Neutrino interactions in dense matter and implications for astrophysics

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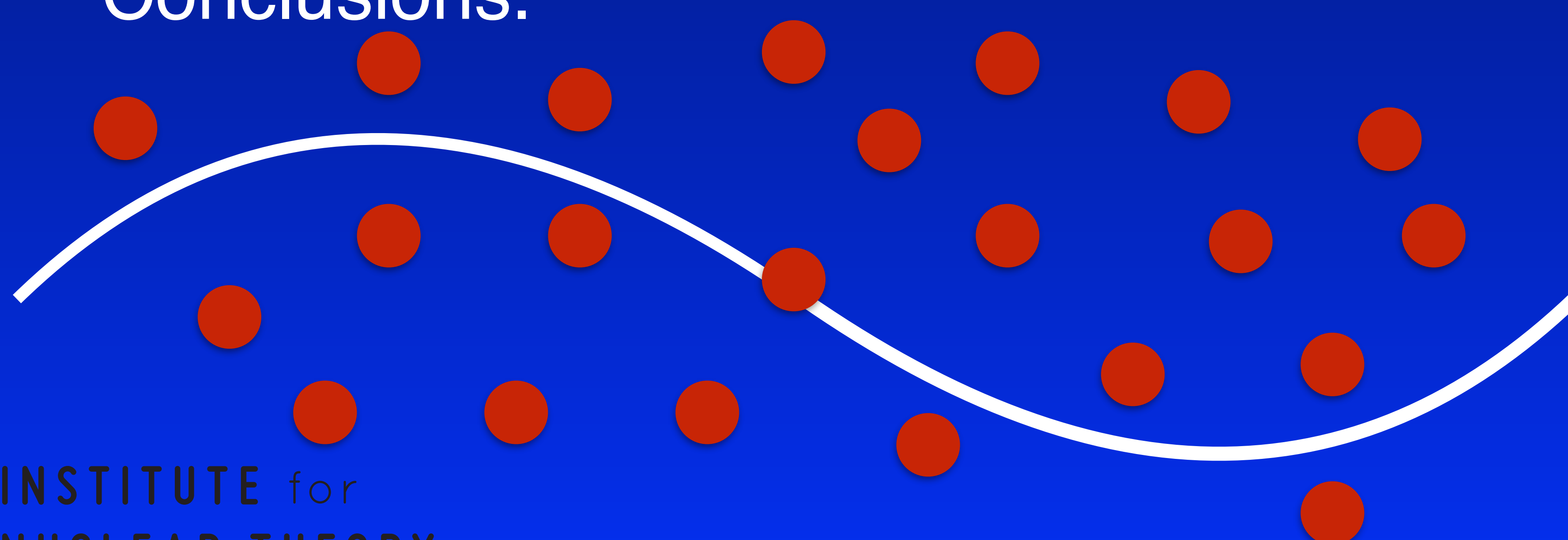
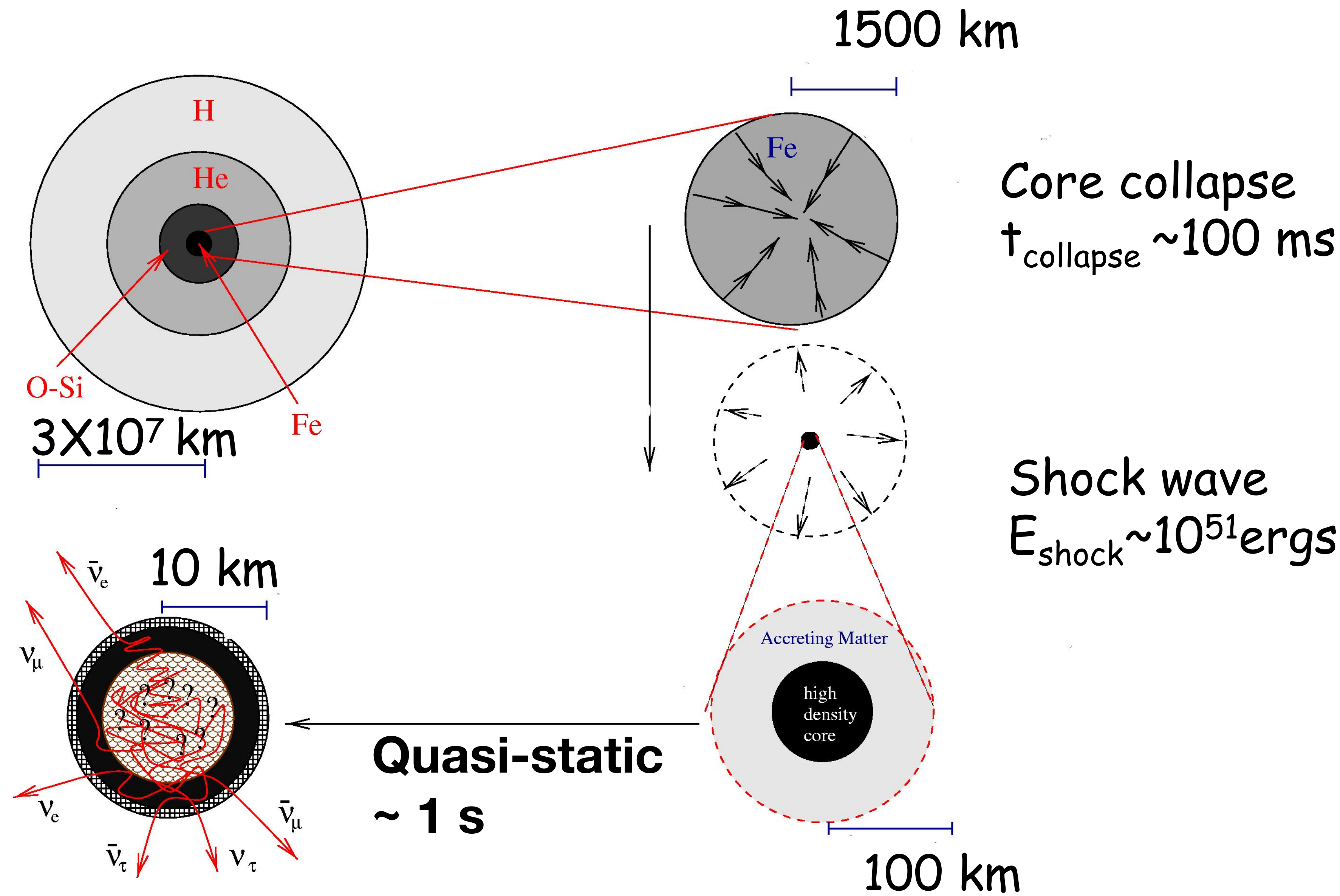
- Motivation
- Preliminaries: linear response & sum rules
- Neutrino scattering at:
  - 10^{12} g/cm^3 (neutrino-sphere)
  - 10^{13} g/cm^3 (role of nuclei and pasta)
  - 10^{14} g/cm^3 (spin response of nuclear matter)
- Conclusions.



INSTITUTE for NUCLEAR THEORY

Core Collapse Supernova



Energy radiated, mostly in neutrinos : $E_{\rm SN} \sim \dfrac{3GM_{\rm pns}^2}{5r_{\rm NS}} \approx 3\times 10^{53}\,{\rm erg}\left(\dfrac{M_{\rm pns}}{M_\odot}\right)^2\left(\dfrac{r_{NS}}{12\,{\rm km}}\right)^{-1}$

Lepton number radiated, also in neutrinos : $N_L \approx 3.4\times 10^{56}\left(\dfrac{M_{\rm pns}}{1.4\,M_\odot}\right)$

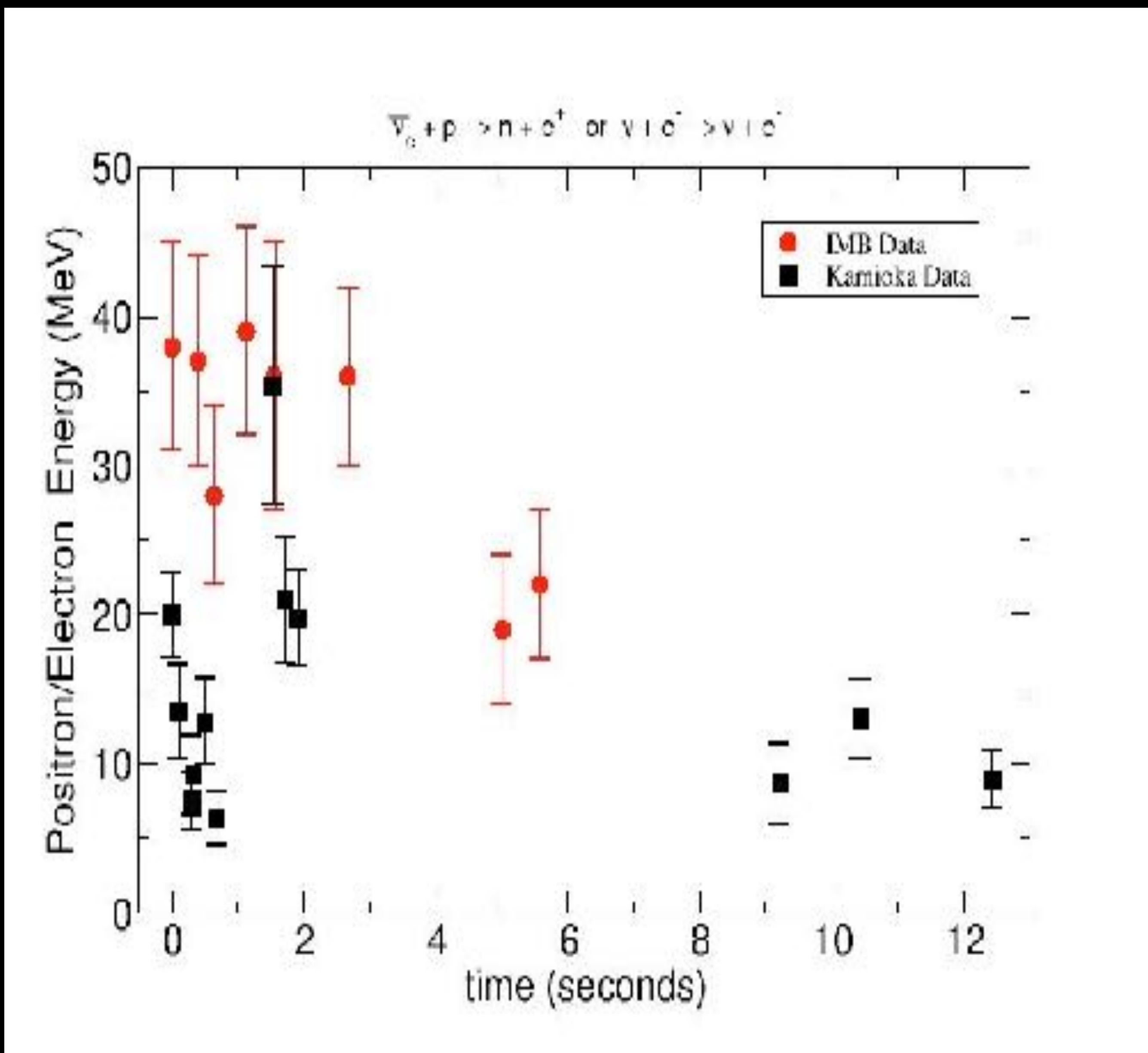
Timescale and energy spectrum of neutrinos is set by neutrino interactions in the hot newly born neutron star

Supernova Neutrinos

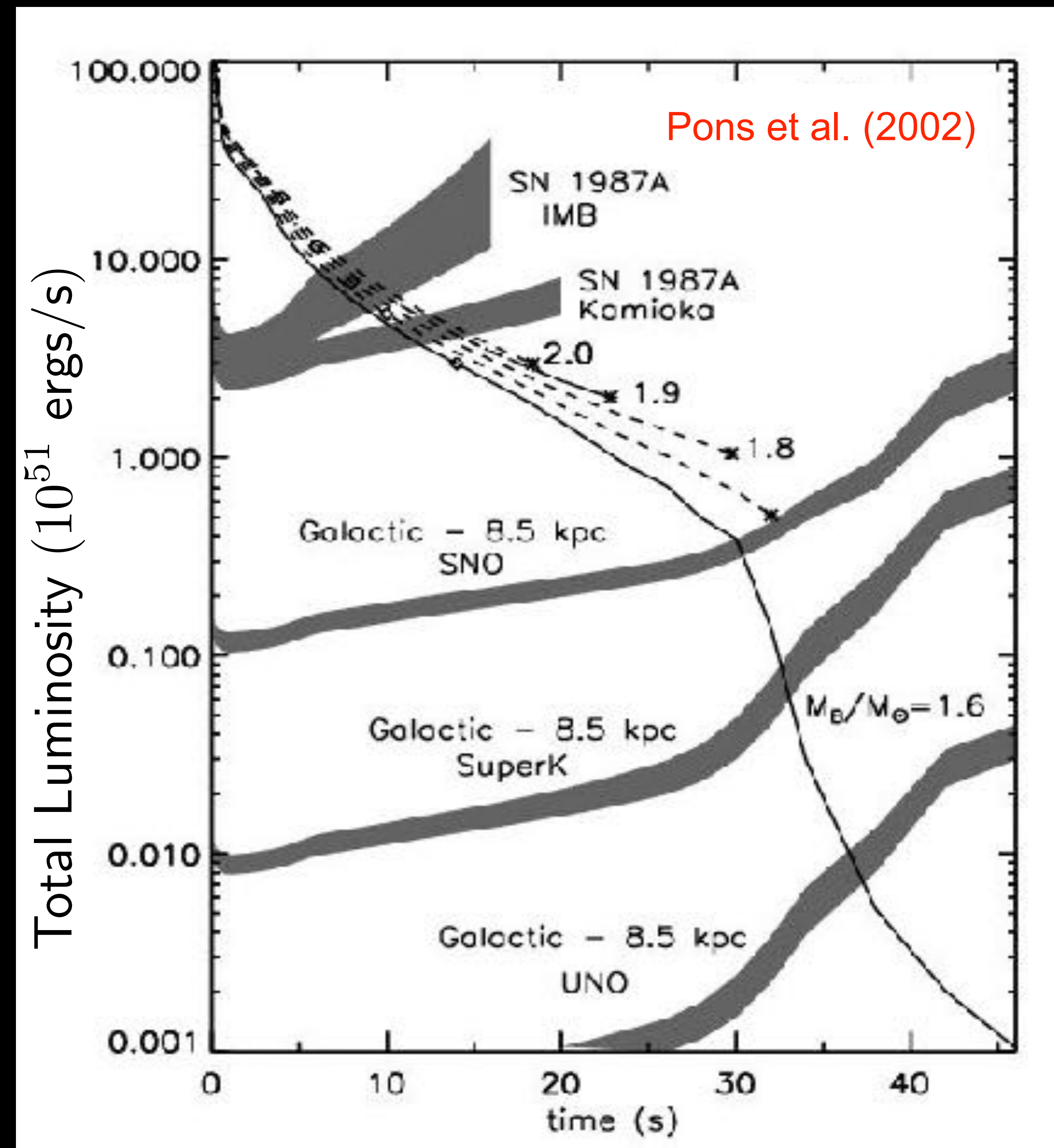
3×10^{53} ergs = $10^{58} \times 20$ MeV Neutrinos

SN 1987a: ~ 20 neutrinos in support of supernova theory

SuperK can detect ~10,000 neutrinos from a galactic supernova at 10 kpc



$$\frac{dN_{\text{detect}}}{dt} \simeq \frac{\sigma_{\text{ref}} \times n_p \times M_{\text{tons}}}{4\pi D^2} \frac{E_\nu^2}{m_e^2} \frac{dN_{emit}}{dt}$$



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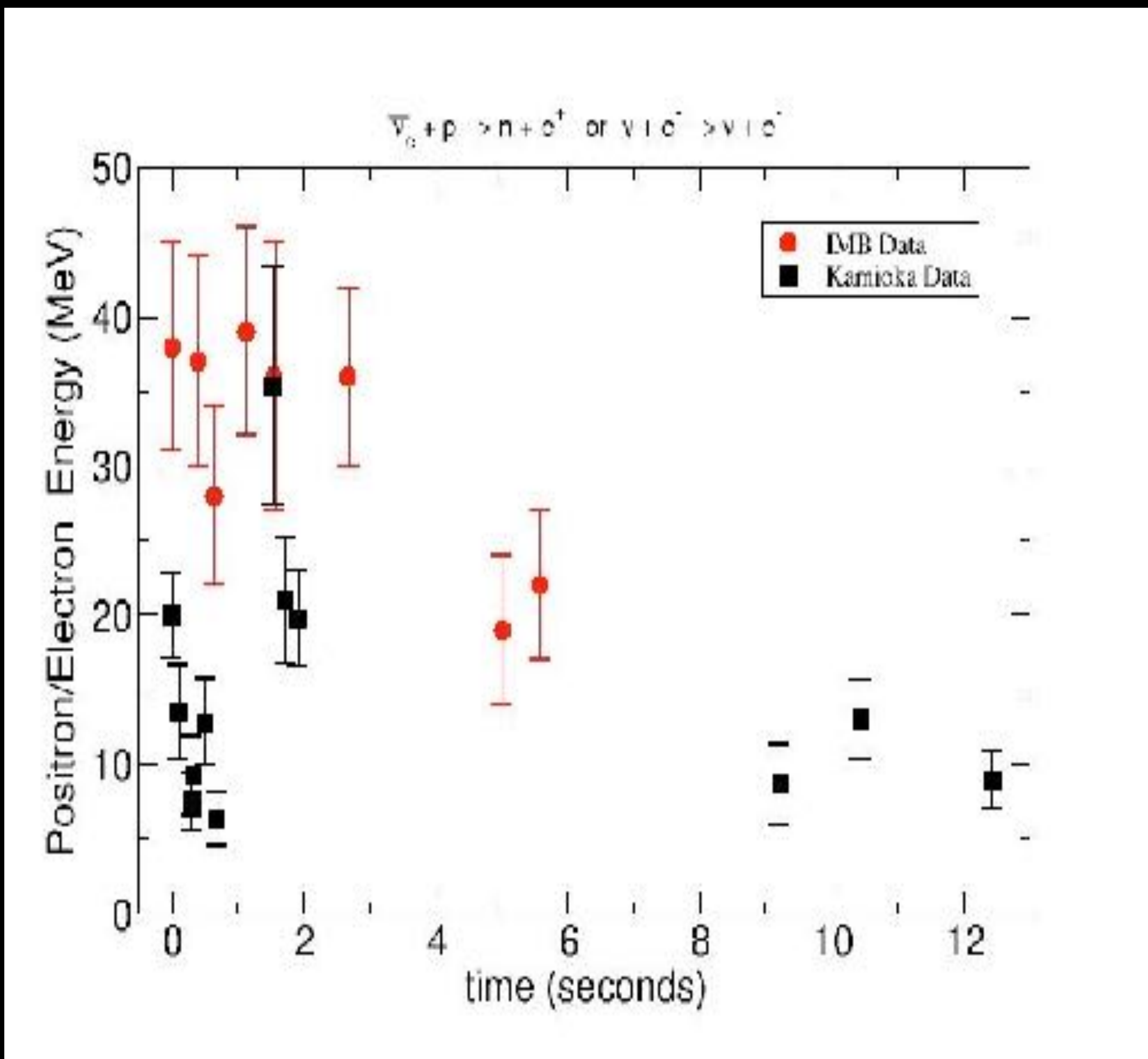

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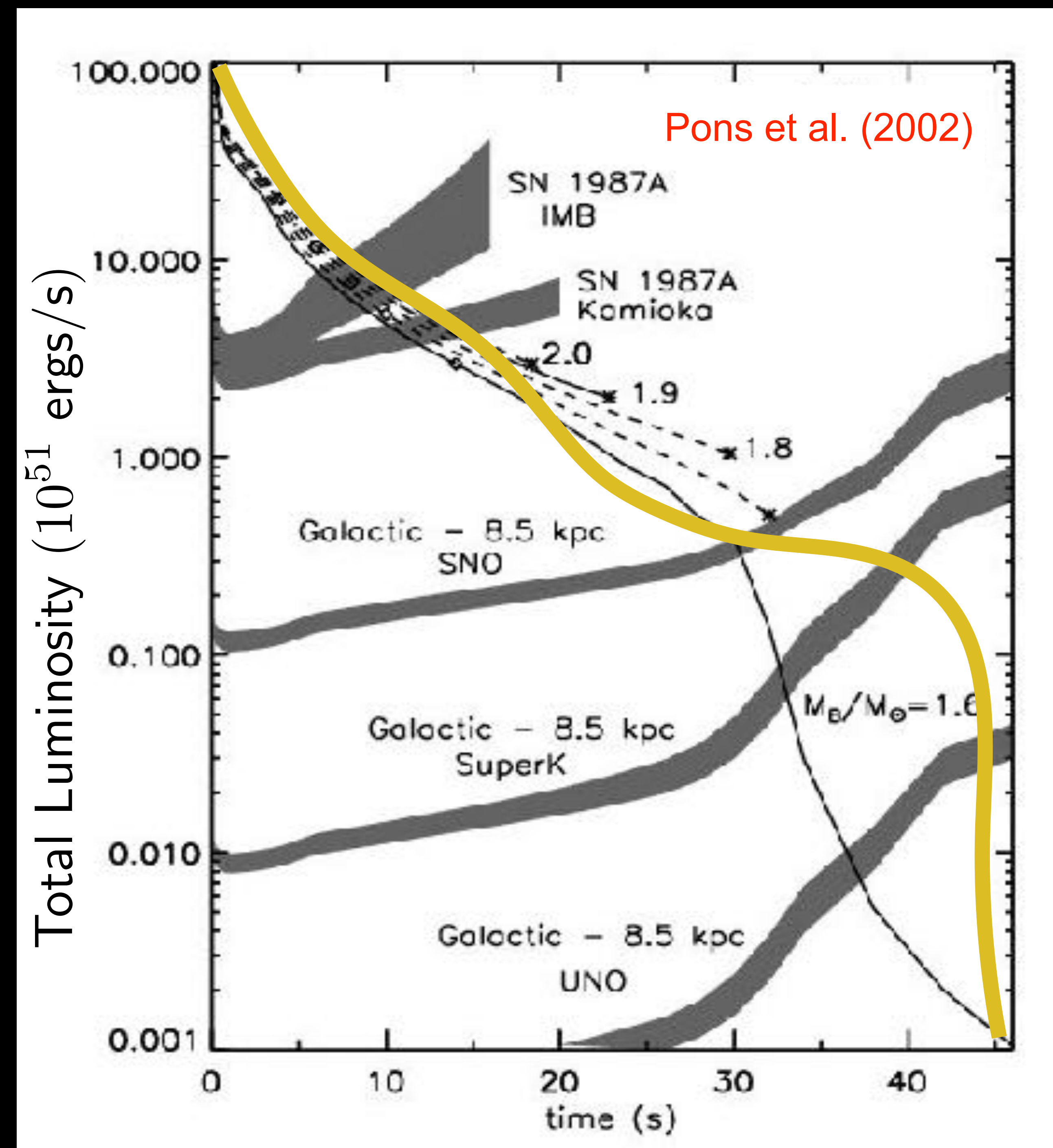

