Nuclear Many-Body Theory for Neutron Star Mergers

The Nuclear EOS at Finite Temperature from MBPT with Chiral Interactions

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Neutron Star Mergers and the Nuclear EOS

- GW170817/GRB170817A: $\mathbf{1.5}M_\odot + \mathbf{1.25}M_\odot \rightarrow (\mathbf{2.75 - 0.025})M_\odot$
- simulations with collections of EOS models → extraction of new constraints, e.g., $R_{1.6} \geq 10.68^{+0.15}_{-0.04}$ km, $R_{\max} \geq 9.60^{+0.14}_{-0.03}$ km

Bauswein et al.; Astrophys.J. 850 (2017)

- nuclear EOS = free energy $F(T, \rho, Y)$; here: $Y = \rho_p/\rho$

systematic approaches to compute EOS:

- classical limit $T/\mu \gg 1$: virial expansion for EOS

Horowitz & Schwenk; Phys.Lett.B 638 (2006)

- degenerate limit $T/\mu \ll 1$: Sommerfeld expansion for T dependence using **statistical** (i.e, not dynamical) version of Fermi liquid theory

Constantinou, Muccioli, Prakash, Lattimer; Ann. Phys. 363 (2015)

requires $T = 0$ EOS + **statistical** quasiparticle energies
(→ details later in "improved formalism")

- intermediate T/μ ? → χ**EFT, MBPT** (but: $\rho \lesssim 2\rho_{\text{sat}}$ for $\Lambda \lesssim 500$ MeV)

Schedule

1. Basic Formalism
2. Basic Results
3. Improved Formalism

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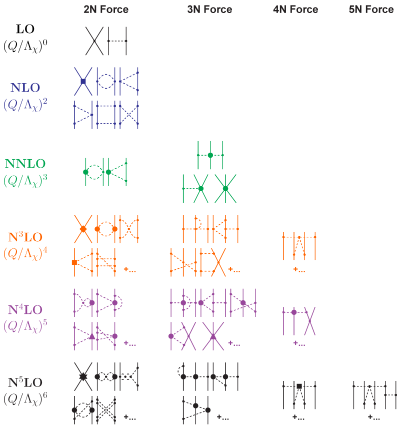
Chiral Effective Field Theory of Nuclear Interactions

- hierarchy of nuclear interactions determined by low-energy expansion in $(Q/\Lambda_\chi)^\nu$
- UV “cutoff” Λ, short-distance effects parametrized by LECs $c_i(\Lambda)$
- LECs $c_i(\Lambda)$ fixed by fits to data



Uncertainty quantification via

- different UV regulators
- different orders in $(Q/\Lambda_\chi)^\nu$
- systematic treatment of LEC fitting ambiguities
- ex- vs. inclusion of Δ isobars
- ...

Be aware of artifacts!

Machleidt, Symmetry **8**,26 (2016)

Chiral Effective Field Theory → MBPT

- hierarchy of nuclear interactions in terms of $(Q/\Lambda_\chi)^\nu$ → N3LO
- UV "cutoff" Λ, short-distance effects parametrized by LECs $c_i(\Lambda) \rightarrow \Lambda \lesssim 500$ MeV
- LECs $c_i(\Lambda)$ fixed by fits to data → few-body sector (scattering data, light nuclei)

↓ construct nuclear potentials $V_{\text{NN}}, V_{\text{3N}}, \ldots$

- interaction Hamiltonian $\mathcal{V}(\Lambda, \{c_i\}) = \frac{1}{2!} \sum_{ij,ab} V_{\text{NN}}^{ij,ab} a_i^\dagger a_j^\dagger a_b a_a + \frac{1}{3!} \sum_{ijk,abc} \ldots$

↓ apply many-body method; **here:**

Many-Body Perturbation Theory: $\mathcal{H} = \mathcal{T}_{\text{kin}} + \mathcal{V} = \underbrace{(\mathcal{T}_{\text{kin}} + \mathcal{U})}_{\substack{\text{reference system} \\ \text{"mean-field theory"}}} + \underbrace{(\mathcal{V} - \mathcal{U})}_{\substack{\text{perturbation} \\ \text{"correlations"}}}$

- expand quantity of interest in terms of $\mathcal{V} - \mathcal{U}$
- matrix elements are evaluated in terms of eigenstates of $\mathcal{T}_{\text{kin}} + \mathcal{U}$

  → $\mathcal{U}$ should be a single-particle Hamiltonian: $\mathcal{U} = \sum_r U_r a_r^\dagger a_r$

  → U_r is a self-consistent single-particle potential ("mean-field")
- usual choices: $U_r = 0$ or $U_r = \sum_i \bar{V}^{ir,ir} n_i \equiv U_{1;r}$ (Hartree-Fock)

  → general case $U_r = \sum_n U_{n;r}$, in "improved formalism"

