

Electromagnetic sum rules and observables from the *ab initio* symmetry-adapted no-core shell model

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... LSU Team ...

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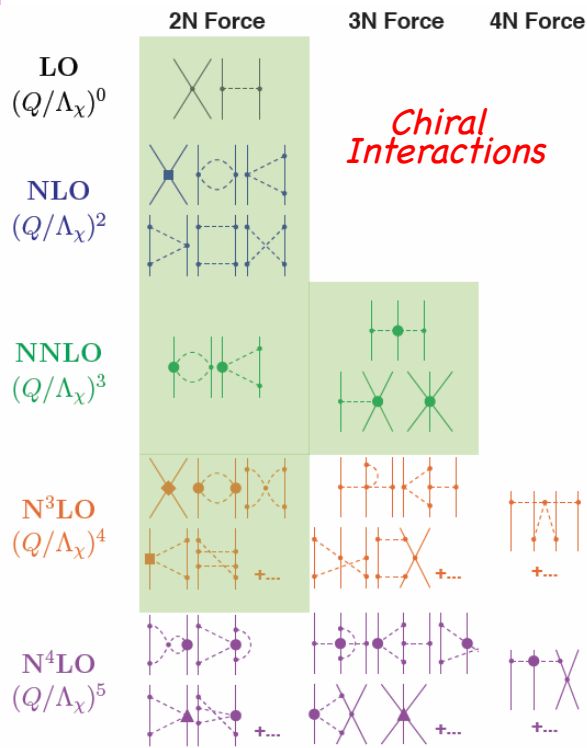
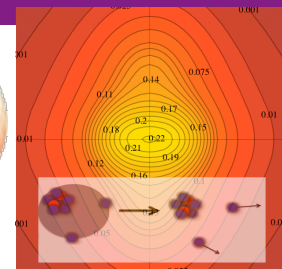
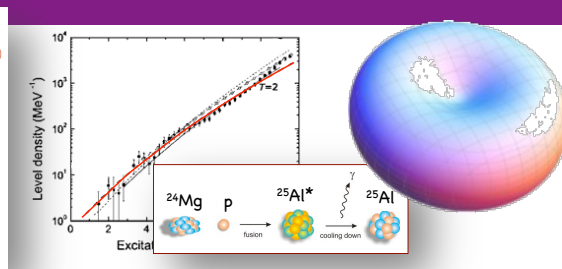
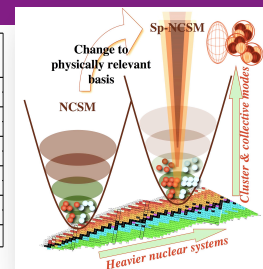
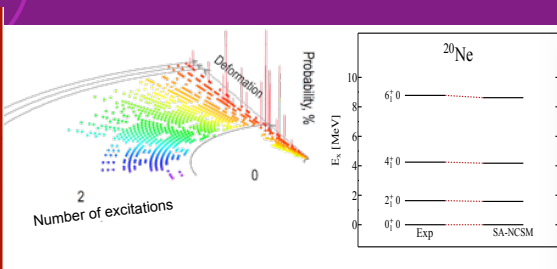
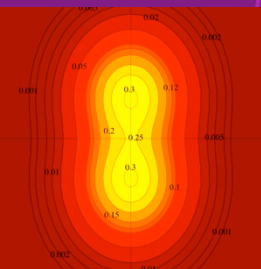
Madeleine Miora

In collaboration with
 Sonia Bacca & Nir Nevo Dinur
 Princeton U. – W. Tang & B. Wang
 Czech Republic – D. Langr & T. Oberhuber

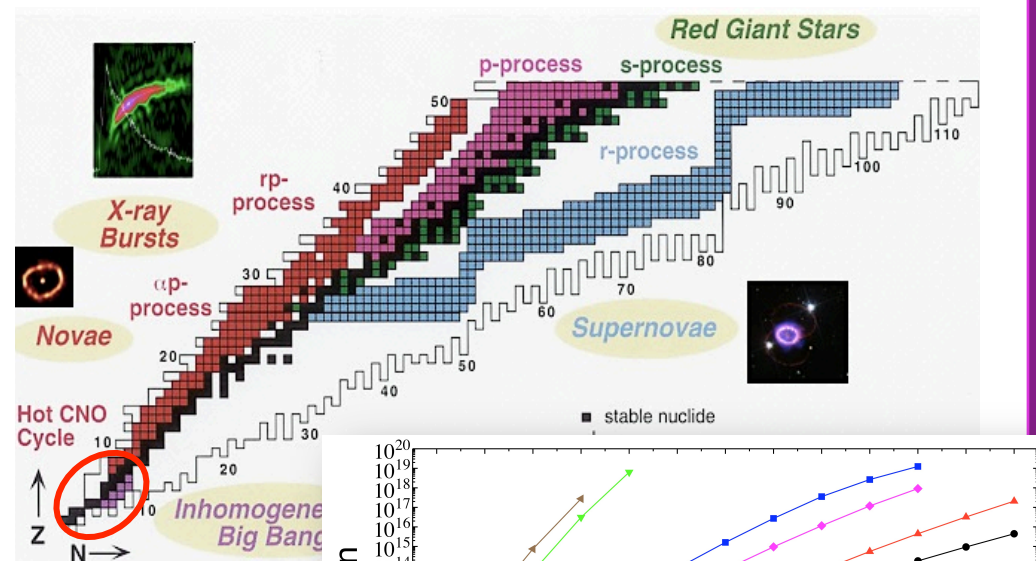
HPC Resources
 NSF/U. of Illinois ...BlueWaters
 LSU...SuperMike-II

Supported by NSF & DOE

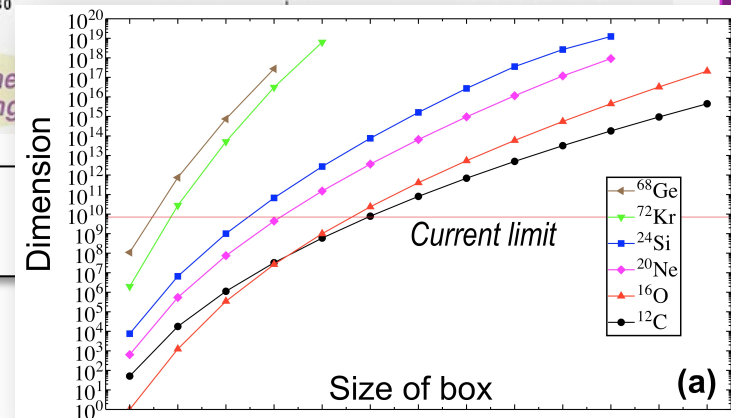


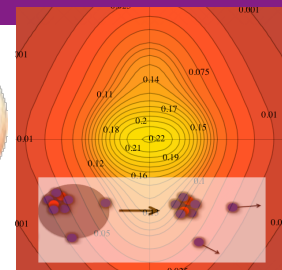
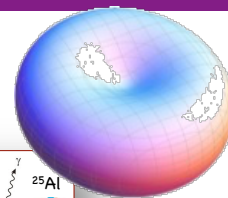
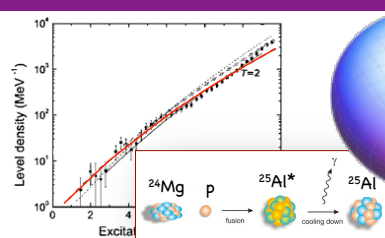
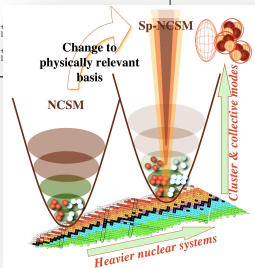
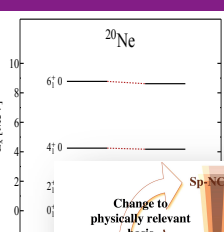
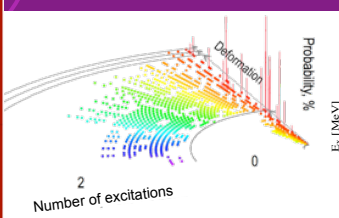
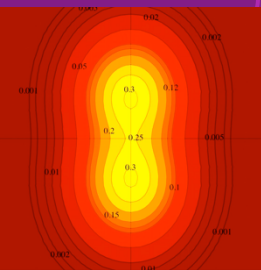


Weinberg, van Kolck, Machleidt, Entem, Meissner, Epelbaum, Krebs, Bernard,...

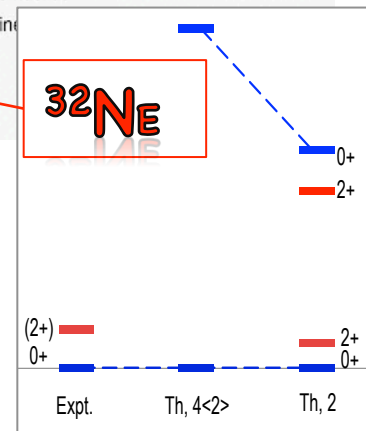
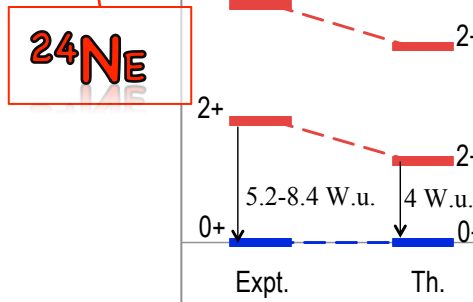
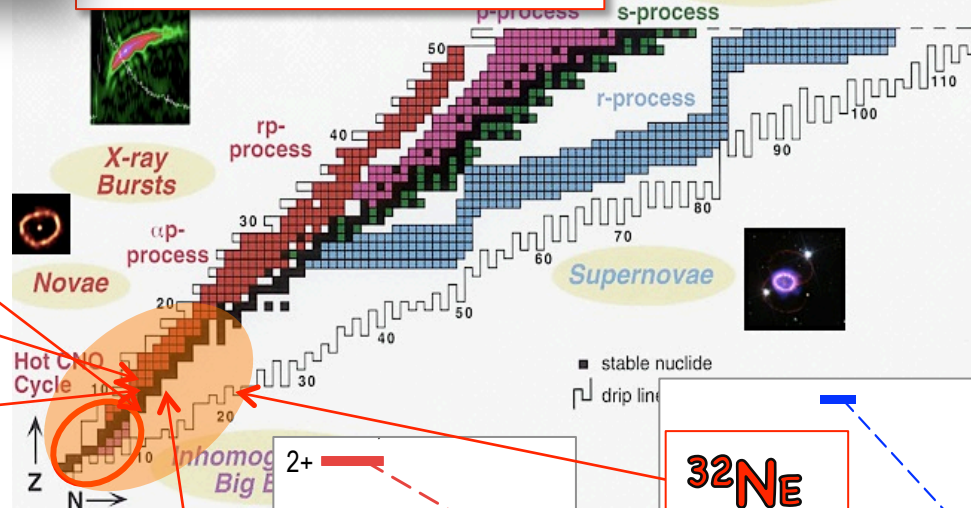
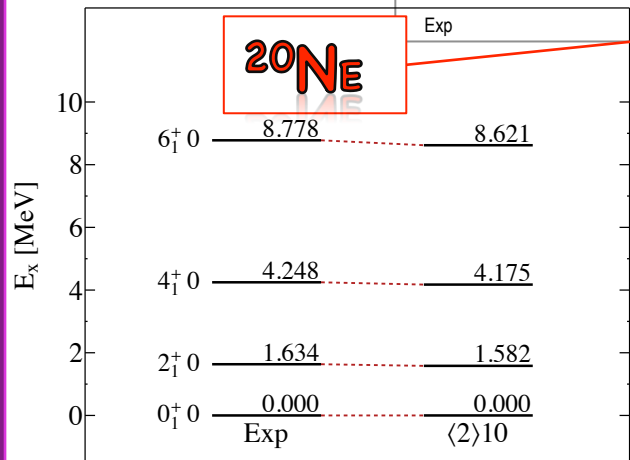
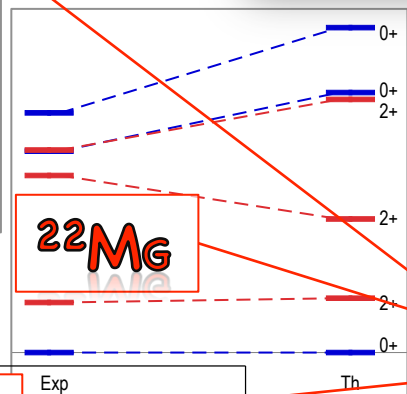
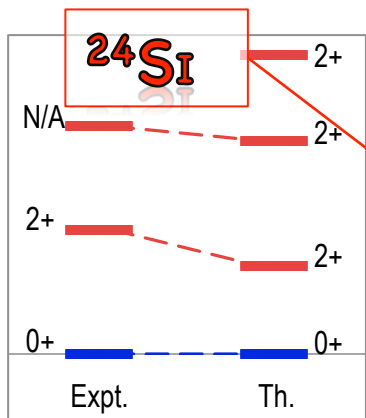


