Light-nuclei spectra and electroweak response: a status report

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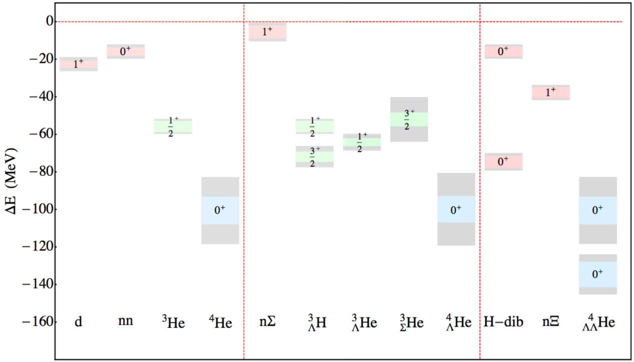
Few-nucleon systems from LQCD

Beane *et al.* (2013); Chang *et al.* (2015); Savage *et al.* (2017)

- NPLQCD spectra calculations ($m_\pi = 806$ MeV)

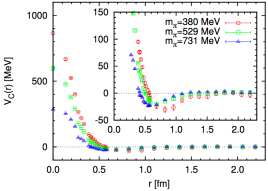


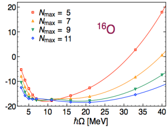
- NPLQCD calculations of magnetic moments and weak transitions in few-nucleon systems also available

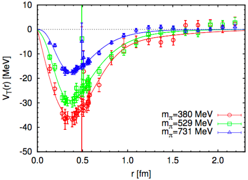
$2N$ potential from LQCD and nuclear spectra

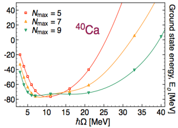
Aoki *et al.* (2012); McIlroy *et al.* (2017)

- LQCD calculation of $2N$ potential by HAL collaboration









Outline

- The basic model of nuclear theory
- Chiral $2N$ and $3N$ potentials and nuclear spectra
- Electroweak currents and (mostly weak) transitions
- Nuclear electroweak response in quasi-elastic regime
- Outlook

The basic model

- Effective potentials:

$$H = \sum_{i=1}^{A} \frac{\mathbf{p}_i^2}{2\,m_i} + \sum_{i<j=1}^{A} \underbrace{v_{ij}}_{\text{th+exp}} + \sum_{i<j<k=1}^{A} \overbrace{V_{ijk}}^{\text{th+exp}} + \cdots$$

- Assumptions:
  - *Quarks in nuclei are in color singlet states close to those of N's (and low-lying excitations: Δ's, . . .)*
  - *Series of potentials converges rapidly*
  - *Dominant terms in v_{ij} and V_{ijk} are due to π exchange*

$$\text{leading } \pi N \text{ coupling} = \frac{g_A}{2\,f_\pi} \tau_a\, \boldsymbol{\sigma} \cdot \boldsymbol{\nabla} \phi_a(\mathbf{r})$$

- Effective electroweak currents:

$$j^{EW} = \sum_{i=1}^{A} j_i + \sum_{i<j=1}^{A} j_{ij} + \sum_{i<j<k=1}^{A} j_{ijk} + \cdots$$

Yukawa potential in classical mechanics

- Connection between meson-exchange interactions and their representation in terms of v_{ij}
- A simple model: a classical scalar field $\phi(\mathbf{r}, t)$ interacting with static particles:

$$\mathcal{L} = \frac{1}{2}\left[\dot{\phi}^2 - |\boldsymbol{\nabla}\phi|^2 - \mu^2\,\phi^2\right] - g\,\phi\sum_{i=1}^{A}\delta\left(\mathbf{r}-\mathbf{r}_i\right)$$

- Lowest-energy configuration occurs in the static limit $\phi(\mathbf{r}, t) \rightarrow \phi(\mathbf{r})$ (Poisson-like equation of electrostatics)

$$\boldsymbol{\nabla}^2\,\phi - \mu^2\,\phi = g\sum_{i=1}^{A}\delta\left(\mathbf{r}-\mathbf{r}_i\right)$$

- Energy of the field (up to self energies) in this limit

$$E_\phi = \frac{1}{2}\int d\mathbf{r}\,\phi(\mathbf{r})\,g\sum_{i=1}^{A}\delta(\mathbf{r}-\mathbf{r}_i) = -\frac{g^2}{4\pi}\sum_{i<j=1}^{A}\frac{\mathrm{e}^{-\mu r_{ij}}}{r_{ij}}$$