Schedule

Interaction Hamiltonians $\mathcal{V}(\Lambda, \{c_i\})$

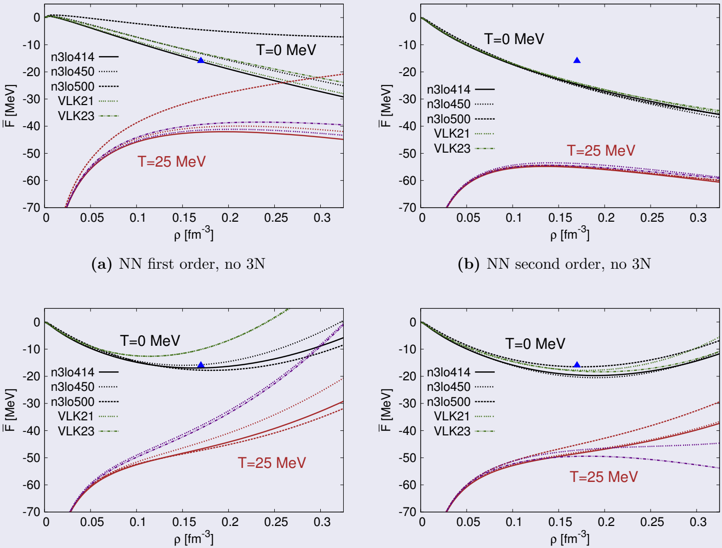
- N3LO two-nucleon + N2LO three-nucleon potential
- nonlocal regulator $f(p, p') = \exp[-(p/\Lambda)^{2n}] - (p'/\Lambda)^{2n}]$
- c_i's from fits to phase shifts, c_D & c_E from fits to ^{3}H binding energy and Gamow-Teller matrix element
- VLK21 & VLK23: NN potential from RG evolution of n3lo450, Nijmegen values for 3N c_i's, c_D & c_E from fits to ^{3}H, ^{3}He, ^{4}He binding energies

	Λ (fm^{-1})	n	c_E	c_D	c_1 (GeV^{-1})	c_3 (GeV^{-1})	c_4 (GeV^{-1})
n3lo414	2.1	10	-0.072	-0.4	-0.81	-3.0	3.4
n3lo450	2.3	3	-0.106	-0.24	-0.81	-3.4	3.4
n3lo500	2.5	2	-0.205	-0.20	-0.81	-3.2	5.4
VLK21	2.1	∞	-0.625	-2.062	-0.76	-4.78	3.96
VLK23	2.3	∞	-0.822	-2.785	-0.76	-4.78	3.96

Coraggio *et al.*; PRC 89 (2014), Entem & Machleidt; PRC 68 (2003), Bogner, Furnstahl, Schwenk, Nogga; NPA 763 (2005), Gazit; Phys.Lett.B 666 (2008) , Nogga, Bogner, Schwenk; PRC 70 (2004)

Results for $U_r = 0$

Isospin-symmetric nuclear matter: $\delta := (\rho_n - \rho_p)/\rho = 0$, $Y := \rho_p/\rho = 1/2$



(a) NN first order, no 3N

(b) NN second order, no 3N

(c) NN second order, 3N first order

(d) NN second order, 3N second order

Wellenhofer, Holt, Kaiser, Weise; PRC 89 (2014)

Results for $U_r = U_{1;r}$

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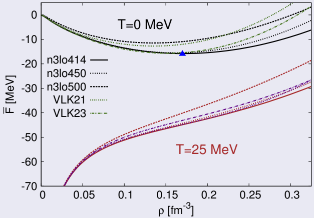
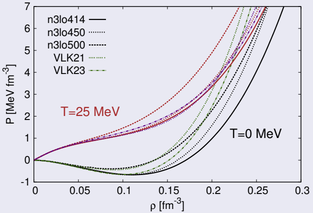
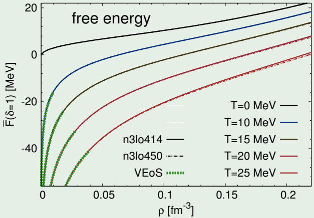
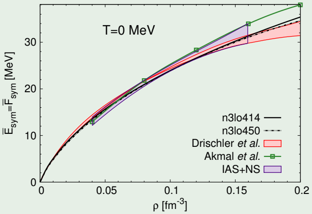




- empirical saturation point: n3lo414, n3lo450, ~~n3lo500~~, ~~VLK21~~, VLK23
- VLK21 & VLK23 ruled out by thermodynamics (pressure isotherm crossing)

Pure neutron matter ($\delta = 1$, $Y = 0$) $\qquad \bar{E}_{sym} := \bar{E}(\delta = 1) - \bar{E}(\delta = 0)$





VEOS: Horowitz & Schwenk, Phys.Lett.B (2006)

Wellenhofer, Holt, Kaiser; PRC 92 (2015)

Isospin-Asymmetric Nuclear Matter

Taylor expansion about $\delta = 0$

$$F(\delta) = F(\delta = 0) + A_2\,\delta^2 + (A_4\,\delta^4 + A_6\,\delta^6 + \ldots) \;\simeq F(\delta = 0) + F_{\text{sym}}\,\delta^2$$

neutron-rich matter: terms beyond δ^2 approximations are important (?)

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neutron-rich matter: terms beyond δ^2 approximations are important (?)

Taylor expansion about $\delta = 0$: **does not exist at $T = 0$**

Exact results (at second order in MBPT) with *S*-wave contact interaction:

$$F_2(T = 0, \rho, \delta) = A_0(0, \rho) + A_2(0, \rho)\,\delta^2 + \sum_{n=2}^{\infty} A_{2n,\text{reg}}(\rho)\,\delta^{2n} + \sum_{n=2}^{\infty} A_{2n,\log}(\rho)\,\delta^{2n} \ln|\delta|$$

Kaiser; PRC 92 (2015), Wellenhofer, Holt, Kaiser; PRC 93 (2016)

- Logarithmic terms also when ladders are resummed to all orders!