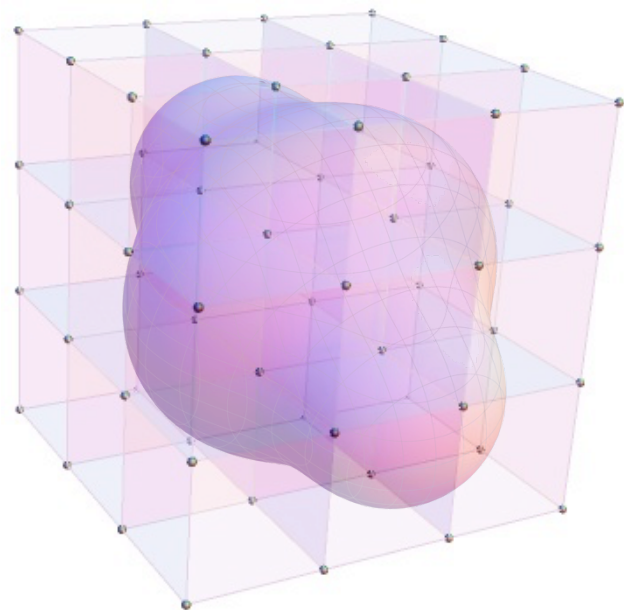
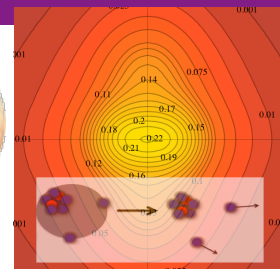
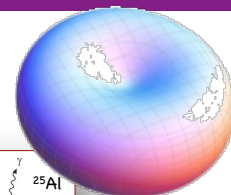
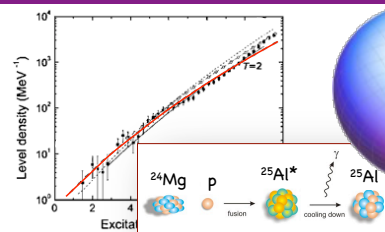
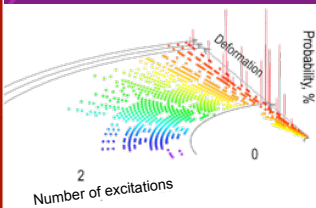
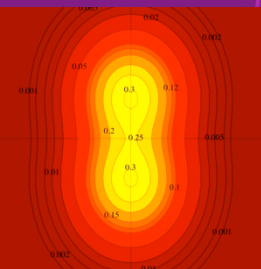
No-core Shell Model (NCSM)



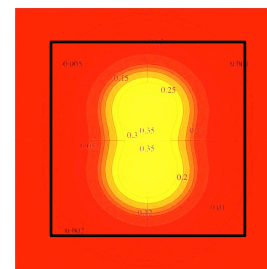


Symmetry-adapted No-core Shell Model (SA-NCSM)

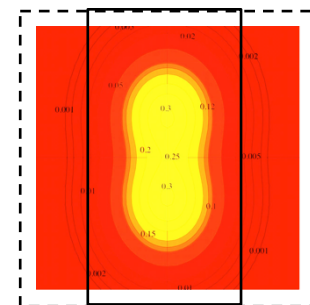




NCSM
Total HO quanta
 N_{\max}

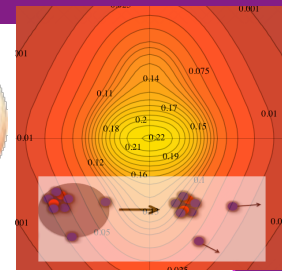
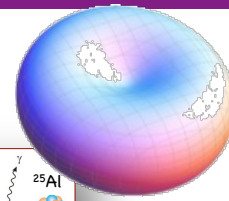
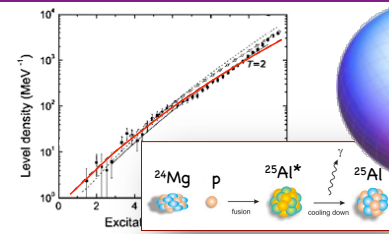
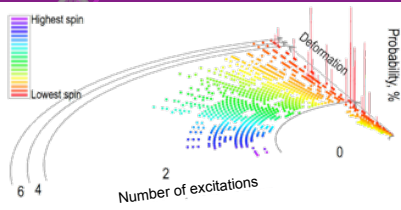


SA-NCSM
Total HO quanta
 $N_{\max} +$
Distribution:
z, x, y

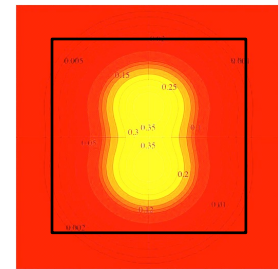


LSU code (LSU3shell): sourceforge.net/projects/lsu3shell
Dytrych et al., Phys. Rev. Lett. 111 (2013) 252501
Launey et al., Prog. Part. Nucl. Phys. 89 (2016) 101

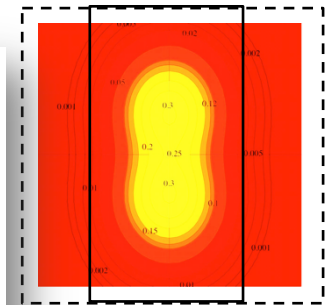
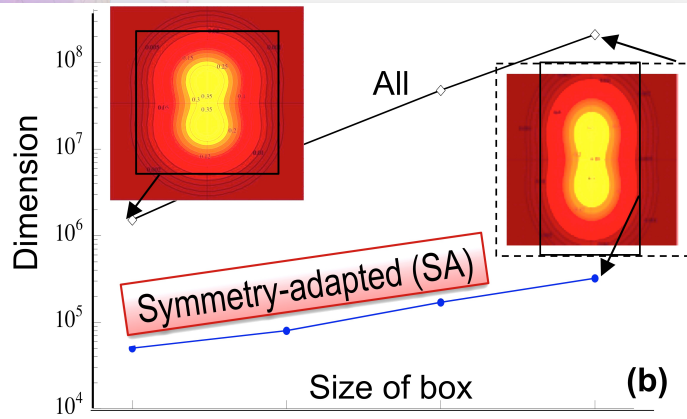
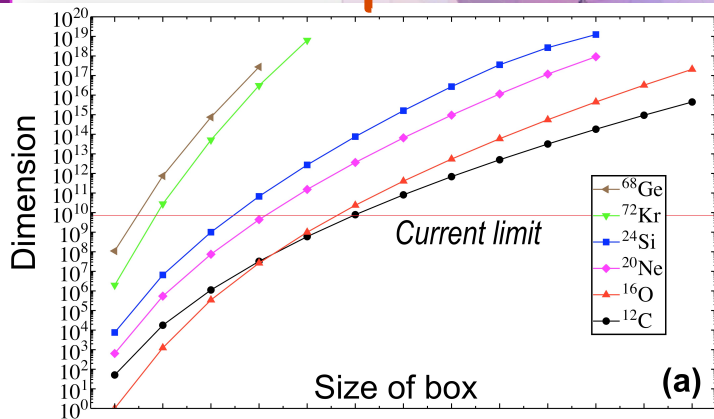




NCSM
Total HO quanta
 N_{\max}



Unitary transformation to collective basis (Sp)



LSU code (LSU3shell): sourceforge.net/projects/lsu3shell

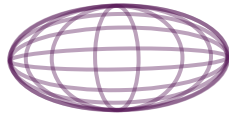
Dytrych et al., Phys. Rev. Lett. 111 (2013) 252501

Launey et al., Prog. Part. Nucl. Phys. 89 (2016) 101

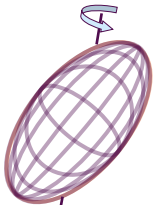


What physics can we learn from Sp basis?

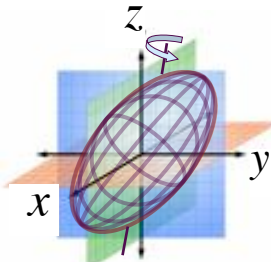
Sp (collective) basis configuration:



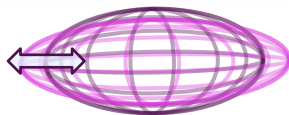
one equilibrium deformation ("shape")



rotations



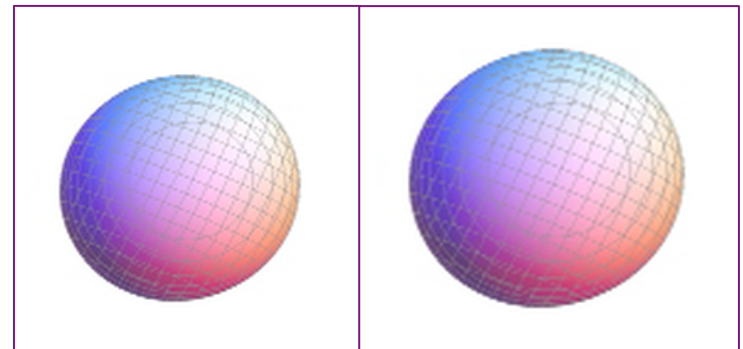
space orientation



Vibrations
(of the giant resonance monopole (r^2)/ quadrupole (Q) type)

All states preserve the equilibrium shape...

Symmetry?



Symplectic Sp(3,R) Symmetry!

Formal definition

All linear canonical transformations of the single-particle phase-space observables

$$x_{i\alpha} \rightarrow \sum_{\beta=x,y,z} a_{\alpha\beta} x_{i\beta} + b_{\alpha\beta} p_{i\beta}$$

$$p_{i\alpha} \rightarrow \sum_{\beta=x,y,z} c_{\alpha\beta} x_{i\beta} + d_{\alpha\beta} p_{i\beta}$$

that **preserve the canonical commutation relation**

$$[x_{i\alpha}, p_{j\beta}] = i\hbar \delta_{ij} \delta_{\alpha\beta}$$

Generators: $Q_{ij} = \sum_n x_{ni} x_{nj},$

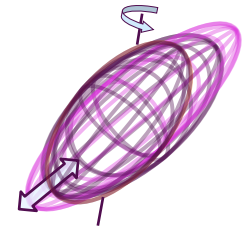
$$S_{ij} = \sum_n (x_{ni} p_{nj} + p_{ni} x_{nj}),$$

$$L_{ij} = \sum_n (x_{ni} p_{nj} - x_{nj} p_{ni}),$$

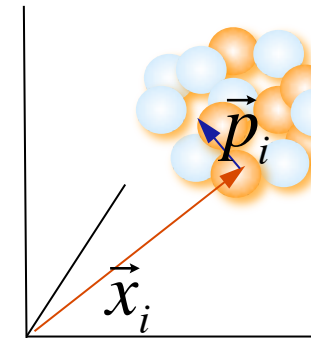
$$K_{ij} = \sum_n p_{ni} p_{nj},$$

SU(3)
in a HO shell
(Elliott, 1958)

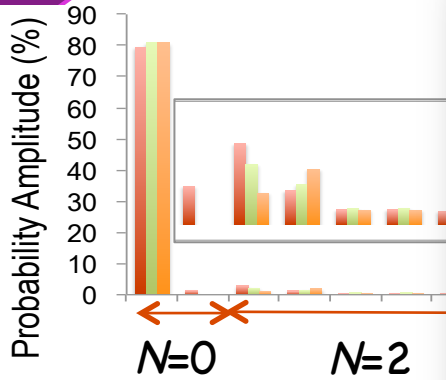
Rowe, Rosensteel, Draayer, Hecht, Suzuki, Escher, Bahri, ...



