Treatment of relativistic effects in lepton-nucleus scattering processes

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Based on:

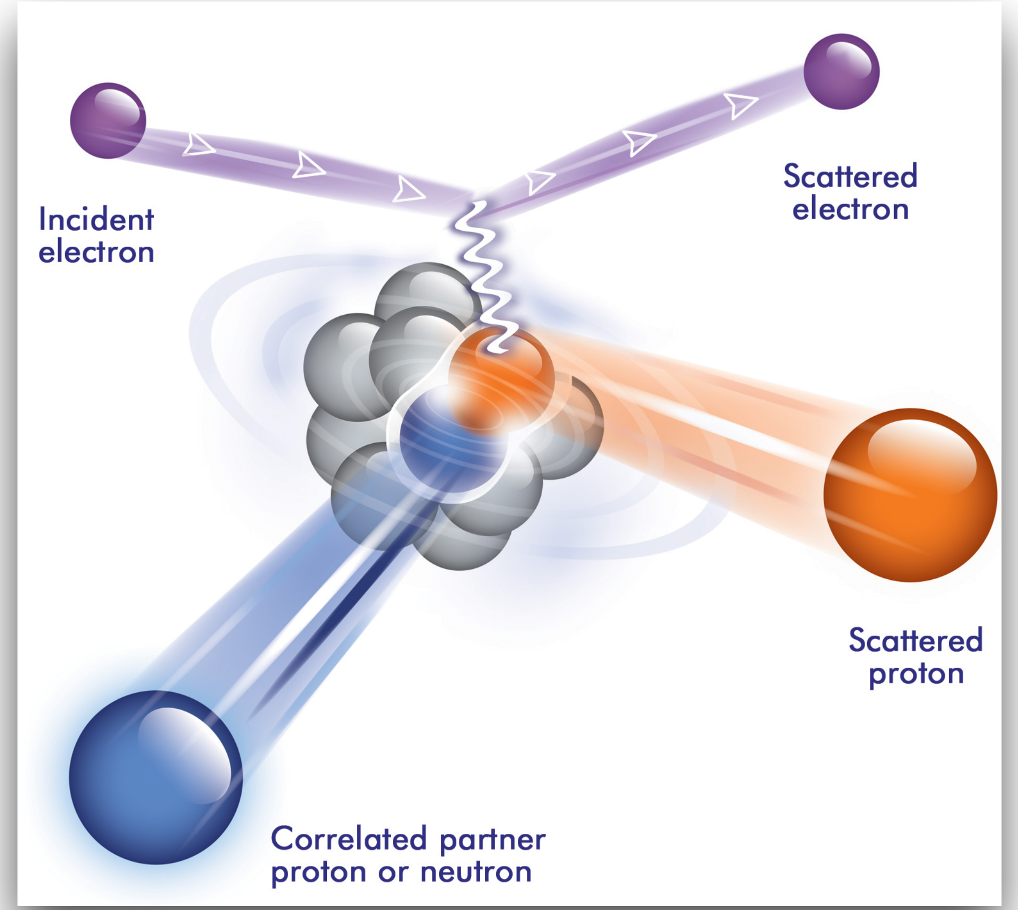
- NR, A. Lovato, and O. Benhar, Phys. Rev. Lett. **116**, 192501
- NR, A. Lovato, L. Alvarez-Ruso, A. Lovato, and J. Nieves, Phys. Rev. C **96**, 015504 (2017)
- J. E. Sobczyk, NR, A. Lovato, J. Nieves, arXiv:1711.06697
- NR, W. Leidemann, A. Lovato, G. Orlandini, arXiv:1801.07111

Motivations

• In electron- scattering experiments the nucleus is mostly seen as a target, as the kinematic of the probe is completely known.

• The first generation of (e,e'p) data in the early 1960s not only established the validity of the nuclear shell model but alse showed its limitations

• More recent measurements, allowed to unveil detailed features of the nuclear wave function, including its high-momentum components.



E07-006 : Short Range Correlation Experiment

• Developing a coherent picture of the electroweak response is also critical for the interpretation of neutrino scattering experiments, such as the Deep Underground Neutrino Experiment

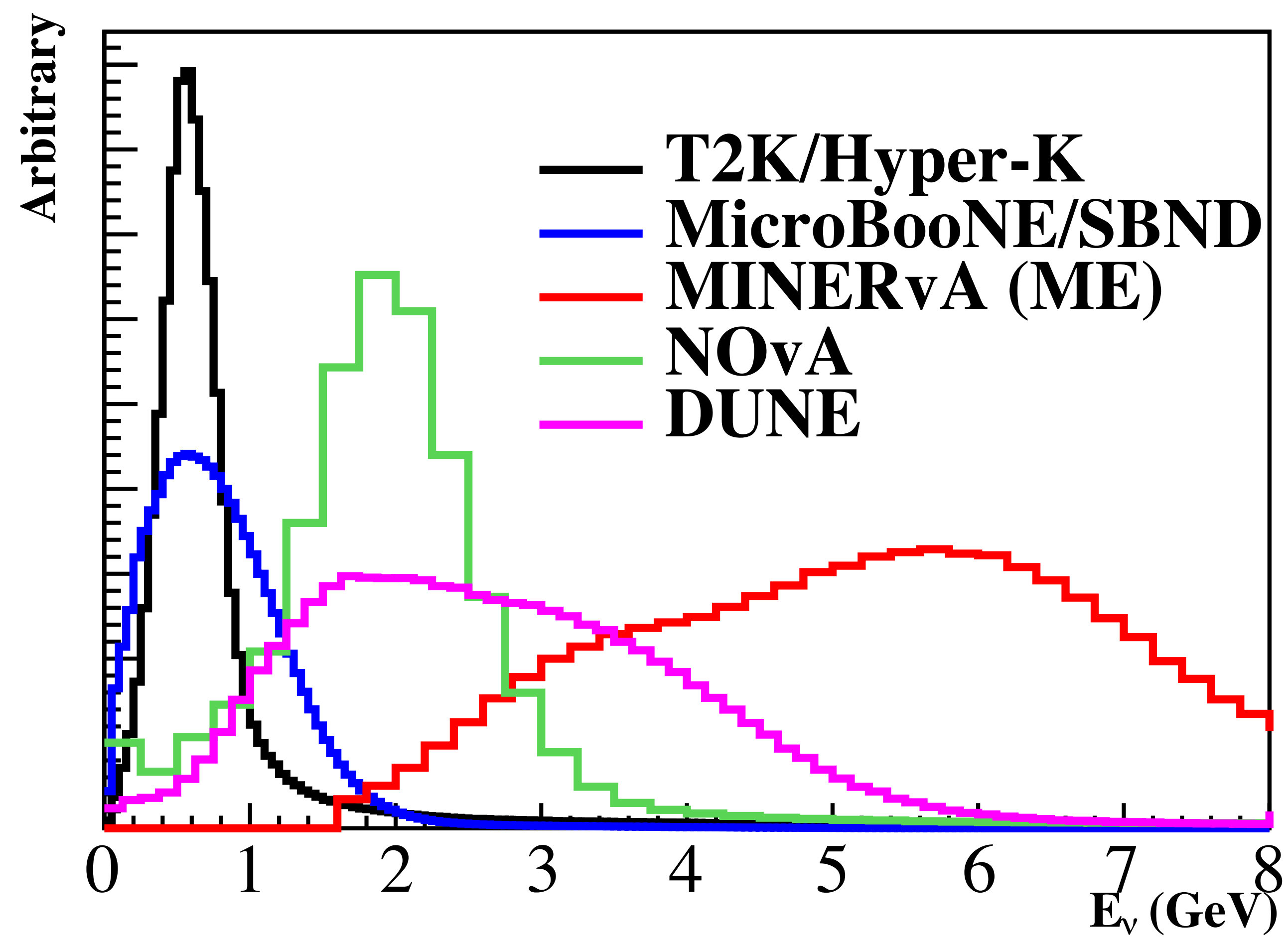
Motivations

“Neutrinos … win the minimalist contest: zero charge, zero radius, and very possibly zero mass.”

—Leon M. Lederman—

- In neutrino-oscillation experiments the use of nuclear target as detectors allows for a substantial increase of the event rate.

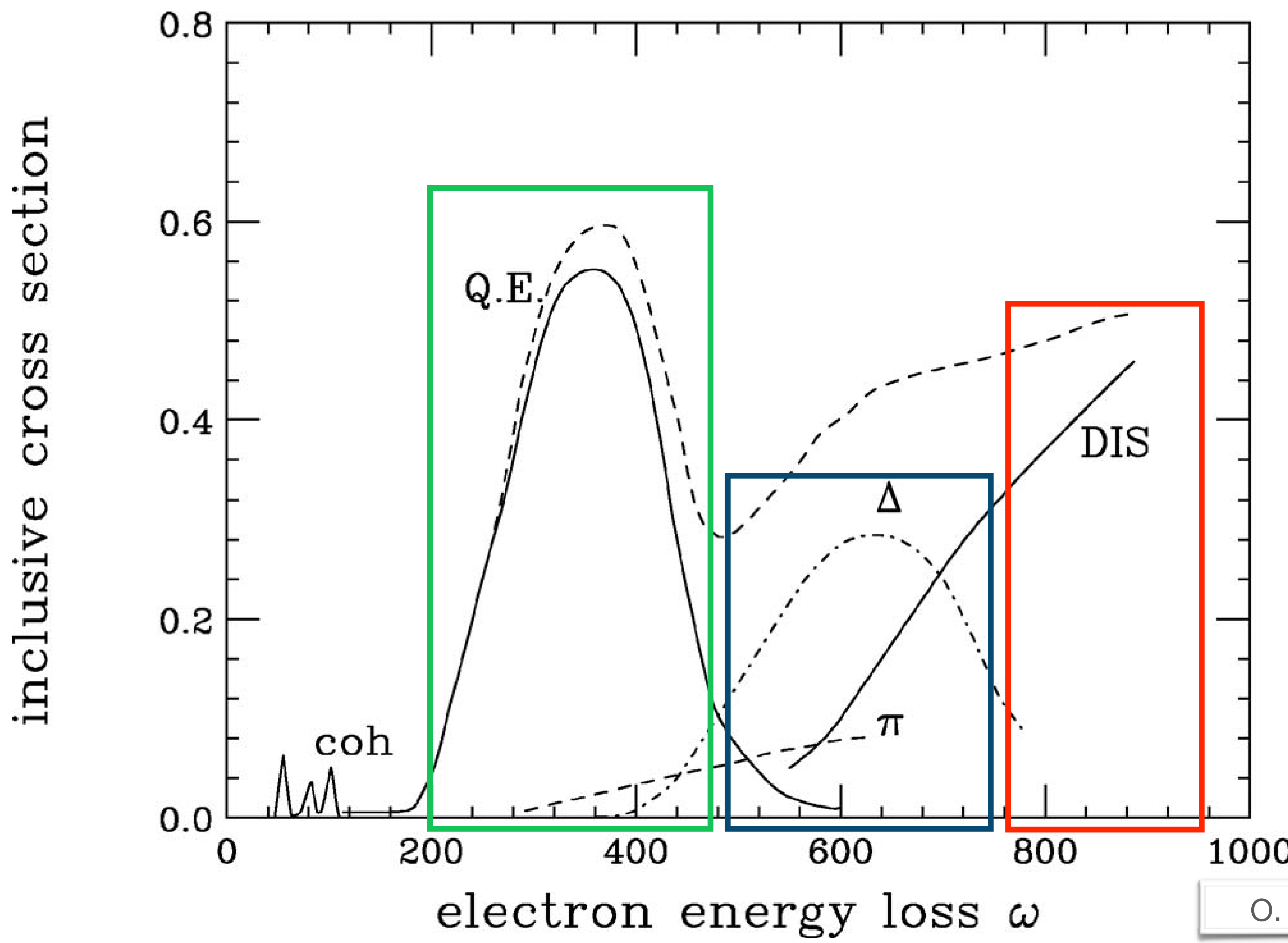
- Understanding neutrino-nucleus interactions in the 1-10 GeV spectrum requires an accurate description of both nuclear dynamics and of the interaction vertex where relativistic effects are accounted for



T. Katori and M. Martini,. arXiv:1611.07770

Lepton-nucleus scattering

In electron-nucleus scattering cross-section the different reaction mechanisms can be easily identified



- Broad peak due to quasi-elastic electron-nucleon scattering.
- Excitation of the nucleon to distinct resonances (like the Δ) and pion production.
- Deep Inelastic Scattering region, productions of hadrons other than protons and neutrons

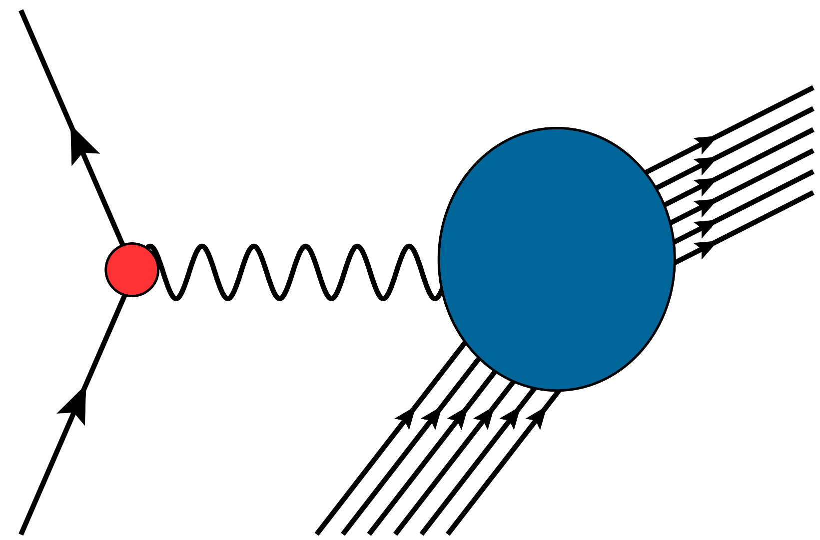
O. Benhar, et al. RMP **80**, 189 (2008)

- In neutrino-nucleus scattering different reaction mechanisms contribute to the cross section for a fixed value of the kinetic energy and scattering angle of the final lepton

Electron-nucleus scattering

The inclusive cross section of the process in which a lepton scatters off a nucleus and the hadronic final state is undetected can be written as

$$\frac{d^2\sigma}{d\Omega_\ell dE_{\ell'}} = L_{\mu\nu} W^{\mu\nu}$$



- The Leptonic tensor is fully specified by the lepton kinematic variables. For instance, in the electron-nucleus scattering case

$$L^{\mathrm{EM}}_{\mu\nu} = 2[k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu}(kk')]$$

- The Hadronic tensor contains all the information on target response

$$W^{\mu\nu} = \sum_f \langle 0|J^{\mu\dagger}(q)|f\rangle\langle f|J^\nu(q)|0\rangle\delta^{(4)}(p_0 + q - p_f)$$

Non relativistic nuclear many-body theory (NMBT) provides a fully consistent theoretical approach allowing for an accurate description of |0>, independent of momentum transfer.

