Small bits of cold, dense matter

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NUCLEI
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Nuclear ab initio Theories and Neutrino Physics
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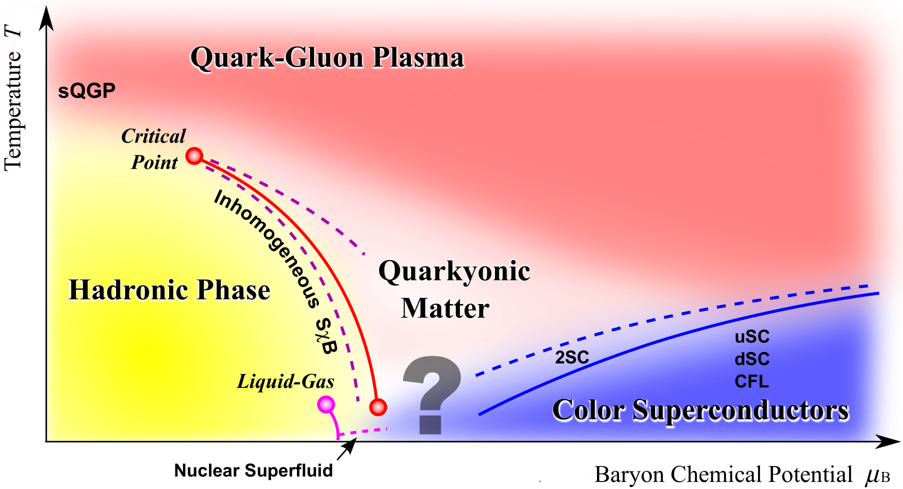
The (conjectured) QCD phase diagram

figure from Fukushima & Hatsuda (2011)



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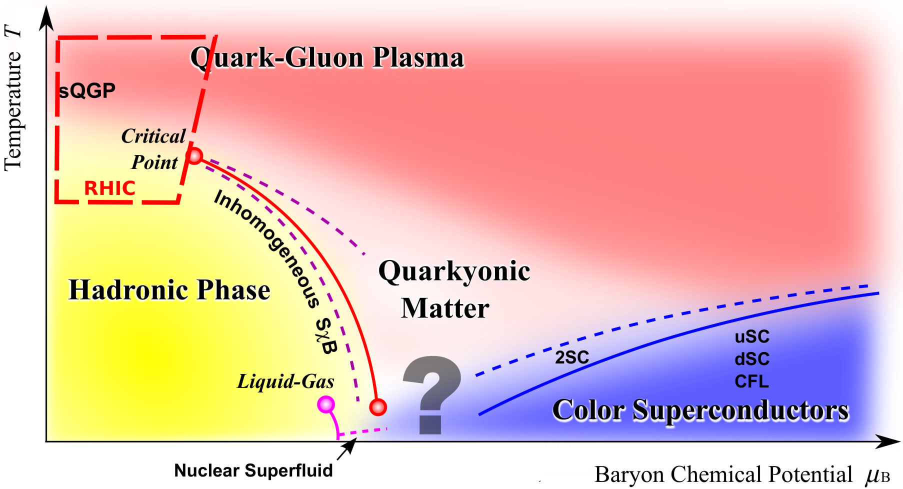
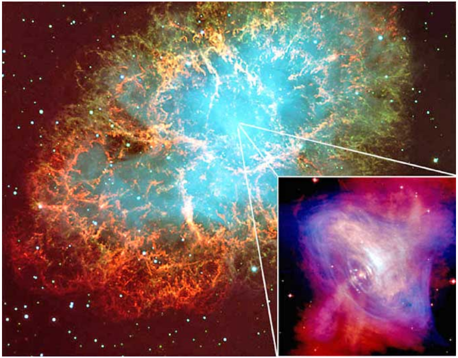
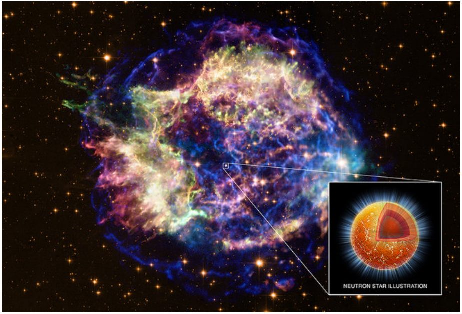


Where to find cold dense QCD matter

Optical + X-ray images from NASA/ESA





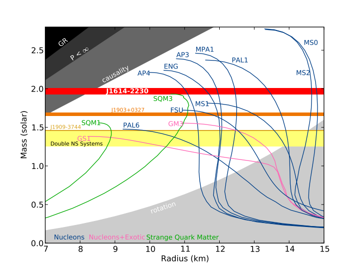
Neutron Stars

- born in the aftermath of core-collapse supernovae
- massive and compact objects: $R \approx 10$ km $M \approx 1 - 2$ $\mathrm{M}_\odot$
- central densities can be several times nuclear density $\rho_0 \approx 10^{14} gr/cm^3$

Can measurements of neutron stars help?