Kaiser; EPJA 48 (2014), Wellenhofer; arXiv:1707.09222

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neutron-rich matter: terms beyond δ^2 approximations are important (?)

Taylor expansion about $\delta = 0$: **does not exist at $T = 0$**

Exact results (at second order in MBPT) with S-wave contact interaction:

$$F_2(T=0,\rho,\delta) = A_0(0,\rho) + A_2(0,\rho)\,\delta^2 + \sum_{n=2}^{\infty} A_{2n,\text{reg}}(\rho)\,\delta^{2n} + \sum_{n=2}^{\infty} A_{2n,\log}(\rho)\,\delta^{2n}\ln|\delta|$$

Kaiser; PRC 92 (2015), Wellenhofer, Holt, Kaiser; PRC 93 (2016)

- Logarithmic terms also when ladders are resummed to all orders!

Kaiser; EPJA 48 (2014), Wellenhofer; arXiv:1707.09222

What is the origin of the logarithmic terms at $T = 0$?

$\rightarrow$ energy denominators in contributions beyond first order, e.g.,

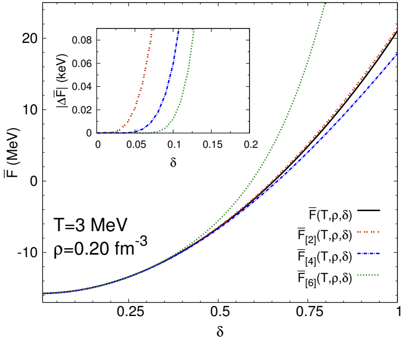
$$E_{0;2} = -\frac{1}{4}\sum_{ijab} \bar{V}_{\text{NN}}^{ij,ab}\,\bar{V}_{\text{NN}}^{ab,ij}\,\frac{\Theta_i^-\Theta_j^-\Theta_a^+\Theta_b^+}{\varepsilon_a+\varepsilon_b-\varepsilon_i-\varepsilon_j}$$

integrand diverges at integral boundary
$\rightsquigarrow E_{0;2} \in C^3$

$$F_2 = -\frac{1}{8}\sum_{ijab} \bar{V}_{\text{NN}}^{ij,ab}\,\bar{V}_{\text{NN}}^{ab,ij}\,\frac{\tilde{f}_i^-\tilde{f}_j^-\tilde{f}_a^+\tilde{f}_b^+ - \tilde{f}_i^+\tilde{f}_j^+\tilde{f}_a^-\tilde{f}_b^-}{\varepsilon_a+\varepsilon_b-\varepsilon_i-\varepsilon_j}$$

smooth integrand
$\rightsquigarrow F_2 \in C^\infty$, but not analytic ($C^\omega$) at low T

Taylor Expansion about $\delta = 0$ at Finite Temperature



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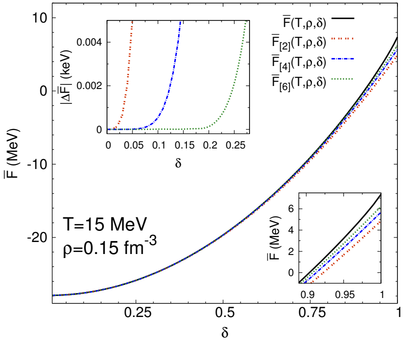

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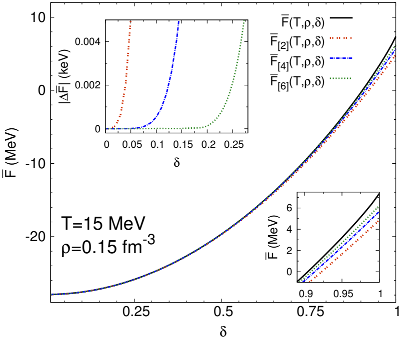
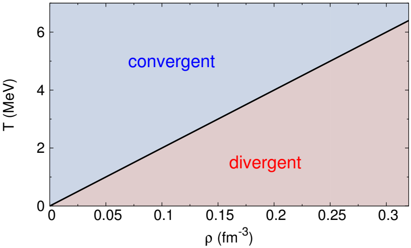
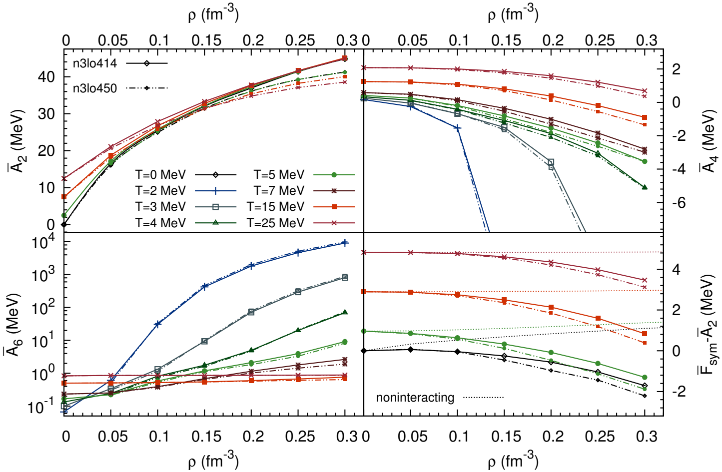