Non relativistic Nuclear Many Body Theory

- Within the NMBT the nucleus is described as a collection of A point-like nucleons, the dynamics of which are described by the non relativistic Hamiltonian

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

v_{ij} → Argonne v18

V_{ijk} → UIX, IL7

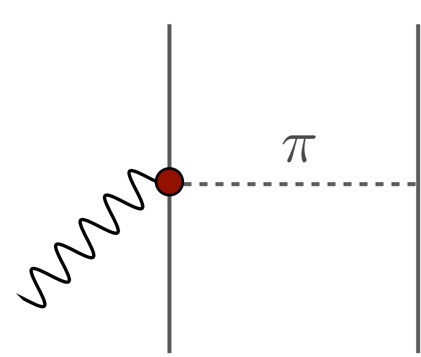
$H|0\rangle = E_0|0\rangle \quad , \quad H|f\rangle = E_f|f\rangle$ can be accurately determined for $A \leq 12$

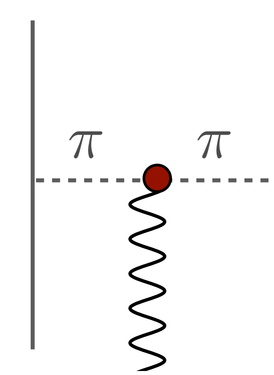
The nuclear electromagnetic current is constrained by the Hamiltonian through the continuity equation

$$\boldsymbol{\nabla} \cdot \mathbf{J}_{\rm EM} + i[H, J^0_{\rm EM}] = 0$$

- The above equation implies that $\mathbf{J}_{\rm EM}$ involves two-nucleon contributions.

- Non relativistic expansion of $\mathbf{J}_{\rm EM}$, powers $|\mathbf{q}|/m$

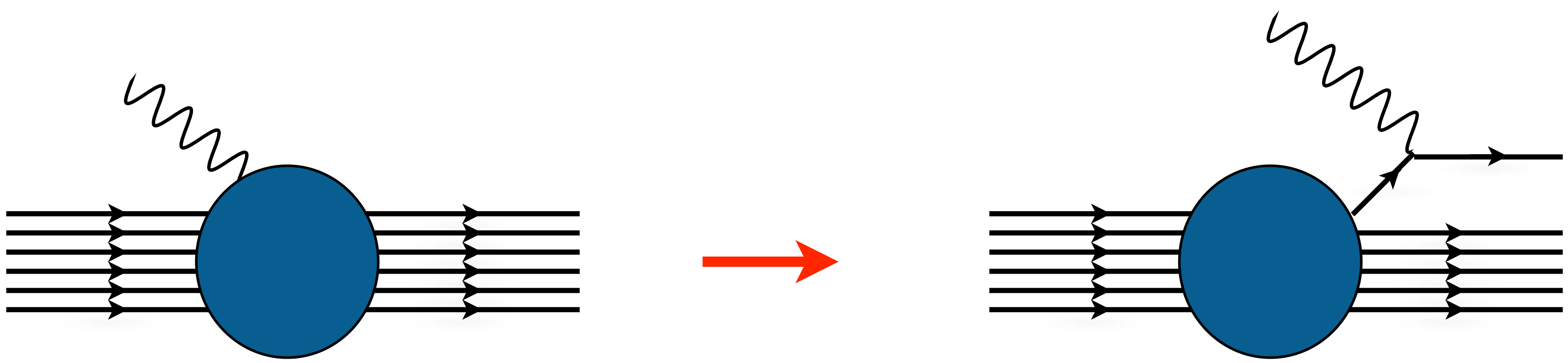




The Impulse Approximation

- For sufficiently large values of |**q**|, the IA can be applied under the assumptions

$$|f\rangle \longrightarrow |p\rangle \otimes |f\rangle_{A-1} \qquad\qquad J_\alpha = \sum_i j^i_\alpha$$



- The matrix element of the current can be written in the factorized form

$$\langle 0|J_\alpha|f\rangle \to \sum_k \langle 0|[|k\rangle \otimes |f\rangle_{A-1}]\langle k|\sum_i j^i_\alpha|p\rangle$$

- The nuclear cross section is given in terms of the one describing the interaction with individual bound nucleons

$$d\sigma_A = \int dE\, d^3k d\sigma_N P(\mathbf{k}, E)$$

- The intrinsic properties of the nucleus are described by the hole spectral function

The one-body Spectral Function of nuclear matter

- The Spectral Function gives the probability distribution of removing a nucleon with momentum **k**, leaving the spectator system with an excitation energy E

$$P_h(\mathbf{k},E) = -\frac{1}{\pi}\mathrm{Im}G_h(\mathbf{k},E) = \sum_f |\langle 0|a^\dagger_{\mathbf{k}}|A-1\rangle_f|^2\delta(E_0+E-E_f^{A-1})$$

- The two points Green's Function describes nucleon propagation in the nuclear medium

$$G(\mathbf{k},E) = \langle 0|a^\dagger_{\mathbf{k}}\frac{1}{H-E_0-E-i\epsilon}a_{\mathbf{k}}|0\rangle + \langle 0|a_{\mathbf{k}}\frac{1}{H-E_0+E+i\epsilon}a^\dagger_{\mathbf{k}}|0\rangle$$
$$= G_h(\mathbf{k},E) + G_p(\mathbf{k},E)$$

- The Correlated Basis Function approach accounts for correlations induced by the nuclear interactions

$$\Phi_n(x_1\dots x_A) \longrightarrow \mathcal{F}\,\Phi_n(x_1\dots x_A)$$

- The correlation operator reflects the spin-isospin dependence of the nuclear interaction

$$\mathcal{F} \equiv \Big(\mathcal{S}\prod_{i<j}F_{ij}\Big) \qquad F_{ij} \equiv \sum_p f^p_{ij}O^p_{ij}$$

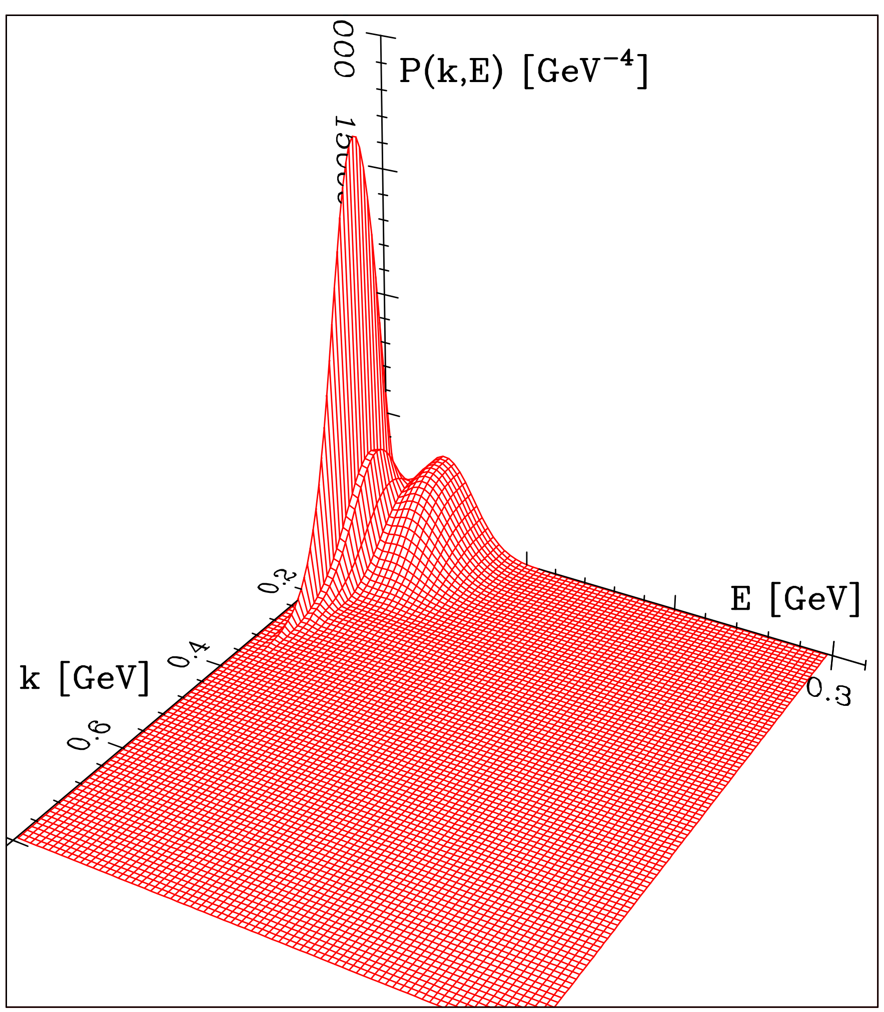
The one-body Spectral Function of finite nuclei

- ^{16}O Spectral Function obtained within CBF and using the Local Density Approximation

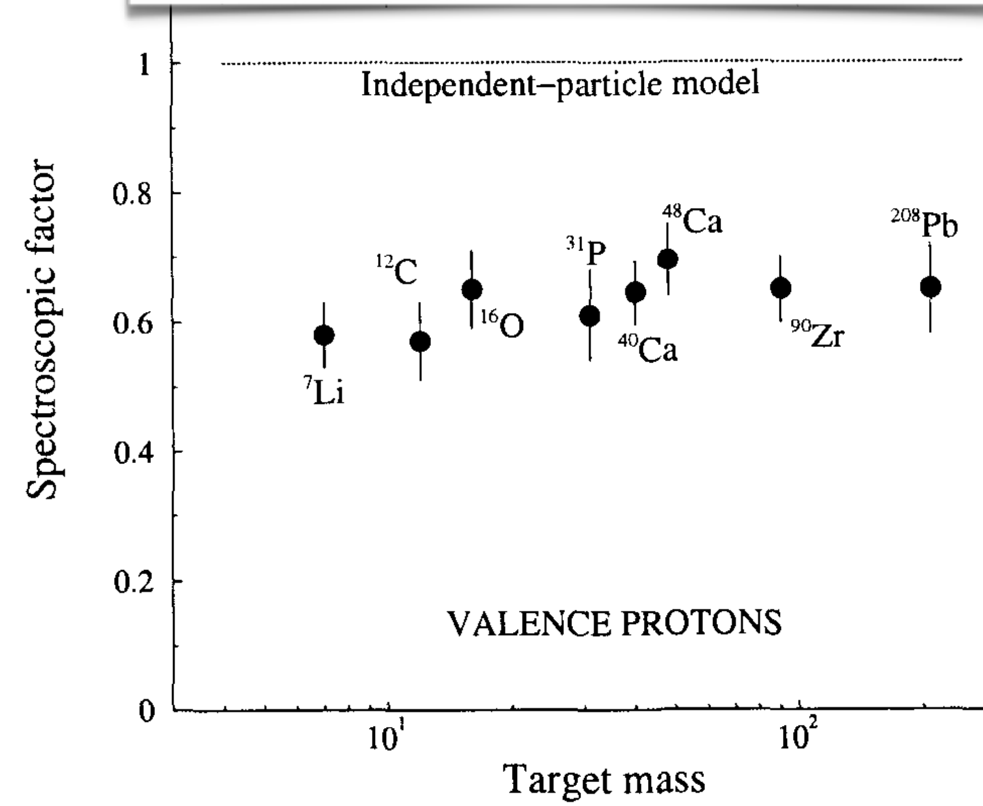
$$P_{LDA}(\mathbf{k},E) = P_{MF}(\mathbf{k},E) + P_{corr}(\mathbf{k},E)$$

$$\sum_n Z_n |\phi_n(\mathbf{k})|^2 F_n(E-E_n)$$

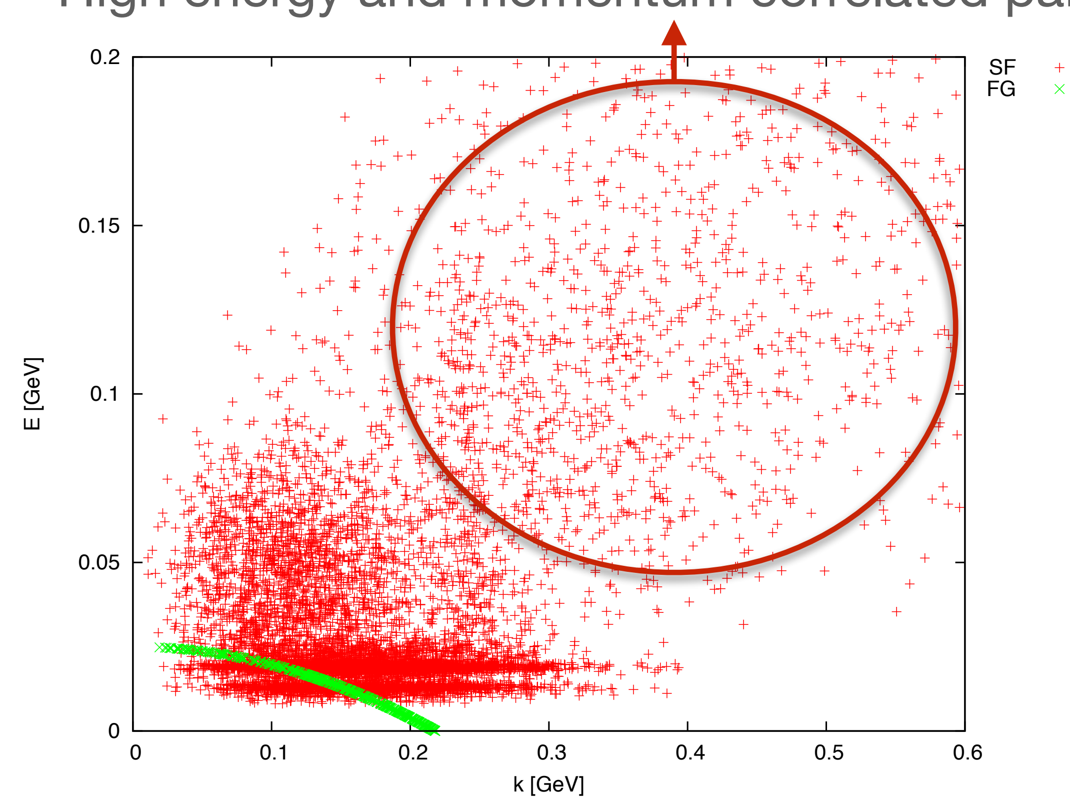
$$\int d^3r P^{NM}_{corr}(\mathbf{k},E;\rho=\rho_A(r))$$



