



Priority Issue 9  
to be Tackled by Using Post K Computer  
“Elucidation of the Fundamental Laws and  
Evolution of the Universe”  
KAKENHI grant 17K05433

“Nuclear *ab initio* Theories  
and Neutrino Physics”  
@ INT, Washington, 2018/03/07

# Double Gamow-Teller transitions and its relation to neutrinoless $\beta\beta$ decay



CENTER for  
NUCLEAR STUDY



Noritaka Shimizu (CNS, U. Tokyo)

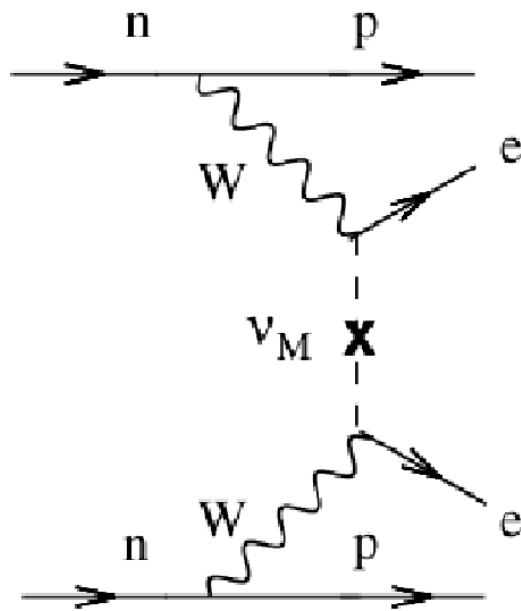
N. Shimizu, J. Menendez, and K. Yako,  
Phys. Rev. Lett. accepted, arXiv:1709.01088 [nucl-th]

# Outline

- Large scale shell model calculations of neutrinoless double beta decay nuclear matrix element ( $0\nu\beta\beta$  NME) of  $^{48}\text{Ca}$
- Double Gamow Teller Resonance and its relation to  $0\nu\beta\beta$  NME of  $^{48}\text{Ca}$
- Relation between double Gamow Teller transition and  $0\nu\beta\beta$  NME, systematic study

# Nuclear Matrix Element (NME) of neutrinoless double-beta decay

Majorana particle or not?  
neutrinoless double beta decay

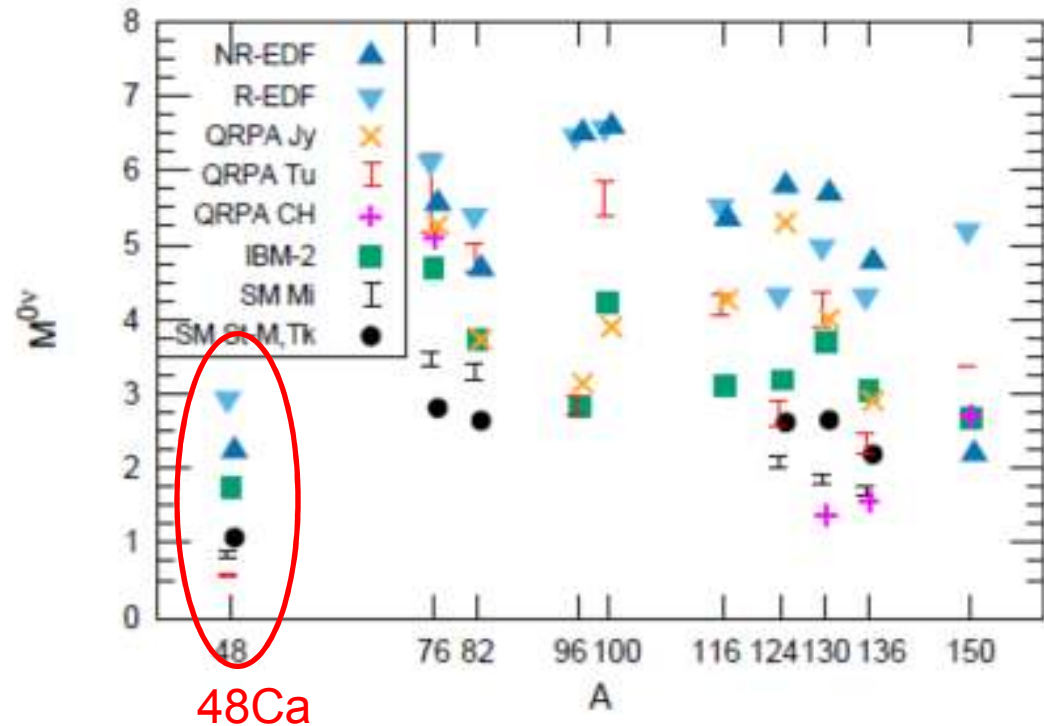


lepton number violation  
(beyond the standard model)

Theoretical prediction on the  $0\nu\beta\beta$  NME varies depending on theoretical models.

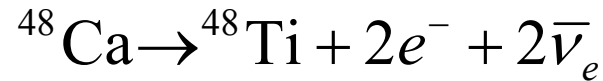
$$[T_{1/2}^{0\nu}]^{-1} = G_1^{0\nu} |M^{0\nu}|^2 \left( \frac{\langle m_{\beta\beta} \rangle}{m_e} \right)^2$$

↑ Half life (exp.)      ↑ NME      ↑ effective neutrino mass

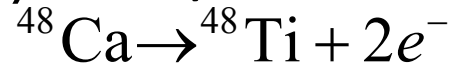


# SM calc. for nuclear matrix element (NME) of $^{48}\text{Ca}$ $0\nu\beta\beta$ decay

$2\nu\beta\beta$  decay



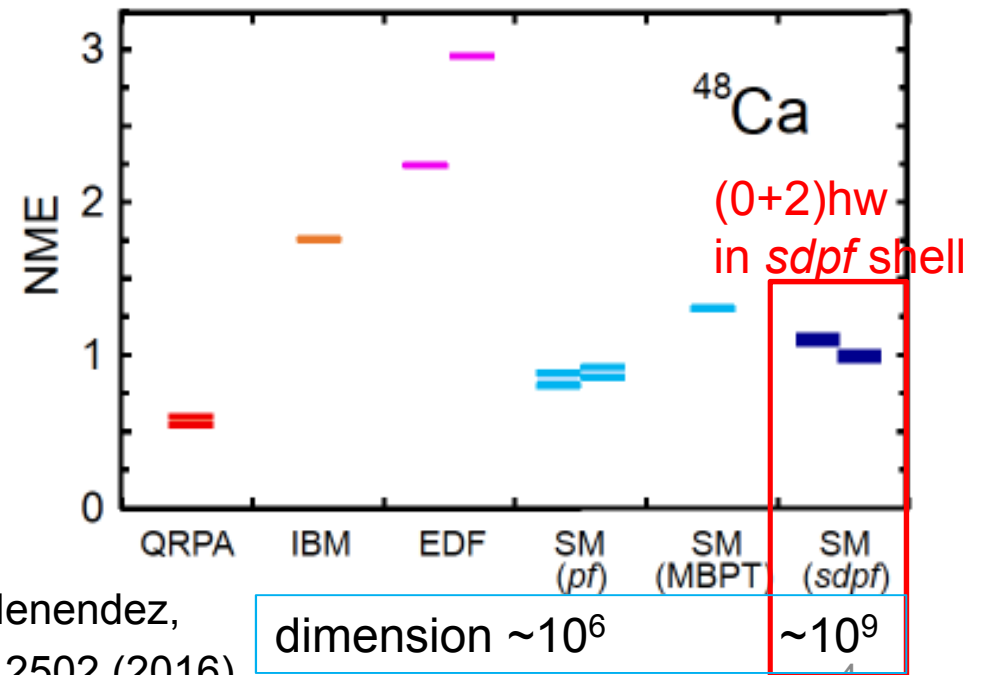
$0\nu\beta\beta$  decay



$$[T_{1/2}^{0\nu}]^{-1} = G_1^{0\nu} |M^{0\nu}|^2 \left( \frac{\langle m_{\beta\beta} \rangle}{m_e} \right)^2$$

↑ Half life (exp.)      ↑ NME      ↑ effective neutrino mass

Large scale shell model calculation including 2hw excitation from  $sd$  shell with closure approximation

