From light-nuclei to neutron stars within chiral dynamics

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The *basic model* of Nuclear Theory

The *basic model* of nuclear theory aims at achieving a comprehensive description of the wealth of data and peculiarities exhibited by nuclear systems. It should simultaneously describe:

- Nucleon-nucleon (NN) scattering data: "thousands" of experimental data available such as differential and total cross sections, polarizations, asymmetries, etc…
- The spectra, properties, and transition of nuclei: binding energies, radii, magnetic moments, beta decays rates, weak/radiative captures, electroweak form factors, etc.
- The nucleonic matter equation of state: neutrons stars with masses of order twice the solar mass

Inputs for the *basic model*:

Many-body interactions between the constituents

$$H = \underbrace{\sum_{i=1}^{A} \frac{\mathbf{p}_i^2}{2m_i}}_{\text{One-body}} + \underbrace{\sum_{i<j=1}^{A} \overbrace{v_{ij}}^{\text{th+exp}}}_{\text{Two-body (NN)}} + \underbrace{\sum_{i<j<k=1}^{A} \overbrace{V_{ijk}}^{\text{th+exp}}}_{\text{Three-body (3N)}} + \underbrace{\ldots\ldots}_{\text{Many-body}}$$

One-body — Kinetic energy; Two-body (NN), Three-body (3N), Many-body — Potential energy

Electroweak current operators:

$$j^{\mathrm{EW}} = \sum_{i=1}^{A} j_i + \sum_{i<j=1}^{A} j_{ij} + \sum_{i<j<k=1}^{A} j_{ijk} + \ldots.$$

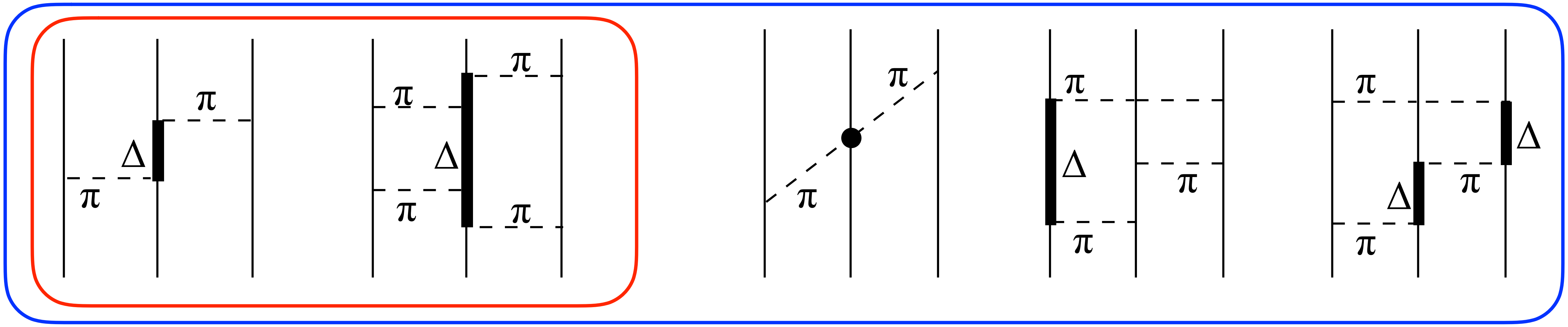
Phenomenological formulation of the *basic model*

❖ NN Argonne V18 Wiringa. Stoks, Schiavilla, PRC **51**, 38 (1995)

$$v_{18}(r_{12}) = v^{\gamma}_{12} + v^{\pi}_{12} + v^{I}_{12} + v^{S}_{12} = \sum_{p=1}^{18} v^{p}(r_{12}) O^{p}_{12}$$

- 42 independent parameters controlled by ~4300 np and pp scattering data below 350 MeV lab energy

❖ 3N Urbana-Illinois J. Carlson et al. NP **A401**, 59 (1983) S. Pieper et al. PRC **64**, 014001 (2001)



Good description for s-shell nuclei (A=3,4) and neutron stars; inadequate description of the absolute binding energies and spin-orbit splitting of heavier nuclei

Good description for light nuclei up to A=12; inadequate description of the neutron star matter equation of state

Pros:
- Very good description of several nuclear observables in particular GFMC binding energies up to A=12 with AV18+IL7 (GFMC energies: uncertainties within 1-2%)

Cons:
- Phenomenological interactions are phenomenological, not clear how to improve their quality
- They do not provide rigorous schemes to consistently derive NN and 3N forces and compatible electroweak currents

χEFT formulation of the *basic model*

S. Weinberg, Phys. Lett. **B251**, 288 (1990); Nucl. Phys. **B363**, 3 (1991); Phys. Lett **B295**, 114 (1992)

❖ In χEFT the symmetries of QCD, including its approximate chiral-symmetry, are employed to constrain the interactions of pions (π) among other pions, baryons (N and Δ-isobars) or external fields (such as electroweak)

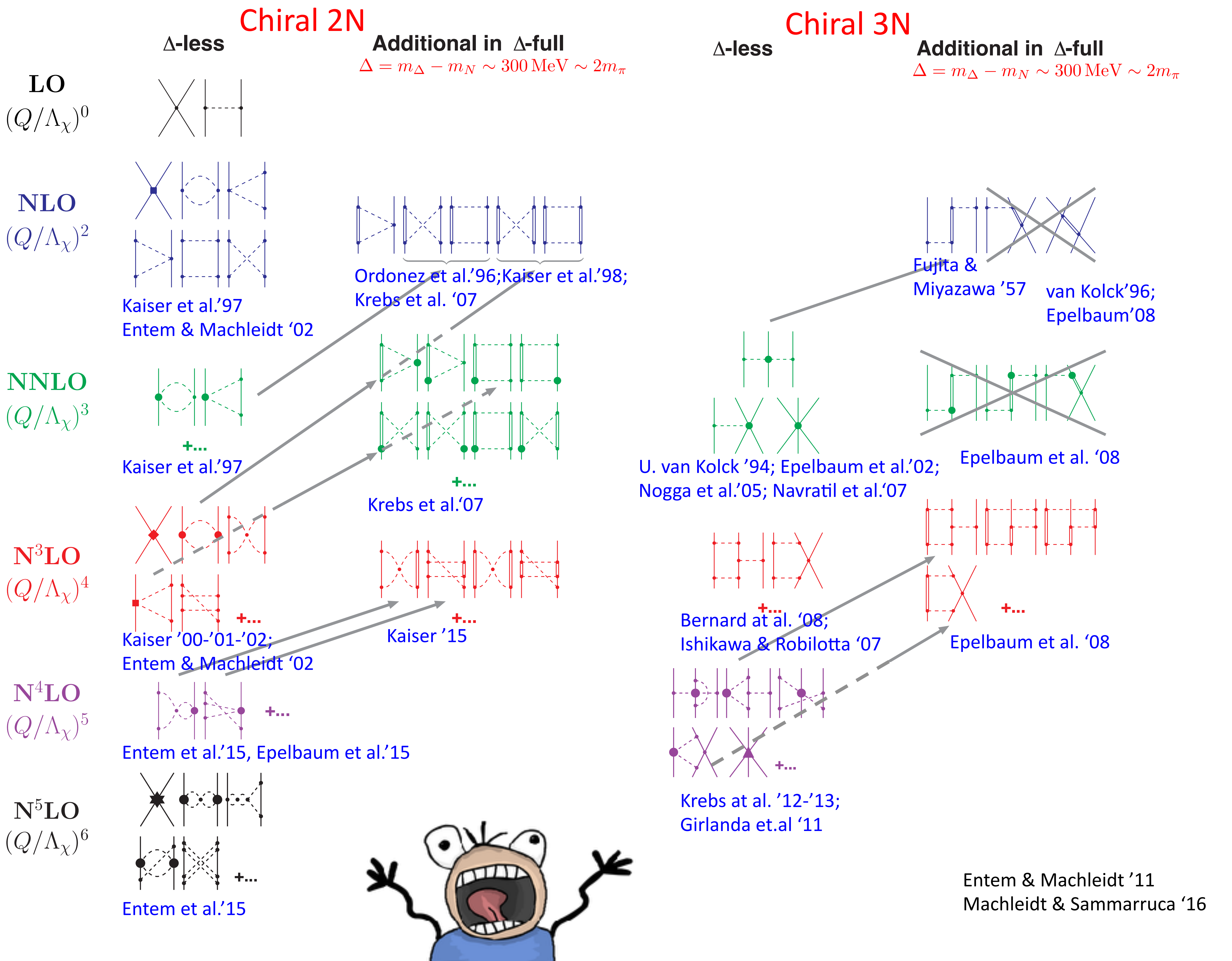
❖ In particular, π's couple to baryons by powers of its momentum *Q*, and the Lagrangian ($\mathscr{L}_{eff}$) can be expanded systematically in powers of *Q*/Λ (according to a power counting scheme); (Q << Λ ≈ 1 GeV is the chiral-symmetry breaking scale and $Q \sim m_\pi$)

$$\mathcal{L}_{eff} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \ldots$$

❖ χEFT allows for a perturbative treatment in terms of powers of a Q—as opposed to a coupling constant— expansion

❖ $\mathcal{L}^{(n)}$ also include contact $(\overline{N}N)(\overline{N}N)$ -type interactions parametrized by low-energy constants (LECs)

