Big| 0 \Big\rangle \delta^{a'b'}\delta^{ab}\,,$$

$$\to S_{11}^{a'b'ab} \sim \delta^{a'b'}\delta^{ab}\langle 0| \left[Y_n^{ce'} Y_{\bar{n}}^{3e'}\right]^\dagger \delta\left[M_\chi^2(1 - z_{\text{soft}})\right] \left[Y_n^{ce} Y_{\bar{n}}^{3e}\right] |0\rangle$$

$$= \delta^{a'b'}\delta^{ab}\delta\left[(M_\chi^2(1 - z_{\text{soft}})\right]$$



One-loop
soft
anomalous dimension

$\Rightarrow$???

Can't live in H_S, because H_S **doesn't know about scale m_W**

COLLINEAR SOFT MODE

Alternate **soft scaling:**

$(p_+,p_-,p_\perp)\sim M(1-z_{cut})(\lambda^2,1,\lambda)$

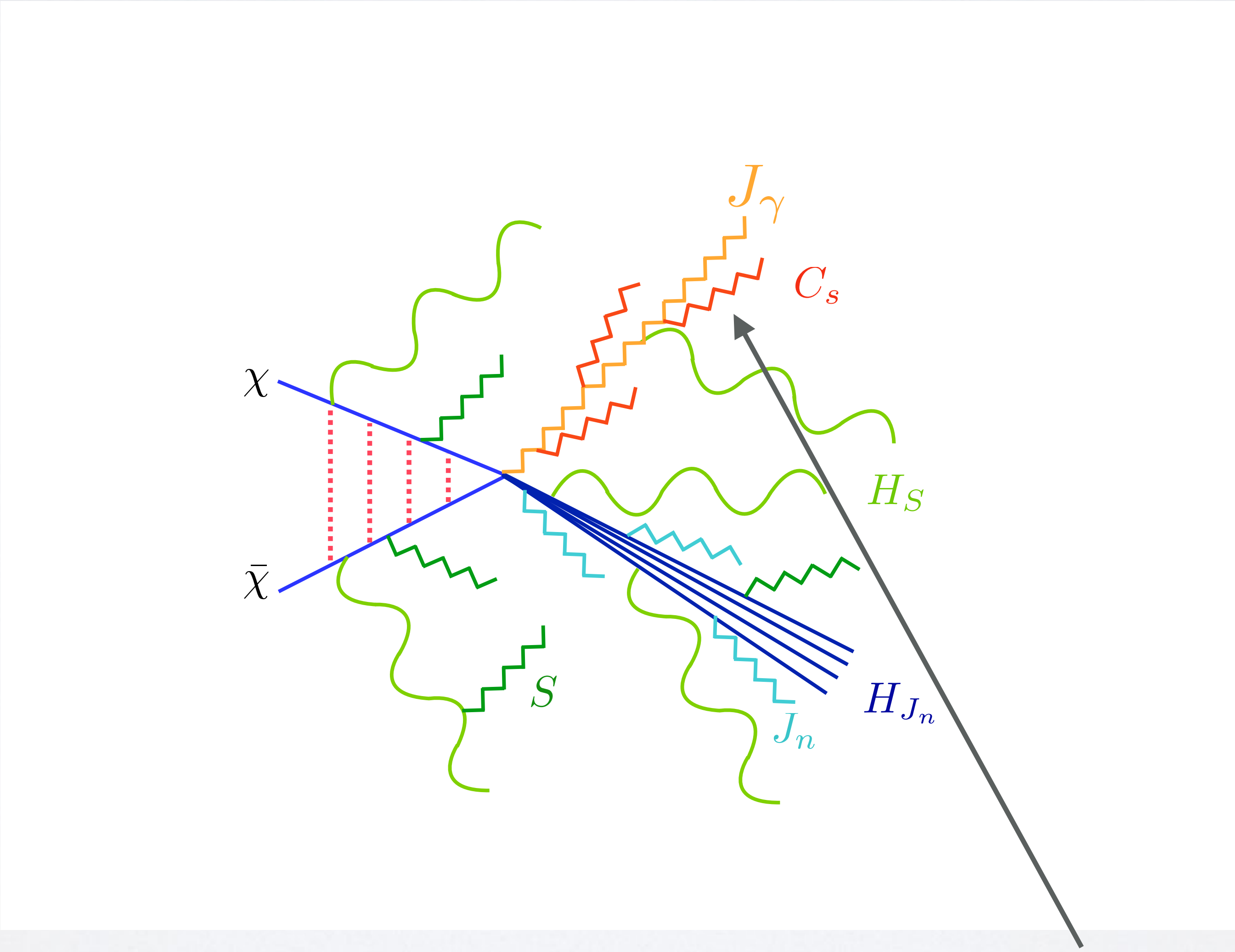
$\lambda = m_W/M_\chi(1-z_{cut})$

$$\tilde{S}_{11}^{aba'b'} = \Big\langle 0 \Big| \left(Y_n^{3f'} Y_{\bar{n}}^{dg'} \right)^\dagger (0)\,\delta(M - \widehat{M}_{C_s}) \\ \times \Big|X_{C_s}\Big\rangle\Big\langle X_{C_s}\Big| \left(Y_n^{3f} Y_{\bar{n}}^{dg} \right)(0)\Big|0\Big\rangle\, \delta^{f'g'}\delta^{a'b'}\delta^{fg}\delta^{ab}\,.$$

Get back pre-refactorized divergence!