Taylor Coefficients at Finite Temperature

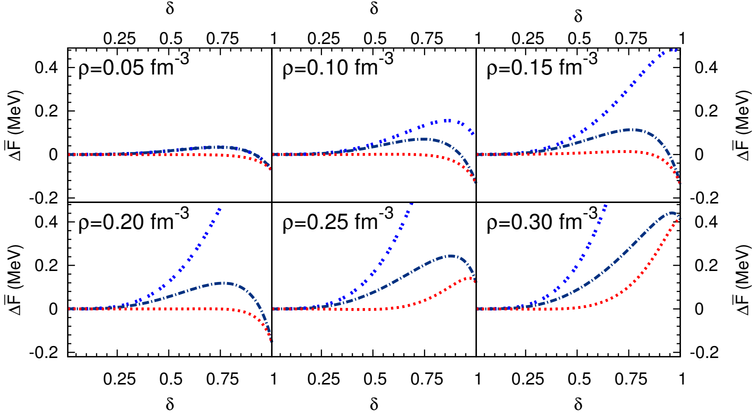
- $A_2 > A_4 > A_6 > \ldots$ at high T, $A_2 \ll A_4 \ll A_6 \ll \ldots$ at low T ($A_{2n\geq4} \xrightarrow{T\to0} \pm\infty$)



- accuracy of parabolic approximation $\sim F_{sym} - A_2$
- dominant contribution to $F_{sym} - A_2$: noninteracting term, 3N interactions

Expansion with Leading Logarithmic Term at $T = 0$

- $E(\delta) \simeq E(0) + A_2\delta^2(+A_4\delta^4)$
- $E(\delta) \simeq E(0) + A_2\delta^2 + A_4\delta^4 + A_{4,\log}\delta^4 \ln|\delta|$



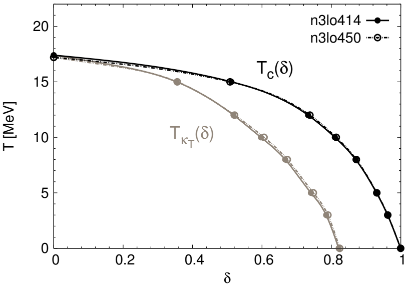
- Question remains: *is the nonanalyticity of the δ dependence a genuine feature of the EOS or only a feature of MBPT?*
- ***Y* dependence** also **nonanalytic**, and this is physical! (entropy of mixing)

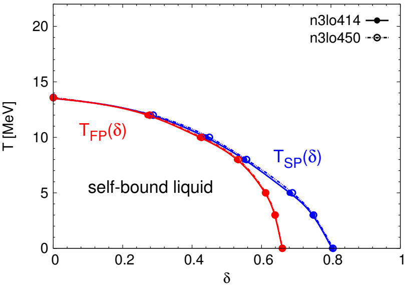
Isospin-Asymmetry Dependence of Nuclear Liquid-Gas Phase Transition

Stability criterion: $\mathcal{F}_{ij} = \frac{\partial^2 F(T,\rho_n,\rho_p)}{\partial\rho_i\partial\rho_j}$ has only positive eigenvalues

- $\delta = 0$: reduces to pure-substance criterion $\partial^2 F/\partial\rho^2 \sim \partial P/\partial\rho > 0$
- isospin distillation in isospin-asymmetric nuclear matter (binary system!)

- endpoint of **critical line** $T_c(\delta)$ at proton fraction $Y = (1-\delta)/2 \simeq 3 \cdot 10^{-4}$
- fragmentation temperature $T_{FP}(\delta)$ endpoint at $Y \simeq 0.17$





→ at large δ: $T_c(\delta)$ strongly influenced by entropy of mixing $\sim T\,Y\ln(Y)$
→ at $T = 0$: terms $\sim Y^{5/3}$ (also from interaction contributions!)

S. Strohmeier; Master's Thesis (2016)

Résumé So Far

- MBPT with chiral low-momentum NN+3N potentials can produce a "realistic" **thermodynamic** nuclear EoS for $\rho \lesssim 2\,\rho_{\text{sat}}$
- accuracy of parabolic δ approximation: decreased for high densities and high temperatures, mainly due to noninteracting term & 3N interactions
- δ dependence is nonanalytic (but smooth) at low T, logarithmic terms (not smooth) at $T = 0$
- Y dependence is nonanalytic (not smooth) $\forall$ T

Prospects/Issues

- thermal pions, hyperons, ... Note: $Y = \rho_p/\rho \xrightarrow{\rho\to 0} 1$ at finite T!!
- better uncertainty quantification:

  → larger set of nuclear potentials, subleading 3N interactions, ...

