Correlation Functions and Nuclear Contact Formalism

Axel Schmidt

MIT

March 28, 2018

Short Range Correlations Collaboration

Massachusetts Institute of Technology

- **Prof. Or Hen**
- Dr. Shalev Gilad
- Dr. Adi Ashkenazi
- Dr. George Laskaris
- Dr. Maria Patsyuk
- Dr. Axel Schmidt
- Barak Schmookler
- Rey Cruz-Torres
- Afro Papadopoulou
- Efrain Segarra

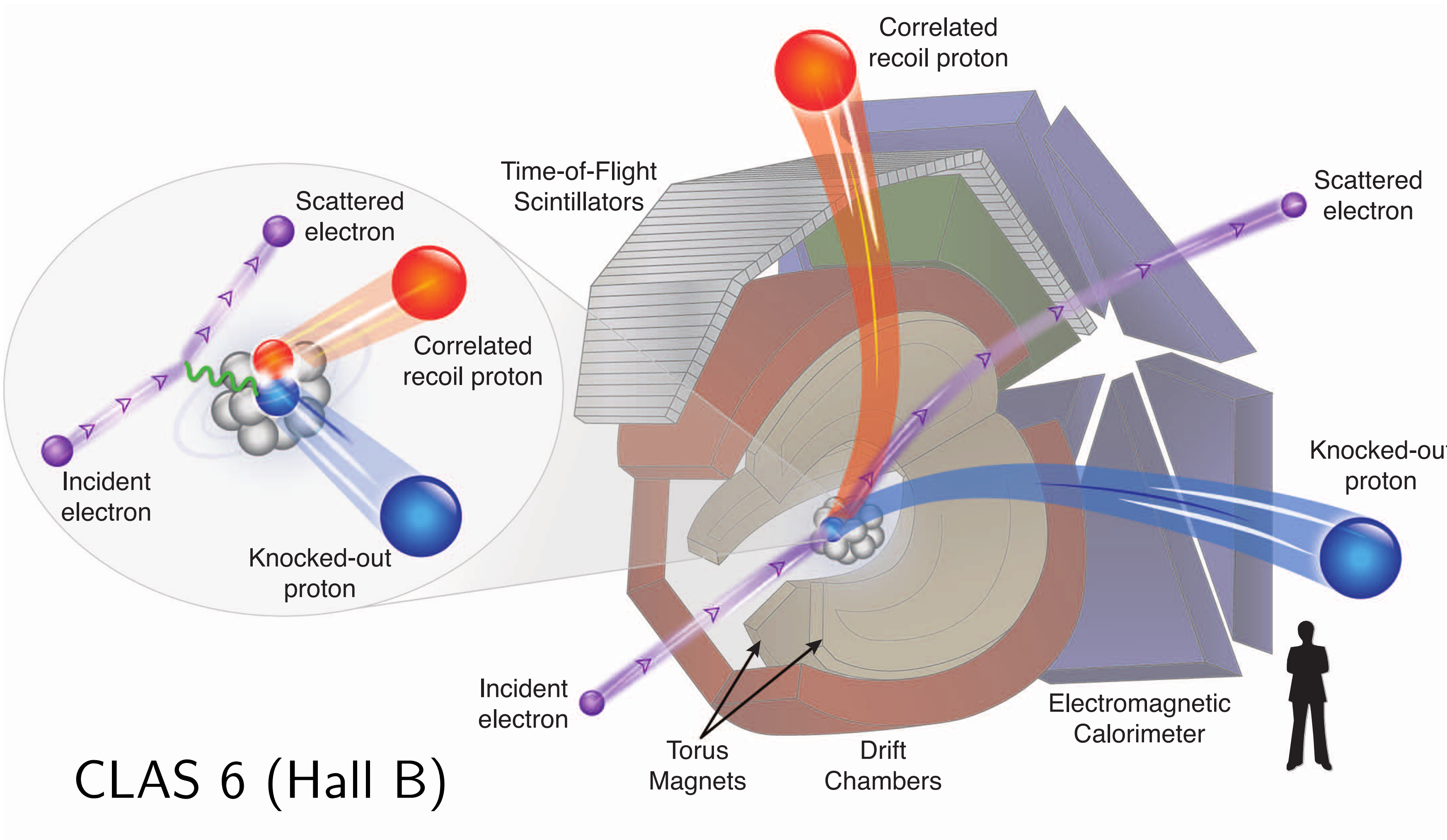
Old Dominion University

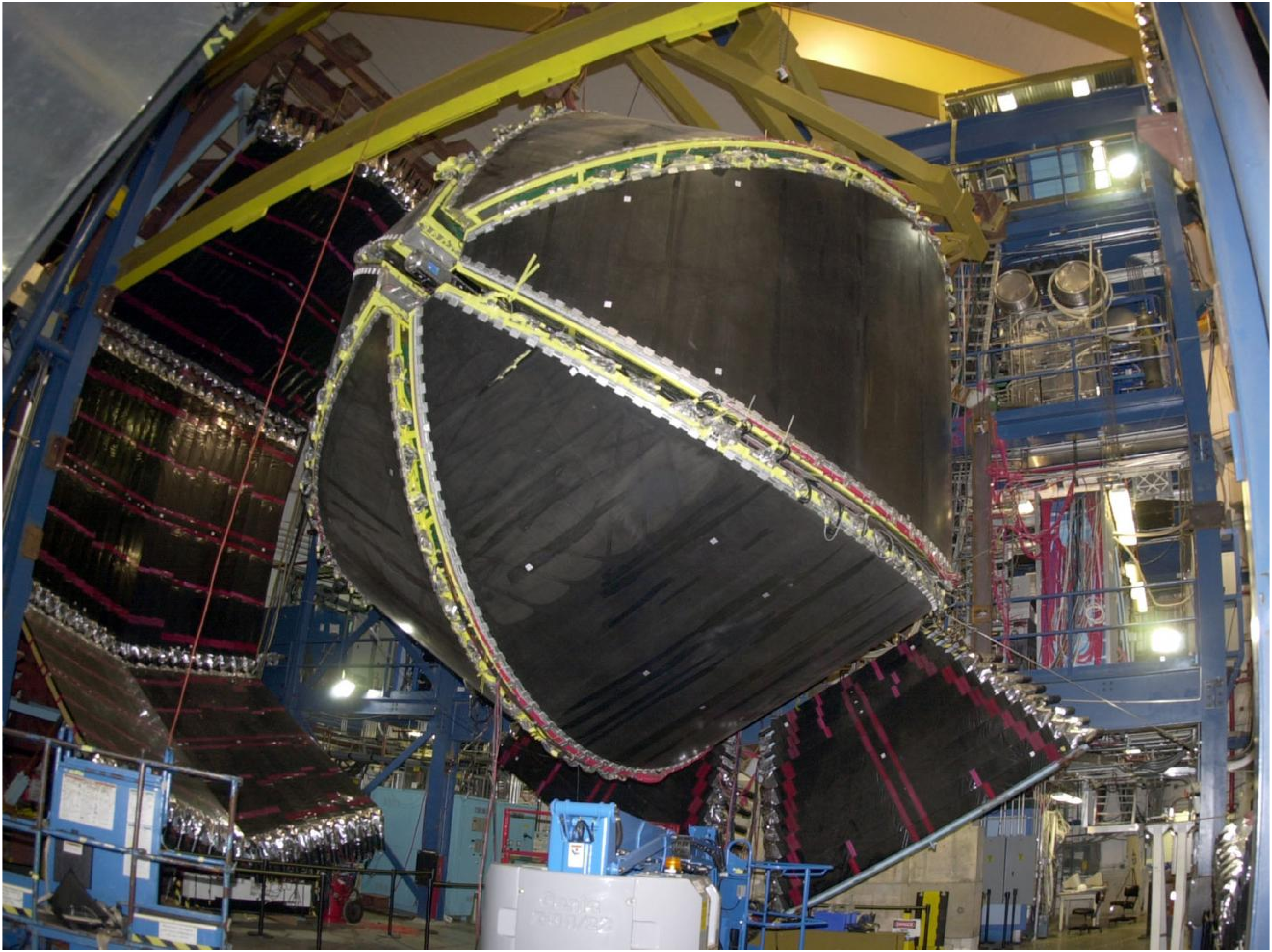
- **Prof. Lawrence Weinstein**
- Dr. Florian Hauenstein
- Mariana Khachatryan

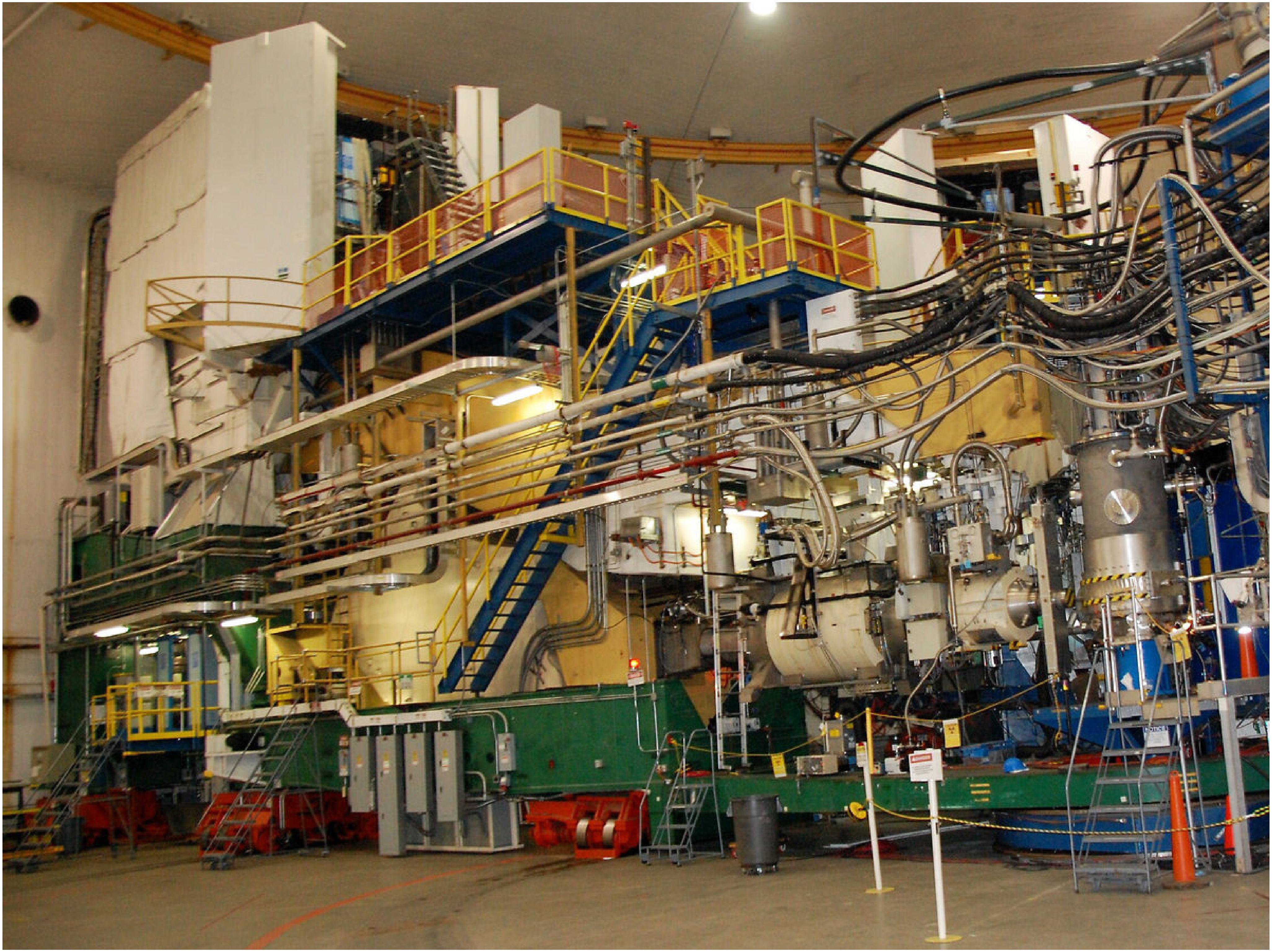
Tel Aviv University

- **Prof. Eli Piasetzky**
- Dr. Igor Korover
- Erez Cohen
- Meytal Duer

Our canonical experiment is the break-up of an SRC pair.







Jefferson Lab Spectrometers

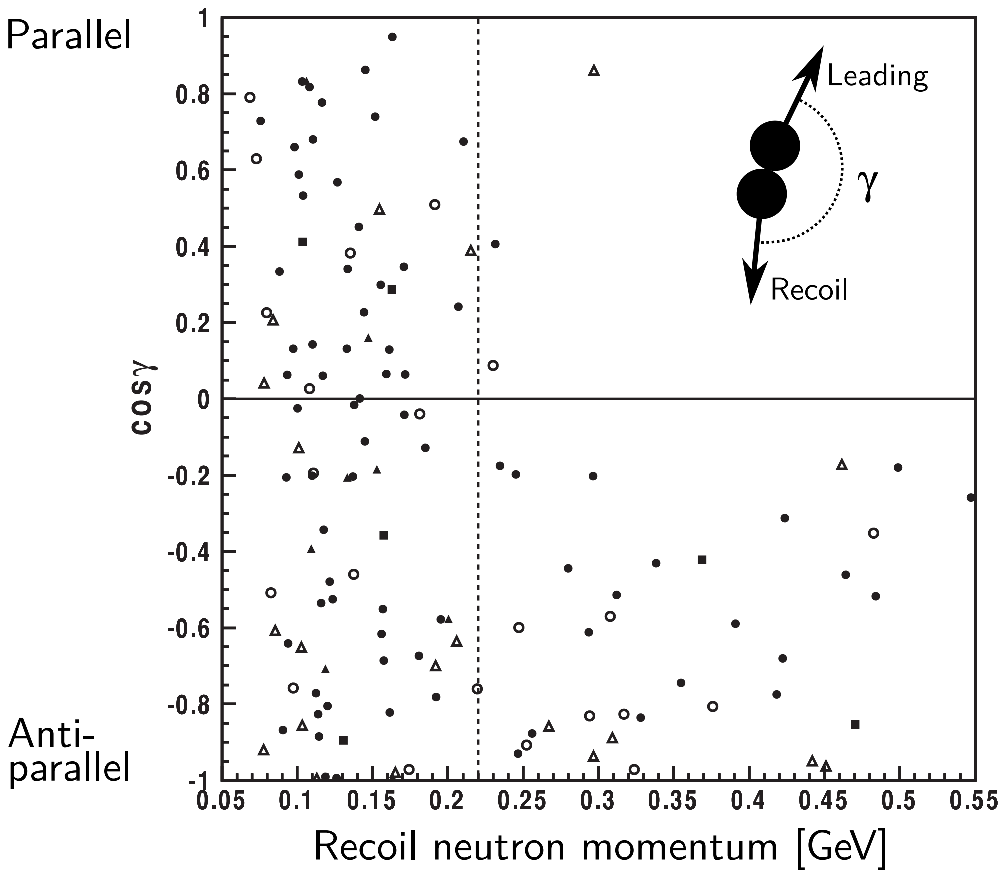
High-Resolution Spectrometer

- Hall A
- Flight path: 23.4 m
- $\delta p/p = 10^{-4}$
- Mom. acceptance: $\pm$4.5%
- Angular acceptance: 6 msr

CEBAF Large Acceptance Spec.

- Hall B
- Flight path: $\approx$ 5 m
- $\delta p/p \approx 1\%$
- Mom. acceptance: 0.3–10 GeV
- Angular acceptance: $> 2\pi$

All high-momentum nucleons have a correlated partner.

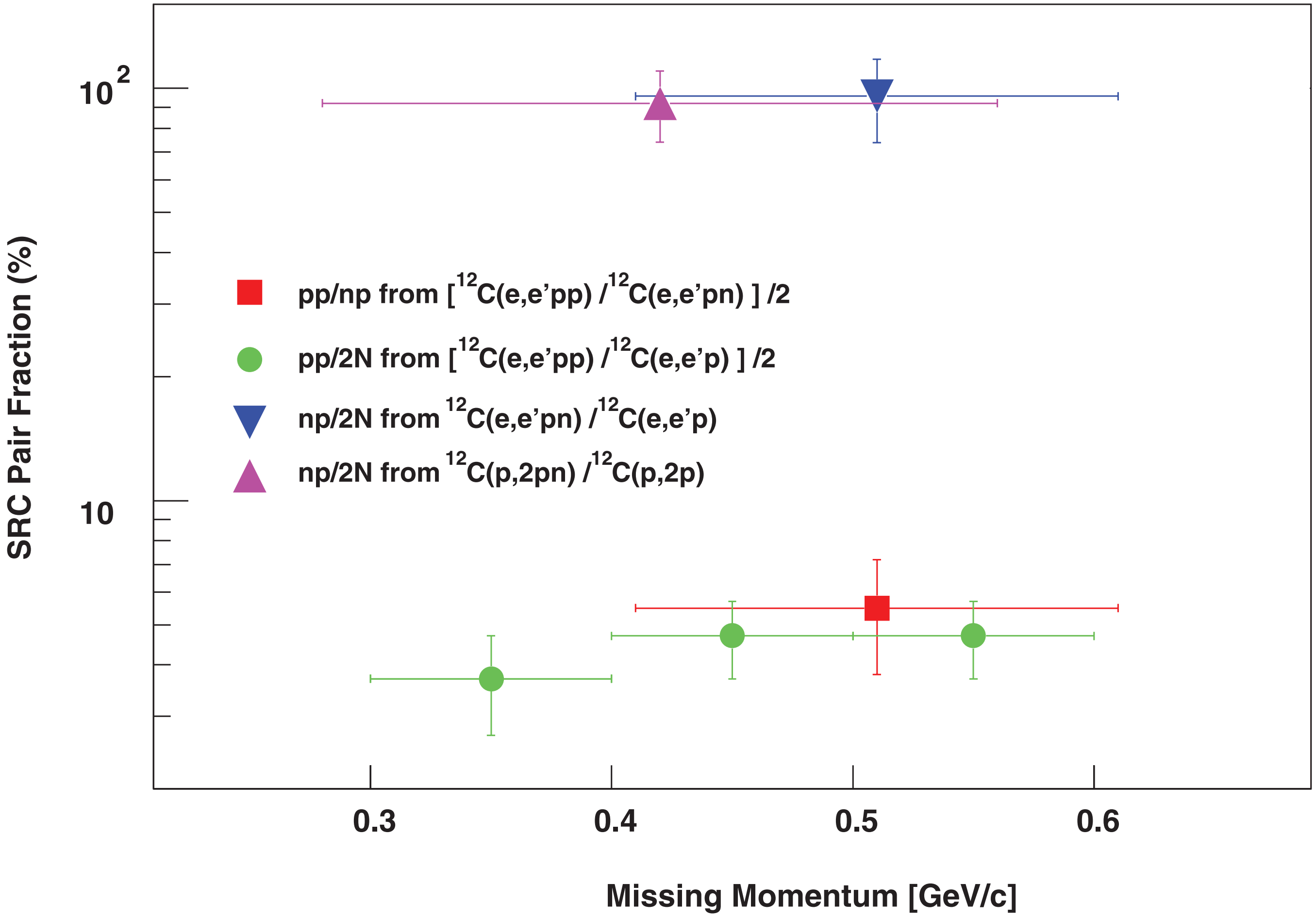


p scattering from Carbon:

- Always a correlated partner
- Anti-parallel momenta

J.L.S. Aclander et al., Phys. Lett. B 453, 211 (1999)
A. Tang et al., Phys. Rev. Lett. 90, 042301 (2003)
E. Piasetzky et al., PRL 97 162504 (2006)

Between 300–600 MeV, *np* pairs predominate.