❖ The χEFT provides a practical scheme to construct potentials and currents, which can be systematically improvements

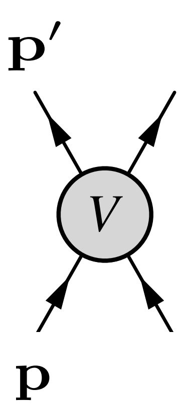

Chiral 2N
Δ-less
Additional in Δ-full
$\Delta = m_\Delta - m_N \sim 300\,\mathrm{MeV} \sim 2m_\pi$
Chiral 3N
Δ-less
Additional in Δ-full
$\Delta = m_\Delta - m_N \sim 300\,\mathrm{MeV} \sim 2m_\pi$
LO
$(Q/\Lambda_\chi)^0$
NLO
$(Q/\Lambda_\chi)^2$
NNLO
$(Q/\Lambda_\chi)^3$
N^{3}LO
$(Q/\Lambda_\chi)^4$
N^{4}LO
$(Q/\Lambda_\chi)^5$
N^{5}LO
$(Q/\Lambda_\chi)^6$
Kaiser et al.'97
Entem & Machleidt '02
Ordonez et al.'96;Kaiser et al.'98;
Krebs et al. '07
Fujita &
Miyazawa '57
van Kolck'96;
Epelbaum'08
+...
Kaiser et al.'97
Krebs et al.'07
U. van Kolck '94; Epelbaum et al.'02;
Nogga et al.'05; Navratil et al.'07
Epelbaum et al. '08
+...
Kaiser '00-'01-'02;
Entem & Machleidt '02
+...
Kaiser '15
Bernard at al. '08;
Ishikawa & Robilotta '07
Epelbaum et al. '08
+...
Entem et al.'15, Epelbaum et al.'15
+...
Krebs at al. '12-'13;
Girlanda et.al '11
+...
Entem et al.'15
Entem & Machleidt '11
Machleidt & Sammarruca '16

Chiral 2N Potentials: some recents developments

- First generation of chiral NN potential up to N3LO:
  Entem-Machleidt PRC **68**, 041001 2003; Epelbaum-Gloeckle-Meissner JNP **A747**, 362 2005
- Optimized N2LO NN potential (πN LECs are tuned to NN peripheral scattering):
  Ekström et al. PRL **110**, 192502 2013; JPG **42**, 034003 2015
- N2LO potential: a simultaneous fit of NN and 3N forces to low NN data (E_{lab}=35 MeV), deuteron BE, BE and CR of hydrogen, helium, carbon and oxygen isotopes;
  Carlsson et al. PRC **91**, 051301(R) 2015
- New generation of chiral NN potentials up to N4LO: improved choice of the regulator, no SFR
  Epelbaum et al. PRL **112**, 102501 2014; EPJ **A51**, 53 2015; PRL **115**, 122301 2015
- Chiral 2π and 3π exchange up to N4LO and up to N5LO in NN peripheral scattering;
  Entem et al. PRC **91**, 014002 2015; PRC **92**, 064001 2015, arXiv:1703.05454 2017
- The LENPIC collaboration arXiv: 1705.01530v1

Note: Many of the available versions of chiral potentials are formulated in momentum-space and are strongly nonlocal: ⇨ $\mathbf{p} \to -i\boldsymbol{\nabla}$ hard to use in QMC methods

Nonlocalities due to contact interactions and to regulator functions



Nonlocal regulator $V_{\rm NN}(\mathbf{p},\mathbf{p}') \to \exp\left[-[(\mathbf{p}^2+\mathbf{p}'^2)/\Lambda^2]^n\right] V_{\rm NN}(\mathbf{p},\mathbf{p}')$

Local regulator $V_{\rm NN}(\mathbf{p},\mathbf{p}') \to \exp\left[-[(\mathbf{p}'-\mathbf{p})^2/\Lambda^2]^n\right] V_{\rm NN}(\mathbf{p},\mathbf{p}')$

- Local NN potentials up to N2LO:
  Gezerlis et al. PRL **111**, 032501 2013; PRC **90**, 054323 2014; Lynn et al. PRL **113**, 192501 2014
- Minimally nonlocal/local NN potentials including N2LO Δ contributions;
  Piarulli et al. PRC **91**, 024003 2015; PRC **94**, 054007 2016

Local chiral NN potential: EM, LR and SR components

- The chiral NN potential we designed can be written as: $v_{12} = v_{12}^{\rm EM} + v_{12}^{\rm L} + v_{12}^{\rm S}$

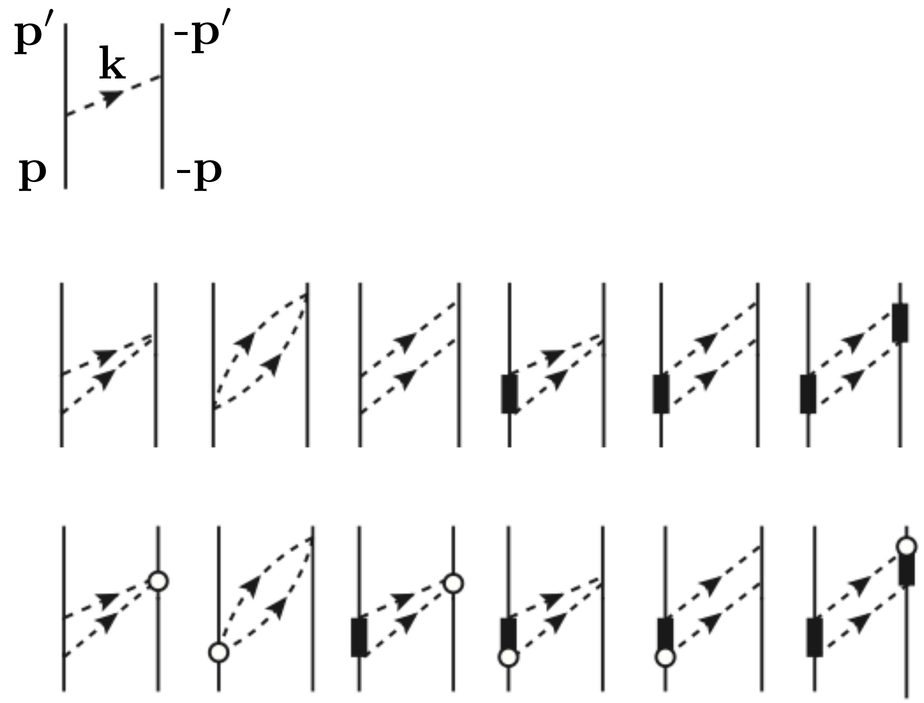
$v_{12}^{\rm EM}$: EM component including corrections up to α^2

$v_{12}^{\rm L}$: long-range component including

LO : Q^0

NLO : Q^2

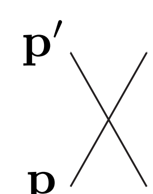
N2LO : Q^3



- dependence only on the momentum transfer **k=p′-p**
- known LECs: g_A, F_π , $h_A = 3\, g_A/\sqrt{2}$
- unknown LECs: $c_1,\ c_2,\ c_3,\ c_4\ (\mathcal{L}^{(2)}_{\pi N})$ $b_3 + b_8\ (\mathcal{L}^{(2)}_{\pi N \Delta})$ taken from π-N scattering (Krebs at al. EPJ **A32,** 127 2007)

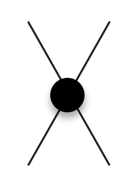
$v_{12}^{\rm S}$: short-range component including

LO : Q^0



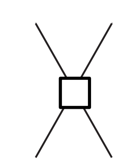
(2)

NLO : Q^2



(7)

N3LO : Q^4



(15)

- At NLO and N3LO strongly nonlocal contact terms: proportional to K^2 and K^4 where **K=(p′+p)**/2; we use Fierz rearrangements to remove these nonlocalities (see also Gezerlis et al. PRL **111**, 032501 2013; PRC **90**, 054323 2014)

- Contact terms of type **k×K** or k^2 K^2 still persist: they can not Fierz-transformed away

