

In Medium Majorana Neutrinos and Double Beta Decay

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- In vacuum Majorana neutrinos and double beta decay (DBD)
	- Classical neutrinoless DBD
	- Effective field theory approach
- In medium Majorana neutrino and neutrinoless DBD
	- Neutrino mixing inside atomic nuclei
	- Neutrinoless DBD of atomic nuclei

Classical Double Beta Decay Problem

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$$
U = \begin{bmatrix}\nU_{e1} & U_{e2} & U_{e3} \\
U_{H1} & U_{H2} & U_{H3}\n\end{bmatrix} = \begin{bmatrix}\nU_{e1} & U_{e2} & U_{e3} \\
U_{H1} & U_{H2} & U_{H3}\n\end{bmatrix} = \begin{bmatrix}\nc_{12}c_{13} & \nS_{13}c_{13} & S_{12}c_{13} & S_{13}c_{13} \\
-S_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & -c_{12}s_{23}s_{13}e^{i\delta} & -c_{22}s_{13}s_{13}e^{i\delta} & -c_{22}s_{13}\n\end{bmatrix} \begin{bmatrix}\ne^{i\alpha_{1}/2} & 0 & 0 \\
0 & e^{i\alpha_{2}/2} & 0 \\
0 & 0 & 1\n\end{bmatrix}
$$
\n
\n**11** - Tritium decay:
\n
$$
U = \begin{bmatrix}\nU_{e1} & U_{e2} & U_{e3} \\
U_{r1} & U_{r2} & U_{r3}\n\end{bmatrix} = \begin{bmatrix}\nc_{12}c_{13} & \nS_{12}c_{13} & S_{12}c_{13} & S_{13}c_{13} & S_{12}c_{13} \\
S_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13}\n\end{bmatrix} \begin{bmatrix}\ne^{i\alpha_{1}/2} & 0 & 0 \\
0 & e^{i\alpha_{2}/2} & 0 \\
0 & 0 & 1\n\end{bmatrix}
$$
\n
\n**12** - Tritium decay:
\n
$$
U = \begin{bmatrix}\nU_{e1} & U_{e2} & U_{e3} \\
U_{e1} & U_{e3} & U_{e4}\n\end{bmatrix} \begin{bmatrix}\nC_{12} & C_{23} & C_{23}c_{23} & C_{23}c_{23} & C_{23}c_{23} & C_{23}c_{23} \\
C_{12} = \cos\theta_{12}, s_{12} = \sin\theta_{12}, etc\n\end{bmatrix}
$$
\n
$$
M_{21} = \sqrt{2
$$

The Black Box Theorems

Black box I (electron neutrino)

- J. Schechter and J.W.F Valle, PRD 25, 2951 (1982)
- E. Takasugi, PLB 149, 372 (1984)
- J.F. Nieves, PLB 145, 375 (1984)

However:

M. Duerr et al, JHEP 06 (2011) 91

 $\left(\delta m_{v_e}\right)_{BB} \sim 10^{-24} eV \ll \sqrt{|\Delta m^2_{32}|} \approx 0.05 eV$

 $0v\beta\beta$ observed at some level

violated by 2 units. (ii) Electron neutrinos are Majorana fermions (with $m > 0$).

(i) Lepton number conservation is

Black box II (all flavors $+$ oscillations)

M. Hirsch, S. Kovalenko, I. Schmidt, PLB 646, 106 (2006)

(i) Lepton number conservation is violated by 2 units.

Regardless of the dominant 0νββ mechanism!

 $0\vee\beta\beta$ observed \Leftrightarrow at some level

(ii) Neutrinos are Majorana fermions.

$$
(iii) \ \left\langle m_{\beta\beta} \right\rangle = \left| \sum_{k=1}^{3} m_k U_{ek}^2 \right| = \left| c_{12}^2 c_{13}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3} \right| > 0
$$

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Other models: Left-Right symmetric model and SUSY R-parity violation

INT 18-1a, March 8, 2018 ial vector coupling constant, ⌘0⌫ = ^h*m*ⁱ

M. Horoi CMU ^(e) **M. Horoi, A. Neacsu, PRD 93, 113014 (2016)** with 18000, 11. 18000, 11. 18000, 11. 18000, 11. 18000, 11. 18000, 11. 18000, 11. 18000, 11. 18000, 11. 180000, 11. 180000, 11. 180000, 11. 180000, 11. 180000, 11. 180000, 11. 180000, 11. 180000, 11. 180000, 11. 180000, 11

QRPA-Jy J. Suhonen, O. Civitarese, Phys. NPA **847** 207–232 (2010).