  → higher orders in MBPT; (state-of-the-art at $T = 0$ is fourth order)

  Drischler, Hebeler, Schwenk; arXiv:1710.08220

  → improved reference state (mean-field): higher-order self-consistent potential $\mathcal{U} = \sum_n \mathcal{U}_n$

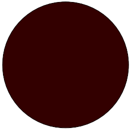
Schedule

1. Basic Formalism
2. Basic Results
3. Improved Formalism

$\mathcal{H} = \mathcal{H}_{\text{mean-field}} = \mathcal{T}_{\text{kin}} + \mathcal{U}$: Ground-State and Excited States

Self-consistent single-particle energies: $\varepsilon_r = \frac{k^2}{2M} + U_r[n_r(\varepsilon_r)]$, $r \equiv \{\vec{k}, \sigma, \tau\}$

Ground-state energy: $E_0 = \sum_r \varepsilon_r n_r^{(0)}$



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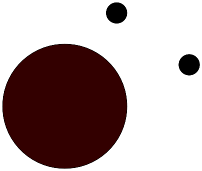

Excited Microstate: $E_i^* = \sum_r \varepsilon_r (n_r^{(0)} + \Delta n_r)_i$

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$\Rightarrow \varepsilon_r \equiv \varepsilon_r[n_r]$: change in distribution function changes spectrum

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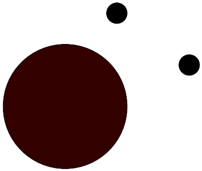

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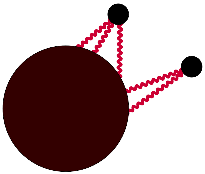
Quasiparticles!



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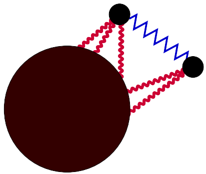

Quasiparticles!

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$= \sum_r \varepsilon_r^{(0)} (n_r^{(0)} + \Delta n_r)_i + \frac{1}{2} \sum_{r,r'} f_{rr'} (\Delta n_r \Delta n_r')_i + \ldots$

Landau Fermi-Liquid Theory (dynamical)

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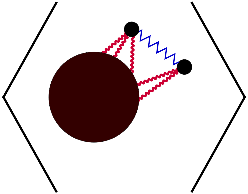

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Landau Fermi-Liquid Theory (statistical)

Macrostate (finite $T = 0 + \Delta T$):

$$E = \sum_i P_i E_i^* \big|_{E-\delta E < E_i^* < E} = \sum_r \varepsilon_r^{(0)} \underbrace{\sum_i P_i (n_r^{(0)} + \Delta n_r)}_{n_r = 1/(1+\exp[(\varepsilon_r^{(0)} - \mu)/T]} + \ldots$$

$\mathcal{H} = \mathcal{H}_{\text{mean-field}} = \mathcal{T}_{\text{kin}} + \mathcal{U}$: Thermodynamic Relations

Expansion about (statistical) reference state works also for $T = T_0 + \Delta T$

$$\Omega = T\sum_r \ln(1 - n_r)$$
$$= T\sum_r n_r \ln n_r + \bar{n}_r \ln \bar{n}_r + \sum_r (\varepsilon_r^{\text{free}} + U_r - \mu) n_r \equiv \Omega'$$

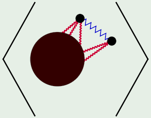
$$\boxed{\frac{\delta \Omega'}{\delta n_r} = 0\,!!}$$

$$N = -\frac{d\Omega}{d\mu}\Big|_T = -\frac{\partial \Omega'}{\partial \mu} - \overbrace{\frac{\delta \Omega'}{\delta n_r}}^{=0} \frac{\partial n_r}{\partial \mu} = \sum_r n_r$$

$$S = -\frac{d\Omega}{dT}\Big|_\mu = \ldots = -\sum_r n_r \ln n_r + \bar{n}_r \ln \bar{n}_r$$

$$F = \mu N + \Omega$$

$$E = F + TS = \sum_r (\varepsilon_r^{\text{free}} + U_r) n_r, \quad \frac{\delta E}{\delta n_r} = \varepsilon_r^{\text{free}} + U_r$$



$\rightarrow$ statistical quasiparticles

Fully interacting system: $\mathcal{H} = \mathcal{H}_{\text{mean-field}} + (\mathcal{V} - \mathcal{U})$

- form of elementary excitations?
- form of thermodynamic relations?
- what is the proper choice of $\mathcal{U}$?

Elementary Excitations of Interacting Fermi Fluids

→ Green's function $iG_r^>(t-t') = \langle a_r(t)a_r^\dagger(t')\rangle_{t>t'} \xrightarrow{\mathcal{H}=\mathcal{H}_{\text{mean-field}}} \bar{n}_r e^{-i\varepsilon_r(t-t')}$
quasiparticle excitation

But: can compute $iG_r^>(t)$ only for imaginary t

→ Fourier transform: $iG_r^>(t) = \int \frac{d\omega}{2\pi} G_r^>(\omega)\, \mathrm{e}^{-i\omega t} = \int \frac{d\omega}{2\pi} \bar{n}(\omega) A_r(\omega)\, \mathrm{e}^{-i\omega t}$

spectral function $A_r(\omega) = \frac{2J_r(\omega)}{[\omega-\varepsilon_r-K_r(\omega)]^2+[J_r(\omega)]^2}$ (Breit-Wigner)

→ compute $\Sigma_r(\omega - i\eta) = K_r(\omega) + iJ_r(\omega)$, where $\Sigma(\zeta)$ (a particular!) analytic cont. of irred. Matsubara self-energy $\Sigma(\zeta_\ell)$, with $\zeta_\ell = T(2\ell+1)\pi i + \mu$

Kadanoff & Baym; "Quantum Statistical Mechanics"