^{40}Ar: JLab E 12-14-012



- High energy and momentum correlated pairs



The Impulse Approximation and convolution scheme

- In the kinematical region in which the interactions between the struck particle and the spectator system can not be neglected, the IA results have to be modified to include the effect of final state interactions (FSI).

$$d\sigma_{FSI} = \int d\omega' f_{\mathbf{q}}(\omega - \omega') d\tilde{\sigma}_{IA} \quad , \quad \tilde{e}(\mathbf{p}) = e(\mathbf{p}) + \mathcal{U}(t_{kin}(\mathbf{p}))$$

Optical Potential

- The theoretical approach to calculate the folding function consists on a generalization of Glauber theory of high energy proton-nucleus scattering

$$f_{\mathbf{q}}(\omega) = \delta(\omega)\sqrt{T_{\mathbf{q}}} + \int \frac{dt}{2\pi} e^{i\omega t} \left[\bar{U}_{\mathbf{q}}^{FSI}(t) - \sqrt{T_{\mathbf{q}}} \right]$$
$$= \delta(\omega)\sqrt{T_{\mathbf{q}}} + (1 - \sqrt{T_{\mathbf{q}}}) F_{\mathbf{q}}(\omega),$$

Glauber Factor

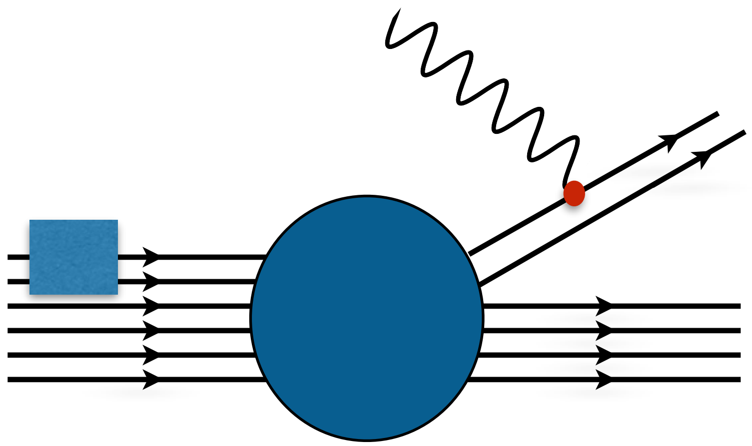
Nuclear Transparency

A.Ankowski et al,Phys. Rev. D91, 033005 (2015)

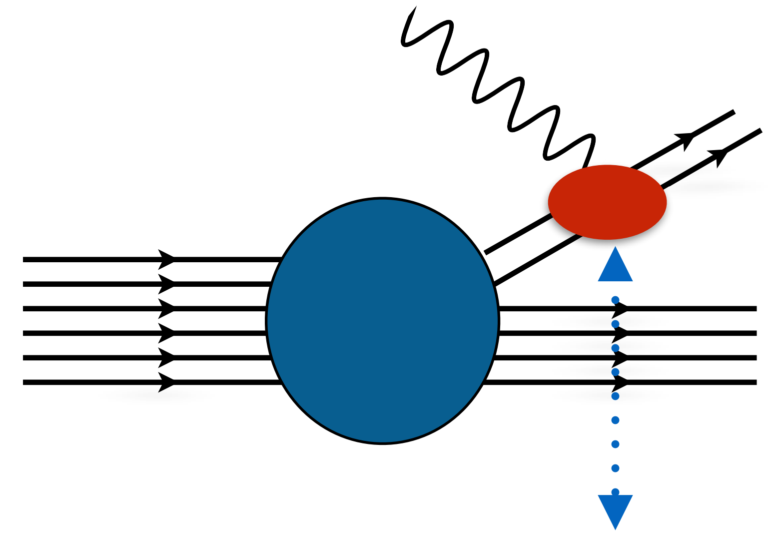
O.Benhar, Phys. Rev. C87, 024606 (2013)

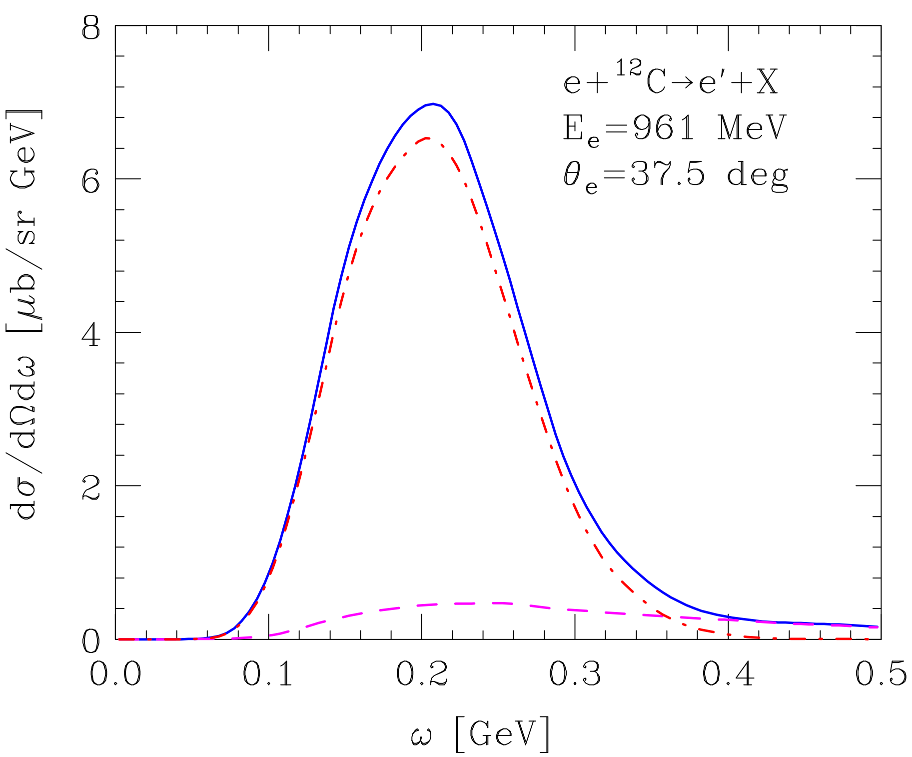
Production of two particle-two hole (2p2h) states

- Initial State Correlations

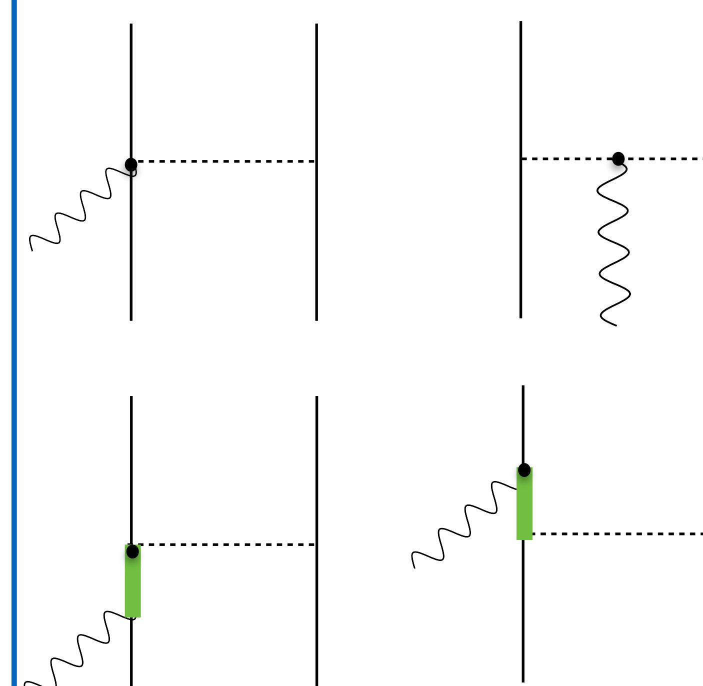


- Meson Exchange currents





- $P_{corr}(\mathbf{k},E)$ accounts for the presence of strongly correlated pairs. Its contribution to the cross section is clearly visible: appearance of a tail in the large energy transfer region



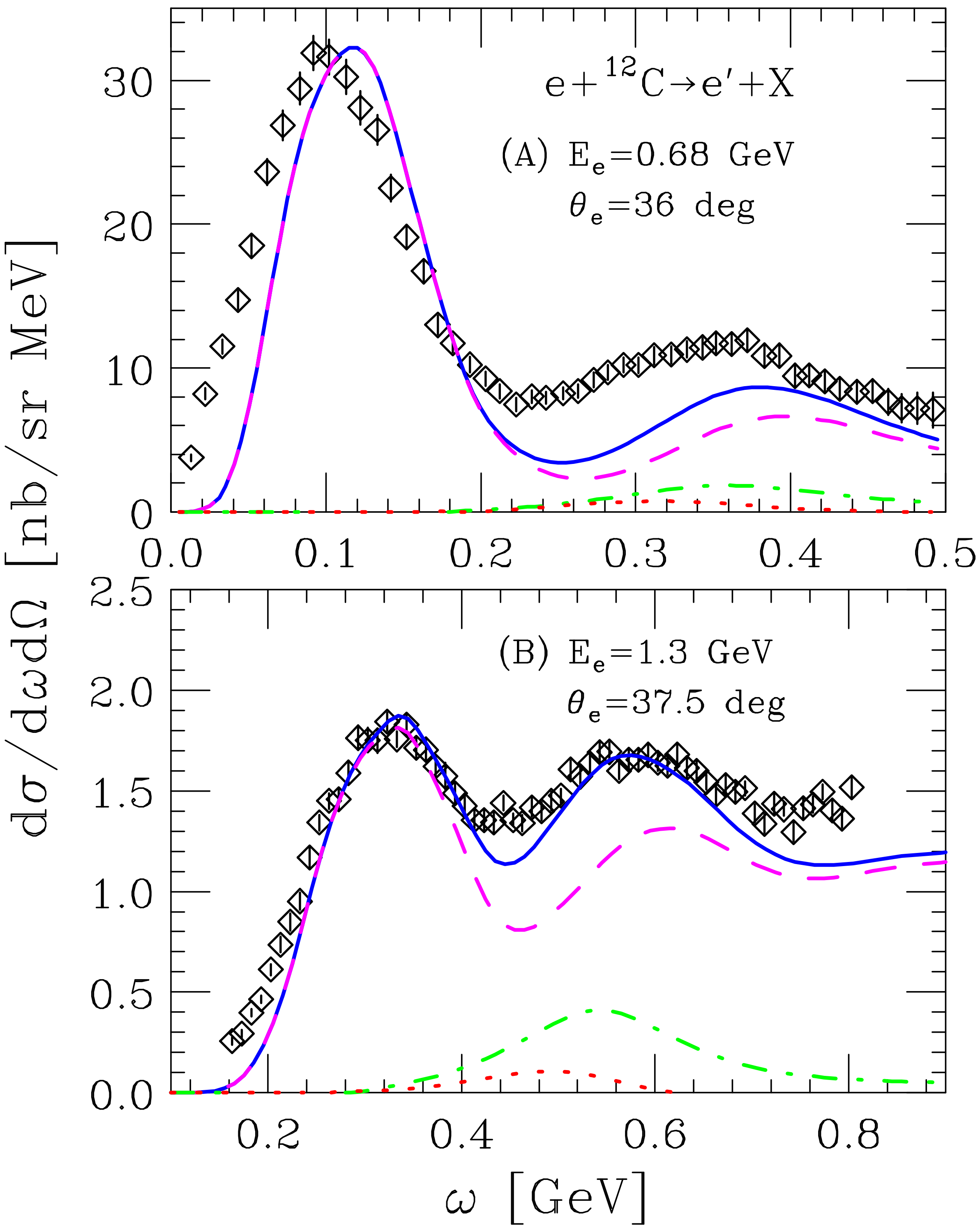
Different contributions to the relativistic two-body currents

- The Spectral Function approach has been generalized:

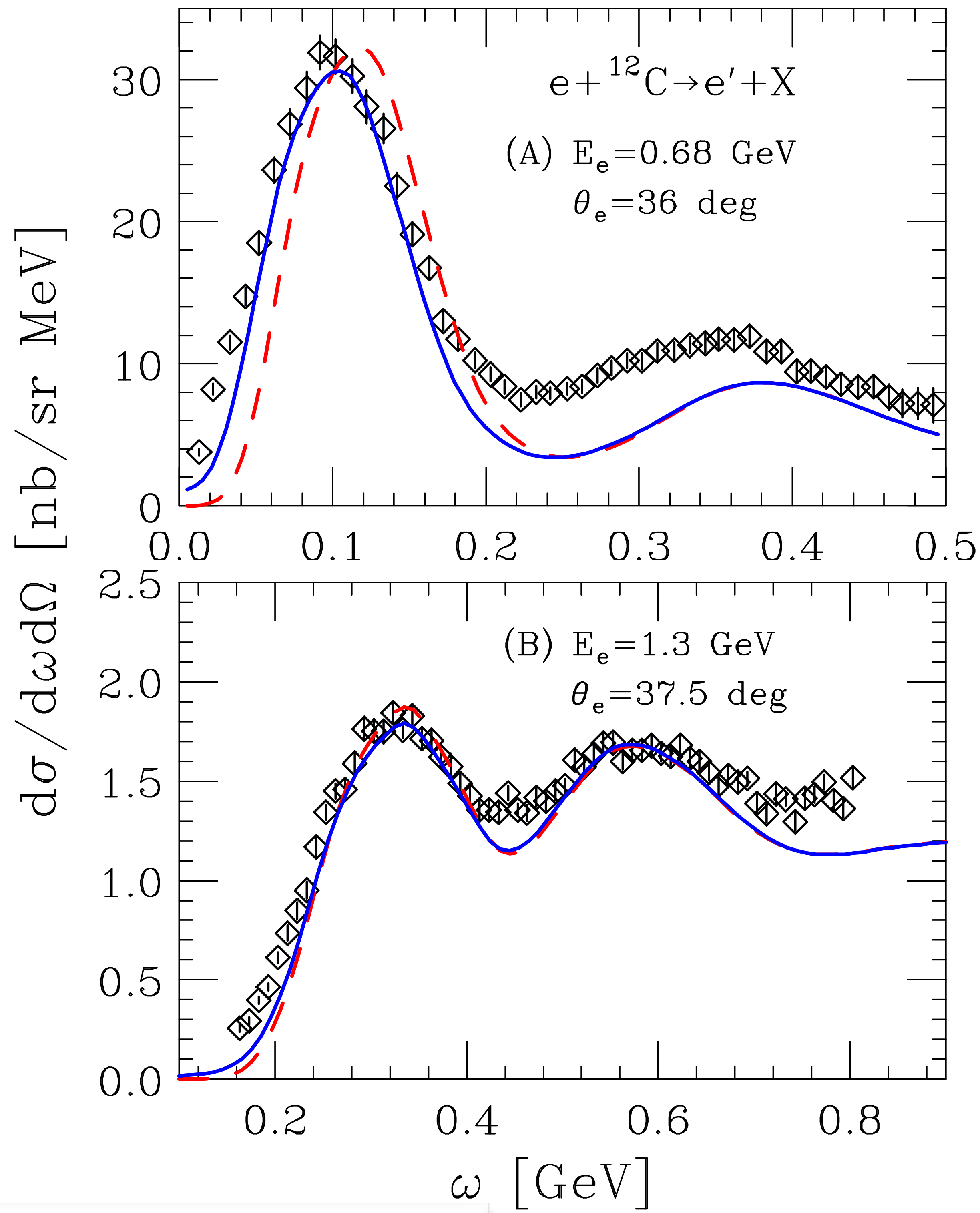
$$W^{\mu\nu}_{2p2h} = W^{\mu\nu}_{ISC} + W^{\mu\nu}_{MEC} + W^{\mu\nu}_{int}$$

Results for electron-^{12}C cross sections

- Separate contributions: IA



- Including FSI in the QE region

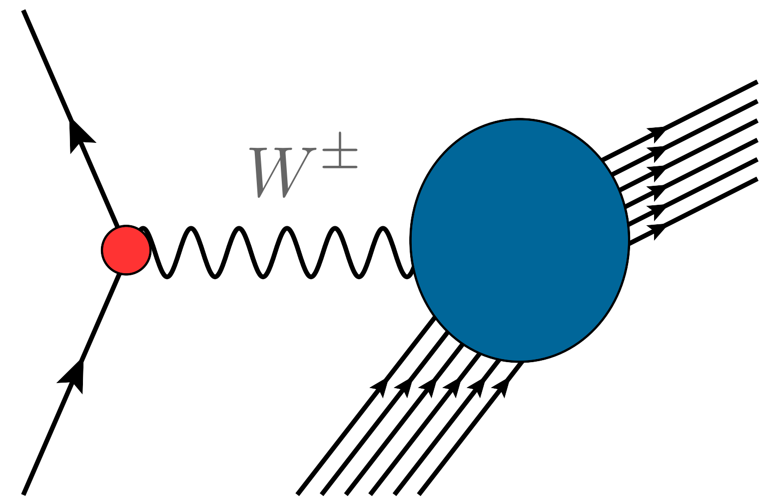


NR, A.Lovato, O.Benhar, PRL 116, 192501 (2016)

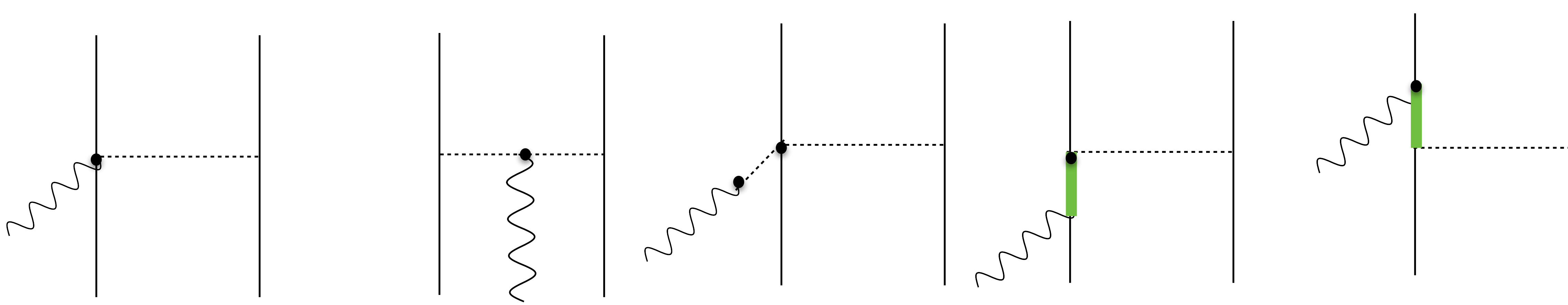
(Anti)neutrino -^{12}C scattering cross sections

The inclusive cross section of the process in which a neutrino or antineutrino scatters off a nucleus can be written in terms of five response functions

$$\frac{d\sigma}{dE_{\ell'}d\Omega_\ell} \propto [v_{00}R_{00} + v_{zz}R_{zz} - v_{0z}R_{0z} + v_{xx}R_{xx} \mp v_{xy}R_{xy}]$$



- The two-body diagrams contributing to the axial and vector responses

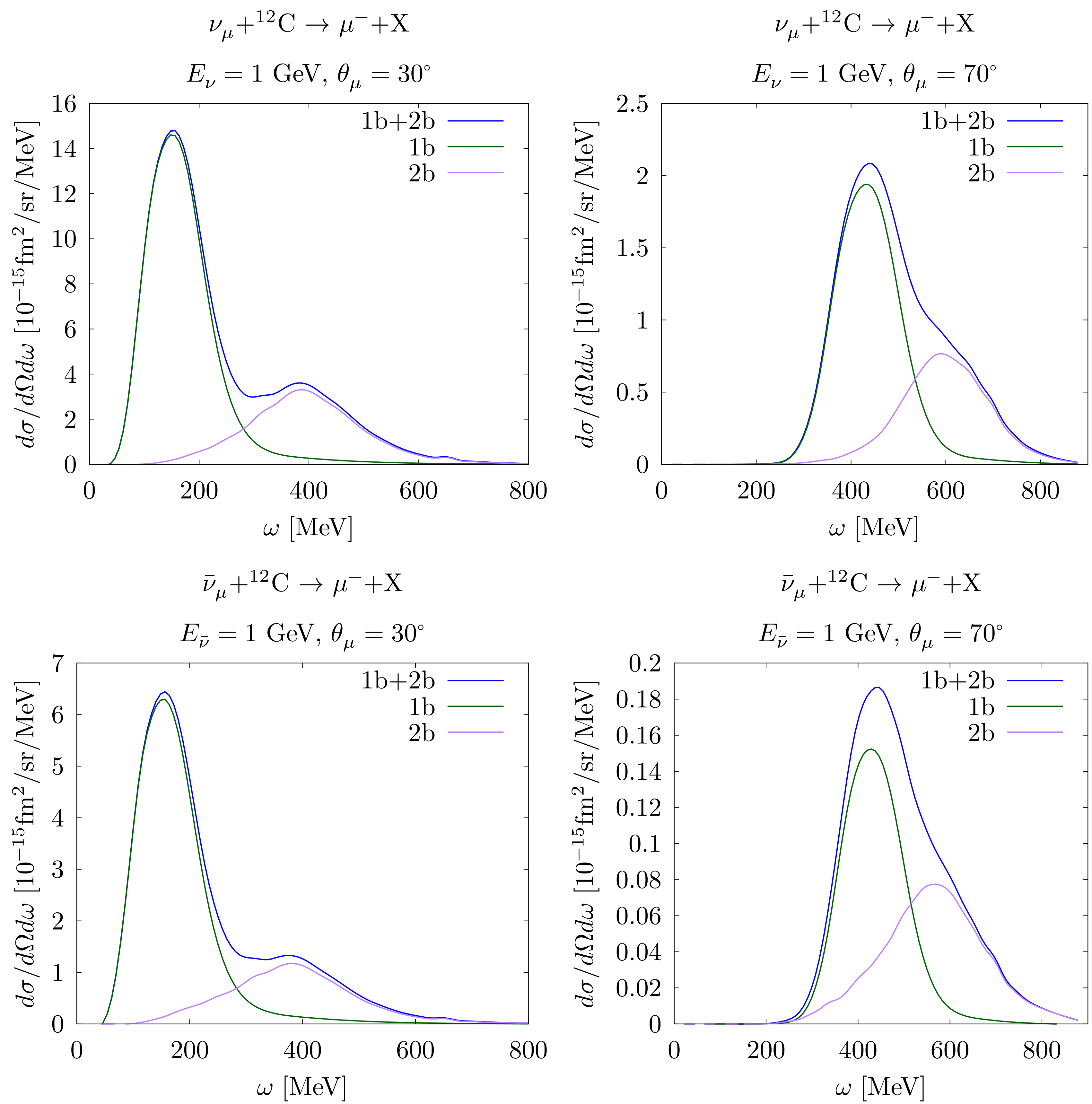


- In the preliminary results we present we only included:

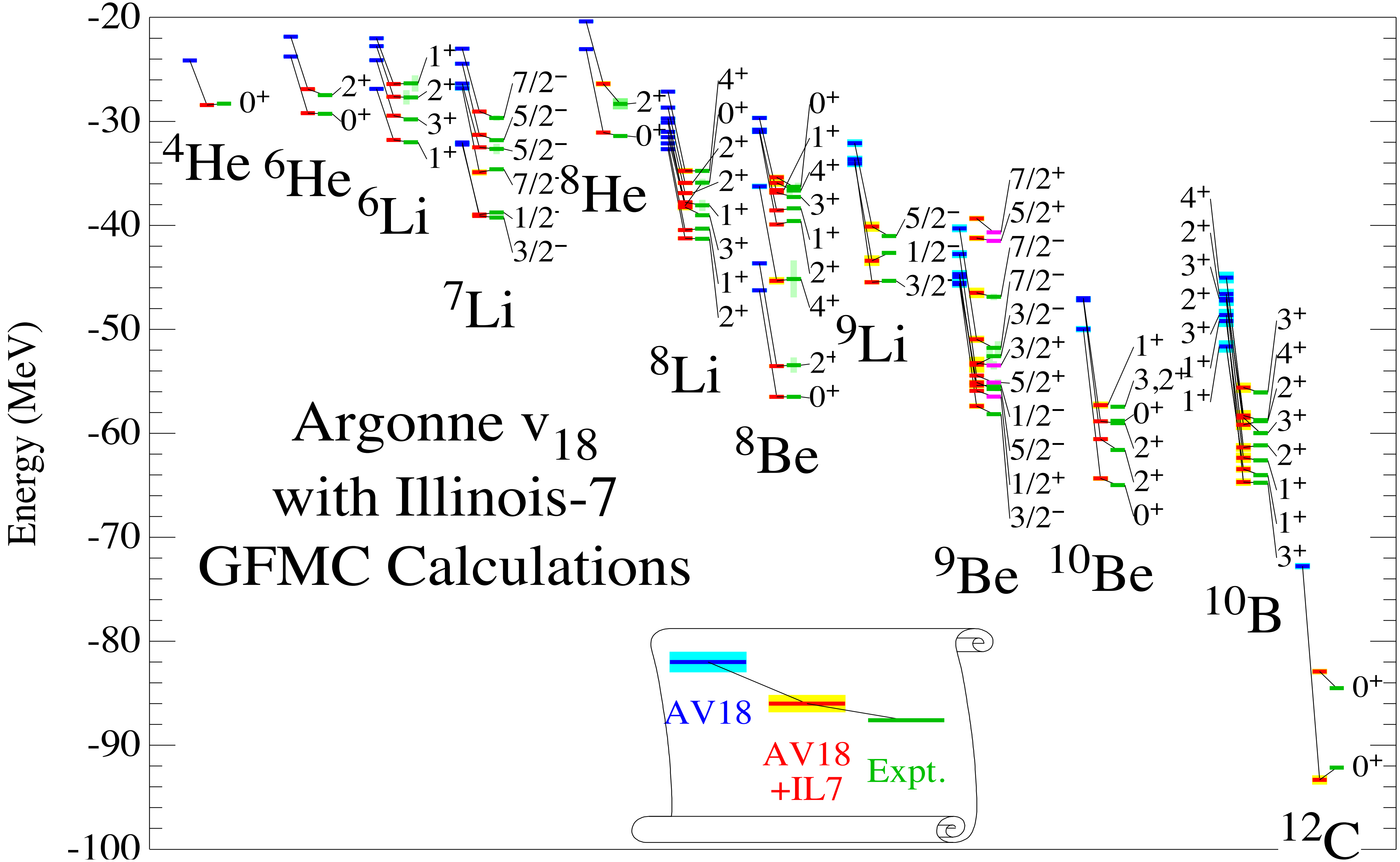
$$W^{\mu\nu}_{2p2h} = W^{\mu\nu}_{ISC} + W^{\mu\nu}_{MEC} + W^{\mu\nu}_{int}$$

(Anti)neutrino -^{12}C scattering cross sections

- The exchange contribution for the 2b is still missing (antisymmetrization of the final two-nucleon state)
- The 2b contribution affects the ‘dip’ region, in analogy with the electromagnetic case
- Meson exchange currents strongly enhance both the neutrino and antineutrino cross section for large values of the scattering angle



The Green's Function Monte Carlo approach



- Green's function Monte Carlo combined with a realistic nuclear hamiltonian reproduces the spectrum of ground and excited states of light nuclei

The Green's Function Monte Carlo approach

• Accurate calculations of the electromagnetic responses of ^{4}He and ^{12}C have been recently performed within GFMC

$$R_{\alpha\beta}(\omega, \mathbf{q}) = \sum_f \langle 0|J^\dagger_\alpha(\mathbf{q})|f\rangle\langle f|J_\beta(\mathbf{q})|0\rangle\delta(\omega - E_f + E_0)$$

• Valuable information on the energy dependence of the response functions can be inferred from the their Laplace transforms

$$E_{\alpha\beta}(\mathbf{q}, \tau) = \int d\omega\, e^{-\omega\tau} R_{\alpha\beta}(\mathbf{q}, \omega) = \langle 0|J^\dagger_\alpha(\mathbf{q}) e^{-(H-E_0)\tau} J_\beta(\mathbf{q})|0\rangle$$

Using the completeness relation for the final states, we are left with ground-state expectations value

Limitations of the original method:

★ It is a nonrelativistic method, can not be safely applied in the whole kinematical region relevant for neutrino experiments

★ The computational effort required by the inversion of $E_{\alpha\beta}$ makes the direct calculation of inclusive cross sections unfeasible

Relativistic effects in a correlated system

- We extend the applicability of GFMC in the quasielastic region to intermediate momentum transfers by performing the calculations in a reference frame that minimizes nucleon momenta.