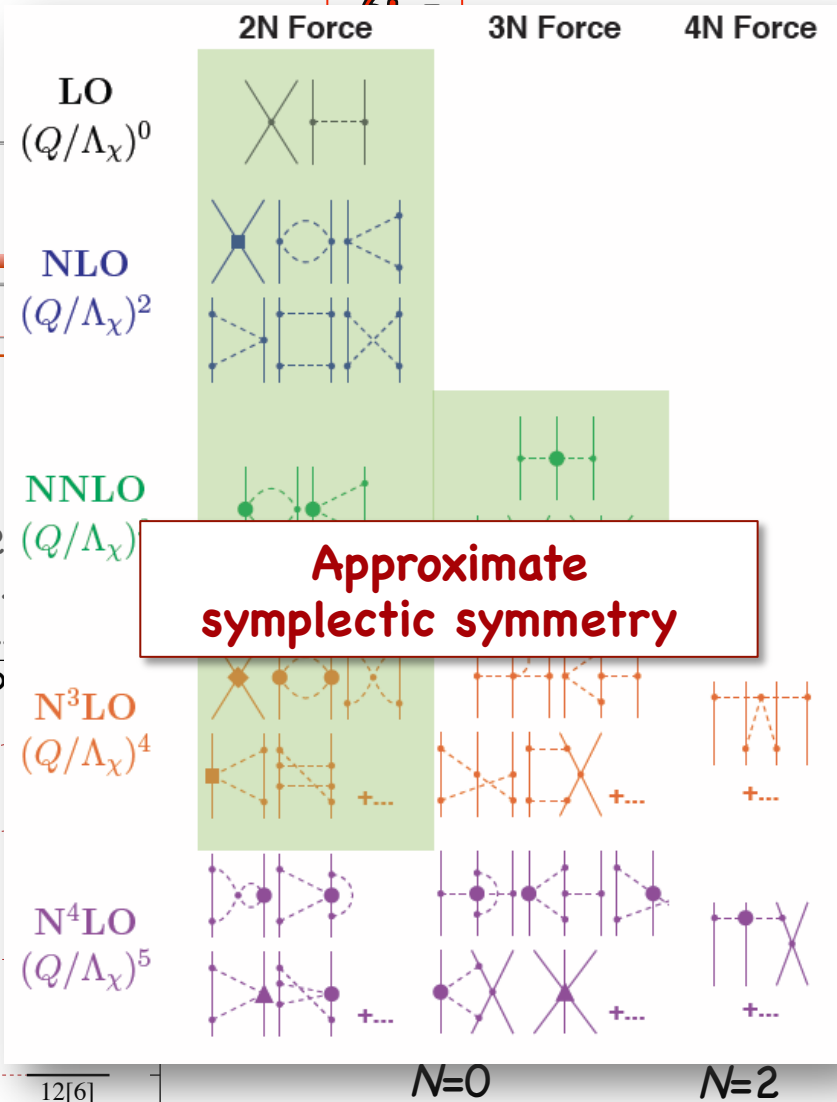
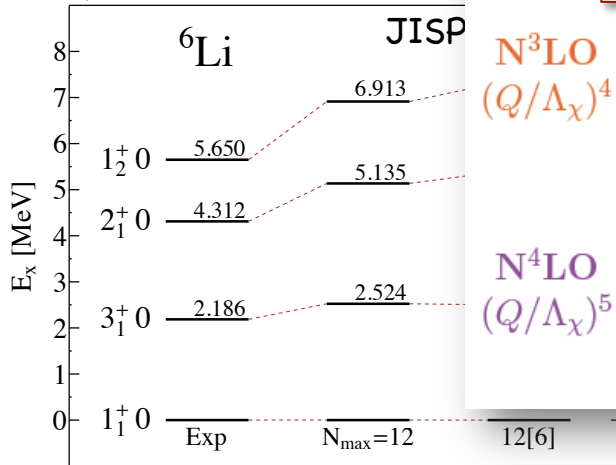
Nucleus with A nucleons



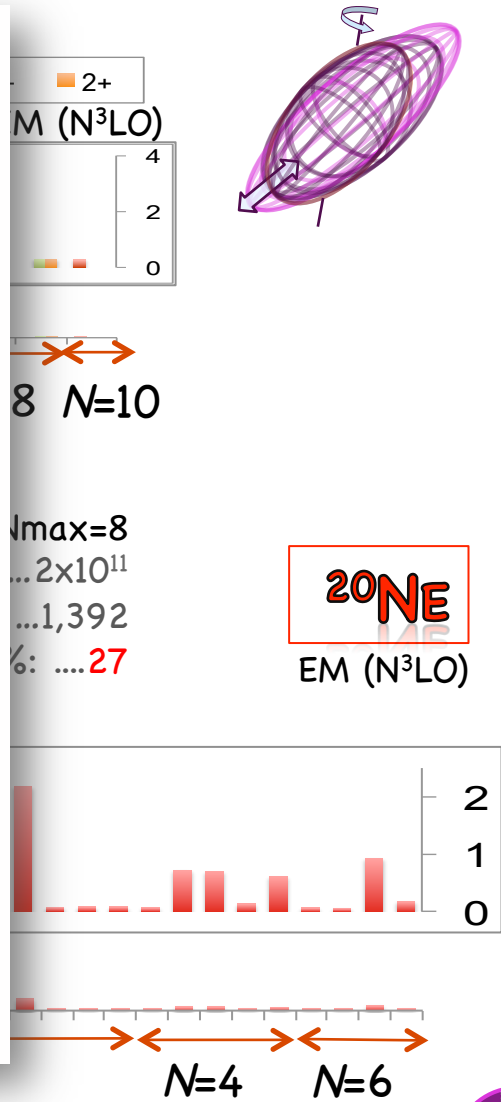
Novel Approximate Symmetry



${}^6\text{Li}$, $N_{\text{max}}=12$
 # $J=1,2,3$ states.....2
 # $\text{Sp}(3,\text{R})$ irreps.....
 # $\text{Sp}(3,\text{R})$ with $P>0.2\%$

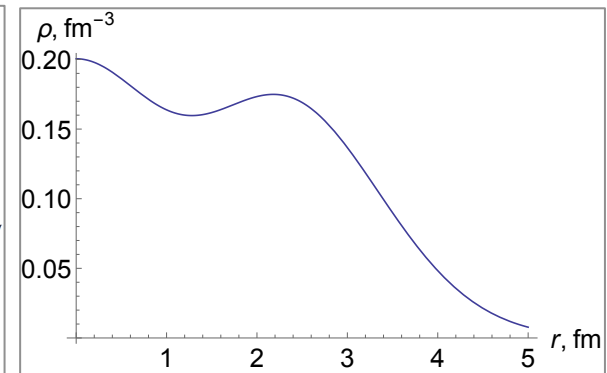
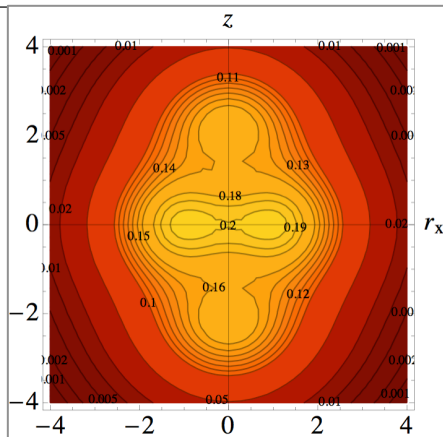
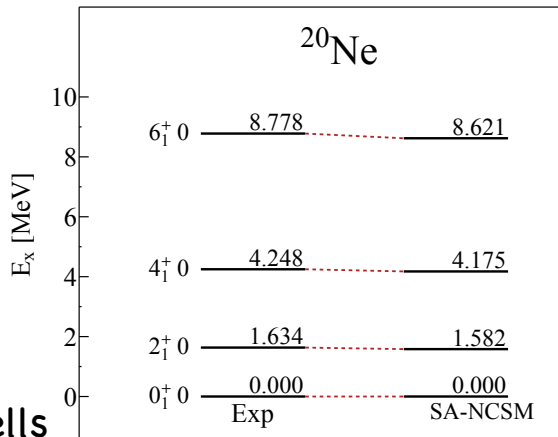


Approximate symplectic symmetry



Collectivity features

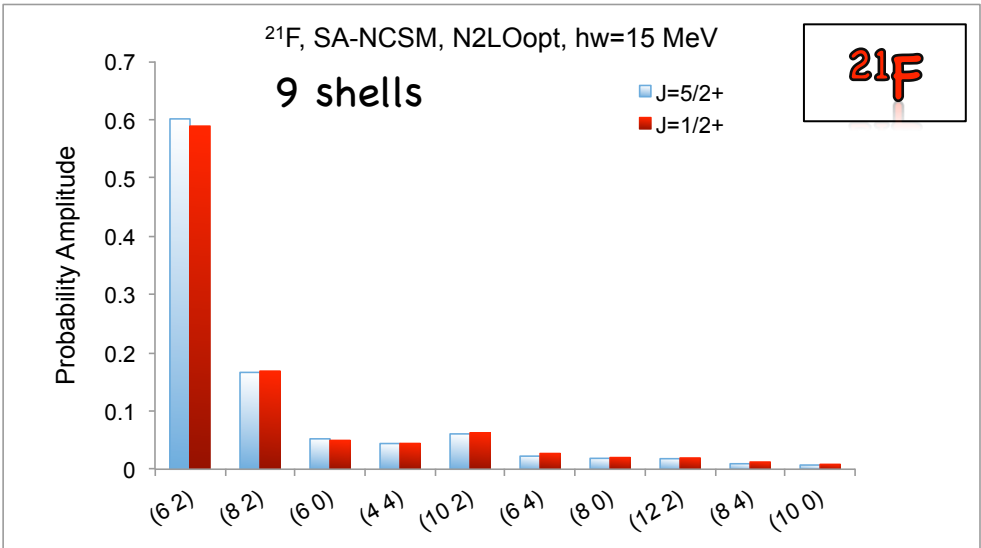
20Ne



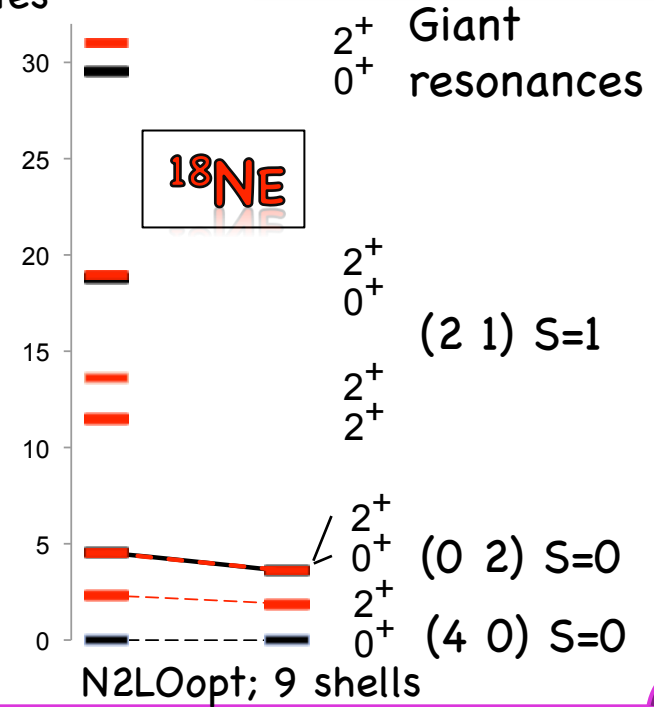
13 shells

SA-NCSM (selected model space): 50 million SU(3) states
 Complete model space: 1000 billion states

Ne & Mg isotopes

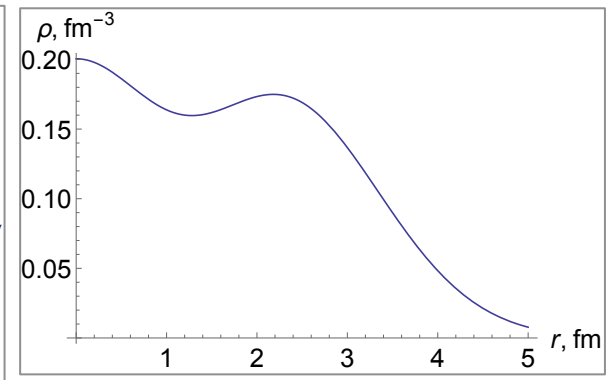
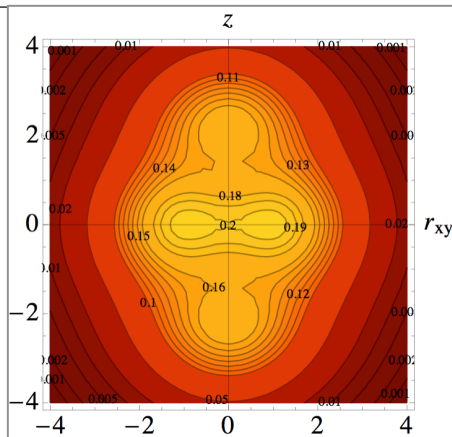
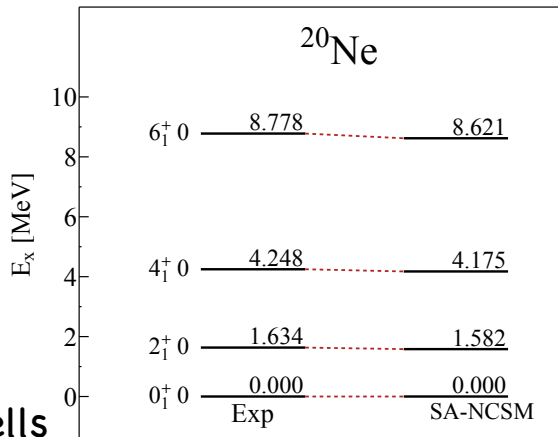


Grigor Sargsyan, PhD student, LSU



Collectivity features

20Ne



13 shells

SA-NCSM (selected model space): 50 million SU(3) states
 Complete model space: 1000 billion states

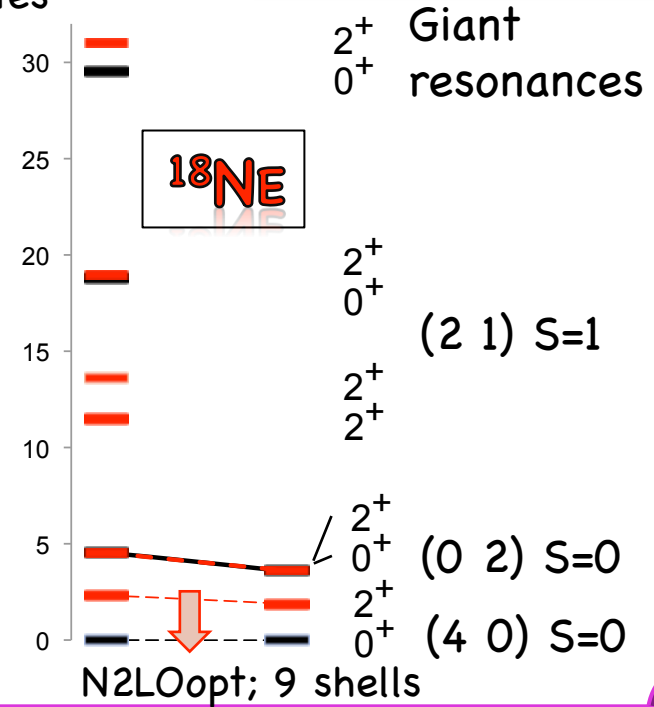
Ne & Mg isotopes

^{18}Ne , $B(E2: 2^+ \rightarrow 0^+)$

 Experiment..... 17.7(18) W.u.