Yukawa potential in quantum mechanics

- Scalar-field Hamiltonian is

$$H = \underbrace{\sum_{\mathbf{k}} \omega_k \, a^\dagger_{\mathbf{k}} a_{\mathbf{k}}}_{H_0} + \underbrace{g \sum_{i=1}^{A} \sum_{\mathbf{k}} \frac{1}{\sqrt{2\,\omega_k\, V}} \left(a_{\mathbf{k}}\, \mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}_i} + a^\dagger_{\mathbf{k}}\, \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{r}_i} \right)}_{H'}$$

- Set of shifted harmonic oscillators; exact eigenenergies of field given by

$$E_{\{n_{\mathbf{k}}\}} = \sum_{\mathbf{k}} \omega_k \, n_{\mathbf{k}} \underbrace{- g^2 \sum_{i<j=1}^{A} \overbrace{\int \frac{d\mathbf{k}}{(2\pi)^3} \frac{\mathrm{e}^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)}}{\omega_k^2}}^{\frac{1}{4\pi} \frac{\mathrm{e}^{-\mu r_{ij}}}{r_{ij}}}}_{\text{energy shift}}$$

- In CM and QM the scalar field energy in the presence of static particles can be replaced by a sum of $v_Y(r_{ij})$

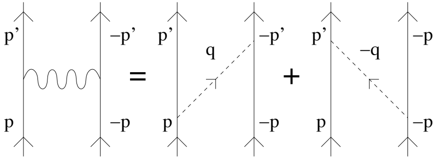
Scattering between slow-moving particles

- In potential theory to leading order

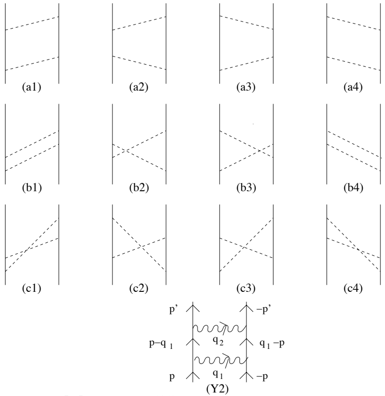
$$T_{fi}^{Y1} = \int d\mathbf{r}\, e^{-i\mathbf{p}'\cdot\mathbf{r}}\, v_Y(r)\, e^{i\mathbf{p}\cdot\mathbf{r}} = \underbrace{-\frac{g^2}{\omega_q^2}}_{\widetilde{v}_Y(q)} \qquad (\mathbf{q} = \mathbf{p} - \mathbf{p}')$$

- In meson-exchange theory (to leading order)

$$T_{fi}^{M1} = \sum_I \frac{\overbrace{\langle \mathbf{p}', -\mathbf{p}'; 0|}^{\text{final state}} H' |I\rangle\langle I| H' \overbrace{|\mathbf{p}, -\mathbf{p}; 0\rangle}^{\text{initial state}}}{E_i - E_I} = -\frac{g^2}{\omega_q^2}$$



Beyond leading order



$$T_{fi}^{M2} = T_{fi}^{Y2} + \text{correction}$$

- Obtain from $T_{fi}^{M2} - T_{fi}^{Y2}$ correction term $\widetilde{v}^{(2)}(\mathbf{q}, \overbrace{\mathbf{Q}}^{\mathbf{p}'+\mathbf{p}})$ such that $\widetilde{v}_Y(q) + \widetilde{v}^{(2)}(\mathbf{q}, \mathbf{Q})$ reproduces T_{fi}^{M2}

Earlier developers of the basic model . . .

χEFT formulation of the basic model

- χEFT is a low-energy approximation of QCD
- Lagrangians describing the interactions of π, N, ... are expanded in powers of Q/Λ_χ ($\Lambda_\chi \sim 1$ GeV)
- Their construction has been codified in a number of papers [1]

$$\mathcal{L} = \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(3)}_{\pi N} + \ldots + \mathcal{L}^{(2)}_{\pi\pi} + \mathcal{L}^{(4)}_{\pi\pi} + \ldots$$

- $\mathcal{L}^{(n)}$ also include contact $(\overline{N}N)(\overline{N}N)$-type interactions parametrized by low-energy constants (LECs)
- Initial impetus to the development of χEFT for nuclei in the early nineties [2,3]

[1] Gasser and Leutwyler (1984); Gasser, Sainio, and Švarc (1988); Bernard *et al.* (1992); Fettes *et al.* (2000)

[2] Weinberg (1990)–(1992); [3] Park, Min, and Rho (1993) and (1996)