  - The importance of relativity emerges in the frame dependence of non relativistic calculations at high values of **q**

  - In a generic reference frame the longitudinal non relativistic response reads

$$R_L^{fr} = \sum_f \left| \langle \psi_i | \sum_j \rho_j(\mathbf{q}^{fr}, \omega^{fr}) | \psi_f \rangle \right|^2 \delta(E_f^{fr} - E_i^{fr} - \omega^{fr})$$

$$\delta(E_f^{fr} - E_i^{fr} - \omega^{fr}) \approx \delta[e_f^{fr} + (P_f^{fr})^2/(2M_T) - e_i^{fr} - (P_i^{fr})^2/(2M_T) - \omega^{fr}]$$

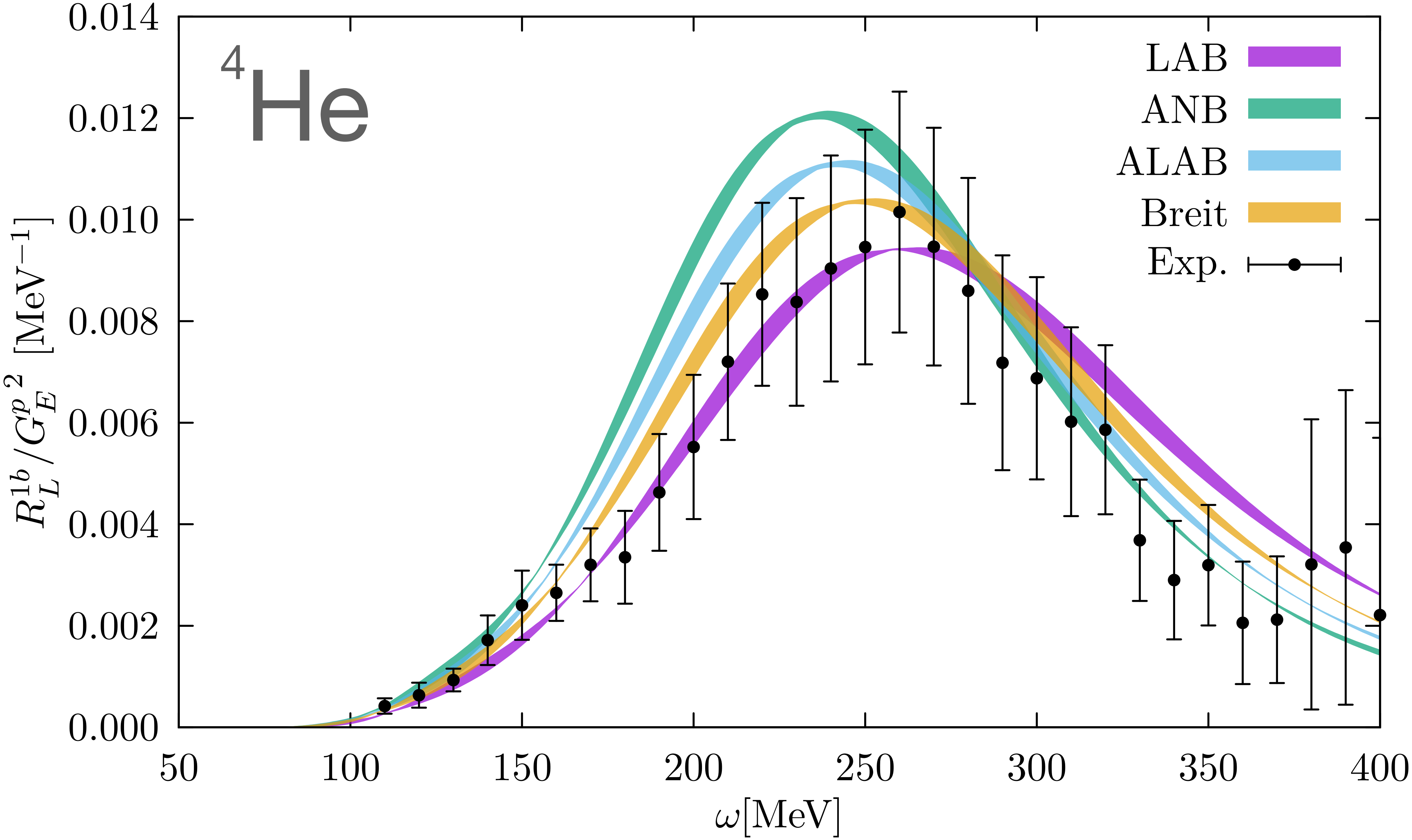
  - The response in the LAB frame is given by the Lorentz transformation

$$R_L(\mathbf{q}, \omega) = \frac{\mathbf{q}^2}{(\mathbf{q}^{fr})^2} \frac{E_i^{fr}}{M_0} R_L^{fr}(\mathbf{q}^{\mathbf{fr}}, \omega^{fr})$$

where

$$q^{fr} = \gamma(q - \beta\omega), \ \omega^{fr} = \gamma(\omega - \beta q), \ P_i^{fr} = -\beta\gamma M_0, \ E_i^{fr} = \gamma M_0$$

Relativistic effects in a correlated system



- Longitudinal responses of ^{4}He for |q|=700 MeV in the four different reference frames. The curves show differences in both peak positions and heights.

Relativistic effects in a correlated system

- The frame dependence can be drastically reduced if one assumes a two-body breakup model with relativistic kinematics to determine the input to the non relativistic dynamics calculation

$$p^{fr} = \mu\Big(\frac{p_N^{fr}}{m_N} - \frac{p_X^{fr}}{M_X}\Big)$$
$$P_f^{fr} = p_N^{fr} + p_X^{fr}$$

$$\mu = \frac{m_N M_X}{m_N + M_X}$$

- The relative momentum is derived in a relativistic fashion

$$\omega^{fr} = E_f^{fr} - E_i^{fr}$$
$$E_f^{fr} = \sqrt{m_N^2 + [\mathbf{p}^{fr} + \mu/M_X \mathbf{P}_f^{fr}]^2} + \sqrt{M_X^2 + [\mathbf{p}^{fr} - \mu/m_N \mathbf{P}_f^{fr}]^2}$$

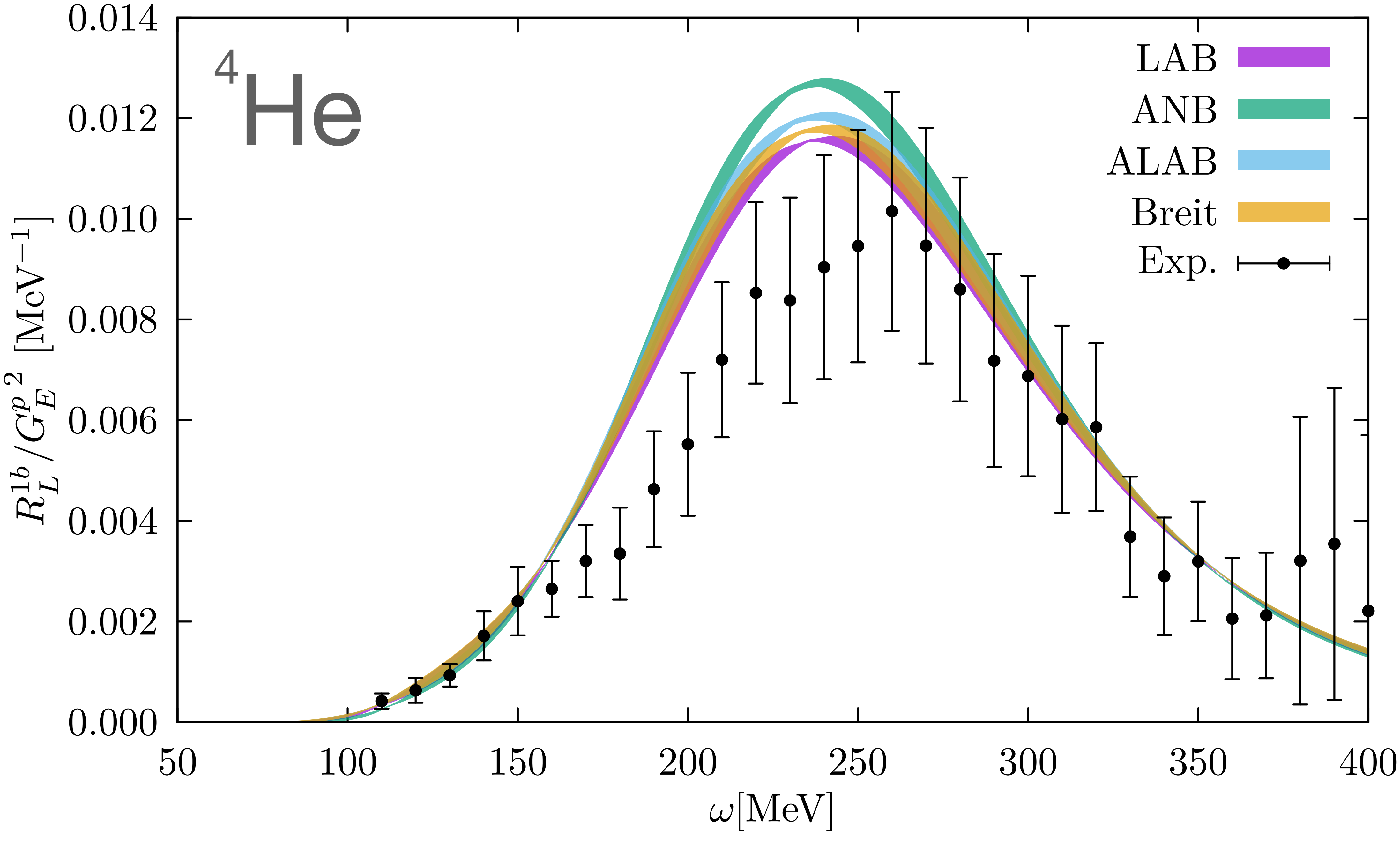
- And it is used as input in the non relativistic kinetic energy

$$e_f^{fr} = (p^{fr})^2/(2\mu)$$

- The energy-conserving delta function reads

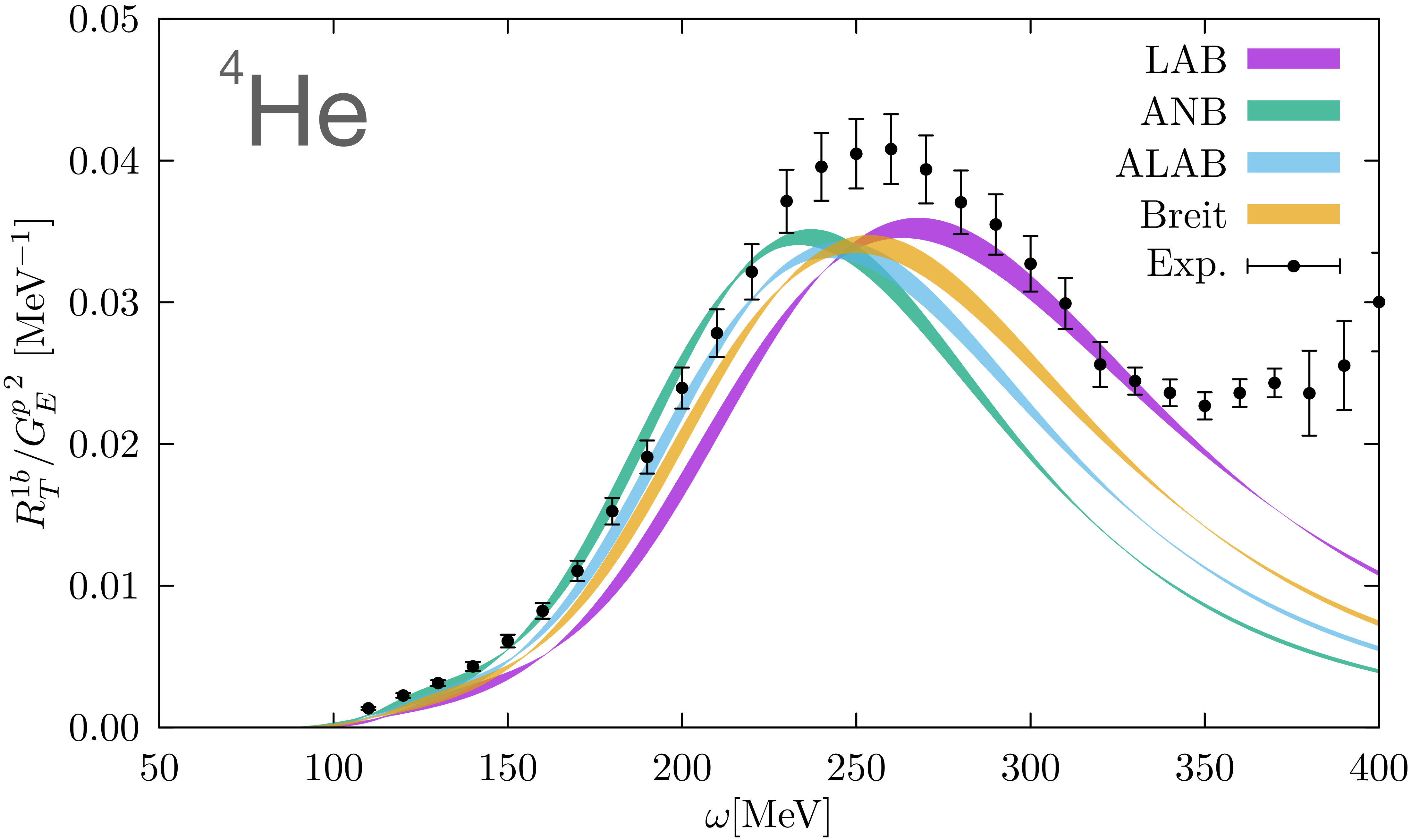
$$\delta(E_f^{fr} - E_i^{fr} - \omega^{fr}) = \delta(F(e_f^{fr}) - \omega^{fr}) = \Big(\frac{\partial F^{fr}}{\partial e_f^{fr}}\Big)^{-1} \delta[e_f^{fr} - e_f^{rel}(q^{fr}, \omega^f)]$$