Local chiral NN potential: coordinate-space formulation

- In coordinate-space the short-range part of the interaction can be written as

$$v_{12} \equiv v_{12}^L + v_{12}^S = \sum_{l=1}^{16} v^l(r) O_{12}^l$$

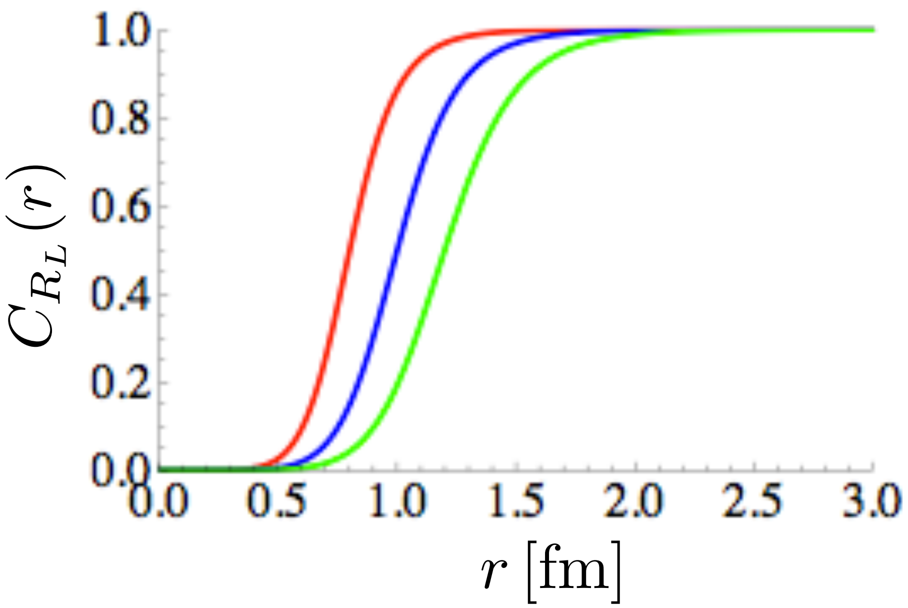
- Static part: $O_{12}^{l=1,\dots,6} = [\mathbf{1}\,,\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\,,\,S_{12}]\otimes[\mathbf{1}\,,\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2]$
- Spin-orbit, (Spin-orbit)², L² part: $O_{12}^{l=7,\dots,11} = \mathbf{L}\cdot\mathbf{S}\,,\,\mathbf{L}\cdot\mathbf{S}\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,,\,(\mathbf{L}\cdot\mathbf{S})^2\,,\,\mathbf{L}^2\,,\,\mathbf{L}^2\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$
- Charge-dependent part: $O_{12}^{l=12,\dots,16} = T_{12}\,,\,(\tau_1^z+\tau_2^z)\,,\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\,T_{12}\,,\,S_{12}\,T_{12}\,,\,\mathbf{L}\cdot\mathbf{S}\,T_{12}$

- The radial functions in v_{12}^L have divergencies of type $1/r^n\;,1\leq n\leq 6$

- $C_{R_\mathrm{L}}(r) = 1 - \dfrac{1}{(r/R_\mathrm{L})^6\,e^{(r-R_\mathrm{L})/a_L}+1}$

$R_L = (0.8, 1.0, 1.2)\ \mathrm{fm}$

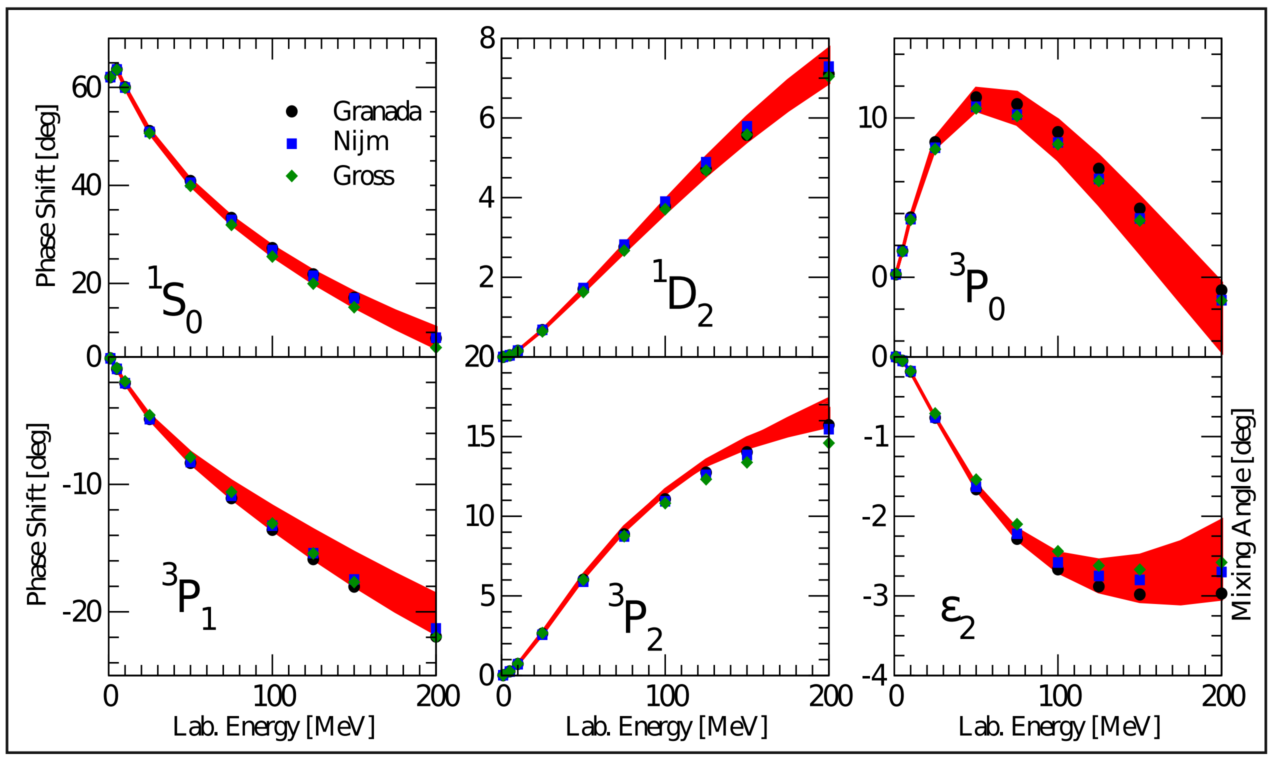
$a_L = R_L/2$

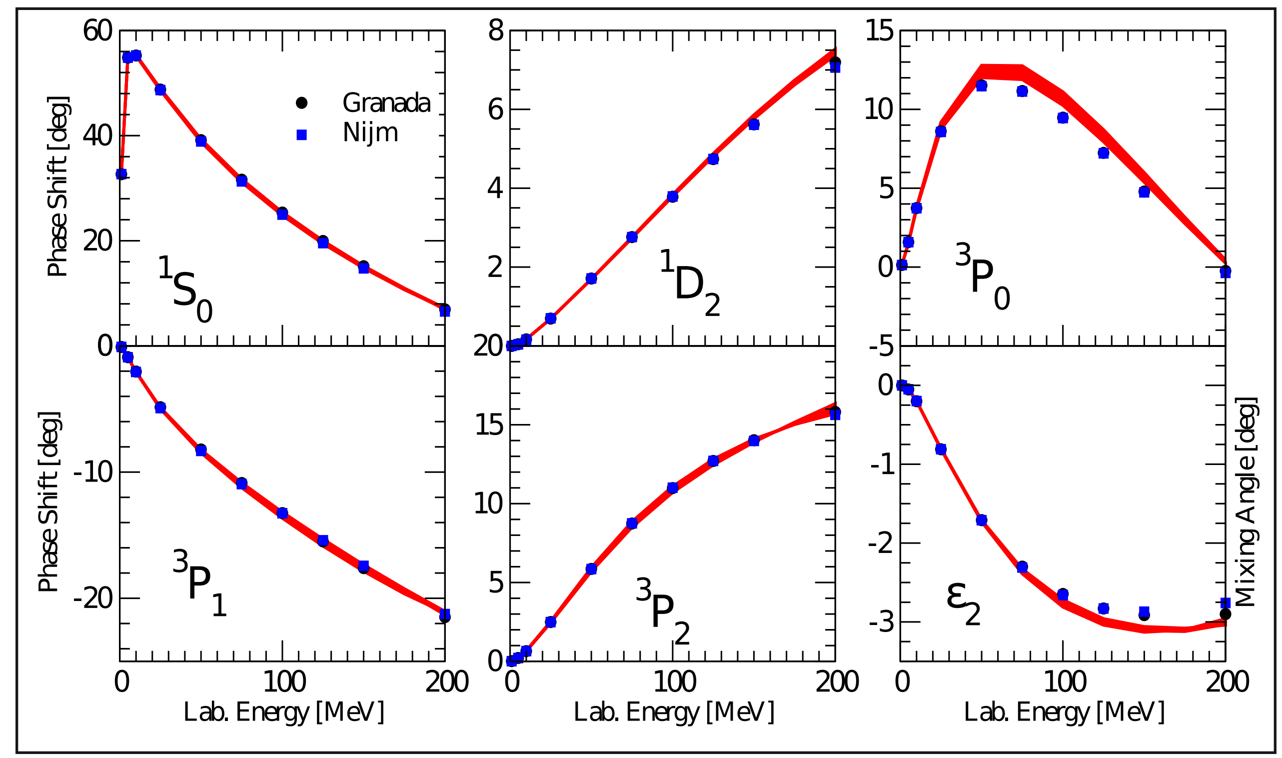


- The FT of the single contact terms is carried out with a Gaussian regulator, depending only on the momentum transfer *k*, such that
  - $\widetilde{C}_{R_\mathrm{S}}(k) = e^{-R_\mathrm{S}^2 k^2/4} \longrightarrow C_{R_\mathrm{S}}(r) = \dfrac{1}{\pi^{3/2}R_\mathrm{S}^3}e^{-(r/R_\mathrm{S})^2}$ $\qquad R_S = (0.6, 0.7, 0.8)\ \mathrm{fm}$