9 shells 1.13 W.u.

33 shells 13.0(7) W.u.
 (no effective charges)



Grigor Sargsyan, PhD student, LSU

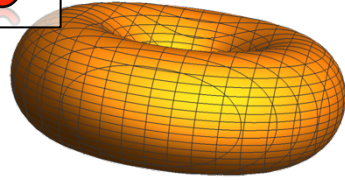
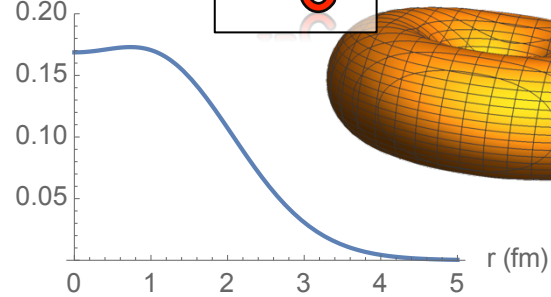


Carbon isotopes

$\rho(r)$ (fm⁻³)

¹²C

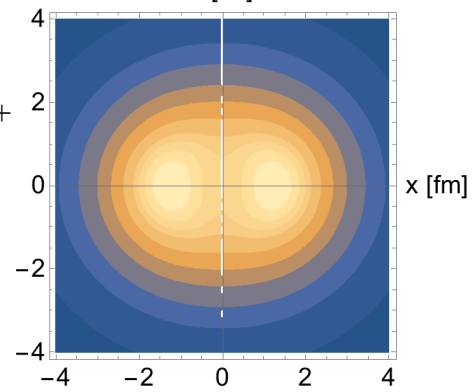
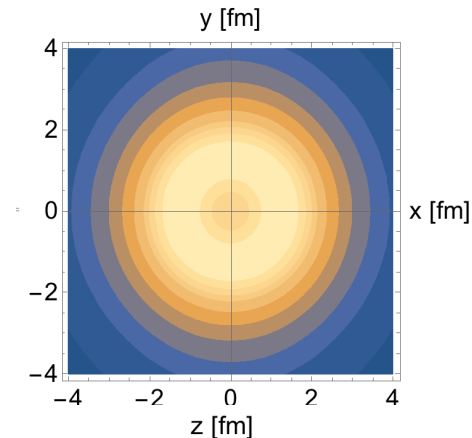
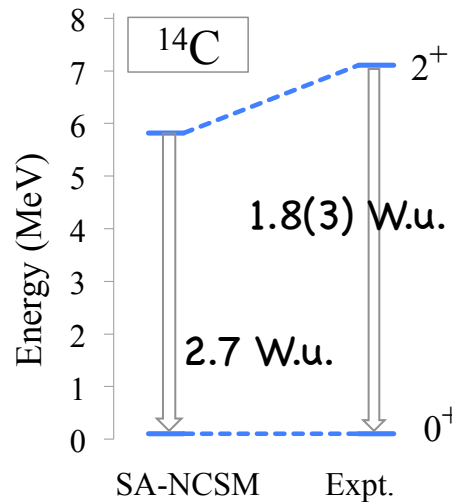
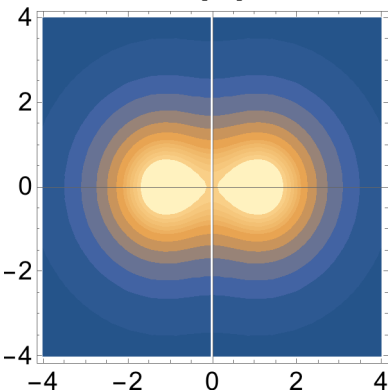
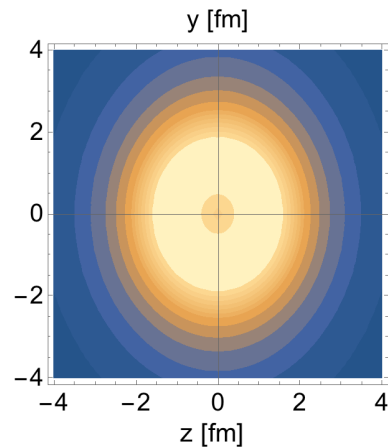
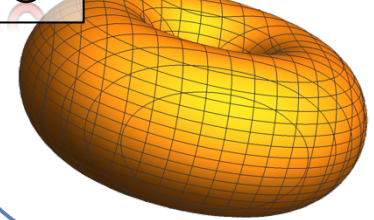
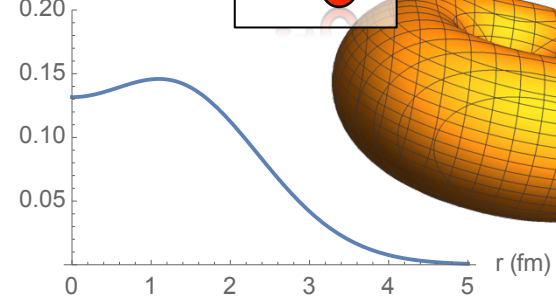
JISP16, hw = 18 MeV



$\rho(r)$ (fm⁻³)

¹⁴C

N2LOopt, hw = 15 MeV



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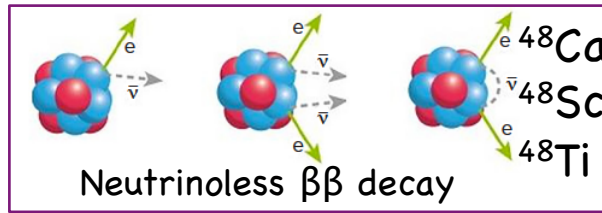
Structure of Ca-48 and Ti-48

^{48}Ca

8 shells, N2LOopt
 0^+

SA-NCSM (selected):966,152
 Complete model space:3,162,511,819

2^+
 SA-NCSM (selected):3,055,554
 Complete model space: ...14,522,234,982



^{48}Ti

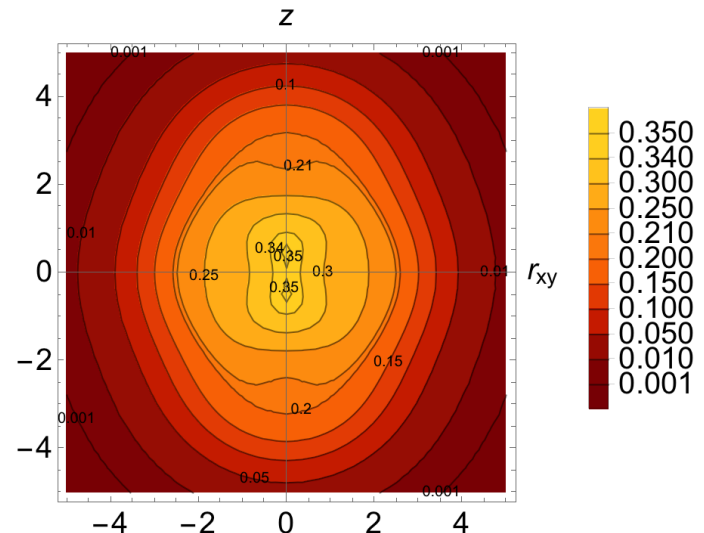
8 shells, N2LOopt
 0^+

SA-NCSM (selected):602,493
 Complete model space:24,694,678,414

2^+
 SA-NCSM (selected):1,178,834
 Complete model space: ...113,920,316,658

^{48}Ti , $Q(2^+)$ [$e \text{ fm}^2$]

 Experiment..... -17.7
 8 shells -19.3
 (no effective charges)

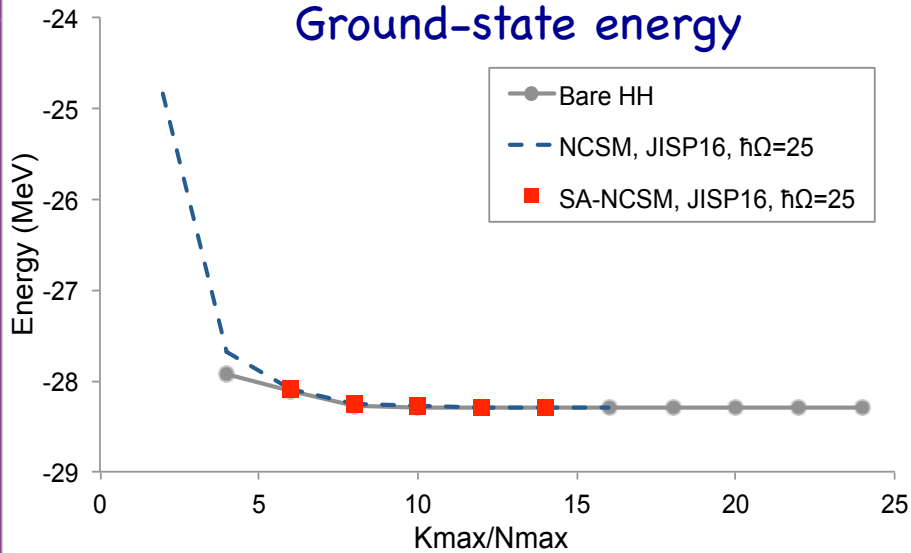


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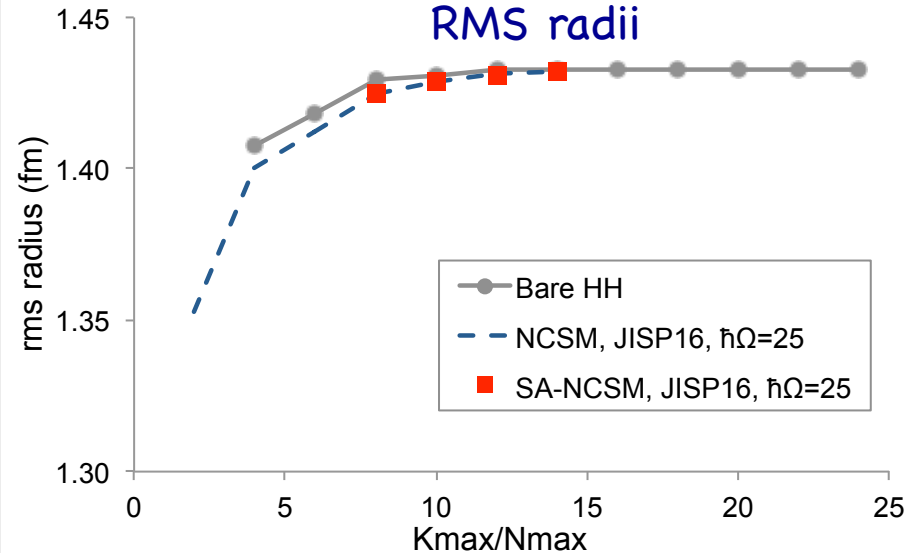