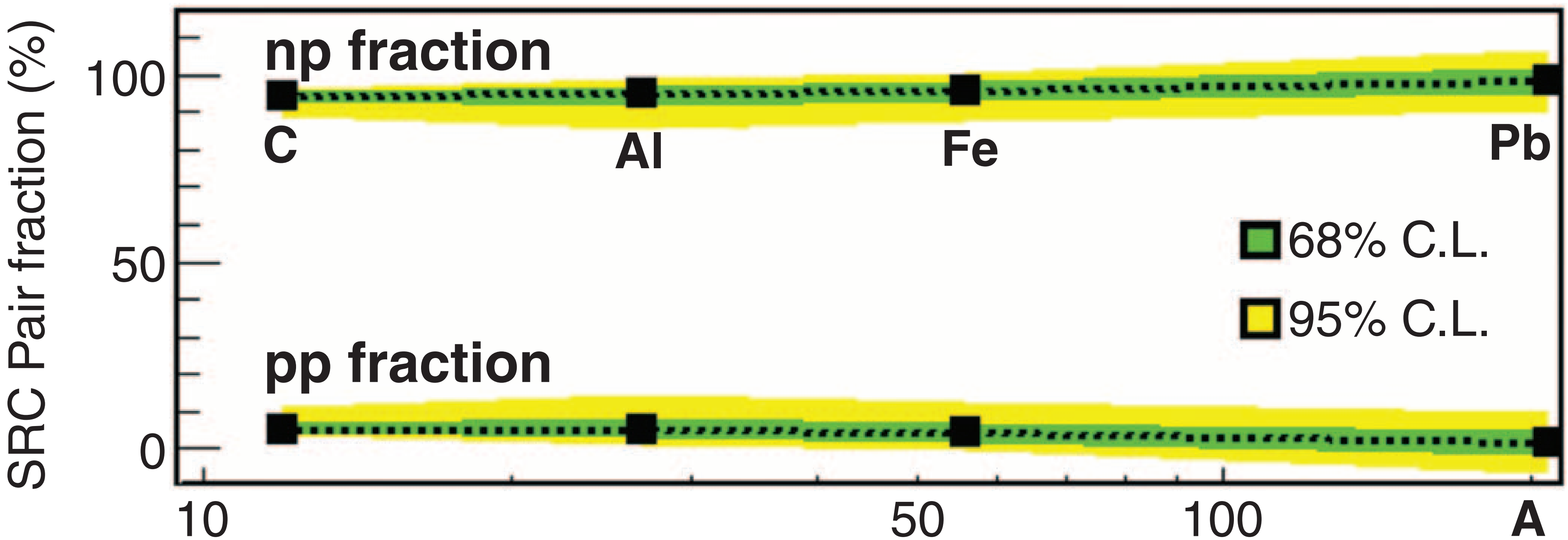


E. Piasetzky et al., PRL 97 162504 (2006)

R. Shneor et al., PRL. 99, 072501 (2007)

R. Subedi et al., Science 320, 1476 (2008)

An absence of *pp* pairs has been verified in a wide-range of nuclei.

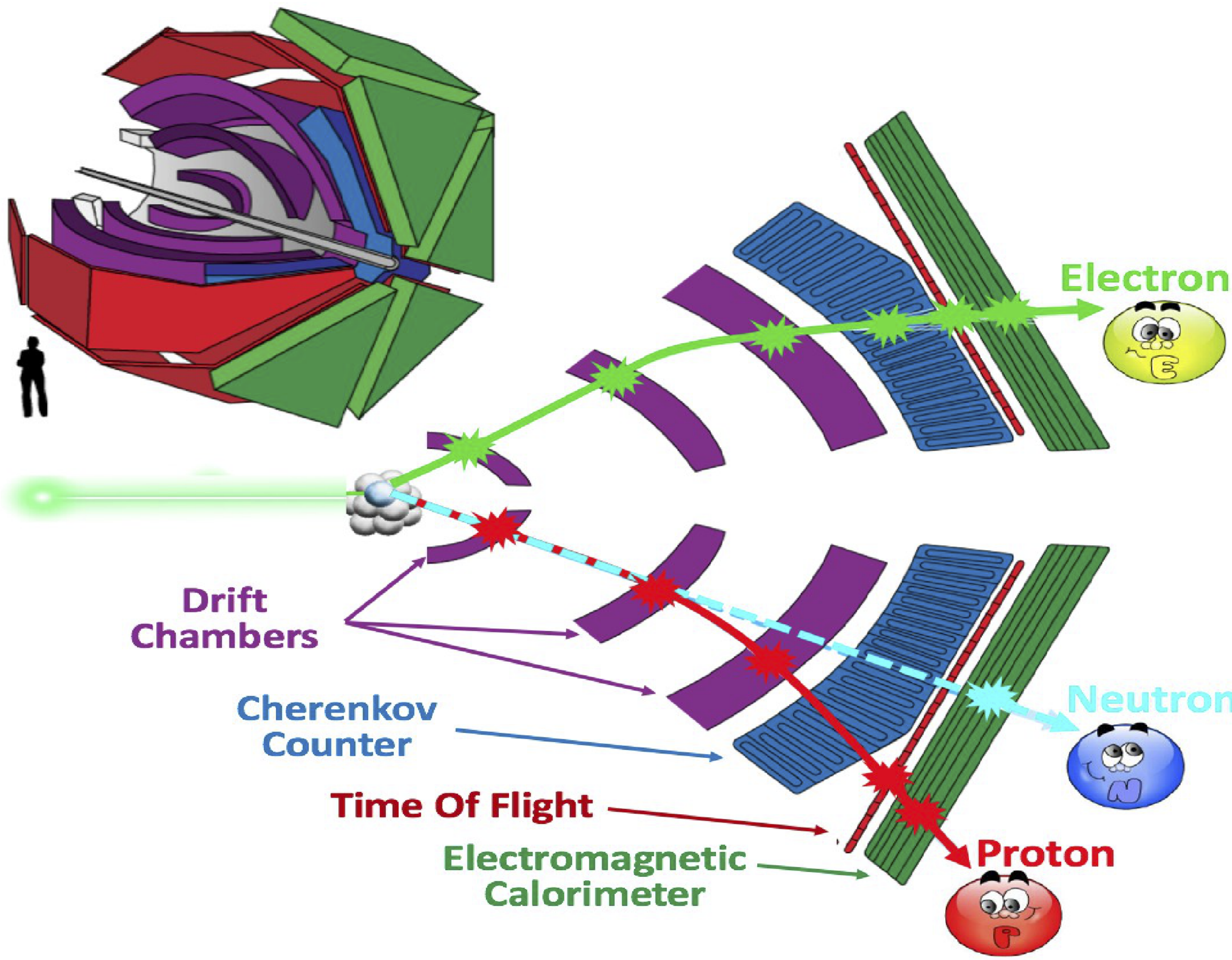


O. Hen et al, Science 346, 614 (2014)

Our current picture of short-range correlations:

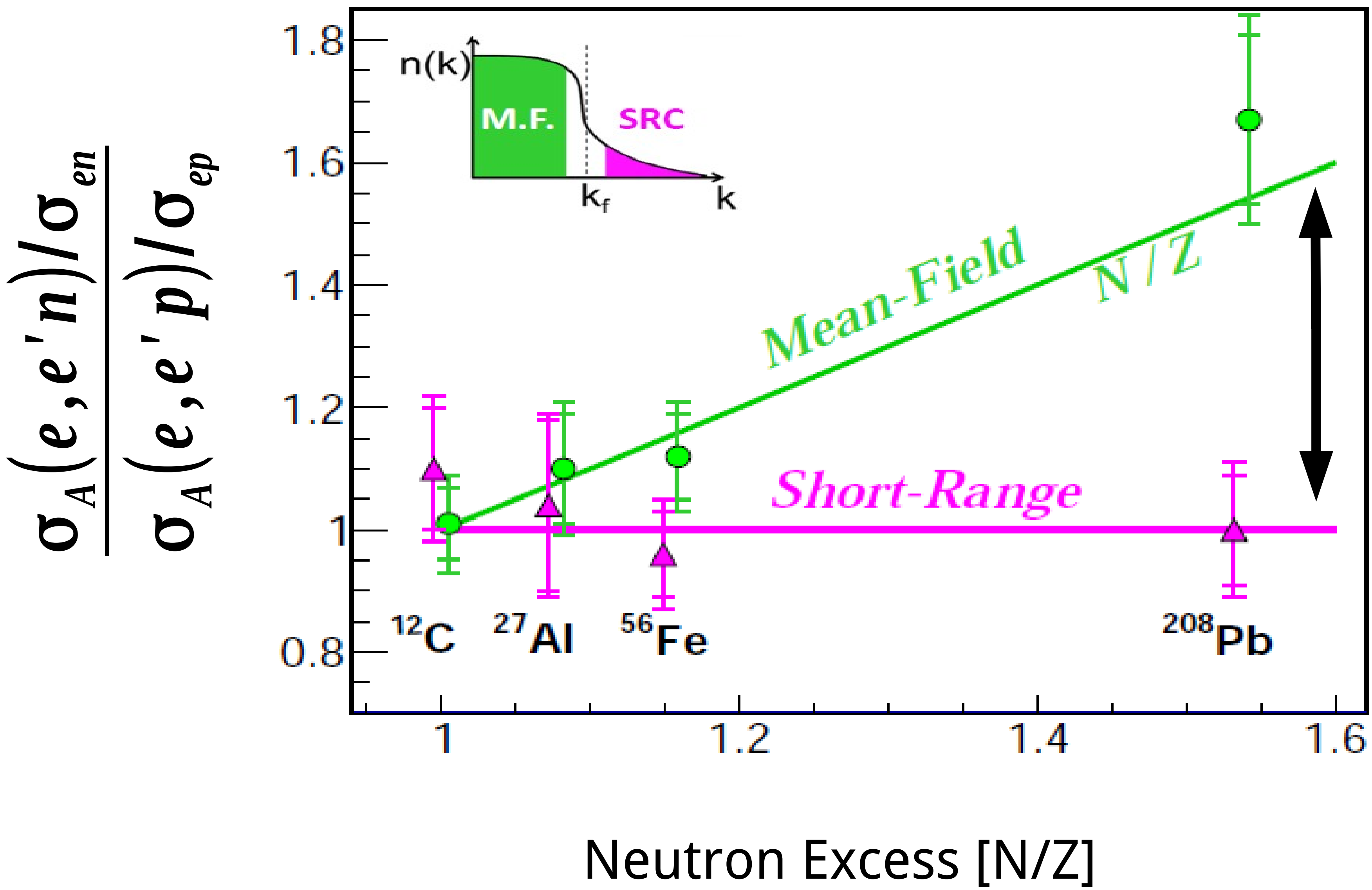
- ≈20% of nucleons are in pairs at any moment.
- All nucleons above k_F have a correlated partner
- High high-relative momentum
- Low center-of-mass momentum
- *np*-dominance ↔ tensor force

Meytal Duer has identified high-momentum neutrons for the first time.



M. Duer et al., submitted to Nature

n/p ratio is constant with asymmetry!



SRC fraction for neutrons saturates.

$$\text{SRC Fraction} \equiv \frac{\sigma^{A}_{\text{SRC}}(e,e'N)}{\sigma^{A}_{\text{MF}}(e,e'N)} \Big/ \frac{\sigma^{C}_{\text{SRC}}(e,e'N)}{\sigma^{C}_{\text{MF}}(e,e'N)}$$

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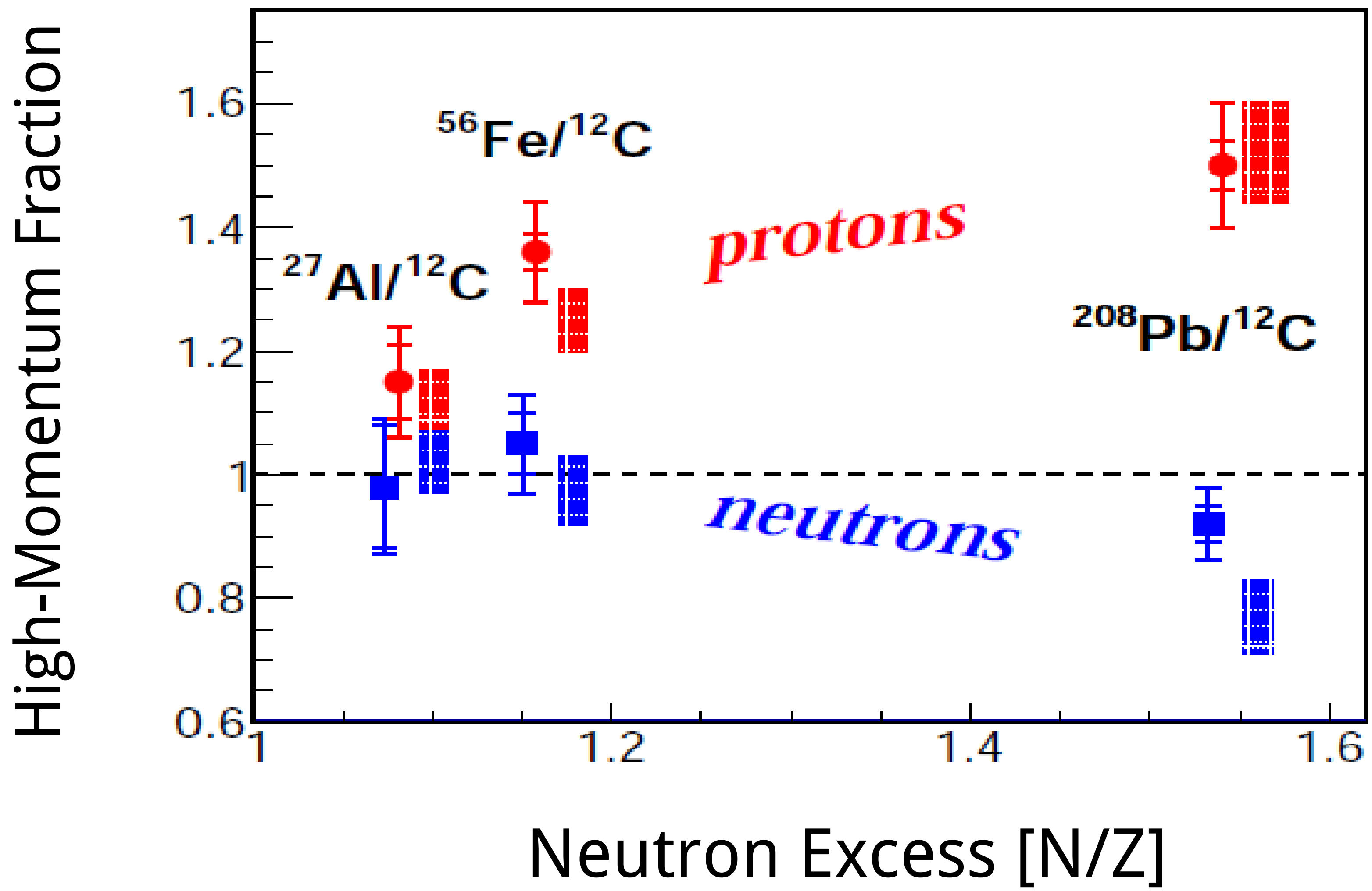
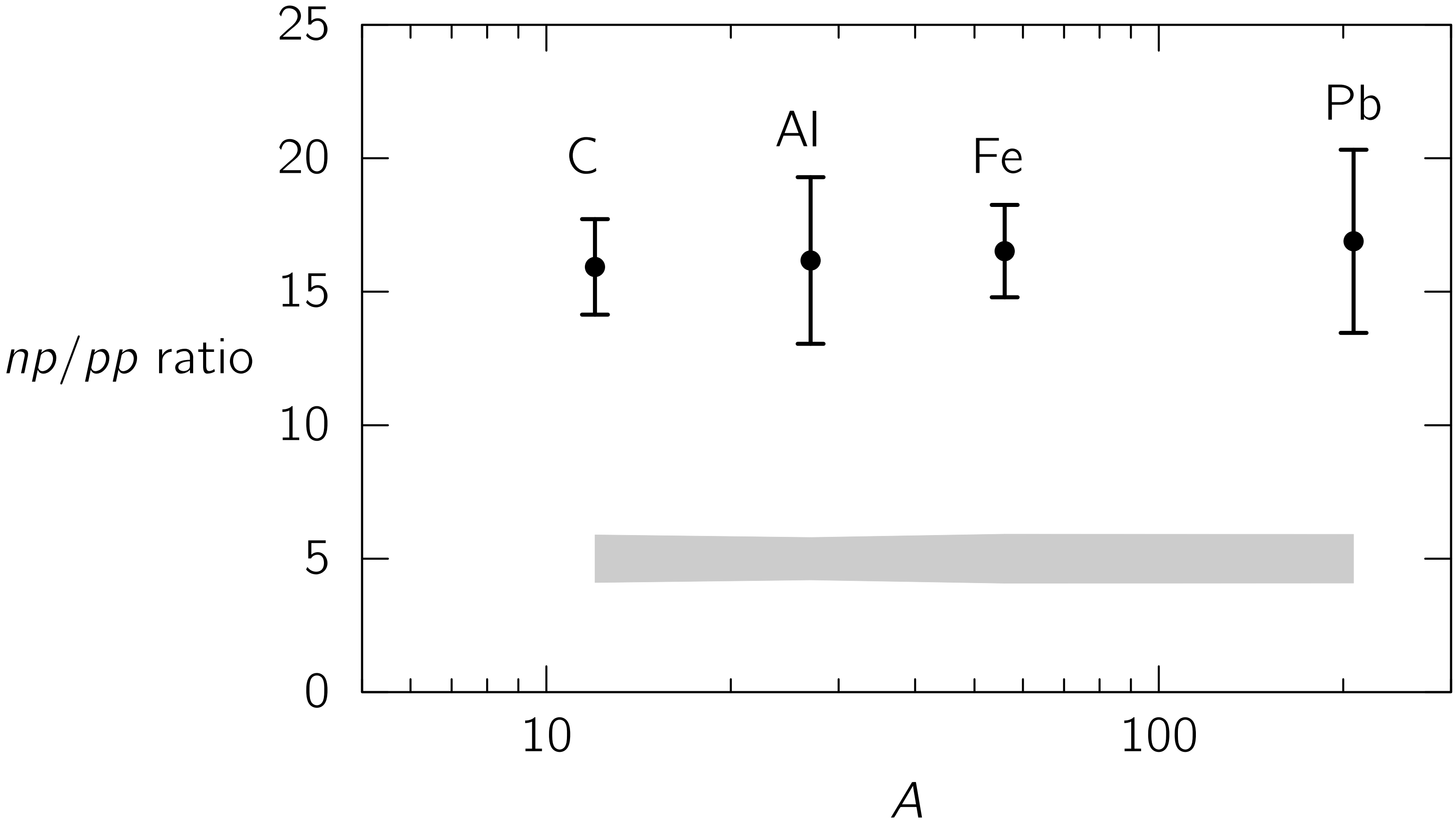


np/pp ratio is constant over all species.