Fitting procedure: NN PWA and database

- The LECs fixed by fitting the pp and np Granada database up to two ranges of E_{lab} = 125 MeV and 200 MeV, the deuteron BE and the nn scattering length: we first fit the partial wave phase shifts then we refine the fit with a direct comparison with the database
- To minimizing χ^2 the we use the Practical Optimization Using No Derivatives (for Squares), POUNDers (M. Kortelainen, PRC **82**, 024313 2010)



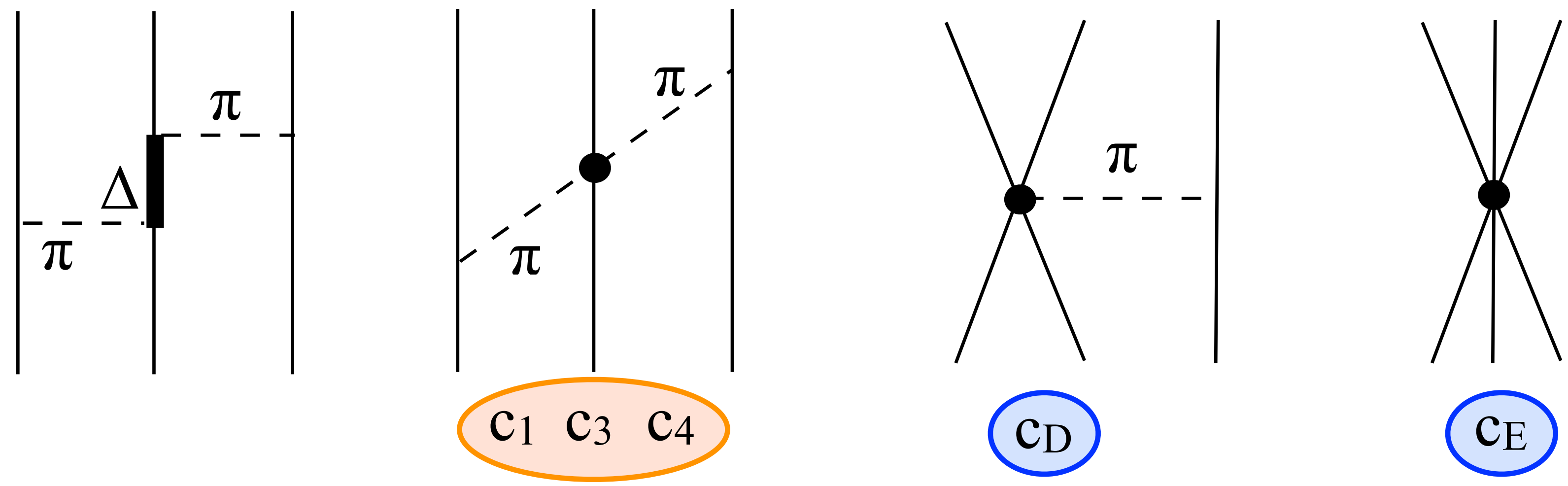


model	order	R_{L} (fm)	R_{S} (fm)	E_{LAB} (MeV)	χ^2/datum
Model b	LO	1.0	0.7	125	59.88
Model b	NLO	1.0	0.7	125	2.18
Model b	N2LO	1.0	0.7	125	2.32
Model b	N3LO	1.0	0.7	125	1.07
Model a	N3LO	1.2	0.8	125	1.05
Model c	N3LO	0.8	0.6	125	1.11
Model $\widetilde{a}$	N3LO	1.2	0.8	200	1.37
Model $\widetilde{b}$	N3LO	1.0	0.7	200	1.37
Model $\widetilde{c}$	N3LO	0.8	0.6	200	1.40

$$\text{Model a}: (R_L, R_S) = (1.2, 0.8)$$
$$\text{Model b}: (R_L, R_S) = (1.0, 0.7)$$
$$\text{Model c}: (R_L, R_S) = (0.8, 0.6)$$

Local chiral 3N potential: LR and SR components

- Inclusion of 3N forces at N2LO:



same as NN interaction | need to be fixed

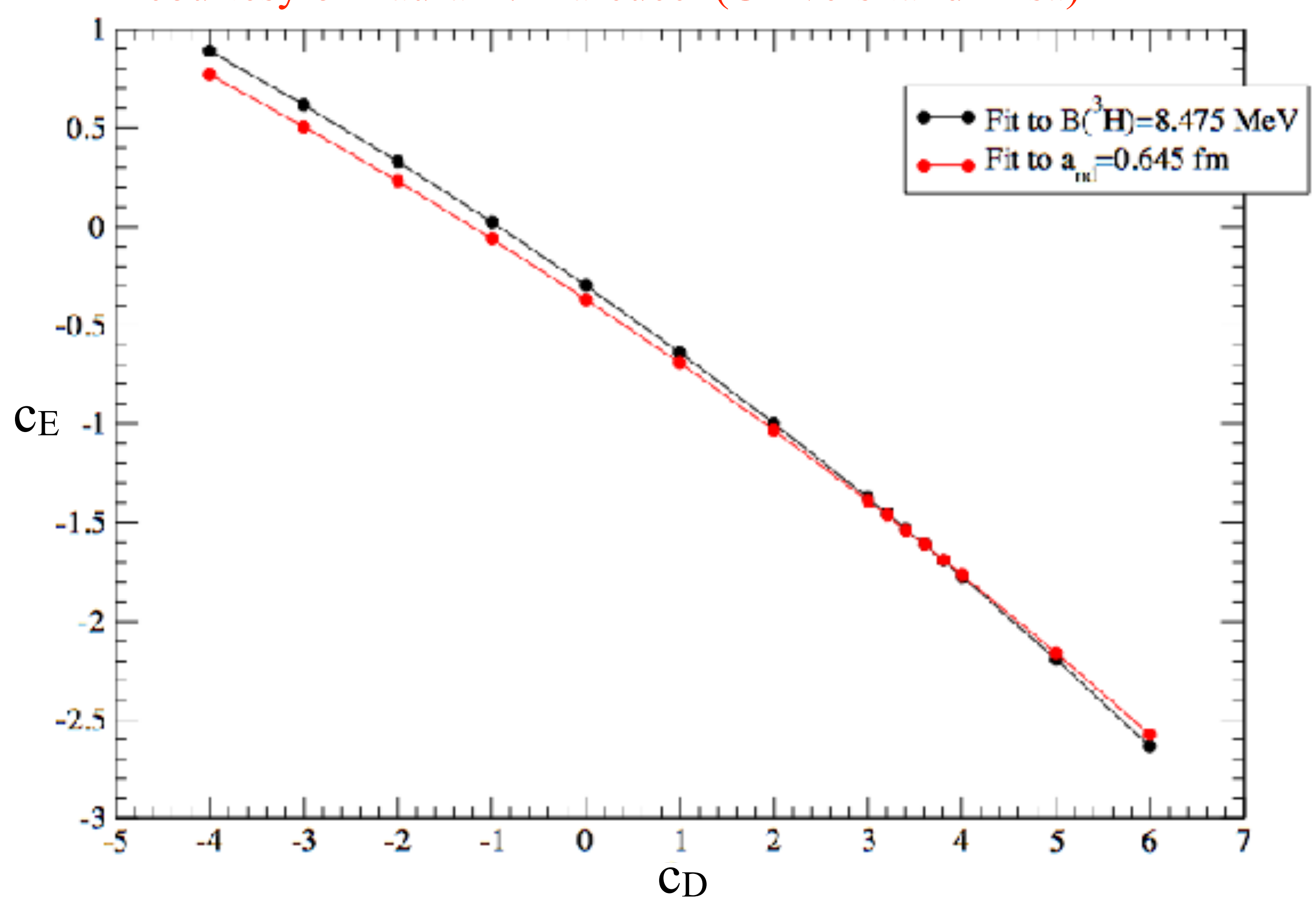
Note:
same regulator functions and cutoff used in the NN interaction

Fit to (for the time being):

- ^{3}H binding energy and nd scattering length

Model	c_D	c_E
Ia	3.666	−1.638
Ib	−2.061	−0.982
IIa	1.278	−1.029
IIb	−4.480	−0.412

courtesy of Laura E. Marcucci (Universita' di Pisa)



The Nuclear Many-Body Problem

❖ We need to solve the many-body Schrödinger equation of the system under consideration

$$H\,\Psi(\mathbf{R}; s_1, .., s_A; t_1, .., t_A) = E\,\Psi(\mathbf{R}; s_1, .., s_A; t_1, .., t_A)$$

3A coordinates in r-space

Nucleon spin

Nucleon isospin (p or n)

Bottom line:

$$2^A \times \frac{A!}{N!Z!}$$

Coupled second order differential equations in 3A dimension

96 for ^{4}He

$17{,}920$ for ^{8}Be

$3{,}784{,}704$ for ^{12}C

Very challenging problem!!!