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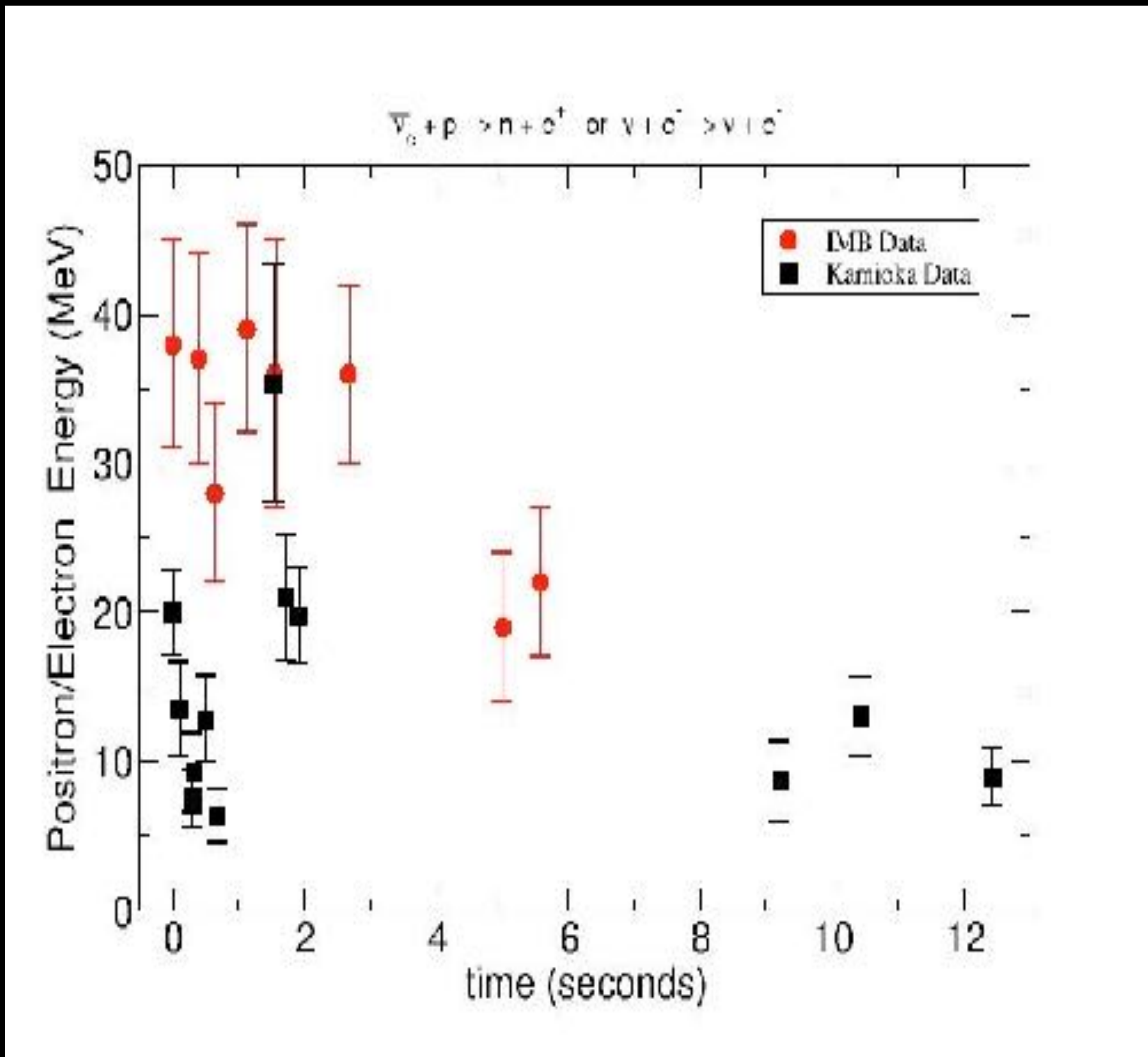

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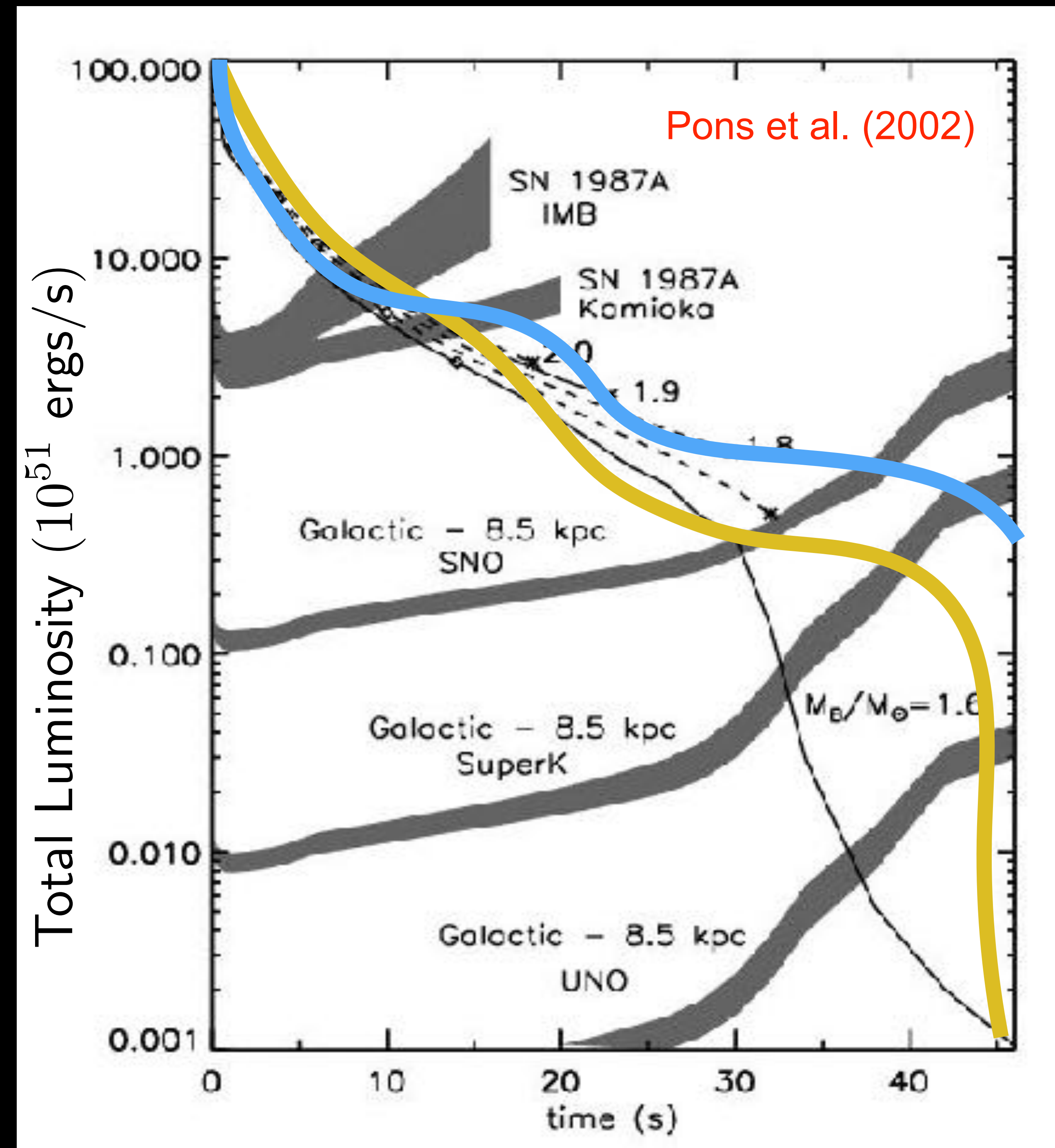

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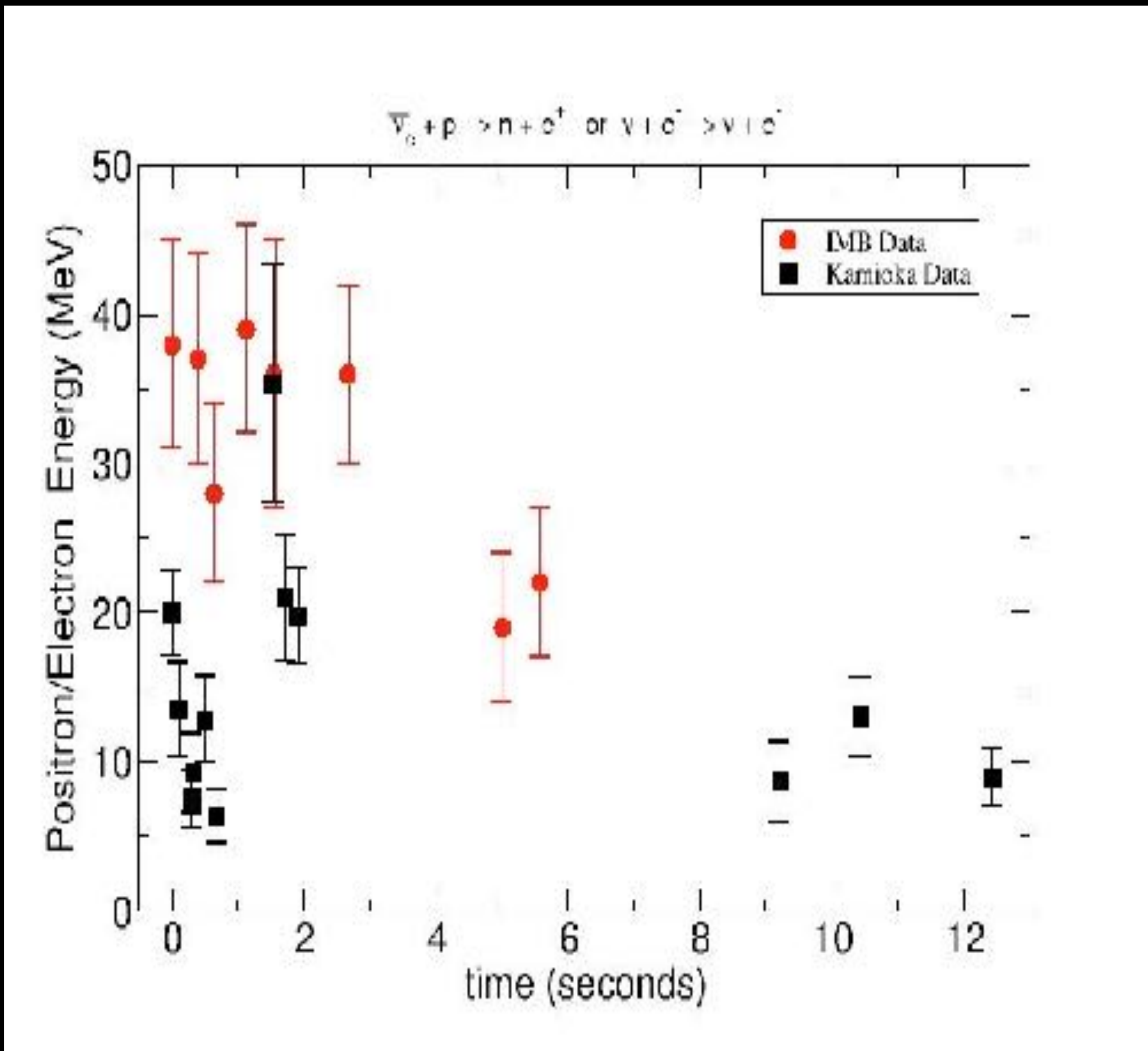

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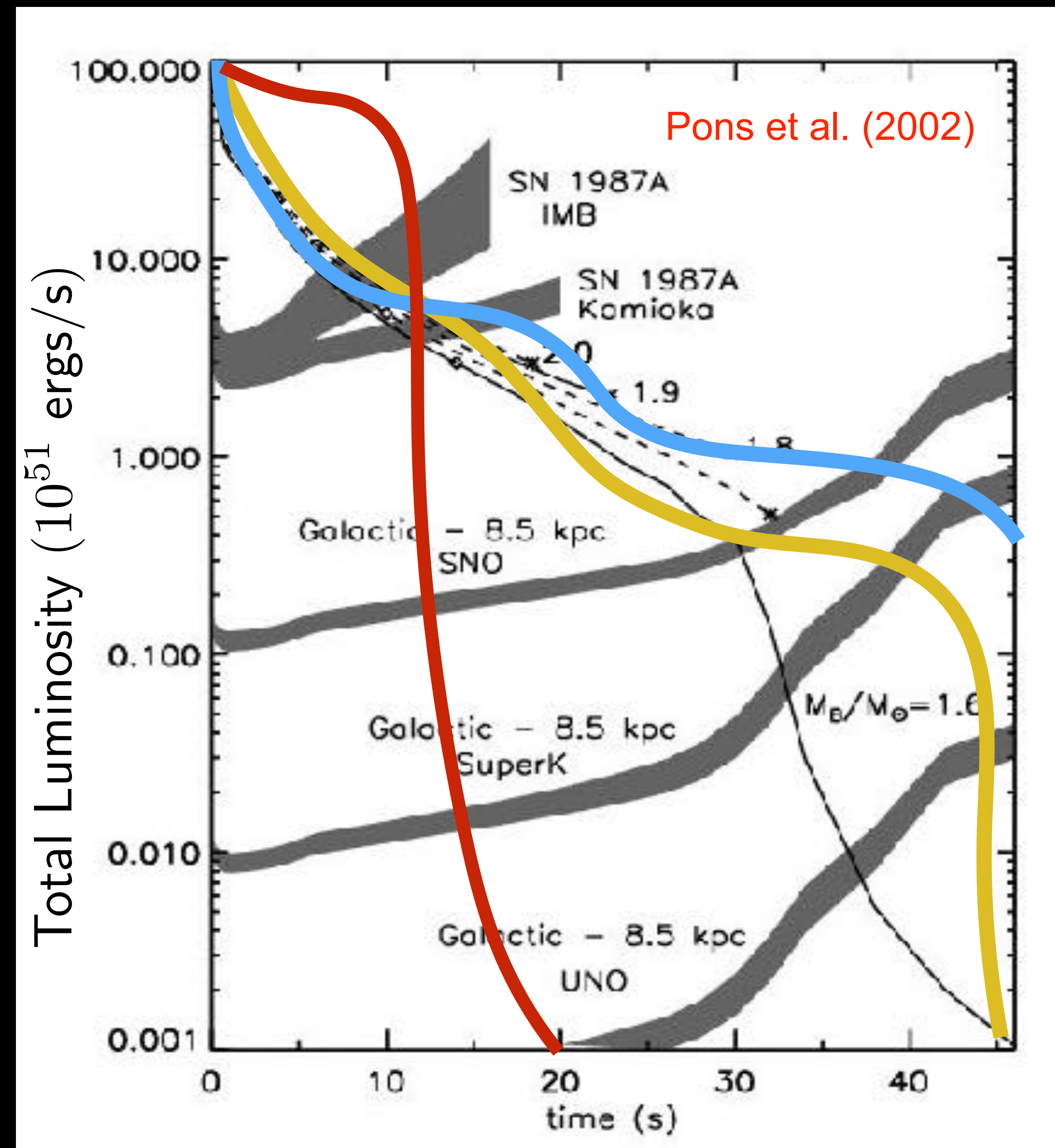

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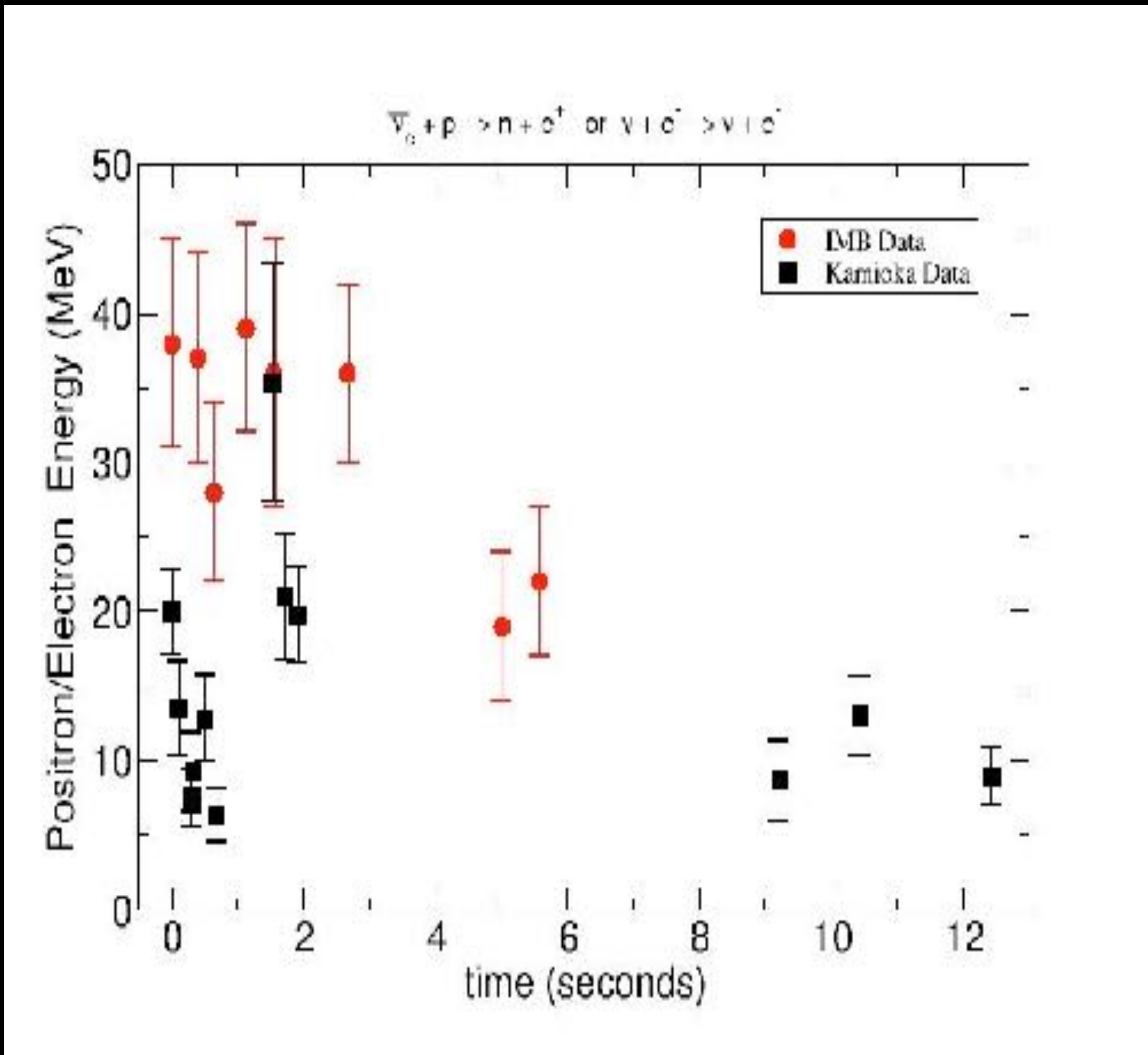