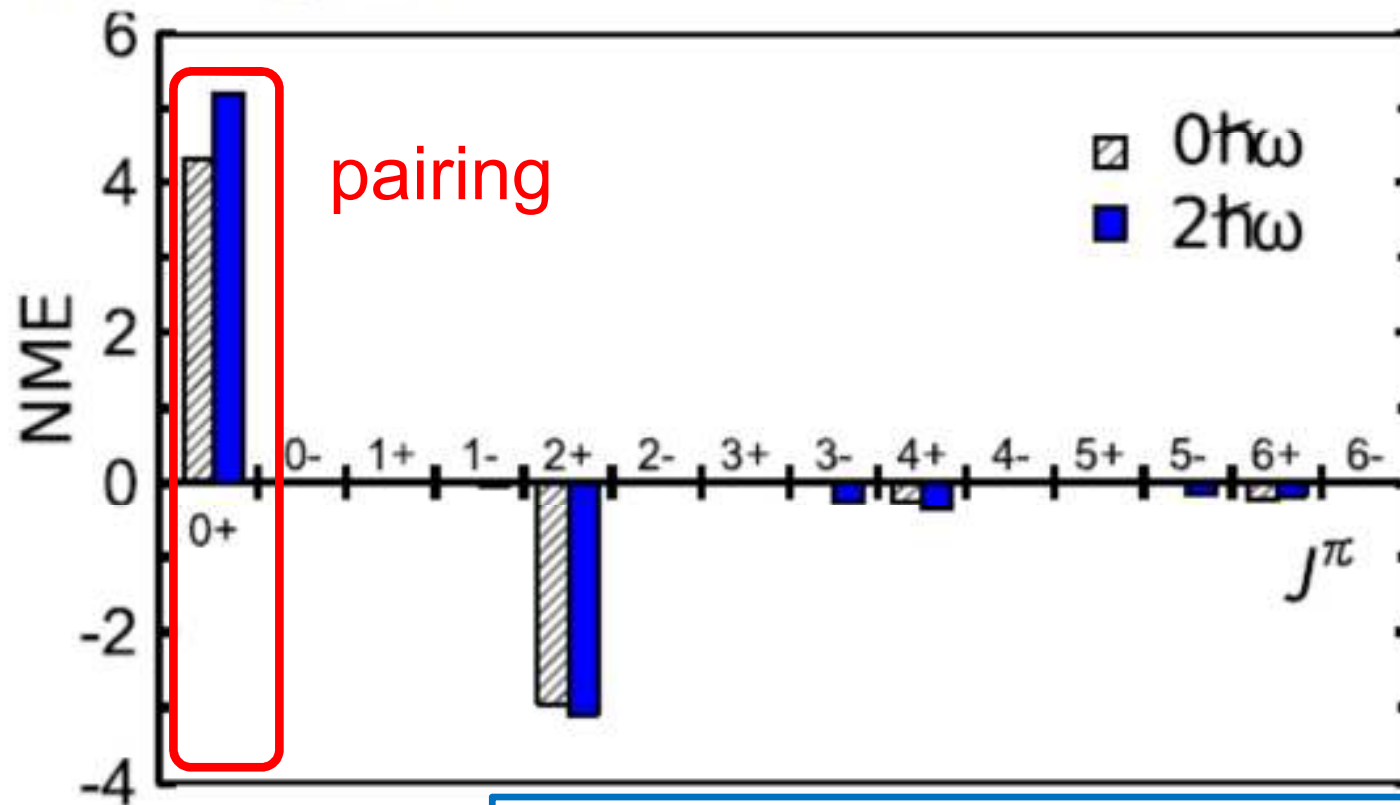


Y. Iwata, N. Shimizu, T. Otsuka, Y. Utsuno, J. Menendez, M. Honma and T. Abe, Phys. Rev. Lett. **116**, 112502 (2016)

# Why does the NME increases by extending the model space?

decompose this sum

$$M^{0\nu} = \sum_J \langle 0_f^+ | \sum_{i \leq j, k \leq l} M_{ij,kl}^J [(\hat{a}_i^\dagger \hat{a}_j^\dagger)^J (\hat{a}_k \hat{a}_l)^J]^0 | 0_i^+ \rangle$$



⇒ Talk by T. Otsuka, tomorrow

# $0\nu\beta\beta$ -decay NME and double Gamow-Teller (DGT) transition

- $0\nu\beta\beta$ -decay nuclear matrix element (NME) with closure approximation

$$M^{0\nu} = M_{GT}^{0\nu} - \left(\frac{g_V}{g_A}\right)^2 M_F^{0\nu} + M_T^{0\nu}$$

$$\mathcal{O}_{GT} = \tau_{1-}\tau_{2-} (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) H_{GT}(r, E_\kappa),$$

$$\mathcal{O}_F = \tau_{1-}\tau_{2-} H_F(r, E_\kappa),$$

$$\mathcal{O}_T = \tau_{1-}\tau_{2-} S_{12} H_T(r, E_\kappa),$$

neutrino potential

$$H_\alpha(r, E_\kappa) = \frac{2R}{\pi} \int_0^\infty \frac{f_\alpha(qr)h_\alpha(q^2)q dq}{q + E_\kappa - (E_i + E_f)/2}$$

N.B. GT-type NME is dominant

- DGT transition

$$\mathcal{O}^\pm = [\sigma t^\pm \otimes \sigma t^\pm]^{(\lambda)} \quad \lambda = 0, 2$$

# Double Gamow-Teller transition

- DGT transition probability

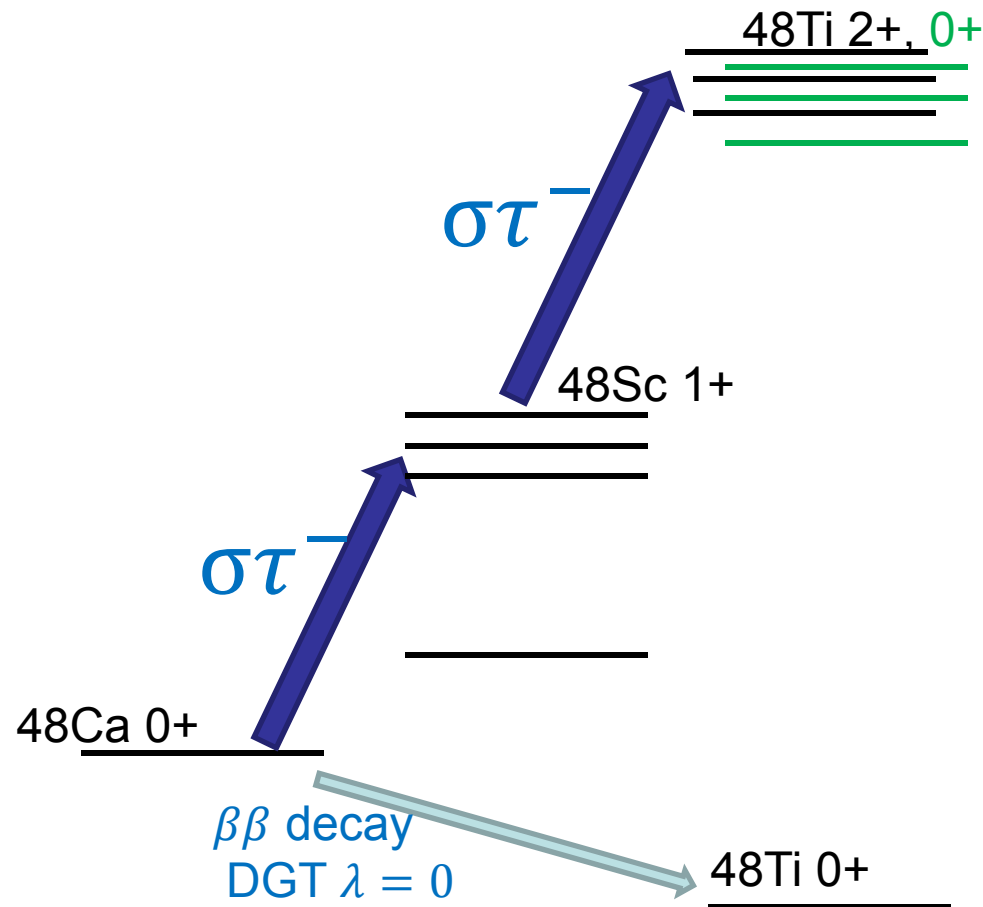
$$- B(DGT; \lambda) = \frac{1}{2J_i+1} \langle J_f || [\sigma t^- \otimes \sigma t^-]^{(\lambda)} || J_i \rangle^2$$