Relativistic effects in a correlated system



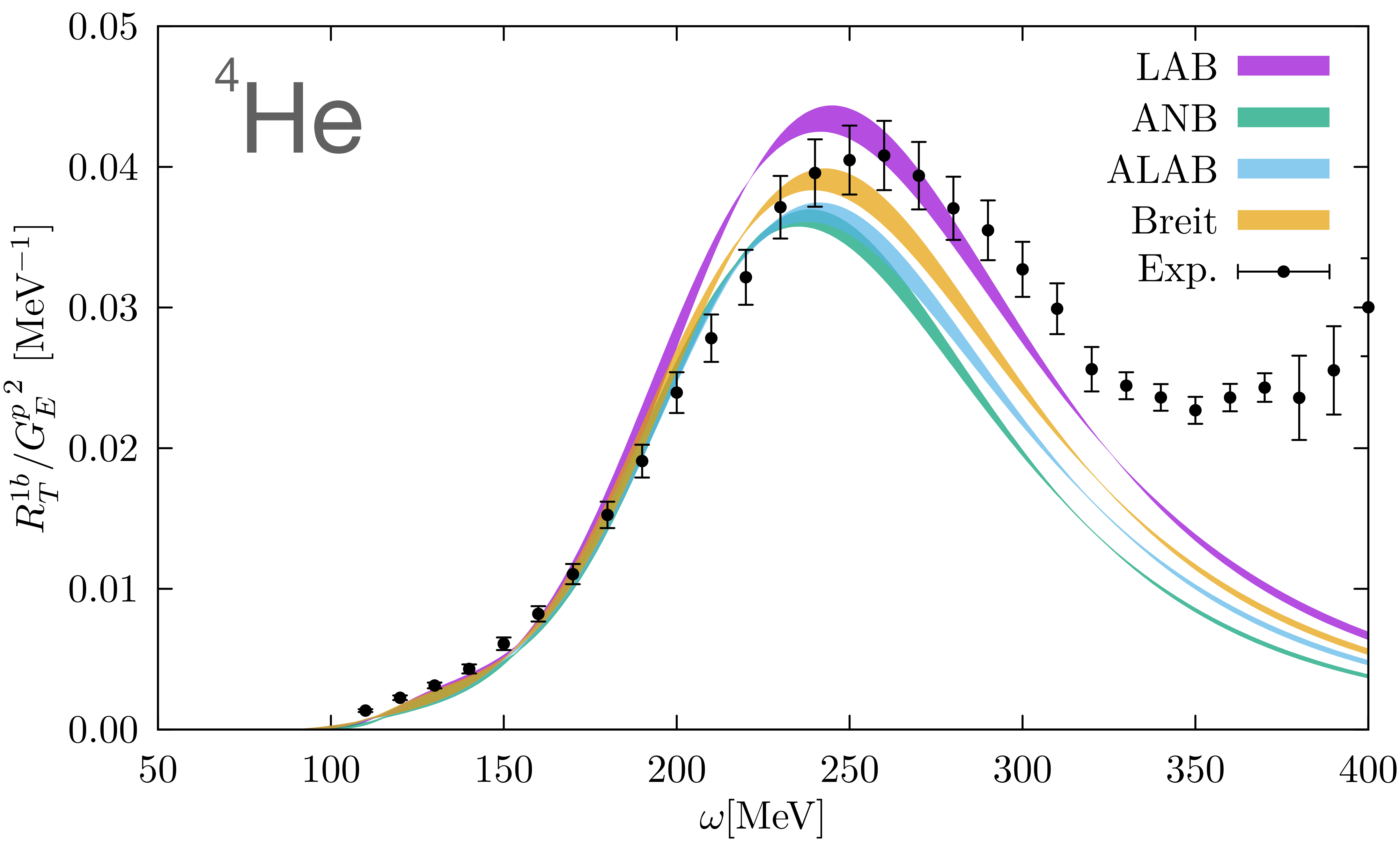
- Longitudinal responses of ^{4}He for |q|=700 MeV in the four different reference frames. The different curves are almost identical.

Relativistic effects in a correlated system



- Transverse responses of ^{4}He for |q|=700 MeV in the four different reference frames. The curves show differences in both peak positions and heights.

Relativistic effects in a correlated system



- Transverse responses of ^{4}He for |q|=700 MeV in the four different reference frames. The position of the quasielastic peak no longer depends on the reference frame. Different heights have to be ascribed to sub-leading relativistic corrections not included in the current operator.

Relativistic effects in a correlated system

- Nonrelativistic reduction of the electromagnetic current
- Charge operator:

$$\langle \rho_i \rangle = \Big[\frac{G_{E,i}}{\sqrt{1+Q^2/(4m^2)}} - i\frac{(2G_{M,i}-G_{E,i})}{4m^2}\mathbf{q}\cdot(\sigma\times\mathbf{p}) \Big] + \mathcal{O}(1/m^3)$$

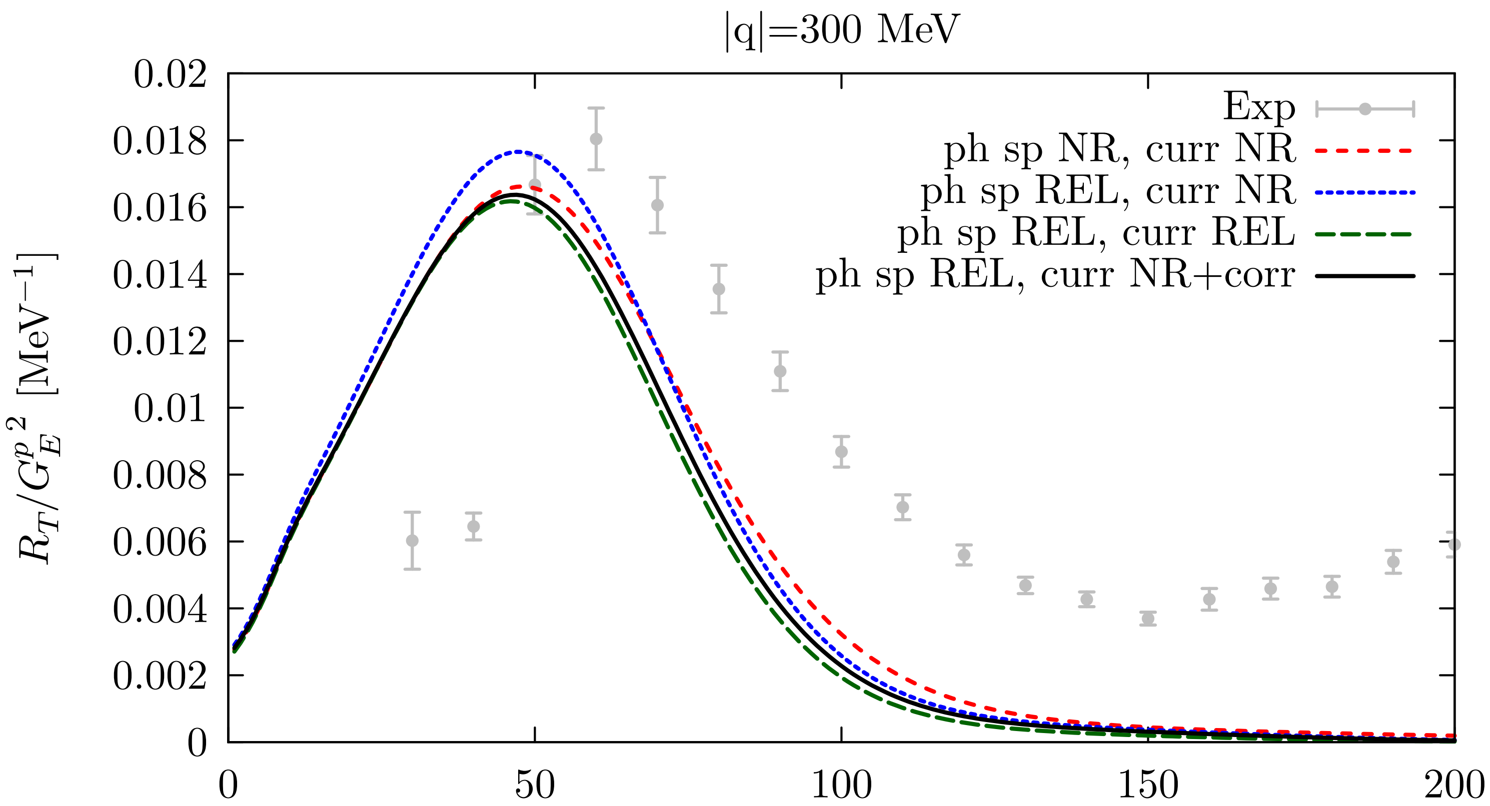
- Current operator:

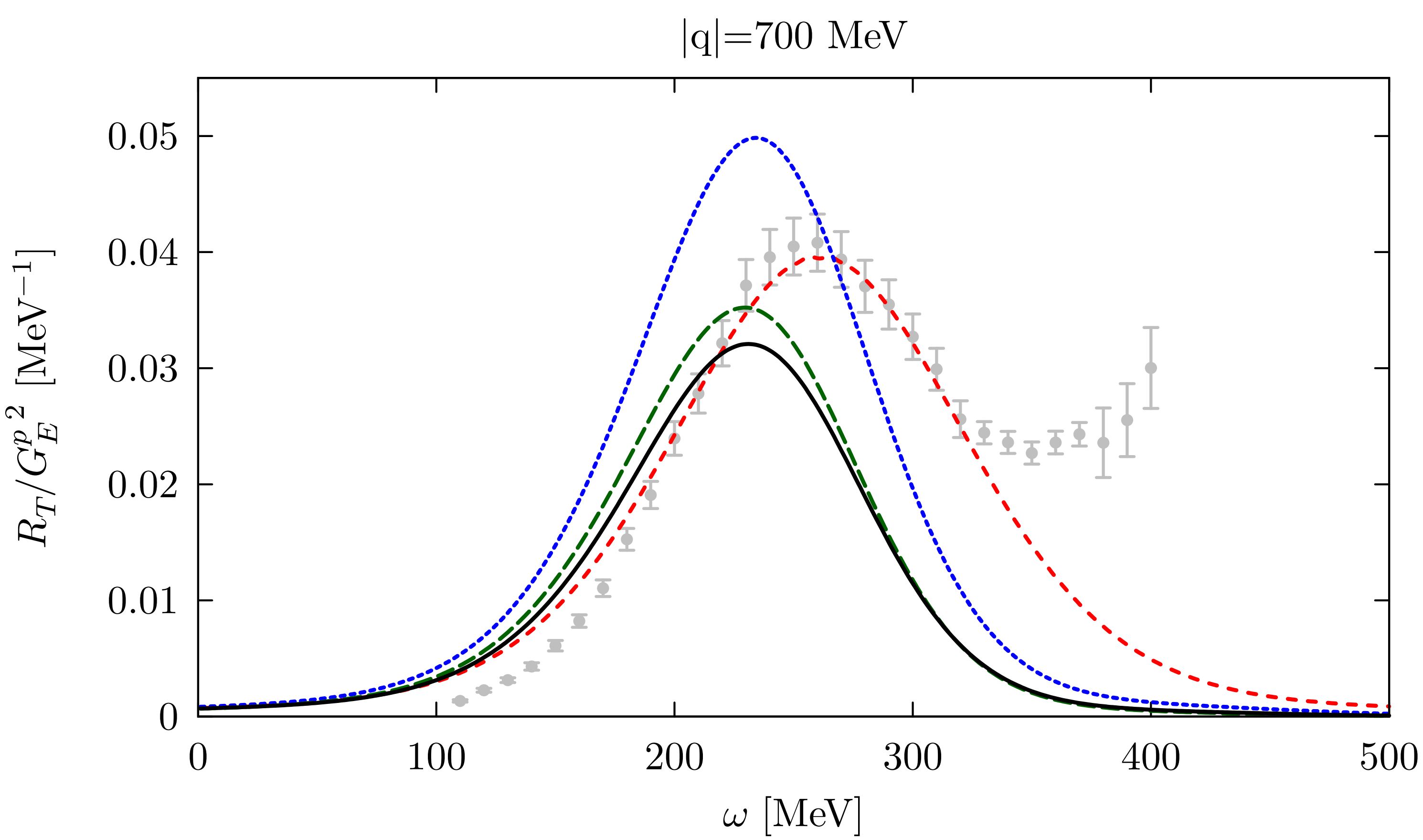
$$\langle j_i^\alpha \rangle = \frac{G_{E,i}}{m}p^\alpha - i\frac{G_{M,i}}{2m}(\mathbf{q}\times\sigma)^\alpha + 0 + \mathcal{O}(1/m^3)$$

- Relativistic corrections turn out to be sub-leading. Higher order terms might be needed in order to improve the accuracy of the results

$$\begin{aligned}\langle j_i^\alpha \rangle =& \Big(G_{E,i} - \tau G_{E,i} + \tau G_{M,i} - \frac{|\mathbf{q}|^2}{4m^2}\Big)\frac{1}{m}p^\alpha + i(\mathbf{p}\times\sigma)^\alpha (G_{M,i}-G_{E,i})\frac{\omega}{2m^2} \\ &+ (\mathbf{q}\times\sigma)^\alpha\Big(-iG_{M,i}\Big(\frac{1}{2m} - \frac{|\mathbf{q}|^2}{16m^3}\Big) + i(G_{M,i}-G_{E,i})\frac{\mathbf{p}\cdot\mathbf{q}}{8m^3}\Big) \\ &+ (\mathbf{p}'\times\sigma)^\alpha\Big(iG_{E,i}\frac{|\mathbf{q}|^2}{8m^3} + i(G_{M,i}-G_{E,i})\frac{\omega}{4m^2}\Big) - (G_{M,i}-G_{E,i})\frac{|\mathbf{q}|^2}{8m^3}\sigma^\alpha(\mathbf{p}\cdot\sigma) \\ &+ \mathcal{O}(1/m^4)\end{aligned}$$