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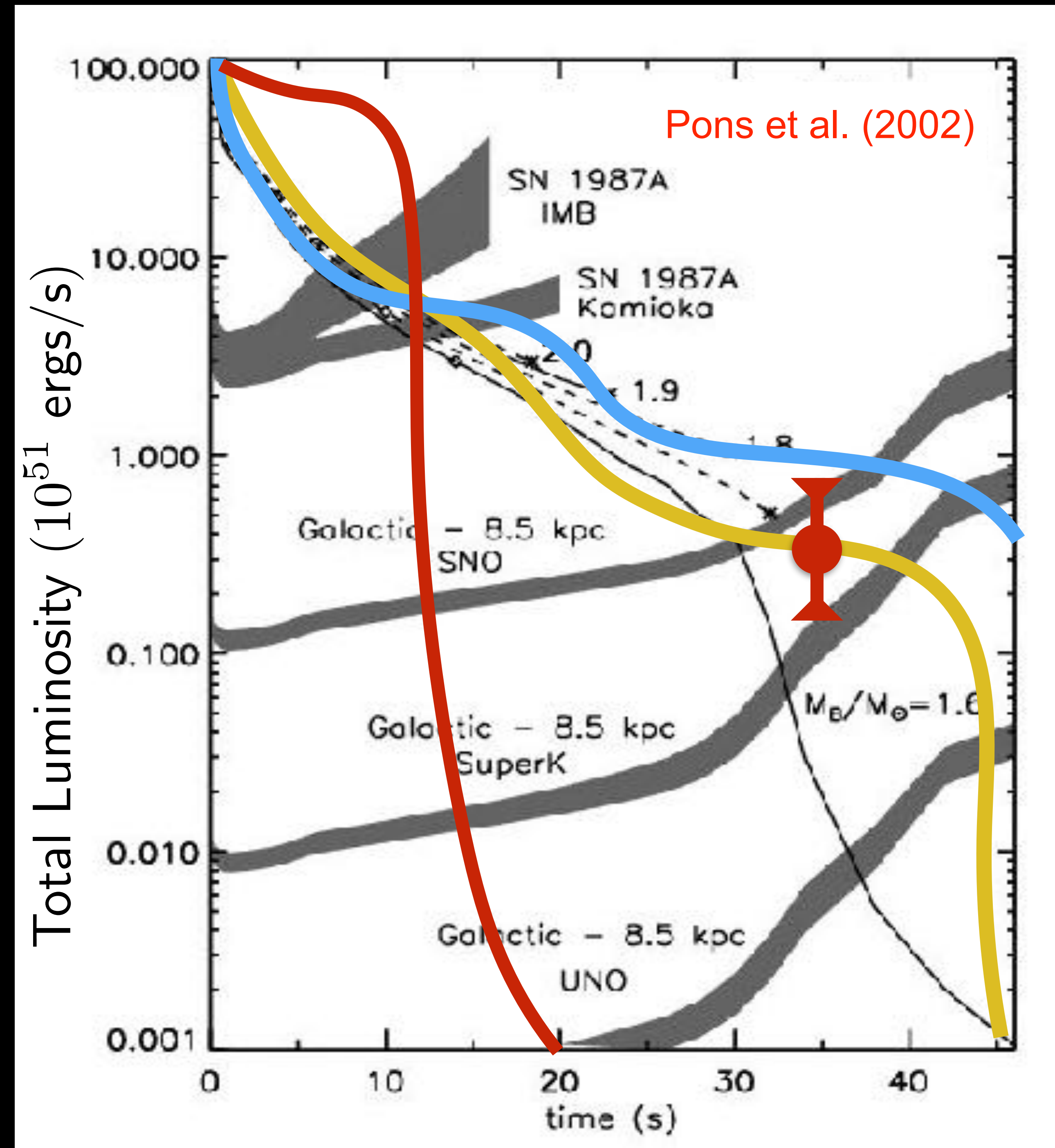
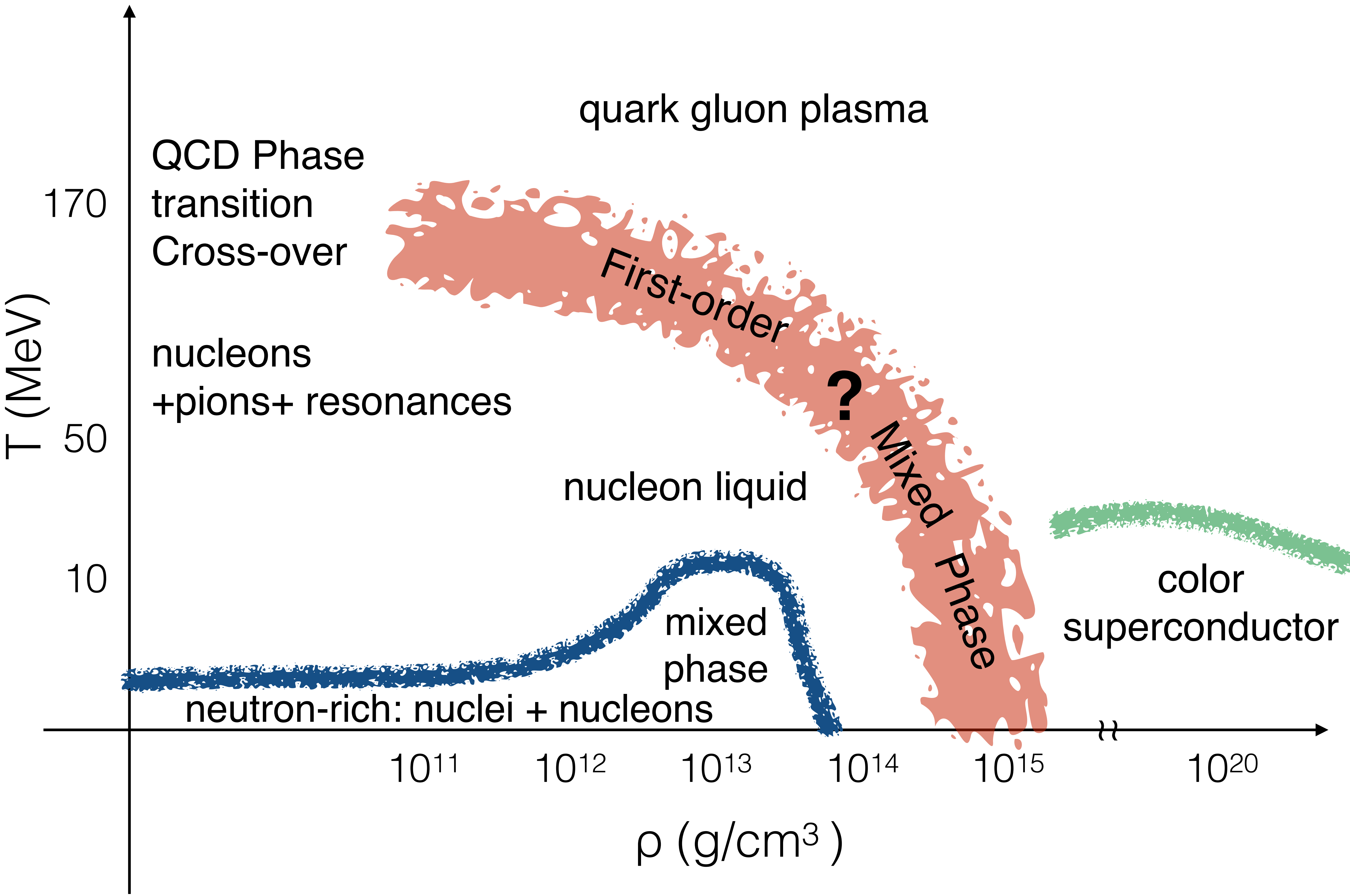
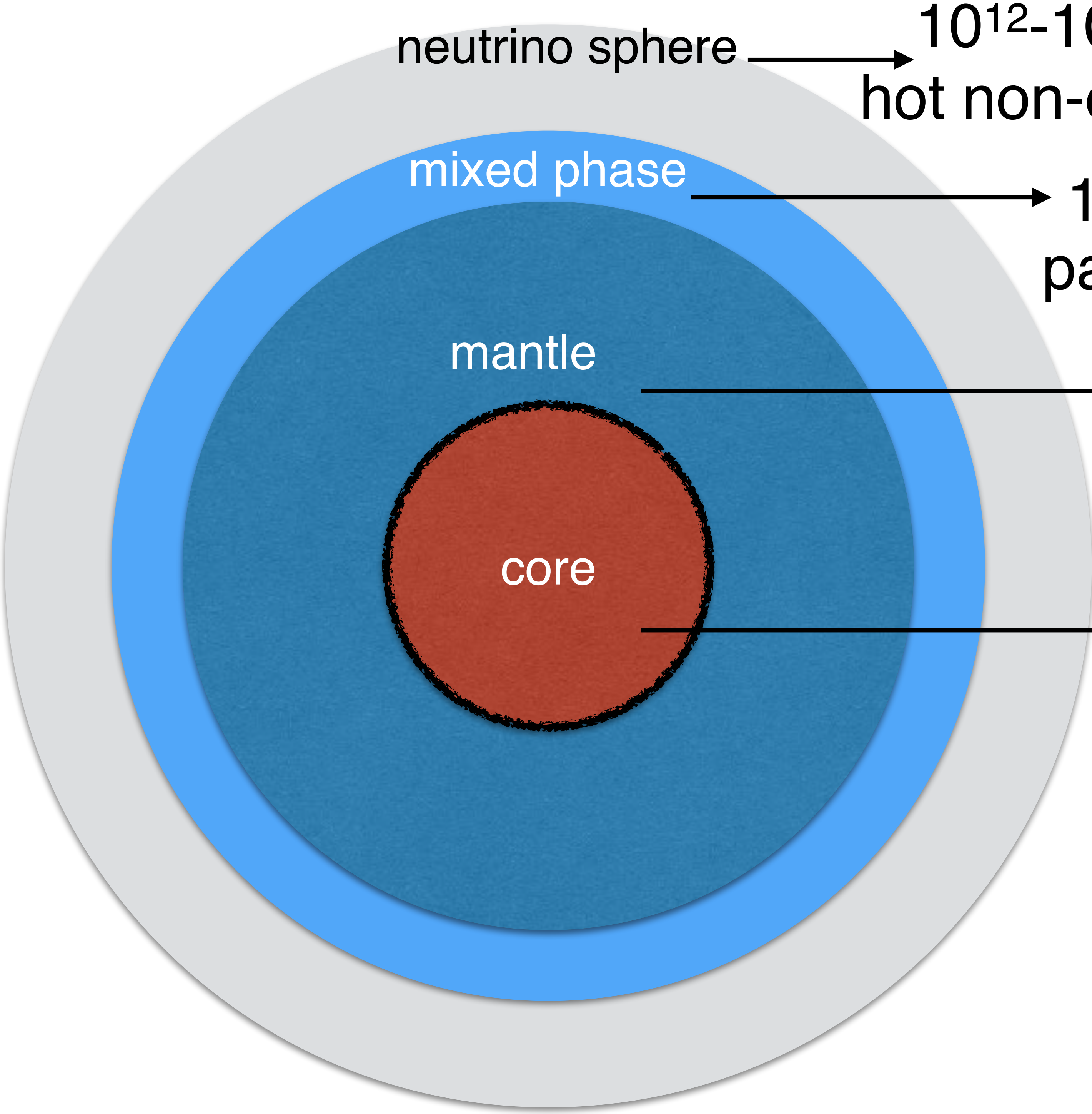


Phase Diagram of Hot and Dense Matter



Neutrino Scattering in Hot Neutron Stars



10^{12}-10^{13} g/cm^3 and T~ 1-20 MeV
hot non-degenerate nucleon gas + few nuclei.

10^{13}-10^{14} g/cm^3 and T~ 5-50 MeV
partially degenerate nucleons + nuclei.

10^{14} - 5x10^{14} g/cm^3 and
T~ 5-50 MeV
nuclear matter - Fermi liquid

> 5x10^{14} g/cm^3

Many possibilities:
Hyperons, pion/kaon
condensation, quark
matter with and without
color superconductivity
..

Neutrino Interactions in Dense Matter

Low energy Lagrangian: $\mathcal{L} = \frac{G_F}{\sqrt{2}} \, l_\mu \; j^\mu$ $\qquad l_1 + N_2 \rightarrow l_3 + N_4$

Absorption:	$l_\mu^{cc} = \bar{l}\gamma_\mu(1-\gamma_5)\nu_l$	$j^\mu_{cc} = \bar{\Psi}_p \left(\gamma^\mu(g_V - g_A\gamma_5) + F_2 \frac{i\sigma^{\mu\alpha}q_\alpha}{2M}\right) \Psi_n$
Scattering:	$l_\mu^{nc} = \bar{\nu}\gamma_\mu(1-\gamma_5)\nu$	$j^\mu_{nc} = \bar{\Psi}_i \; (\gamma^\mu(C_V^i - C_A^i\gamma_5) + F_2^i \frac{i\sigma^{\mu\alpha}q_\alpha}{2M}) \; \Psi_i$

Rate: $$\frac{d\Gamma(E_1)}{dE_3 d\mu_{13}} = \frac{G_F^2}{32\pi^2}\frac{p_3}{E_1}(1 - f_3(E_3))L_{\mu\nu}\; \mathcal{S}^{\mu\nu}(q_0, q)$$

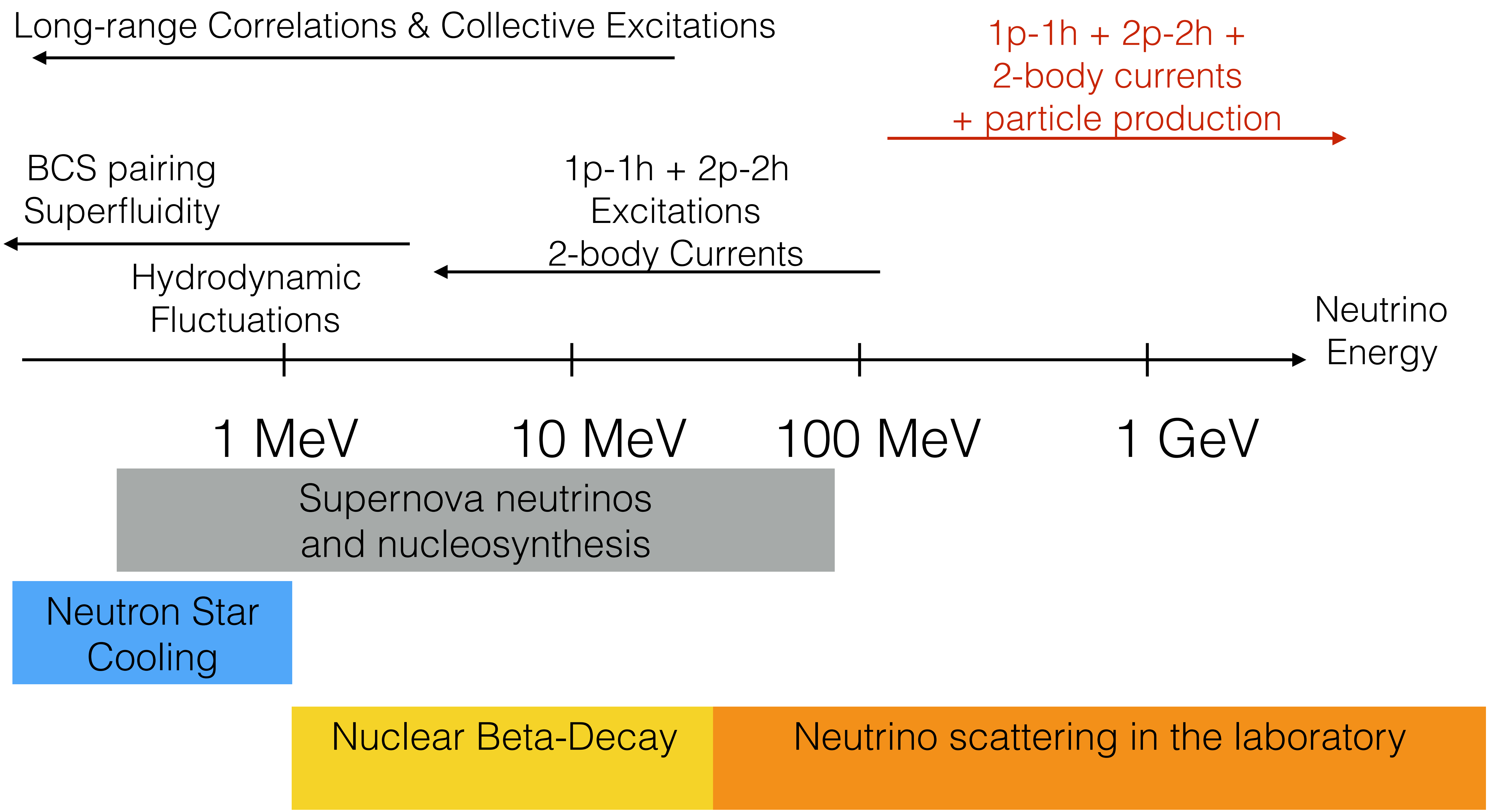
Dynamic structure function: $$\mathcal{S}^{\mu\nu}(q_0,q) = \frac{-2 \;\mathrm{Im}\; \mathbf{\Pi}^{\mu\nu}(q_0,q)}{1-\exp\left(-(q_0+\Delta\mu)/T\right)}$$

Current-current correlations functions: $$\mathbf{\Pi}^{\mu\nu}(q_0,q) = -i\int dt\; d^3x\; \theta(t)\; e^{i(q_0 t - \vec{q}\cdot\vec{x})}\langle\; |[j_\mu\;(\vec{x},t), j_\nu(\vec{0},0)]|\;\rangle$$

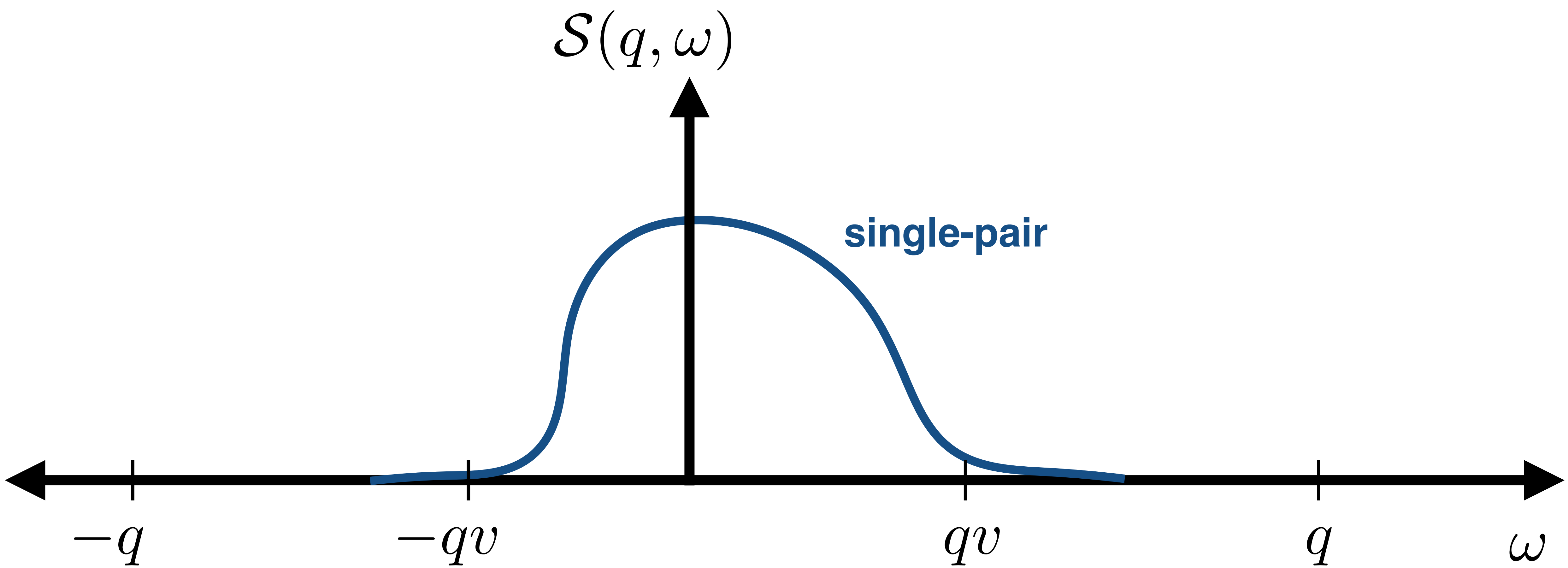
difficult to calculate in general due to the non-perturbative nature of strong interactions.

Sawyer (1970s), Iwamoto & Pethick (1980s),
Burrows & Sawyer, Horowitz & Wehrberger, Raffelt et al., Reddy et al. (1990s),
Benhar, Carlson, Gandolfi, Horowitz, Lavato, Pethick, Reddy, Roberts, Schwenk, Shen, and others (2000s)

Correlations in Neutrino Interactions in Nuclear Matter & Nuclei



General Structure of the Dynamic Response



In hot and dense nuclear matter single-pair, multi-particle and collective modes all contribute to low energy response.

- At small ω response is governed by hydrodynamic.
- Single-pair response dominates for $|\omega\tau_{\mathrm{coll}}| > 1$ and $|\omega| < qv$.
- Multi-particle response dominates for $|\omega| > qv$.
- Collective modes arise due to phase transitions or repulsive interactions.

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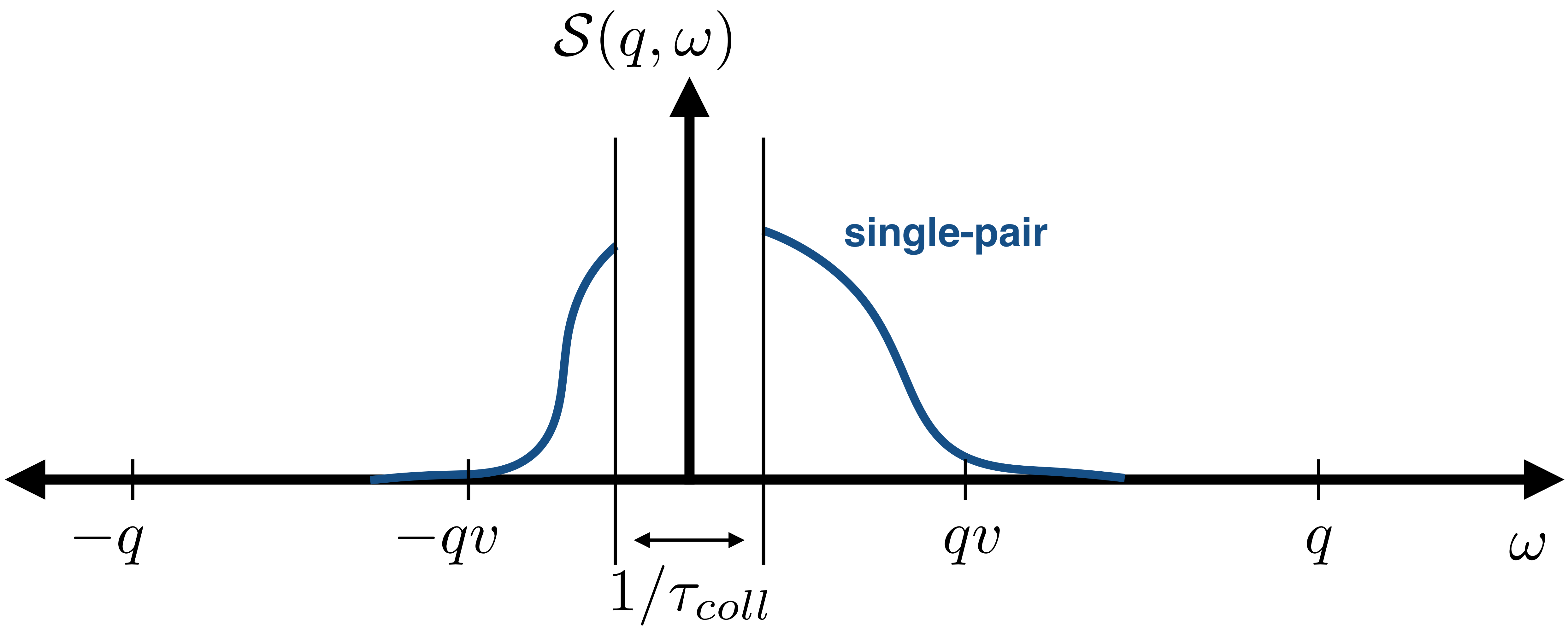

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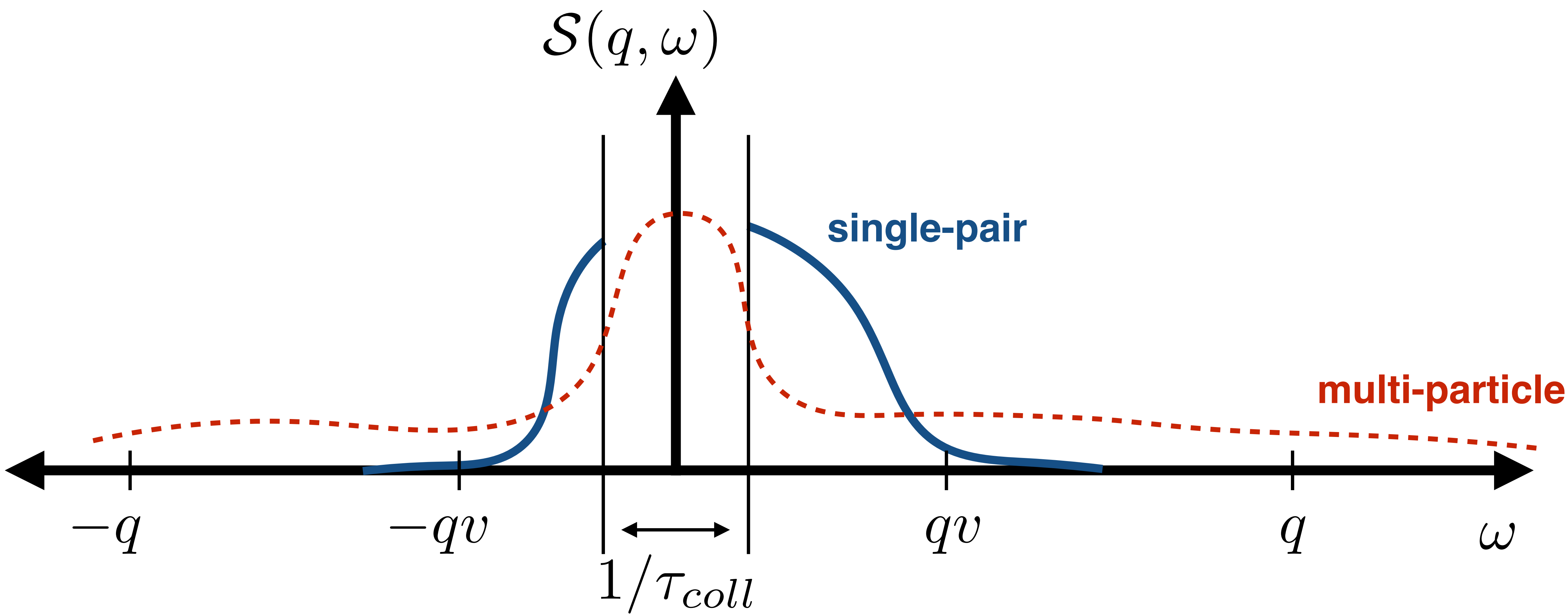