FULLY FACTORIZED THEORY

$$\frac{d\sigma}{dz} = H(m_\chi, \mu) \cdot H_{J_n}(m_\chi, (1-z), \mu) \cdot H_S(m_\chi, (1-z), \mu)$$
$$\cdot J_\gamma(m_W, \mu, \nu) \cdot S(m_W, \mu, \nu) \cdot C_S(m_\chi, (1-z), m_W, \mu, \nu) \cdot J_n(m_W, \mu, \nu)$$



Collinear soft modes account for **radiation along photon direction**, but contribute to recoil jet mass

RAPIDITY RG

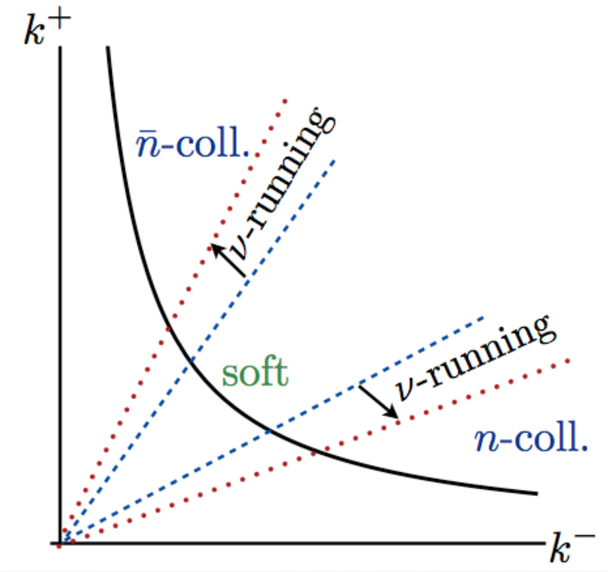
- SCET is a "modal" theory

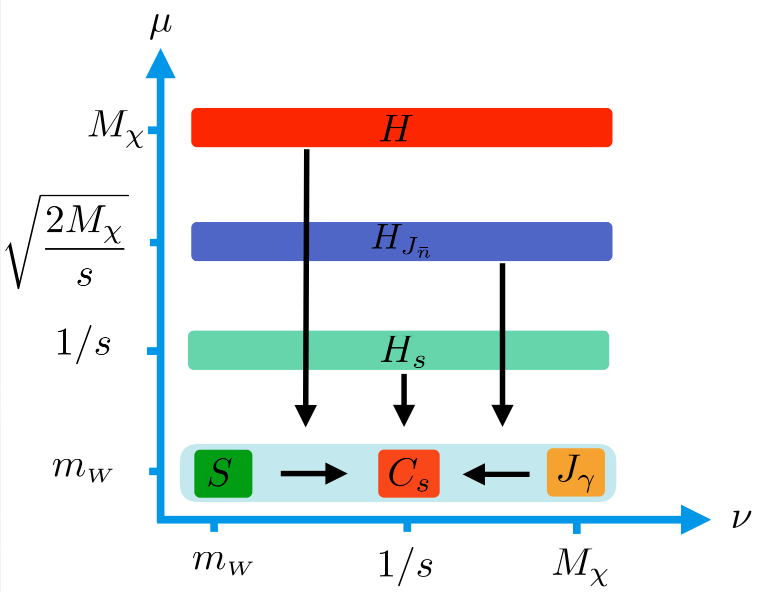
$$A_\mu = A_\mu^{c,n} + A_\mu^{c,\bar{n}} + A_\mu^{soft} + \dots$$

- We can get divergences when integrals invade other sectors. Soft-collinear overlap requires boost-violating regulator

- Regulating sets up RG for resumming these rapidity logs

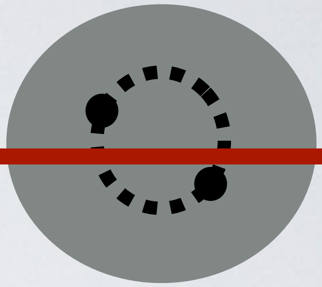
$$W_n = \sum_{\text{perms}} \exp\left[-\frac{g}{\bar{n}\cdot P}\frac{\nu^\eta}{|\bar{n}\cdot P|^\eta}\bar{n}\cdot A_n\right]$$



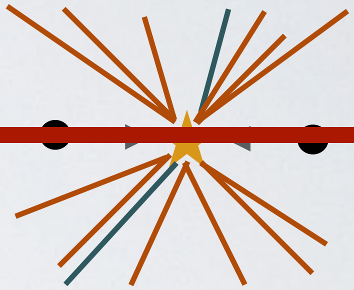


Above from Chiu et al. 1202.0814: In SCETII, soft and collinear modes have same virtuality
ν-running lets us minimize log between soft & collinear scales

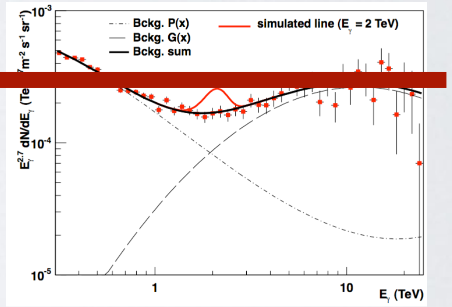
- 3 separate threats to perturbation theory!



- ~~$M_\chi/m_W >> 1$ → Long range force~~



- ~~$M_\chi/m_W >> 1$ → Electroweak shower~~



- ~~Log(1-zcut) → Phase space restriction~~
- Proliferation of scales → Effective Field Theory

RESUMMED PHOTON SPECTRUM

$$\frac{d\sigma}{dz} = \frac{\pi\alpha_W^2 \sin^2\theta_W}{2M_\chi^2 v} e^{\left[-2C_2(W)\frac{\alpha_W}{\pi}\log^2\left(\frac{2M_\chi}{M}\right)\right]} \Bigg\{ (F_0+F_1)\delta(1-z)$$

$$+ \left(C_2(W)\frac{\alpha_W}{\pi}\log\left(\frac{4M_\chi^2(1-z)}{M^2}\right) \frac{e^{\left[C_2(W)\frac{\alpha_W}{2\pi}\log^2\left(\frac{M^2}{4M_\chi^2(1-z)}\right)\right]}}{1-z}\right)_+ F_0$$