Theory: Auerbach 1989, Zheng 1989, Muto 1981, Sagawa 2016

- DGTR itself attracts attention as an exotic collective motion
- In the first half of this talk, focus on  $^{48}\text{Ca}$ 
  - one of  $\beta\beta$  decay nuclei with large Q value
  - shell model calc. is a suitable theoretical method
  - DGT resonance (DGTR) was/will be measured experimentally

Takaki at RCNP/Osaka, plan: RIBF/RIKEN, INFN/Catania

c.f. Cappuzello et al., Euro Phys. J. A. 51 145 (2015)



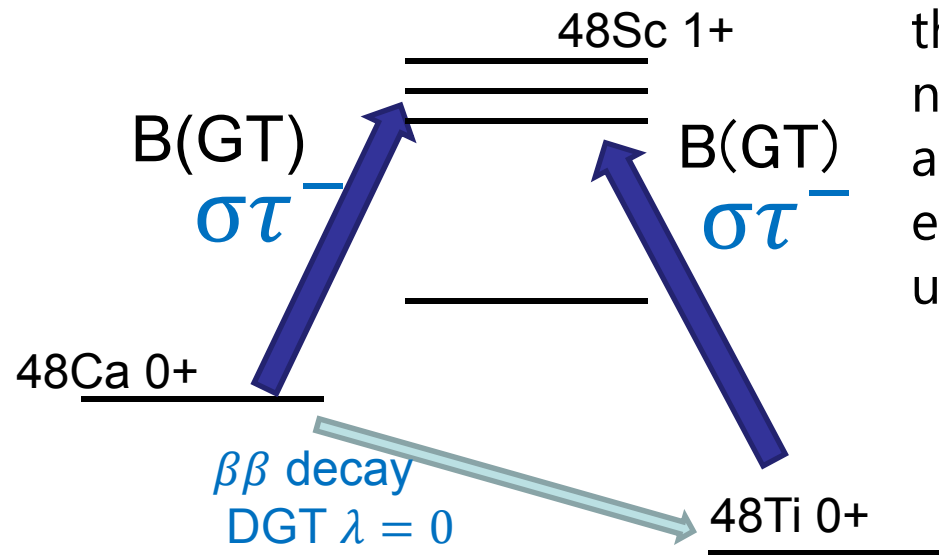
DGTR  $\lambda = 0, 2$

By studying the stronger DGT transitions experimentally (...), theoretically, one may be able to “calibrate” the calculations of  $2\beta$ -decay nuclear elements.

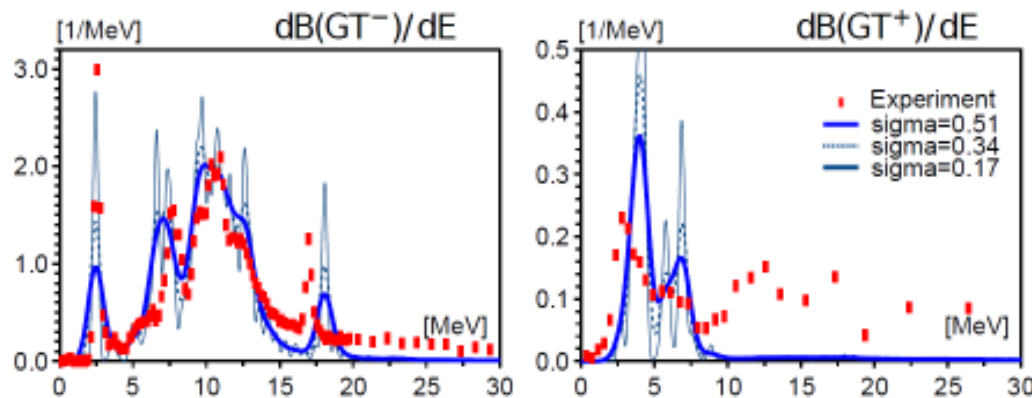
N. Auerbach, L. Zamick and D.C.Zheng  
Ann. Phys. **192**, 197 (1989)

“smearing” the Fermi surface. The matrix element, however, still remains very small and accounts for only a  $10^{-4}$  to  $10^{-3}$  fraction of the total DGT sum rule [13]. A precise calculation of such hindered transitions is, of course, very difficult and is inherently a subject of large percent uncertainties. At the present there is no direct way to “calibrate” such complicated nuclear structure calculations involving miniature fractions of the two-body DGT transitions. By studying the stronger DGT transitions and, in particular, the giant DGT states experimentally and as we do here, theoretically, one may be able to “calibrate” the calculations of  $2\beta$ -decay nuclear elements.





Both sides of the Gamow-Teller transitions are also useful for the "calibration" of the  $\beta\beta$ -decay nuclear elements. However only absolute values can be measured experimentally. (relative phase unknown)

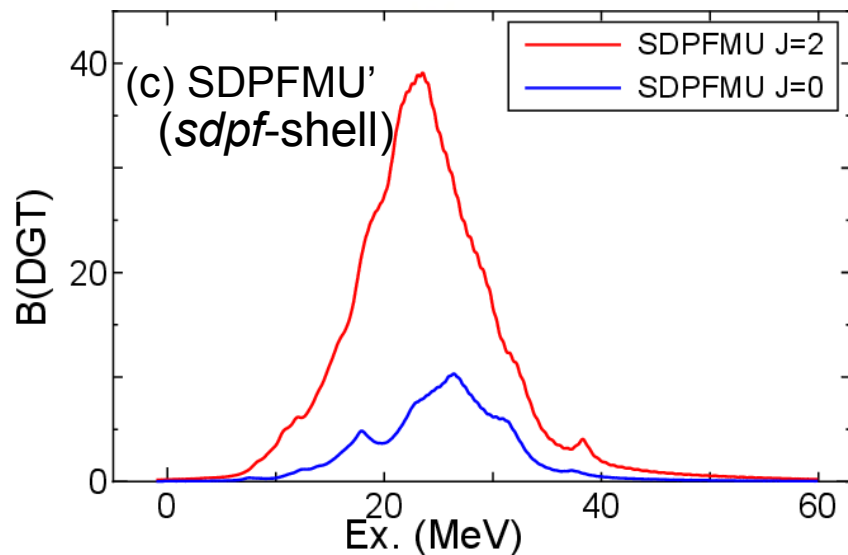
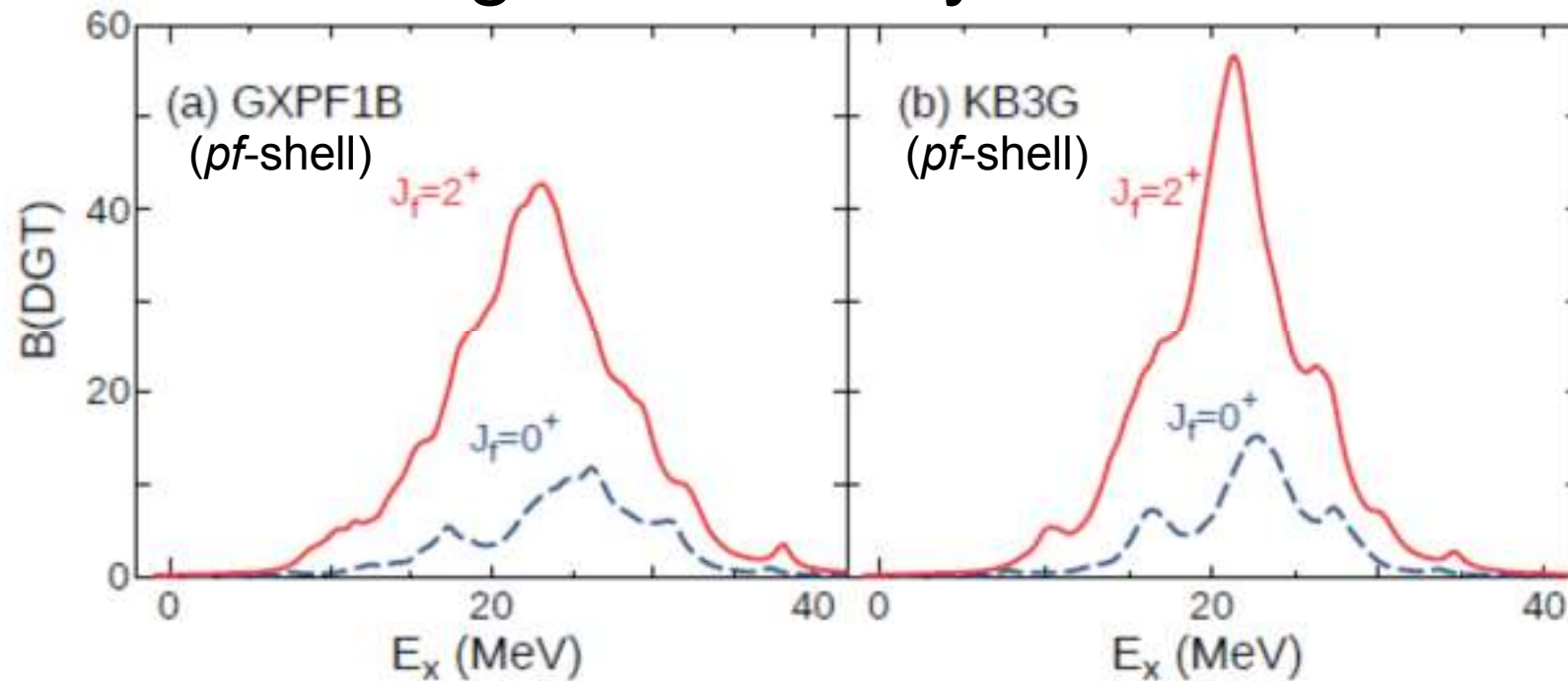