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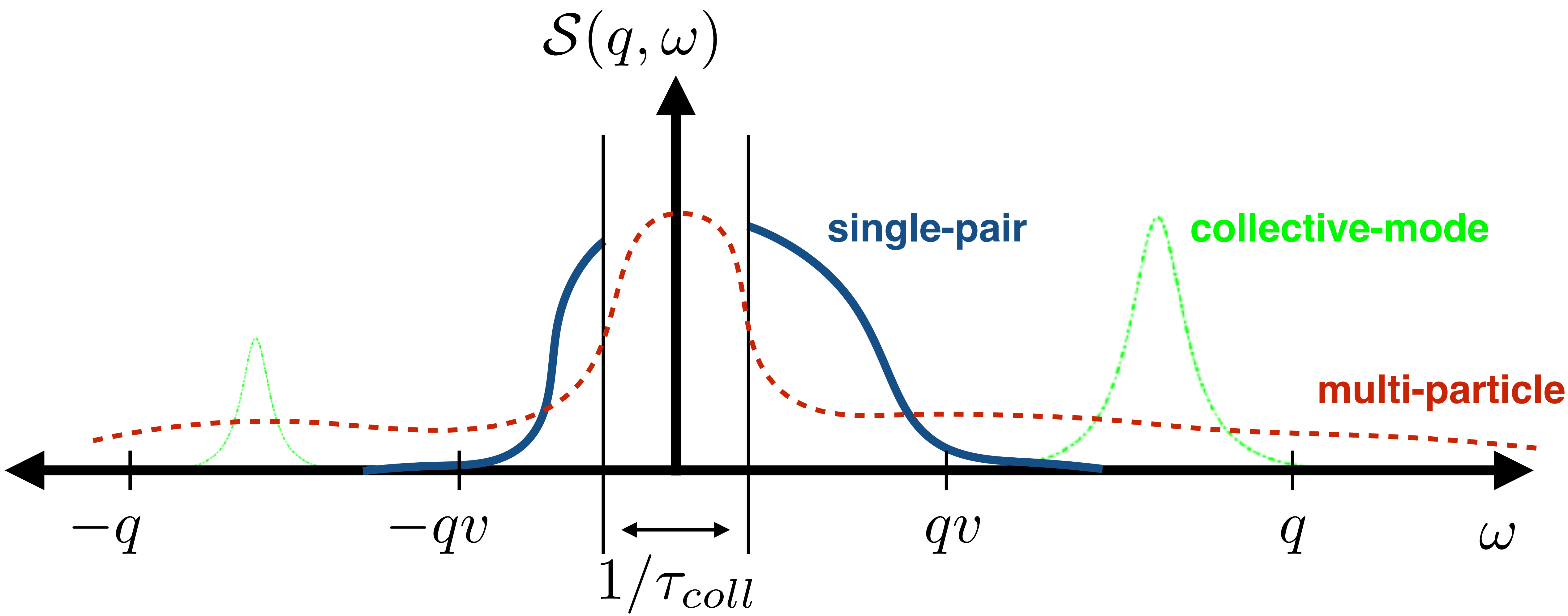

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Linear Response

Perturbation:

$$\mathcal{H}_{int} = \int d^3x \; \mathcal{O}(x) \; \phi_{ext}(x,t)$$

Response:

$$\delta\rho(\vec{q},\omega) = \Pi^R(\vec{q},\omega) \; \phi_{ext}(\vec{q},\omega)$$

Response function:
Polarization function
or Generalized Susceptibility

$$\Pi^R(\vec{q},\omega) = \frac{-i}{\hbar} \int dt \; e^{i\omega t} \; \theta(t) \; \langle [\mathcal{O}(-\vec{q},t), \mathcal{O}(\vec{q},0)] \rangle$$

Response to static and uniform perturbations is related to thermodynamic derivatives.

$$\phi_{ext}(\vec{q} \to 0, \omega = 0) = \delta\mu$$

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Compressibility sum-rule:

$\Pi^R(0,0) = \left(\frac{\partial n}{\partial\mu}\right)_T$ where $n = \langle\mathcal{O}(0,0)\rangle$ is the associated density.

Dynamic Structure Factor

A simpler correlation function $\mathcal{S}(\vec{q},\omega) = \int dt\ e^{i\omega t}\ \langle \mathcal{O}(-\vec{q},t)\mathcal{O}(\vec{q},0)\rangle$

$$= 2\pi\hbar \sum_{m,n} \frac{e^{\beta K_n}}{\mathcal{Z}}\ |\langle n|\mathcal{O}_q|m\rangle|^2\ \delta(K_n - K_m - \hbar\omega)$$

where K_n are eigenvalues of $K = \mathcal{H} - \mu\ N$ (grand canonical Hamiltonian)

Fluctuation-dissipation theorem: $\mathcal{S}(\vec{q},\omega) = \dfrac{-2\hbar\ \mathrm{Im}\ \Pi^R(\vec{q},\omega)}{1 - e^{-\beta\hbar\omega}}$

The dynamic structure factor incorporates all of the many-body effects into the neutrino scattering and absorption rates.

Sum Rules

Static structure factor: $\mathcal{S}_q = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi}\, \mathcal{S}(q,\omega')$

F-sum rule: $\boxed{\int_{-\infty}^{\infty} d\omega'\ \omega'\ \mathrm{Im}\ \Pi^R(q,\omega') = \langle [[\mathcal{H},\mathcal{O}_q],\mathcal{O}_q] \rangle}$

Compressibility sum-rule: $\boxed{-\int_{-\infty}^{\infty} \frac{d\omega'}{\pi}\ \frac{\mathrm{Im}\ \Pi^R(0,\omega')}{\omega'} = \mathrm{Re}\ \Pi^R(0,0) = \left(\frac{\partial n}{\partial \mu}\right)_T}$

At T=0:

$$\int_{-\infty}^{\infty} \frac{d\omega'}{2\pi}\ \frac{(1-e^{-\beta\hbar\omega'})\mathcal{S}(q\to 0,\omega')}{\hbar\omega'} = \boxed{\int_0^{\infty} \frac{d\omega'}{2\pi}\ \frac{\mathcal{S}(q\to 0,\omega')}{\hbar\omega'} = \left(\frac{\partial n}{\partial \mu}\right)_{T=0}}$$

At high temperature: $(1-e^{-\beta\hbar\omega'}) \simeq \beta\hbar\omega'$

$$\boxed{\mathcal{S}_{q=0} = \mathrm{lt}_{q\to 0} \int_{-\infty}^{\infty} d\omega'\ \mathcal{S}(q,\omega) = T\ \left(\frac{\partial n}{\partial \mu}\right)_T}$$

Neutrino-nucleon scattering

Nucleon currents simplify in the non-relativistic limit:

$$j^{\mu}_{nc} = \underbrace{\Psi^{\dagger}\Psi}_{\text{density}}\ \delta^{\mu}_{0} + \underbrace{\Psi^{\dagger}\sigma_k\Psi}_{\text{spin-density}}\ \delta^{\mu}_{k} + \mathcal{O}[\frac{p}{M}]$$

$$\frac{d\Gamma(E_1)}{d\Omega dE_3} = \frac{G_F^2}{4\pi^2}\ E_3^2\ \left[C_V^2(1+\cos\theta_{13})S_{\rho}(\omega,q) + C_A^2(3-\cos\theta_{13})S_{\sigma}(\omega,q)\right]$$

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density

spin-density

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Integrate over the final neutrino energy:

$$\frac{d\Gamma(E_1)}{dq} = \frac{G_F^2}{\pi}\ q\ \left(C_V^2 I_{\rho}(q) + C_A^2 I_{\sigma}(q)\right)$$

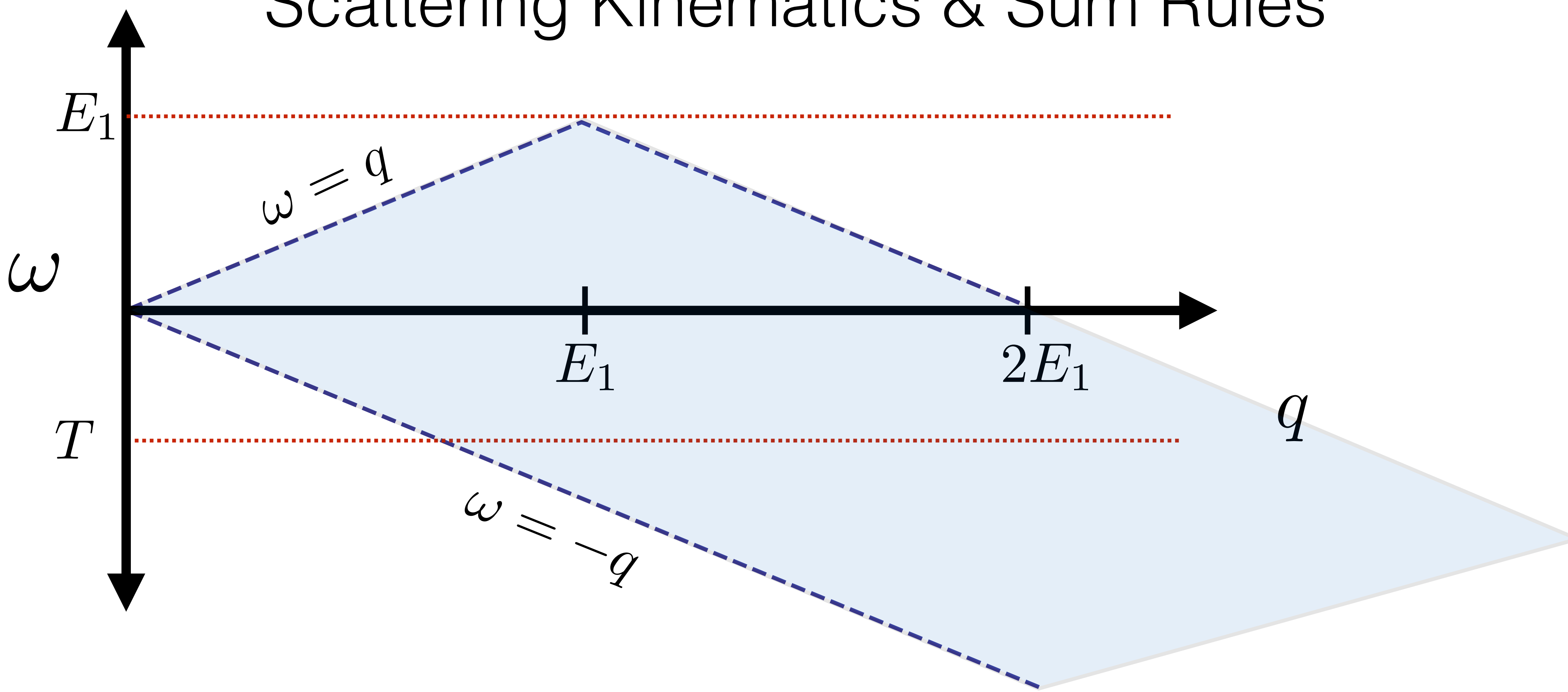
The "static" response functions are :

$$I_{\rho}(q) = \tilde{S}_{\rho}(q)\ \left(1 - \frac{q^2}{4E_1^2} - \frac{\langle\omega_{\rho}(q)\rangle}{E_1} + \frac{\langle\omega_{\rho}^2(q)\rangle}{4E_1^2}\right)$$

$$I_{\sigma}(q) = \tilde{S}_{\sigma}(q)\ \left(1 + \frac{q^2}{4E_1^2} - \frac{\langle\omega_{\sigma}(q)\rangle}{E_1} - \frac{\langle\omega_{\sigma}^2(q)\rangle}{4E_1^2}\right)$$

$$\tilde{S}_{\alpha}(q) = \int_{-q}^{\omega_{max}} d\omega\ S_{\alpha}(\omega,q) \qquad \langle\omega_{\alpha}^n\rangle = \frac{\int_{-q}^{\omega_{max}} d\omega\ \omega^n\ S_{\alpha}(\omega,q)}{\tilde{S}_{\alpha}(q)}$$

Scattering Kinematics & Sum Rules



Neutrinos only probe the space-like region with $|\omega| < q$

In general $\tilde{S}_\alpha(q) = \int_{-q}^{\omega_{max}} d\omega \; S_\alpha(\omega, q) \; < \quad S_\alpha(q) = \int_{-\infty}^{\infty} d\omega \; S_\alpha(\omega, q)$

In practice for conserved currents at long-wavelengths:

$$\tilde{S}_\rho(q \to 0) = S_\rho(q \to 0)$$
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At high temperature recall that $S_\rho(q \to 0) = T \left(\frac{\partial n}{\partial \mu}\right)_T$

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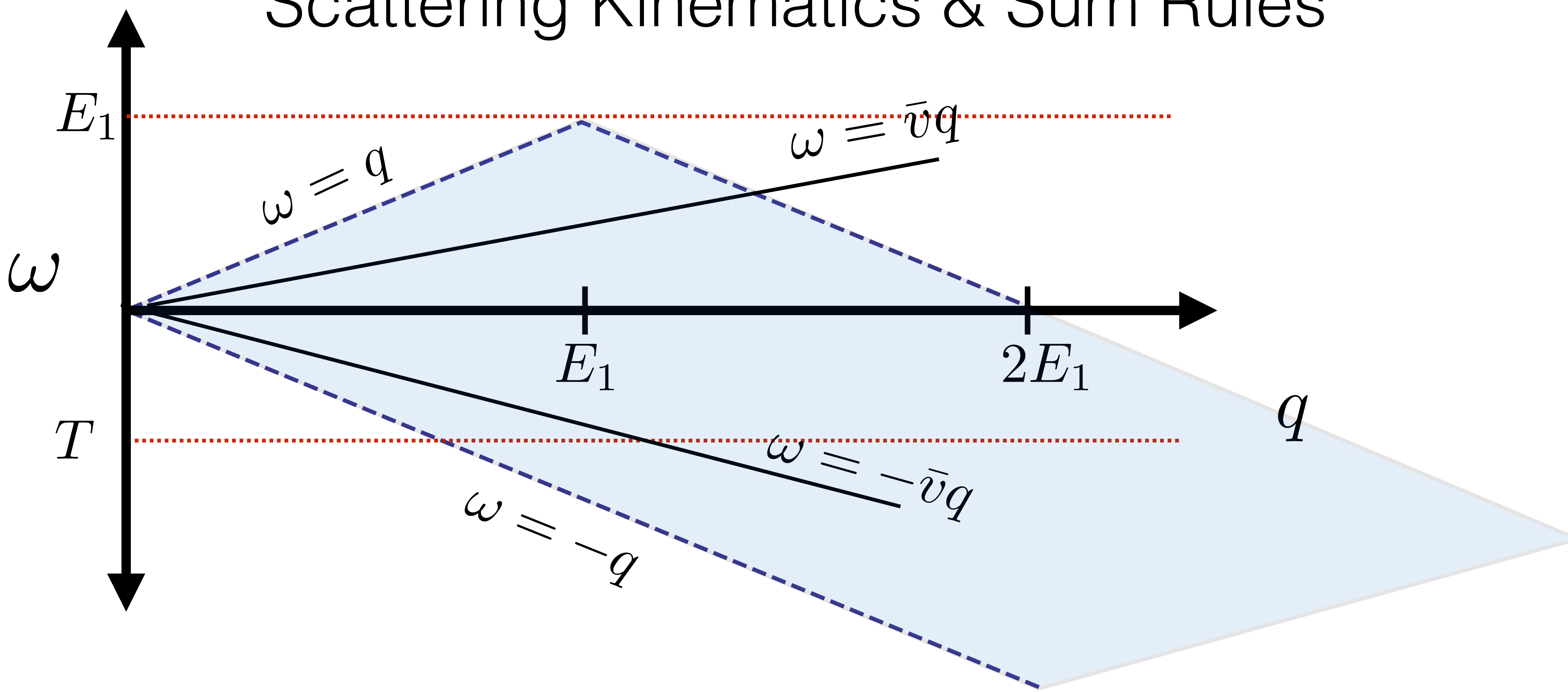

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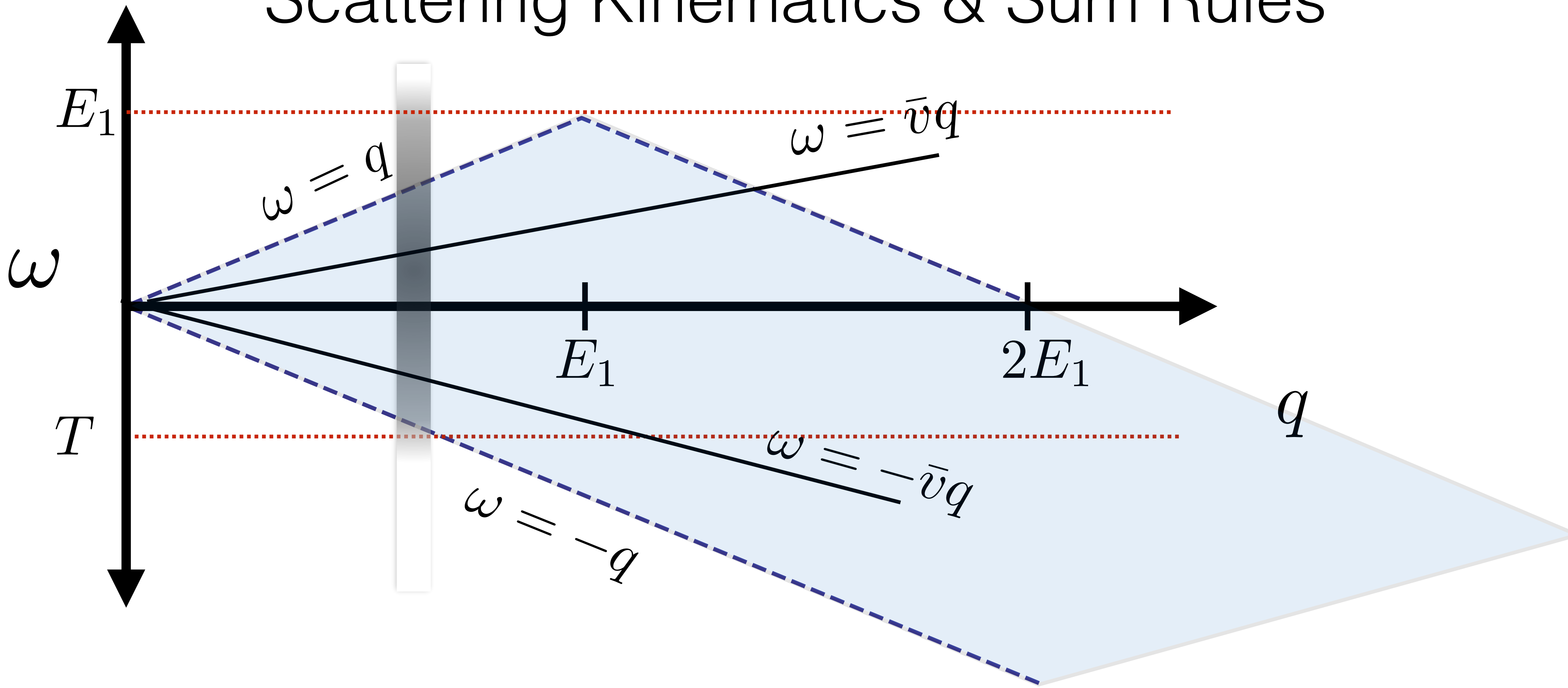

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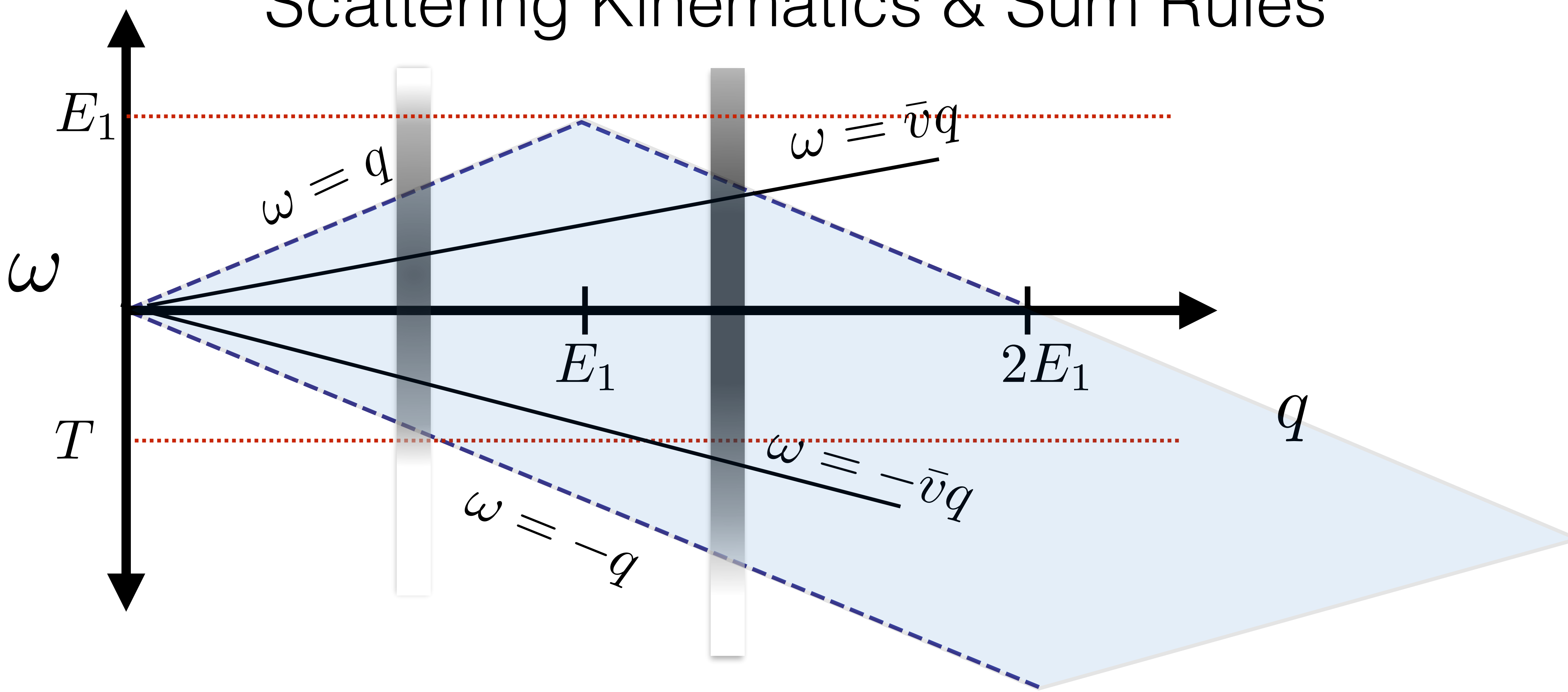

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Spin is not conserved by strong interactions

F-sum Rule: $$F_\alpha(q) = \int d\omega\ \omega S_\alpha(\omega, q) = \frac{1}{2}\langle[\mathcal{O}_\alpha^\dagger, [\mathcal{O}_\alpha, H]]\rangle$$

$$F_\rho(q) = n\ \frac{q^2}{2m} \qquad\qquad F_\sigma(q) = C + \tilde{n}\ \frac{q^2}{2m}$$

- Thermodynamic derivates may not be adequate to accurately describe the long wavelength spin response.
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2p-2h contribution
Finite time-like response.