HH and SA-NCSM benchmark: ${}^4\text{He}$

Ground-state energy



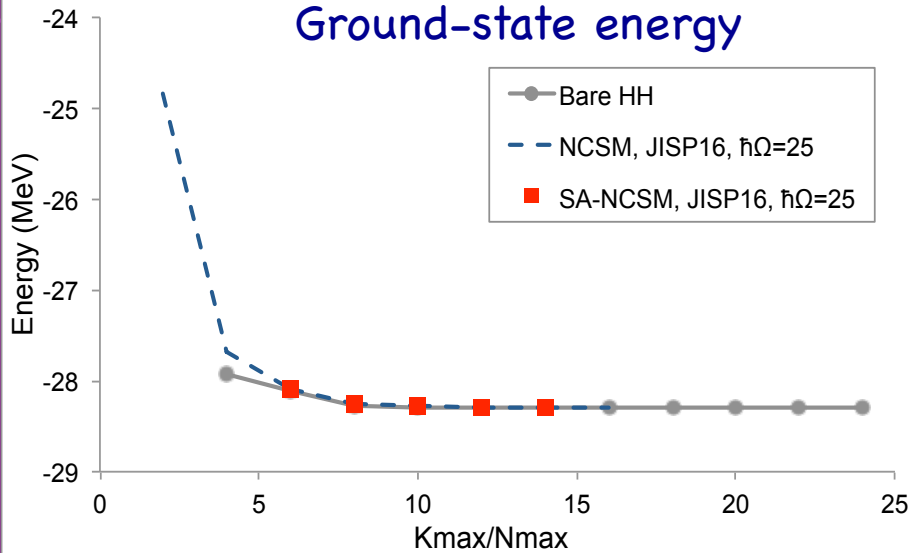
RMS radii



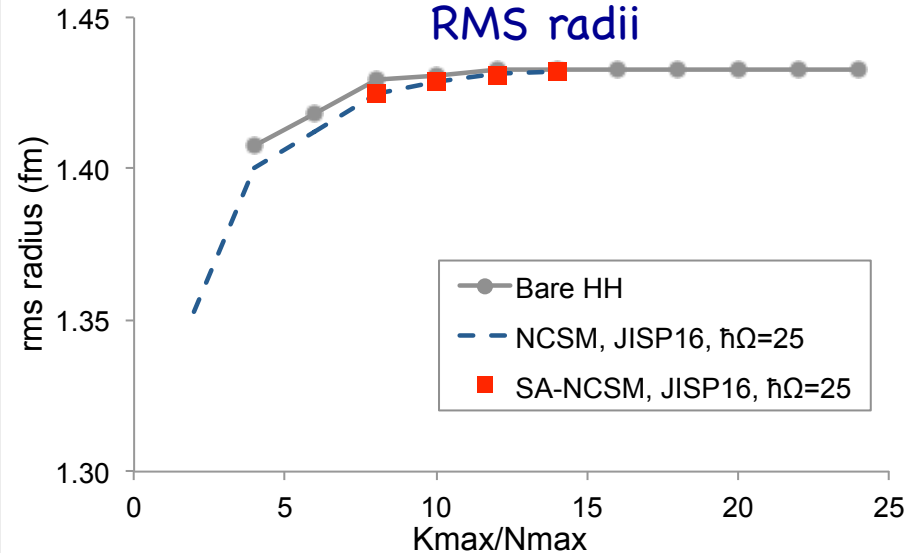
Baker et al., in preparation (2018)

HH and SA-NCSM benchmark: ${}^4\text{He}$

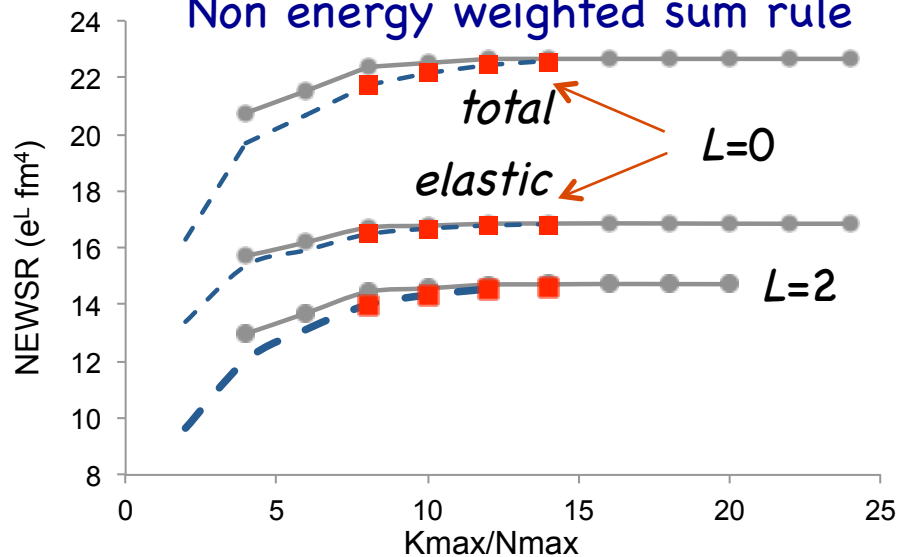
Ground-state energy



RMS radii



Non energy weighted sum rule



Response function

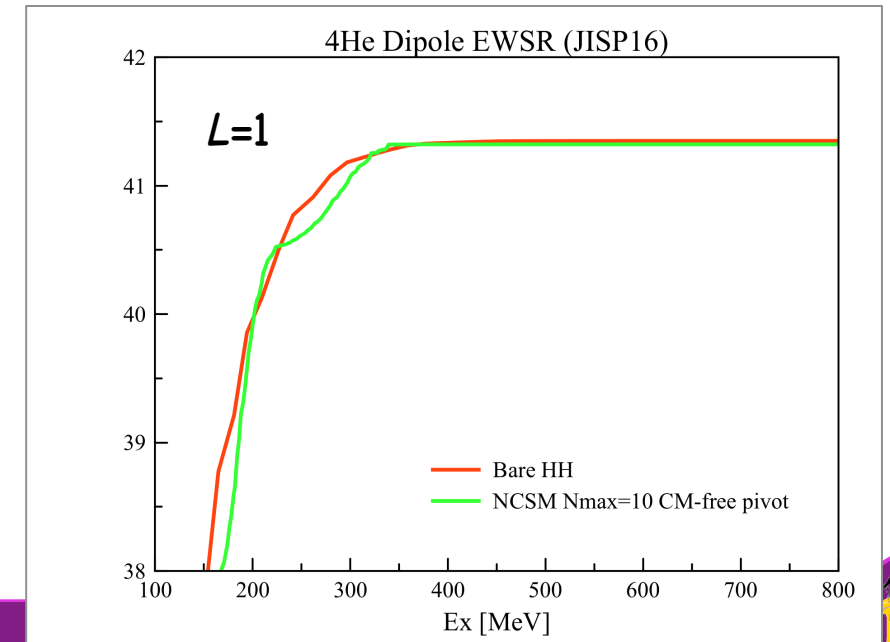
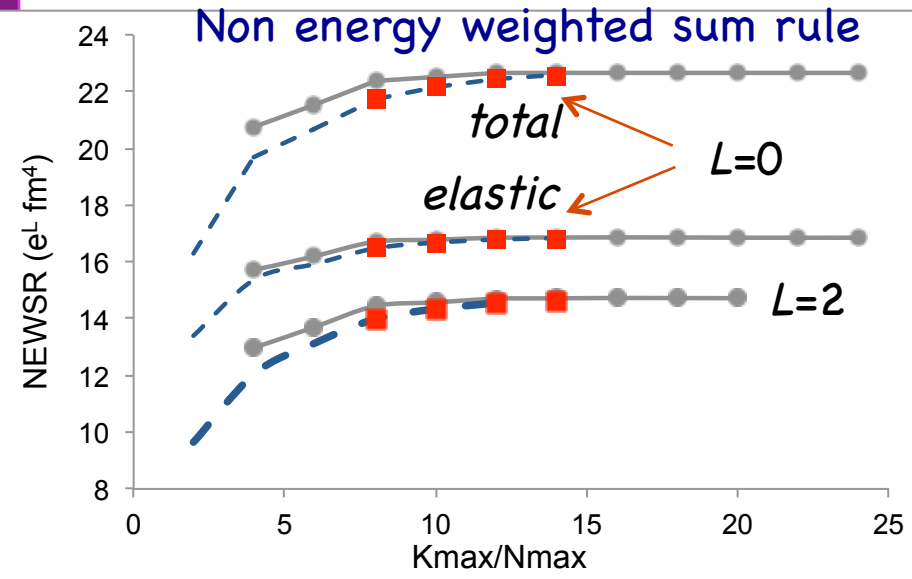
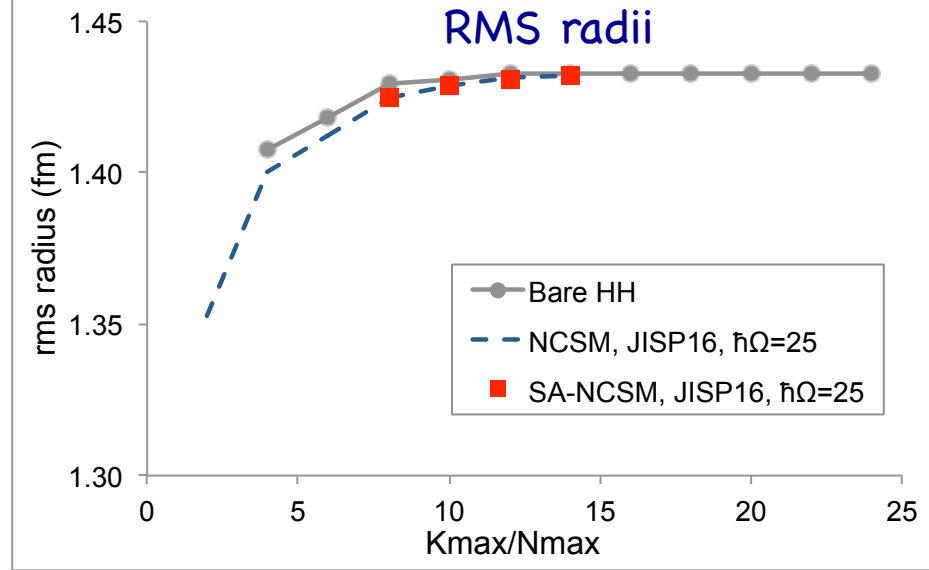
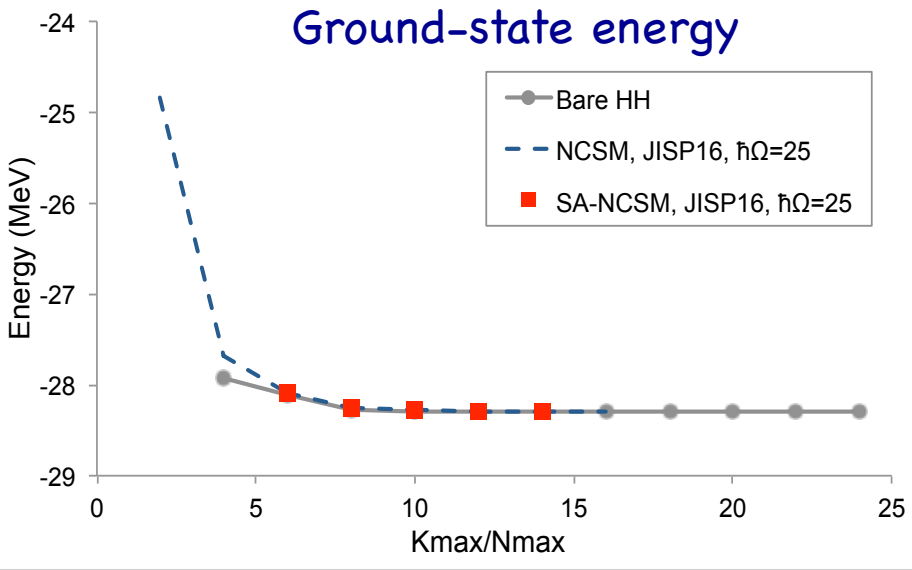
$$R(\omega) = \sum_f \left| \langle \psi_f | \Theta | \psi_0 \rangle \right|^2 \delta(E_f - E_0 - \omega)$$