General considerations

- Time-ordered perturbation theory (TOPT):

$$\langle f \mid T \mid i \rangle = \langle f \mid H_1 \sum_{n=1}^{\infty} \left(\frac{1}{E_i - H_0 + i\,\eta} H_1 \right)^{n-1} \mid i \rangle$$

- Momentum scaling of contribution

$$\underbrace{\left(\prod_{i=1}^{N} Q^{\alpha_i - \beta_i/2} \right)}_{H_1 \text{ scaling}} \times \underbrace{Q^{-(N-N_K-1)}\, Q^{-2N_K}}_{\text{denominators}} \times \underbrace{Q^{3L}}_{\text{loop integrations}}$$

- Each of the N_K energy denominators involving only nucleons is of order Q^{-2}
- Each of the other $N - N_K - 1$ energy denominators involving also pion energies is expanded as

$$\frac{1}{E_i - E_I - \omega_\pi} = -\frac{1}{\omega_\pi} \left[1 + \frac{E_i - E_I}{\omega_\pi} + \frac{(E_i - E_I)^2}{\omega_\pi^2} + \dots \right]$$

- Power counting:

$$T = T^{LO} + T^{NLO} + T^{N^2LO} + \dots\,, \quad \text{and} \quad T^{N^nLO} \sim (Q/\Lambda_\chi)^n\, T^{LO}$$

From amplitudes to potentials

Pastore *et al.* (2009); Pastore *et al.* (2011)

- Construct v such that when inserted in LS equation

$$v + v\, G_0\, v + v\, G_0\, v\, G_0\, v + \dots \qquad G_0 = 1/(E_i - E_I + i\,\eta)$$

leads to T-matrix order by order in the power counting

- Assume

$$v = v^{(0)} + v^{(1)} + v^{(2)} + \dots \qquad v^{(n)} \sim (Q/\Lambda_\chi)^n\, v^{(0)}$$

- Determine $v^{(n)}$ from

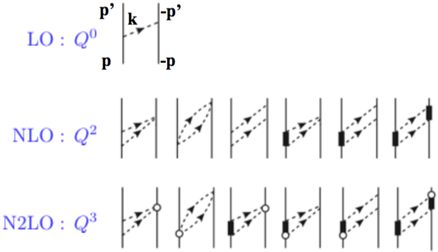
$$\begin{aligned} v^{(0)} &= T^{(0)} \\ v^{(1)} &= T^{(1)} - \left[v^{(0)}\, G_0\, v^{(0)}\right] \\ v^{(2)} &= T^{(2)} - \left[v^{(0)}\, G_0\, v^{(0)}\, G_0\, v^{(0)}\right] - \left[v^{(1)}\, G_0\, v^{(0)} + v^{(0)}\, G_0\, v^{(1)}\right] \end{aligned}$$

and so on, where

$$v^{(m)}\, G_0\, v^{(n)} \sim (Q/\Lambda_\chi)^{m+n+1}$$

Chiral $2N$ potentials with Δ's

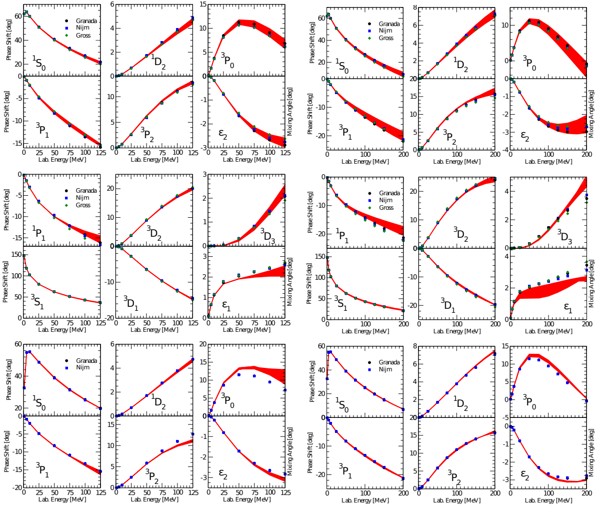
- Two-nucleon potential: $v = v^{\mathrm{EM}} + v^{\mathrm{LR}} + v^{\mathrm{SR}}$
- EM component v^{EM} including corrections up to α^2
- Chiral OPE and TPE component v^{LR} with Δ's