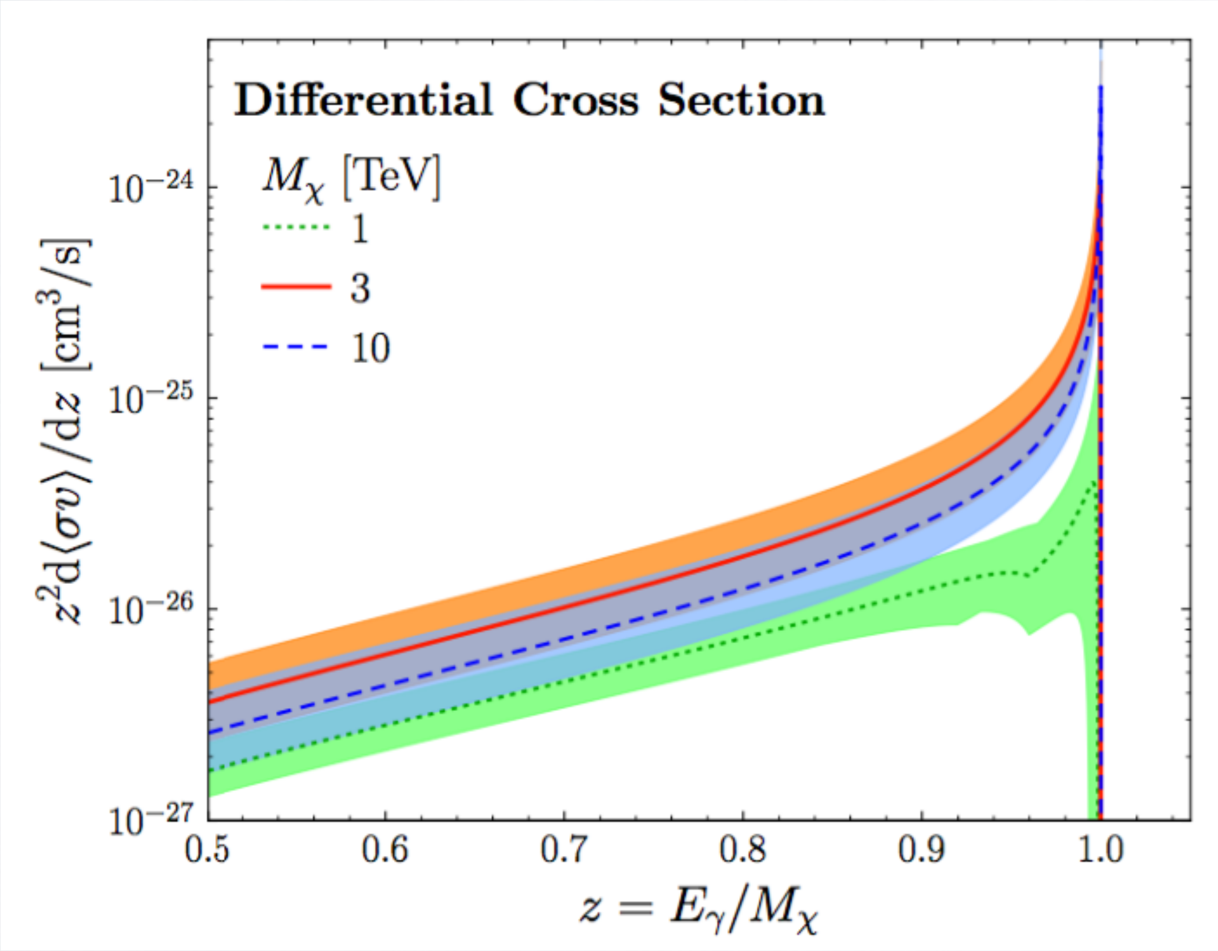
$$+ \Bigg[\Bigg(C_2(W)\frac{\alpha_W}{\pi}\log\left(\frac{4M_\chi^2(1-z)}{M^2}\right) + 3C_2(W)\frac{\alpha_W}{\pi}\log\left(\frac{M}{2M_\chi(1-z)}\right)\Bigg)$$