Red symbol : exp.  
Blue line: shell-model calc.

Y. Iwata *et al.*, JPS Conf. Proc. **6**, 030057 (2015)  
Exp. K. Yako *et al.*, Phys. Rev. Lett. **103**, 012503 (2009).

Double Gamow-Teller  
Resonance in  $^{48}\text{Ca}$   
by shell-model calculations

# DGTR strength of $^{48}\text{Ca}$ by shell-model calc.



Lanczos strength function  
smeared out by Lorentzian  $\Gamma = 1$  MeV

focus on GXPF1B and *pf* shell  
hereafter

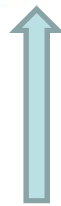
# Dependence of isoscalar pairing

We artificially add the isoscalar pairing interaction

$$H' = H + G^{10} P^{J=1, T=0}$$

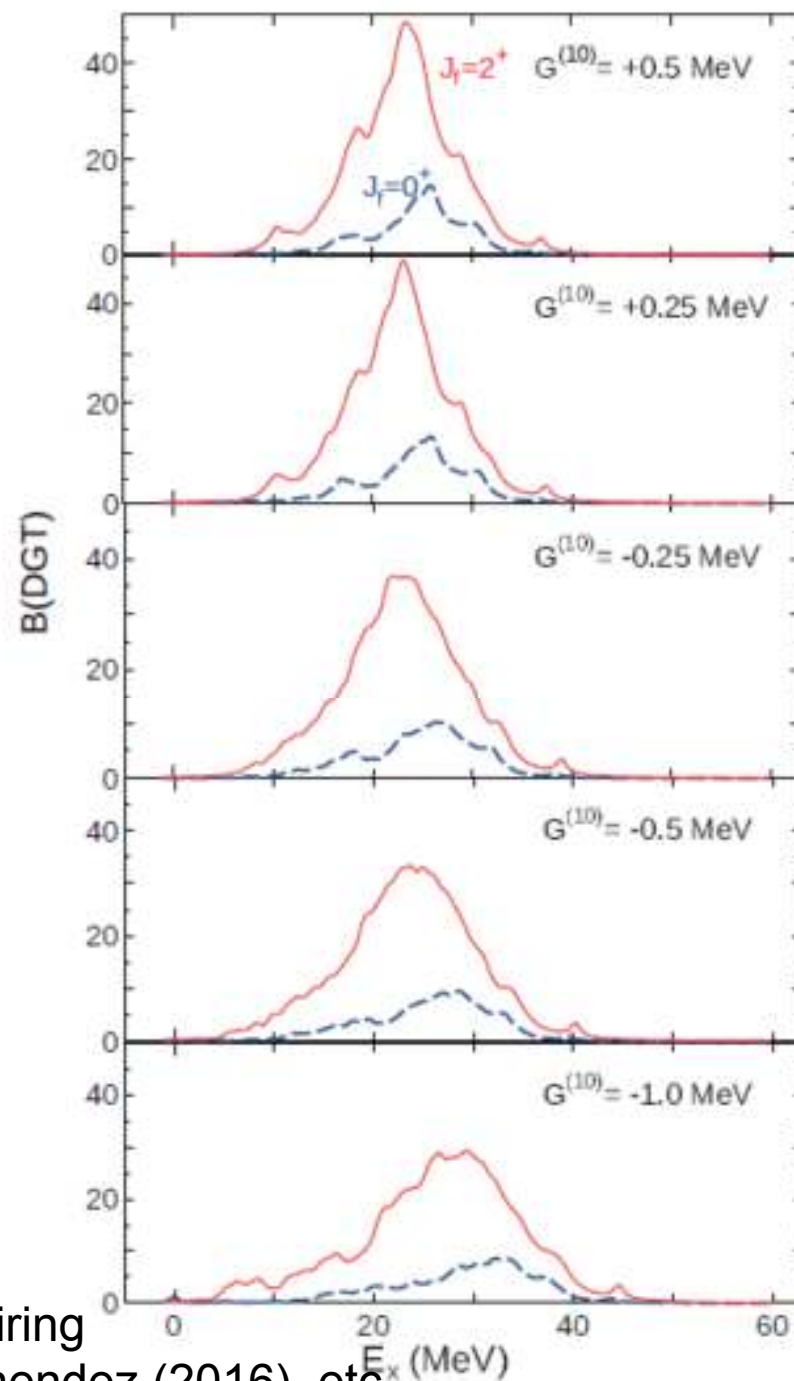


GXPF1B



Isoscalar pairing int.

The NME is sensitive to the J=1 proton-neutron matrix element, or isoscalar pairing  
 Vogel (1986), Muto (1991), Rodin (2003) Menendez (2016), etc.



# Isoscalar pairing dependence: $0\nu\beta\beta$ decay NME and DGT

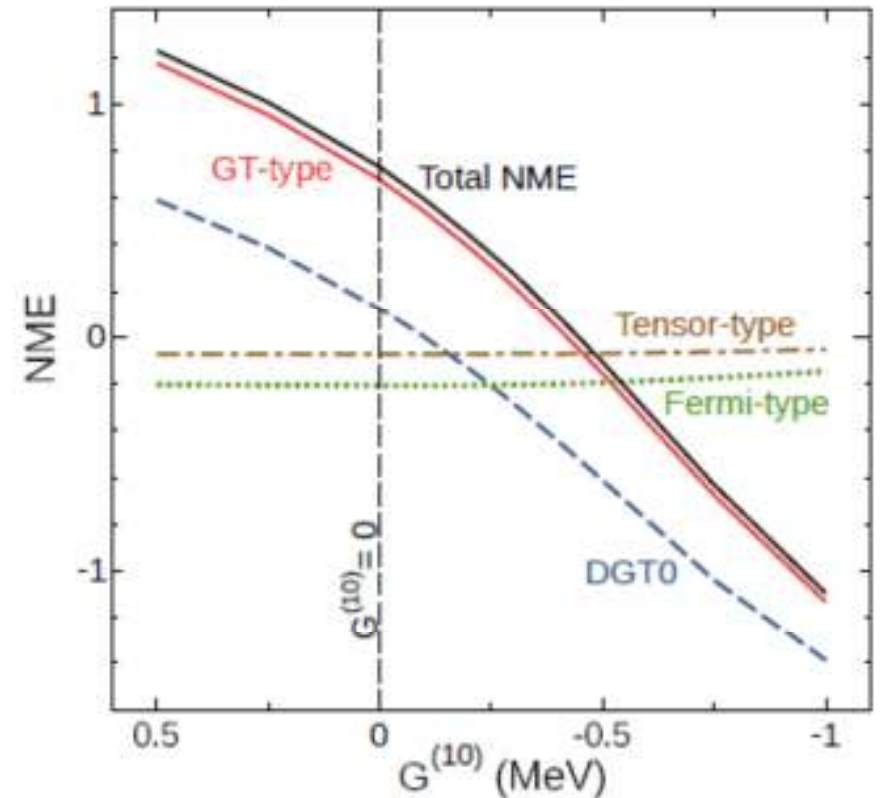
$$M^{0\nu} = M_{GT}^{0\nu} - \left(\frac{g_V}{g_A}\right)^2 M_F^{0\nu} + M_T^{0\nu}$$