- Short-range contact component v^{SR} up to order Q^4 parametrized by (2+7+11) IC and (2+4) IB LECs
- v^{SR} functional form taken as $C_{R_S}(r) \propto \mathrm{e}^{-(r/R_S)^2}$ with R_S=0.8 (0.7) fm for a (b) models

np ($T=0$ and 1) and pp phase shifts

Ia-Ib: $E_{\rm lab} = 125$ MeV   IIa-IIb: $E_{\rm lab} = 200$ MeV



Ab initio methods utilized by our group

- Hyperspherical harmonics (HH) expansions for $A = 3$ and 4 bound and continuum states

$$|\psi_V\rangle = \sum_\mu c_\mu \underbrace{|\phi_\mu\rangle}_{\text{HH basis}} \quad \text{and} \quad c_\mu \ \text{from} \ E_V = \frac{\langle\psi_V|H|\psi_V\rangle}{\langle\psi_V|\psi_V\rangle}$$

- Quantum Monte Carlo for $A > 4$ bound states

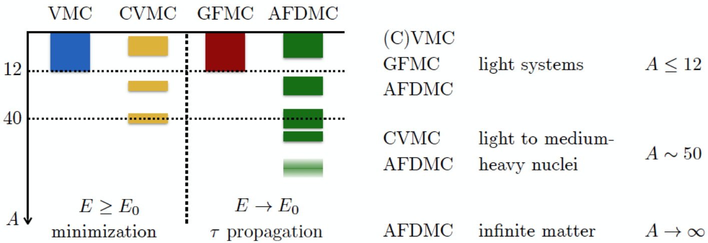
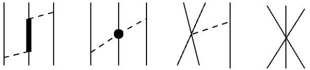


Figure by Lonardoni

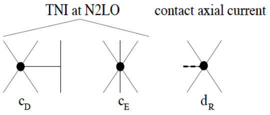
Chiral $3N$ potentials with Δ's

Piarulli *et al.* (2018)

- $3N$ potential up to N2LO[1]: 
- c_D and c_E fixed by fitting $E_0^{\rm exp}(^3{\rm H}) = -8.482$ MeV and nd doublet scattering length $a_{nd}^{\rm exp} = (0.645 \pm 0.010)$ fm

	without $3N$						with $3N$	
Model	c_D	c_E	$E_0(^3{\rm H})$	$E_0(^3{\rm He})$	$E_0(^4{\rm He})$	$^2a_{nd}$	$E_0(^3{\rm He})$	$E_0(^4{\rm He})$
Ia	3.666	–1.638	–7.825	–7.083	–25.15	1.085	–7.728	–28.31
Ib	–2.061	–0.982	–7.606	–6.878	–23.99	1.284	–7.730	–28.31
IIa	1.278	–1.029	–7.956	–7.206	–25.80	0.993	–7.723	–28.17
IIb	–4.480	–0.412	–7.874	–7.126	–25.31	1.073	–7.720	–28.17

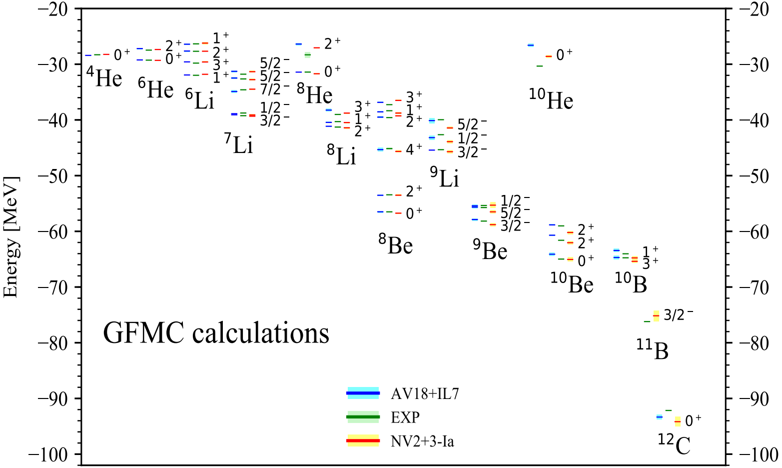
- Alternate strategy: fix c_D and c_E by reproducing $E_0^{\rm exp}(^3{\rm H})$ and the GT$^{\rm exp}$ matrix element in ^{3}H β-decay



[1] Epelbaum *et al.* (2002)

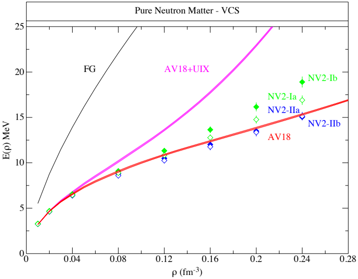
Spectra of light nuclei

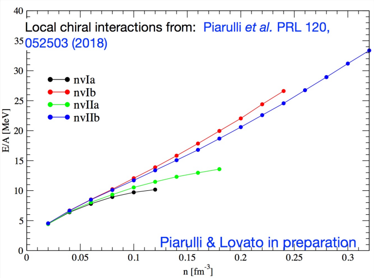
Piarulli *et al.* (2018)