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- Thermodynamic derivates may not be adequate to accurately describe the long wavelength spin response.
- Some dynamical information is needed to calculate neutrino scattering rates in the medium.

Supernova Neutrino Spectra and Nucleosynthesis

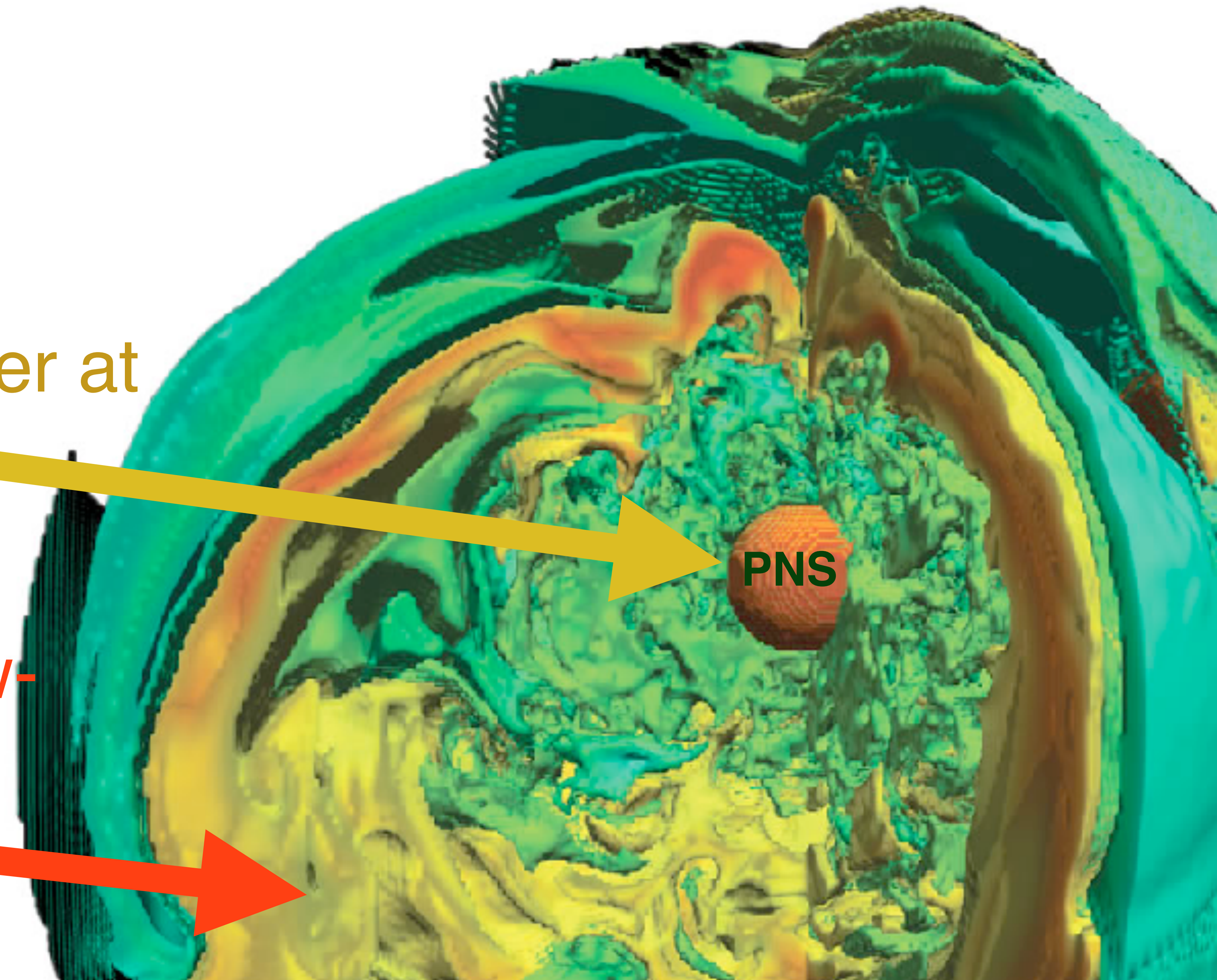
Electron and anti-electron neutrinos play a crucial role in supernova. Their energy spectrum impacts:

1. Explosion mechanism
2. Nucleosynthesis
3. Detection

$$\begin{cases} \bar{\nu}_e + p \rightarrow n + e^+ \\ \nu_e + n \rightarrow p + e^- \end{cases}$$

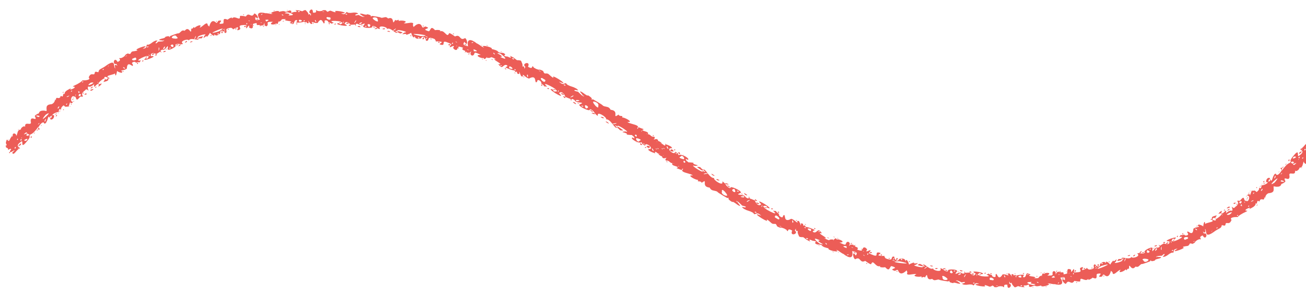
Neutrino-sphere at high density. Neutron-rich matter at moderate entropy.
R ~ 10-20 km

Neutrino driven wind at low-density and high entropy.
R ~ 10^3-10^4 km



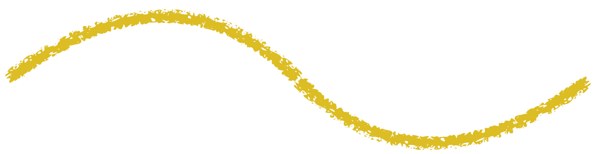
10^{12} g/cm^3 : Neutrino Processes in the Neutrino-sphere

Hierarchy of length scales at T=5 MeV and $\rho = 10^{12}$ g/cm^3

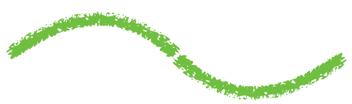


Neutrino Wavelength

$$\Lambda_\nu = \frac{2\pi\hbar}{\langle E_\nu \rangle} \approx 100 \text{ fm}$$



Inter-nucleon distance $a = \left(\frac{3}{4\pi n_n}\right)^{1/3} \approx 10 \text{ fm}$



De Broglie wavelength $\Lambda_D = \frac{2\pi\hbar}{\langle p_n \rangle} \approx 4 \text{ fm}$

Range of the n-n interaction $r_{nn} \approx 2 \text{ fm}$

- Matter is dilute, but interactions are strong and non-perturbative.
- Nucleon-nucleon scattering length is large ~ 20 fm.
- Small expansion parameter is the fugacity $z=e^{\mu/T}$ - virial expansion.
- Horowitz & Schwenk developed a virial EoS in this regime.

Long-wavelength Response using the Virial EoS

Assumes that scattering is nearly elastic to include all many-body correlations through the static structure factors.

$$\tilde{S}_\rho(q) = \int_{-q}^{\omega_{max}} d\omega \; S_\rho(\omega, q) \simeq S_\rho(q)$$

$$\tilde{S}_\rho(q \to 0) = S_\rho(q \to 0)$$

Calculate the static structure factors using the compressibility or thermodynamic sum rule

$$S_\rho(q \to 0) = T \left(\frac{\partial n}{\partial \mu}\right)_T$$

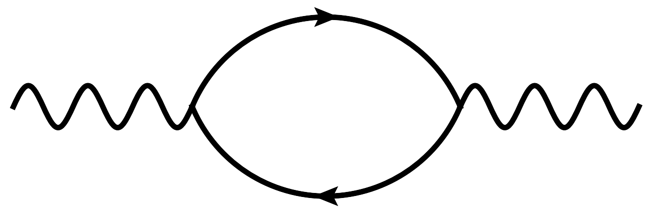
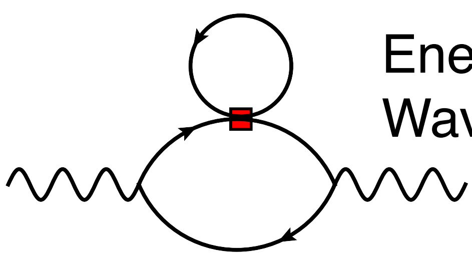
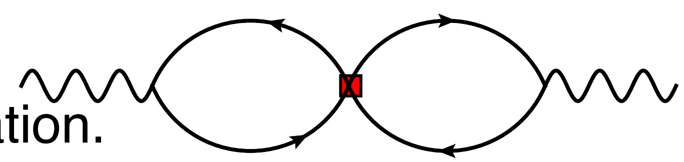
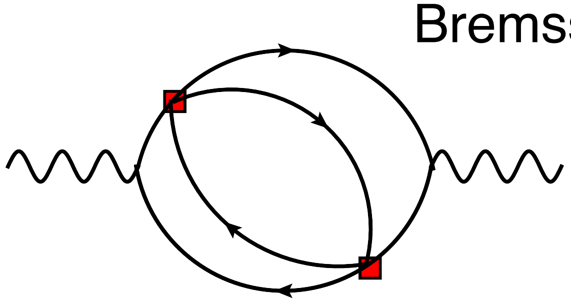
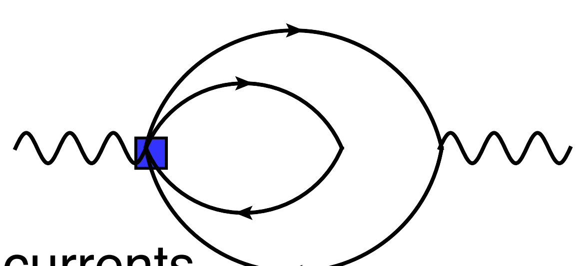
Sawyer (1975, 1979)
Horowitz and Schwenk (2005), Horowitz et al. (2017)

This is an excellent approximation for the density response relevant to neutral current reactions in the neutrino sphere.

The spin response and charged current reactions require some dynamical input.

Pseudo-potential for Hot & Dilute Nuclear Matter

The dynamic structure factor calculable using standard diagrammatic "perturbation" theory - with a twist.
Interactions represented by a pseudo-potential: $\mathcal{V}_{ps} \propto \frac{\delta(p_{rel})}{p_{rel}\ M}$

	Leading order diagram neglects interactions.	$\mathcal{O}[z]$
 Energy and density shifts. Wave-function renormalization. Screening. Vertex renormalization. 	Includes interactions at leading order. Consistent with the viral expansion.	$\mathcal{O}[z^2\ \mathcal{V}_{ps}]$
Bremsstrahlung processes   2-body or meson-exchange currents	Includes 2p-2h excitations and 2-body currents. These corrections are beyond the leading order viral expansion.	$\mathcal{O}[z^2\ \mathcal{V}_{ps}^2]$

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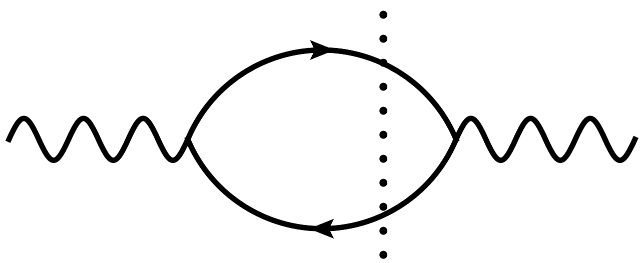
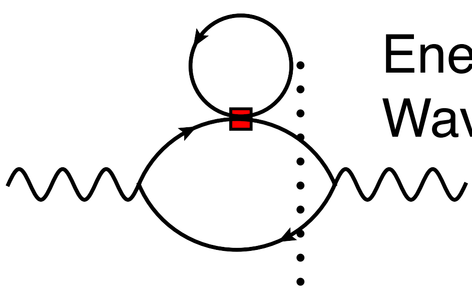
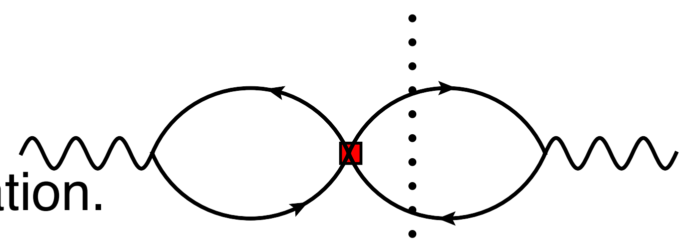
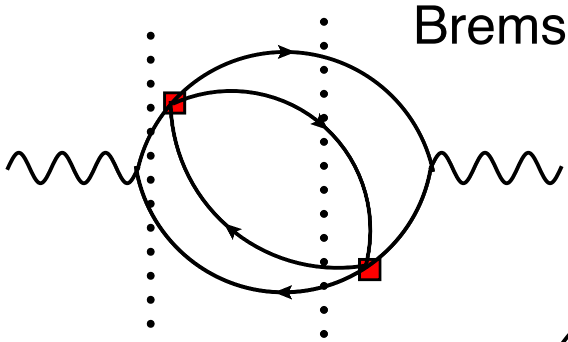
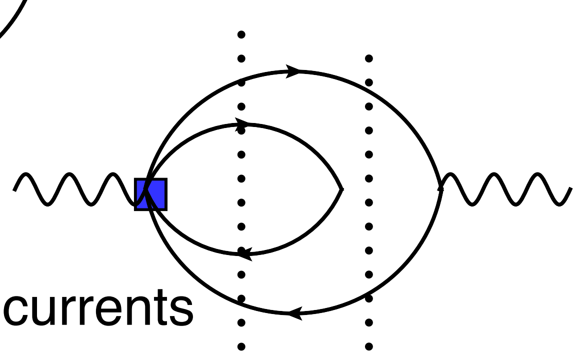
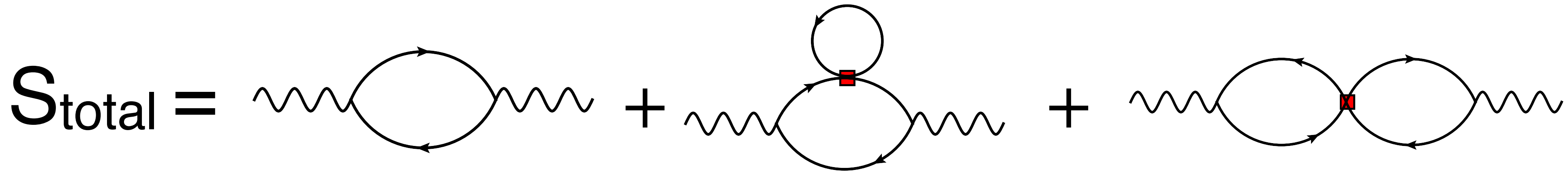
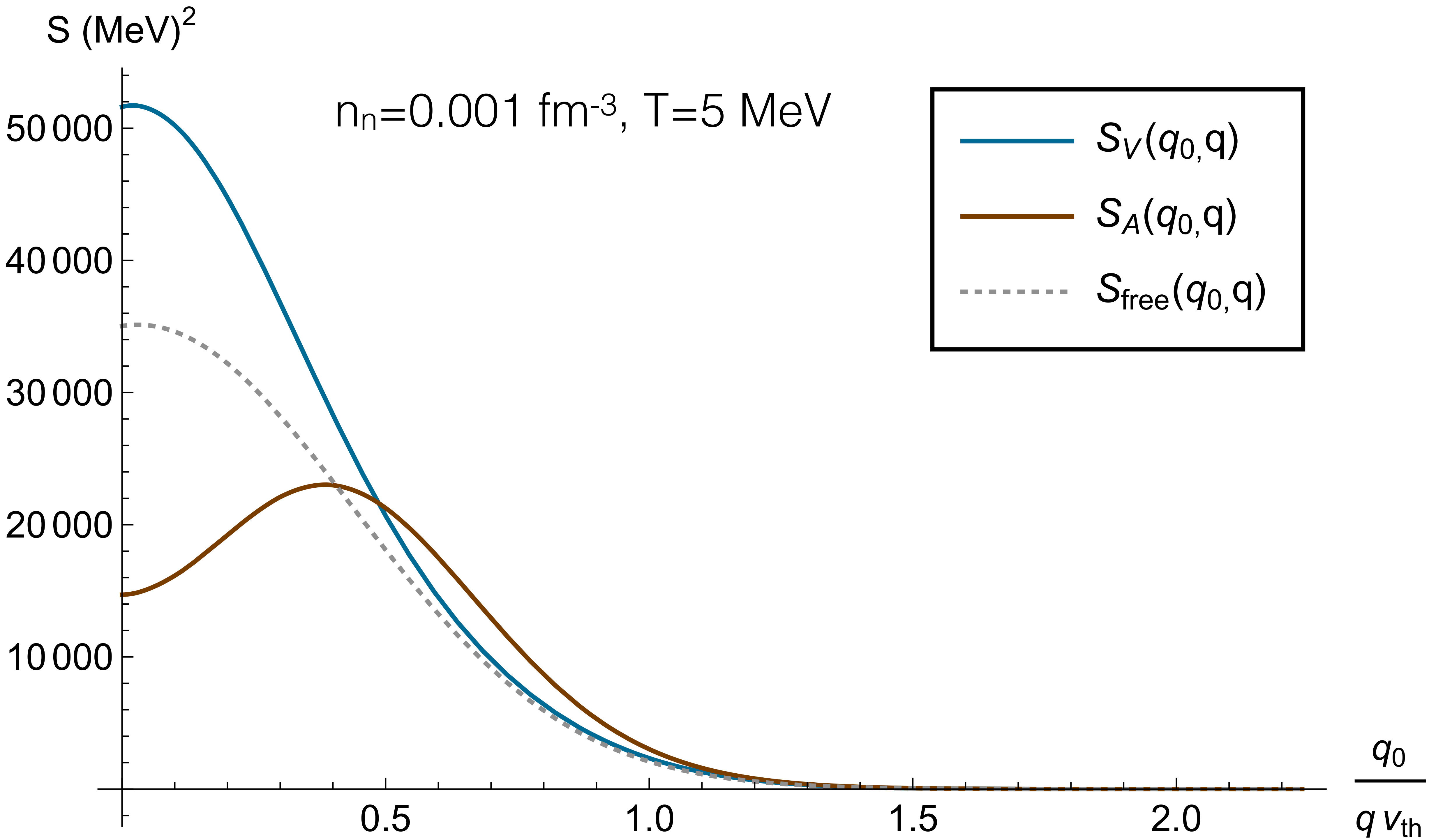
	Leading order diagram neglects interactions.	$\mathcal{O}[z]$
 Energy and density shifts. Wave-function renormalization. Screening. Vertex renormalization. 	Includes interactions at leading order. Consistent with the viral expansion.	$\mathcal{O}[z^2\ \mathcal{V}_{ps}]$
 Bremsstrahlung processes 2-body or meson-exchange currents 	Includes 2p-2h excitations and 2-body currents. These corrections are beyond the leading order viral expansion.	$\mathcal{O}[z^2\ \mathcal{V}_{ps}^2]$

Dynamic Structure Factor with Pseudo-potential

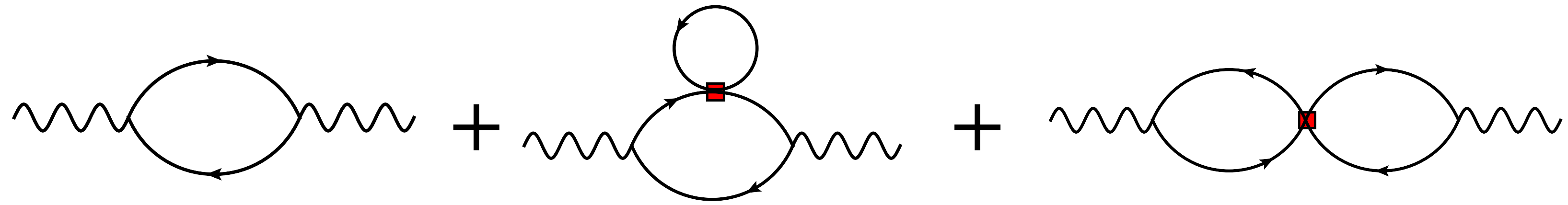


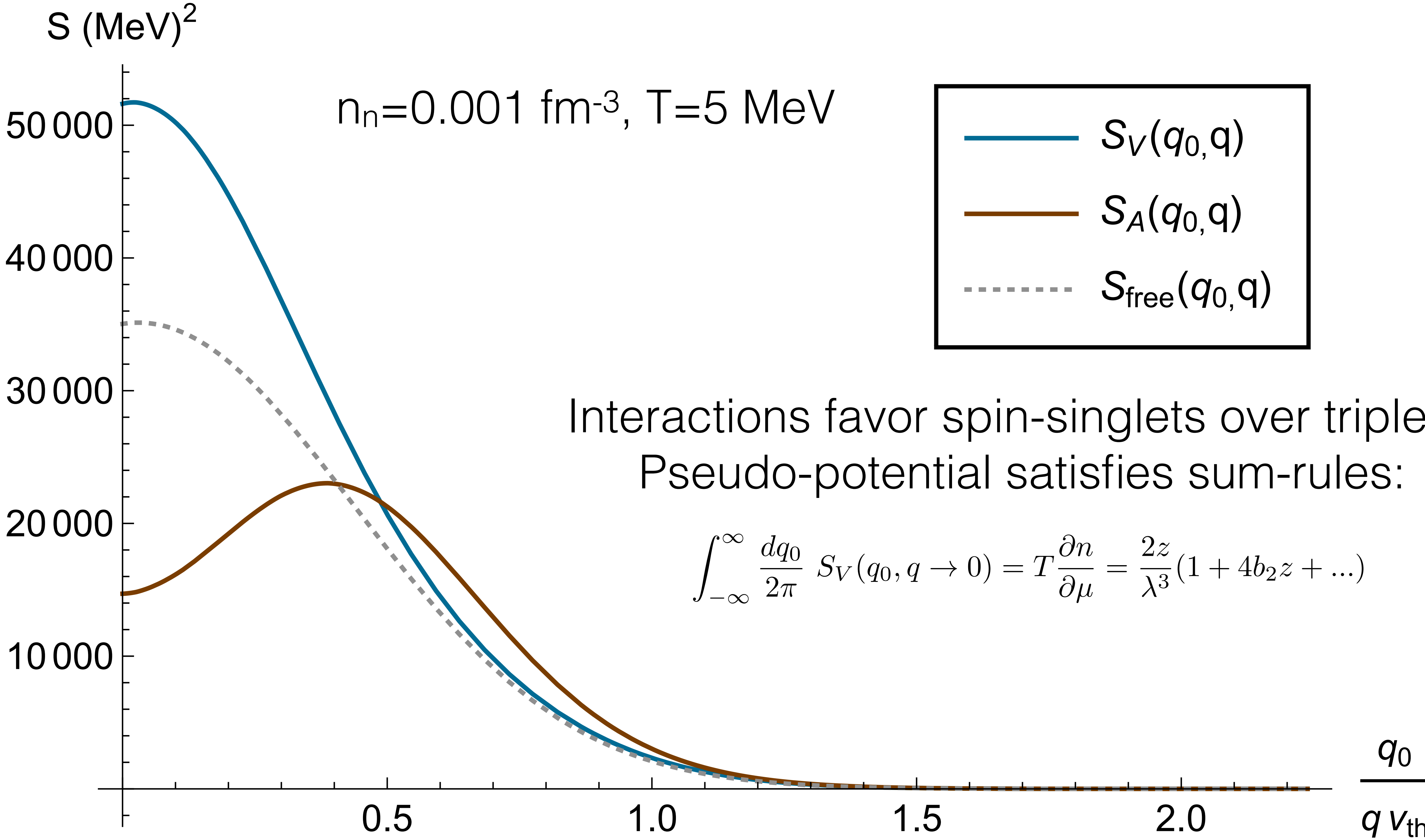


Bedaque, Reddy, Sen & Warrington (2018)