Our current picture of short-range correlations:

- $\approx$20% of nucleons are in pairs at any moment.
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- High high-relative momentum
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- *np*-**dominance** $\leftrightarrow$ **tensor force**

My talk today:

1. Scale Separation and Nuclear Contact Formalism

2. Correlation Functions

3. Short-range correlations and the EMC effect

Scale separation

At short distance scales, long-range structure can factor out.

At high momentum scales, low-momentum structure can factor out.

Contact formalism (as given by S. Tan)

In a dilute spin-1/2 system:

$$r_{\text{eff.}} << a, d$$

a is the scattering length, d is the average inter-particle distance

Wave function at short distances:

$$\Psi(r_{ij} \to 0) = \left(\frac{1}{r_{ij}} - \frac{1}{a} \right) \times \phi(\vec{R}_k, \vec{r}_{i \neq j})$$

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In a dilute spin-1/2 system:

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Wave function at short distances:

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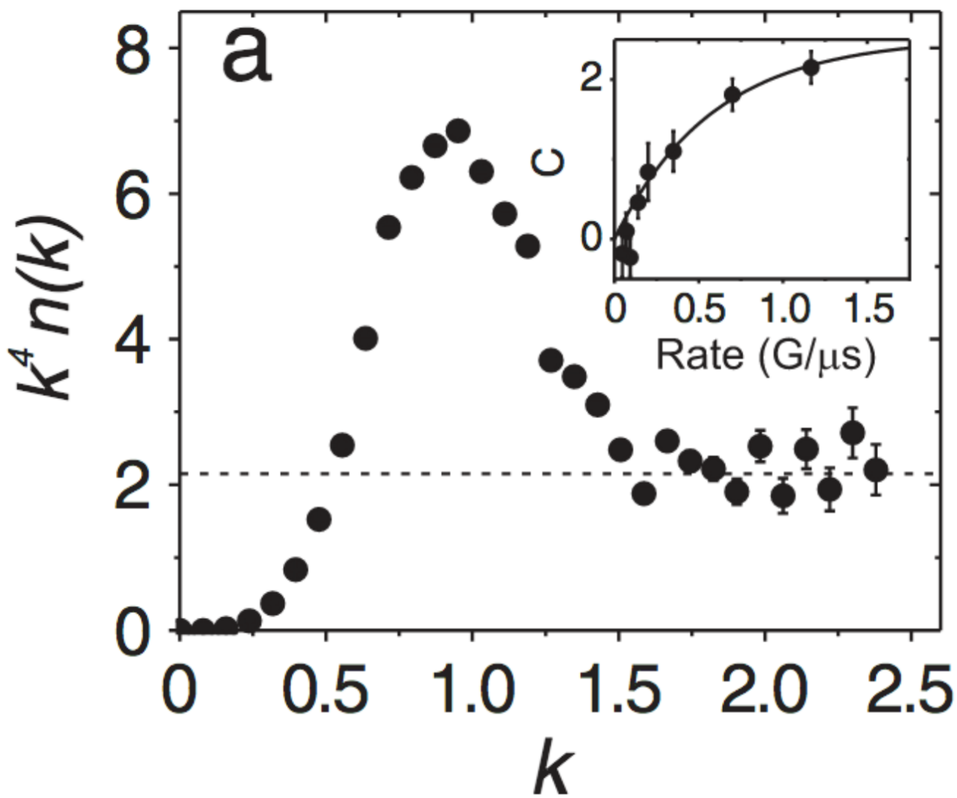
Wave function at short distances:

$$\Psi(k_{ij} \to \infty) = \frac{1}{k_{ij}^4} \times C$$

Contact behavior has been confirmed in atomic systems.

Dilute ^{40}K atoms



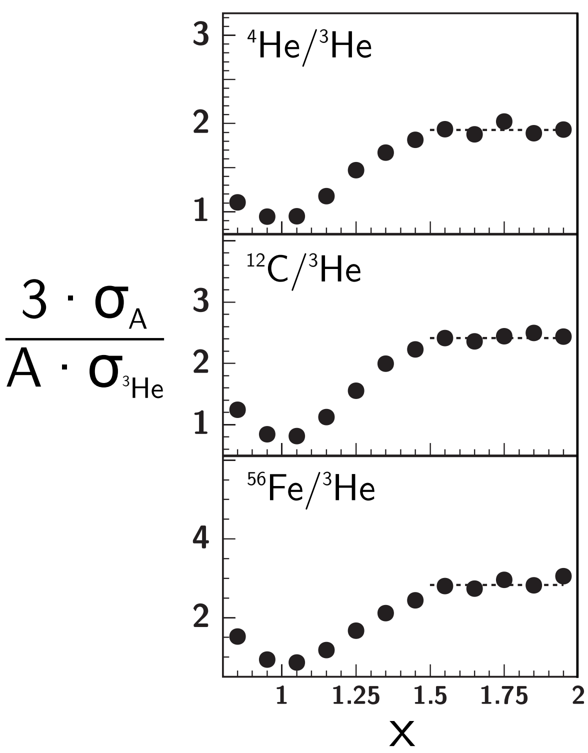
J.T. Stewart et al., PRL 104 235301 (2010)

A nucleus is not dilute.
Can we see scale separation?

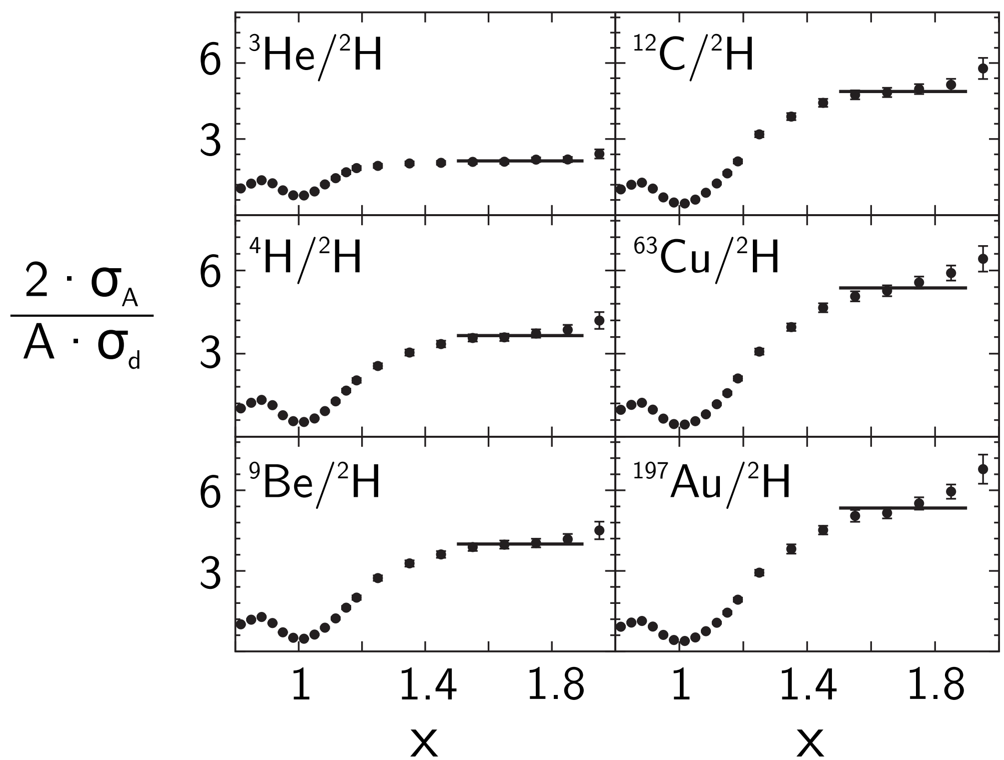
$$r_{\text{eff.}} << a, d$$

- $r_{\text{eff.}} \approx 1/2m_\pi \approx 0.7$ fm
- $d \approx (2/\rho)^{1/3} \approx 2.3$ fm
- $a \approx 5.4$ fm

Inclusive e^- scattering at $x > 1$ shows scale separation.

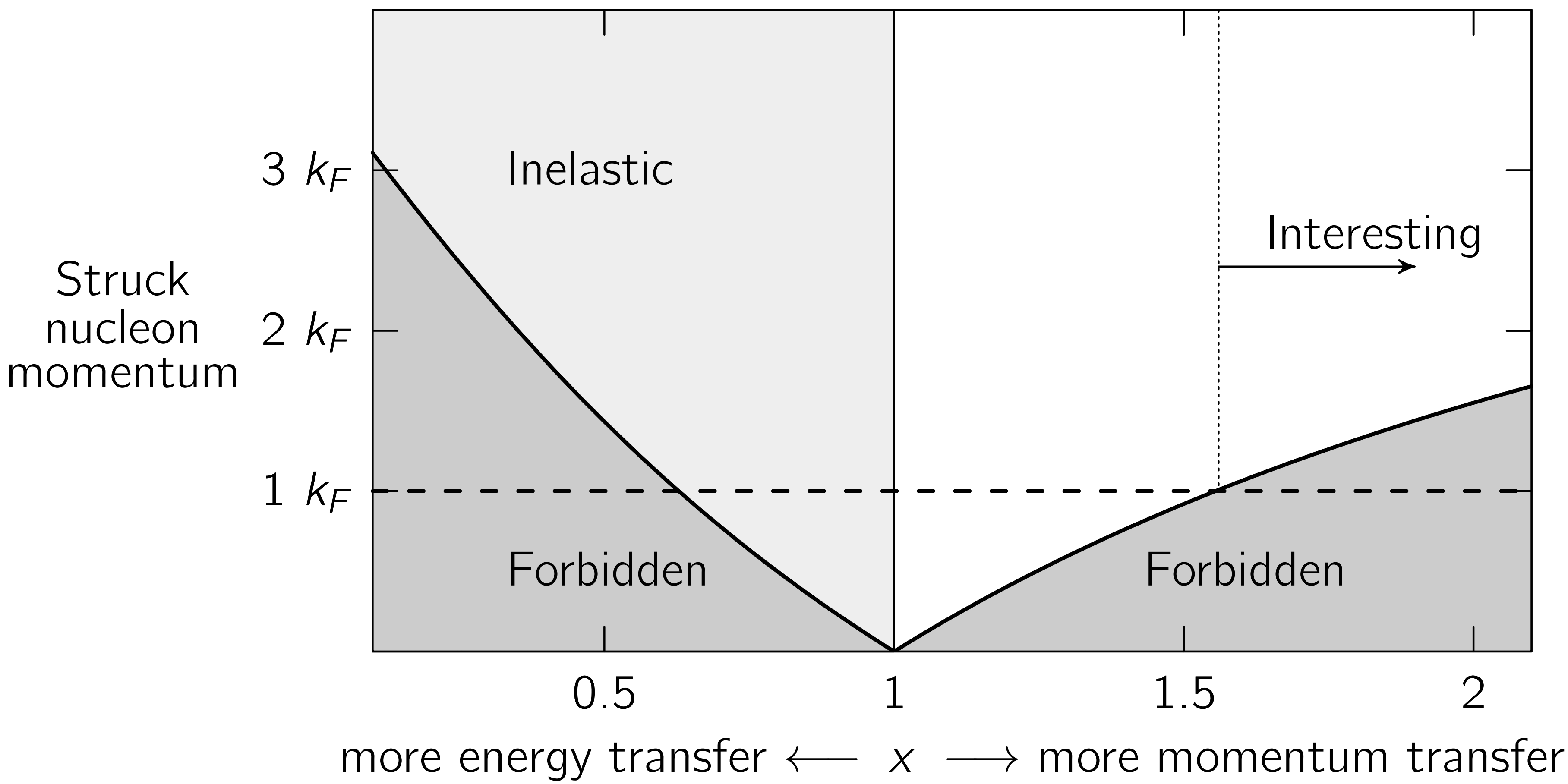


K.S. Egiyan et al. PRL 96, 082501(2006)



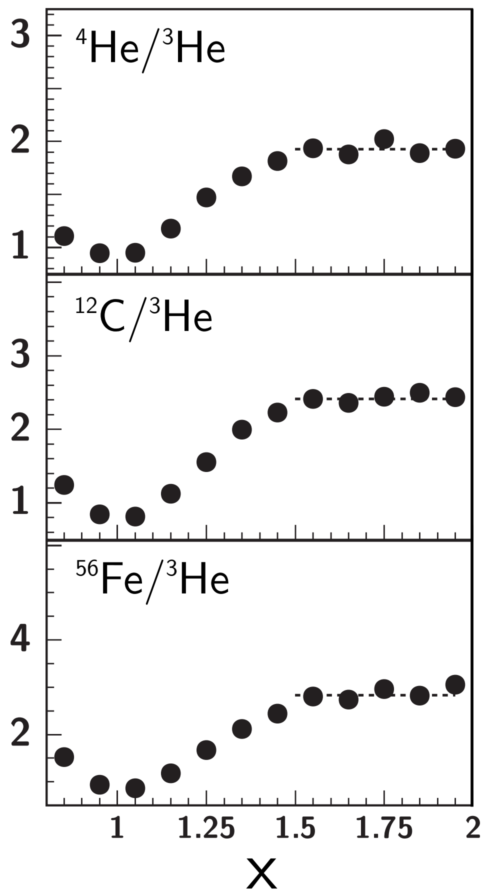
Fomin et al., PRL 108, 092502 (2012)

At high x, quasielastic scattering can only proceed from a high-momentum nucleon.



a_2 plateaus tell us that high-momentum tails are universal.

$$\frac{3 \cdot \sigma_A}{A \cdot \sigma_{^3He}}$$



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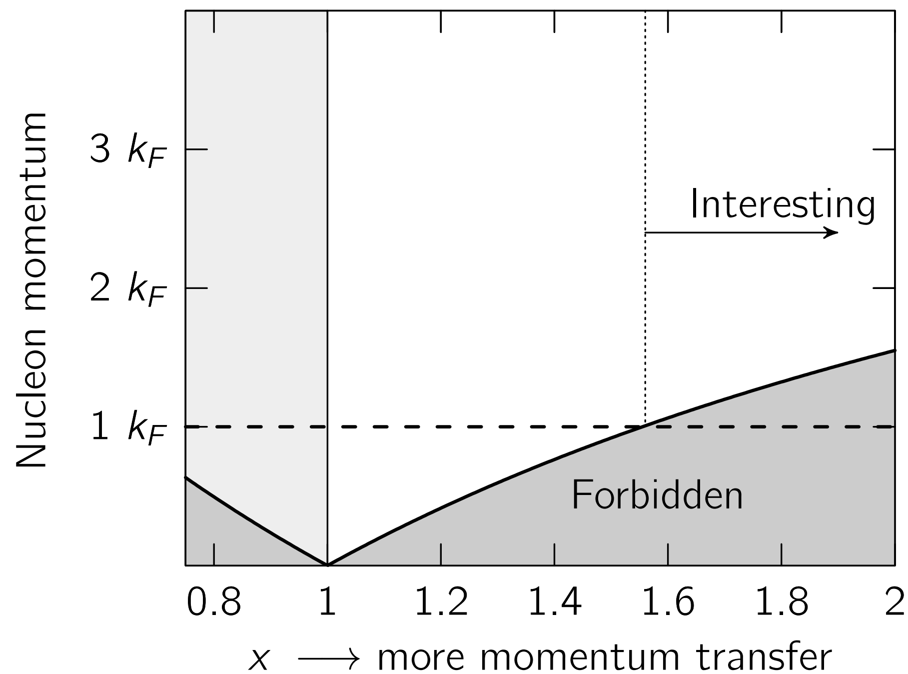
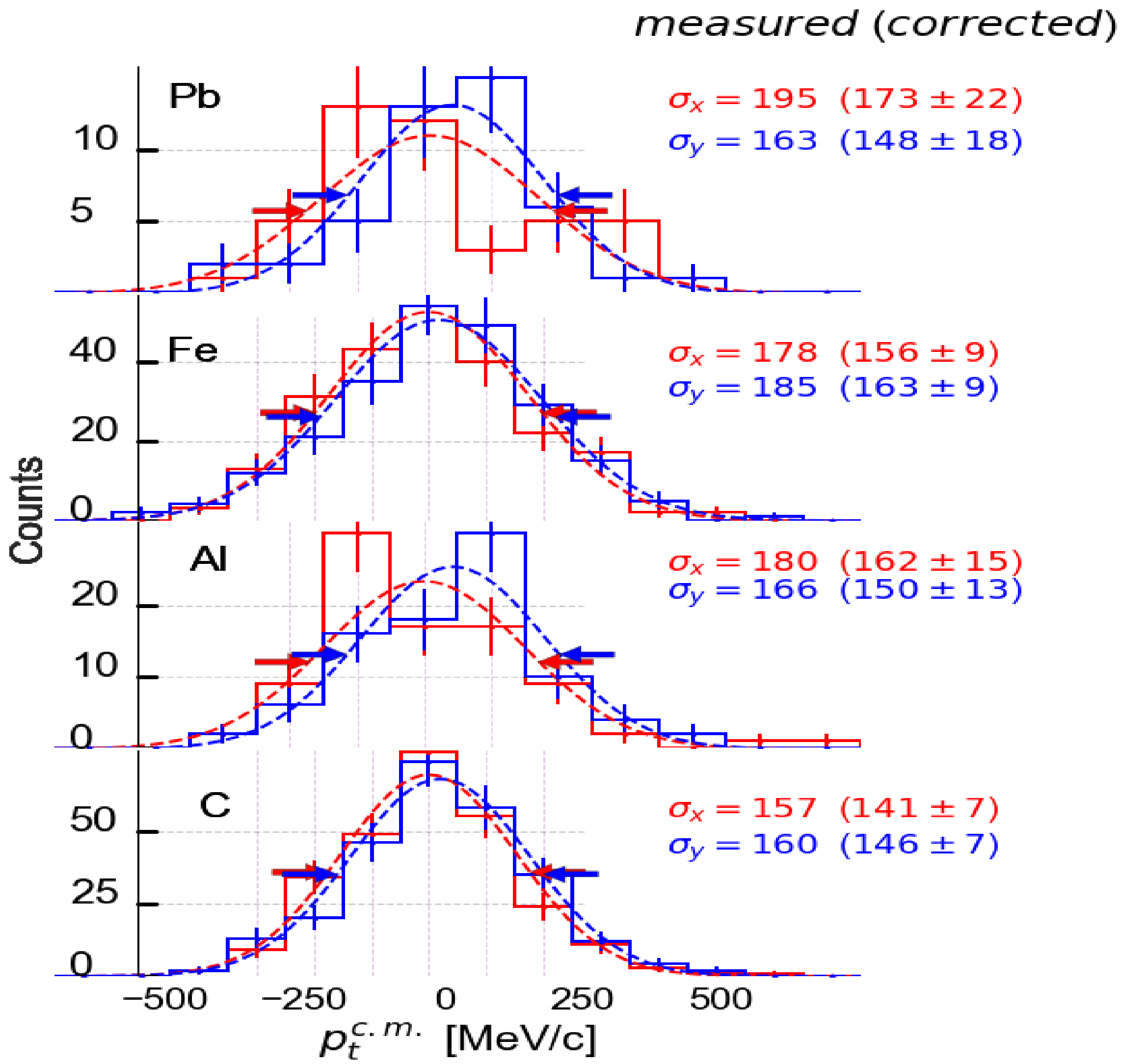