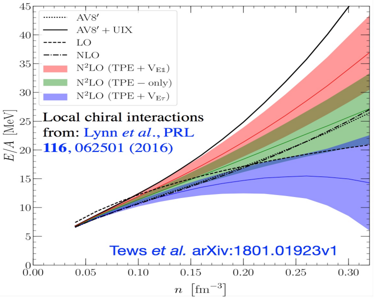


Neutron matter equation of state

Piarulli *et al.*, private communication

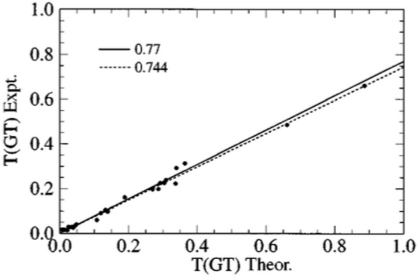




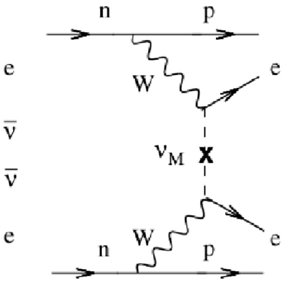


- Sensitivity to $3N$ contact term:
  - $c_E < 0$ repulsive in $A \leq 4$
  - but attractive in PNM
- Cutoff sensitivity:
  - *modest in NV2 models*
  - *large in NV2+3 models*

- Shell model in agreement with exp if $g_A^{\rm eff} \simeq 0.7\, g_A$
- Understanding "quenching" of g_A in nuclear β decays
- Relevant for neutrinoless 2β-decay since rate $\propto\ g_A^4$



Martinez-Pinedo *et al.* (1996)



0ν–2β amplitude

Including electroweak (ew) interactions

Pastore *et al.* (2009,2011); Piarulli *et al.* (2013); Baroni *et al.* (2017)

- Power counting of ew interactions (treated in first order)

$$T_{\rm ew} = T_{\rm ew}^{(-3)} + T_{\rm ew}^{(-2)} + T_{\rm ew}^{(-1)} + \dots \qquad T_{\rm ew}^{(n)} \sim (Q/\Lambda_\chi)^n \, T_{\rm ew}^{(-3)}$$

- For $v_{\rm ew}^{(n)} = A^0 \, \rho_{\rm ew}^{(n)} - \mathbf{A} \cdot \mathbf{j}_{\rm ew}^{(n)}$ to match $T_{\rm ew}$ order by order

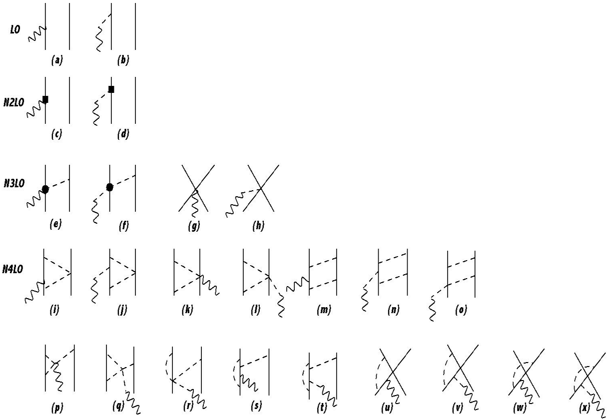
$$\begin{aligned}
v_{\rm ew}^{(-3)} &= T_{\rm ew}^{(-3)} \\
v_{\rm ew}^{(-2)} &= T_{\rm ew}^{(-2)} - \left[v_{\rm ew}^{(-3)} \, G_0 \, v^{(0)} + v^{(0)} \, G_0 \, v_{\rm ew}^{(-3)} \right] \\
v_{\rm ew}^{(-1)} &= T_{\rm ew}^{(-1)} - \left[v_{\rm ew}^{(-3)} \, G_0 \, v^{(0)} \, G_0 \, v^{(0)} + \text{permutations} \right] \\
&\quad - \left[v_{\rm ew}^{(-2)} \, G_0 \, v^{(0)} + v^{(0)} \, G_0 \, v_{\rm ew}^{(-2)} \right]
\end{aligned}$$

and so on up to $n = 1$

- $\rho_{\rm ew}^{(n)}$ and $\mathbf{j}_{\rm ew}^{(n)}$ (generally) depend on off-the-energy shell prescriptions adopted for $v^{(\leq n)}$ and $v_{\rm ew}^{(\leq n)}$