Masses and radii

Demorest et al. (2010)



- pure quark stars disfavored
- hybrid stars still compatible

Alford et al. (2004)
Zdunik & Hansel (2013)
Lastowiecki et al. (2015)
Alford & Han (2016)
Baym et al. (2017)

Neutrino cooling

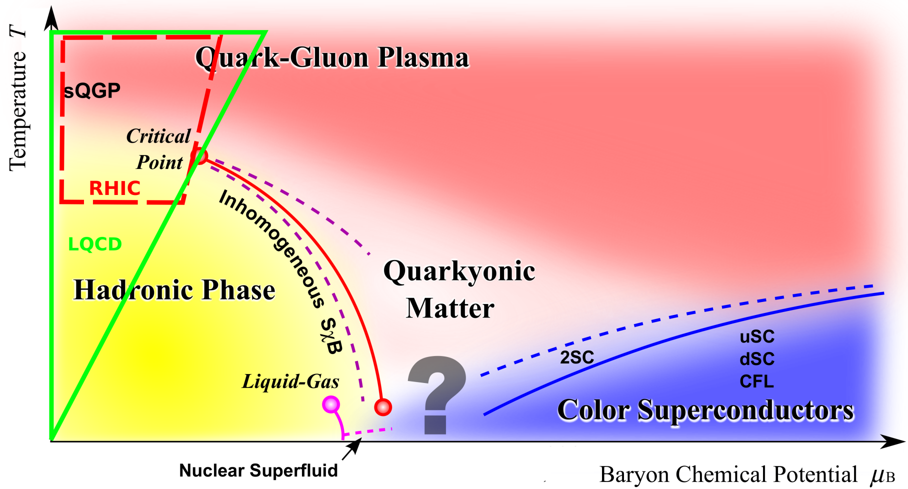
- direct URCA produce fast cooling in the core (Iwamoto (1980))

$$d \to u + e^- + \bar{\nu}_e \quad u + e^- \to d + \nu_e$$

- quark pairing can drastically reduce emissivity (Page et al. (2000))

Ab initio calculations with Lattice QCD

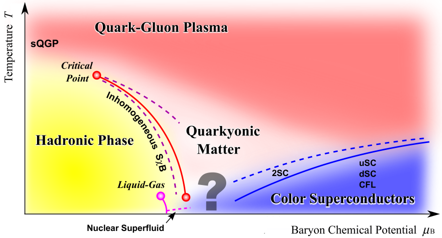
figure from Fukushima & Hatsuda (2011)



Using Lattice QCD at fixed A and $T = 0$

NPLQCD,PACS,HAL,CalLat

Fukushima & Hatsuda (2011)



Sign problem in LQCD

$$E_G^{LQCD} = E_G + \mathcal{O}\left(e^{-(E_1 - E_G)\tau}\right)$$

$$\Delta E / E \sim e^{A\left(M_N - \frac{3}{2} m_\pi\right)\tau}$$

Systems in small volumes with low A are becoming possible

What can we learn from these small systems?

- nucleon interactions by matching to EFT (eg. P.Klos et al. (2016))
- signatures of high-density phase transitions (this talk)

Detecting change in degrees of freedom in small boxes

IDEA

- perform simulations in both nucleonic and quark models
- study energy spectrum as a function of A
- reduce box size L to reach higher densities (few times n_0)

Features in the spectra determined by the high momenta (short distances) arising in small periodic volumes. In particular we look for:

- shell closures (large gaps in single-particle energies)
- pairing effects in open-shell systems

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CAVEATS

- large corrections from long-range physics for small box size L
  - qualitatively similar to those in LQCD if pions are consistently included
  - can study effect by using calculations for larger m_π (easier)
- small systems won't capture critical behavior (only qualitative)

Nucleonic models

- for now limited to systems of non-relativistic neutrons only

$$H_N = -\frac{\hbar^2}{2M_N}\sum_i \nabla_i^2 + \sum_{i<j} V_{ij}$$

We choose 3 different realistic NN interactions (match exp. phase-shifts)