$$M_{GT}^{0\nu} = \langle f | \sum_{jk} \tau_j \sigma_j \tau_k \sigma_k V_{GT}(r_{jk}) | i \rangle$$

$$M_F^{0\nu} = \langle f | \sum_{jk} \tau_j \tau_k V_F(r_{jk}) | i \rangle$$

$$M_T^{0\nu} = \langle f | \sum_{ik} \tau_j \tau_k S_{jk} V_T(r_{jk}) | i \rangle,$$

$$M^{\text{DGT}} = -\langle {}^{48}\text{Ti}, 0_1^+ || \mathcal{O}_-^{(\lambda=0)} || {}^{48}\text{Ca}, 0_1^+ \rangle$$



isoscalar pairing strength

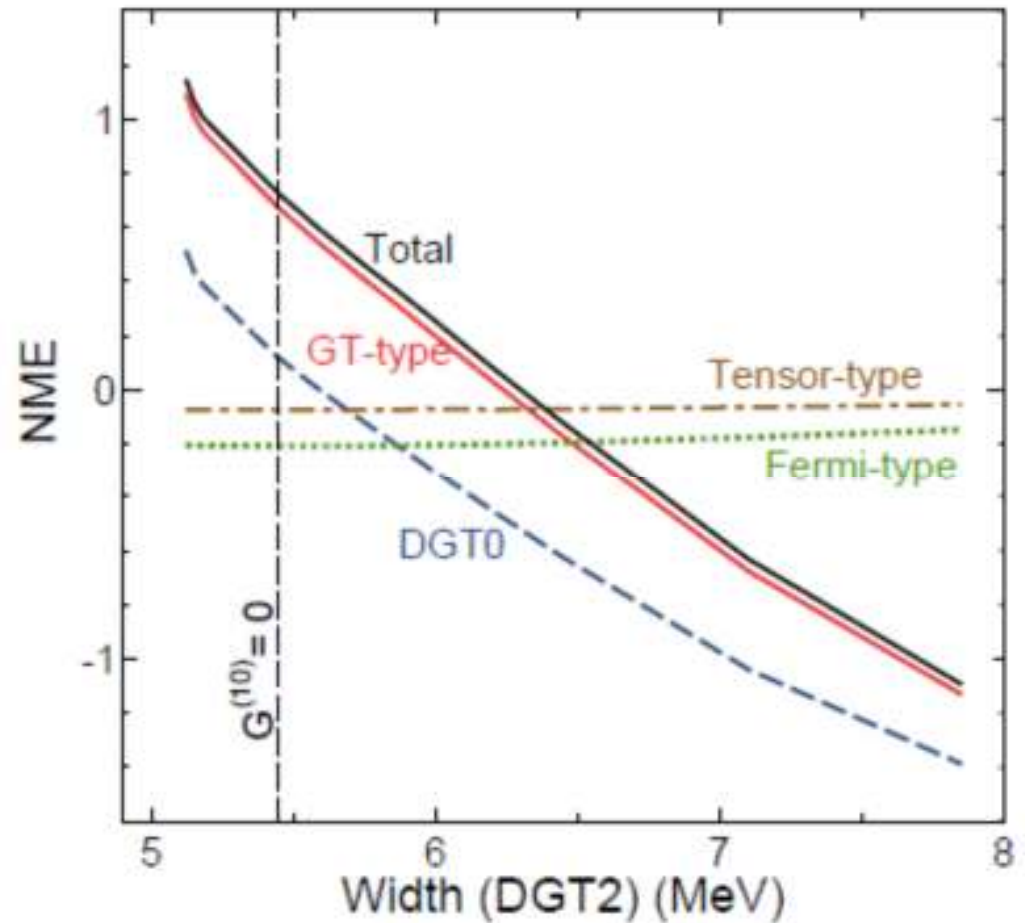
The NME is sensitive to the J=1 proton-neutron matrix element,  
or isoscalar pairing

Vogel (1986), Muto (1991), Rodin (2003) Menendez (2016), etc.

# DGTR width vs NME

$$\sigma = \sqrt{\frac{\sum_f (E_f - E_c)^2 B(\text{DGT2}, f)}{\sum_f B(\text{DGT2}, f)}}$$