Nuclear axial currents at one loop

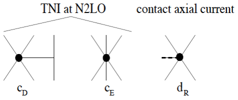
Park *et al.* (1993,2003); Baroni *et al.* (2016); Krebs *et al.* (2017)



- Some of the contributions—panels (m) and (s)—differ in the Baroni *et al.* and Krebs *et al.* derivations
- 1 unknown LEC in $\mathbf{j}_5$ (4 unknown LECs in ρ_5)

β decays in light nuclei

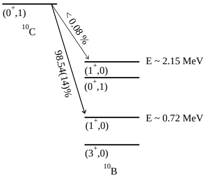
Pastore *et al.* (2017)

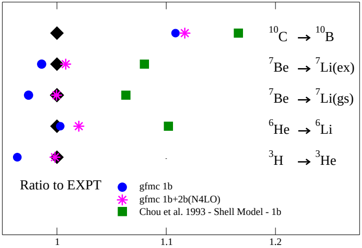


- Correct relation between c_D and d_R

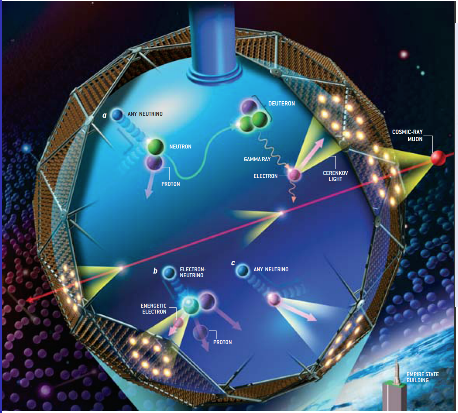
$$d_R = -\frac{m}{4\, g_A\, \Lambda_\chi}\, c_D + \frac{m}{3}\,(c_3 + 2\, c_4) + \frac{1}{6}$$

- GT m.e.'s in $A = 6$–10 nuclei (AV18/IL7 potential with χEFT axial current)

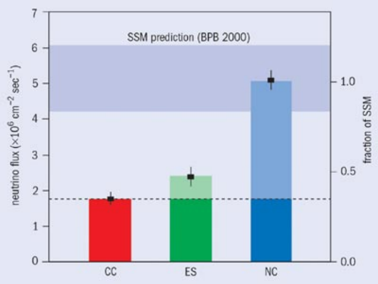




Low-energy neutrinos



SNO experiment



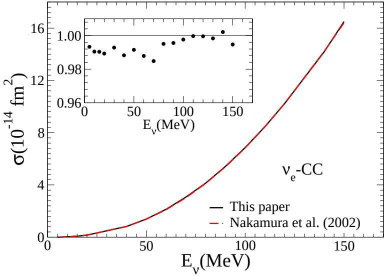
CC from $d + \nu_e \longrightarrow p + p + e^-$

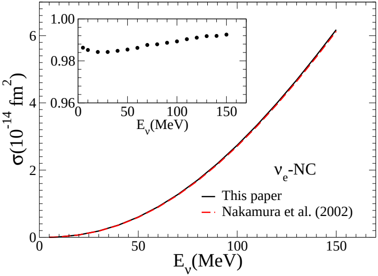
ES from (mostly) $e^- + \nu_e \longrightarrow e^- + \nu_e$

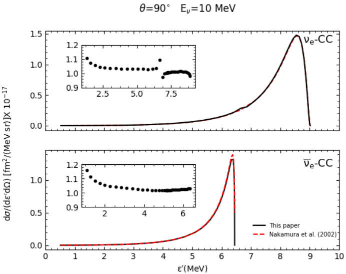
NC from $d + \nu_x \longrightarrow p + n + \nu_x$

Low-energy inclusive ν-d scattering in χEFT

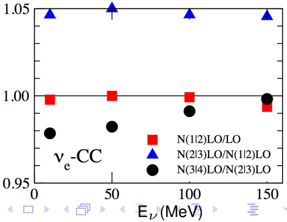
Baroni and Schiavilla (2017)







convergence pattern



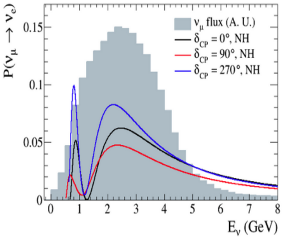
CC and NC ν-A scattering

- Large program in accelerator ν physics (MicroBooNE, NOνA, T2K, Minerνa, DUNE, ...)