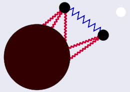
$T = 0$: $J_r(\omega) = -C_r(\omega-\mu)|\omega-\mu|$, with $\mu = \varepsilon_{k_F} + K_{k_F}(\mu) \equiv \mathcal{E}_{k_F}$

Luttinger; PR 121 (1960)

→ $k \simeq k_F$: $A_k(\omega)$ has strong peak at $\mathcal{E}_k = \varepsilon_k + K_k(\mathcal{E}_k) \simeq \mu$

→ contribution from $\omega = \mathcal{E}_k$: $iG_r^>(t) \sim \mathrm{e}^{-i\mathcal{E}_k t}\, \mathrm{e}^{-\gamma t}$

low-lying elementary excitations have quasiparticle form (approximately) on short time scales

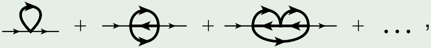


→ quasiparticle decay $\rightsquigarrow$ collective excitation

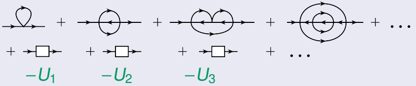
Dynamical-Quasiparticle Constraint on $\mathcal{U}$

→ compute $\Sigma_r(\omega - i\eta) = K_r(\omega) + iJ_r(\omega)$, where $\Sigma(\zeta)$ (a particular!) analytic cont. of irred. Matsubara self-energy $\Sigma(\zeta_\ell)$, with $\zeta_\ell = T(2\ell + 1)\pi i + \mu$

Self-Consistent Green's Function Approach

$\Sigma(\zeta_\ell) =$  , $\longrightarrow \equiv \mathcal{G}_r(\zeta_{\ell'}) = \int \frac{d\omega}{2\pi} \frac{A_r(\omega)}{\zeta_{\ell'} - \omega}$

Perturbative Approach, $\mathcal{H} = \mathcal{H}_{\text{mean-field}} + (\mathcal{V} - \mathcal{U})$

$\Sigma(\zeta_\ell) =$  , $\longrightarrow \equiv \mathcal{G}^0_r(\zeta_{\ell'}) = \frac{1}{\zeta_{\ell'} - \omega}$

$$\Sigma_{1;r}(\zeta) = 0$$

$$\Sigma_{2;r}(\zeta) = -\frac{1}{2} \sum_{jkl} |\bar{V}^{rjkl}|^2 \frac{n_j \bar{n}_k \bar{n}_l + \bar{n}_j n_k n_l}{\zeta + \varepsilon_j - \varepsilon_k - \varepsilon_l} - U_{2;r} \xrightarrow{\zeta = \omega - i\eta} \overbrace{\tilde{K}_{2;r}(\omega) - U_{2;r}}^{K_{2;r}(\omega)} + iJ_{2;r}(\omega)$$

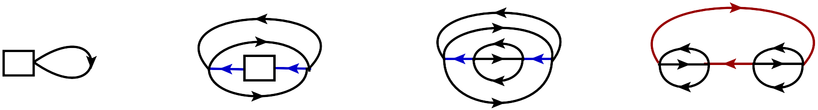
with $J_{2;r}(\omega) = 0$ for $\omega = \varepsilon_{k_F}$, but: $J_r^{(2)}(\omega) \stackrel{!}{=} 0$ for $\omega = \varepsilon_{k_F} + K_{k_F}(\mu)$

Luttinger; PR 121 (1960)

$$\Rightarrow \boxed{U_{2;k_F} \stackrel{!}{=} \tilde{K}_{2;k_F}(\mu)}$$

Anomalous Contributions and Statistical Quasiparticles

Perturbation series for free energy F: "closed" diagrams, e.g., at 4th order



normal ($\notin \tilde{F}$) normal ($\in \tilde{F}$) normal ($\in \tilde{F}$) anomalous ($\in \tilde{F}$)

- $U^{(I)}_{n;r} = \text{Re}\big[\Sigma_{n;r}(\varepsilon_r - i\eta)\big] = \dfrac{\delta \tilde{F}_n}{\delta n_r}\Big|_{r \notin \{\text{articulation lines}\}}$ satisfies $U_{n;k_F} \overset{!}{=} \tilde{K}_{n;k_F}(\mu)$
- $U^{(II)}_{n;r} = \dfrac{\delta \mathscr{D}_n}{\delta n_r}$ does not satisfy $U_{n;k_F} \overset{!}{=} \tilde{K}_{n;k_F}(\mu)$

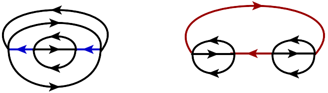
where $\mathscr{D}$ given by (reduced & disentangled & regularized) normal part of $\tilde{F}$

$$F = F_0 + \tilde{F}_1 + \tilde{F}_2 + \tilde{F}_3 + \tilde{F}_4 + \ldots + \tilde{F}_N - \sum_{n=1}^{N} \sum_r U_{n;r} n_r$$

$$\downarrow U_{n;r} = U^{(II)}_{n;r}$$

$$= \underbrace{F_0 + \tilde{F}_1 + \tilde{F}_2 + \tilde{F}_3 + \tilde{F}_{4,\text{normal}} + \ldots + \tilde{F}_{N,\text{normal}}}_{\mu N + \Omega' + \mathscr{D}} - \sum_{n=1}^{N} \sum_r U^{(II)}_{n;r} n_r$$