$$E_c = \frac{\sum_f E_f B(\text{DGT2}, f)}{\sum_f B(\text{DGT2}, f)}$$

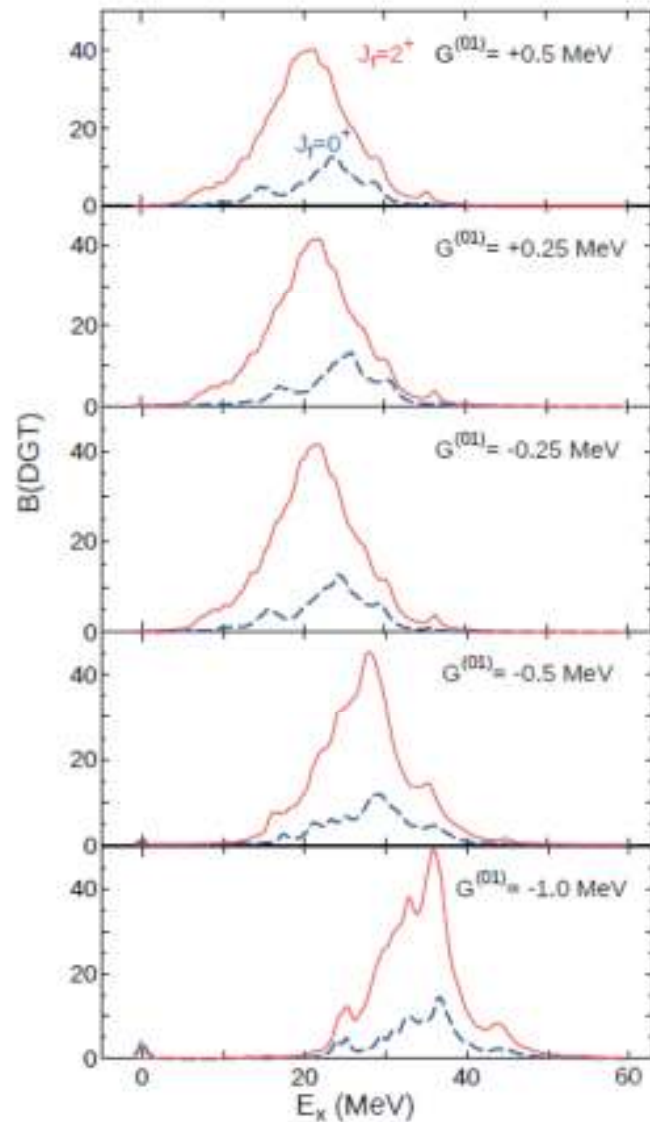


N. B. width is independent  
of quenching factor.

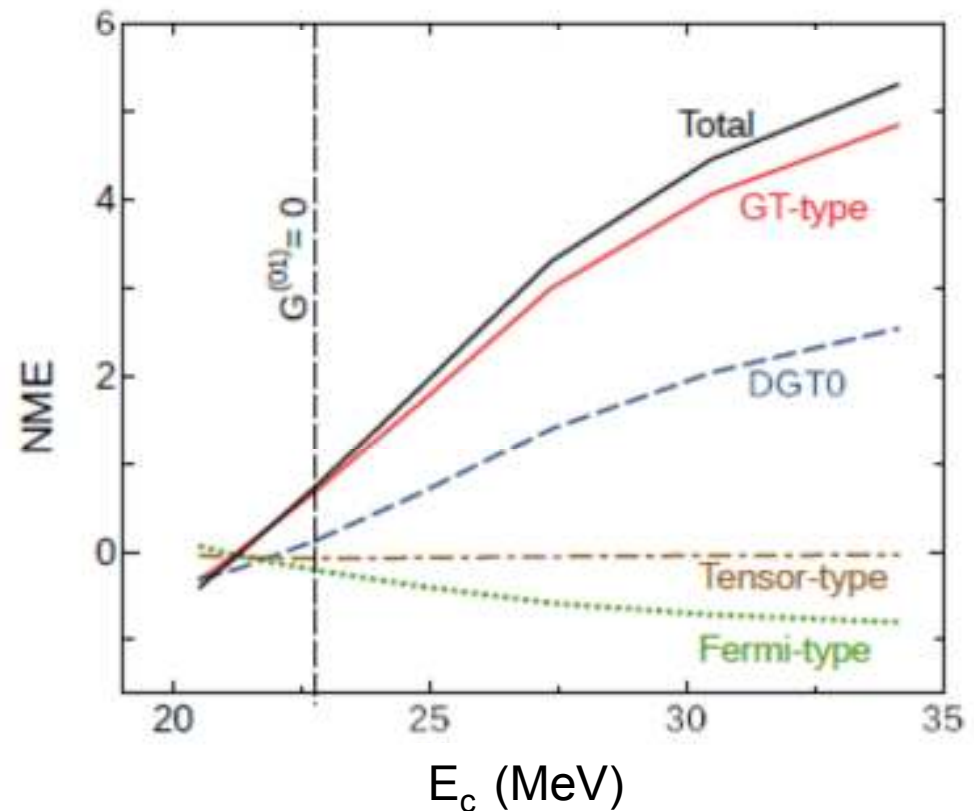
# DGTR and Isovector pairing

$$H' = H + G^{01} P^{J=0, T=1}$$

$$E_c = \sum_f E_f B(\text{DGT2}, f) / \sum_f B(\text{DGT2}, f).$$



DGTR centroid energy vs. NME

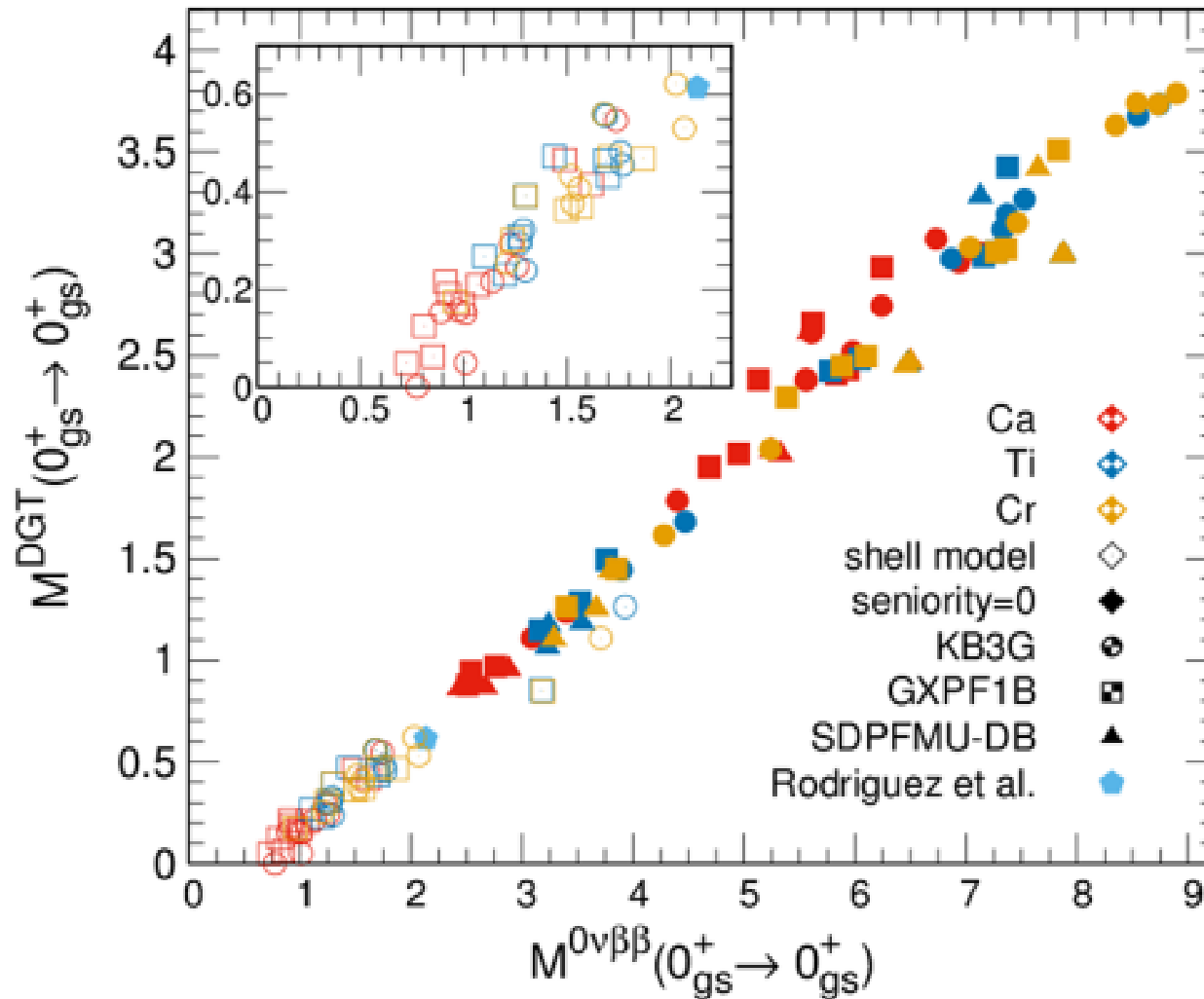


DGT transition between the ground states

See the relation between  $DGT(\lambda = 0)$  and  $0\nu\beta\beta$  NME (initial and final states are common)



# DGT( $\lambda = 0$ ) transition vs. $0\nu\beta\beta$ decay NME



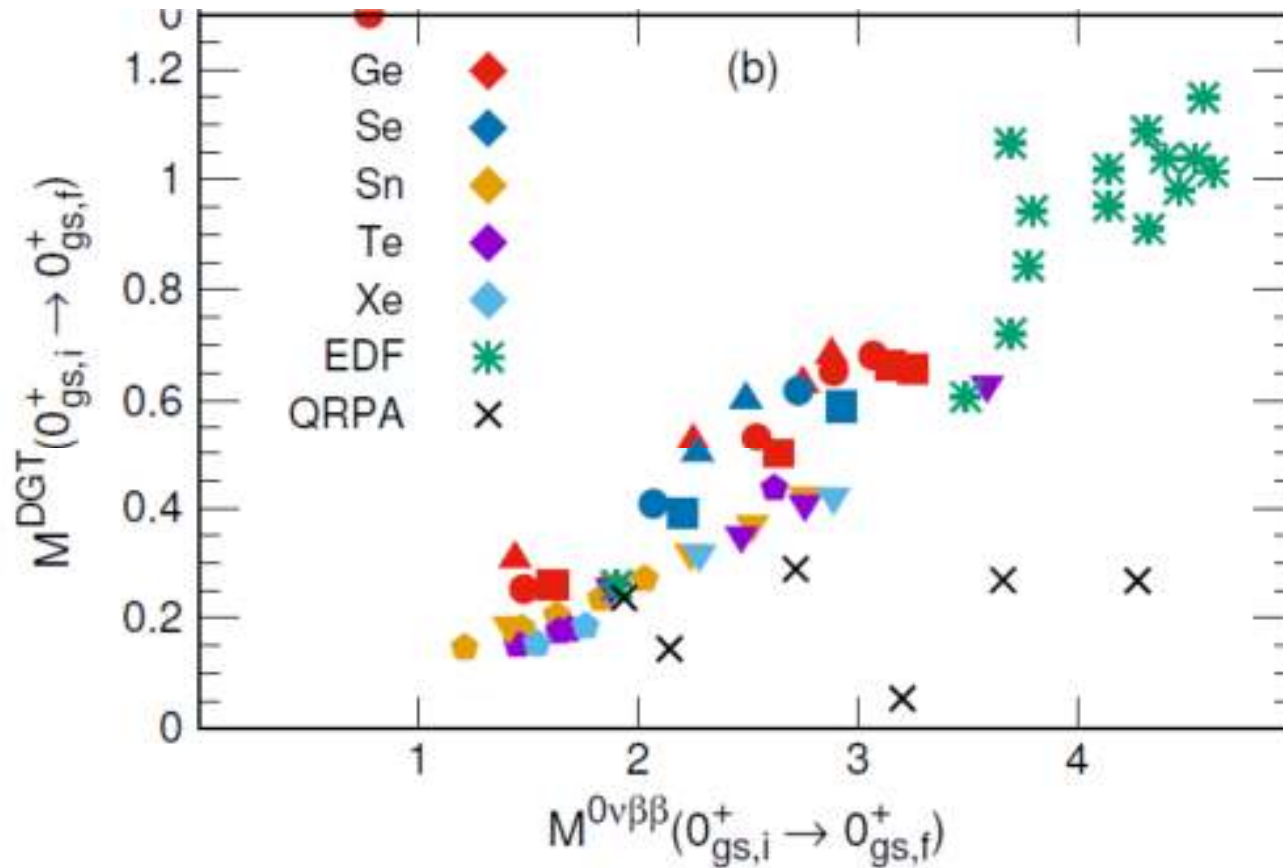
Ca, Ti, Cr isotopes  
( $N=22, 24, \dots, 36$ )