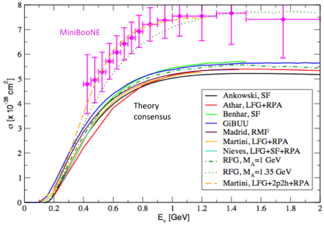
$$\text{rate} \propto \int dE\, \Phi_\alpha(E)\, P(\nu_\alpha \to \nu_\beta; E)\, \sigma_\beta(E, E')$$

- Determination of oscillation parameters depends crucially on our understanding of
  - ν *flux* $\Phi_\alpha(E)$
  - ν-A *cross section* $\sigma_\beta(E, E')$

DUNE ν_μ flux



MiniBooNE



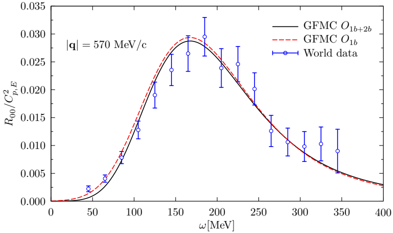
GFMC calculation of EM response in ^{12}C

Carlson and Schiavilla (1992,1994); Lovato *et al.* (2013–2016)

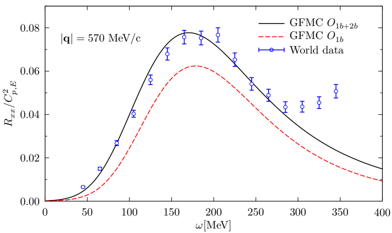
$$\int_0^\infty \mathrm{d}\omega\, \mathrm{e}^{-\tau\omega}\, R_{\alpha\beta}(q,\omega) = \langle i \,|\, j_\alpha^\dagger(\mathbf{q})\, \mathrm{e}^{-\tau(H-E_i)}\, j_\beta(\mathbf{q})\, |\, i\rangle$$

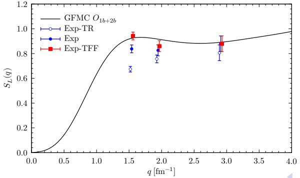
- Inversion back to $R_{\alpha\beta}(q,\omega)$ by maximum entropy methods

longitudinal



transverse





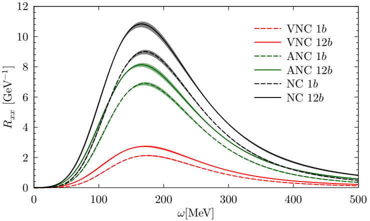
$\leftarrow$ Coulomb sum

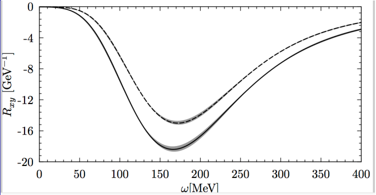
NC responses and cross sections in ^{12}C

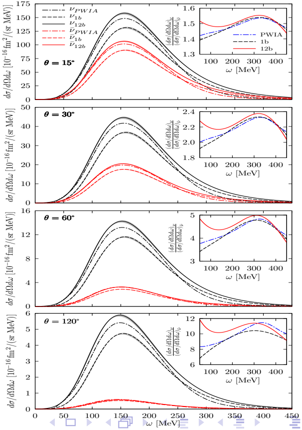
Lovato *et al.* (2018)

- Inclusive $\nu/\overline{\nu}$ $(-/+)$ cross section given in terms of five response functions

$$\frac{d\sigma}{d\epsilon_l' d\Omega_l} \propto \left[v_{00}\, R_{00} + v_{zz}\, R_{zz} - v_{0z}\, R_{0z} + \overbrace{v_{xx}\, R_{xx} \mp v_{xy}\, R_{xy}}^{\text{dominant}} \right]$$

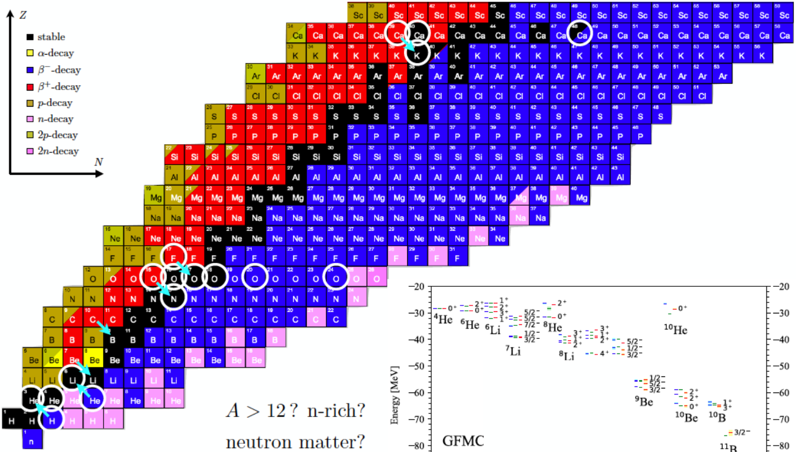






Outlook

Basic model
Nuclear χEFT
Chiral $2N$ potentials
Chiral $3N$ potentials
EW interactions
EW QE response
Outlook



$A > 12$? n-rich?

neutron matter?

nuclear matter?