Sum rules

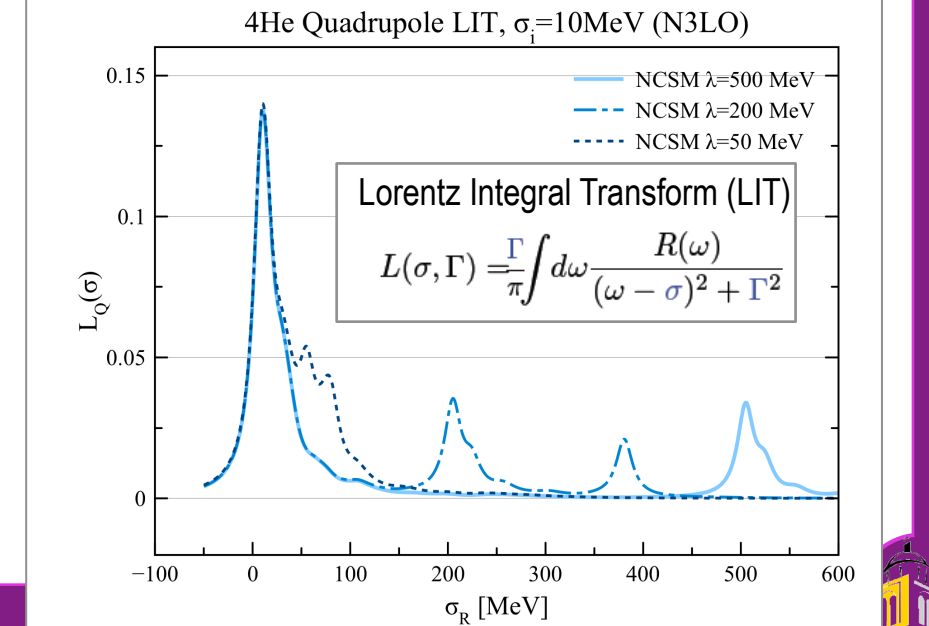
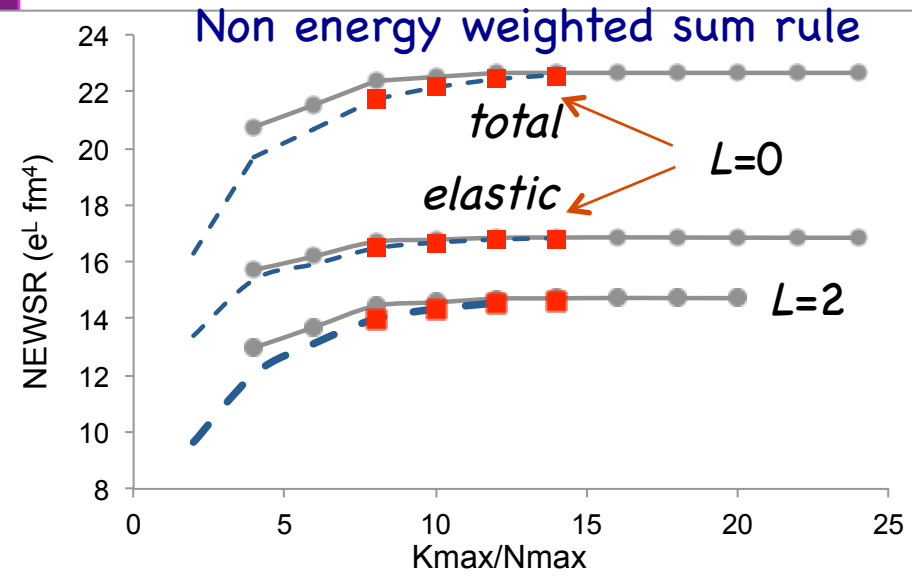
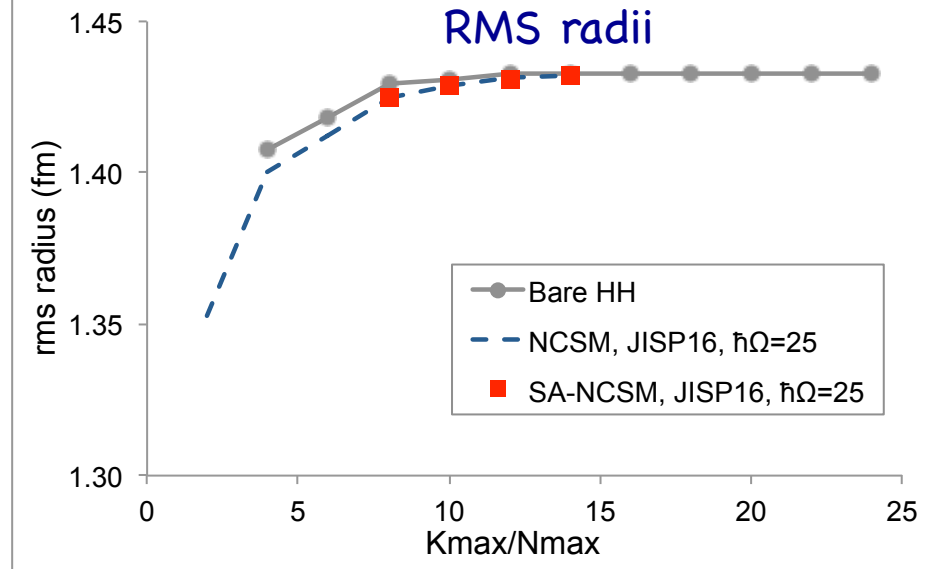
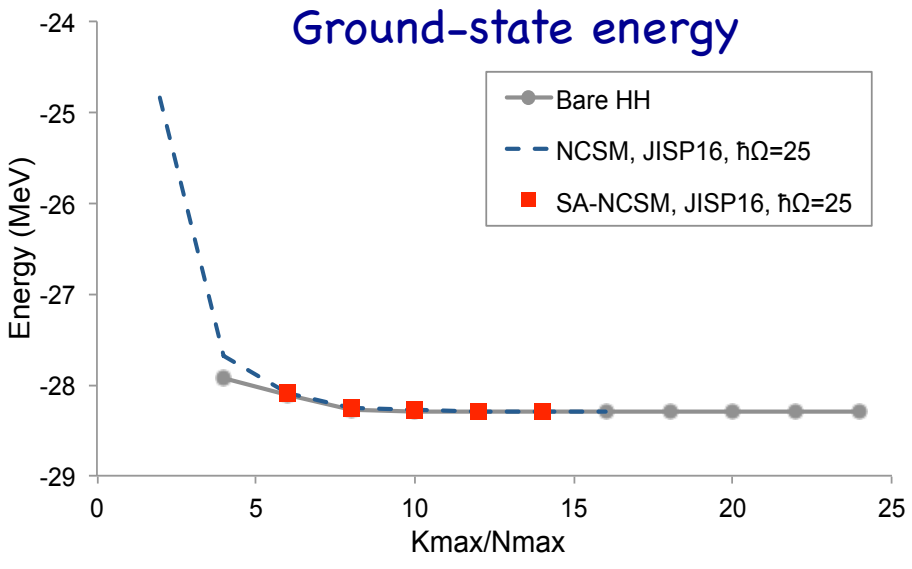
$$m_n = \int_0^\infty d\omega \omega^n R(\omega)$$

Baker et al., in preparation (2018)

Sum rules for ^4He : HH and SA-NCSM benchmark



Sum rules for ^4He : HH and SA-NCSM benchmark



Efficacy of SA-NCSM

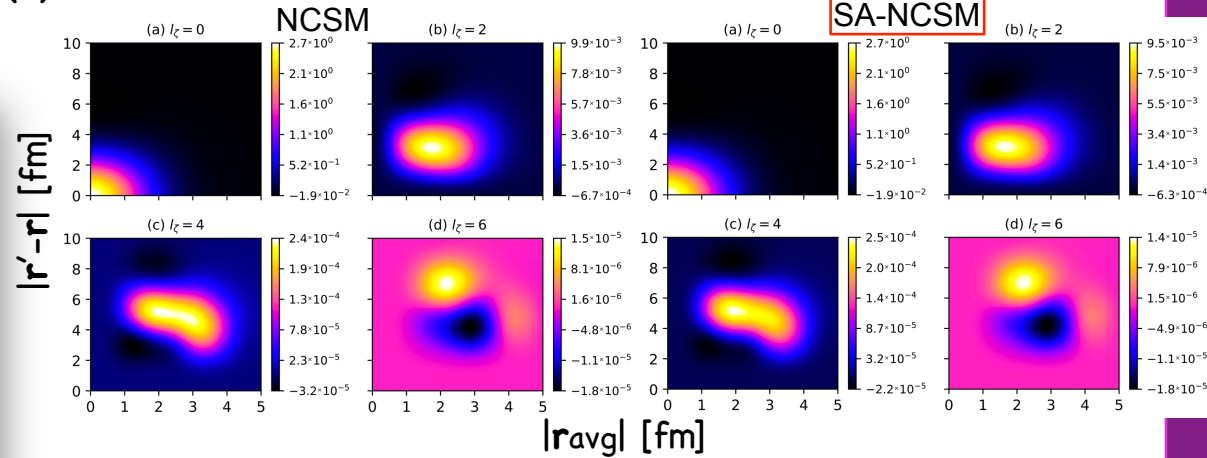
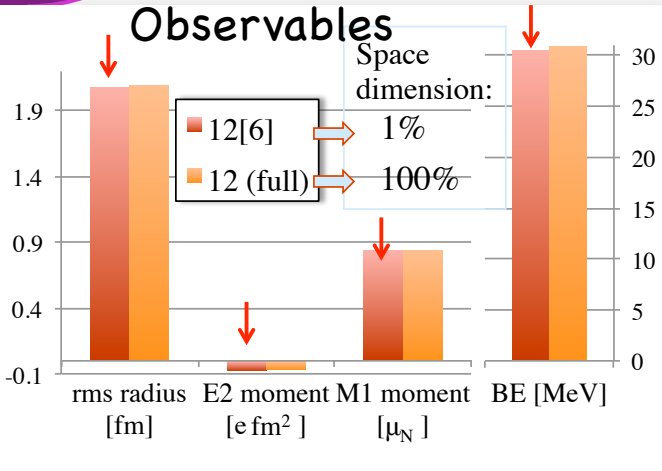
${}^6\text{Li}$

SA-NCSM, SU(3)

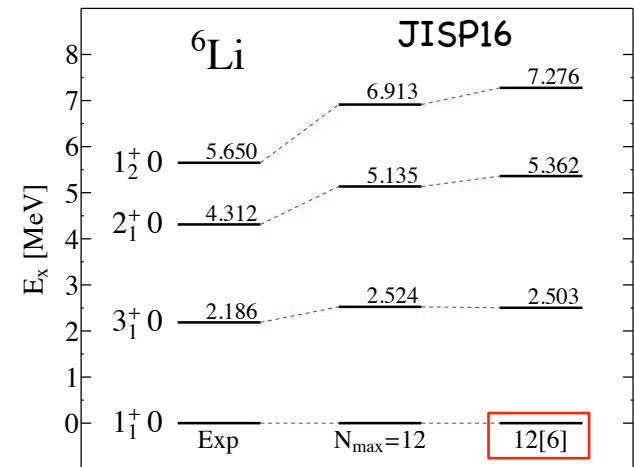
Non-local densities

SA-NCSM

N2LOopt



Burrows, Elster, Popa, Launey, Nogga, Maris, Phys. Rev. C 97 (2018) 024325



Launey et al., Prog. Part. Nucl. Phys. 89 (2016) 101;
 Dytrych et al., Phys. Rev. C 91 (2015) 024326



Efficacy of SA-NCSM

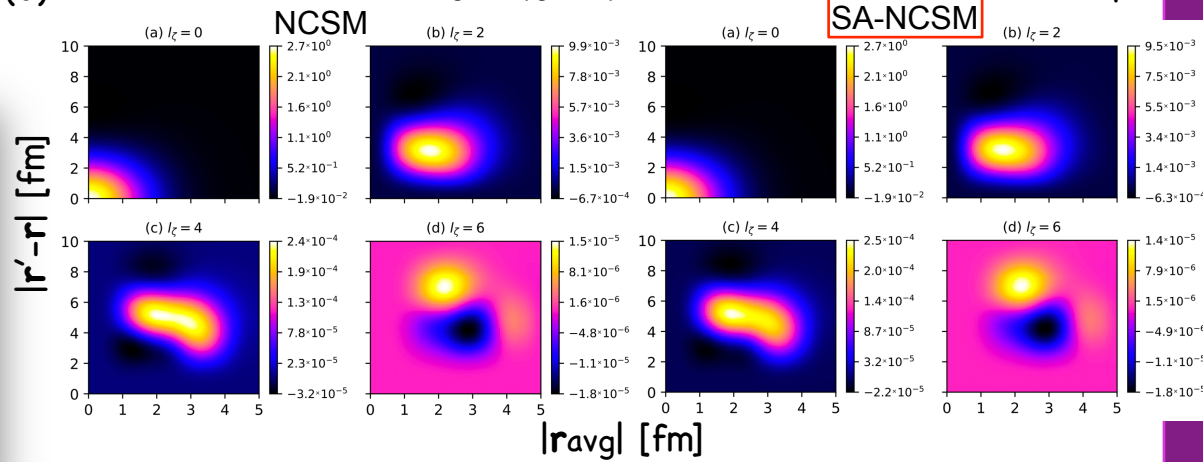
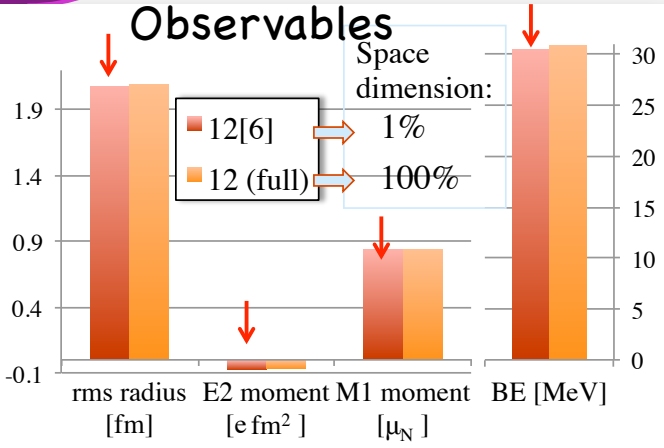
⁶Li

SA-NCSM, SU(3)