Ab initio Methods: HH and QMC

- Hyperspherical Harmonics (HH) expansion for A=3 and 4 bound and continuum states

$$|\Psi\rangle = \sum_\mu c_\mu \underbrace{|\Phi_\mu\rangle}_{\text{HH basis}} \qquad c_\mu \quad \text{from} \quad E = \frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle}$$

Kievsky *et al.*, JPG: NPP **35,** 063101 (2008)

- Quantum Monte Carlo (QMC) methods encompass a large family of computational methods whose common aim is the study of complex quantum systems

VMC, GFMC: sampling in coordinate space

→ limited number of nucleons A=12 (new developments for A=13)

R.B. Wiringa, PRC **43**, 1585 (1991)
Carlson, *et al.*, Rev. Mod. Phys. **87**, 1067 (2015)

AFDMC: sampling in coordinate space + spin-isospin coordinate

→ larger nuclei A~50 & neutron matter

Schmidt and Fantoni, Phys. Lett. B **446**, 99 (1999)
Carlson, *et al.*, Rev. Mod. Phys. **87**, 1067 (2015)

CVMC: sampling in coordinate space + cluster expansion

→ closed shell nuclei (+/- 1): A=40

Pieper, *et al.*, Phys. Rev. C **46**, 1741 (1992)
Lonardoni, *et al.*, arXiv:1705.04337

QMC: Variational Monte Carlo (VMC)

R.B. Wiringa, PRC **43**, 1585 (1991)

- Minimize the expectation value of H:

$$E_T = \frac{\langle \Psi_T | H | \Psi_T \rangle}{\langle \Psi_T | \Psi_T \rangle} \geq E_0$$

- Trial wave function (involves variational parameters):

$$|\Psi_T\rangle = \left[1 + \sum_{i<j<k} U_{ijk}\right] \left[S \prod_{i<j} (1 + U_{ij})\right] |\Psi_J\rangle$$

$|\Psi_J\rangle = \left[\prod_{i<j} f_c(r_{ij})\right] |\Phi(JMTT_z)\rangle$ (s-shell nuclei): Jastrow wave function, fully antisymmetric

$S \prod_{i<j}$: represents a symmetrized product

$U_{ij} = \sum_{p=2,6} u_p(r_{ij})\, O^p_{ij}$: pair correlation operators

$U_{ijk} = \sum_x \epsilon_x\, V^x_{ijk}$: three-body correlation operators

$|\Psi_T\rangle$ are spin-isospin vectors in 3A dimension with $2^A \begin{pmatrix} A \\ Z \end{pmatrix}$

- The search in the parameter space is made using COBYLA (Constrained Optimization BY Linear Approximations) algorithm available in NLopt library

- The typical number of variational parameters for s-shell nuclear wave functions is about two dozen for a two-body potential; four to six parameters are added if a three-body potential is included in the Hamiltonian

QMC: Diffusion Monte Carlo (DMC)

J. Carlson et al., Rev. Mod. Phys. **87**, 1067 (2015)

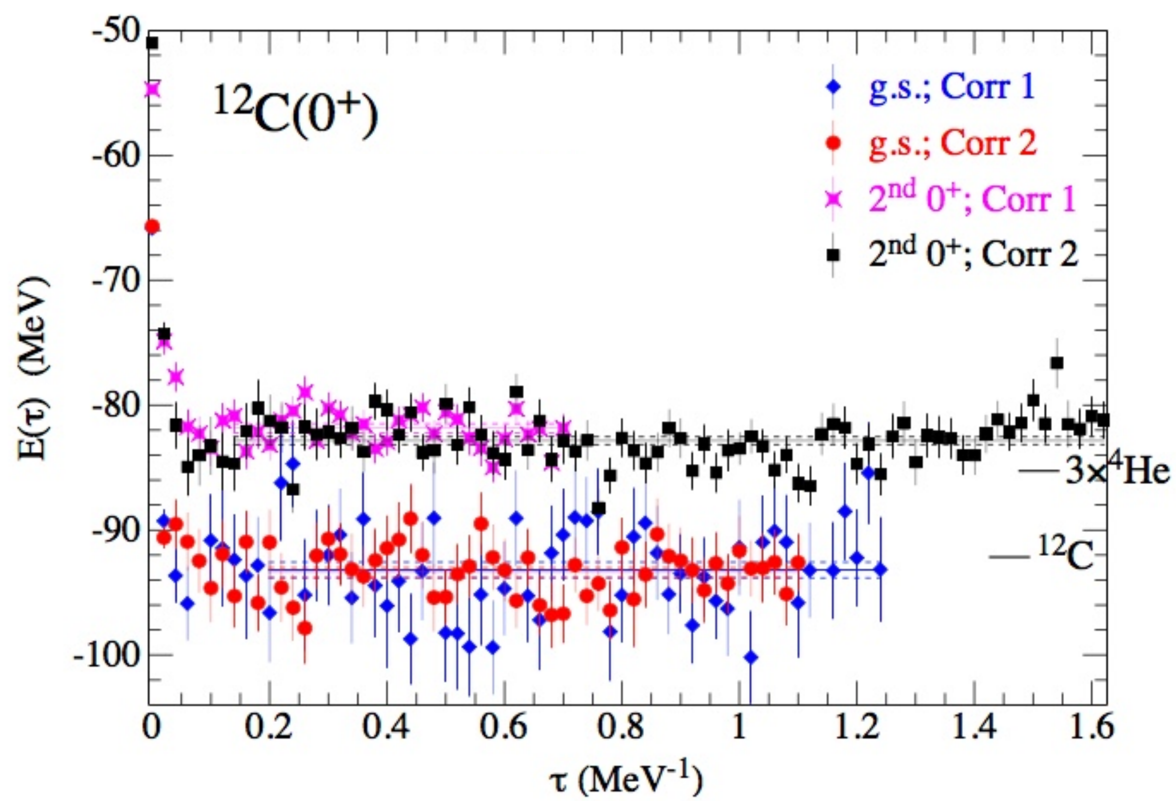
- The diffusion Monte Carlo (DMC) method (ex. GFMC or AFDMC) overcomes the limitations of VMC by using a projection technique to determine the true ground-state
- The method relies on the observation that Ψ_T can be expanded in the complete set of eigenstates of the Hamiltonian according to

$$|\Psi_T\rangle = \sum_n c_n |\Psi_n\rangle \qquad H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

$$\lim_{\tau\to\infty} |\Psi(\tau)\rangle = \lim_{\tau\to\infty} e^{-(H-E_0)\,\tau}\,|\Psi_T\rangle = c_0|\Psi_0\rangle \qquad |\Psi(\tau=0)\rangle = |\Psi_T\rangle$$

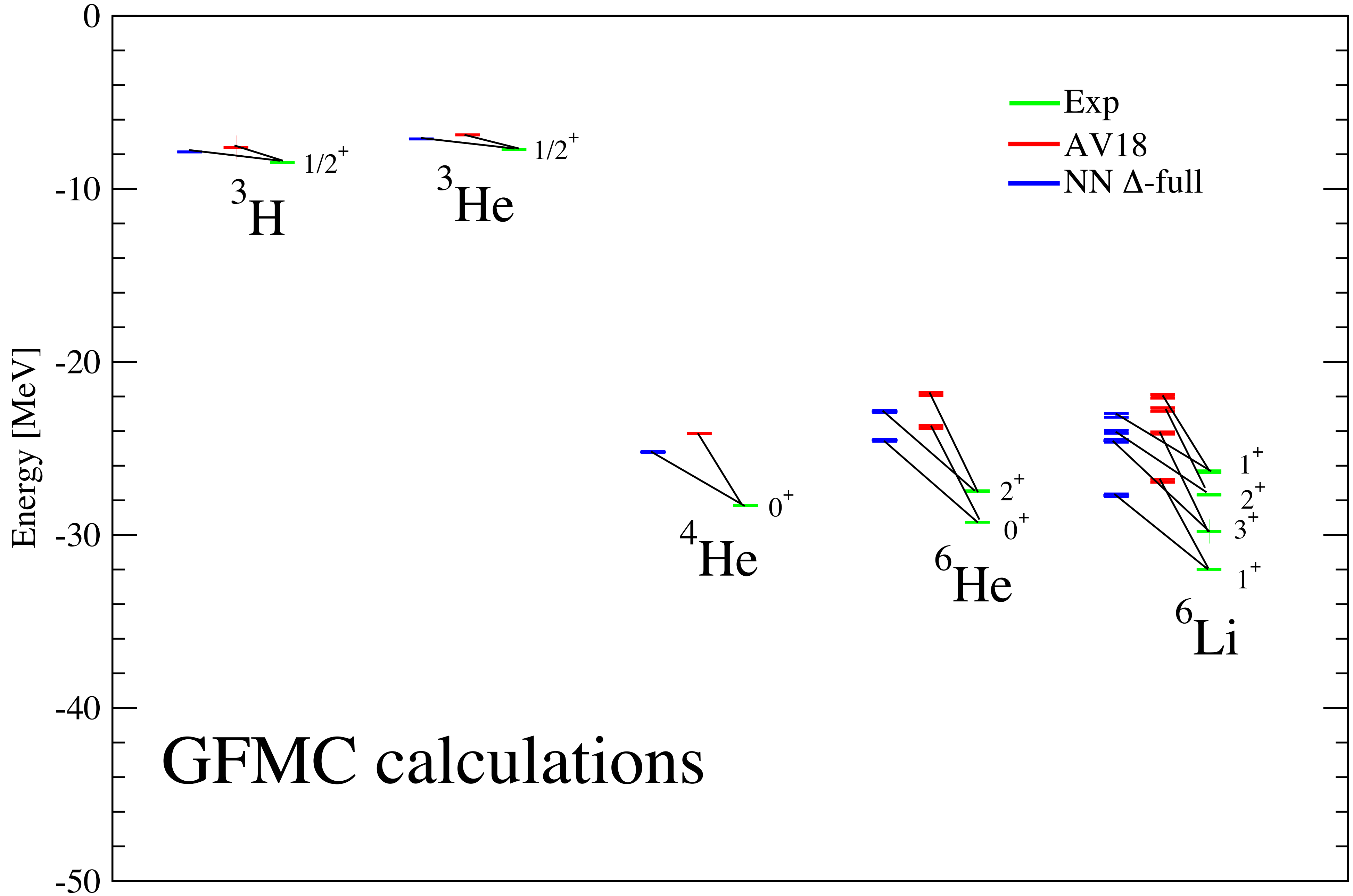
where τ is the imaginary time

- The evaluation of $\Psi(\tau)$ is done stochastically in small time steps $\Delta\tau$ ($\tau = n\,\Delta\tau$) using a Green's function formulation
- Propagator does not contain p^2, L^2, $(\mathbf{L}\cdot\mathbf{S})^2$: it is carried out with a simplified version H' of the full Hamiltonian H



QMC for A≤6 with only local NN chiral potential

- The A≤6 ground- and excited state energies with only local NN chiral interaction compared with the corresponding GFMC results obtained with AV18 and experimental values



3N interactions are needed!!