Scaling constant a_2:

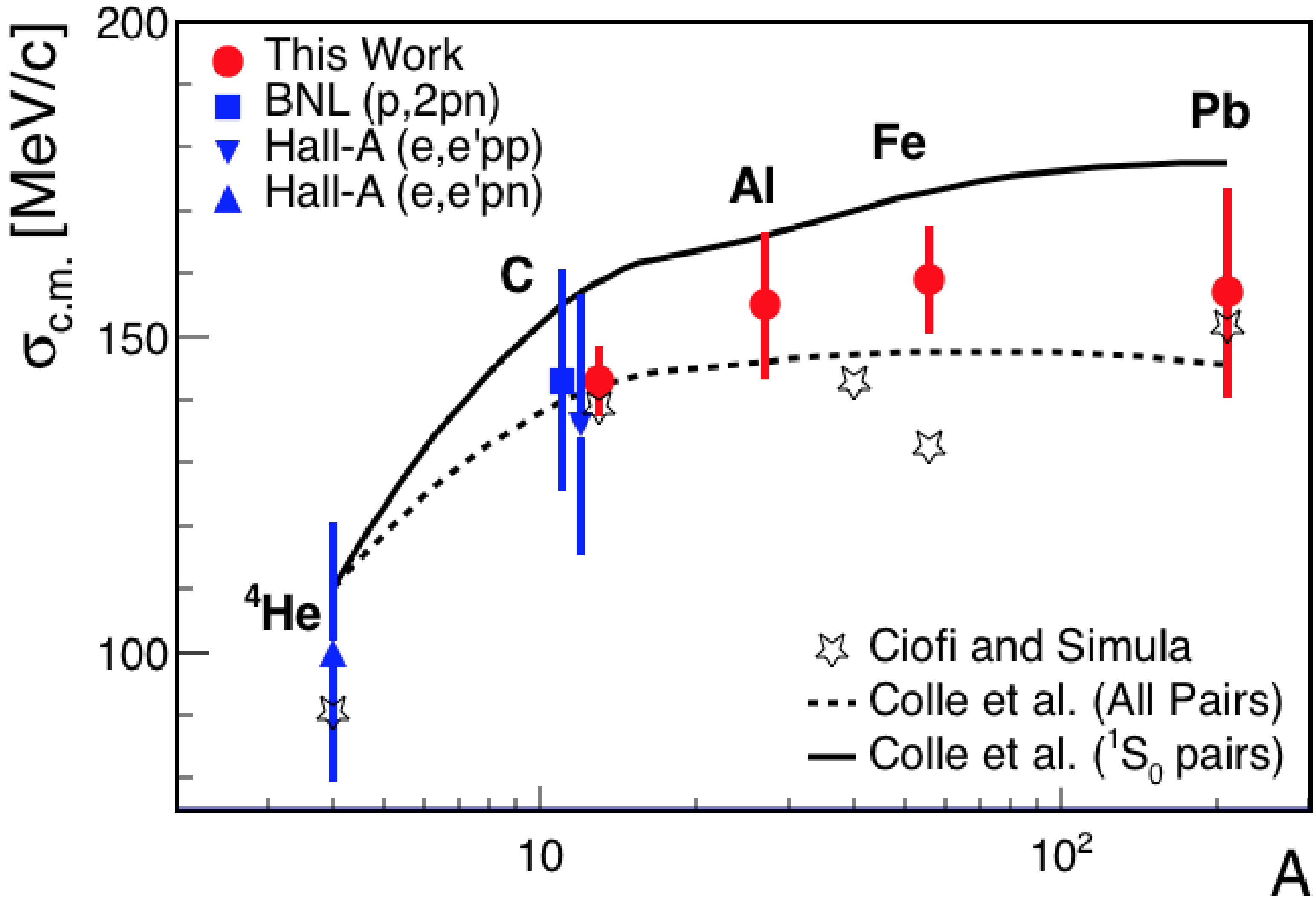
$$\sigma_A = \mathbf{a}_2 \times \frac{A}{2}\sigma_d$$

Nucleus-dependence should appear in the center-of-mass momentum distribution.

Erez Cohen
in preparation



CoM momentum distribution width saturates with nuclear size.



Even though a nucleus is not dilute, we can see scale separation.

$$r_{\text{eff.}} << a, d$$

- $r_{\text{eff.}} \approx 1/2m_\pi \approx 0.7$ fm
- $d \approx (2/\rho)^{1/3} \approx 2.3$ fm
- $a \approx 5.4$ fm

Replace the contact-force with a two-body *NN* force.

Ronen Weiss, Rey Cruz-Torres et al.
"The nuclear contacts and short range correlations in nuclei"
Phys. Lett. B 780 211–215 (2018)

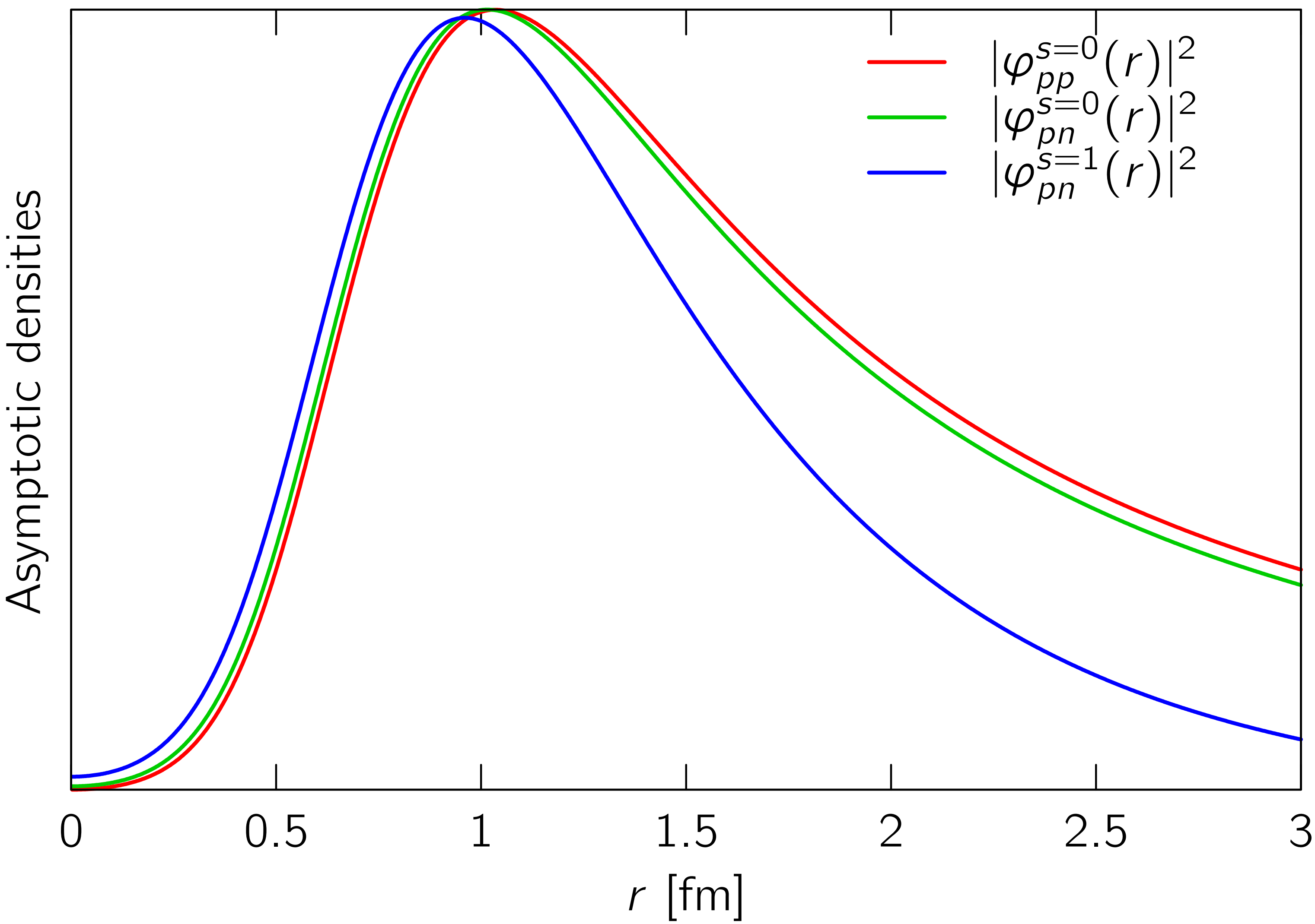
Replace the contact-force with a two-body *NN* force.

For $r \to 0$, approximate two-body densities:

$$\begin{aligned}
\rho_2^{pp}(r) &= C_{pp}^{s=0}|\varphi_{pp}^{s=0}(r)|^2 \\
\rho_2^{pn}(r) &= C_{pn}^{s=0}|\varphi_{pn}^{s=0}(r)|^2 + C_{pn}^{s=1}|\varphi_{pn}^{s=1}(r)|^2 \\
\rho_2^{nn}(r) &= C_{nn}^{s=0}|\varphi_{nn}^{s=0}(r)|^2
\end{aligned}$$

$\varphi_{ij}^{s}(r)$ are zero-energy Schrödinger eq. solutions with *NN* potential.

Asymptotic distributions



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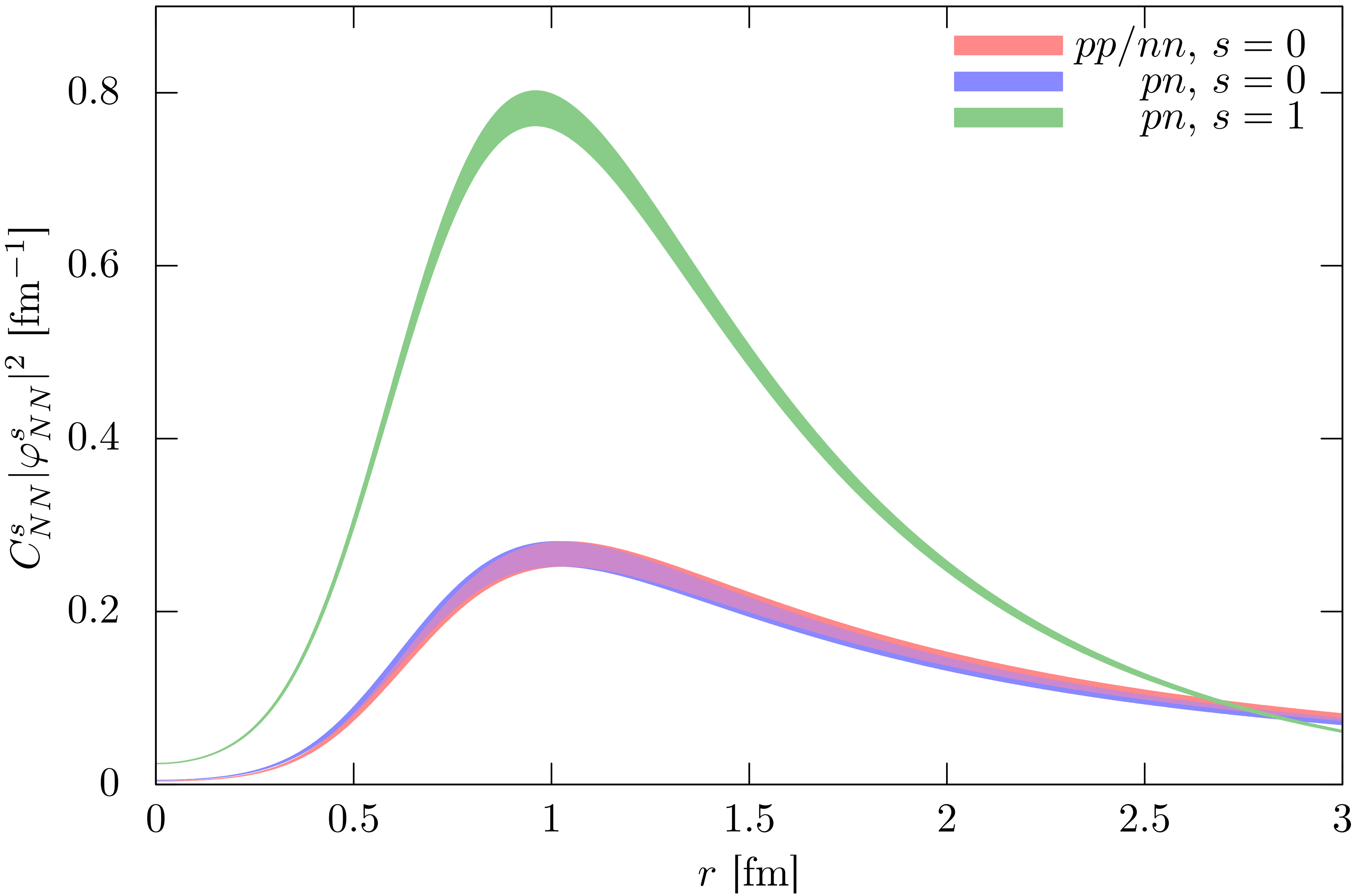

Replace the contact-force with a two-body NN force.

For $k \to \infty$, approximate two-body momentum densities:

$$\tilde{\rho}_2^{pp}(k) = C_{pp}^{s=0} |\tilde{\varphi}_{pp}^{s=0}(k)|^2$$
$$\tilde{\rho}_2^{pn}(k) = C_{pn}^{s=0} |\tilde{\varphi}_{pn}^{s=0}(k)|^2 + C_{pn}^{s=1} |\tilde{\varphi}_{pn}^{s=1}(k)|^2$$
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\end{aligned}$$

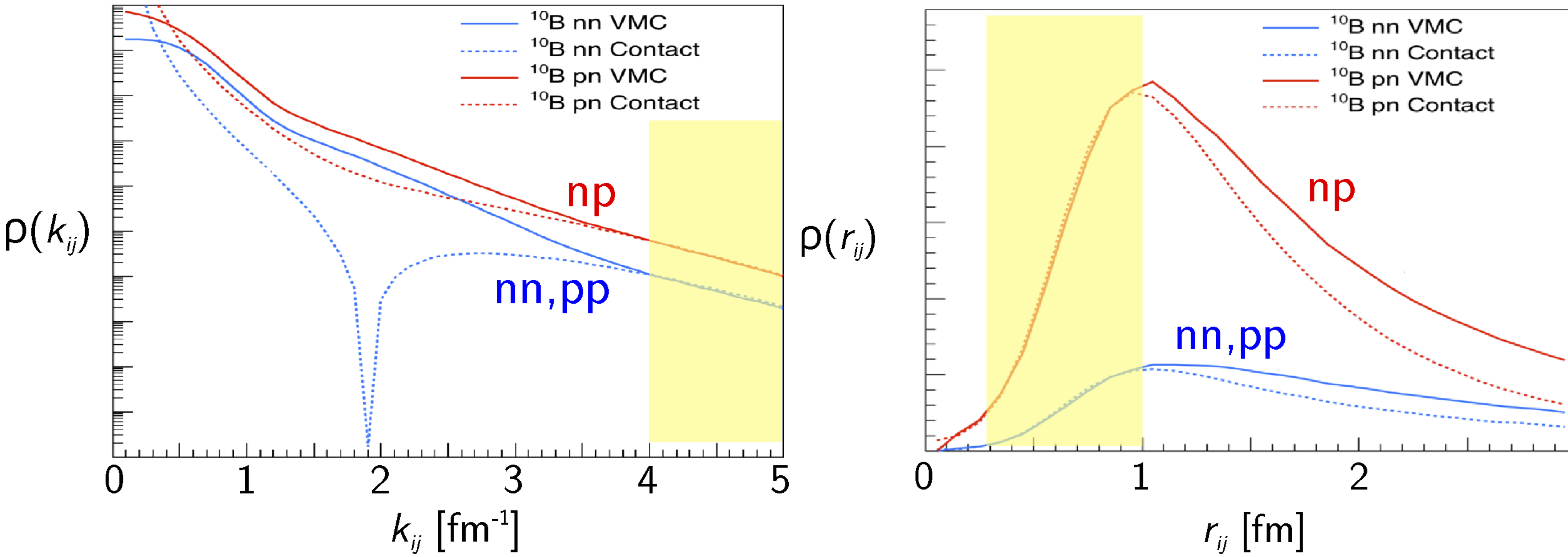
$\tilde{\varphi}_{ij}^{s}(k)$ are zero-energy Schrödinger eq. solutions with NN potential.

In symmetric nuclei:

$$C_{pp}^{s=0} \approx C_{nn}^{s=0}$$

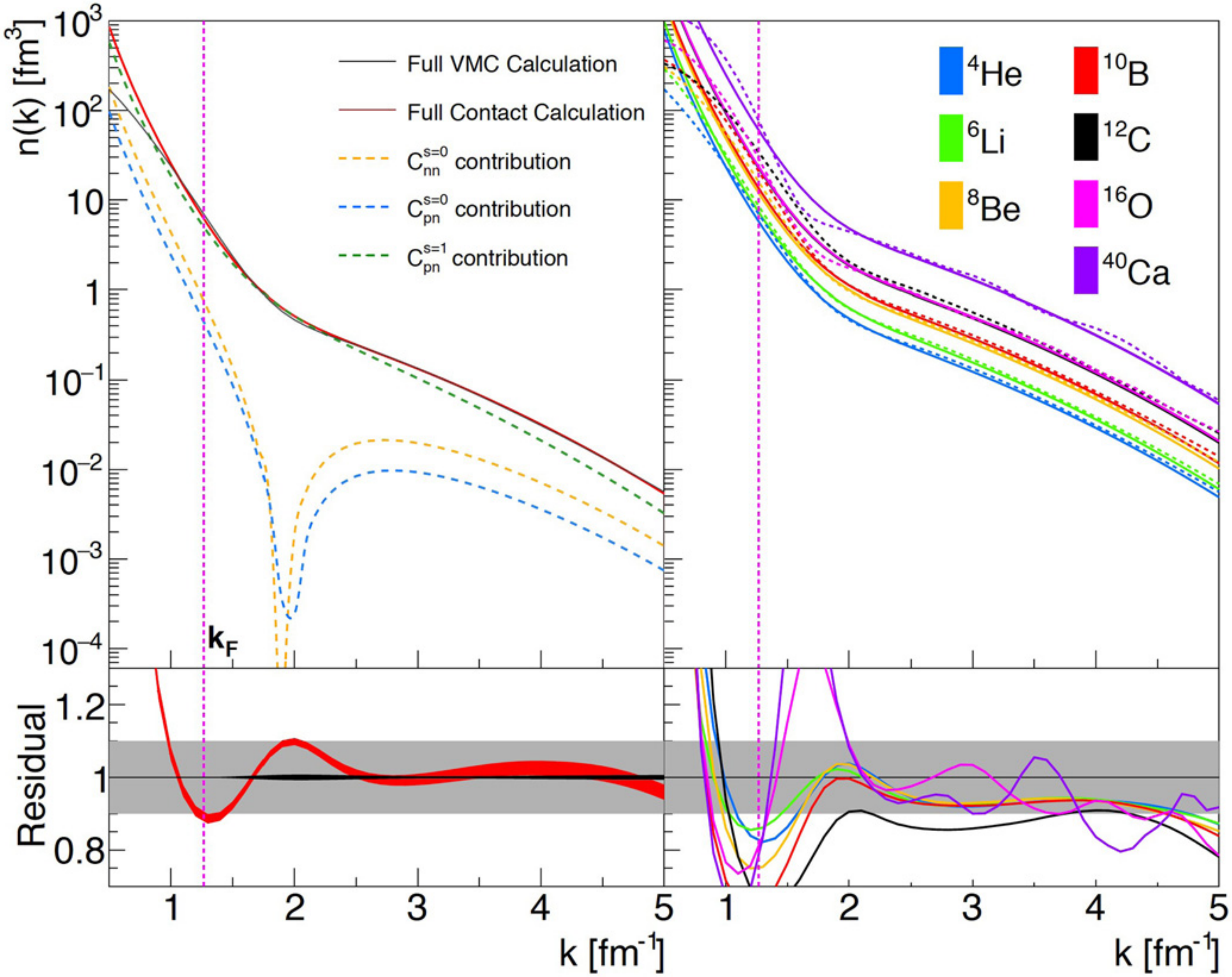
Contacts can be bench-marked against ab initio calculations.