Dynamic Structure Factor with Pseudo-potential

$S_{\text{total}} =$



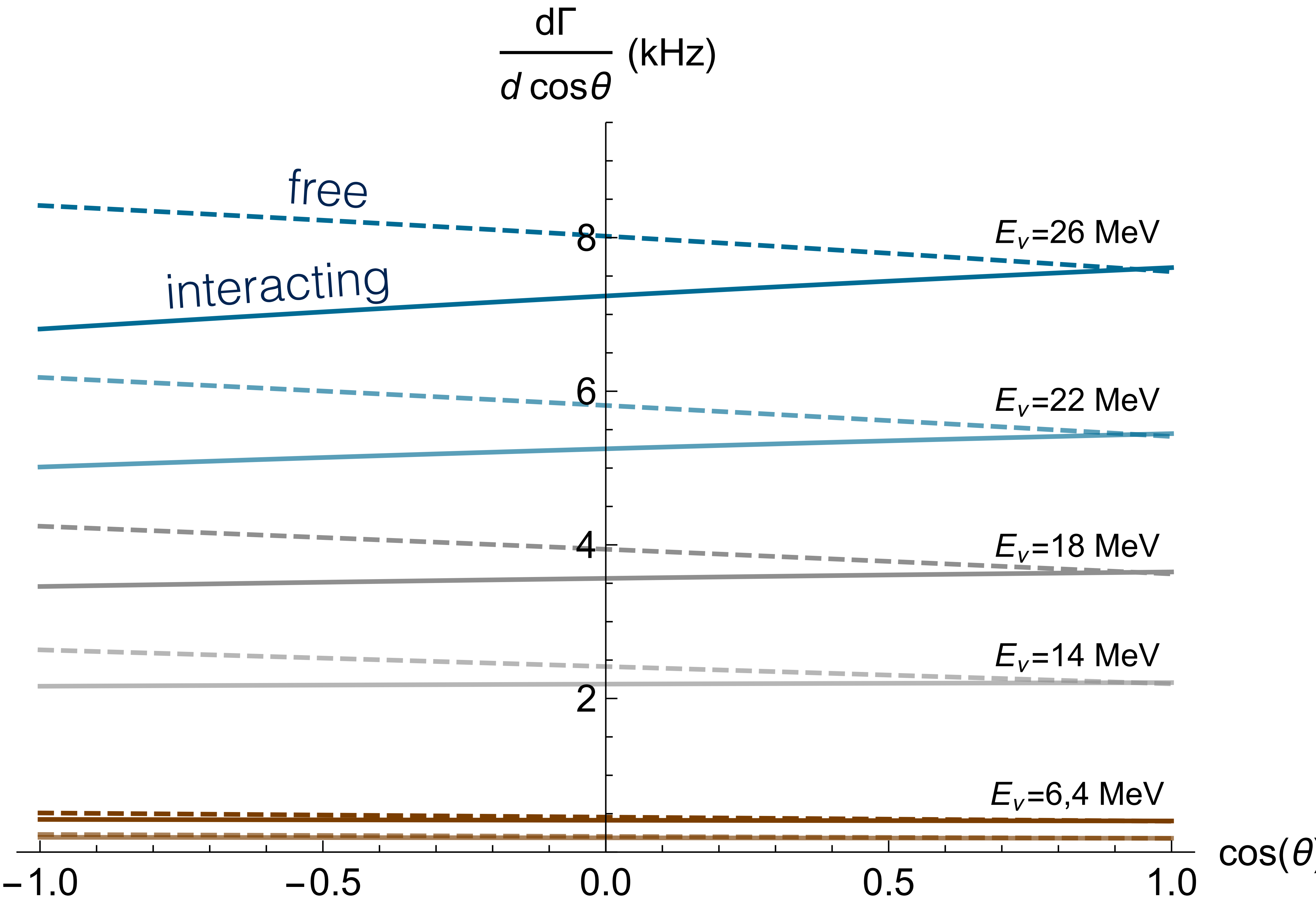


Interactions favor spin-singlets over triplets.
Pseudo-potential satisfies sum-rules:

$$\int_{-\infty}^{\infty}\frac{dq_0}{2\pi}\;S_V(q_0,q\to 0)=T\frac{\partial n}{\partial\mu}=\frac{2z}{\lambda^3}(1+4b_2z+...)$$

Bedaque, Reddy, Sen & Warrington (2018)

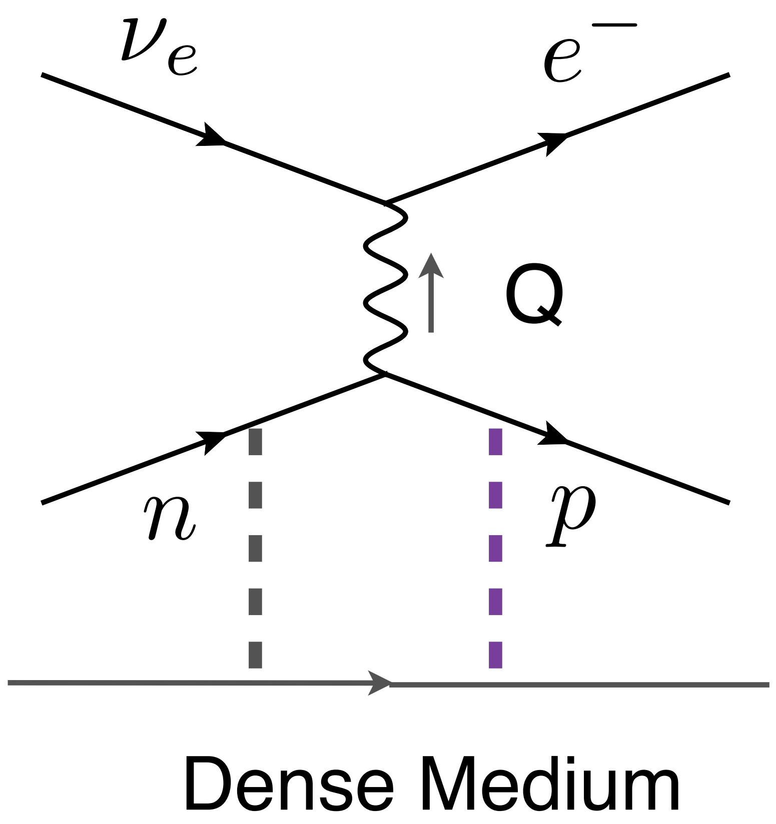
Back-scattering is suppressed



A reduced axial response implies a reduced scattering at backward angles : Important for transport and spectra.

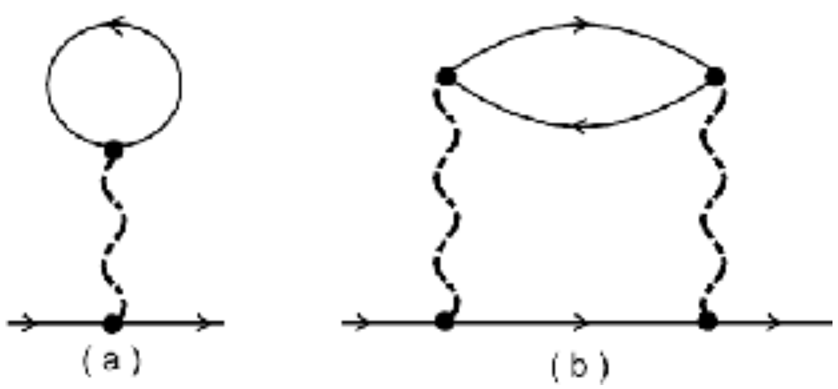
Bedaque, Reddy, Sen & Warrington (2018)

Charged Current Reactions in Neutron-Rich Matter



Potential energy difference between neutrons and protons is large - and related to the low density symmetry energy.

The pseudo-potential is suitable to calculate the nucleon self-energy.



$$E_n(p) \approx m_n + \frac{p^2}{2m_n^*} + U_n + i\,\Gamma_n$$

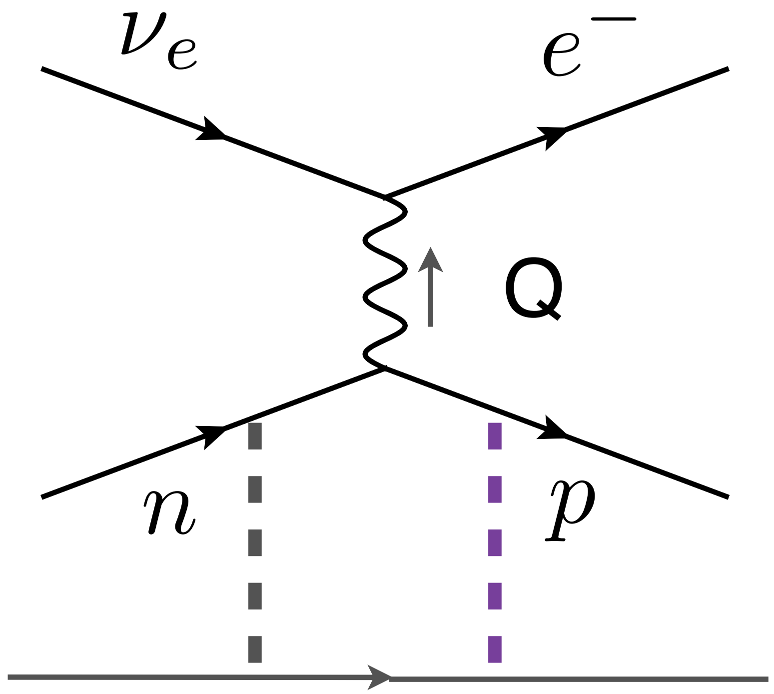
$$E_p(p+q) \approx m_p + \frac{(p+q)^2}{2m_n^*} + U_p + i\,\Gamma_p$$

$$\mathsf{Q} = \varepsilon_n(\vec{k}) - \varepsilon_p(\vec{k} - \vec{q})$$

$$= M_n - M_p + \Sigma_n(k) - \Sigma_n(k-q)$$

Reddy, Prakash & Lattimer (1998), Martinez-Pinedo et al. (2012), Roberts & Reddy (2012), Rrapaj, Bartl, Holt, Reddy, Schwenk (2015)

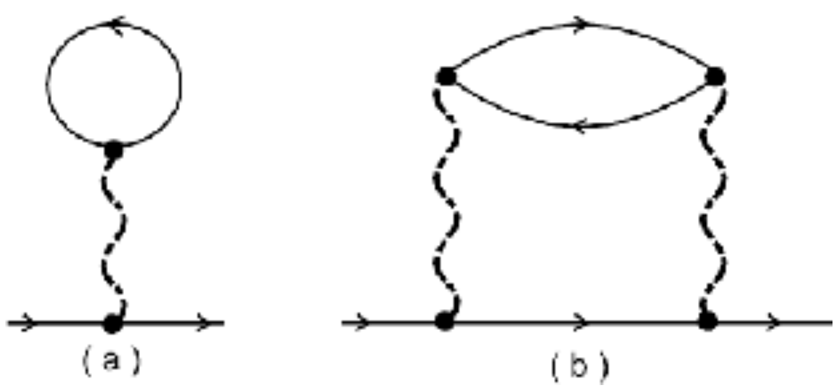
Charged Current Reactions in Neutron-Rich Matter



Dense Medium

Potential energy difference between neutrons and protons is large - and related to the low density symmetry energy.

The pseudo-potential is suitable to calculate the nucleon self-energy.



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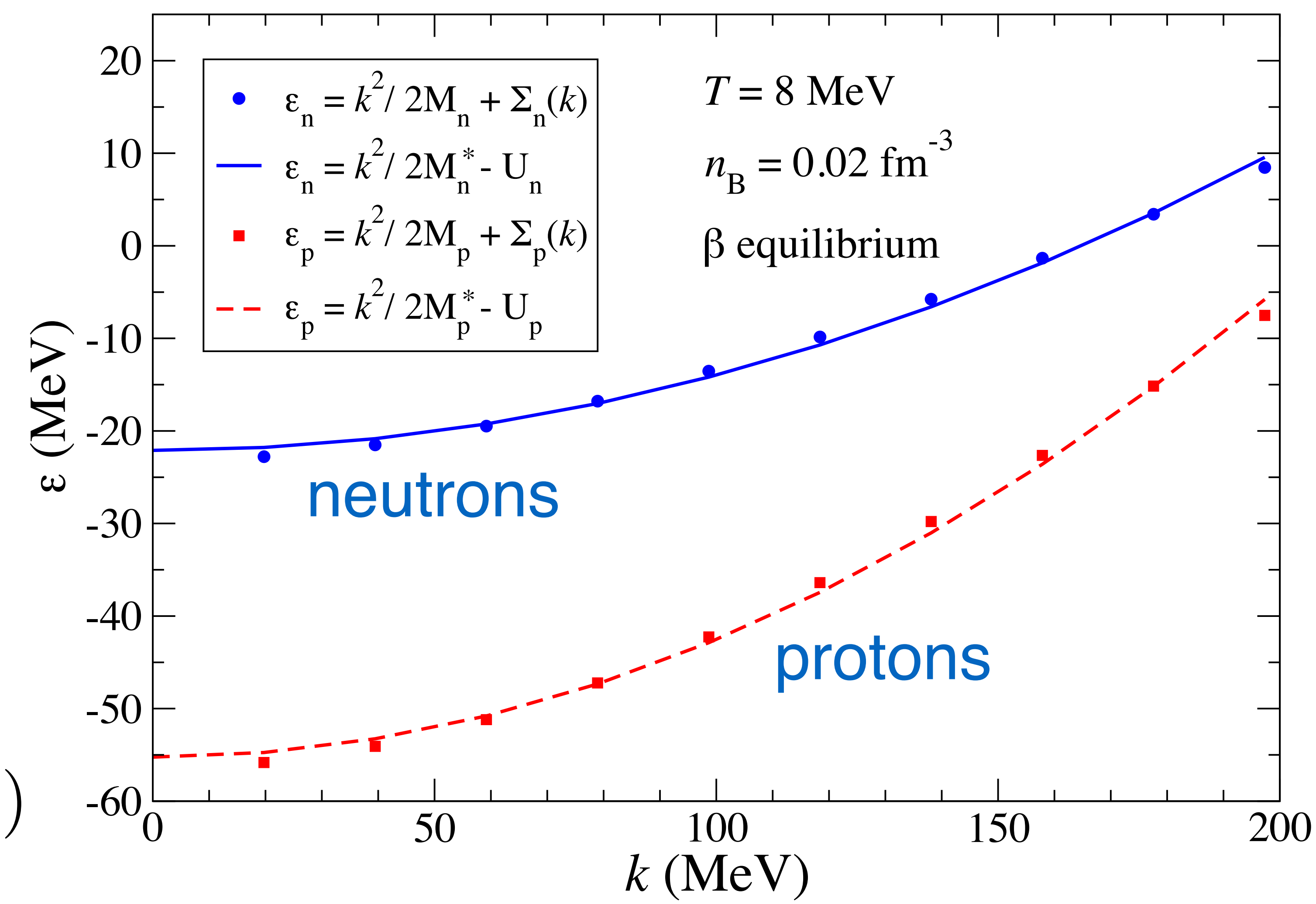


Reddy, Prakash & Lattimer (1998), Martinez-Pinedo et al. (2012), Roberts & Reddy (2012), Rrapaj, Bartl, Holt, Reddy, Schwenk (2015)

Mean Field & Collisional Broadening

Ansatz for the spin-isospin charge-exchange response function in hot matter:

$$S_{\sigma\tau^-}(q_0,q) = \frac{1}{1-\exp\left(-\beta(q_0+\mu_n-\mu_p\right)}\mathrm{Im}\left[\frac{\tilde{\Pi}(q_0,q)}{1-V_{\sigma\tau}\tilde{\Pi}(q_0,q)}\right]$$

$$V_{\sigma\tau} \simeq 200-220\ \mathrm{MeV/fm}$$

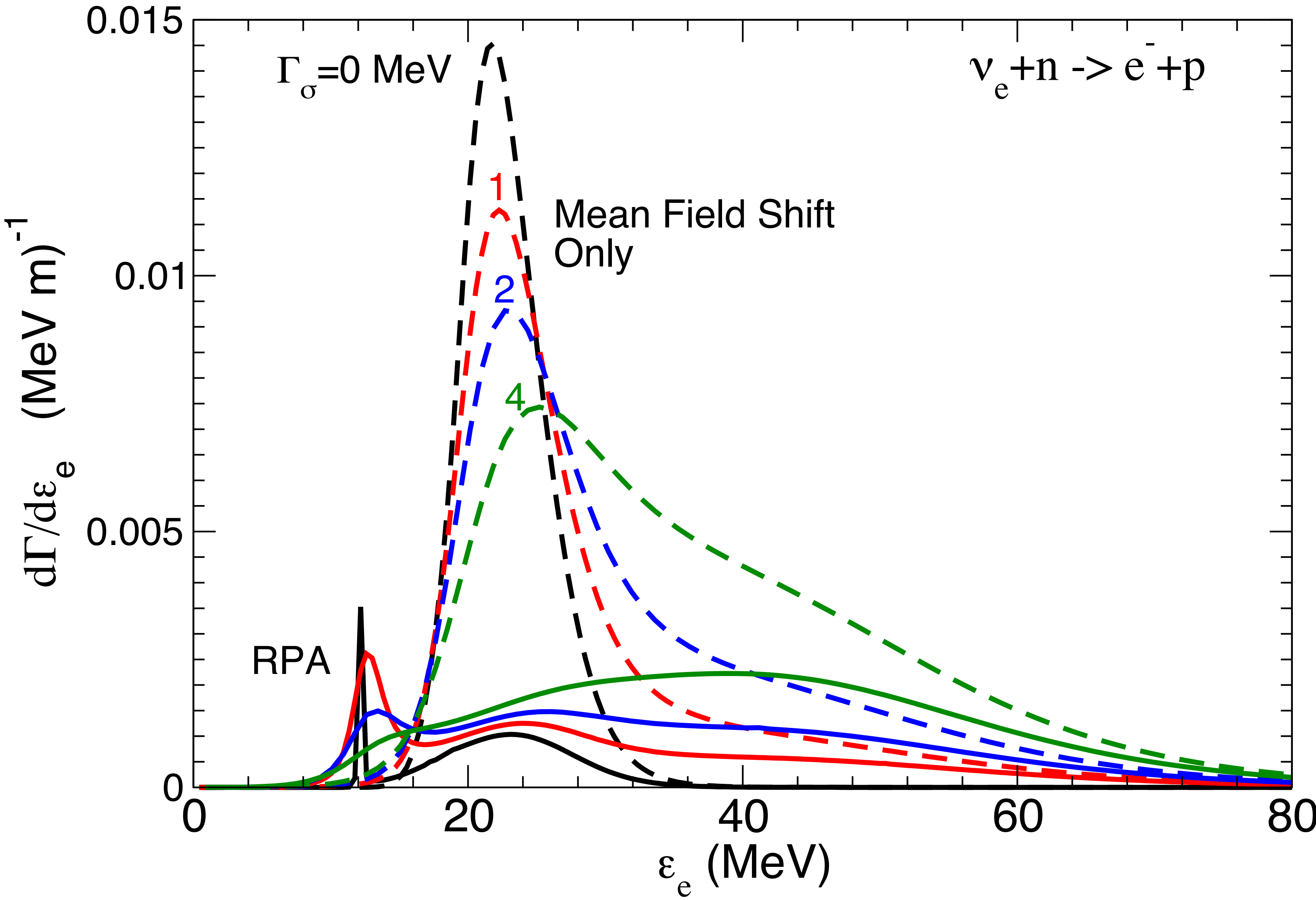
G. Bertsch, D. Cha, and H. Toki (1984)

Collisional broadening (finite lifetime) introduced in the relaxation time approximation: $\Gamma = \tau_\sigma^{-1}$

$$\mathrm{Im}\tilde{\Pi}(q_0,q) = \frac{1}{\pi}\int \frac{d^3p}{(2\pi)^3}\frac{f_p(\epsilon_{p+q})-f_n(\epsilon_p)}{\epsilon_{p+q}-\epsilon_p+\hat{\mu}}\,\mathcal{I}(\Gamma)$$

$$\mathcal{I}(\Gamma) = \frac{\Gamma}{(q_0+\Delta U-(\epsilon_{p+q}-\epsilon_p))^2+\Gamma^2}$$

Energy shifts, the Gamow-Teller resonance and collisional broadening all play a role.