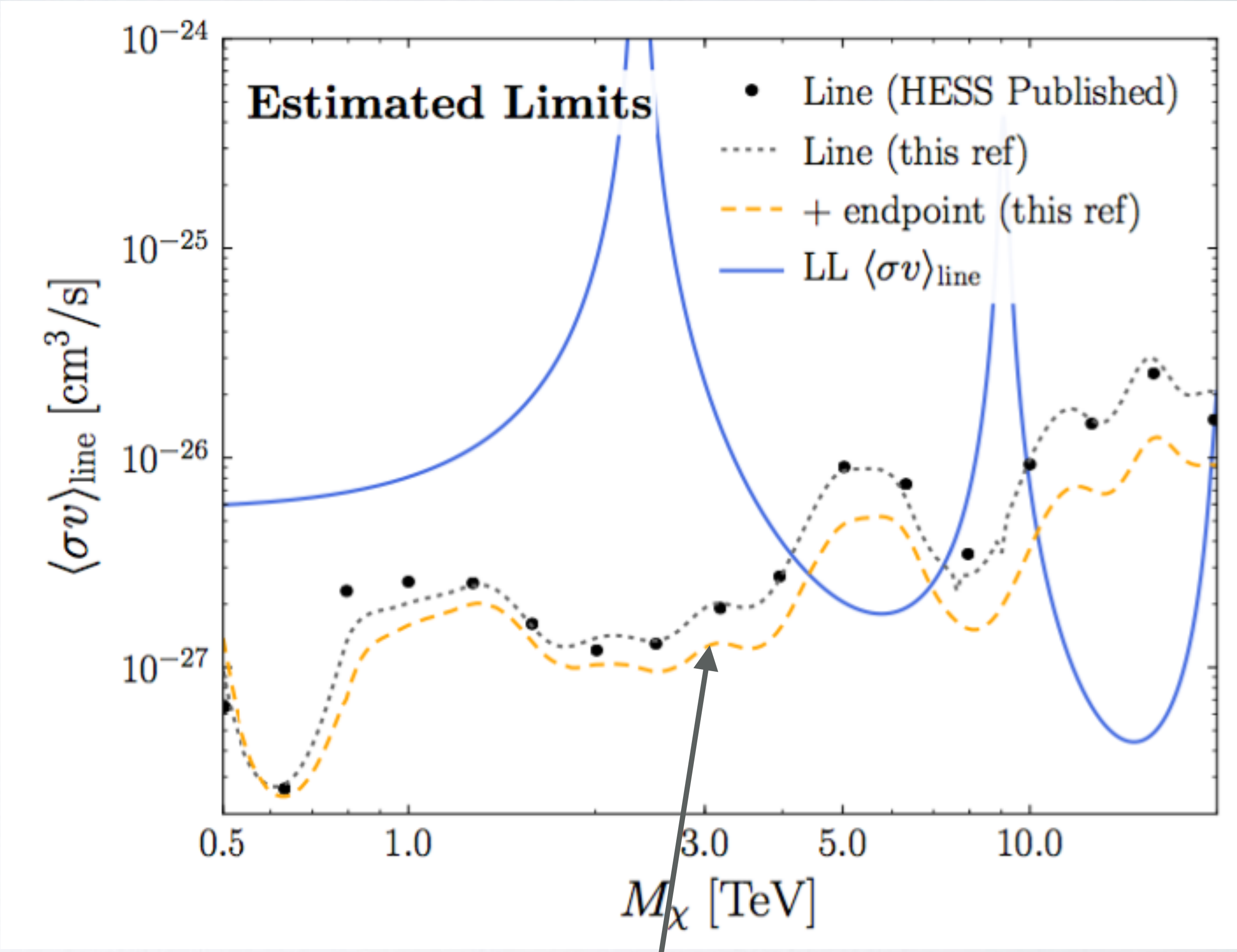
$$\times \left(\frac{e^{\left[-\frac{3}{2}C_2(W)\frac{\alpha_W}{\pi}\log^2\left(\frac{M}{2M_\chi(1-z)}\right) + C_2(W)\frac{\alpha_W}{2\pi}\log^2\left(\frac{M^2}{4M_\chi^2(1-z)}\right)\right]}}{1-z}\right)\Bigg]_+ F_1\Bigg\}$$

Squared Wilson Coefficient for wino annihilation

Linear combination of Sommerfeld factors

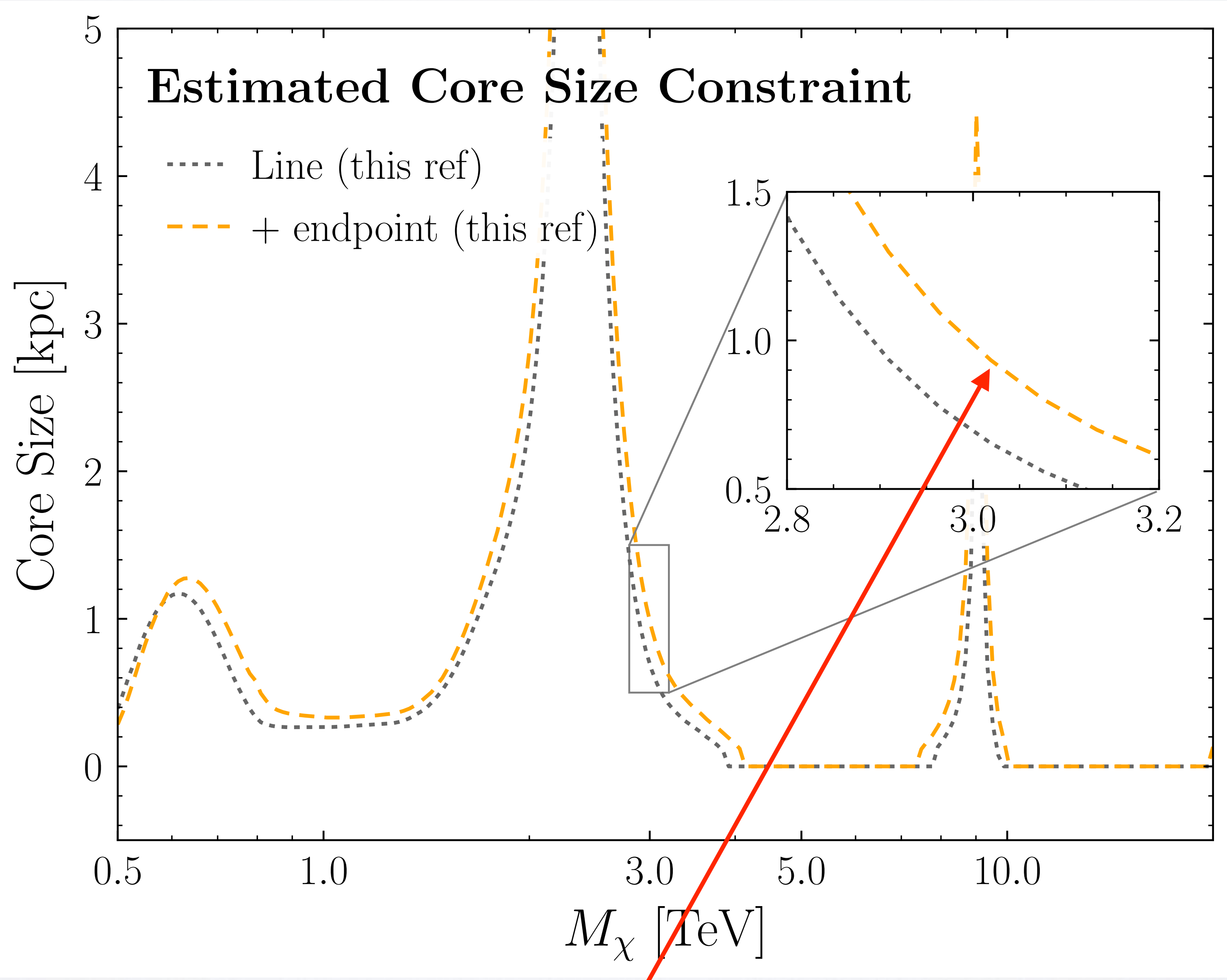
WINO SPECTRUM & LIMITS





Estimate **~30% strengthening**
of HESS limit from tree + Sommerfeld
(Einasto profile)