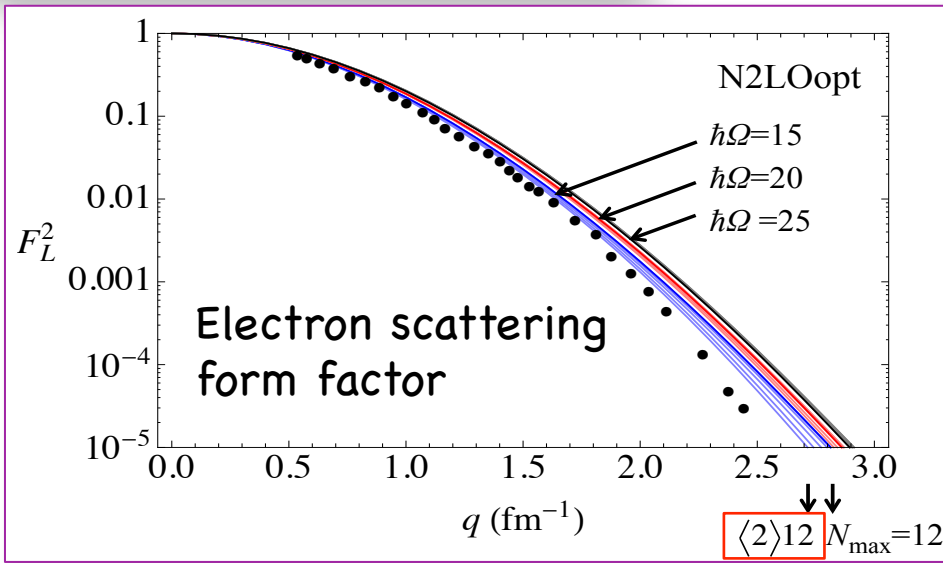
Non-local densities

N2LOopt

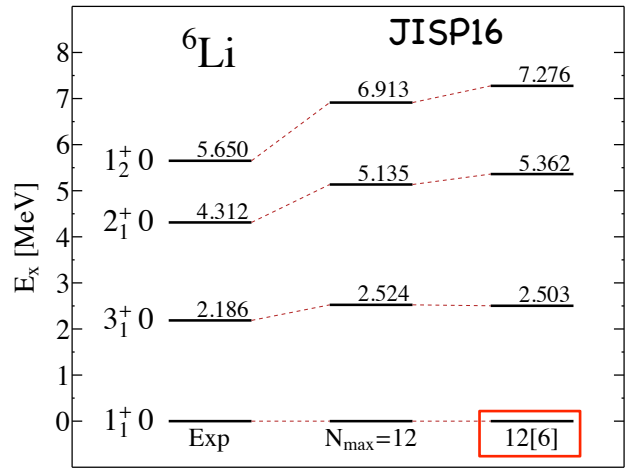
SA-NCSM



Burrows, Elster, Popa, Launey, Nogga, Maris, Phys. Rev. C 97 (2018) 024325



Launey et al., Prog. Part. Nucl. Phys. 89 (2016) 101; Dytrych et al., Phys. Rev. C 91 (2015) 024326



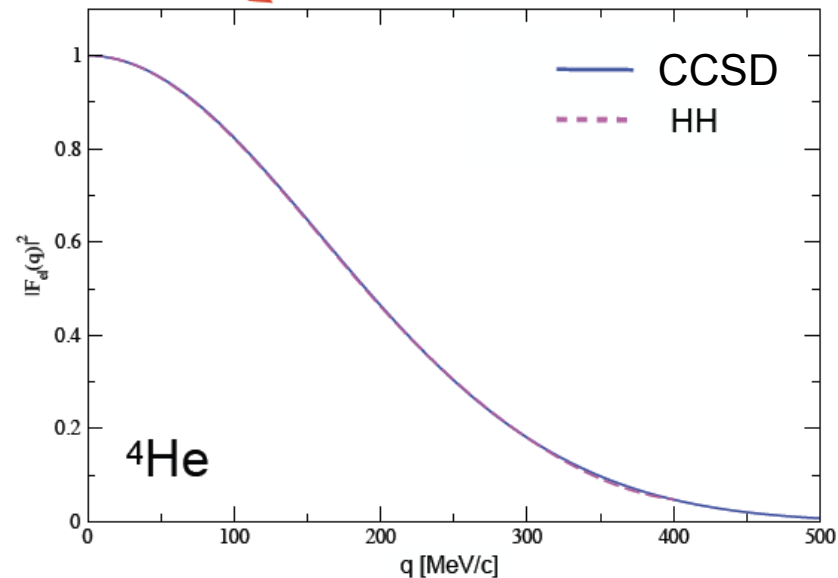
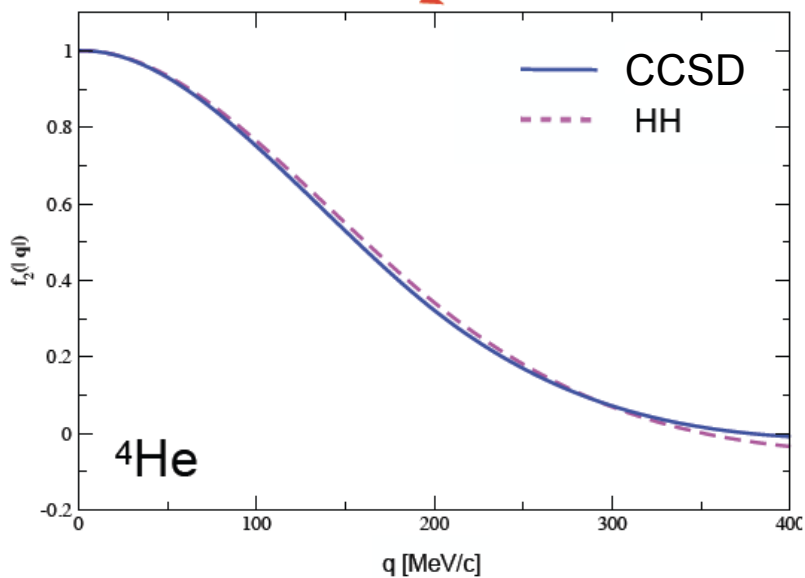
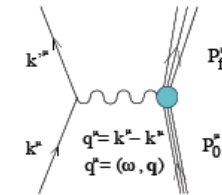
Coulomb sum rule

Total strength of inelastic longitudinal response function

$$\text{CSR}(q) = \int d\omega R_L^{in}(\omega, \mathbf{q}) \quad R_L^{in}(\omega, \mathbf{q}) = \sum_f |\langle f | \rho(\mathbf{q}) | 0 \rangle|^2 \delta(\omega - \mathbf{E}_f + \mathbf{E}_0)$$

$$\text{CSR}(q) = Z + \langle 0 | \sum_{i \neq j} e^{i\mathbf{q} \cdot (\mathbf{r}_i - \mathbf{r}_j)} | 0 \rangle - |F_{el}(\mathbf{q})|^2 Z^2$$

||
Z(Z - 1)f₂(|q|)



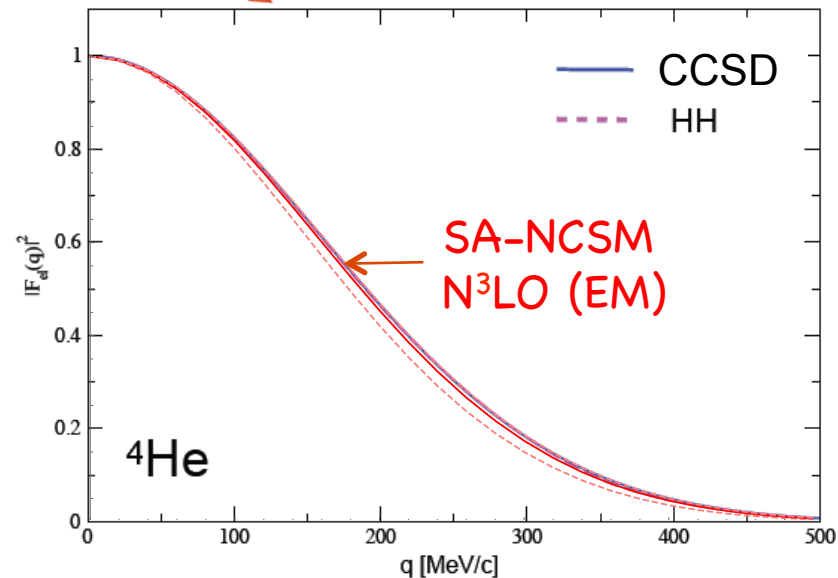
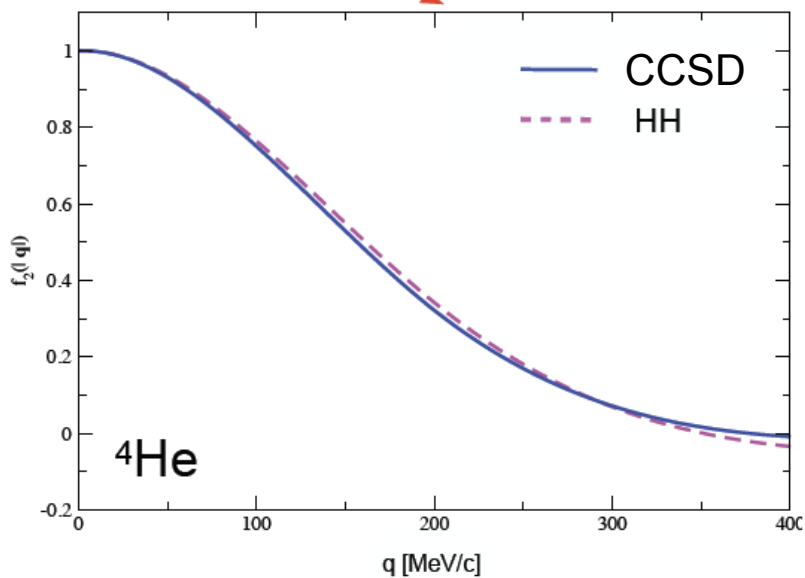
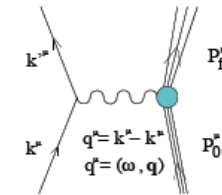
Coulomb sum rule

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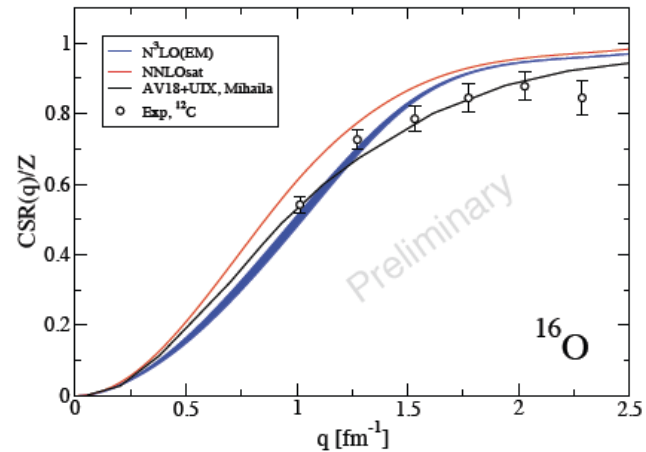
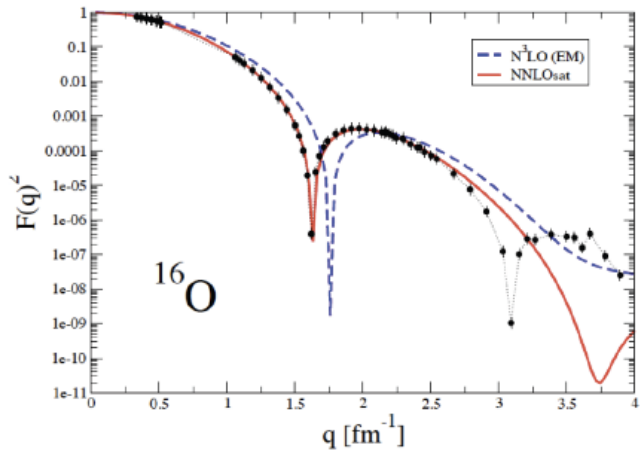
$$\text{CSR}(q) = Z + \langle 0 | \sum_{i \neq j} e^{i\mathbf{q} \cdot (\mathbf{r}_i - \mathbf{r}_j)} | 0 \rangle - |F_{el}(\mathbf{q})|^2 Z^2$$

||
Z(Z-1)f₂(|q|)