VMC calculations by the Argonne group
Wiringa, Pieper, Lonardoni et al.

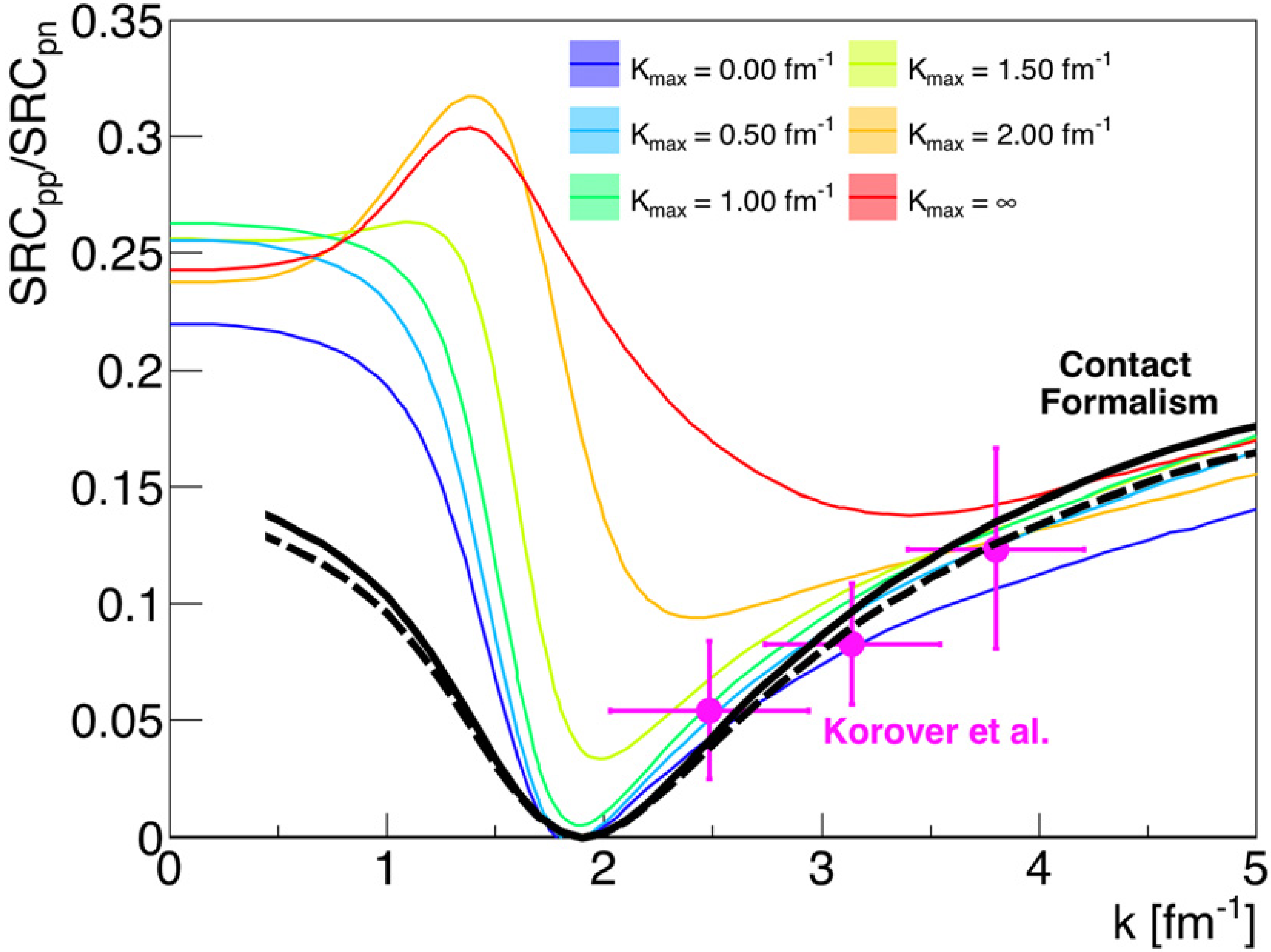


Extraction in $k-$space must be performed at very large momentum.

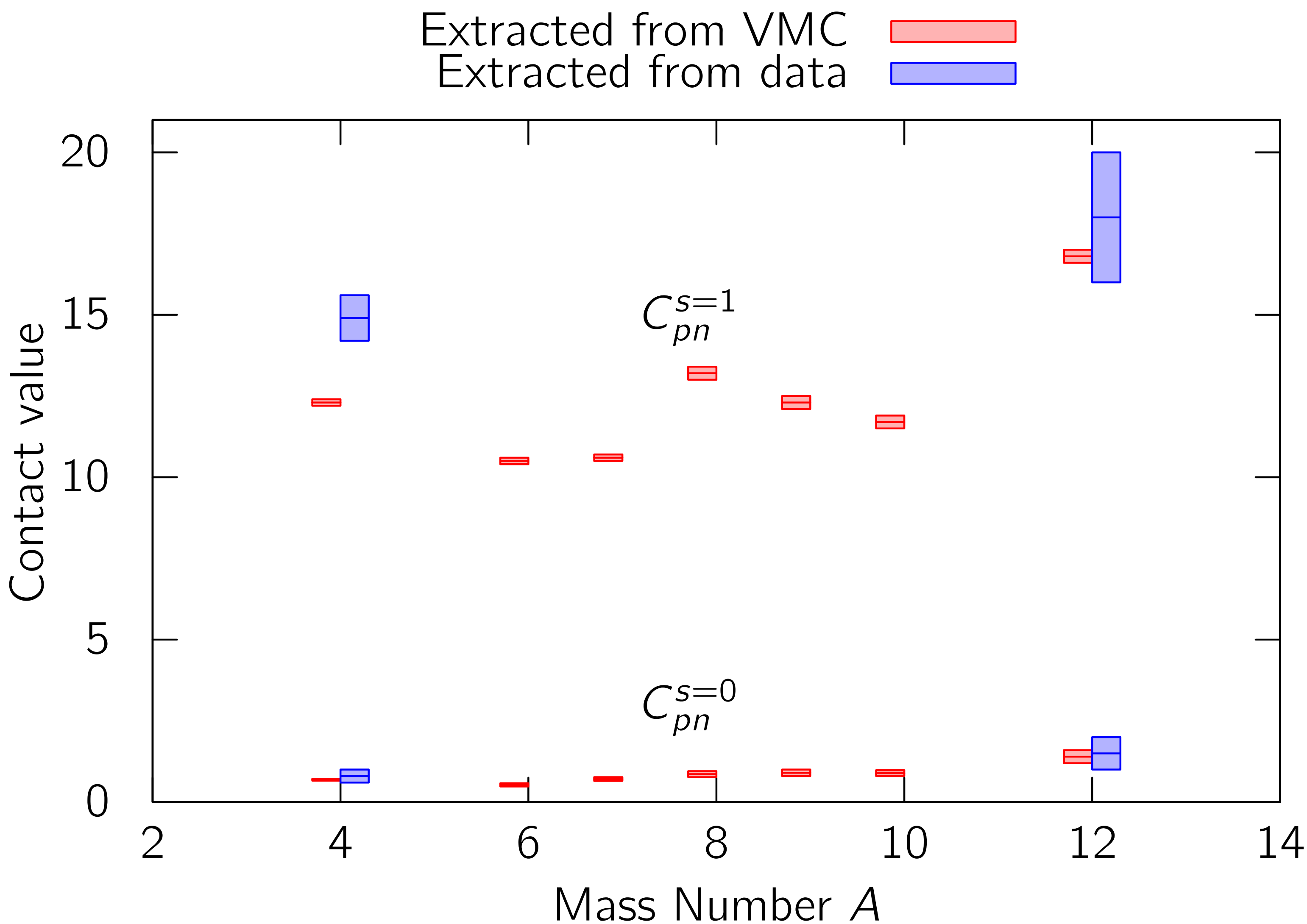
Extractions in position space can predict momentum-space distributions.



Small pair center-of-mass momentum is required to select correlated pairs.

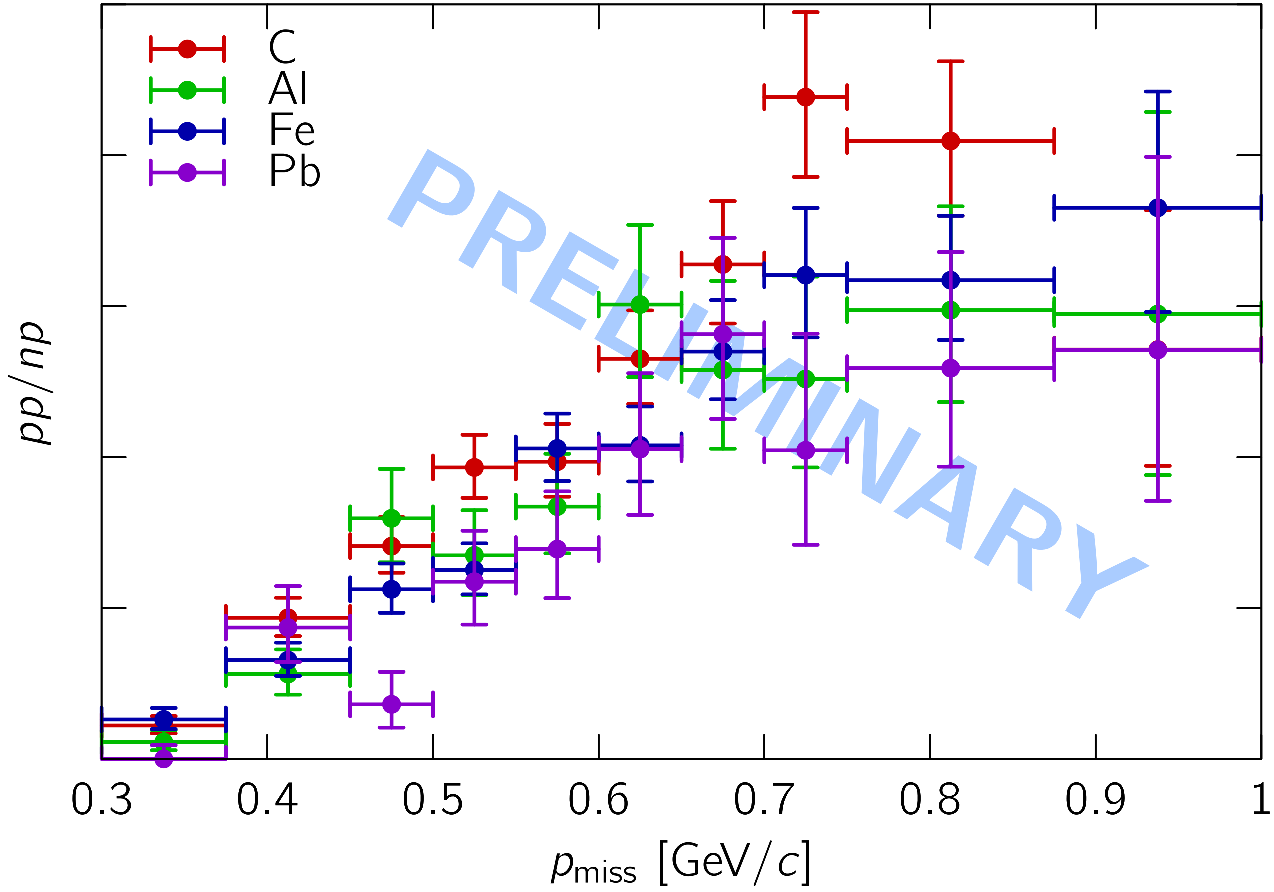


Contact extraction from data would allow us to go beyond small nuclei.



Key ingredient would be pp/np ratio.

Adin Hrnjic
A. Schmidt
in preparation



My talk today:

1. Scale Separation and Nuclear Contact Formalism
2. **Correlation Functions**
3. Short-range correlations and the EMC effect

Applying contact formalism to nuclear correlation functions.

"Short range correlations and the isospin dependence of nuclear correlation functions"
R. Cruz-Torres, A. Schmidt, G. A. Miller et al.
arXiv:1710.07966, submitted to Phys. Lett. B

Applying contact formalism to nuclear correlation functions.

$$\rho_2(\vec{x}, \vec{y}) = F(\vec{x}, \vec{y}) \times \rho(\vec{x})\rho(\vec{y})$$

Applying contact formalism to nuclear correlation functions.

$$\rho_2(\vec{x},\vec{y}) = F(\vec{x},\vec{y}) \times \rho(\vec{x})\rho(\vec{y})$$

$$\rho_2(r) = F(r) \int \mathrm{d}^3\vec{R}\rho(\vec{R}+\vec{r}/2)\rho(\vec{R}-\vec{r}/2)$$

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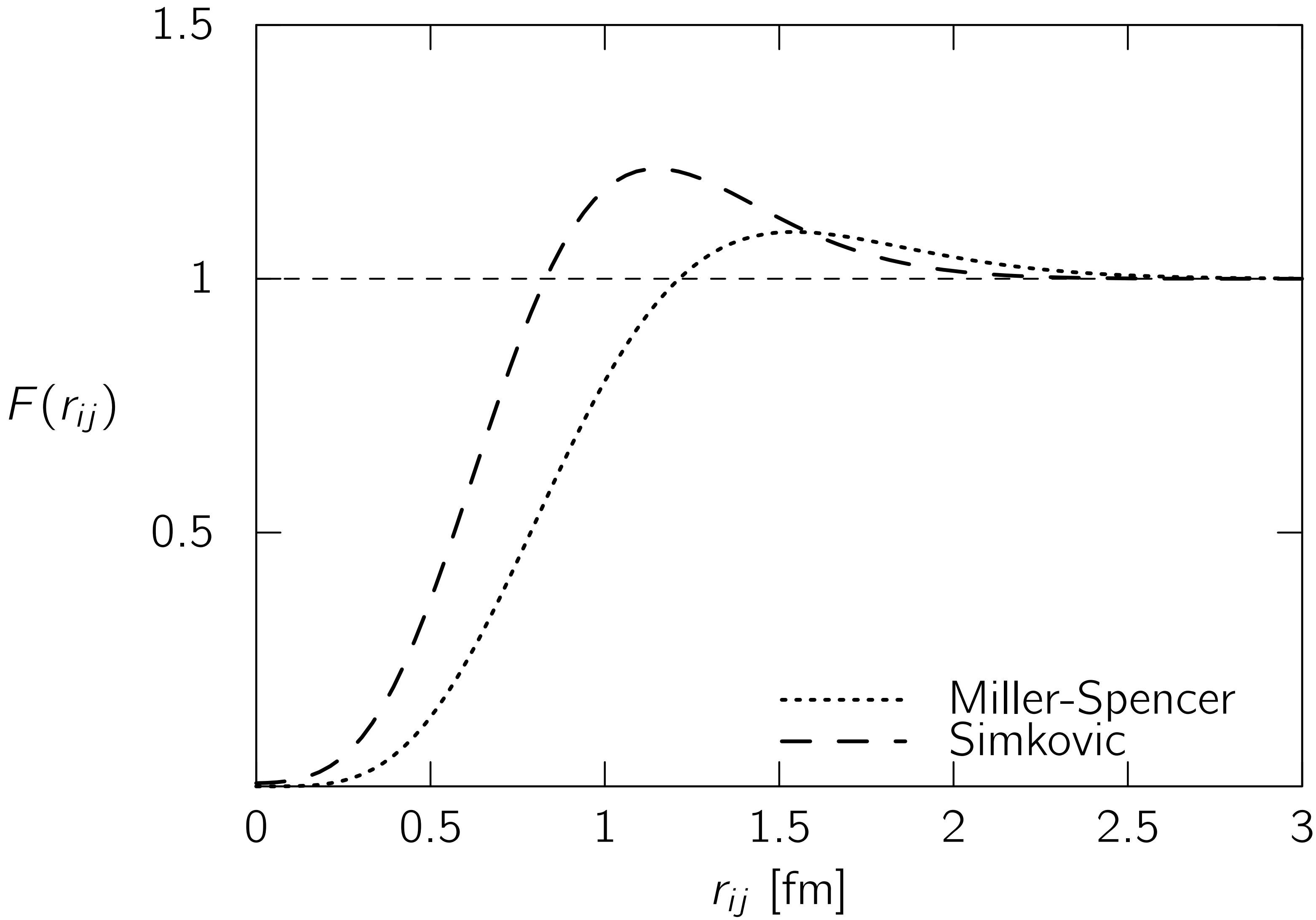
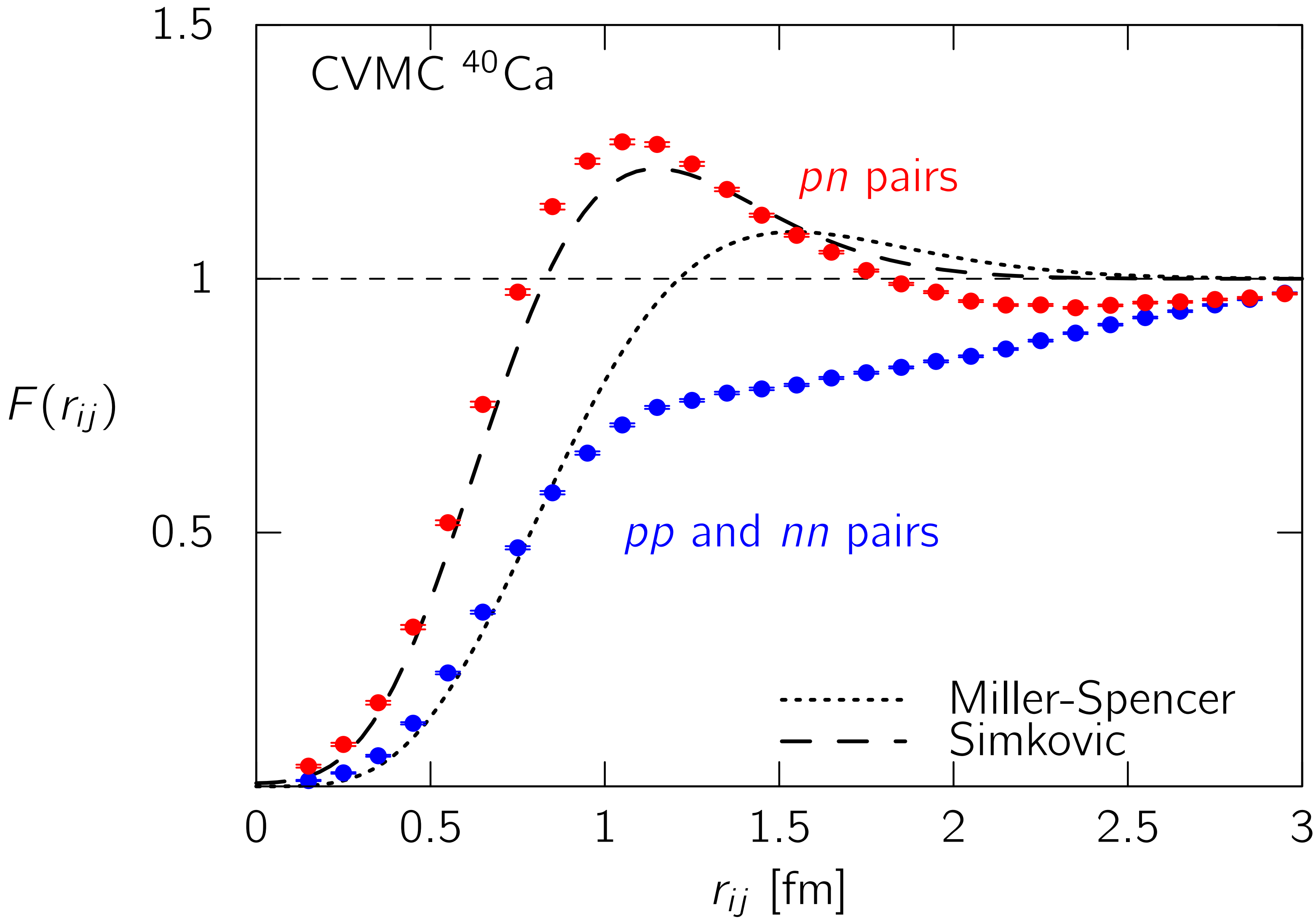
$$\rho_2(r) = F(r)\int \mathrm{d}^3\vec{R}\rho(\vec{R}+\vec{r}/2)\rho(\vec{R}-\vec{r}/2)$$