A MORE USEFUL LIMIT



(Cutoff NFW)

Preliminary indications
show thermal relic wino
weakened by quantum corrections

~1 kpc core to save wino

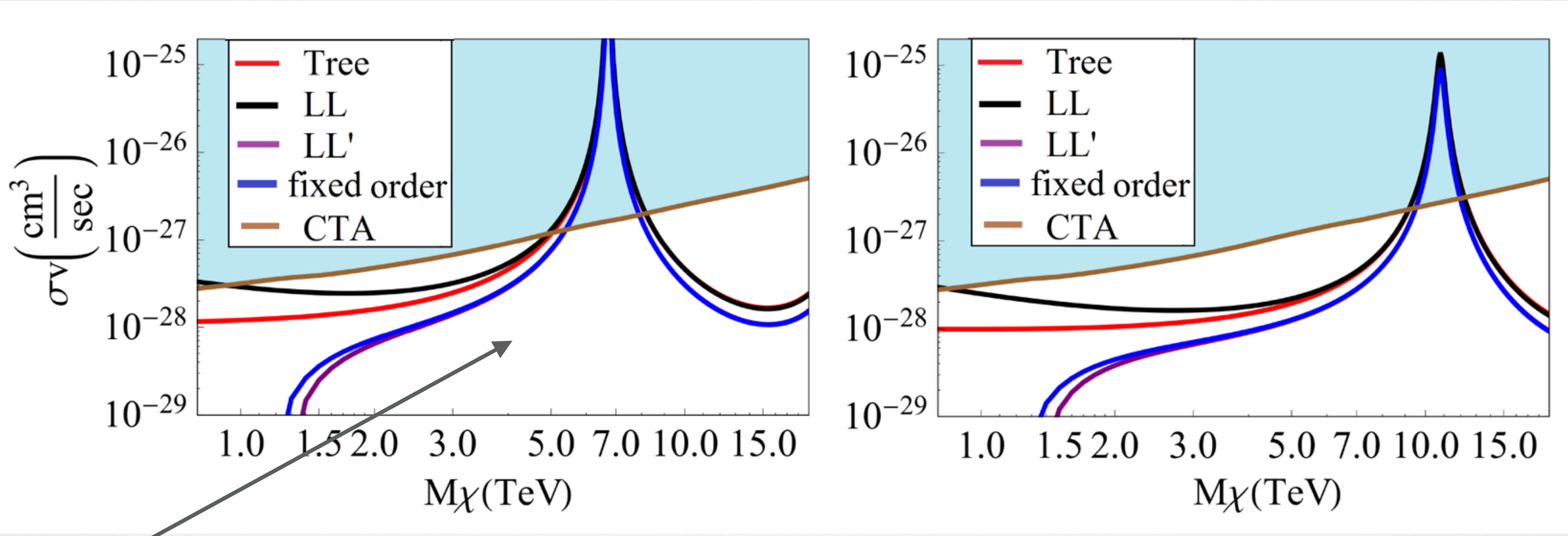
Core limits:

Simulation: <1 kpc [1507.02282]

Observation: <2 kpc [1608.00003]