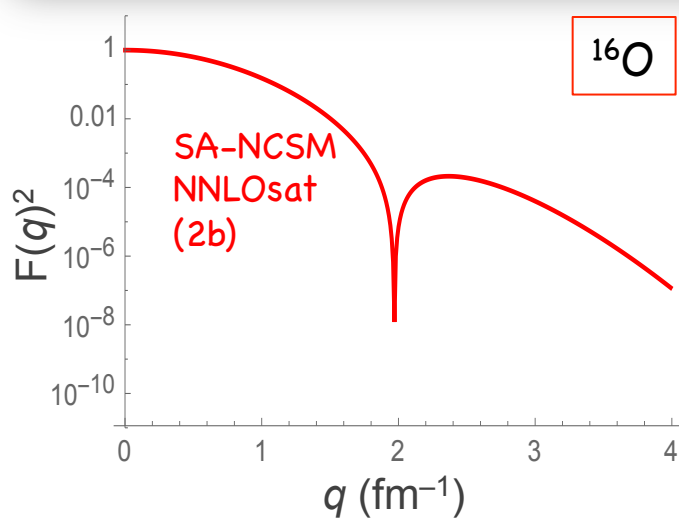
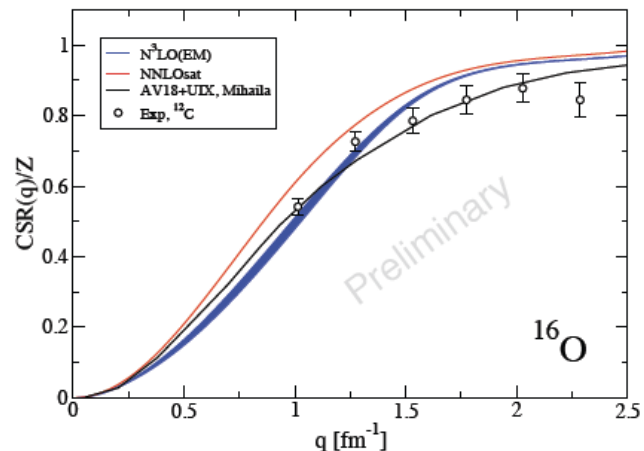
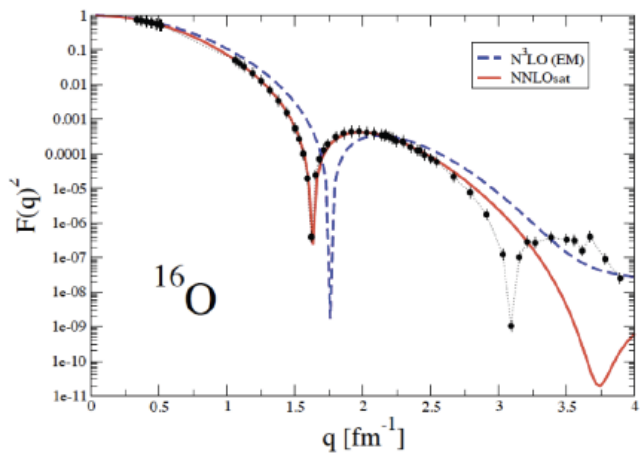
Coulomb sum rule

S. Bacca et al., in preparation (2018)



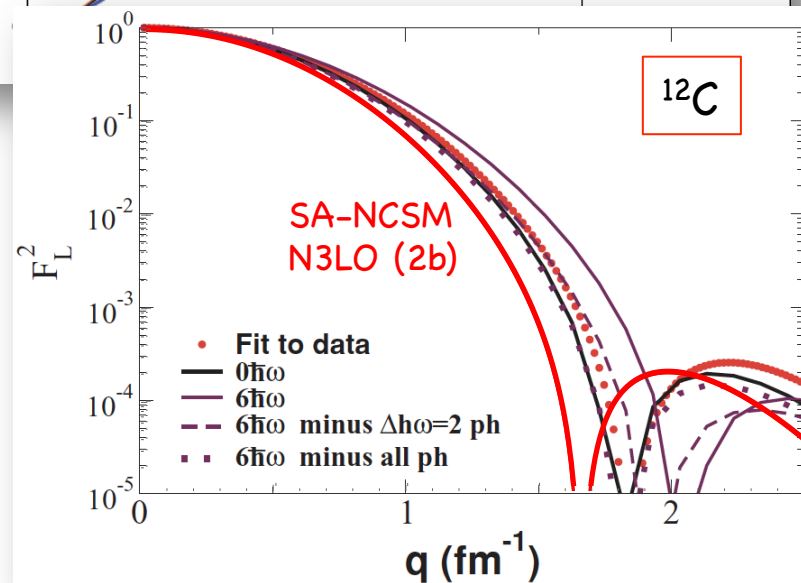
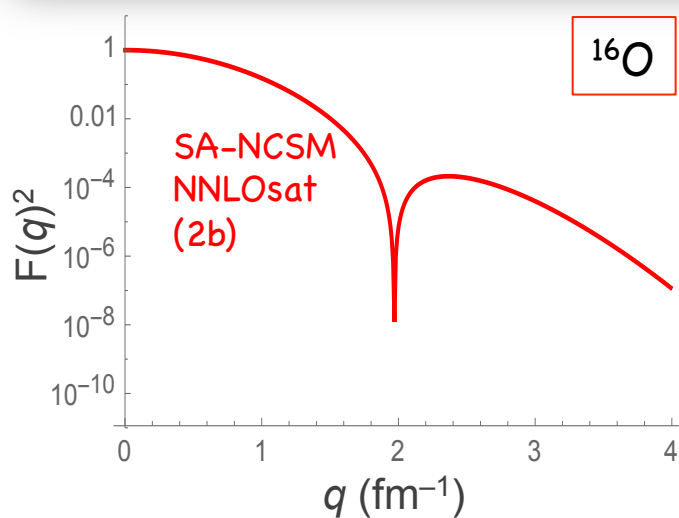
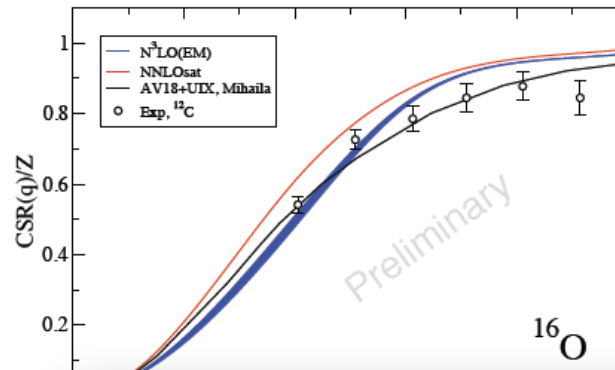
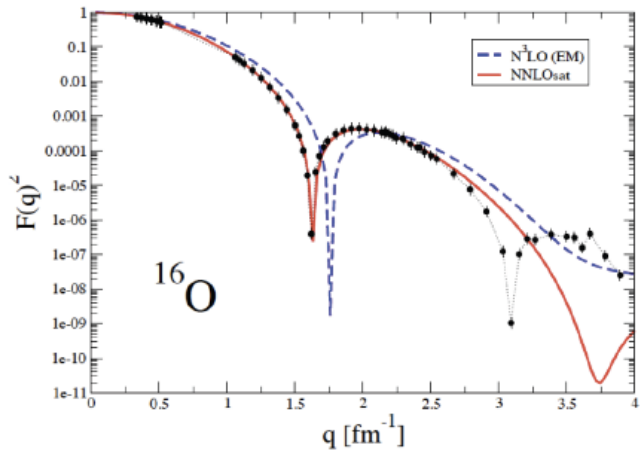
Coulomb sum rule

S. Bacca et al., in preparation (2018)



Coulomb sum rule

S. Bacca et al., in preparation (2018)

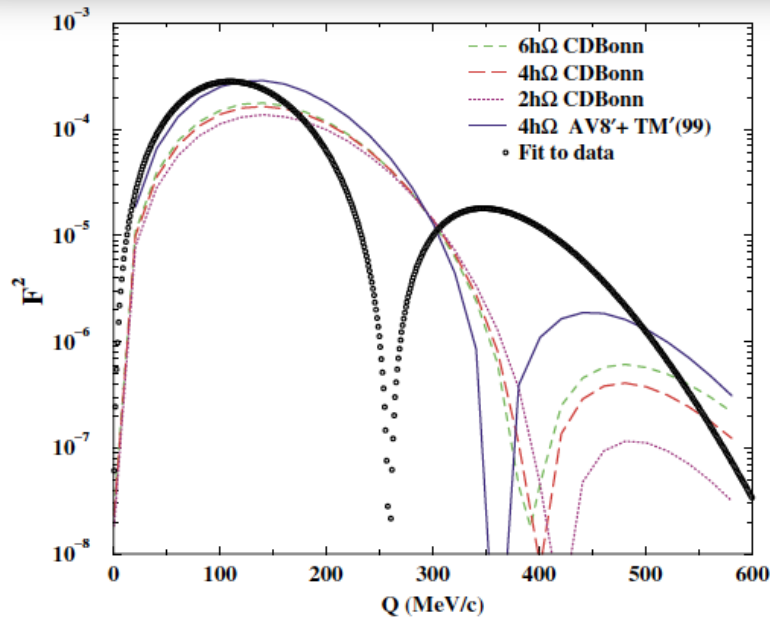


Hayes et al., PRC 81, 054301 (2010)

Outlook

TABLE II. Predicted weak interaction rates for the $^{12}\text{C} \rightarrow T = 1\ 1^+$ transitions. The units are 10^{-42} cm^2 for the (ν_e, e^-) DAR cross section, 10^{-40} cm^2 for the (ν_μ, μ^-) DIF cross section, and 10^3 sec^{-1} for muon capture.

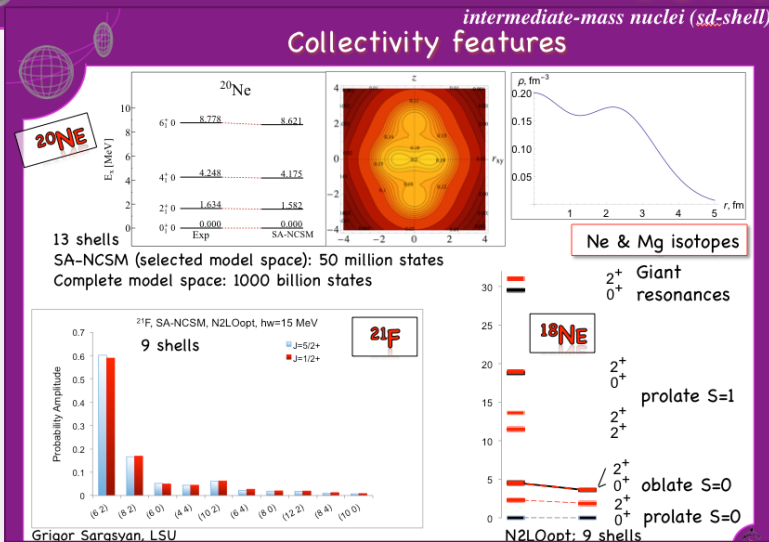
Interaction	CD-Bonn			AV8' + TM'(99)	Experiment
	$2\hbar\Omega$	$4\hbar\Omega$	$6\hbar\Omega$	$4\hbar\Omega$	
(ν_e, e^-)	2.27	3.2	3.69	6.8	$8.9 \pm 0.3 \pm 0.9$ [19]
(ν_μ, μ^-)	0.168	0.275	0.312	0.537	$0.56 \pm 0.08 \pm 0.1$ [20]
μ -capture	1.46	2.07	2.38	4.43	6.0 ± 0.4 [21]



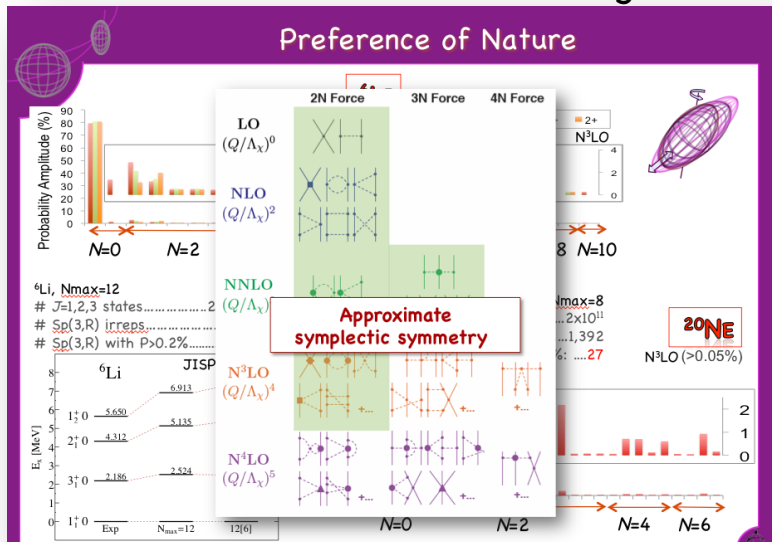
Hayes et al., PRL 91, 012502(2003)

- Improve earlier NCSM studies, use bare chiral potentials
- ^{40}Ar SA-NCSM calculations
- Calculate response functions from SA-NCSM with LIT

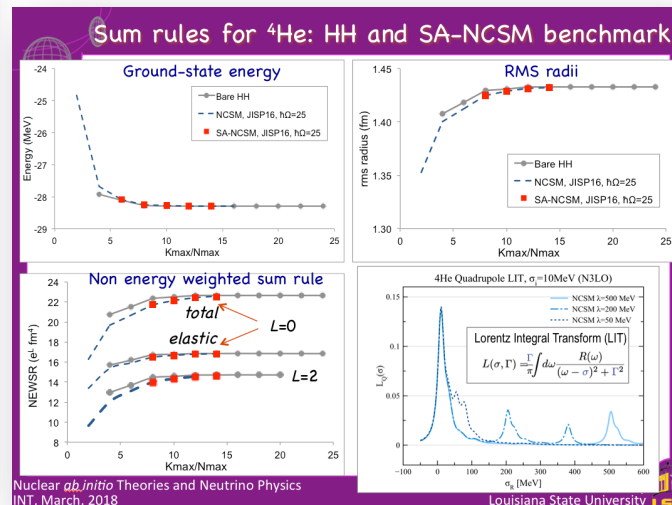
Conclusions



Collective and alpha clustering features in nuclei



Simple physics: "shape" + vibrations + rotations



SA-NCSM+LIT (with S. Bacca):
sum rules and responses