SM: KB3G, GXPF1B,  
SDPFMU-DB  
interactions

filled symbol: SM w/  
seniority-zero  
approximation

EDF:  $^{48}\text{Ca}$  Gogny+GCM  
Rodriguez *et al.*,  
PLB719 174 (2013)

# DGT transition vs $0\nu\beta\beta$ decay NME



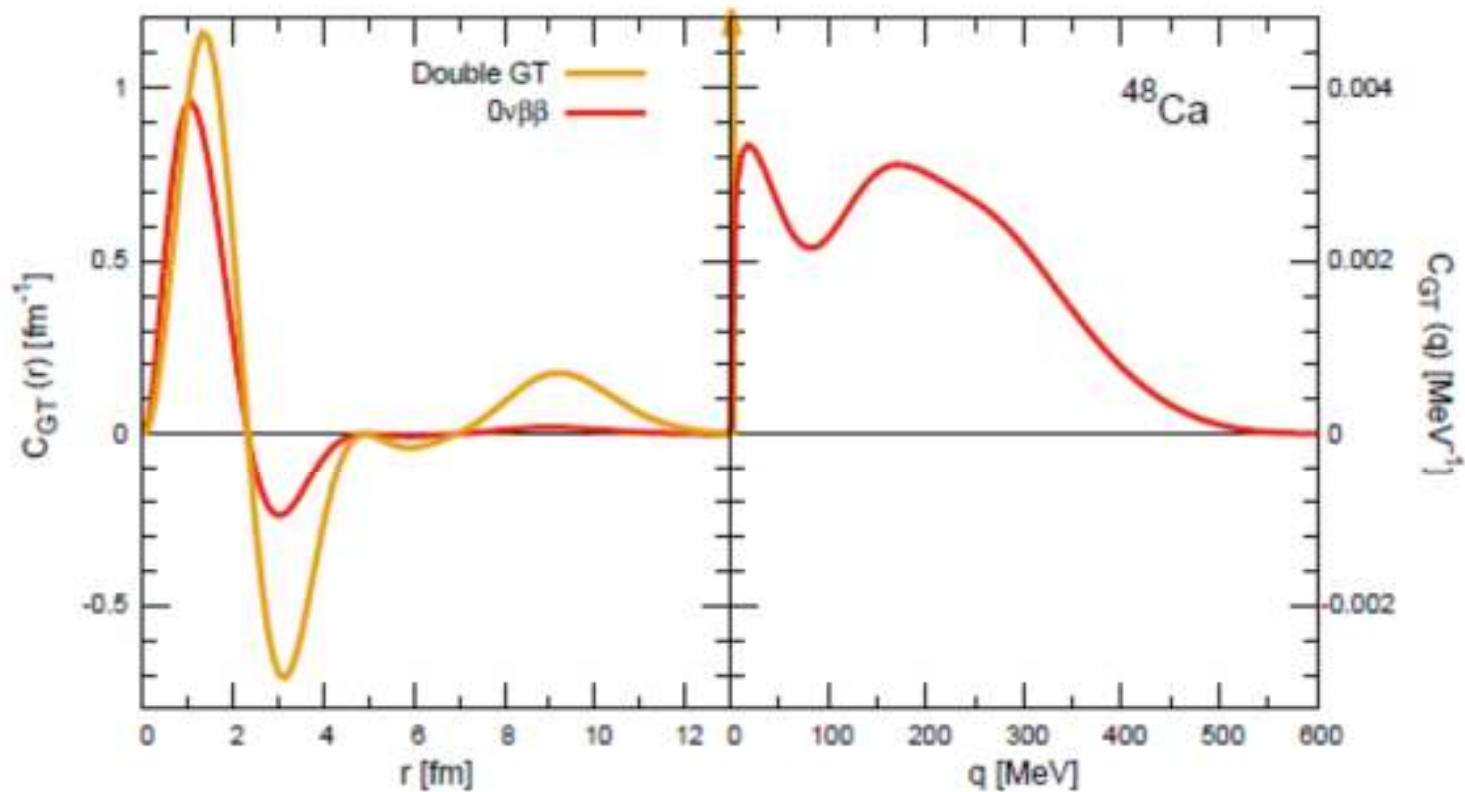
$^{74-82}\text{Ge}$ ,  $^{74,76}\text{Se}$ ,  $^{124-132}\text{Sn}$ ,  
 $^{128-130}\text{Te}$ ,  $^{134,136}\text{Xe}$

SM: shell model  
 GCN2850, jj44b,  
 JUN45,  
 GCN5082, QX

EDF: Gogny+GCM  
 Rodriguez *et al.*,  
 PLB719 174 (2013)

QRPA: AV18+G-matrix  
 F. Simkovic *et al.*,  
 PRC83, 015502 (2011).