HIGGSINO RATE AT LL´

I TeV higgsino needs proper treatment of endpoint region



ΔM_0= 200 keV;
ΔM_+= 350 MeV

ΔM_0= 2 GeV;
ΔM_+= 480 MeV

Minimally split Higgsino.*
Purest viable doublet

*Cheung et al: hep-ph:0512192

Thermal Higgsino unconstrained by current/future tests

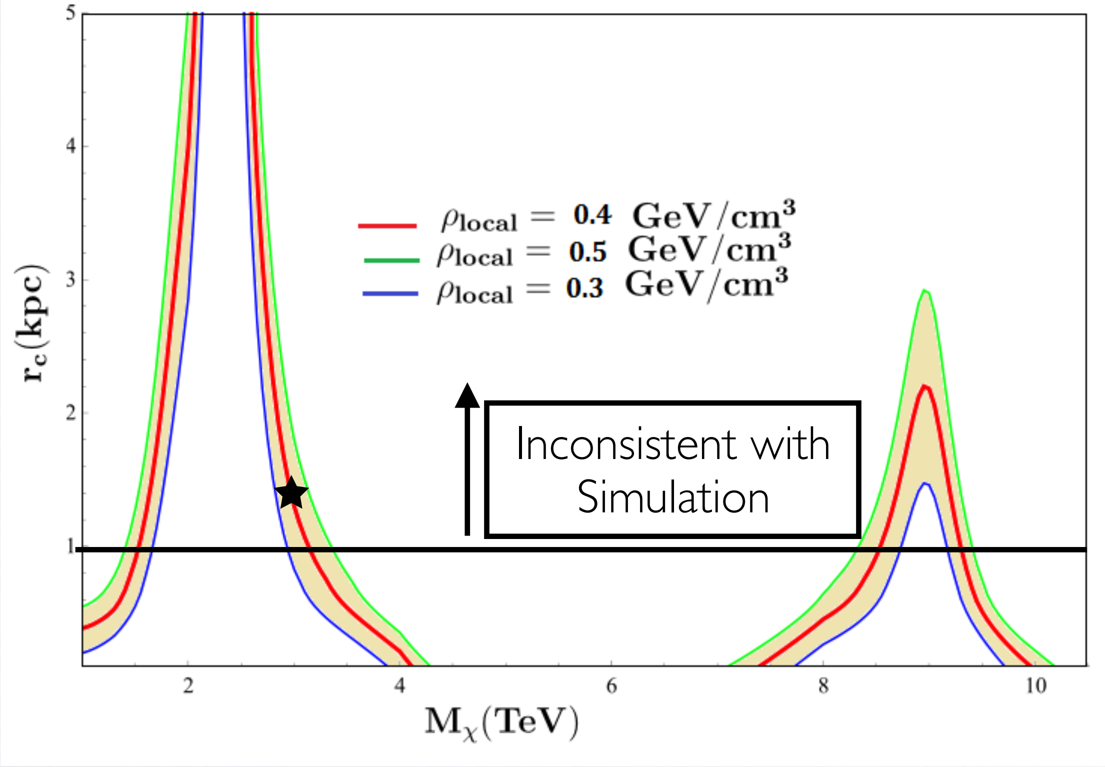
MB, Vaidya, V.: 1510.02470

CONCLUSION

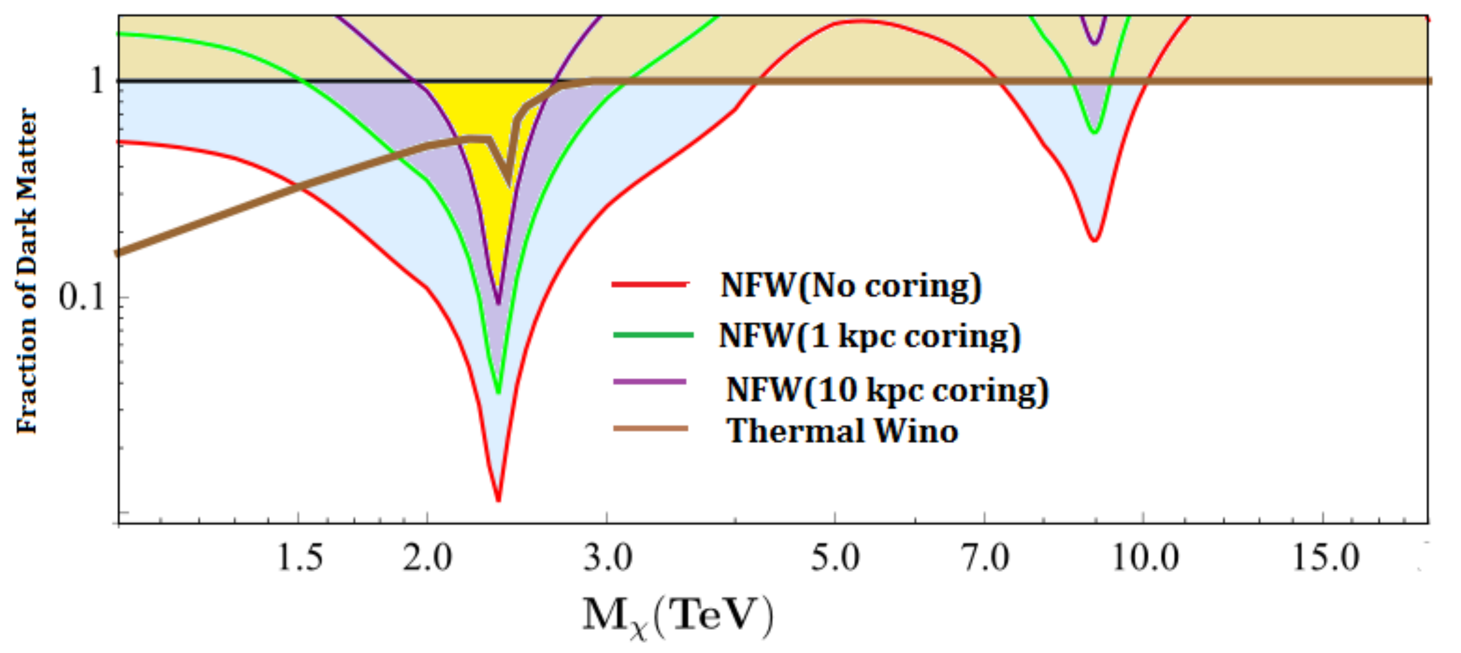
- Despite simple model and straightforward experiment, **proper calculation has rich theoretical structure**
- EFT combines **NRQCD, SCET-I, SCET-II, and SCET$_+$** (collinear-soft mode)
- Techniques **generic for WIMPs coupled to lighter states** (other representations of electroweak group, e.g. 5 of SU(2) @ 9 TeV IN PROGRESS!)
- Put in next set of corrections, Log(1-z) and Log(M_χ/m_W) IN PROGRESS!
- Test alternate simple model, Higgs portal that requires electroweak resummation (χ^2)(h^2) IN PROGRESS!
- **Wino is in tension with Milky Way like simulations**, new data on the way (VERITAS, HESS, CTA, WFIRST, LSST, GAIA) **HESS projected improvement by 5x, pushing core out to 9kpc!**
- **Can anyone find the higgsino**, the most elusive motivated particle in high energy physics? NEXT UP AFTER CURRENT PAPERS ARE OUT

WINO VIABILITY

Initial motivation for Wino stressed its simplicity,
but perhaps its role in Dark Matter is more involved, including
a non-thermal history, multi-component DM, mixing with other electroweak states



Imposing a constant density below a given radius for NFW (core), at what point does wino become viable total DM?