$$\rho_2(r) = F(r)\rho_2^{\text{uncorr.}}(r)$$

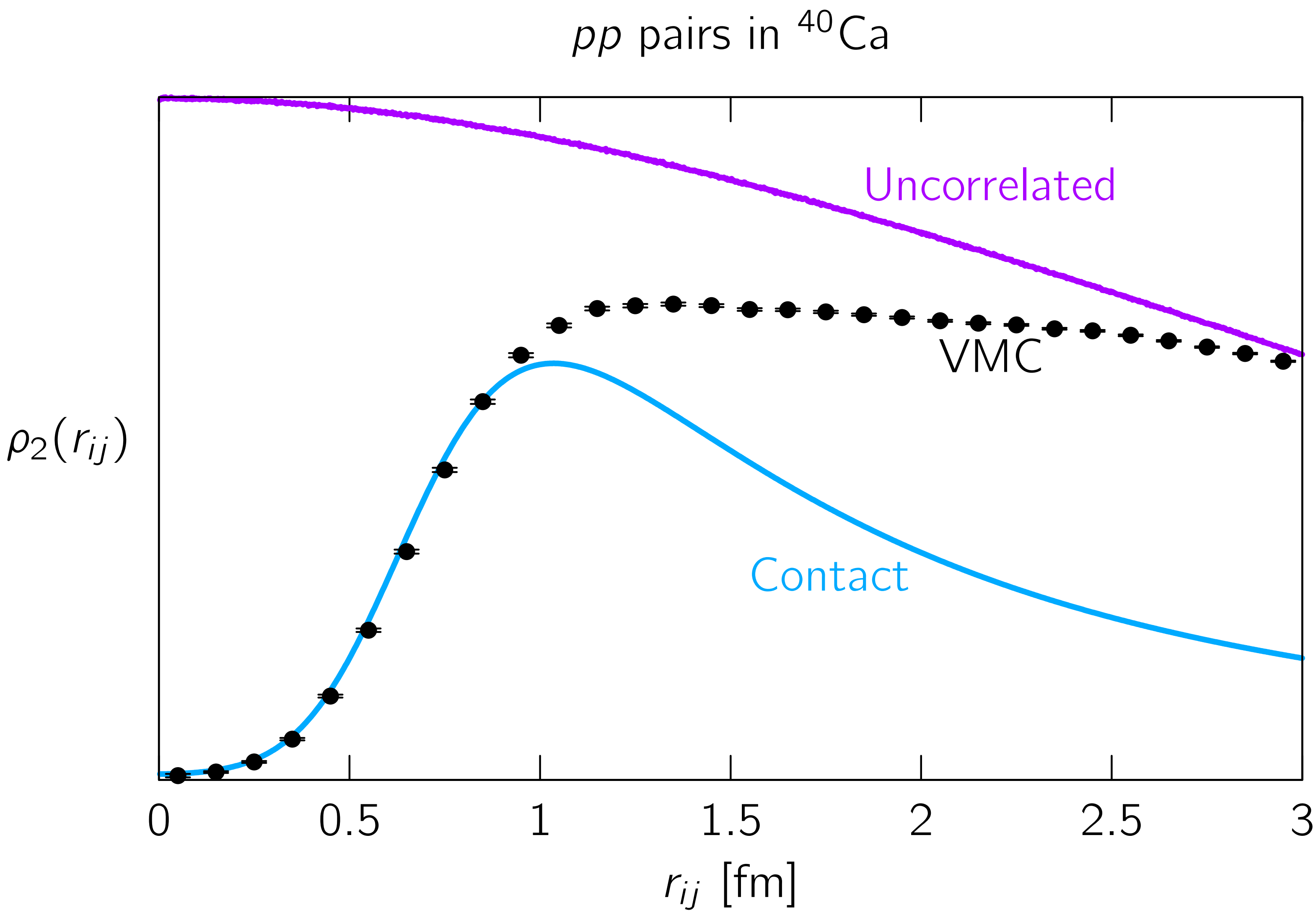
Correlation functions from VMC



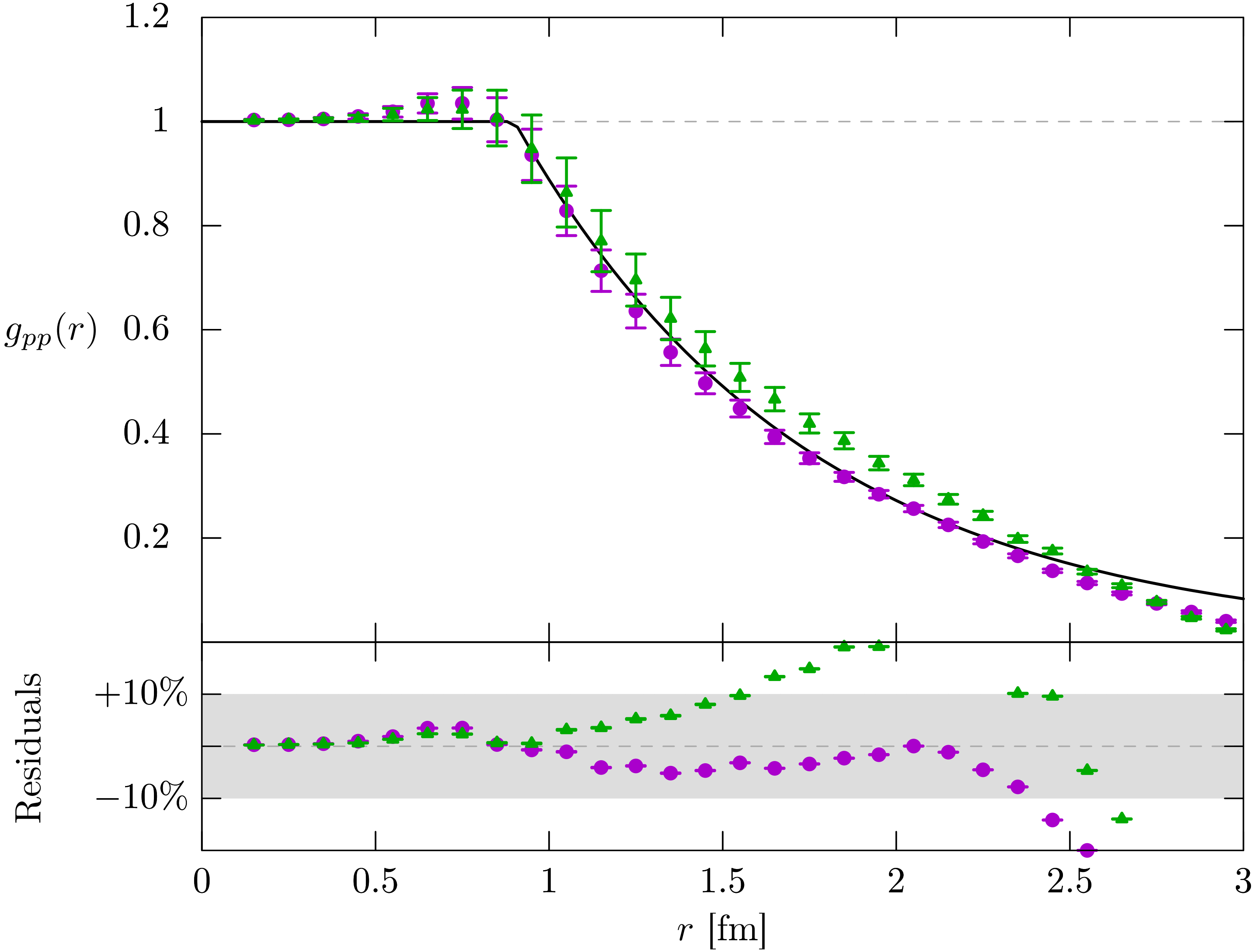
Correlation functions from VMC



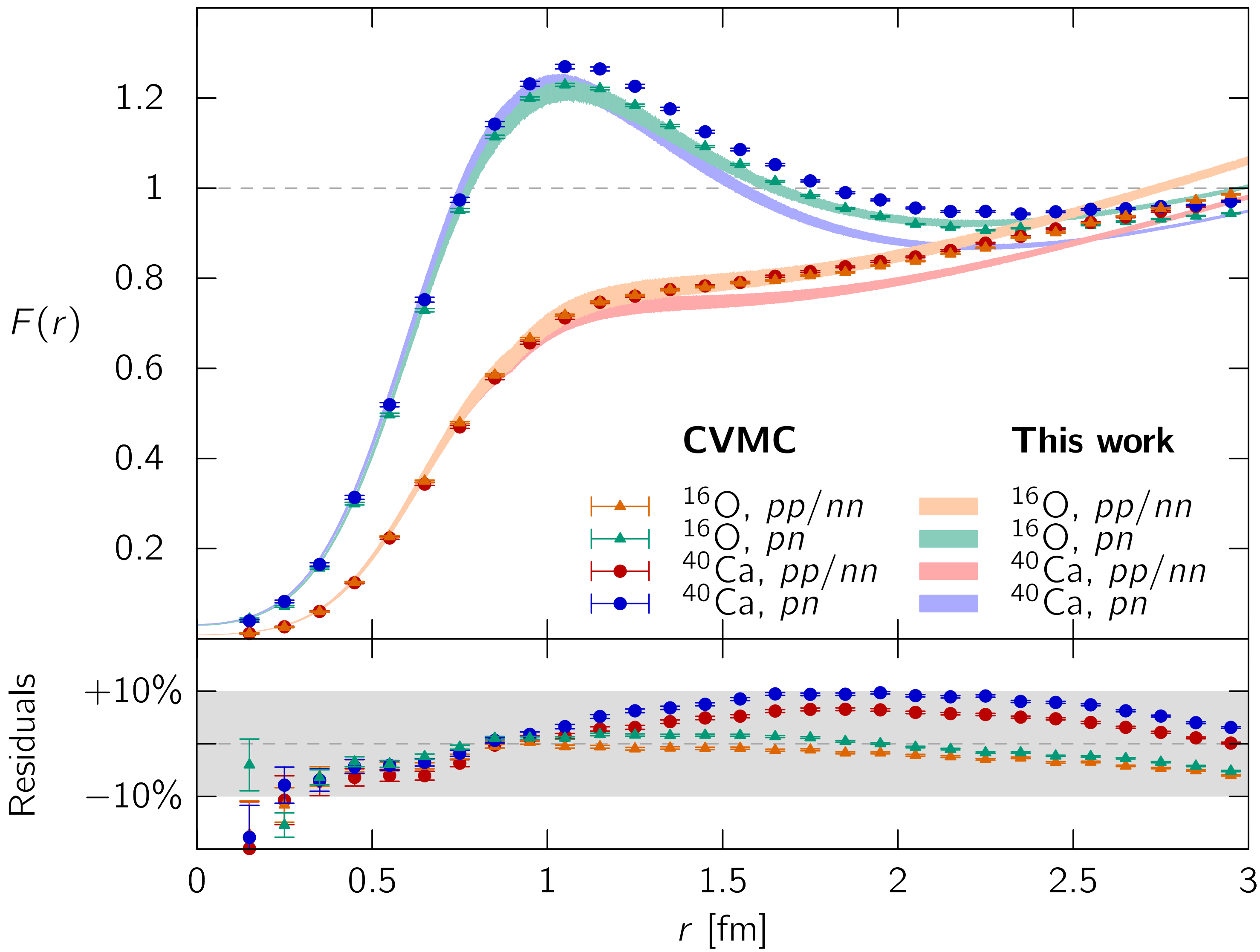
Short-range and long-range behavior can be blended with the same scheme for *pp* and *pn*



Blending function $g(r)$



The isospin-dependent correlations are driven entirely by short-range interactions.



Would it make more sense to account for Pauli exchange?

Classical:

$$\rho_2^{\text{uncorr.}}(r) \equiv \int \mathrm{d}^3 R \rho(\vec{R} + \vec{r}/2)\rho(\vec{R} - \vec{r}/2)$$

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Classical:

$$\rho_2^{\text{uncorr.}}(r) \equiv \int \mathrm{d}^3 R \rho(\vec{R} + \vec{r}/2)\rho(\vec{R} - \vec{r}/2)$$

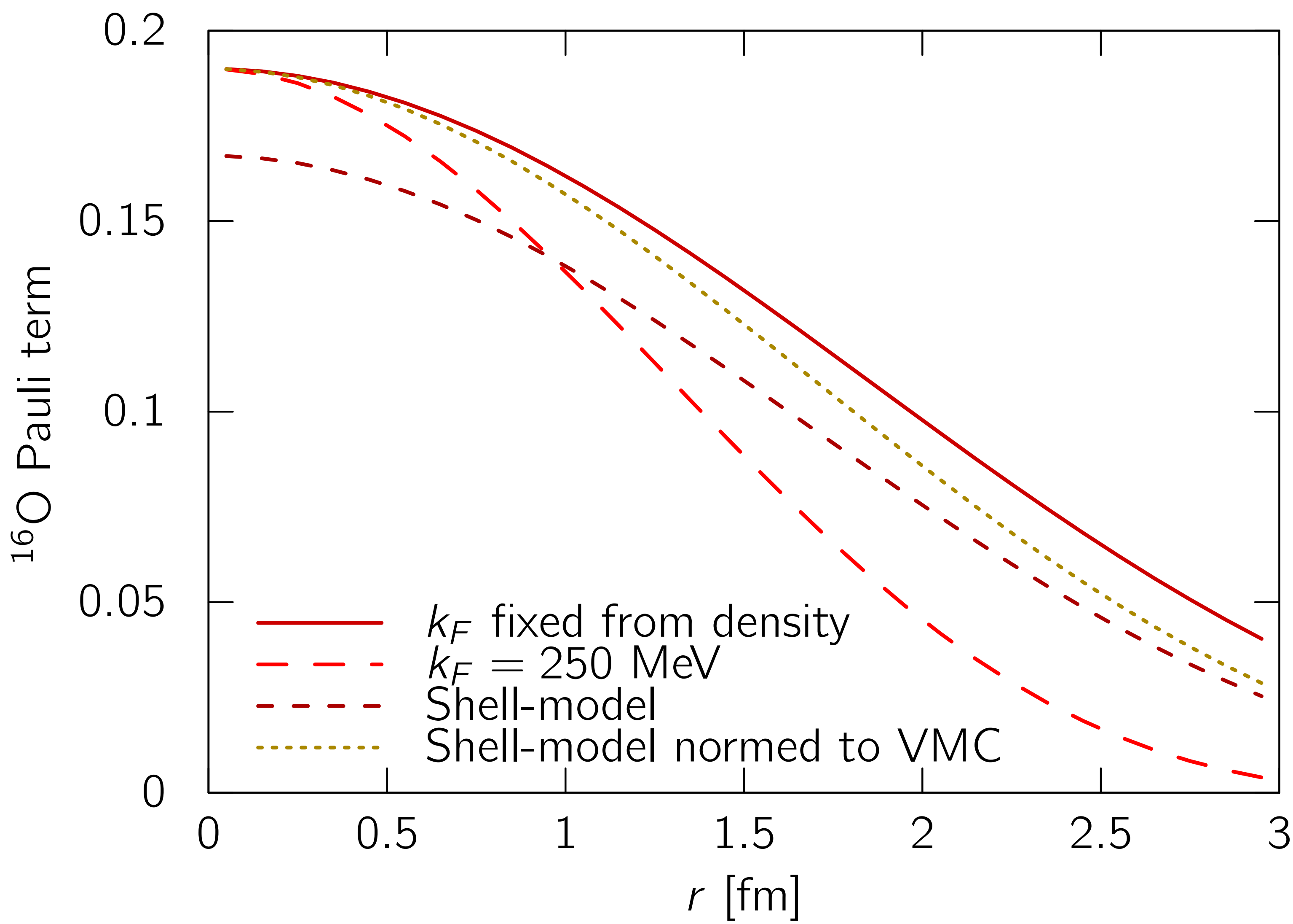
Accounting for exchange:

$$\rho_2^{\text{uncorr.}}(r) \equiv \sum_{i,j} \int \mathrm{d}^3 R \psi_i^*(\vec{R} + \vec{r}/2)\psi_j^*(\vec{R} - \vec{r}/2)$$
$$\times \left[\psi_i(\vec{R} + \vec{r}/2)\psi_j(\vec{R} - \vec{r}/2) - \psi_i(\vec{R} - \vec{r}/2)\psi_j(\vec{R} + \vec{r}/2)\right]$$