“Reduced & Disentangled & Regularized”



(a) (b)

These two diagrams are “entangled” (→ cyclic permutations):

$$F^{\text{cyclic}}_{a+b} = -\frac{1}{4}\sum_{xjaklmn} n_{xxja}\,\bar{n}_{klmn}\,[V]\left\{\frac{1}{\varepsilon_1^2(\varepsilon_1+\varepsilon_2)} - \frac{e^{-(\varepsilon_1+\varepsilon_2)/T}}{\varepsilon_2^2(\varepsilon_1+\varepsilon_2)} + \frac{e^{-\varepsilon_1/T}(-\beta\varepsilon_1\varepsilon_2+\varepsilon_1-\varepsilon_2)}{\varepsilon_1^2\varepsilon_2^2}\right\}$$

No energy-denominator poles (*good!*), “double-indices” mix normal-anomalous (*not so good*)

- **“reduced”**: normal diagrams similar to $T = 0$ formalism (note: $F_a^{\text{reduced}} = \infty$ for $T \neq 0$)

$$F_a^{\text{reduced}} = -\frac{1}{4}\sum_{xjaklmn} n_{xxja}\,\bar{n}_{klmn}\,[V]\left\{\frac{1}{\varepsilon_1^2(\varepsilon_1+\varepsilon_2)}\right\} \xrightarrow{T\to 0} E_a$$

- **“disentangled”**: normal without “double-indices”, anomalous factorized!!

(but still $F_a^{\text{reduced,disentangled}} = \infty$ for $T \neq 0$)

$$F_a^{\text{reduced,disentangled}} = -\frac{1}{4}\sum_{xjaklmn} n_{xja}\,\bar{n}_{klmn}\,[V]\left\{\frac{1}{\varepsilon_1^2(\varepsilon_1+\varepsilon_2)}\right\} \xrightarrow{T\to 0} E_a$$

$$F_b^{\text{reduced,disentangled}} = -\frac{1}{4}\sum_{xjaklmn} n_{xakl}\,\bar{n}_{xjmn}\,[V]\left\{\frac{-\beta}{\varepsilon_1\varepsilon_2}\right\} = -\beta\sum_x \frac{\delta F_2}{\delta n_x}\, n_x\bar{n}_x \underbrace{\frac{\delta F_2}{\delta n_x}}_{U_{2,x}}$$

- **“regularized”**: finite part $\mathcal{P}$ **plus** cyclic permutations of integration order (poles!)

$$F_a^{\text{reduced,disentangled,regularized}} = -\frac{1}{4}\frac{1}{|C[xjaklmn]|}\sum_{C[xjaklmn]} n_{xja}\,\bar{n}_{klmn}\,[V]\left\{\frac{\mathcal{P}}{\varepsilon_1^2(\varepsilon_1+\varepsilon_2)}\right\} \xrightarrow{T\to 0} E_a$$

$\mathcal{H} = \mathcal{H}_{\text{mean-field}} = \mathcal{T}_{\text{kin}} + \mathcal{U}$: Thermodynamic Relations

Expansion about (statistical) reference state works also for $T = T_0 + \Delta T$

$$\Omega = T \sum_r \ln(1 - n_r)$$
$$= T \sum_r n_r \ln n_r + \bar{n}_r \ln \bar{n}_r + \sum_r (\varepsilon_r^{\text{free}} + U_r - \mu) n_r \equiv \Omega'$$

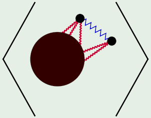
$$\frac{\delta \Omega'}{\delta n_r} = 0 \,!!$$

$$N = -\frac{d\Omega}{d\mu}\Big|_T = -\frac{\partial \Omega'}{\partial \mu} - \overbrace{\frac{\delta \Omega'}{\delta n_r}}^{=0} \frac{\partial n_r}{\partial \mu} = \sum_r n_r$$

$$S = -\frac{d\Omega}{dT}\Big|_\mu = \ldots = -\sum_r n_r \ln n_r + \bar{n}_r \ln \bar{n}_r$$

$$F = \mu N + \Omega$$

$$E = F + TS = \sum_r (\varepsilon_r^{\text{free}} + U_r) n_r, \quad \frac{\delta E}{\delta n_r} = \varepsilon_r^{\text{free}} + U_r$$



→ statistical quasiparticles

Fully interacting system: $\mathcal{H} = \mathcal{H}_{\text{mean-field}} + (\mathcal{V} - \mathcal{U})$

- form of elementary excitations?
- form of thermodynamic relations?
- what is the proper choice of $\mathcal{U}$?

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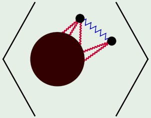


$\rightarrow$ statistical quasiparticles

Fully interacting system: $\mathcal{H} = \mathcal{H}_{\text{mean-field}} + (\mathcal{V} - \mathcal{U})$

- form of elementary excitations?
- form of thermodynamic relations?
- what is the proper choice of $\mathcal{U}$?