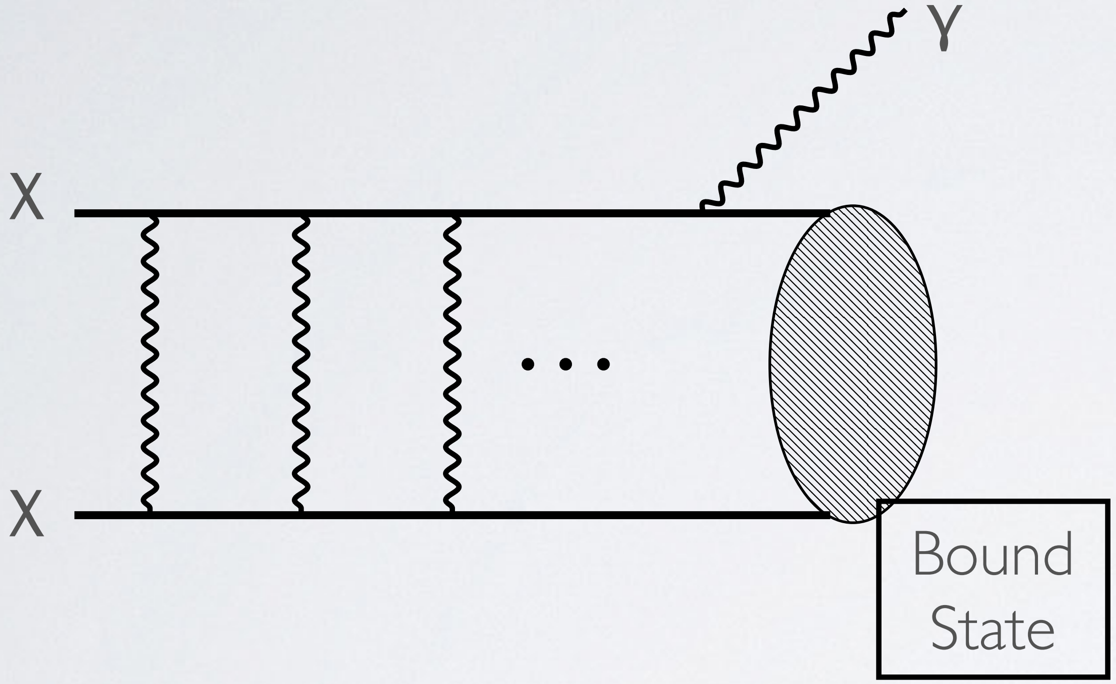


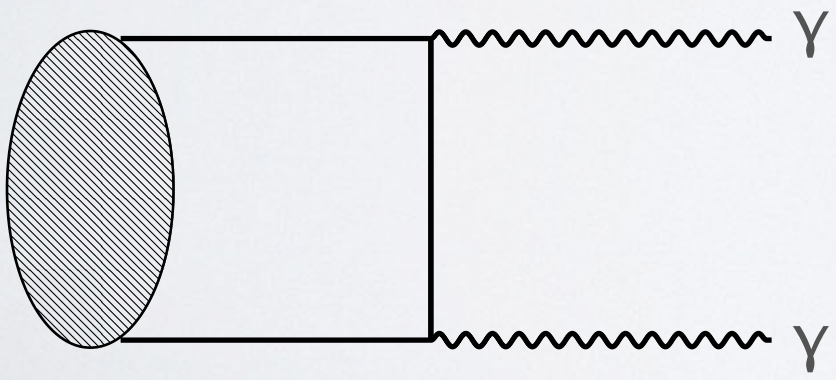
Possibility for the wino to make up some fraction of DM with NFW profile flattened to a constant core at some radius

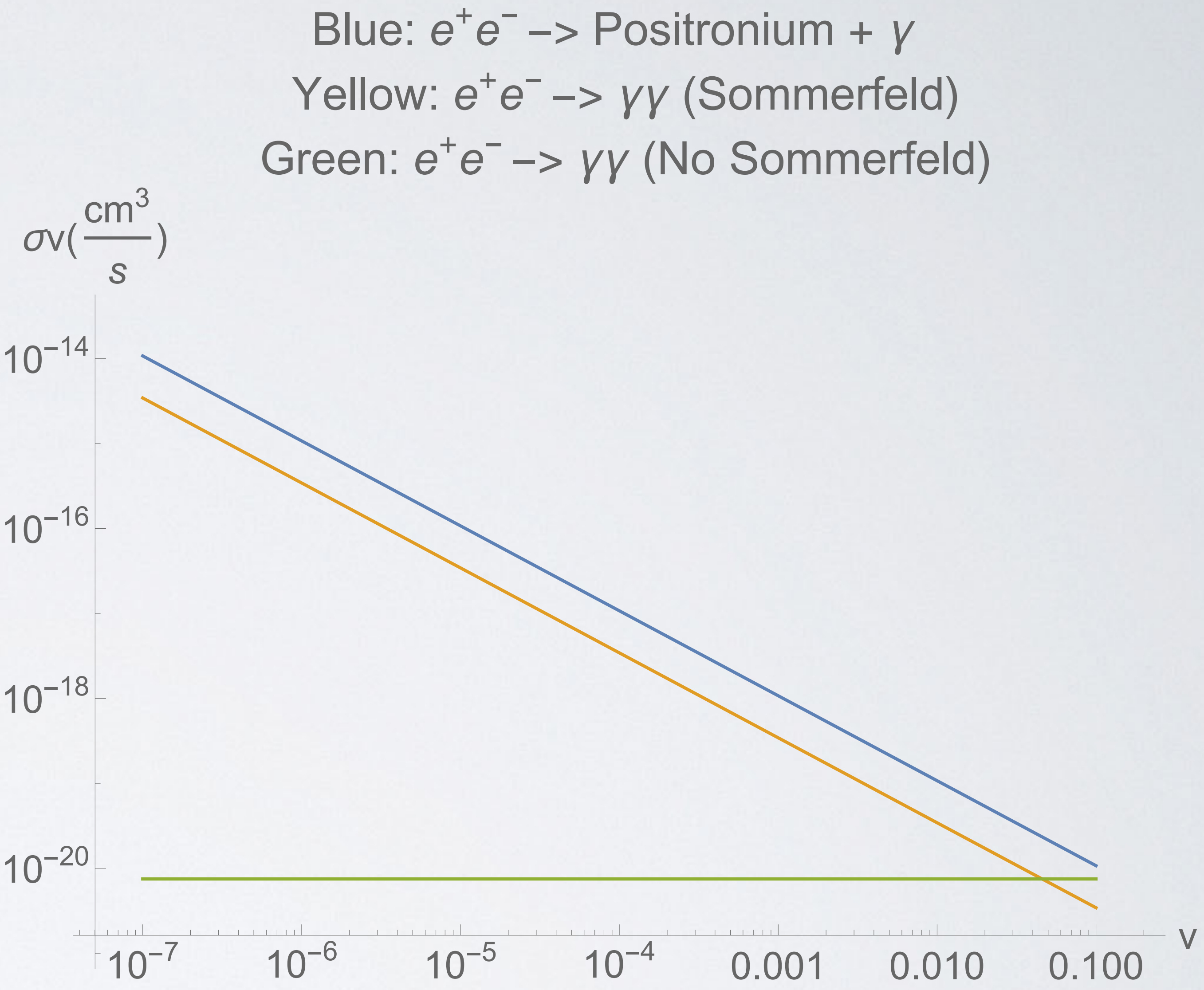
*__MB__, Rothstein, I., Vaidya, V.: 1412.8698