QRPA-Tu A. Faessler, M. Gonzalez, S. Kovalenko, and F. Simkovic, arXiv:**1408.6077**

ISM-Men J. Menéndez, A. Poves, E. Caurier, F. Nowacki, NPA **818** 139–151 (2009).

SM M. Horoi et. al. PRC **88**, 064312 (2013), PRC **89**, 045502 (2014), PRC **89**, 054304 (2014), PRC **90**, 051301(R) (2014), PRC **91**, 024309 (2015), PRL **110**, 222502 (2013), PRL **113**, 262501(2014).

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IBA-2 J. Barea, J. Kotila, and F. Iachello, Phys. Rev. C **87**, 014315 (2013).

QRPA-Tu A. Faessler, M. Gonzalez, S. Kovalenko, and F. Simkovic, arXiv:**1408.6077.**

QRPA-Jy J. Hivarynen and J. Suhonen, PRC 91, 024613 (2015), **ISM-StMa** J. Menendez, private communication.

ISM-CMU M. Horoi et. al. PRC **88**, 064312 (2013), PRC **90**, PRC **89**, 054304 (2014), PRC **91**, 024309 (2015), PRL **110**, 222502 (2013).

INT 18-1a, March 8, M. Ho 2019 2018 *^V ^A,* ✏ *^S±^P ,* ✏*T R T L,* ✏*T R V* +*A ^V* ⁺*A,* ✏ *^S±^P ,* ✏*T R* 2018 scribed in Eqs. (8, 10, 12, 14, and 16) (see also Eq.(18) in the Appendix for the individial NME) and integrated \mathcal{N} and integrated \mathcal{N} number of observables is not enough to extract all cou p

 $[1]$ ₂ M₂ $\frac{1}{2}$

\blacksquare In terms of the effective of the effective of the effective or Ω UNIVERSITY CONSEQUENCES: - SCAIES TOP \overline{a} at the loop level; in such cases, the additional behavior \overline{a} Consequences: - scales for new physics \mathbb{R}^n ew pnysics

couplings are normalized with respect to the Fermi coupling GF and the proton mass mass mass mass mass mass much mass \sim baryogen $\mathbf{S} = \mathbf{S} \mathbf{S}$. This will make it unlikely that such contributions of the such contr - baryogenesis via leptogenesis

PHYSICAL REVIEW D 92, 036005 (2015)

$$
\mathcal{L}_D=\frac{g}{\Lambda_D^{D-4}}\mathcal{O}_D
$$

 $g \approx 1$ v = 174 GeV (Higgs expectation value) P_{max} and P_{max} improved by function P_{max} vaccum expectation value, *^G^F* = 1*.*¹⁶⁶ ⇥ ¹⁰⁵ GeV²

limit *^T*1*/*² *>* ¹*.*¹ ⇥ ¹⁰²⁶ years.

$$
\frac{\mathcal{O}_D}{\mathcal{O}_5} \quad \frac{\bar{\epsilon}_D}{2.8 \times 10^{-7}} \quad \Lambda_D \left(\text{GeV} \right)
$$
\n
$$
\mathcal{O}_7 \quad 2.0 \times 10^{-7} \quad 3.75 \times 10^4
$$
\n
$$
\mathcal{O}_9 \quad 1.5 \times 10^{-7} \quad 2.48 \times 10^3
$$
\n
$$
\mathcal{O}_{11} \quad 1.5 \times 10^{-7} \quad 1.16 \times 10^3
$$

INT 18-1a, March 8, M. Horoi CMU 2018 \mathbf{r} (a), \mathbf{r} , \mathbf{r} (a), \mathbf{r} , \mathbf{r} and \mathbf{r} , \mathbf{r} and \mathbf{r} 2018 \overline{P} be smaller for loop-induced diagrams. As \overline{P} $\begin{bmatrix} 1N1 & 10-1a, \text{ [NaCl]} 0 \\ 2019 & \text{ [Fe]} \end{bmatrix}$ 2010