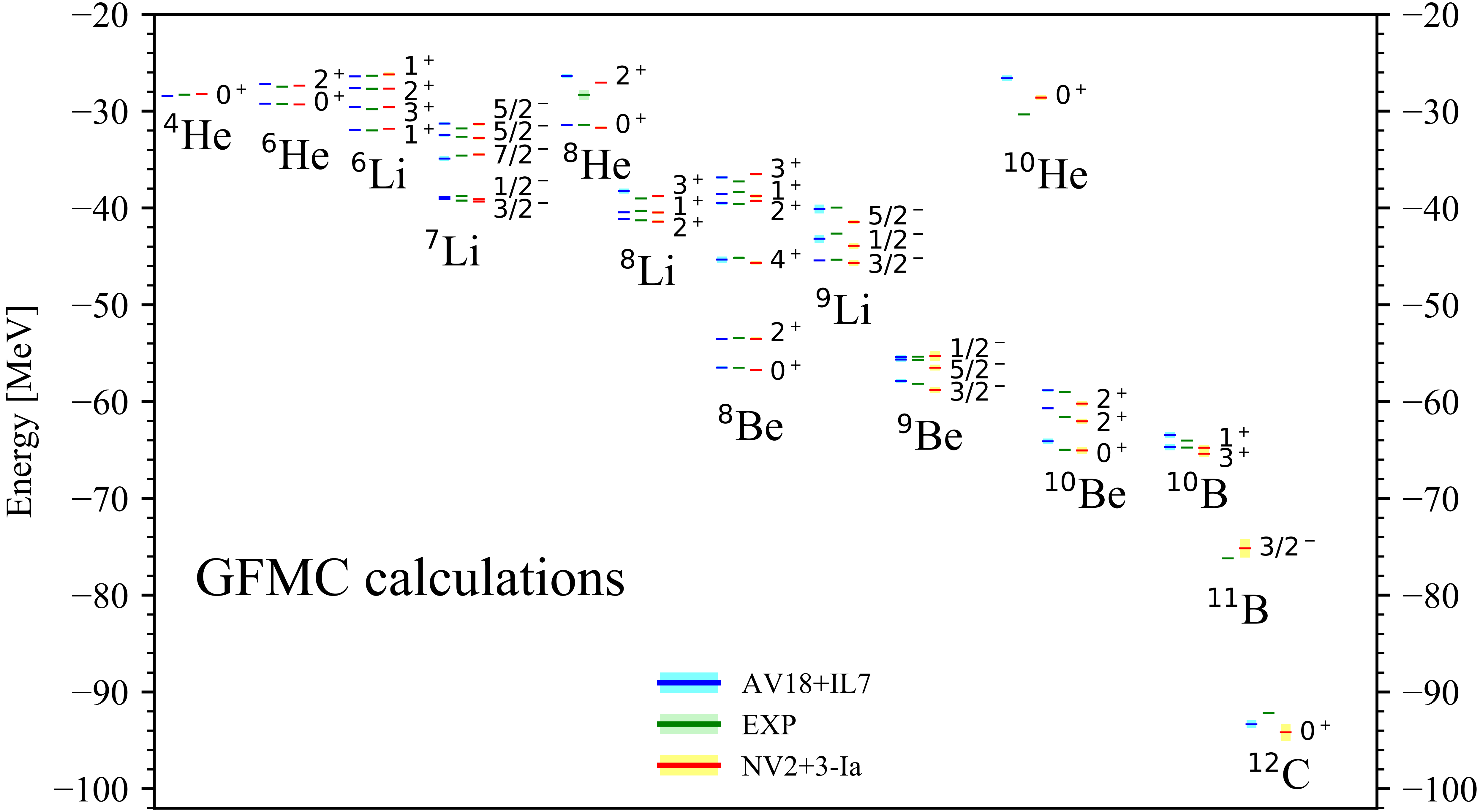
Piarulli et al. PRC **94**, 054007 2016

- For A=3, 4 benchmark with the HH calculations

- For A=3, 4, and 6 the energies differ by about 0.2-0.3, 1.0, and 0.5-1.3 MeV, respectively, from the corresponding ones obtained using the AV18

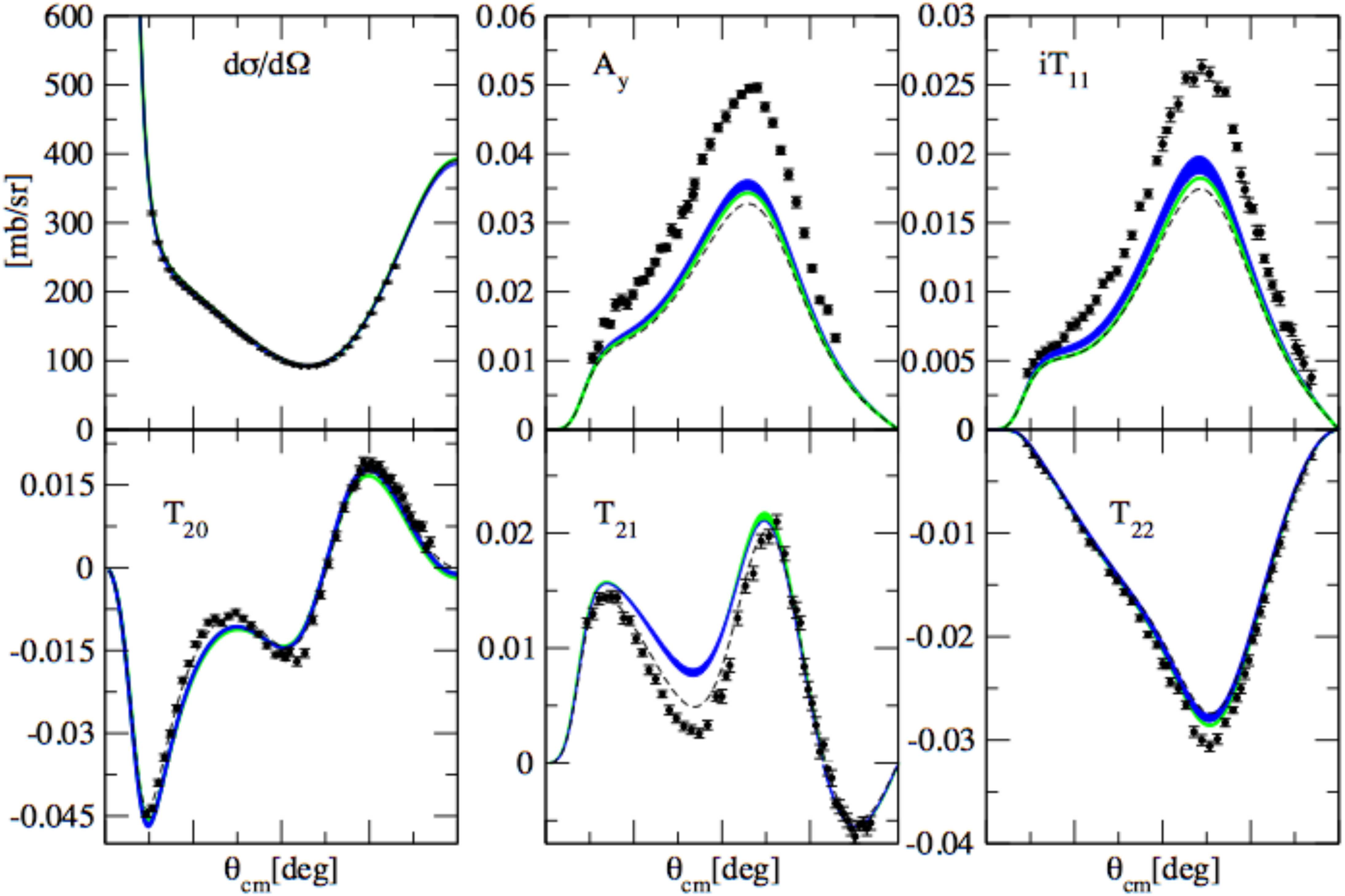
Spectra of Light Nuclei: Phenomenology vs χEFT