# DGT and $0\nu\beta\beta$ NMEs: distance and momentum dependences : $^{48}\text{Ca}$



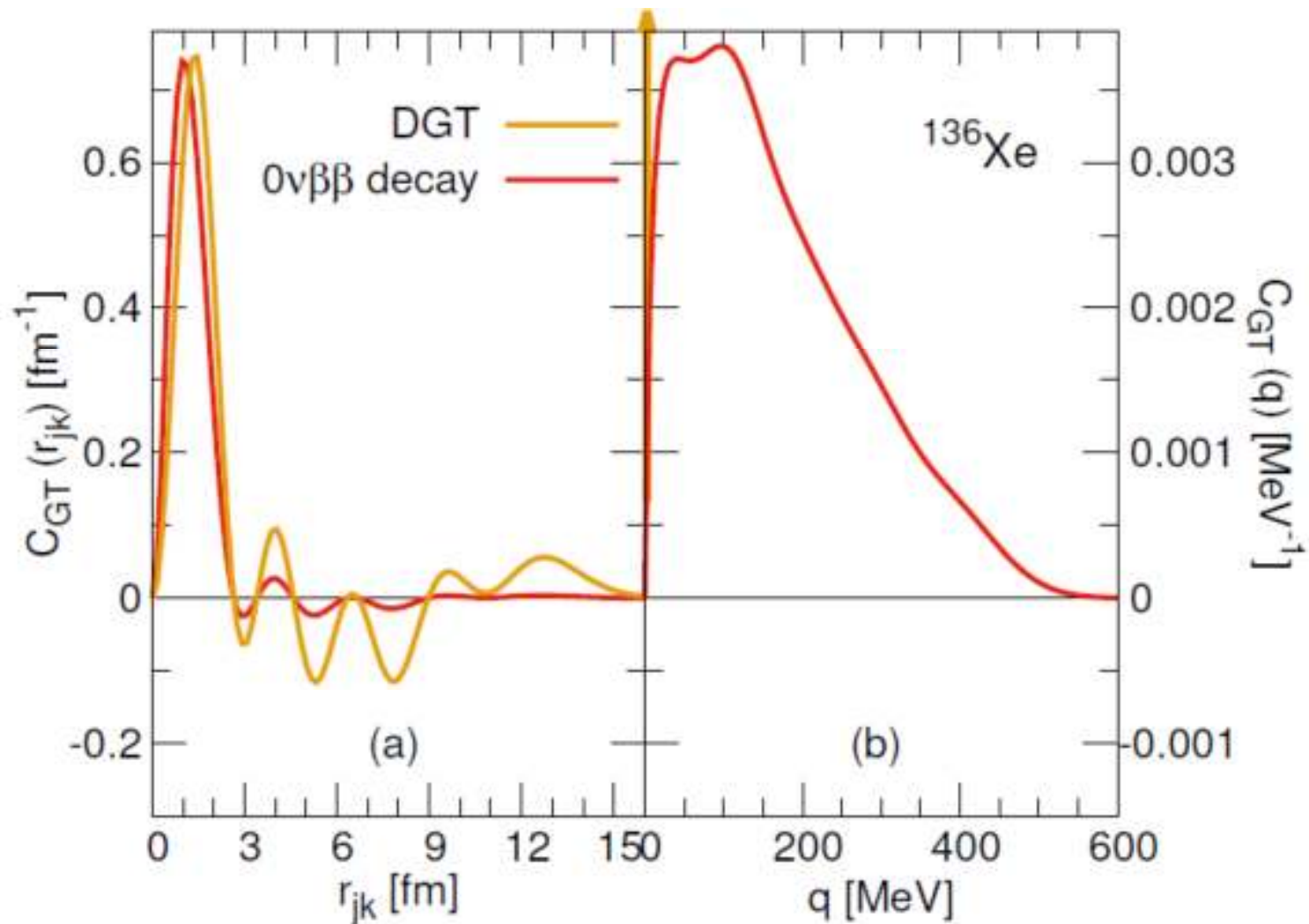
$$M = \int C(r_{ab}) dr_{ab}$$

internucleon distance

$$M = \int C(|\mathbf{q}|) d|\mathbf{q}|$$

momentum transfer

# DGT and $0\nu\beta\beta$ NMEs: distance and momentum dependences : $^{136}\text{Xe}$

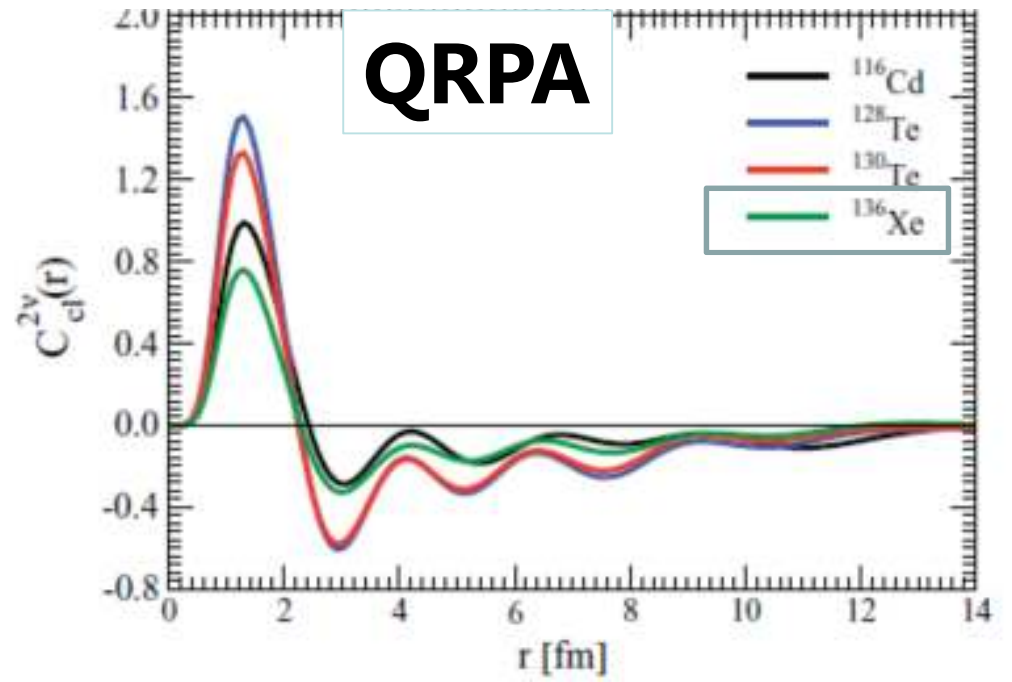
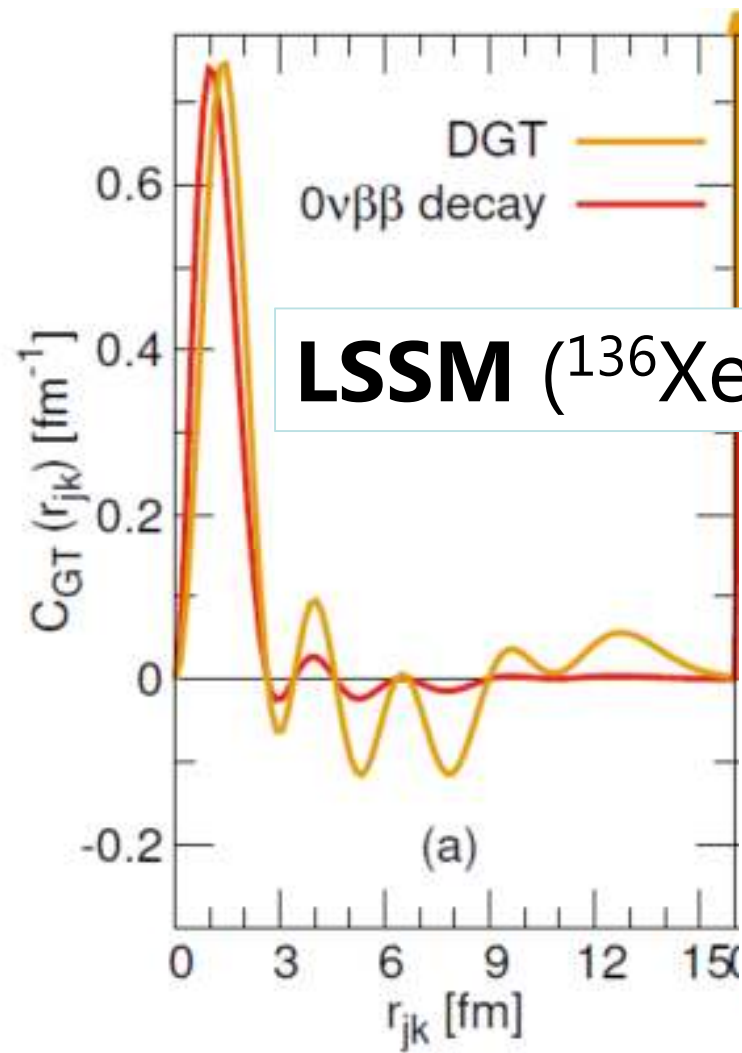


$$M = \int C(r_{ab}) dr_{ab}$$

$$M = \int C(|\mathbf{q}|) d|\mathbf{q}|$$

# Why linear correlation between DGT and $0\nu\beta\beta$ NMEs?

- Similar dependence in distance dependence, contrary to momentum dependence
- Intermediate and long-range parts show cancellation, resulting small contribution to the NME. The short-range character dominates
- factorization: short-distance details decouple from long-distance dynamics
  - E. R. Anderson, S. K. Bogner, R. J. Furnstahl, and R. J. Perry, Phys. Rev. C **82**, 054001 (2010)
  - S. K. Bogner and D. Roscher, Phys. Rev. C **86**, 064304 (2012).



NO cancellation  
 $\Rightarrow$  small  $M(\text{DGT})$

# Summary

- Using the shell-model calculations, double Gamow-Teller Resonance of  $^{48}\text{Ca}$  and its relation to  $0\nu\beta\beta$  NME is studied.
  - DGTR is correlated to the  $0\nu\beta\beta$  NME via isovector and isoscalar pairing correlations.
- DGT and  $0\nu\beta\beta$  NMEs show clear linear correlation. They are dominated by the short-range character.
- The HIDCX reaction may be useful to “calibrate” theoretical studies of  $0\nu\beta\beta$  NME. Challenges remain: reaction theory of HIDCX, ...