- Argonne AV18 (R.Wiringa, V.Stoks and R.Schiavilla (1995))
- Argonne AV8' (R.Wiringa & S.Pieper (2002))
- local chiral EFT at N^{2}LO (A.Gezerlis et al. (2014))

Many-body forces will play a role but:

- the features we identify depend predominantly on the single-particle states and some of them are present even for non-interacting nucleons
- the structure of 3-,4- and many-body forces will be very different

Quark models

- we consider both free and interacting $SU(2)_f$ quarks

$$H_q = \sum_i T_i + \sum_{i<j} V_{ij} + V_c$$

- for free quarks we use both relativistic and non-relativistic dispersion
- for interacting quarks we assume χ-symmetry is not restored and use $m_q = 0.3GeV$ with non-relativistic dispersion

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- we impose baryon number and color, charge and spin neutrality

DISCLAMER: model is <u>not</u> intended to give realistic description of QCD in this regime but to be qualitatively correct: ie. to reproduce the 2SC color superconducting state (M.Alford et al. (2008))

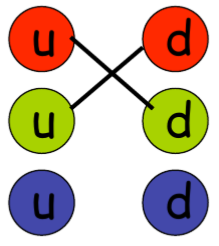
Pairing in quark matter

Attractive one-gluon interaction destabilize Fermi surface ⇒ Cooper pairs

intermediate densities: 2SC

Rapp et al. (1998), Alford et al. (1998)

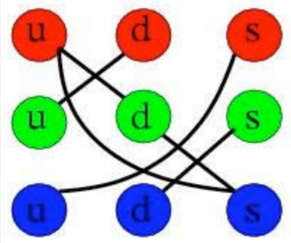
- 2-flavor pairing



large densities: CFL

Alford et al. (1999)

- 9 quark pairing: $\mu \gg M_s^2/\Delta$



figures from Reddy (2004)

- structure of 2SC ground state: $|\Psi_{2SC}\rangle = |\Psi_B^{free}\rangle \otimes |\Psi_{RG}^{SF}\rangle$

Quantum Monte Carlo calculations

for reviews see Foulkes et al. (2001), Carlson et al. (2015)

BASIC IDEA

any quantum system relaxes to its ground-state at low-enough temperature

Given hamiltonian H: $E(\beta) = \frac{Tr\left[He^{-\beta H}\right]}{Tr\left[e^{-\beta H}\right]} \xrightarrow{\beta\gg 1} E_0 + \mathcal{O}\left(e^{-\beta(E_1-E_0)}\right)$

- map quantum problem to a classical one with more degrees of freedom
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solution: modify expectation value using good reference state $|\Phi_r\rangle$

$$\widetilde{E}(\beta) = \frac{Tr\left[He^{-\beta H}|\Phi_r\rangle\langle\Phi_r|\right]}{Tr\left[e^{-\beta H}|\Phi_r\rangle\langle\Phi_r|\right]}$$

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$$\xrightarrow{\beta \gg 1} \frac{\langle\Psi_0|\Phi_r\rangle\langle\Phi_r|H|\Psi_0\rangle + \mathcal{O}\left(e^{-\beta(E_1-E_0)}\right)}{|\langle\Phi_r|\Psi_0\rangle|^2 + \mathcal{O}\left(e^{-\beta(E_1-E_0)}\right)} = E_0 + \mathcal{O}\left(e^{-\beta(E_1-E_0)}\right)$$

Quantum Monte Carlo calculations: fermions and pairing

Fermionic states have non trivial phases (entanglement)

sign problem: statistical noise grows exponentially with β (cf. LQCD)

$$\widetilde{E}(\beta) \to E_0 + \mathcal{O}\left(e^{-\beta(E_1 - E_0)}\right) \qquad \Delta\widetilde{E}(\beta) \to \mathcal{O}\left(e^{\beta\left(E_0^F - E_0^B\right)}\right)$$

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- phase constrained Quantum Monte Carlo (Ortiz et al. (1993))

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Restricted traces over states $|\psi\rangle$ such that: $\langle\psi|\Phi_r\rangle > 0 \Rightarrow |\psi\rangle$ fermionic

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- Systems with superfluid pairing can be described by appropriate $|\Psi_r\rangle$

$$|\Phi_r^{SF}\rangle = \mathcal{A}|\Psi_{12}^A\rangle\otimes|\Psi_{34}^B\rangle\otimes|\Psi_{56}^C\rangle\otimes\cdots \qquad \text{(eg. Carlson et al. (2003))}$$