Piarulli *et al.* PRL **120**, 052503 (2018)



❖ The rms energy deviation from experiment for these states is 0.72 MeV for NV2+3-Ia compared to 0.80 MeV for AV18+IL7

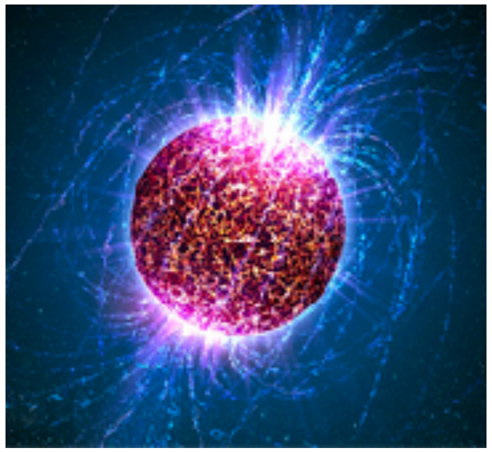
❖ Polarization observables in pd elastic scattering at 3 MeV, obtained in HH calculations with the NV2+3 models Ia-Ib (IIa-IIb), are shown by the green (blue) band. The black dashed line are results obtained with only the two-body interaction NV2-Ia



subleading contact terms in 3N interaction??? Additional 10 LECs

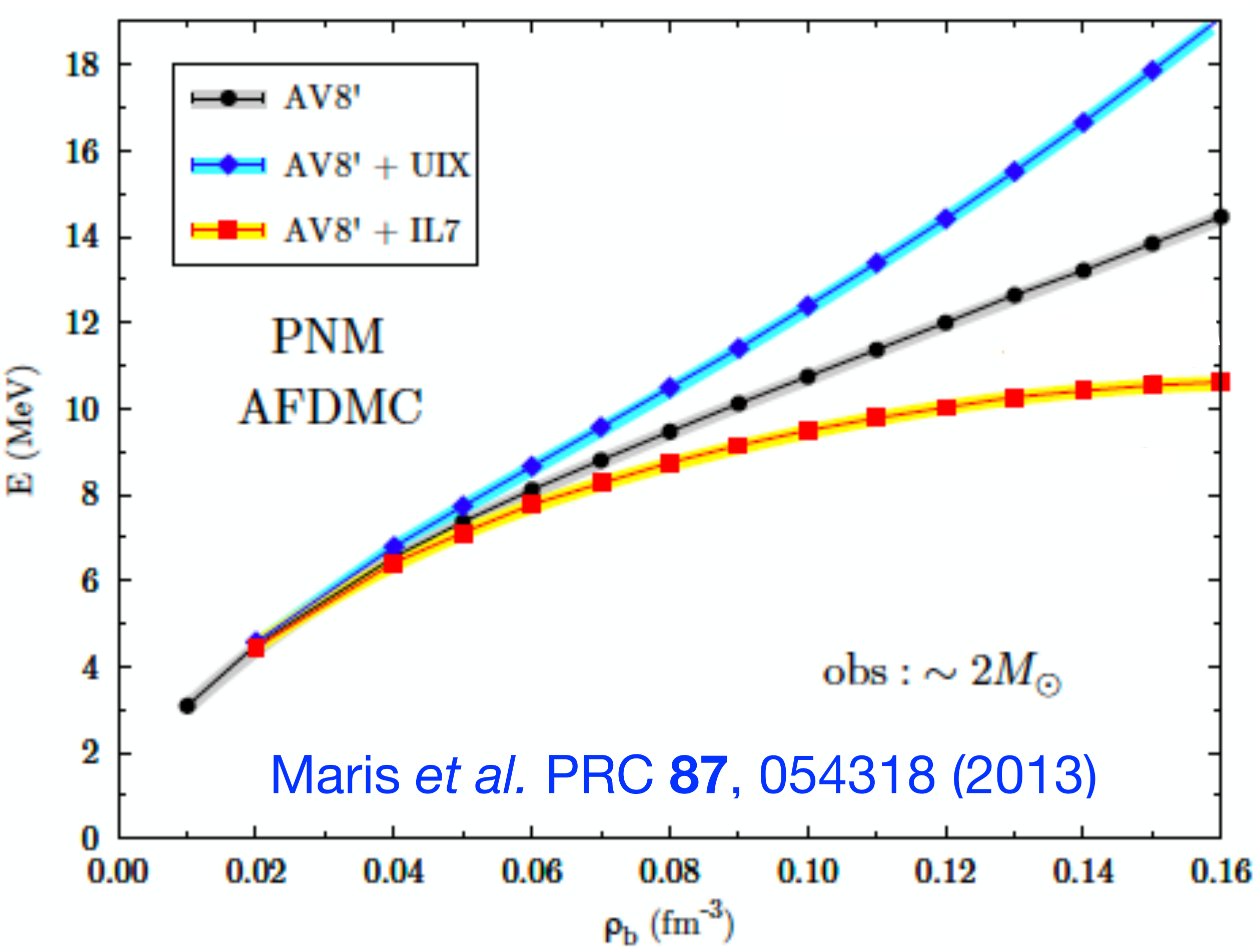
Equation of State of Pure Neutron Matter

❖ The EoS of pure neutron matter (PNM): neutrons stars



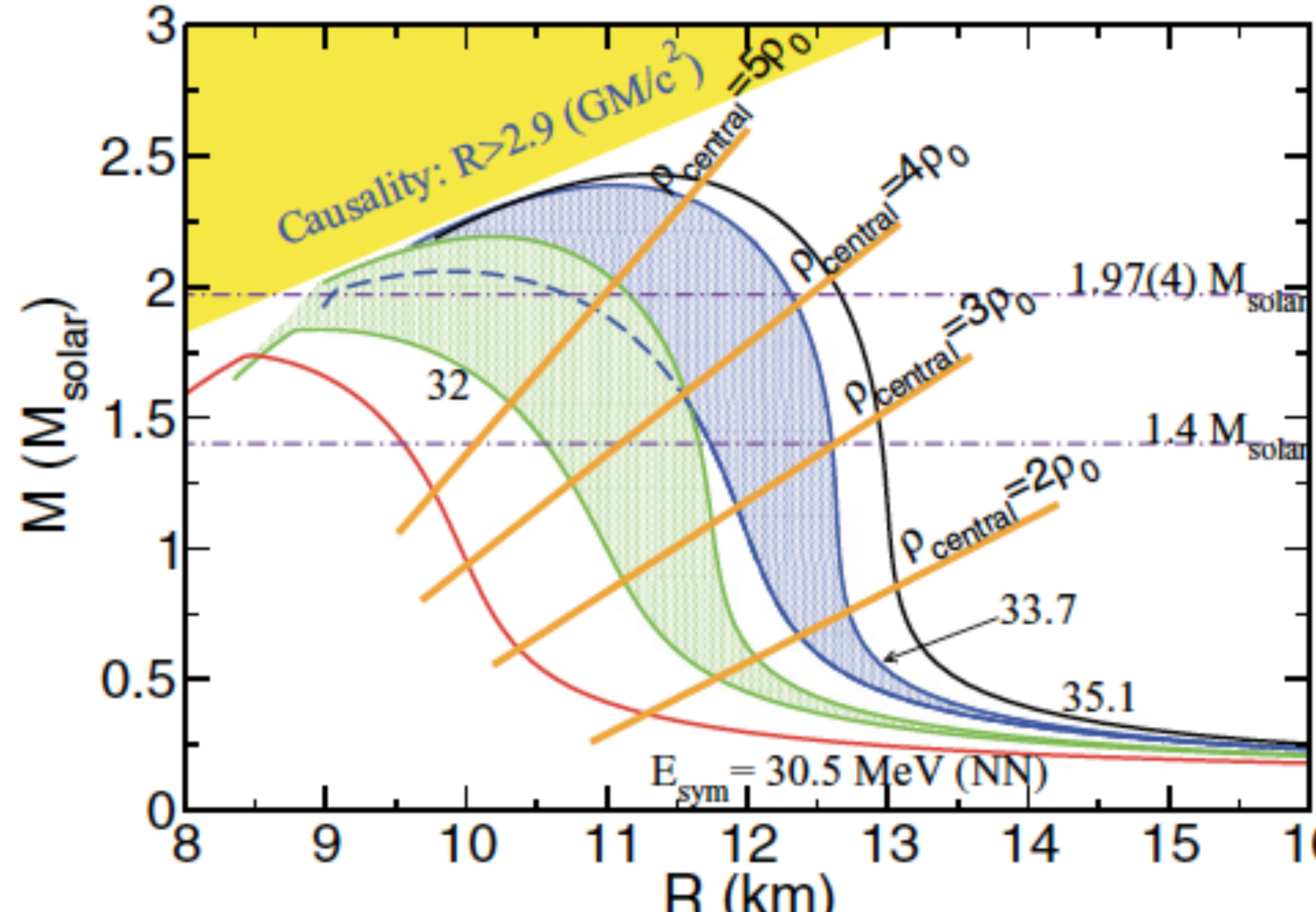
- Compact objects: R ~ 10km, $M_{\rm max}^{\rm obs} \sim 2M_{\odot}$
- Composed predominantly of neutrons between the inner crust and the outer core
- NS from gravitational collapse of a massive star after a supernova explosion