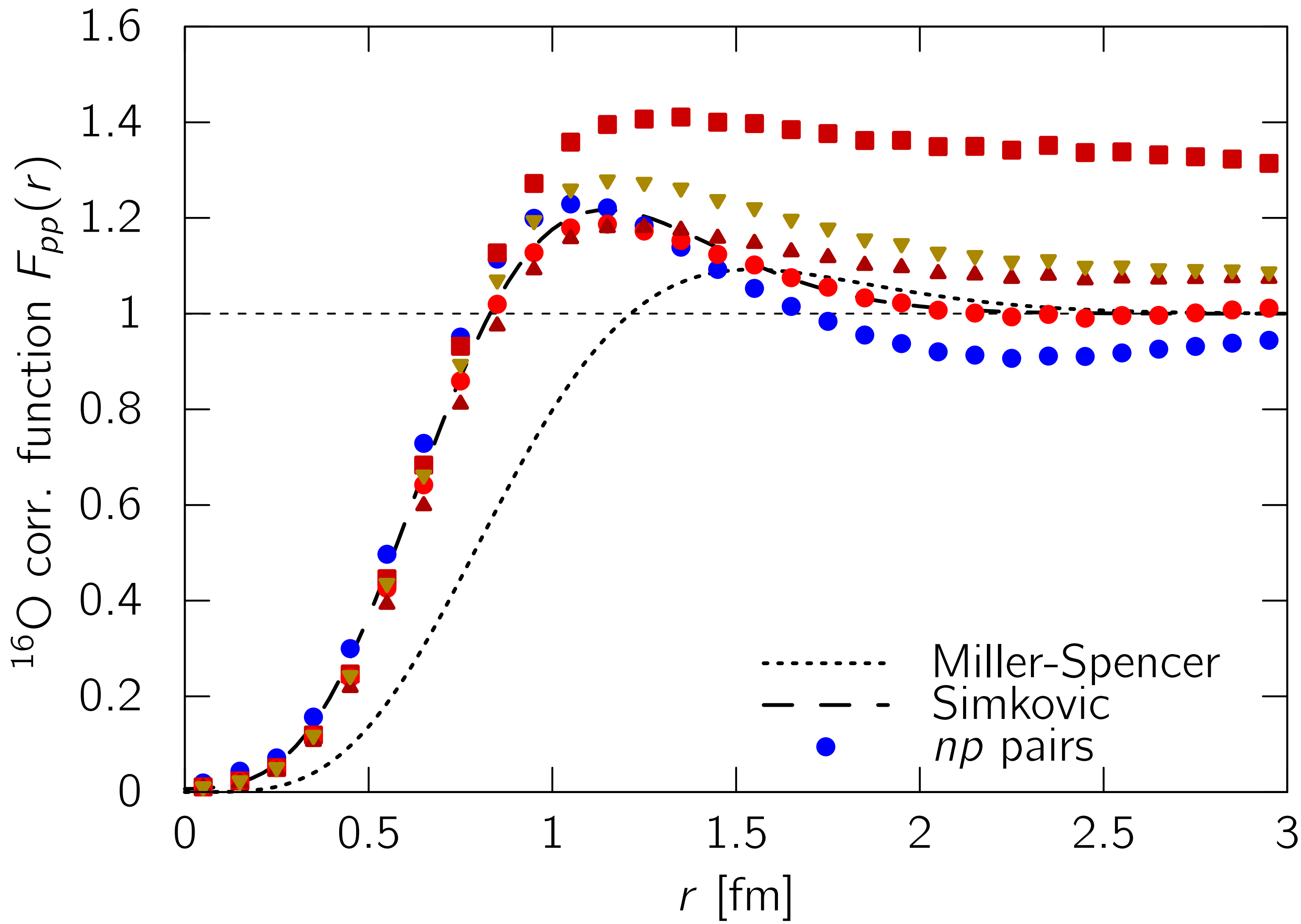
There are a few ways to calculate the exchange term.

1. Infinite nuclear matter approximation
   - $\rho_2^{\text{exch.}} = a[3j_1(k_F r)/(k_F r)]^2$
   - Fix a, k_F?
2. Shell-model calculation
   - Evaluate integrals over single-particle states

There are a few ways to calculate the exchange term.



There are a few ways to calculate the exchange term.



Conclusions

- Isospin differences are driven by short-range physics.
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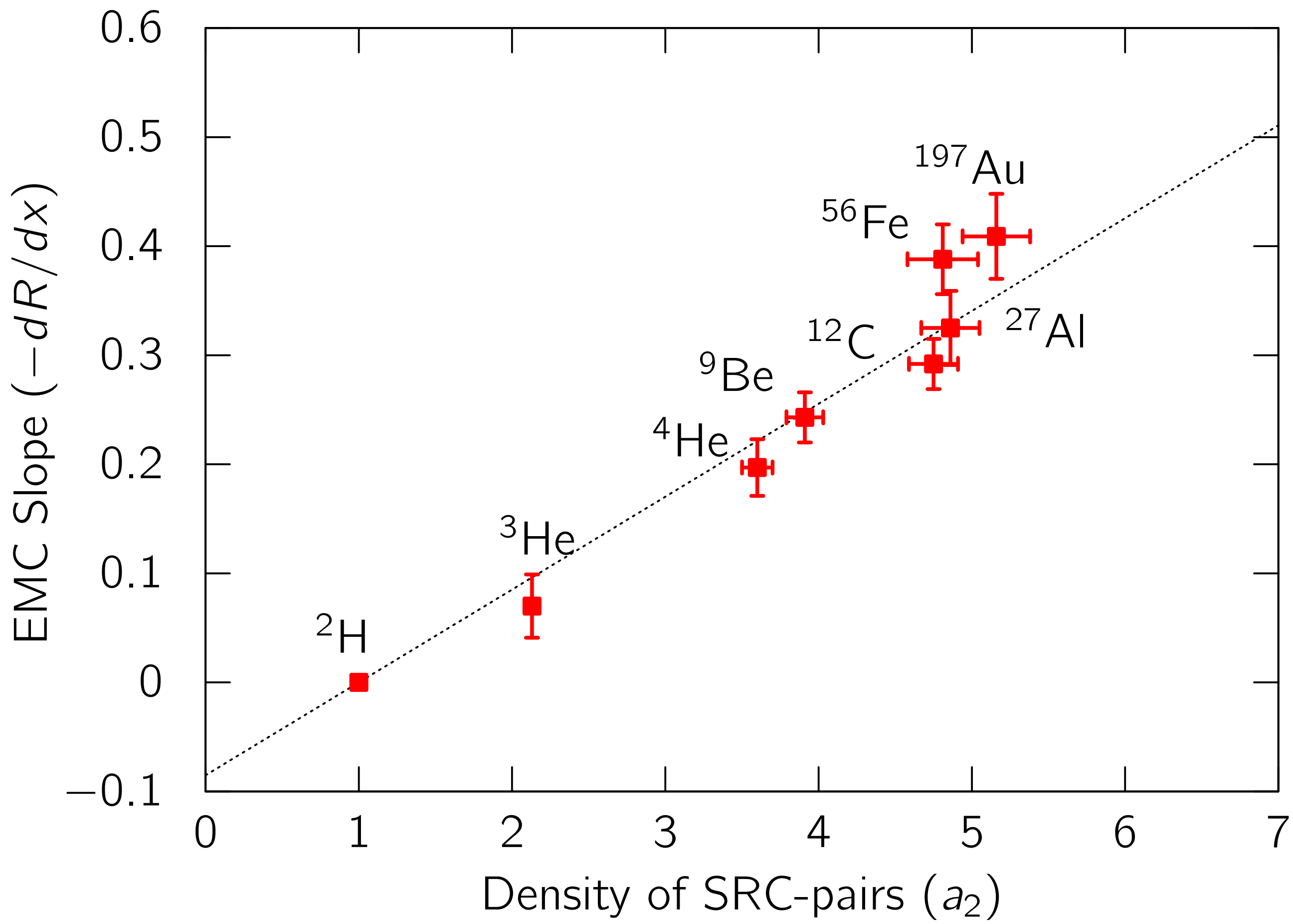
Conclusions

- Isospin differences are driven by short-range physics.
  - Mostly Pauli exchange
- VMC calculations show same behavior as Simkovic.
  - Taking difference to Miller-Spencer as systematic may not be appropriate.
- Contact formalism gives us new insight into Simkovic
  - Spin-isospin decomposition
  - Universal short range $\longleftrightarrow$ nucleus-dependent long-range

My talk today:

1. Scale Separation and Nuclear Contact Formalism
2. Correlation Functions
3. **Short-range correlations and the EMC effect**

There is a suggestive correlation between SRC pairing and the EMC effect.



Hen, Miller, Piasetzky, Weinstein, RMP (2017)

Nuclear contact in EFT and the EMC effect

Chen, Detmold, Lynn, Schwenk, PRL 119, 262502 (2017)

$$\frac{F_2^A(x, Q^2)}{A} \simeq F_2^N(x, Q^2) + g_2(A, \Lambda) f_2(x, Q^2, \Lambda)$$

$$g_2(A, \Lambda) = \frac{1}{2A} \left\langle A \middle| : \left(N^\dagger N\right)^2 : \middle| A \right\rangle_\Lambda$$

g_2 **IS** the nuclear contact!

We tried to model the modification of a single *np*-SRC pair.

Work in collaboration with Barak Schmookler

We tried to model the modification of a single *np*-SRC pair.

$$F_2^A = (Z - n_{\mathrm{SRC}}^A)F_2^p + (N - n_{\mathrm{SRC}}^A)F_2^n + n_{\mathrm{SRC}}^A(F_2^{p*} + F_2^{n*})$$

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$$F_2^A = ZF_2^p + NF_2^n + n_{\text{SRC}}^A(\Delta F_2^p + \Delta F_2^n)$$

$$F_2^d = F_2^p + F_2^n + n_{\text{SRC}}^d(\Delta F_2^p + \Delta F_2^n)$$

We tried to model the modification of a single *np*-SRC pair.

$$\frac{n^d_{\mathrm{SRC}}}{F_2^d}(\Delta F_2^p + \Delta F_2^n) = \frac{\frac{F_2^A}{F_2^d} - (Z-N)\frac{F_2^p}{F_2^d} - N}{\frac{n^A_{\mathrm{SRC}}}{n^d_{\mathrm{SRC}}} - N}$$

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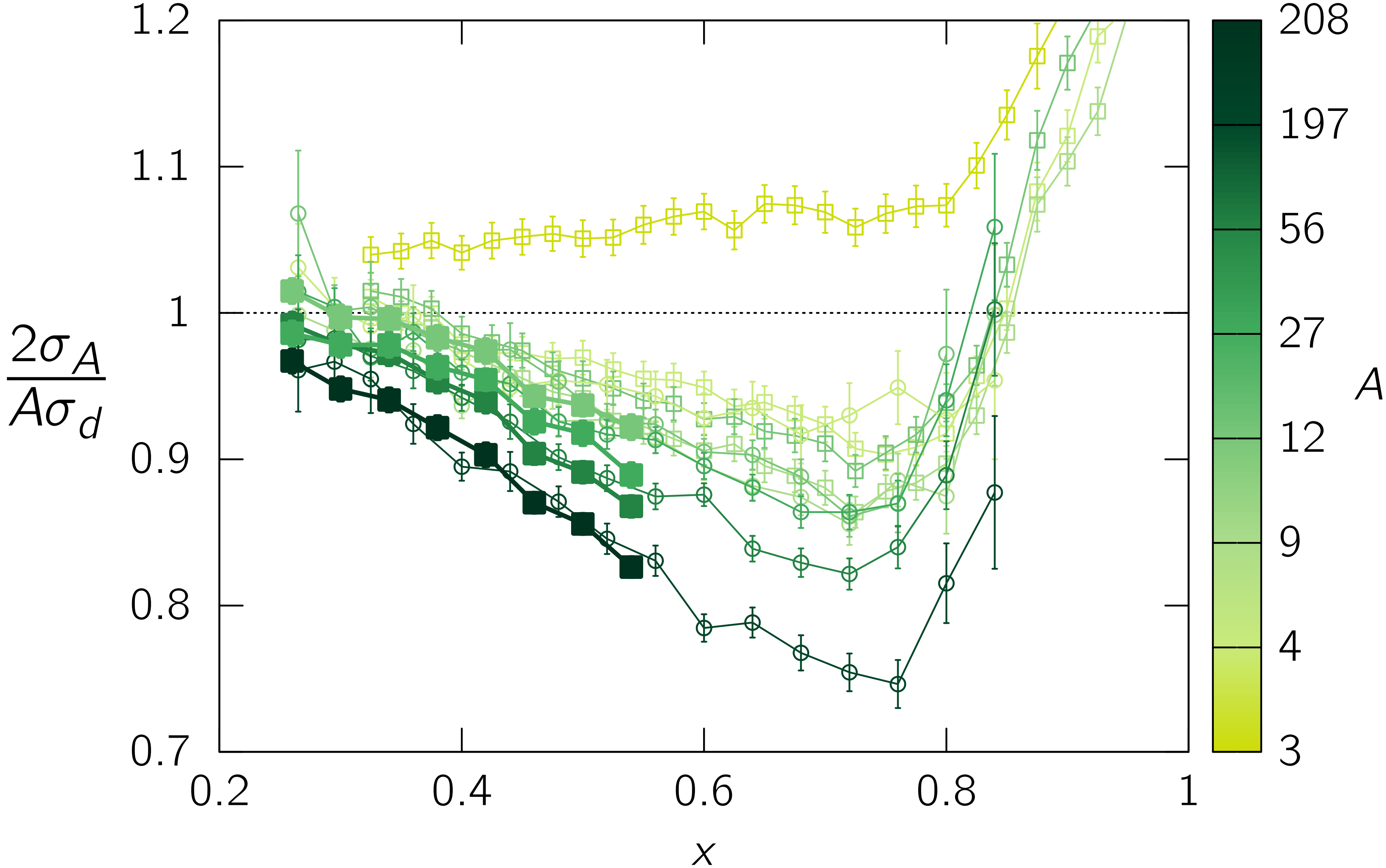
$$\frac{n^d_{\text{SRC}}}{F^d_2}(\Delta F^p_2 + \Delta F^n_2) = \frac{\frac{F^A_2}{F^d_2} - (Z-N)\frac{F^p_2}{F^d_2} - N}{\frac{n^A_{\text{SRC}}}{n^d_{\text{SRC}}} - N}$$

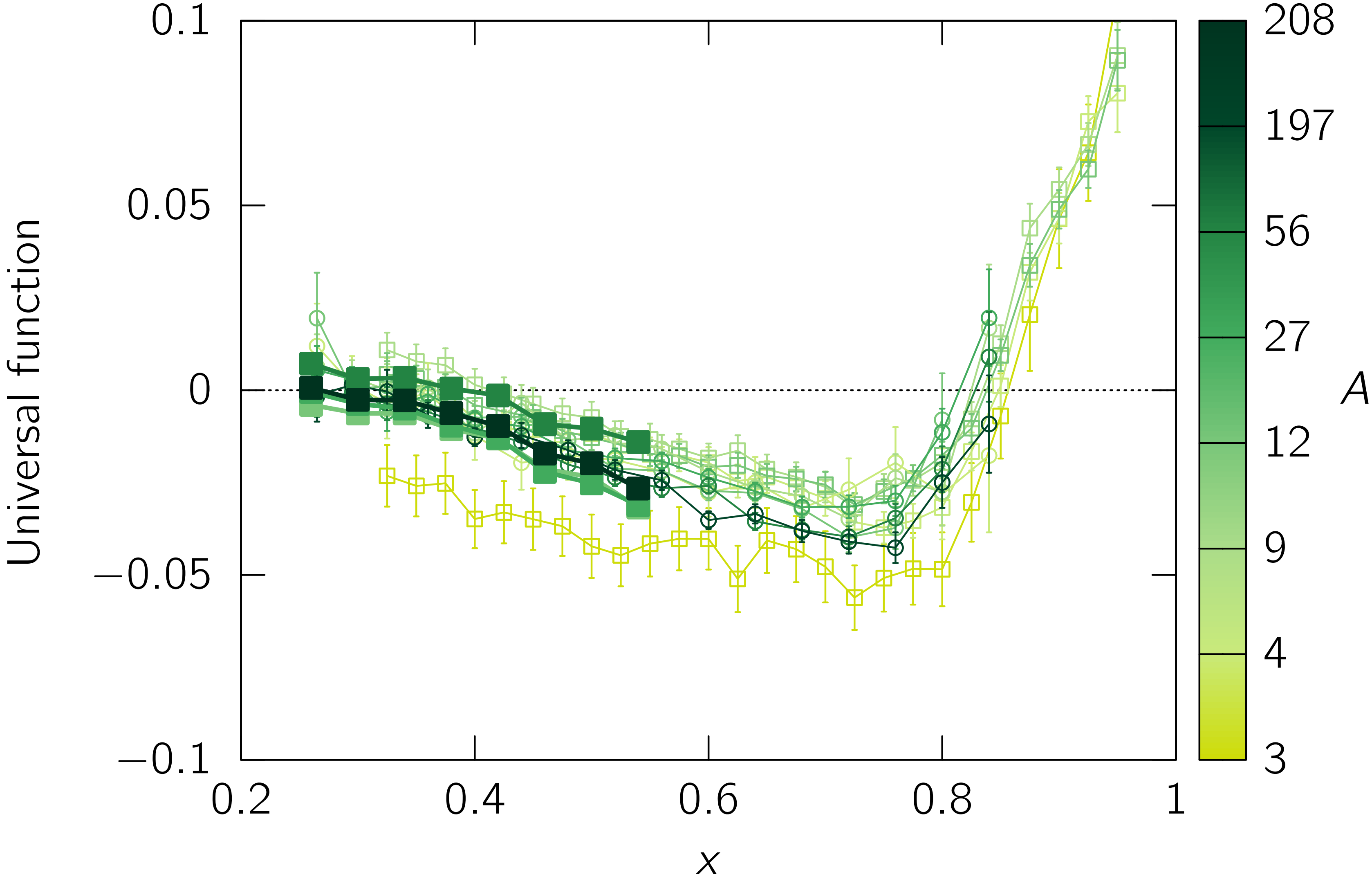
Universal function Nucleus-dependent

EMC data vary significantly by nucleus.