Relativistic effects in a correlated system





- We use the PWIA to investigate the role played by the different relativistic corrections

"phase space"

$$R_\alpha(|\mathbf{q}|,\omega) = \int d^3p\, n(\mathbf{p})\delta(\omega + e(\mathbf{p}) - e(\mathbf{p}')) \times \sum_i \langle \mathbf{p}|j^\dagger_{\alpha,i}(\mathbf{q},\omega)|\mathbf{p}'\rangle\langle\mathbf{p}'|j_{\alpha,i}(\mathbf{q},\omega)|\mathbf{p}\rangle$$

"curr: REL, NR, NR+corr"

- Using a relativistic vs non relativistic kinematics: compare blue and red curves

- Using OLD and NEW currents: compare blue and black curves

- Using OLD and NEW currents: compare blue and black curves

- Using OLD and NEW nonrelativistic with relativistic calculation: compare red and black with green curves

Scaling in the Fermi gas model

- Scaling of the first kind: the nuclear electromagnetic responses divided by an appropriate function describing the single-nucleon physics no longer depend on the two variables ω and **q**, but only upon $\psi(\mathbf{q},\omega)$

Adimensional variables:

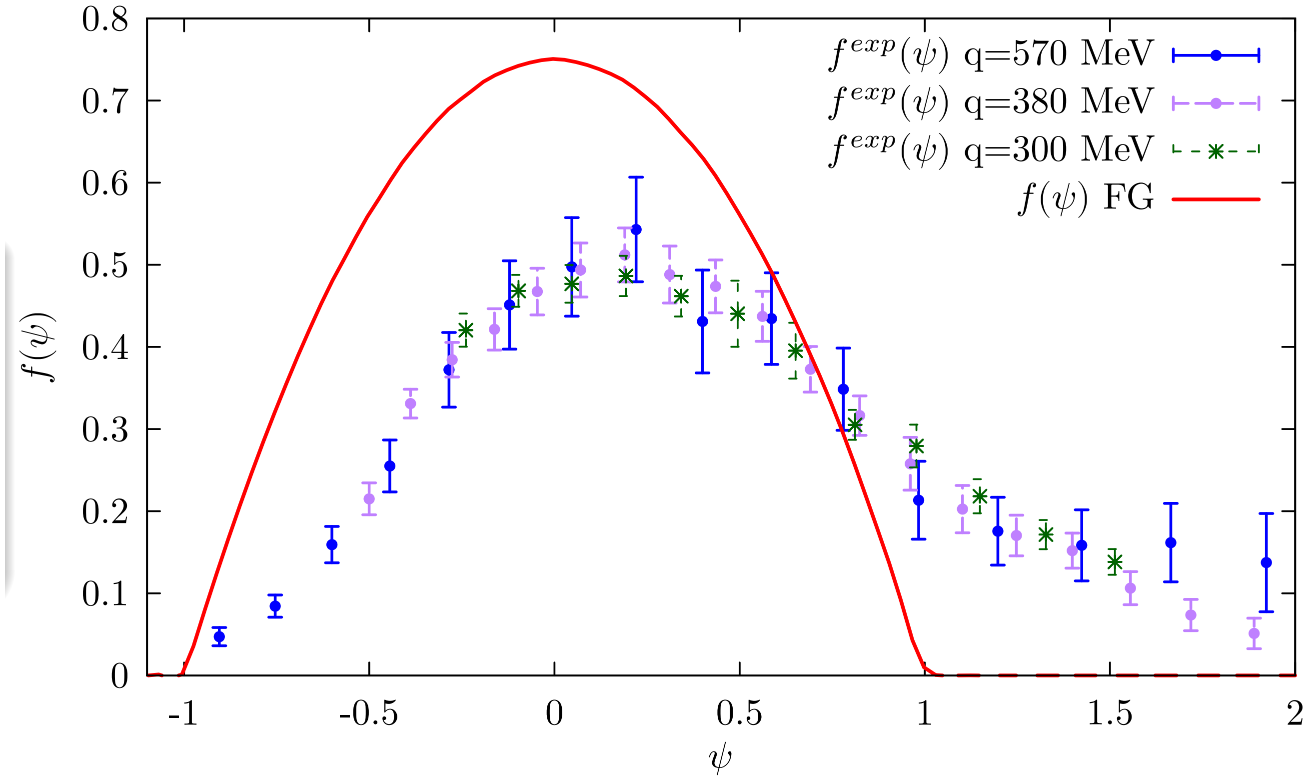
$$\lambda = \omega/2m$$
$$\kappa = |\mathbf{q}|/2m$$
$$\tau = \kappa^2 - \lambda^2$$
$$\eta_F = p_F/m$$
$$\xi_F = \sqrt{p_F^2 + m^2}/m - 1$$

Scaling function:

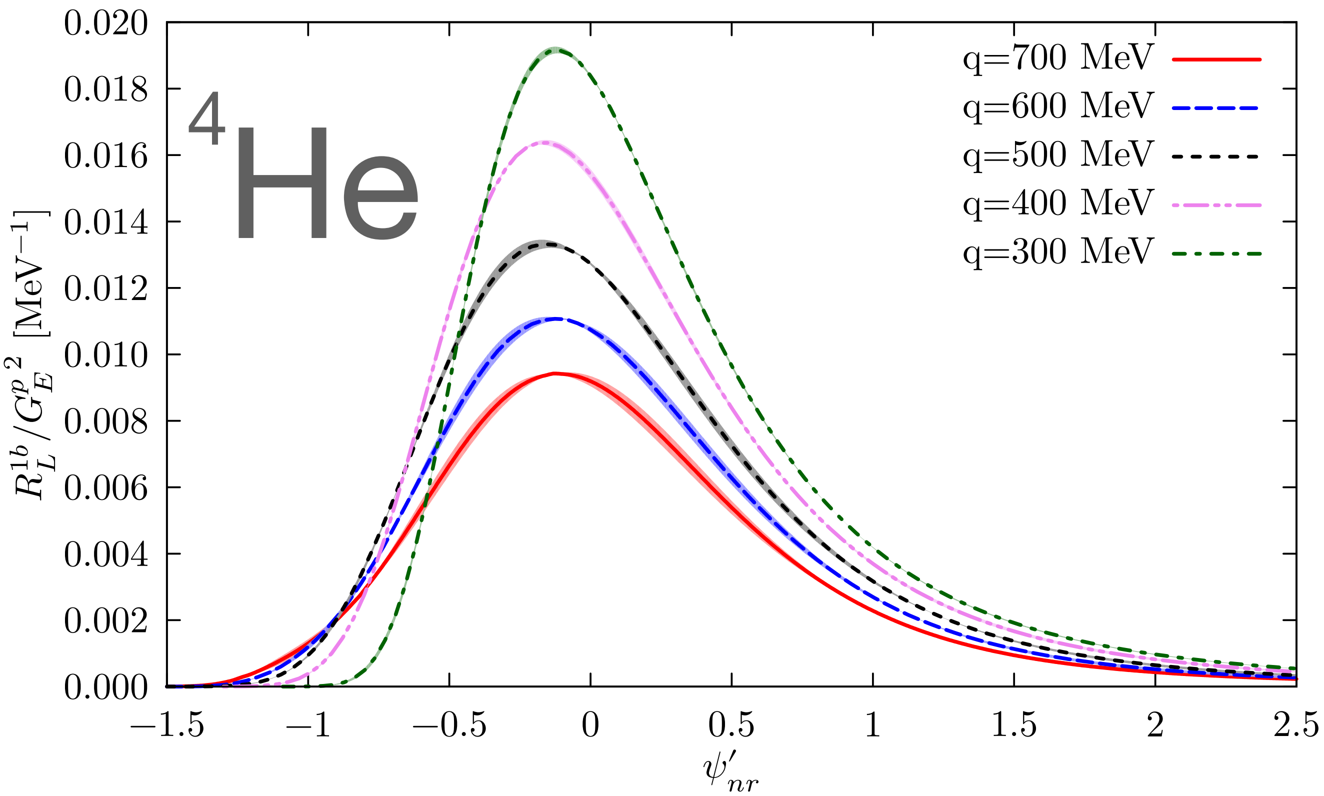
$$\psi = \frac{1}{\xi_F} \frac{\lambda - \tau}{\sqrt{(1+\lambda)\tau + \kappa\sqrt{\tau(1+\tau)}}}$$

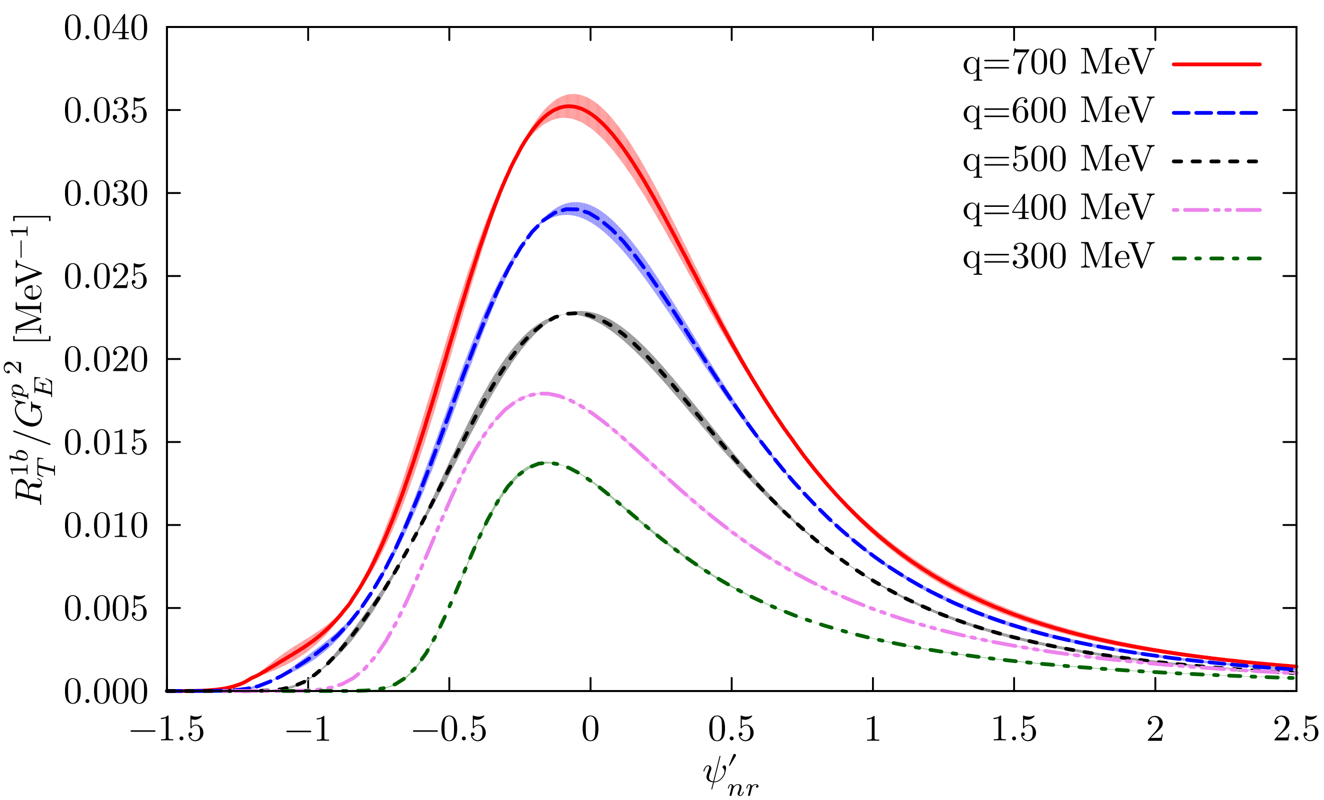
In the FG the L and T responses have the same functional form :

$$R_{L,T} = (1-\psi^2)\theta(1-\psi^2) \times G_{L,T}$$



Scaling as a tool to interpolate the responses





- In order to obtain the GFMC inclusive electron-nucleus cross sections we developed a novel interpolation algorithm based on the scaling of the nuclear responses.