CAPTURE TO BOUND STATE

Sommerfeld Resonances
reveal **bound states in spectrum**





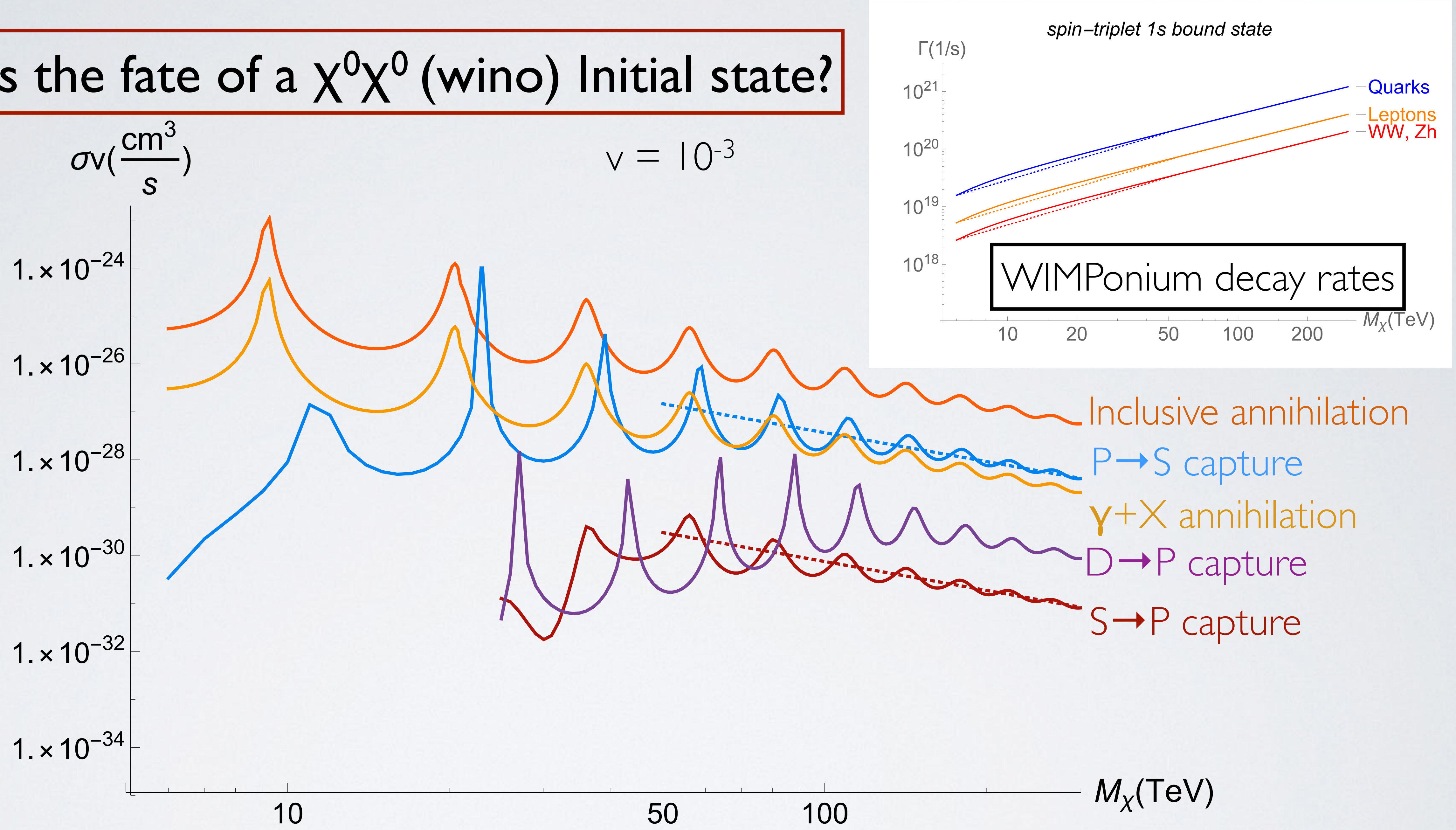


$$\sigma v|_{\text{capture}} = \frac{2^{10}\pi^2}{3} e^{-4}\,\alpha^3 \frac{1}{m_e^2}\frac{1}{v_{\text{rel}}} \qquad \sigma v|_{\text{ann.}} = \frac{2\pi^2\alpha^3}{m_e^2\,v_{\text{rel}}}$$

For postitronium
Capture ≈ 3x Annihilation

CAPTURE TO BOUND STATE

What is the fate of a $\chi^0\chi^0$ (wino) Initial state?



Capture loses to annihilation, but
new signature (capture photons)
Importance depends on identity of WIMP

1610.07617: Asadi, P., **MB**, Fitzpatrick P,. Krupczak, E., Slatyer, T.