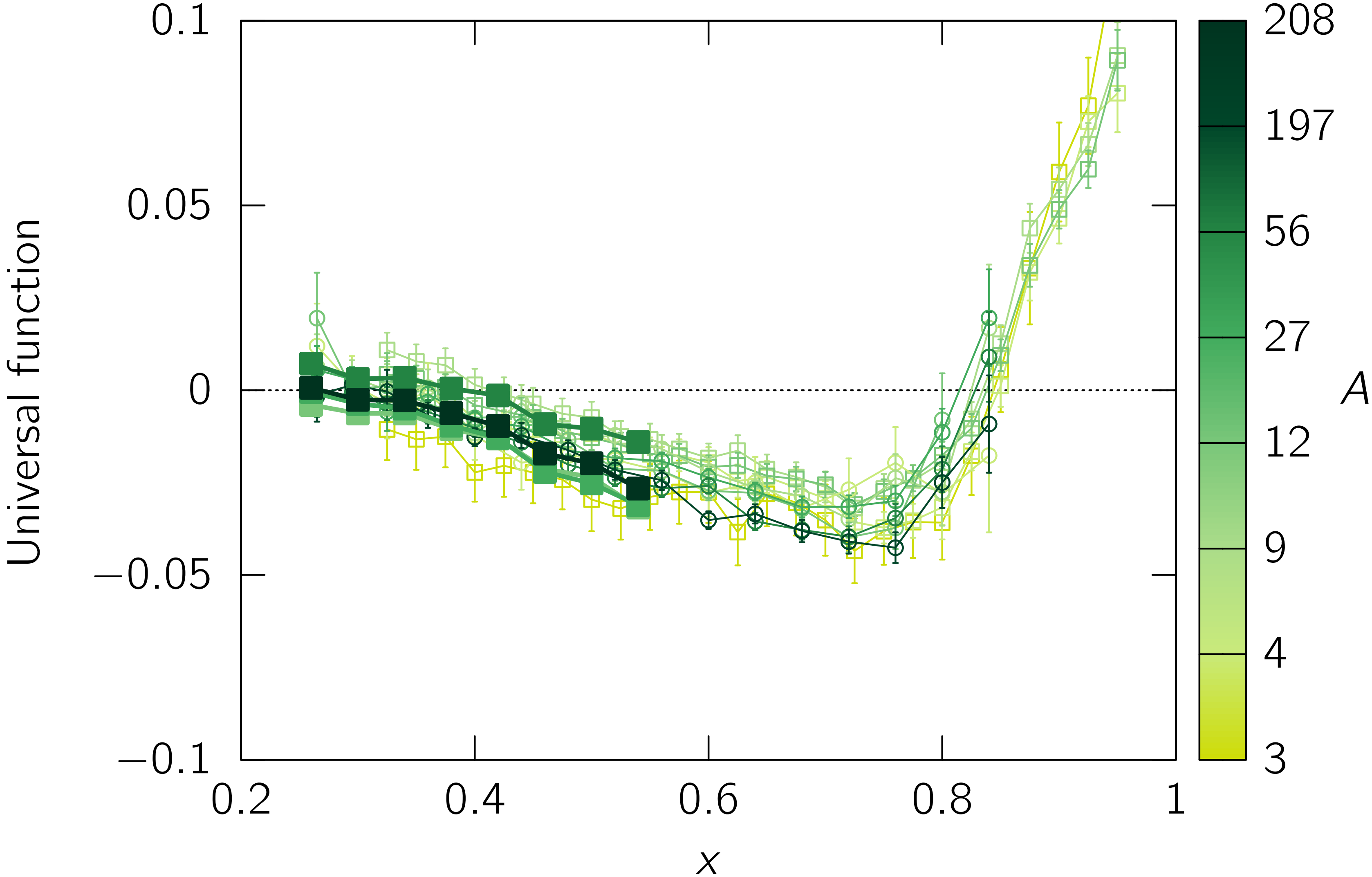


Submitted for publication

The SRC-modification function seems universal.



The SRC-modification function seems universal.



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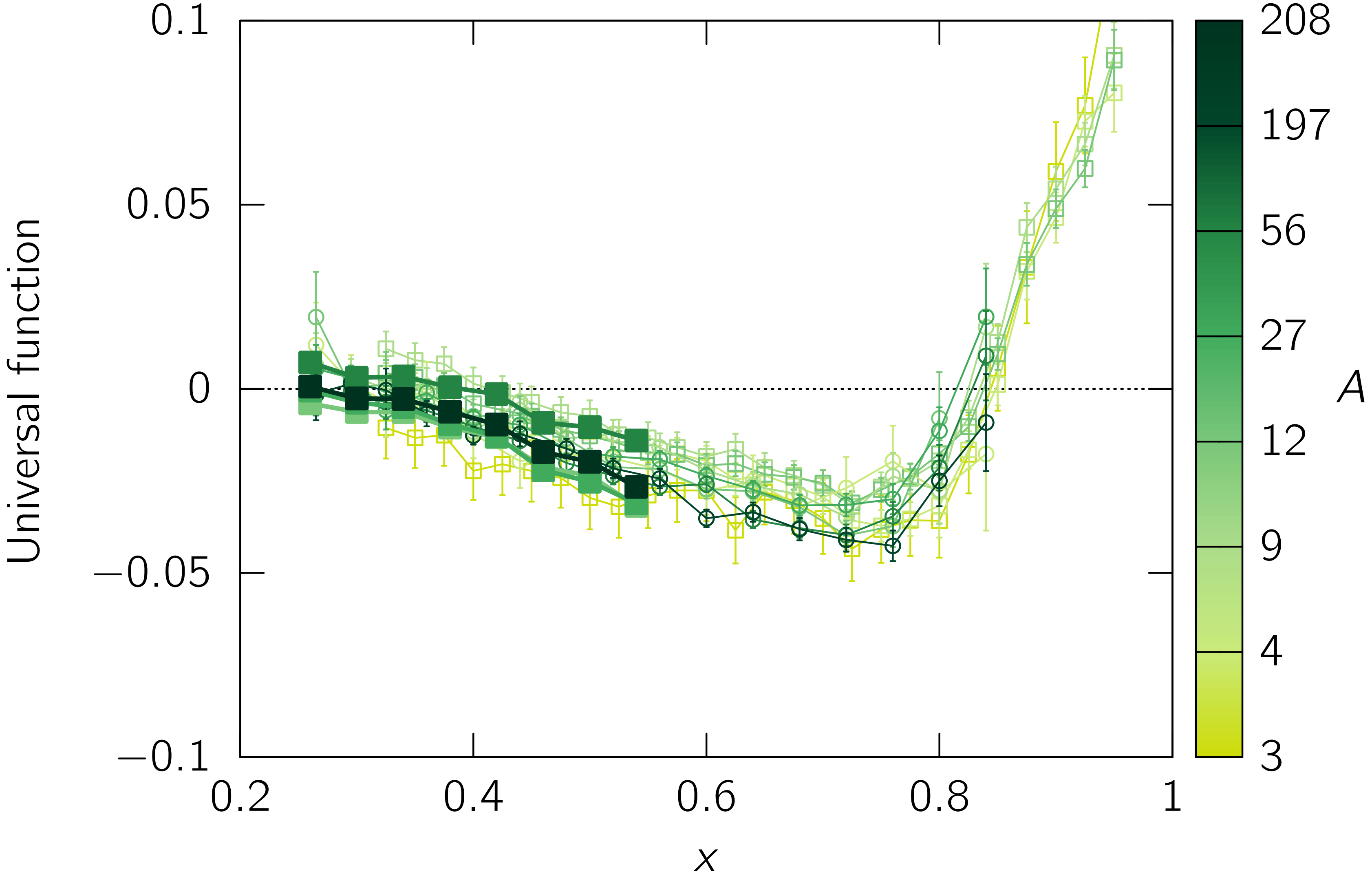
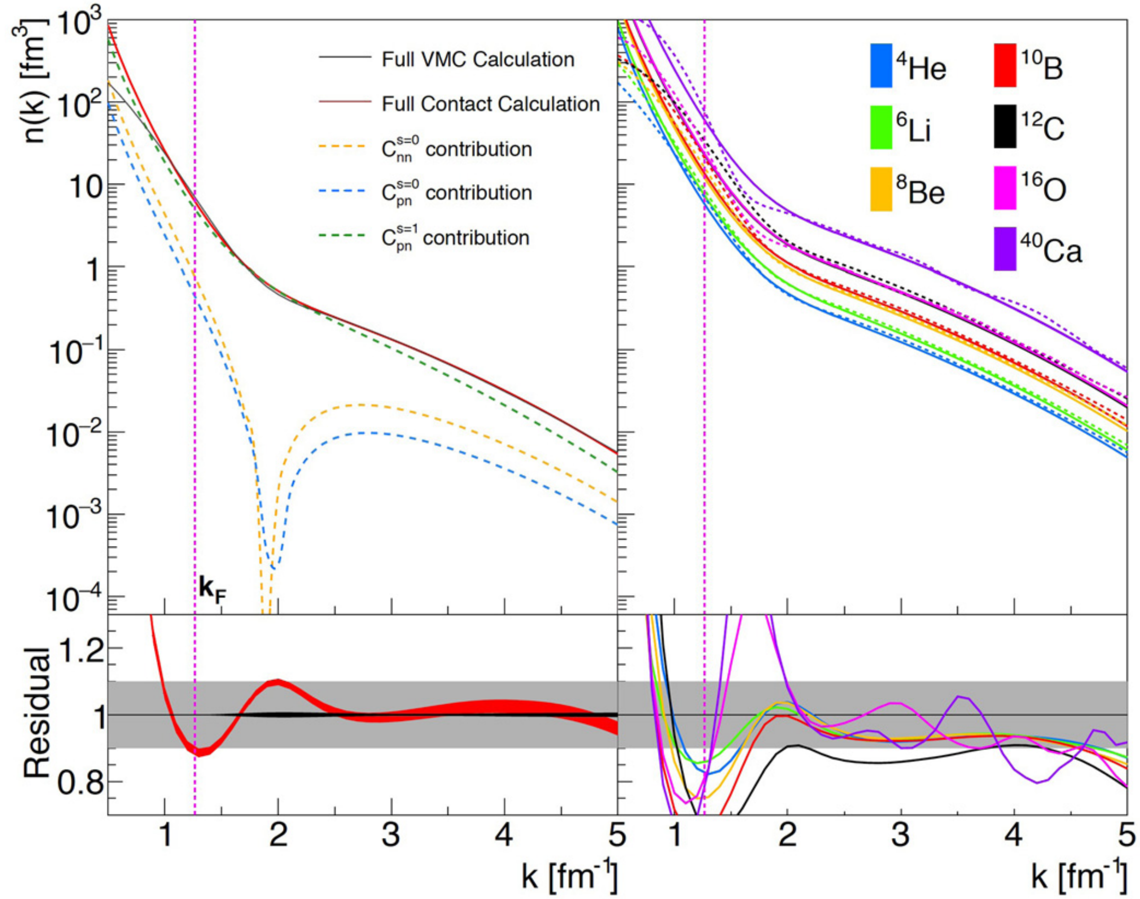


See Kulagin and Petti, PRC 82 054614 (2010)

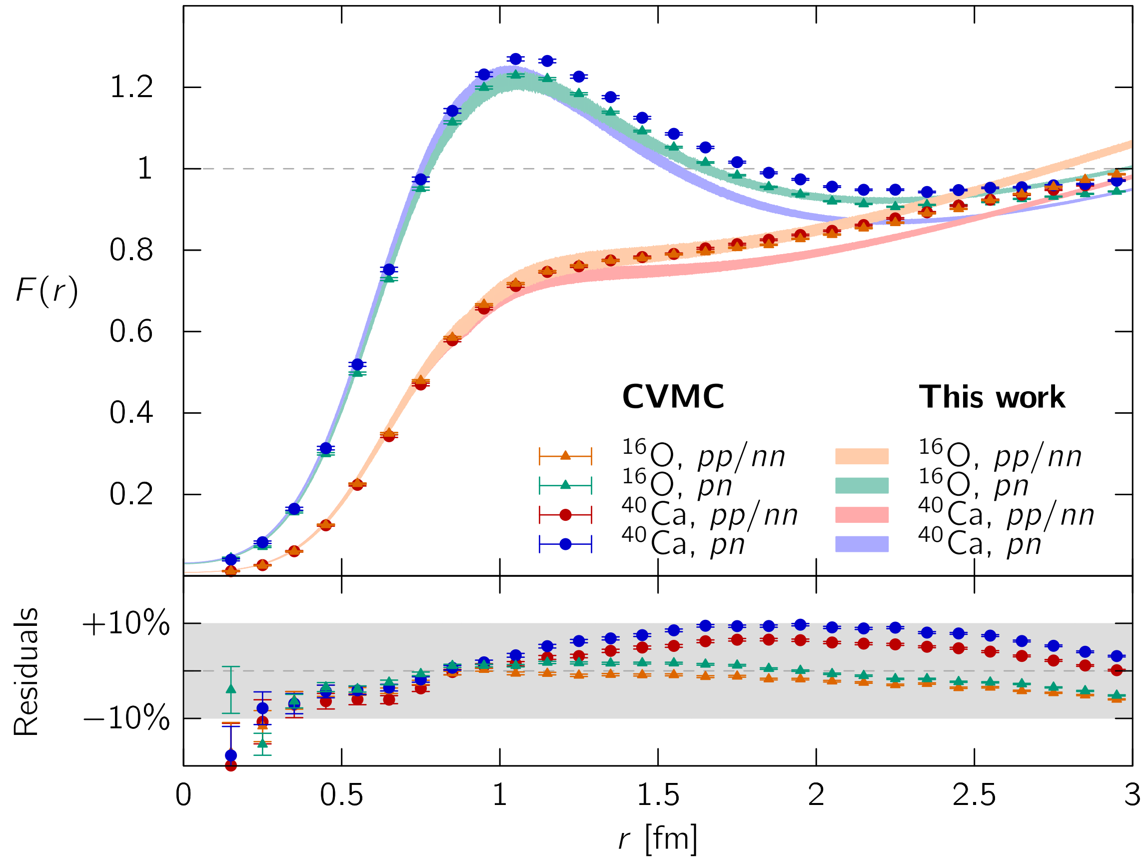
To recap:

- Nuclear contacts



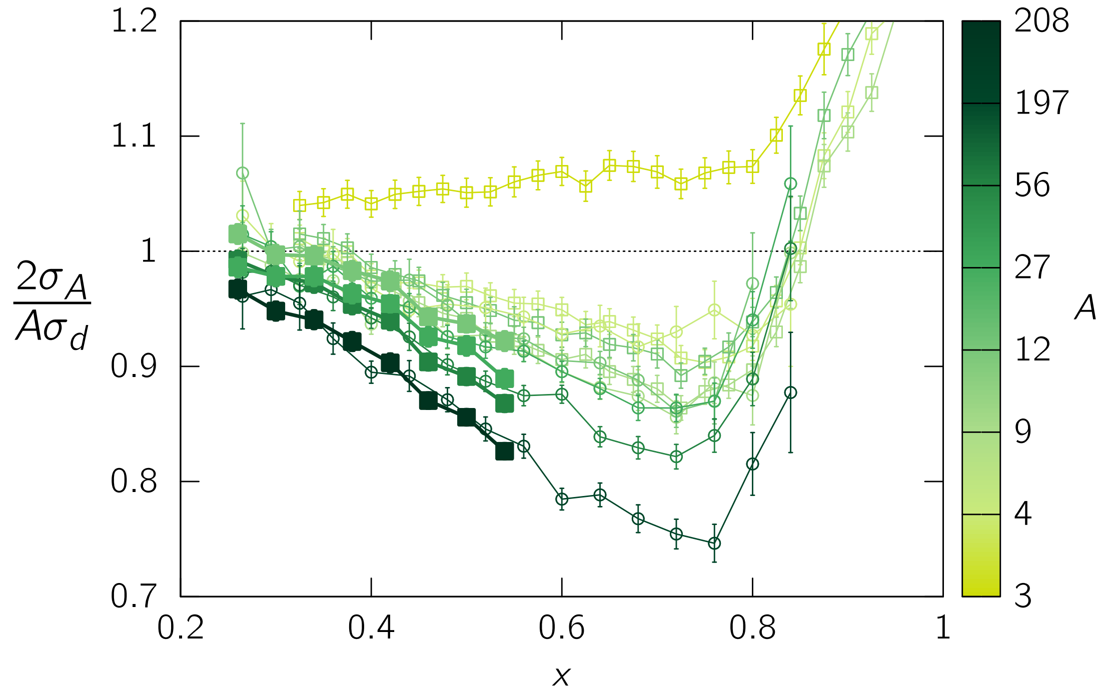
To recap:

- Nuclear contacts
- Correlation functions



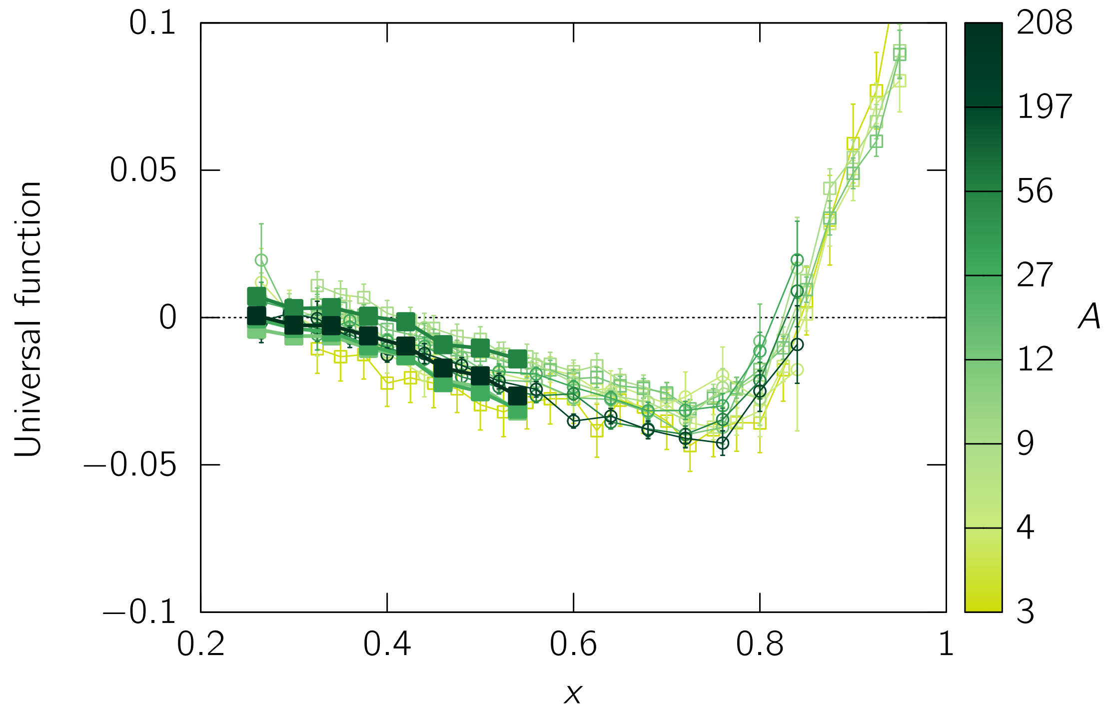
To recap:

- Nuclear contacts
- Correlation functions
- SRCs in the EMC effect



To recap:

- Nuclear contacts
- Correlation functions
- SRCs in the EMC effect



Conclusions

- Scale separation
  - Valuable tool for attacking problems
  - New insight into underlying physics
- We are looking for new applications for nuclear contacts.