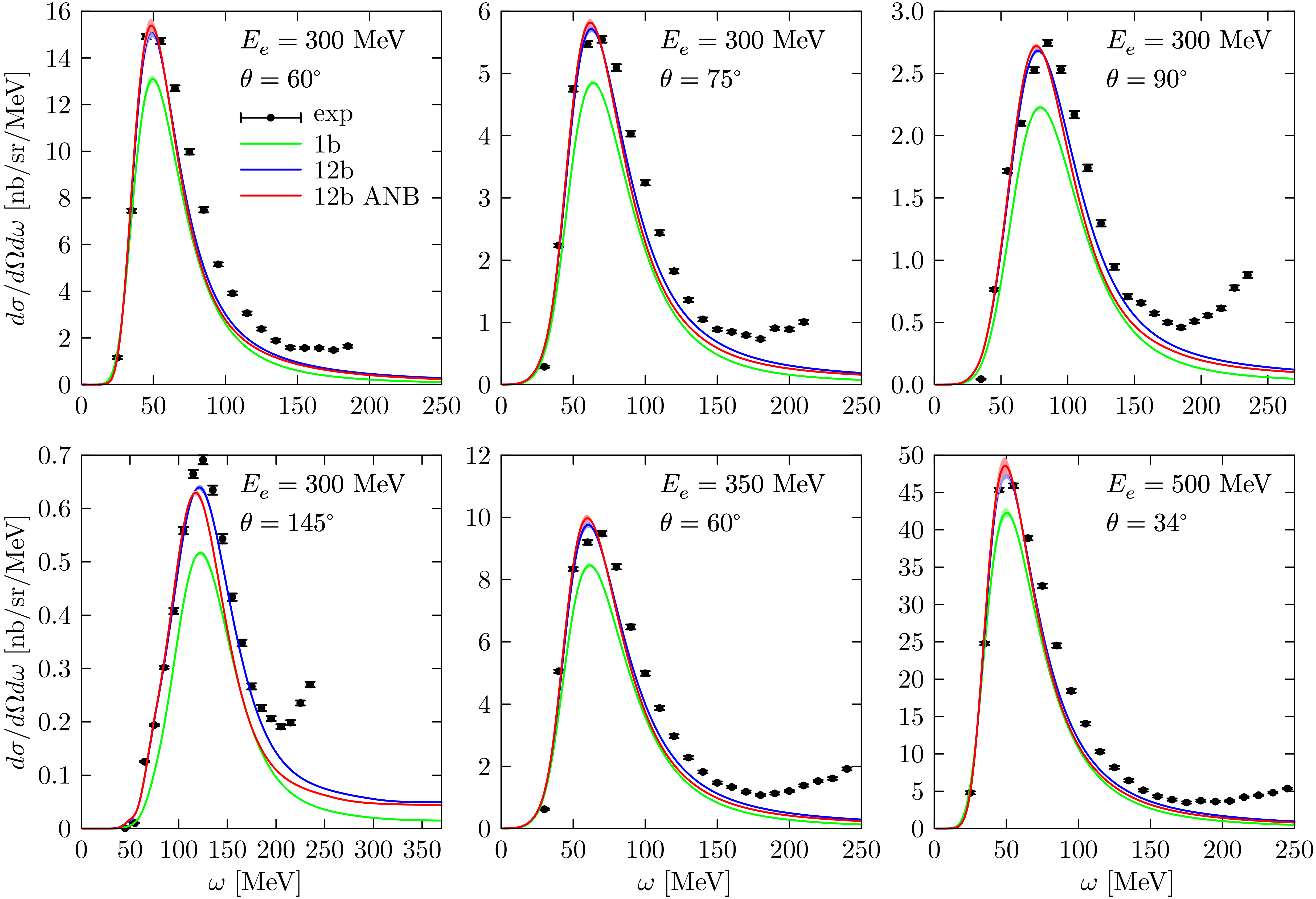
- For a fixed value of E_e and θ_e

$$Q^2 = 4E_e(E_e - \omega)\sin^2\frac{\theta_e}{2}\ ,\ |\mathbf{q}| = \sqrt{Q^2 + \omega^2}$$

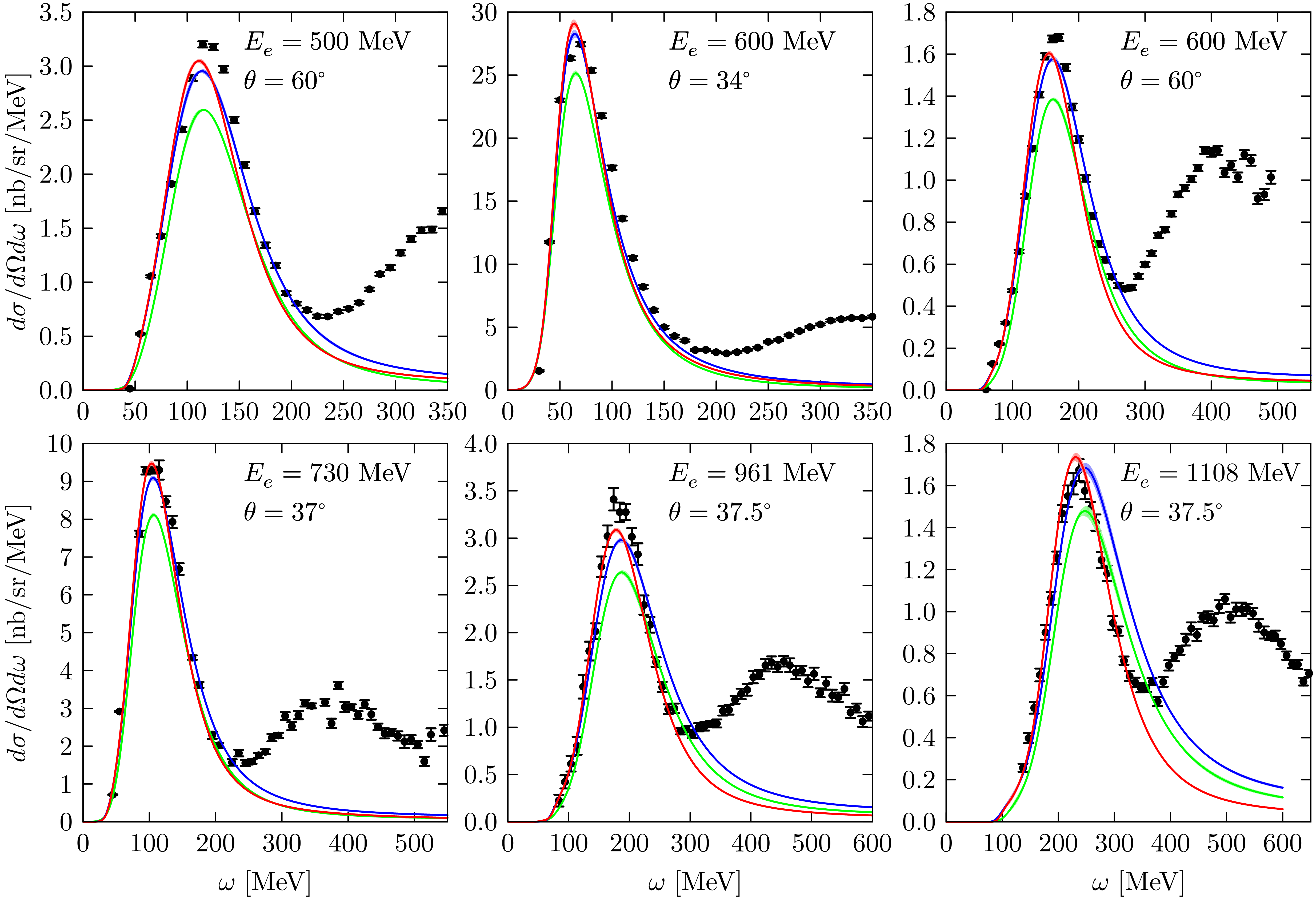
- We first compute ψ'_{nr} then the set of $R_{L,T}(\psi'_{nr}, \mathbf{q})$ is interpolated in |**q**|.

- For a given value of ψ'_{nr} the curves corresponding to different values of |**q**| are almost perfectly aligned and monotonic functions of |**q**|. Using the concept of scaling, largely improves the accuracy of the interpolation procedure and reduces the computational cost

Scaling as a tool to interpolate the responses



Scaling as a tool to interpolate the responses



Summary and conclusions

- The Correlated Basis Function approach :

Accurate calculations are available for symmetric nuclear matter

The extension to both low- and medium-mass nuclei has been performed using the Local Density Approximation

Using the generalized factorization ansatz we are able do describe all the different reaction mechanisms contributing to the lepton-nucleus scattering cross sections

Final State Interactions are included in an approximate fashion

- The Green's Function Monte Carlo approach:

Accurate results for electroweak responses of ^{4}He and ^{12}C

The main limitations of this method comes from its nonrelativistic nature and its computational cost

The two- fragment model, suitable for realistic models of nuclear dynamics, has been employed to account for relativistic kinematics. Double-differential cross sections has be extracted through an accurate interpolation of the response functions.

Back up slides

Relativistic aspects of nuclear dynamics

LAB:

$$P_i^{fr} = 0$$
$$P_f^{fr} = \mathbf{q}^{fr}$$
$$p_{Nf}^{fr} = \mathbf{q}$$
$$\mathbf{q}^{fr} = \mathbf{q}$$
$$\beta = \frac{q}{M_0 + \omega}$$

- In the LAB frame, the momentum of the active nucleon is the largest

Anti-LAB:

$$P_i^{fr} = -\mathbf{q}^{ALAB}$$
$$P_f^{fr} = 0$$
$$p_{Nf}^{fr} = \frac{A-1}{A}\mathbf{q}^{ALAB}$$
$$\beta = \frac{q^{ALAB}}{M_0 + \omega}$$

- The momentum of the active nucleon is $\approx q$

Breit:

$$P_i^{fr} = -\frac{\mathbf{q}^B}{2}$$
$$P_f^{fr} = \frac{\mathbf{q}^B}{2}$$
$$p_{Nf}^{fr} = \frac{2A-1}{2A}\mathbf{q}^B$$
$$\beta = \frac{q^B}{2M_0 + \omega}$$

- The Breit frame minimizes the sum of the center of mass kinetic energies of the initial and final state

Active nucleon Breit:

$$P_i^{fr} = -\frac{A\mathbf{q}^{ANB}}{2}$$
$$P_f^{fr} = -\frac{(A-2)\mathbf{q}^{ANB}}{2}$$
$$p_{Nf}^{fr} = \frac{\mathbf{q}^{ANB}}{2}$$
$$\beta = \frac{q^{ANB}}{2M_0/A + \omega}$$

- ω^{ANB} at the QE peak is 0. This applies both to the relativistic and non relativistic case

Extension of the factorization scheme to two-nucleon emission amplitude

$$|X\rangle \longrightarrow |\mathbf{p}\ \mathbf{p}'\rangle \otimes |n_{(A-2)}\rangle = |n_{(A-2)}; \mathbf{p}\ \mathbf{p}'\rangle \ ,$$

We can introduce the two-nucleon Spectral Function...

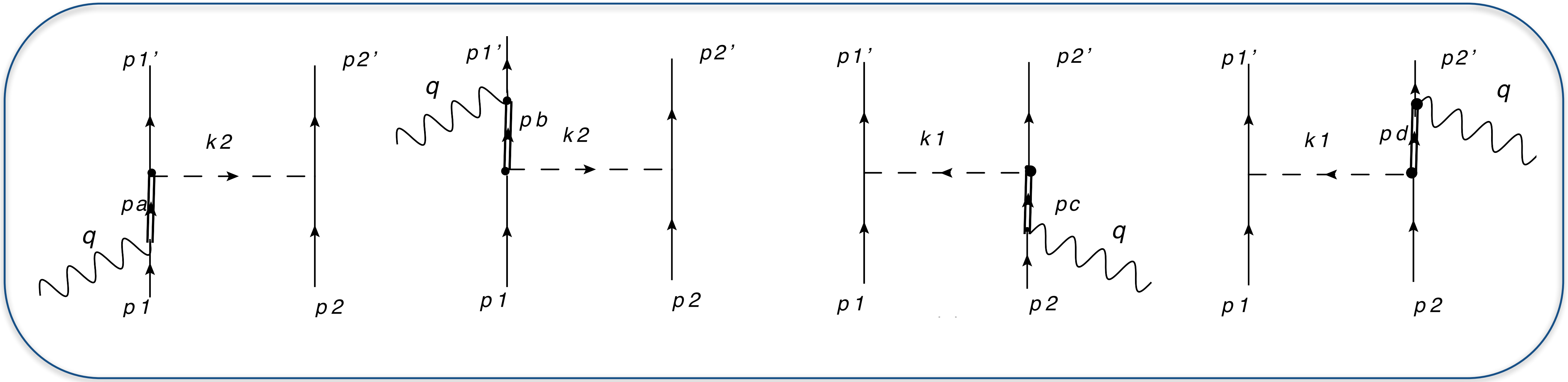
$$P(\mathbf{k}, \mathbf{k}', E) = \sum_n |\langle n_{(A-2)}; \mathbf{k}\ \mathbf{k}'|0\rangle|^2 \delta(E + E_0 - E_n)$$

probability of removing two nucleons leaving the A-2 system with energy E

The pure 2-body & the interference contribution to the hadron tensor read

$$W^{\mu\nu}_{2p2h,22} \propto \int d^3k d^3k' d^3p d^3p' \int dE\ P_{2h}(\mathbf{k}, \mathbf{k}', E) \langle \mathbf{k}\mathbf{k}'|j^{\mu}_{12}|\mathbf{p}\mathbf{p}'\rangle \langle \mathbf{p}\mathbf{p}'|j^{\nu}_{12}|\mathbf{k}\mathbf{k}'\rangle$$

$$W^{\mu\nu}{}_{2p2h,12} \propto \int d^3k\ d^3\xi\ d^3\xi'\ d^3h\ d^3h' d^3p\ d^3p' \phi^{hh'\,*}_{\xi\xi'} \langle \mathbf{p}, \mathbf{p}'|j^{\nu}_{12}|\boldsymbol{\xi}, \boldsymbol{\xi}'\rangle$$
$$\left[\Phi^{hh'p'}_{k} \langle \mathbf{k}|j^{\mu}_{1}|\mathbf{p}\rangle + \Phi^{hh'p}_{k} \langle \mathbf{k}|j^{\mu}_{2}|\mathbf{p}'\rangle\right]$$



The Rarita-Schwinger (RS) expression for the Δ propagator reads

$$S^{\beta\gamma}(p, M_\Delta) = \frac{\not{p} + M_\Delta}{p^2 - M_\Delta^2}\left(g^{\beta\gamma} - \frac{\gamma^\beta\gamma^\gamma}{3} - \frac{2p^\beta p^\gamma}{3M_\Delta^2} - \frac{\gamma^\beta p^\gamma - \gamma^\gamma p^\beta}{3M_\Delta}\right)$$

WARNING

If the condition $p_\Delta^2 > (m_N + m_\pi)^2$ the real resonance mass has to be replaced by $M_\Delta \longrightarrow M_\Delta - i\Gamma(s)/2$ where $\Gamma(s) = \frac{(4f_{\pi N\Delta})^2}{12\pi m_\pi^2}\frac{k^3}{\sqrt{s}}(m_N + E_k)$.

Hadronic monopole form factors

$$F_{\pi NN}(k^2) = \frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - k^2}$$

$$F_{\pi N\Delta}(k^2) = \frac{\Lambda_{\pi N\Delta}^2}{\Lambda_{\pi N\Delta}^2 - k^2}$$

and the EM ones

$$F_{\gamma NN}(q^2) = \frac{1}{(1 - q^2/\Lambda_D^2)^2} \; ,$$

$$F_{\gamma N\Delta}(q^2) = F_{\gamma NN}(q^2)\left(1 - \frac{q^2}{\Lambda_2^2}\right)^{-1/2}\left(1 - \frac{q^2}{\Lambda_3^2}\right)^{-1/2}$$

where $\Lambda_\pi = 1300$ MeV, $\Lambda_{\pi N\Delta} = 1150$ MeV, $\Lambda_D^2 = 0.71\mathrm{GeV}^2$, $\Lambda_2 = M + M_\Delta$ and $\Lambda_3^2 = 3.5\ \mathrm{GeV}^2$.