$\mathcal{H} = \mathcal{H}^{(II)}_{\text{mean-field}} + \mathcal{V} - \mathcal{U}^{(II)}$: Thermodynamic Relations

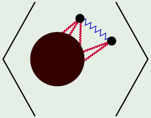
Expansion about (statistical) reference state works also for $T = T_0 + \Delta T$

$$\Omega = T \sum_r \ln(1 - n_r) + \mathscr{D} - \sum_r U_r^{(II)} n_r$$
$$= T \sum_r n_r \ln n_r + \bar{n}_r \ln \bar{n}_r + \sum_r (\varepsilon_r^{\text{free}} - \mu) n_r + \mathscr{D} \equiv \Omega'$$

$$\boxed{\frac{\delta \Omega'}{\delta n_r} = 0\,!!}$$

$$N = -\frac{d\Omega}{d\mu}\Big|_T = -\frac{\partial \Omega'}{\partial \mu} - \overbrace{\frac{\delta \Omega'}{\delta n_r}}^{=0} \frac{\partial n_r}{\partial \mu} = \sum_r n_r$$

$$S = -\frac{d\Omega}{dT}\Big|_\mu = \ldots = -\sum_r n_r \ln n_r + \bar{n}_r \ln \bar{n}_r$$



$$F = \mu N + \Omega$$

$$E = F + TS = \sum_r \varepsilon_r^{\text{free}} n_r + \mathscr{D}, \qquad \frac{\delta E}{\delta n_r} = \varepsilon_r^{\text{free}} + U_r$$

$\rightarrow$ statistical quasiparticles

Balian, Bloch, de Dominicis; various (1958-1971), Horwitz, Brout, Englert, PR 120 (1961), PR 130 (1963); Wellenhofer; arXiv:1707.09222

Fully interacting system: $\mathcal{H} = \mathcal{H}_{\text{mean-field}} + (\mathcal{V} - \mathcal{U})$

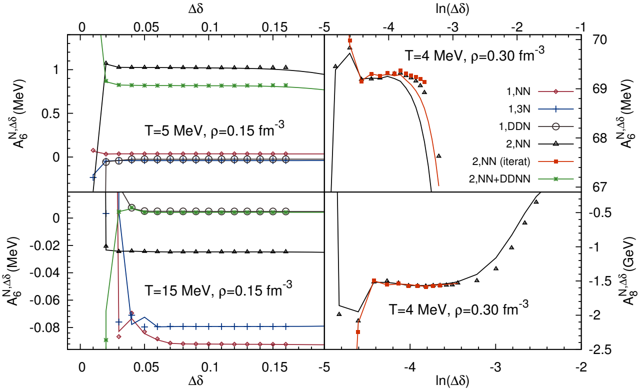
- form of elementary excitations? $\rightarrow$ quasiparticles at short time-scales and low energies
- form of thermodynamic relations? $\rightarrow$ quasiparticle relations for S, N, and $\delta E/\delta n_r$
- what is the proper choice of $\mathcal{U}$? $\rightarrow$ $\mathcal{U}^{(I)}$ (dynamical) or $\mathcal{U}^{(II)}$ (statistical)

A1: Extraction of Isospin-Asymmetry Coefficients

General of $2N+1$ point central finite difference approximation for $\bar{A}_{2n}(T,\rho)$

$$\bar{A}_{2n}(T,\rho) \simeq \bar{A}_{2n}^{N,\Delta\delta}(T,\rho) = \frac{1}{(2n)!\,(\Delta\delta)^{2n}} \sum_{k=0}^{N} \omega_{2n}^{N,k}\, \bar{F}(T,\rho,k\Delta\delta).$$

Fornberg; Math.Comp 51 (1988)



- stepsize ($\Delta\delta$) and grid length (N) variations as accuracy checks
- systematically increase precision of numerical integration routine

A2: Extraction of Leading Logarithmic Term at Zero Temperature

- finite differences of zero-temperature logarithmic series ($\sim \delta^{2n\geq 4}\ln|\delta|$):

$$\bar{A}_4^{N,\Delta\delta} = \bar{A}_{4,\mathrm{reg}} + C_1^4(N)\bar{A}_{4,\log} + \bar{A}_{4,\log}\ln(\Delta\delta) + C_2^4(N)\bar{A}_{6,\log}\Delta\delta^2 + \mathcal{O}(\Delta\delta^4), \tag{3.1}$$

$$\bar{A}_6^{N,\Delta\delta} = \bar{A}_{6,\mathrm{reg}} + C_1^6(N)\bar{A}_{4,\log}\Delta\delta^{-2} + \bar{A}_{6,\log}\ln(\Delta\delta) + C_2^6(N)\bar{A}_{6,\log} + \mathcal{O}(\Delta\delta^2). \tag{3.2}$$

- extract leading logarithmic term via:

$$\Xi_4(N_1,N_2) := \frac{\bar{A}_4^{N_1,\Delta\delta} - \bar{A}_4^{N_2,\Delta\delta}}{C_4^1(N_1) - C_4^1(N_2)} \simeq \bar{A}_{4,\log}, \tag{3.3}$$

$$\Xi_6(N_1,N_2) := \frac{\bar{A}_6^{N_1,\Delta\delta} - \bar{A}_6^{N_2,\Delta\delta}}{C_6^1(N_1) - C_6^1(N_2)}\Delta\delta^2 \simeq \bar{A}_{4,\log}, \tag{3.4}$$

- benchmark against analytical results for *S*-wave contact interaction