By choosing the same (s–wave) state for every pair $|\Phi_r^{SF}\rangle = |\Phi_r^{BCS}\rangle$

4 neutrons in a box: free case

Gandolfi, Carlson, Roggero, Lynn, Reddy (2017)

Particle	mass (GeV)	N	ρ (fm^{-3})	E (k=1, GeV)
Nucleon	0.94	4	0.16	0.096
rel q	0.0	4	0.16	0.424
rel q	0.3	4	0.16	0.219
non-rel q	0.3	4	0.16	0.299
Nucleon	0.94	4	0.32	0.152
rel q	0.0	4	0.32	0.534
rel q	0.3	4	0.32	0.313
non-rel q	0.3	4	0.32	0.476

nuclear case: two nucleons in the $k = 0$ shell and two in $|k| = 1$ shell
quark case: all quarks in $k = 0$ shell costs twice $M_N - M_\Delta$ ($\approx 300MeV$)

- at low density (large volumes) 4 neutron system is favored
- at large density (small volume) all quarks in $k = 0$ shell preferable

4 neutrons in a box: surprise in the interacting case

- ref. states with different symmetry: $|\Phi_r^{4N}\rangle = \mathcal{A}|\psi_{12}^{S,k=0}\rangle \otimes |\psi_{34}^{X,|k|=1}\rangle$

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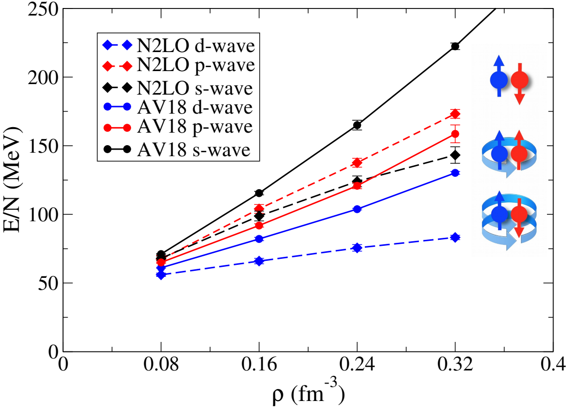
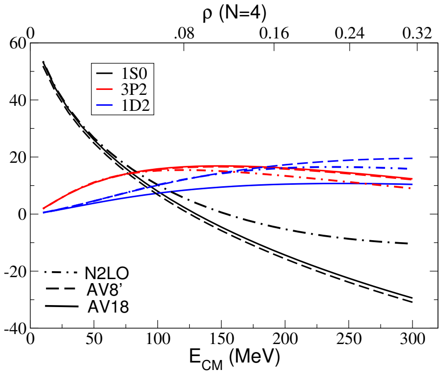
NN interaction favor d-wave pairing for the 2 neutrons in $|k| = 1$ state

Gandolfi, Carlson, Roggero, Lynn, Reddy (2017)



4 neutrons in a box: interacting case (phase shifts)

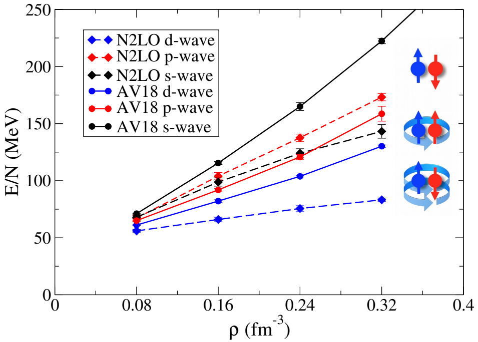
Gandolfi, Carlson, Roggero, Lynn, Reddy (2017)



- s-wave interaction turns repulsive at large momenta (small volumes)
- d-wave state favored over p-wave due to spatial symmetry of wf
  - d-wave state is symmetric across periodic boundaries → attractive
  - interference between periodic images for p-wave state

4 neutrons in a box: comparison with quarks models

Gandolfi, Carlson, Roggero, Lynn, Reddy (2017)



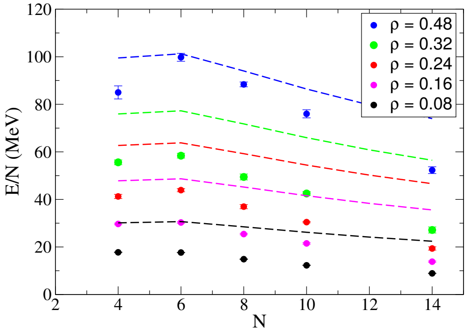
nuclear case: at $\rho = 2\rho_0$ s-d gap is $\gtrapprox 75$ MeV for both interactions

quark case: at $\rho = 2 - 3\rho_0$ s-wave is still $\approx 30 - 40$ MeV lower in energy