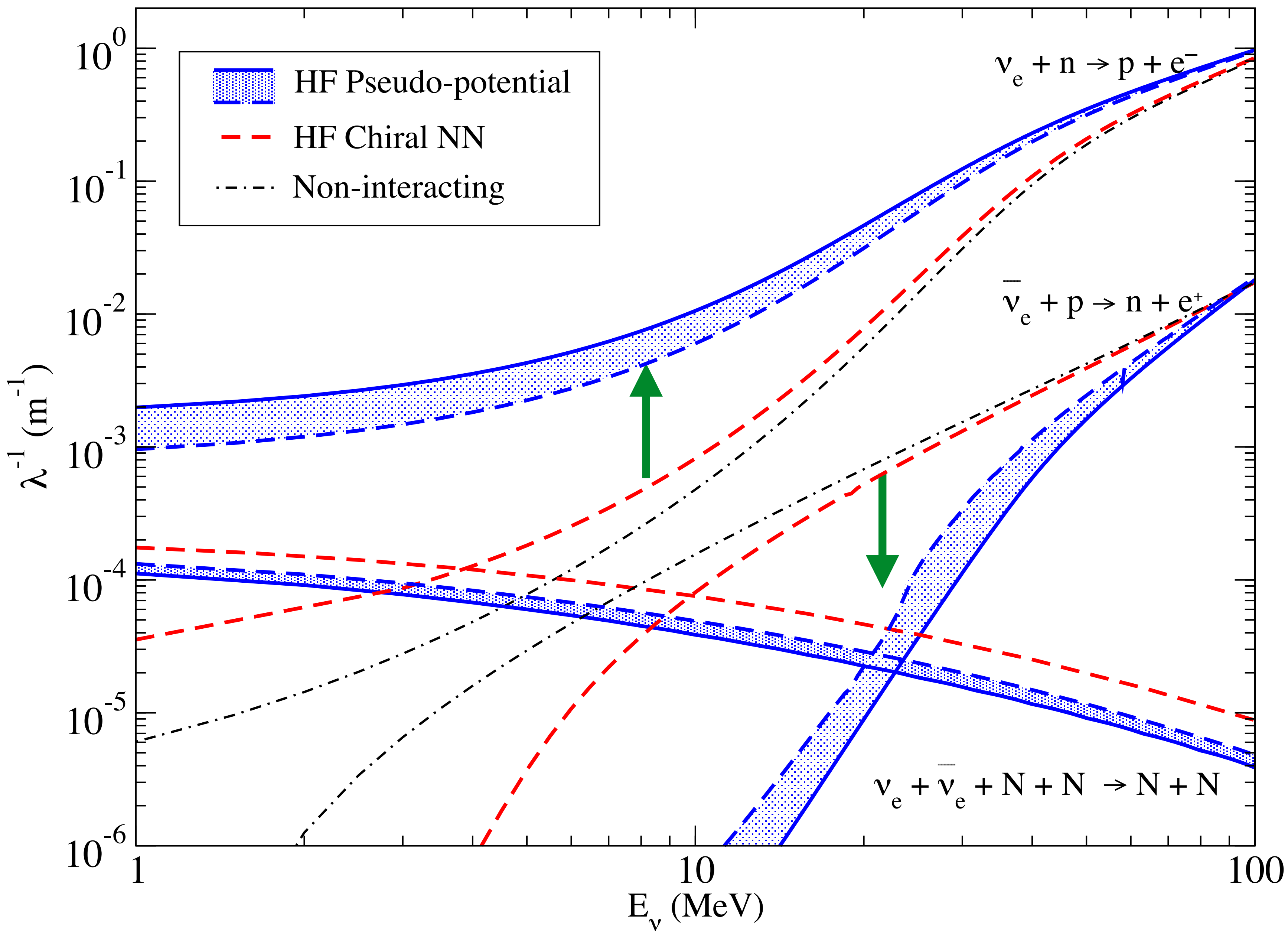


Shen, Roberts, Reddy (2013)

Charged Currents at Low Density with the Pseudo-potential

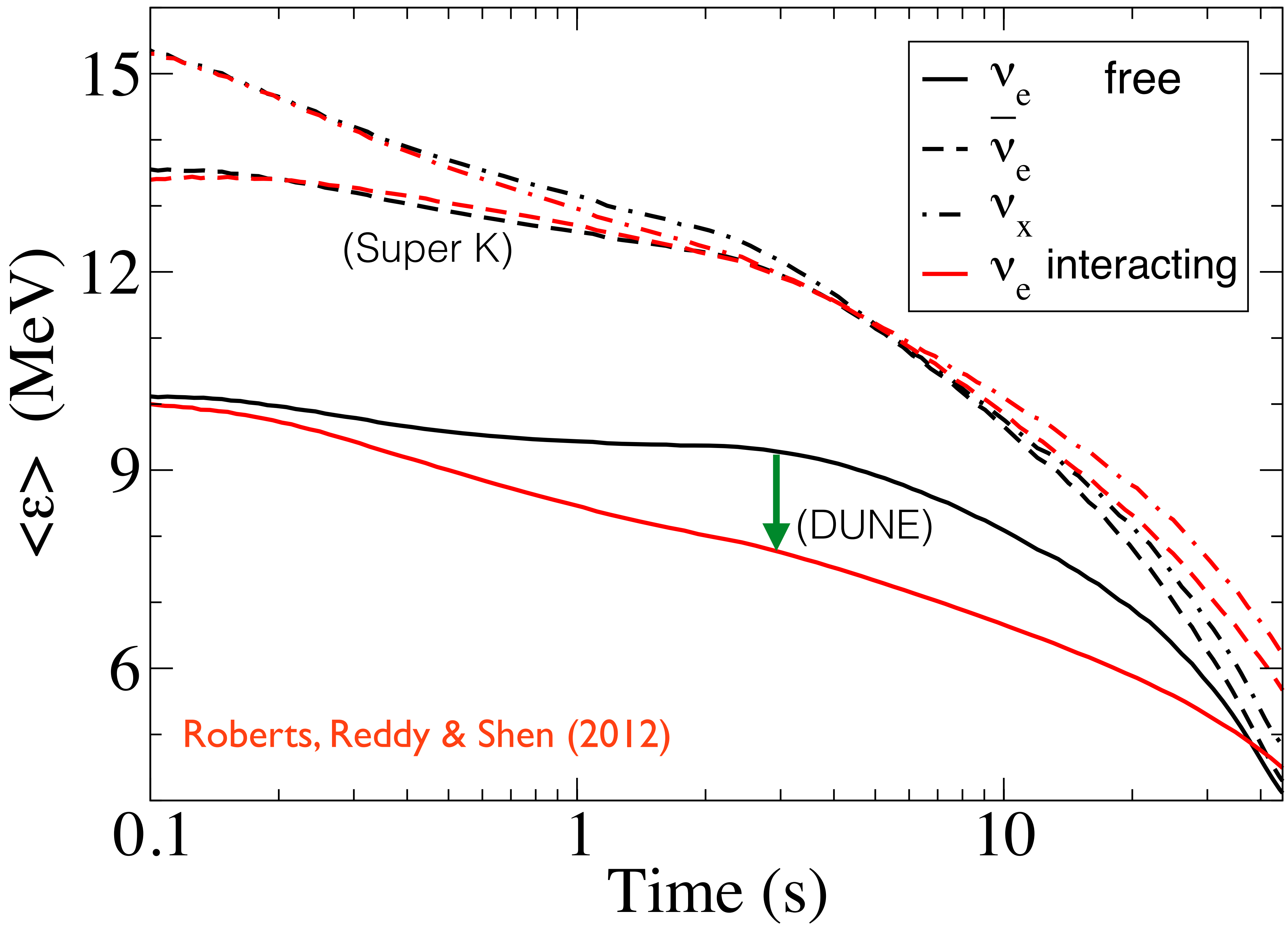
$$\frac{d\Gamma}{\cos\theta dE_e} = \frac{G_F^2}{2\pi} p_e\ E_e\,(1 - f_e(E_e)) \times \left[(1+\cos\theta)S_\tau(q_0,q) + g_A^2(3-\cos\theta)S_{\sigma\tau}(q_0,q)\right]$$

- Energy shift helps overcome electron final state blocking.
- Enhances ν_e absorption
- Larger energy needed to produce neutrons suppresses anti-ν_e absorption.



Rrapaj, Holt, Bartl, Reddy & Schwenk (2015)

Neutrino Spectra are Sensitive to Symmetry Energy

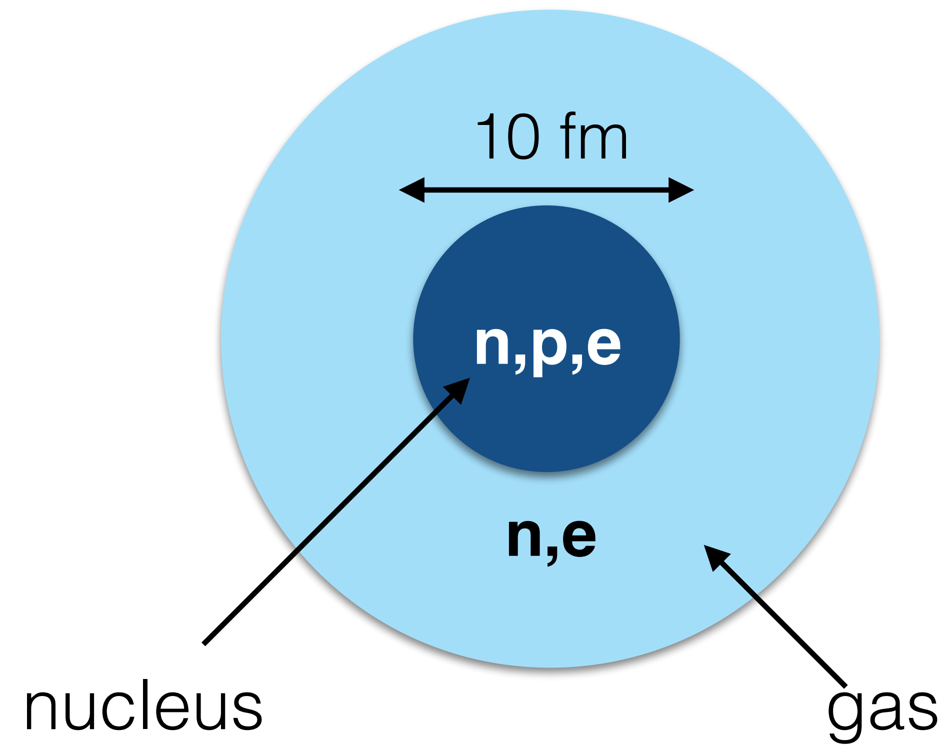


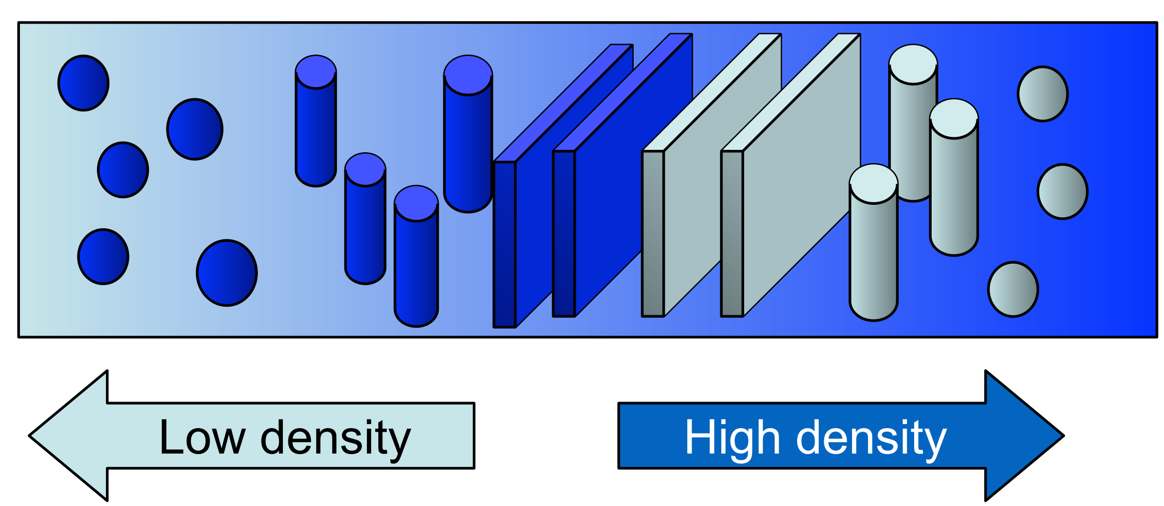
Time evolution of electron neutrino spectrum could be a useful diagnostic. Larger difference between electron and anti-electron neutrino energies is good for the r-process.

10^{13}-10^{14} g/cm^3 and T~3-20 MeV

- Interesting regime where matter behaves as a dense heterogenous liquid.
- Virial expansion fails. No small expansion parameter.
- Coexistence between neutron-rich nuclei and neutron-rich matter is favored at lower temperature - favors large density fluctuations.
- Neutrinos can coherently scatter of the heterogenous structures.

$$\frac{d\Gamma_{\rm coh}}{d\cos\theta} = \frac{G_F^2\ E_\nu^2}{8\pi}\ n_A\ (1+\cos\theta)\ S(q)\ N_{\rm w}^2\ F_A^2(q)$$



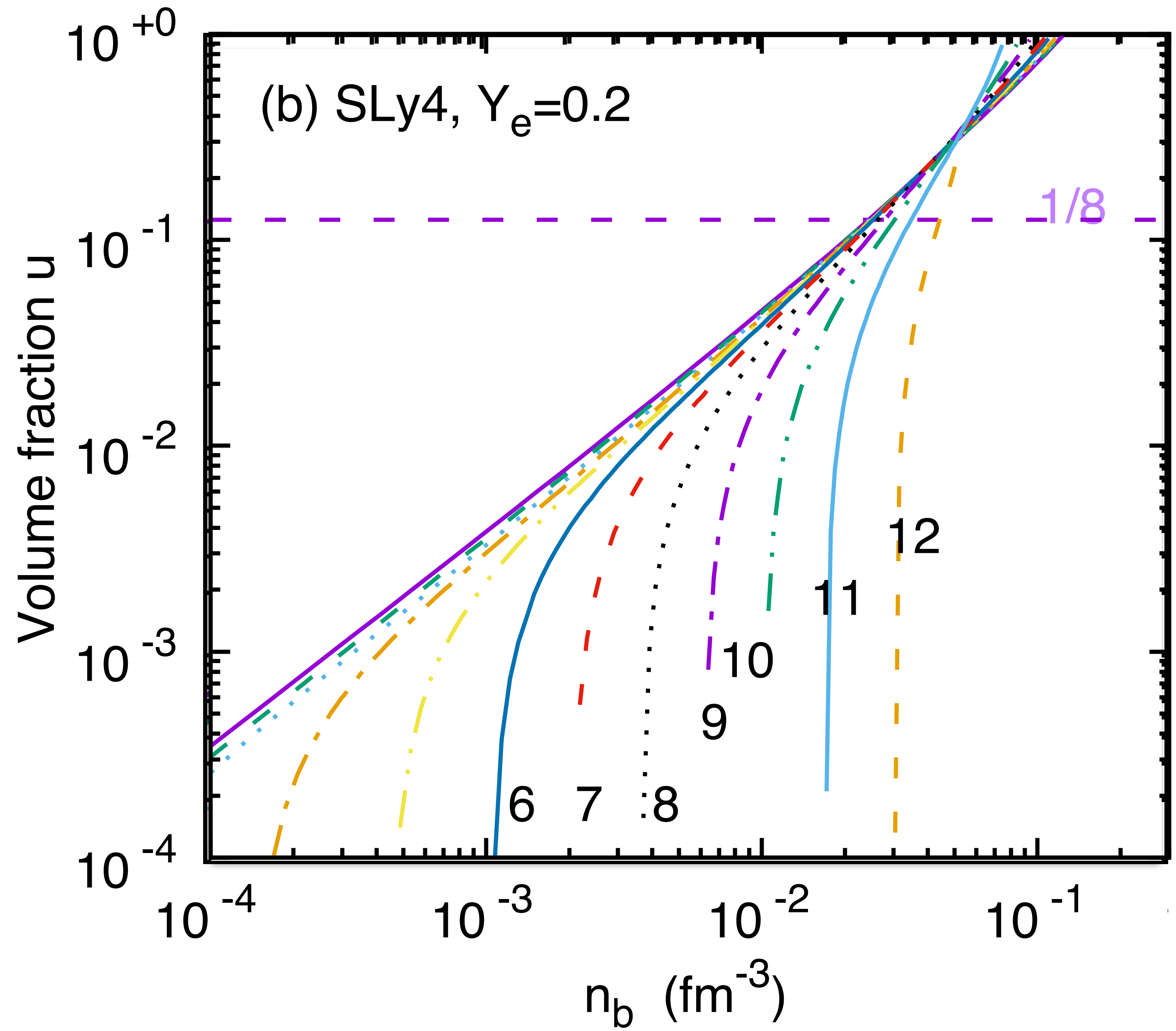


Pasta in Beta-Equilibrium Dissolves at Low Temperature

For large Y_e the volume fraction of nuclei denoted by **u** is large near the transition density. For small Y_e **u** decreases rapidly with T as protons leak out of nuclei.

Gibbs equilibrium is altered due to thermal protons in the low-density phase.

Pasta configurations favored at large Y_e at T=0, are not realized at T > 1 MeV for matter close to beta-equilibrium.



Roggero, Margueron, Reddy, & Roberts & (2017)

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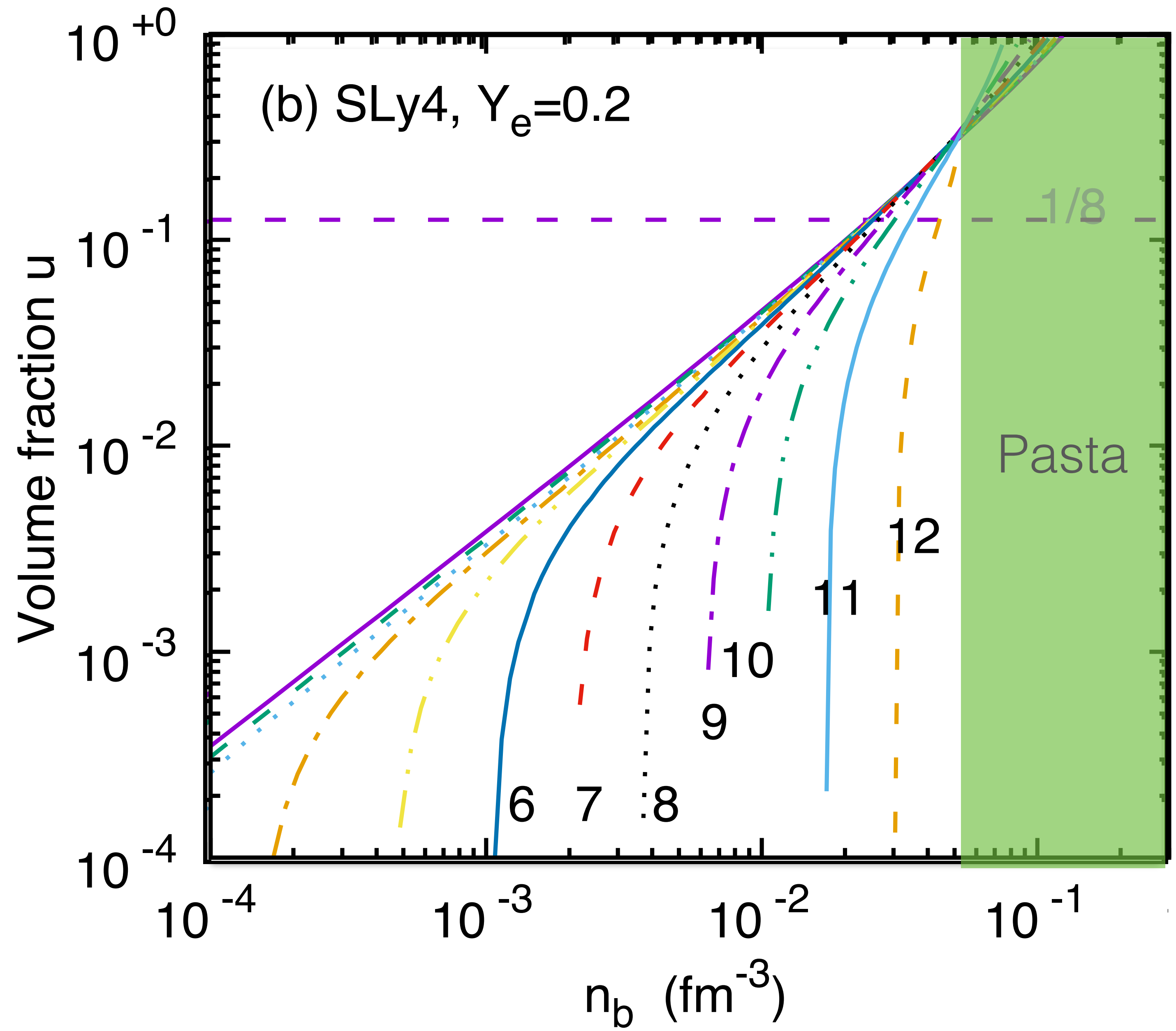

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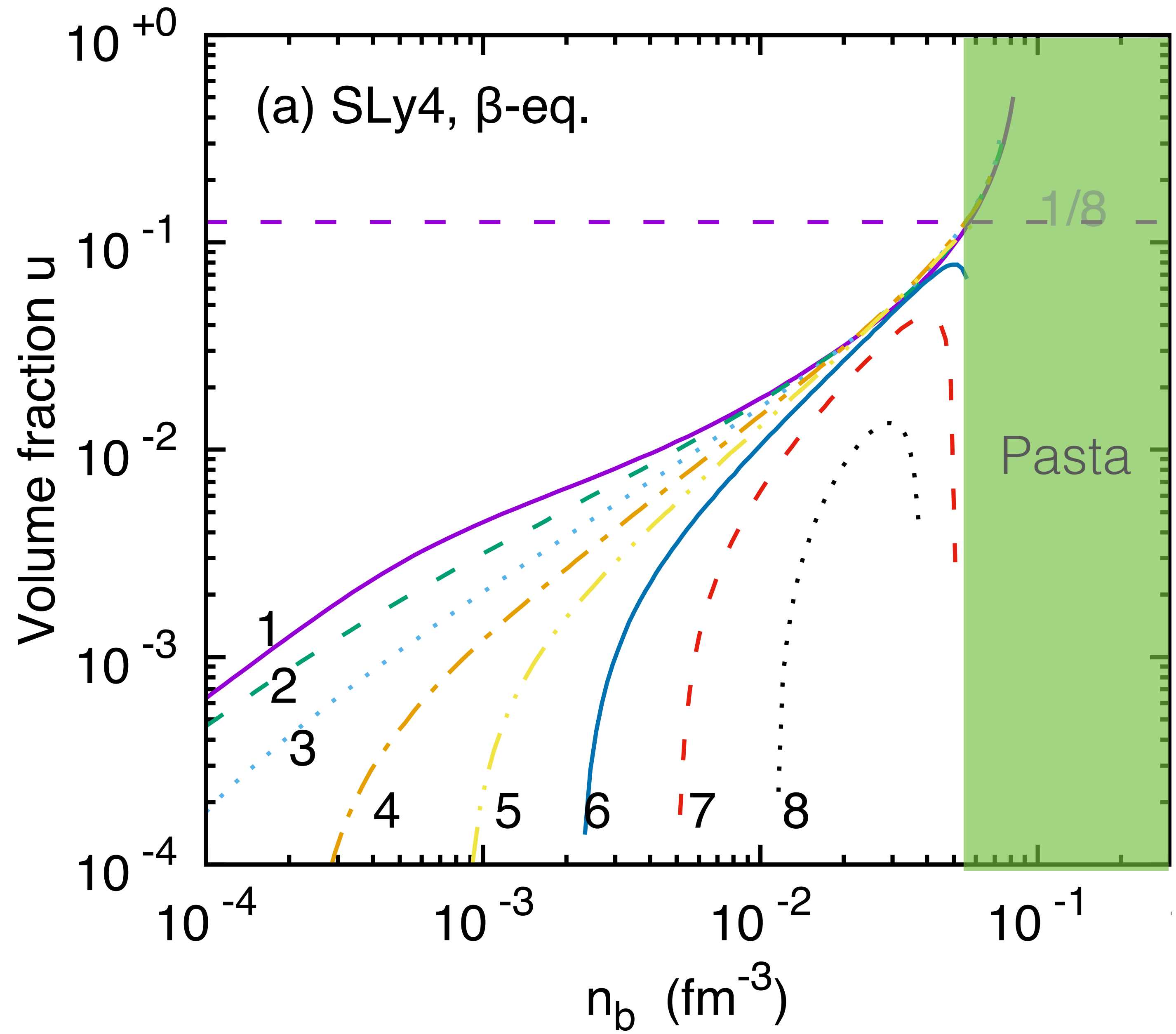