(error estimate?)

M. Piarulli *et al.*, arXiv:1707.02883

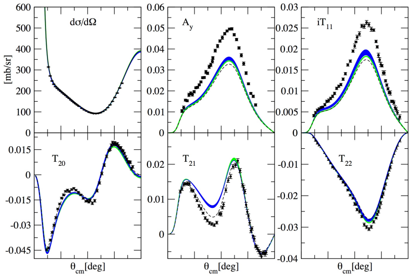
Slide by Lonardoni

Leading and subleading $3N$ potentials

Girlanda *et al.* (2011) and private communication

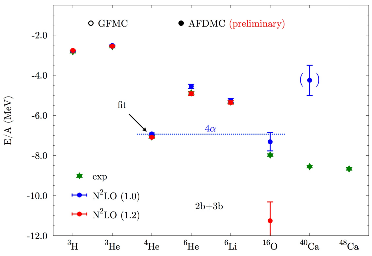
- Fix c_D by reproducing measured ^{3}H GT matrix element
- Possible strategies for constraining c_E and the (10 in principle) LECs in subleading contact $3N$ potential:
  - *Nd scattering observables at low energies*
  - *Spectra of light- and medium-weight nuclei and properties of nuclear/neutron matter*

pd scattering at 3 MeV



Piarulli *et al.* (2017)

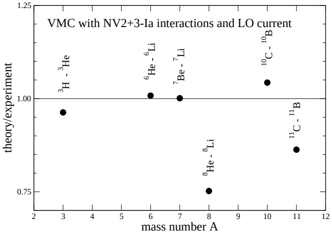
AFDMC for $A = 16$ and 40

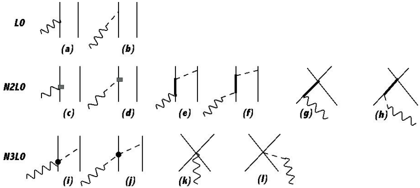


Lynn *et al.* (2016); Lonardoni *et al.* (2017)

Weak transitions with χEFT in $A \geq 3$ nuclei

- Simple at tree level (and calculations are in progress); still a single LEC in the axial current





- A major task at N4LO as there are a great many two- and three-body contributions at that order

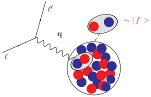
Approximate methods for ν-A scattering

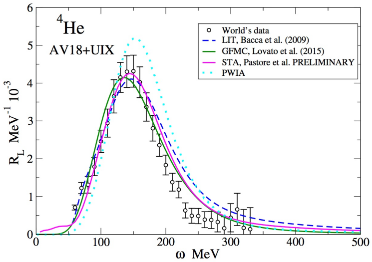
Pastore *et al.* private communication; Rocco *et al.* private communication

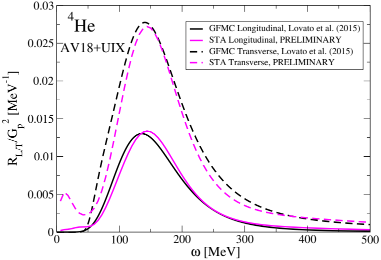
- Beyond PWIA: including two-body physics in the short-time approximation (STA)

$$R(q,\omega) \sim \int dt \, \langle 0| \underbrace{O^\dagger(\mathbf{q}) \, \overbrace{e^{-i(\omega-H)t}}^{\text{expand } P(t)} \, O(\mathbf{q})}_{\text{keep up to 2b terms in } O} |0\rangle$$

$$O_i^\dagger P(t) O_i + O_i^\dagger P(t) O_j + O_i^\dagger P(t) O_{ij} + O_{ij}^\dagger P(t) O_{ij}$$







- STA applicable to heavier targets (^{16}O and ^{40}Ar) and can accommodate relativity and pion production

The HH/QMC team

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- Computational resources from *ANL LCRC*, *LANL Open Supercomputing*, and *NERSC*