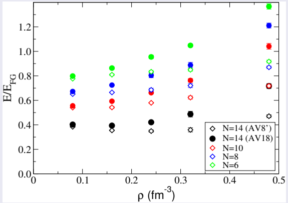
- conclusions are not altered by inclusion of spin dependent gluon exchange interactions (Carlson, Kogut and Pandharipande (1983))

Evolution of the spectrum with particle number

Gandolfi, Carlson, Roggero, Lynn, Reddy (2017)



Model dependence



dashed lines free neutrons

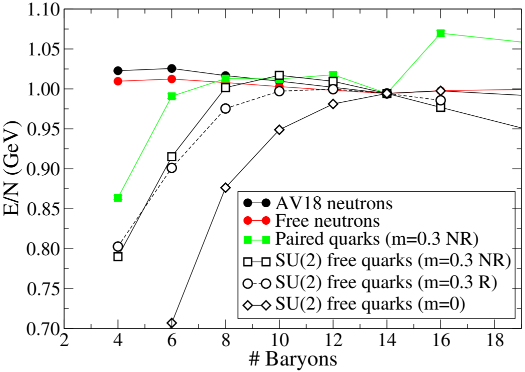
points AV18 interaction

- $N = 14$ corresponds to closed neutron shell with $|k| = 0, 1$
- maximum at $N = 6$ always present
- at high density large ≈ 10 MeV differences among neighboring N
- very large gap of ≈ 50 MeV between $N = 6$ and $N = 14$

Spectra for nucleons and quarks at $\rho = 2\rho_0$

Gandolfi, Carlson, Roggero, Lynn, Reddy (2017)

- full energies containing the rest mass
- confinement energy (assumed constant with ρ) to match result for $N = 14$ neutrons



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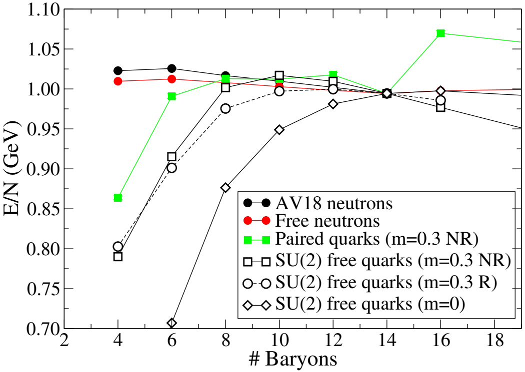
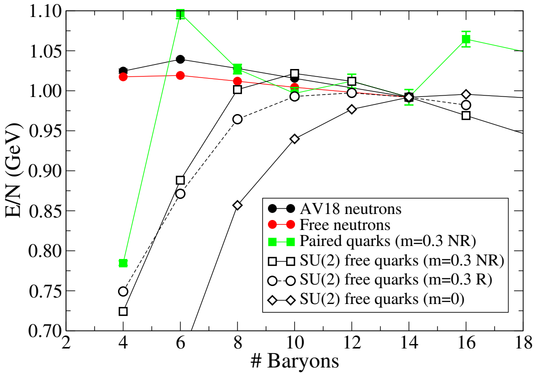


- for both quarks and nucleons no qualitative change from interactions
  - exception: interacting quarks show shell closure at $N = 14$ (blue-q)
- sizable $N = 4 - 6$ gap comes from pairs filling $|k| = 1$ quark shell

Spectra for nucleons and quarks at $\rho = 3\rho_0$

Gandolfi, Carlson, Roggero, Lynn, Reddy (2017)



- even larger effect for $N = 6$ due to larger energy of $|k| = 1$ states

	quarks		nucleons	
	free	interacting	free	interacting
$E_{46}(2\rho_0)$	120 MeV	130 MeV	3 MeV	4 MeV
$E_{46}(3\rho_0)$	170 MeV	310 MeV	2 MeV	15 MeV

Summary

Useful signatures to tell apart relevant degrees of freedom

- pairing pattern for $N = 4$: $E_N(s) > E_N(d)$ vs. $E_q(s) \leq E_q(d)$
- $N = 4 - 6$ energy gap much larger for quarks (seems robust)
- shell closure at $N = 14$ if quarks are free (difficult for near term)

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Future directions

- explore effects of heavier pion masses
  - LQCD can be extended to larger A (much) more easily
  - nuclear interactions become (much) more controllable with EFT($\not\pi$)
- add protons and/or hyperons to the system

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