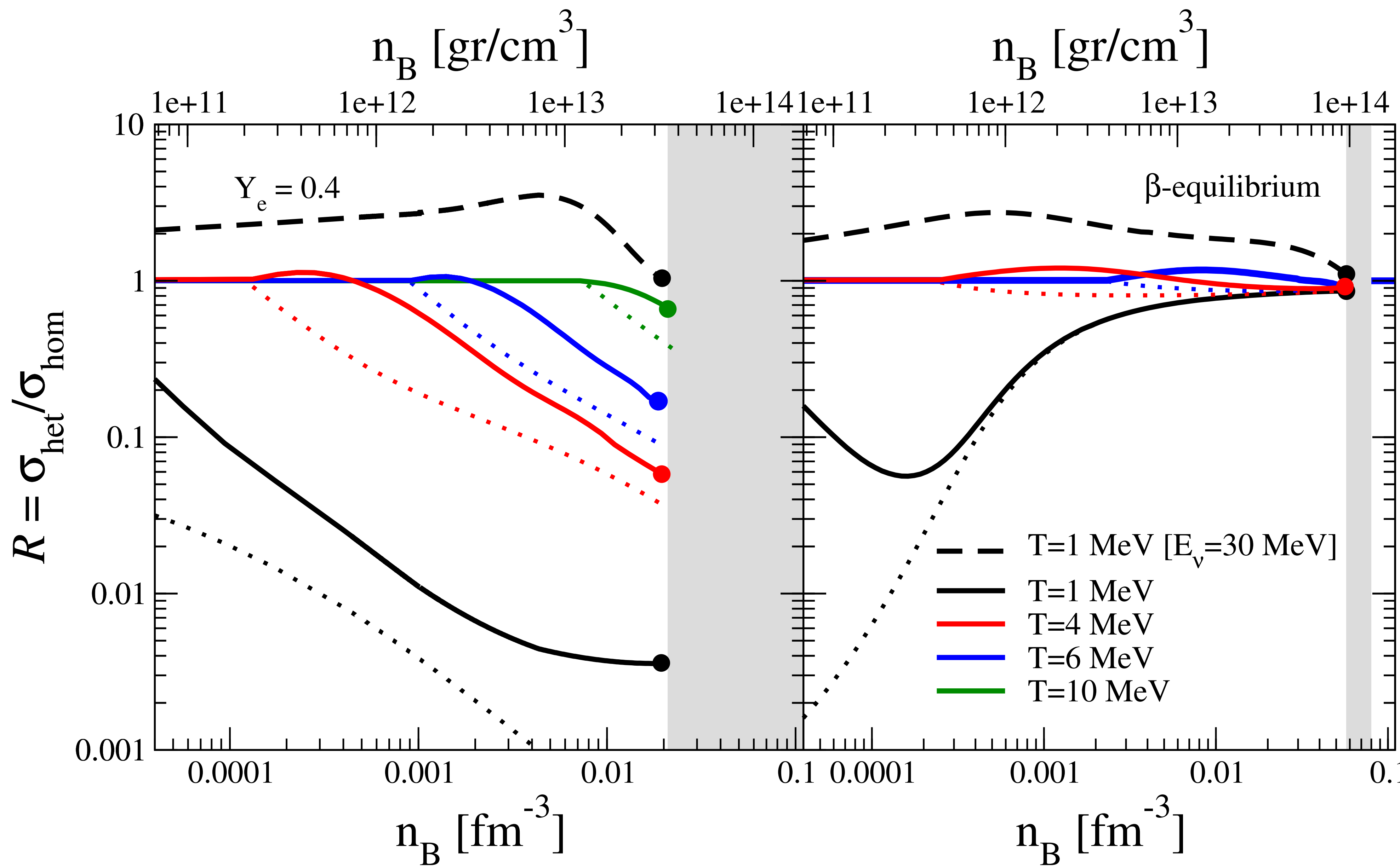
Roggero, Margueron, Reddy, & Roberts & (2017)

Paucity of Large Nuclei & Reduced Coherent Scattering

Coherent scattering makes a modest contribution to the total opacity at sub-nuclear density

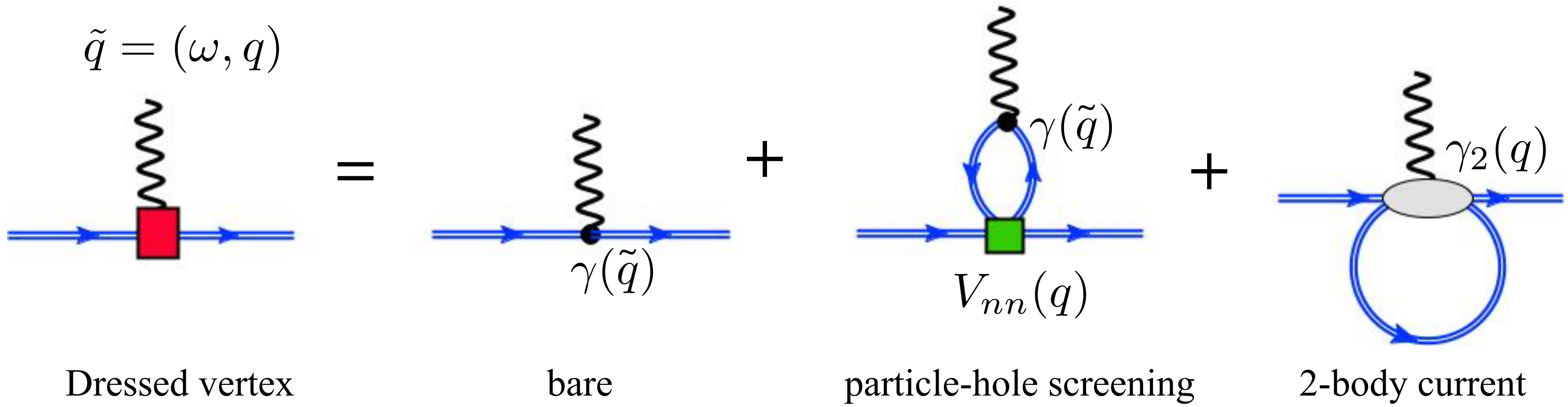
$$\mathcal{R} = \frac{\sigma^{\rm tran}_{het}(E_\nu)}{\sigma^{\rm tran}_{hom}(E_\nu)} = \frac{\sigma^{\rm tran}_{coh}(E_\nu) + \sigma^{\rm tran}_{gas}(E_\nu)}{\sigma^{\rm tran}_{hom}(E_\nu)} = \frac{n_A N_{\rm w}^2 \langle S_{cl}(E_\nu)\rangle + (1+5(C_A^n)^2) S_{gas}(\mu_n^{het},T) + 5(C_A^p)^2 S_{gas}(\mu_p^{het},T)}{(1+5(C_A^n)^2) S_{gas}(\mu_n^{hom},T) + 5(C_A^p)^2 S_{gas}(\mu_p^{hom},T)}$$

Coulomb Correlations between nuclei / pasta reduces the response. Heterogeneous phase is a low-pass filter !



Margueron, Reddy, Roberts & Roggero (2017)

10^{14} g/cm^3 uniform neutron-rich matter



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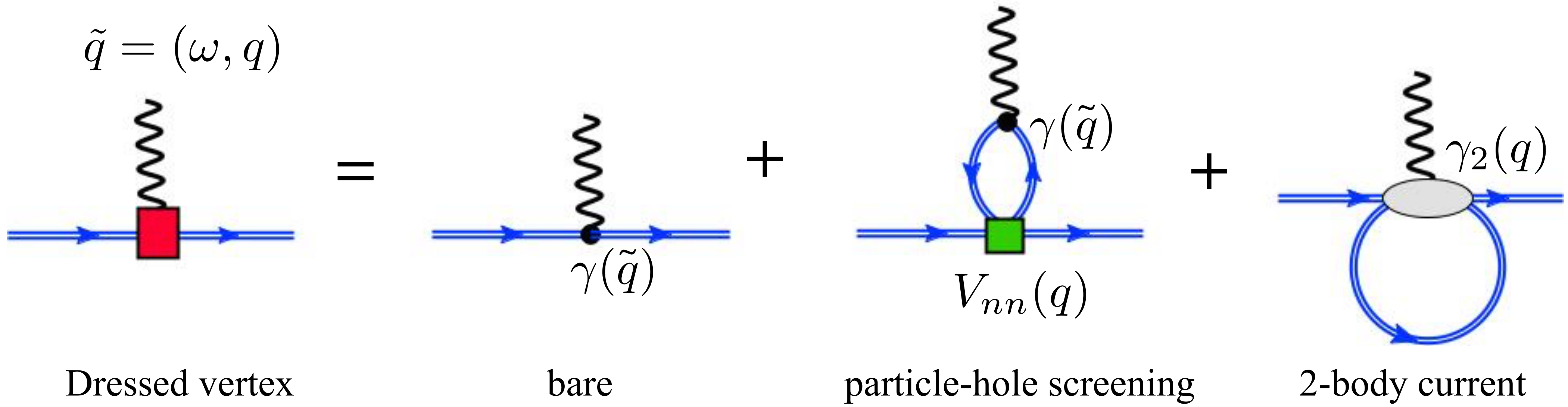

- Corrections due to screening, 2-body currents and 2p-2h excitations are all large. No expansion parameter - results rely on (uncontrolled) many-body approximations.

- Need re-summations - Random Phase Approximation or RPA.

- A lot of work in this direction suggests that both the density and soon response is reduced by factors of 2-4.

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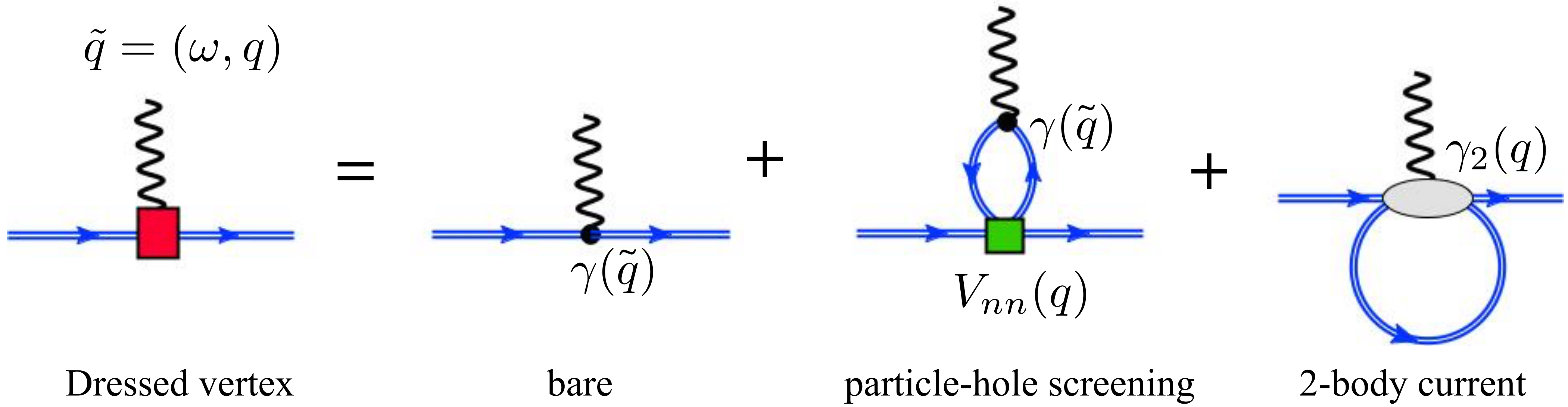

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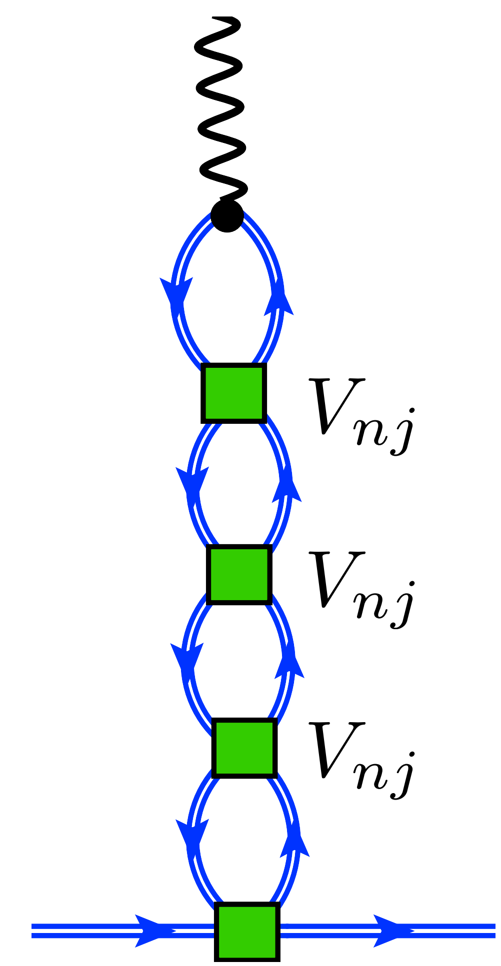

Spin-Response of Neutron Matter: Guidance from Quantum Monte Carlo

Going beyond RPA: Sum-rules can be calculated with QMC.

$$S_\sigma^n = \int_{-\infty}^{\infty} d\omega\ \omega^n\ S(\omega, q \simeq 0) \qquad \bar{\omega}_0 = \frac{S_\sigma^0}{S_\sigma^{-1}} \qquad \bar{\omega}_1 = \frac{S_\sigma^1}{S_\sigma^0}$$

Density (fm^{-3})	S_σ^{-1} (MeV^{-1})	S_σ^0	S_σ^{+1} (MeV)	$\bar{\omega}_0$ (MeV)	$\bar{\omega}_1$ (MeV)
$n = 0.12$	0.0057(9)	0.20(1)	8(1)	35(9)	40(8)
$n = 0.16$	0.0044(7)	0.20(1)	11(1)	46(11)	55(8)
$n = 0.20$	0.0038(6)	0.18(1)	14(1)	47(12)	78(10)

Shen, Gandolfi , Carlson, Reddy (2012)

In the vicinity of nuclear density QMC sum-rules indicate significant strength at

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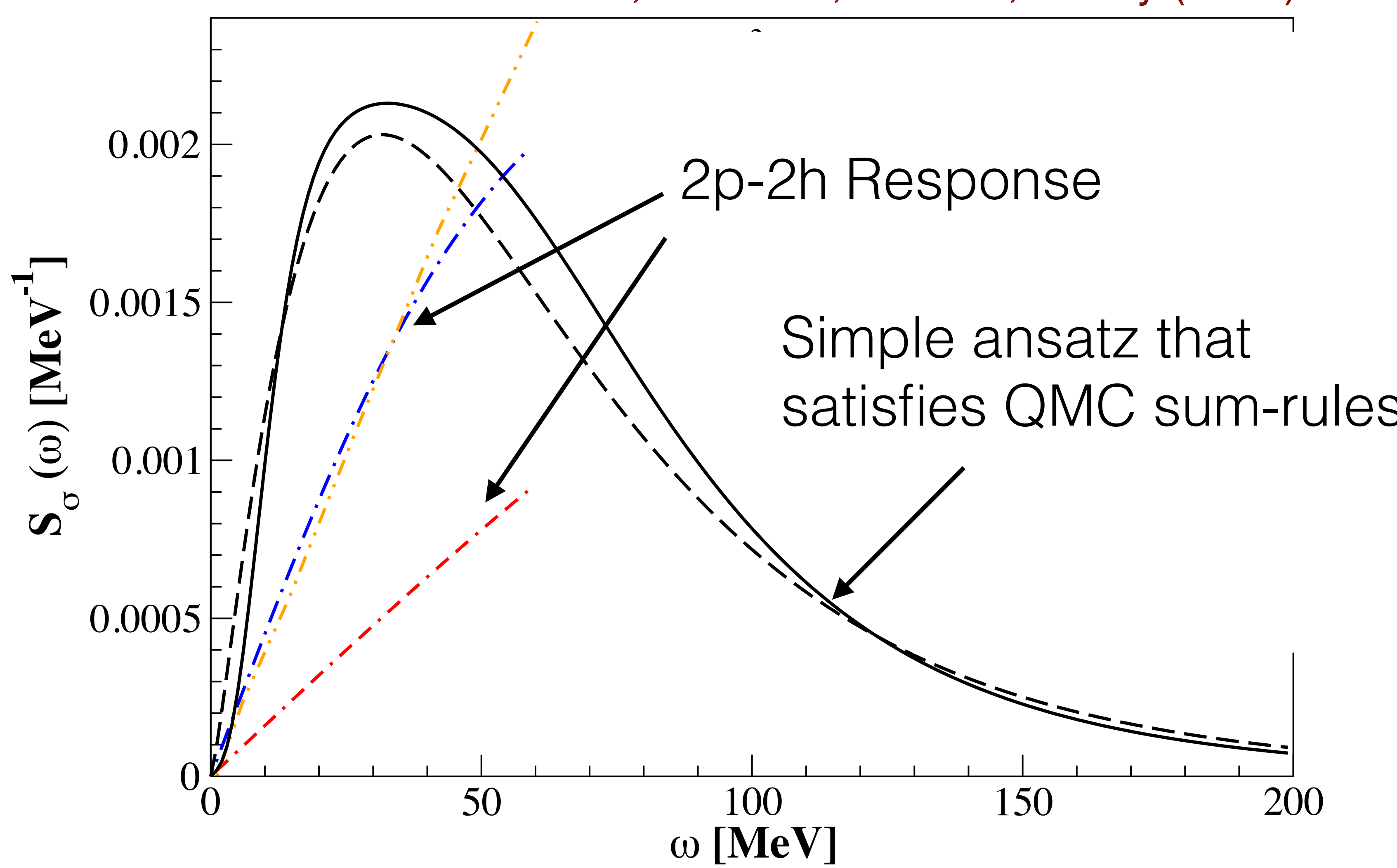

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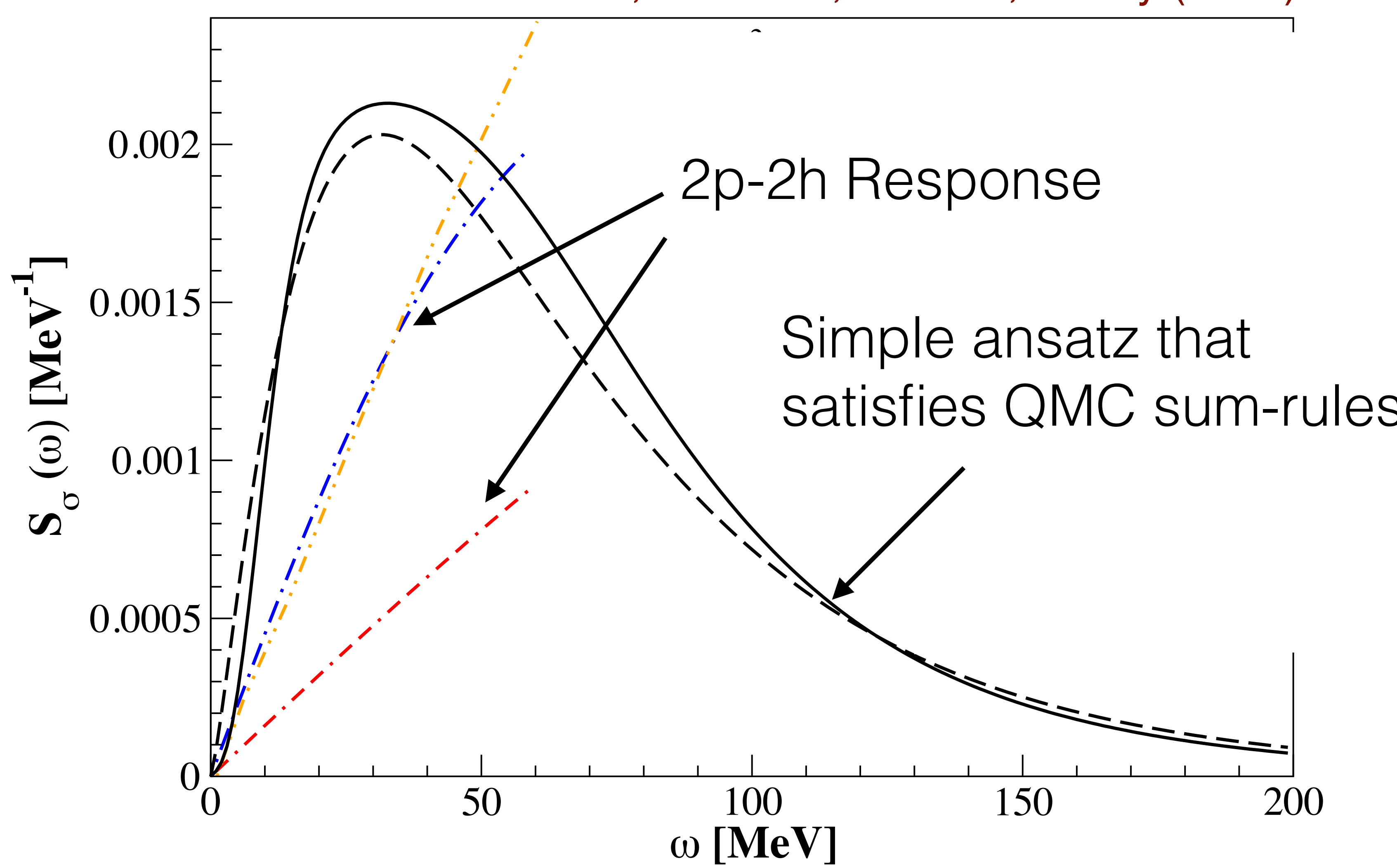
$$\omega \simeq 30 - 50\ \mathrm{MeV}$$

Energy scale is large compared to

$$\frac{q^2}{2m} \quad \text{or} \quad q \times v_F$$

Shen, Gandolfi , Carlson, Reddy (2012)



Conclusions

- Effects due nuclear interactions on the density, spin and isospin susceptibility impacts neutrino transport and spectra in supernovae and mergers. Affects SN and BNS mergers: explosion mechanism, nucleosynthesis, mass ejection, and detections.
- First steps towards an ab-inito approach to calculating the dynamic structure factors at densities and temperatures of interest to the neutrino-sphere are encouraging.
- Nuclei and coherent neutrino scattering are reduced in hot neutron-rich matter at small Y_e. Pasta dissolves rapidly.
- At high density sum rules from ab initio theory can be useful to construct reliable models for the dynamic response.
- It is essential to ensure consistency between the EoS and neutrino opacities in simulations.