❖ Using AFDMC to obtain EoS of PNM: finite number of particles in a box imposing periodic boundary conditions (66 particles); sum on different boxes to reduce finite-size effects



Maris *et al.* PRC **87**, 054318 (2013)

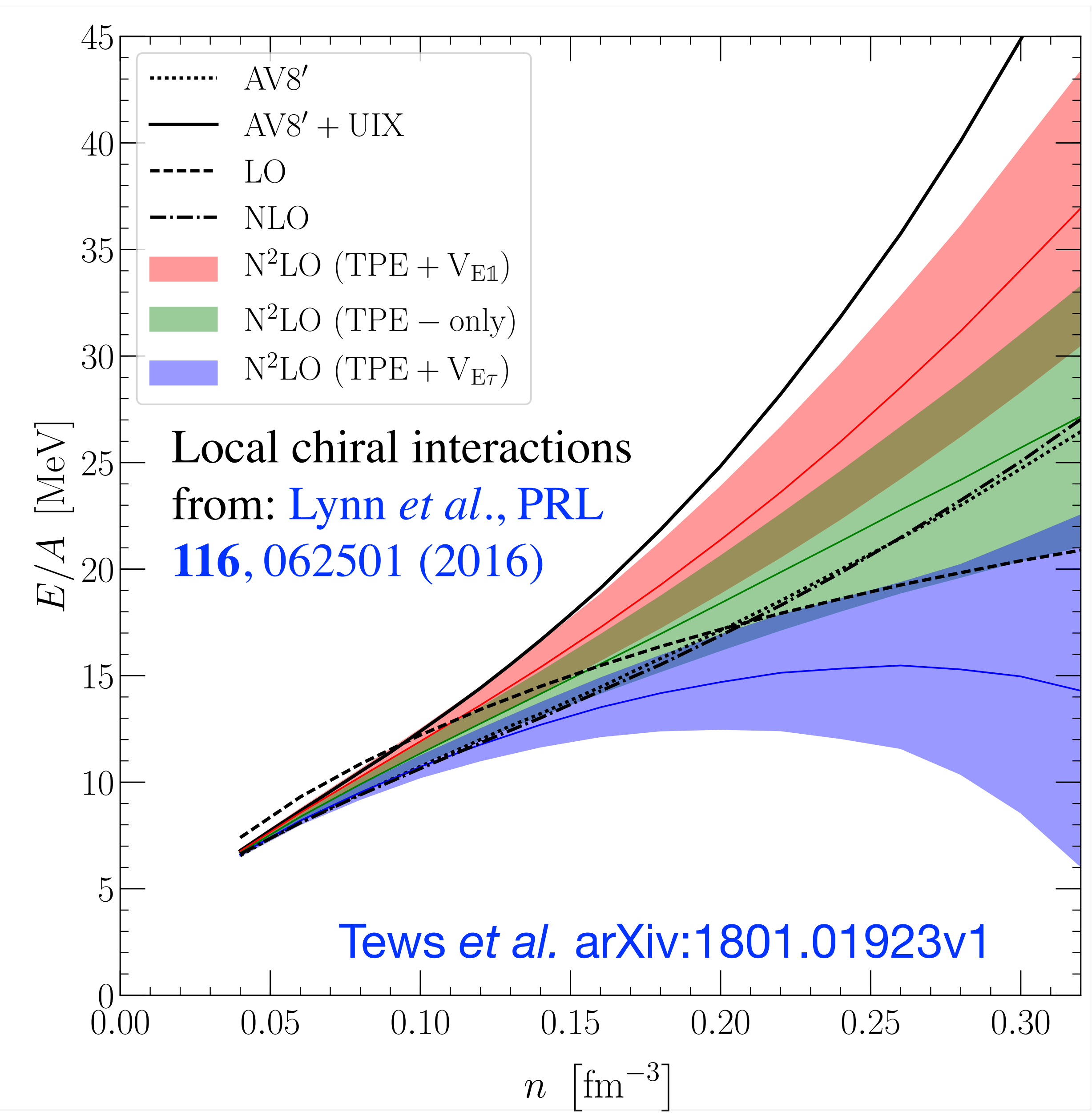
Gandolfi *et al.* PRC **85**, 032801(R 2012)

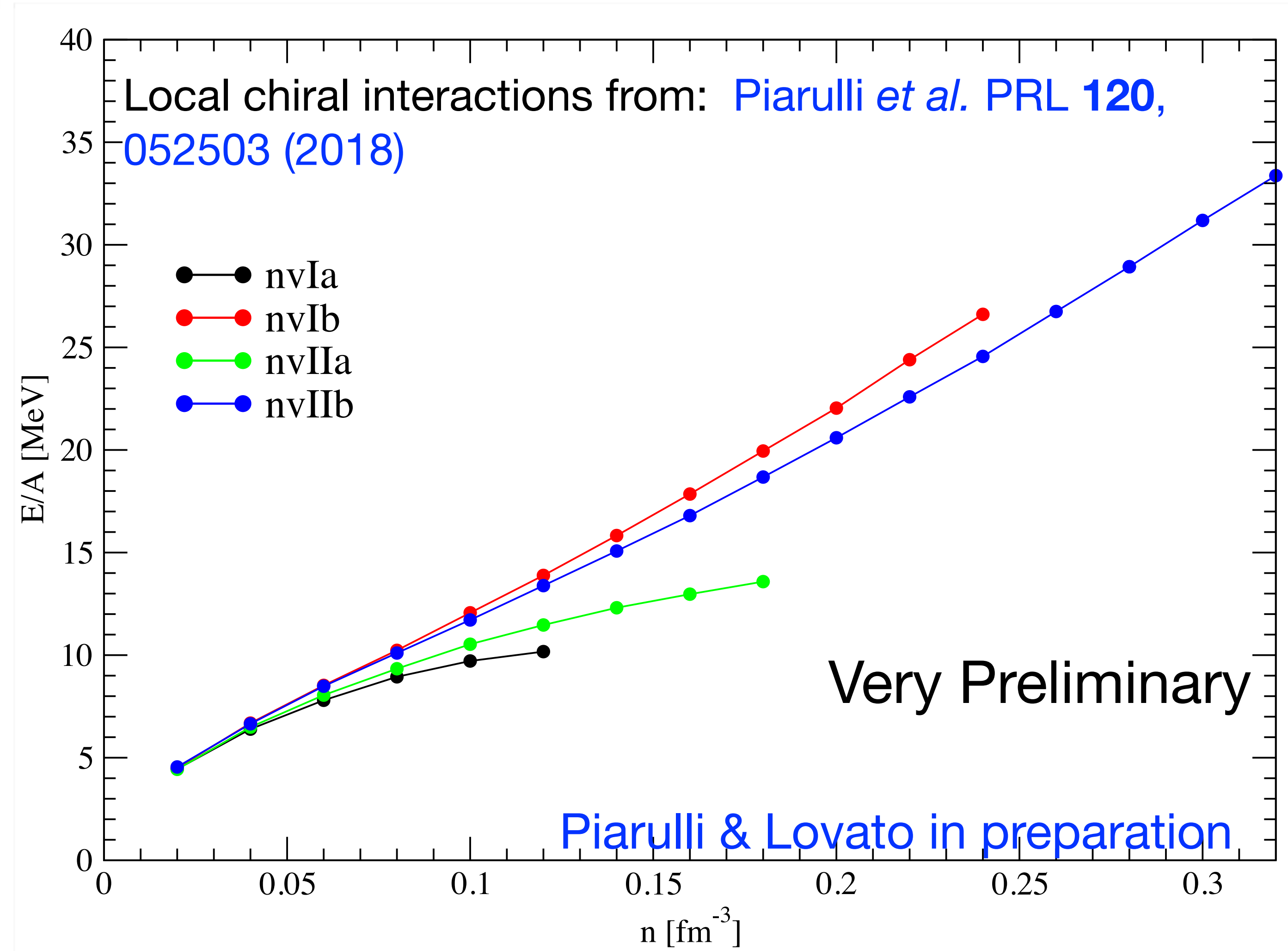


Equation of State of Pure Neutron Matter in χEFT

❖ EoS of PNM is very sensitive to the choice of the 3N force; particularly the short-range part of the 3N which is the less understood

EoS using Local chiral forces in AFDMC





NN only at saturation density~ 15-17 MeV

Conclusions

- We have developed a family of local NN potential with chiral TPE including Δ-isobar up to N2LO and contact interactions up to N3LO in the chiral expansion
- Different versions of this NN chiral potential have been developed with good fits to np and pp Granada database
- Corresponding local 3N chiral interaction up N2LO have been also developed; they involve two new LECs fixed by fitting the binding energy of ^{3}H and nd scattering length
- A subset of these local NN and NN+3N chiral interactions have been used to in HH and QMC calculations of binding energies and rms proton radii for some nuclei with A≤12 and more recently for EoS of PNM

Outlook

- Test other versions of NV2+3 with different energy fits and regulators and compare
- Different strategies to fit 3NI
- Studies of the effect of subleading 3N contact interactions in light nuclei
- Comprehensive treatment of radii, moments, electroweak transitions in VMC/GFMC including exchange currents
- EoS of neutron matter testing the different parametrization for the 3NF