$Consequences: -scal$ \sim 5 16 meeting meeting meeting meeting me, whereas the other mass me, whereas the other mass me, whereas the other measureme, whereas the other measureme, whereas the other measureme, whereas the other measureme, wh $\frac{1}{2}$ $\frac{1}{2}$ CENTRAL MICHIGAN Consequences: - scales for new physics <u>CONSQUENCES. SCAICS FOR HEW PHYSICS.</u> C ENTRAL MICHIGAN C_{α} and α and α and α and α Consequences. Sea *^L^D* ⁼ *^g*

couplings are not the Fermi contributions of the Fermi contr $\frac{\partial a}{\partial x}$ che be observed in the programs. - baryogenesis via leptogenesis stringent upper-limits than "1. With the exception of ¹ 1*.*08 0*.*75 2*.*81 1*.*98 1*.*63 $\frac{1}{2}$ *[|]*"1*[|]* ¹*.*⁴ *·* ¹⁰⁵ ³*.*² *·* ¹⁰⁷ ²*.*⁴ *·* ¹⁰⁶ ⁷*.*¹ *·* ¹⁰⁷ ¹*.*⁵ *·* ¹⁰⁷ *•* baryogenesis via leptogenesis $\overline{1}$ of \overline{C}

PHYSICAL REVIEW D 92, 036005 (2015) $\frac{1}{2}$ example the $\frac{1}{2}$ limit is $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ $\frac{1}{2}$ F1 is signal keview D 92, 0500 PHYSICAL REVIEW D 92, 036005 (2015)

$$
\mathcal{L}_{D}=\frac{g}{\left(\Lambda_{D}\right)^{D-4}}\mathcal{O}_{D}
$$

$$
m_e \bar{\epsilon}_5 = \frac{g^2 (yv)^2}{\Lambda_5}, \qquad \frac{G_F \bar{\epsilon}_7}{\sqrt{2}} = \frac{g^3 (yv)}{2(\Lambda_7)^3},
$$

$$
\frac{G_F^2 \bar{\epsilon}_9}{2m_p} = \frac{g^4}{(\Lambda_9)^5}, \qquad \frac{G_F^2 \bar{\epsilon}_{11}}{2m_p} = \frac{g^6 (yv)^2}{(\Lambda_{11})^7}
$$

TABLE VIII. The BSM effective scale $\left(\text{in GeV}\right)$ for different dimension-D operators at the present ¹³⁶Xe half-life limit (Λ_D^0) and for $T_{1/2} \approx 1.1 \times 10^{28}$ years (Λ_D) . *BLE VIII.* The BSM effective scale (in GeV) for differthe Higgs value of $\frac{136}{8}$ Xe half-life limit) and for $T_{1/2} \approx 1.1 \times 10^{26}$ years (Λ_D) .

\mathcal{O}_D	$\bar{\epsilon}_D$	$\Lambda_D^0(y=1)$ $(\Lambda_D^0(y=y_e)$ $\Lambda_D(y=y_e))$	
	\mathcal{O}_5 $2.8 \cdot 10^{-7}$ $2.12 \cdot 10^{14}$	1904	19044
	\mathcal{O}_7 $2.0 \cdot 10^{-7}$ $3.75 \cdot 10^4$	541	1165
	\mathcal{O}_9 $1.5 \cdot 10^{-7}$ $2.47 \cdot 10^3$	2470	3915
	\mathcal{O}_{11} $1.5 \cdot 10^{-7}$ $1.16 \cdot 10^{3}$	31	43

$$
\eta_N \propto \frac{1}{m_{W_R}^4 m_N} \qquad \qquad g \approx 1 \quad v = 17
$$

 $\eta_N \propto \frac{1}{m^4 m}$ $g \approx 1$ $v = 174 \text{ GeV}$ $v_s = 3 \times 10^{-6}$ electron mass Yukawa y_e \rightarrow \rightarrow \sim 10 electron mass fund was *^g* ≈*1 ^v* =*174 GeV* $\eta_{N} \propto \frac{1}{m_{W}^{4} m_{N}}$ $g \approx 1$ $v = 174 \text{ GeV}$ $y_{e} = 3 \times 10^{-6}$ electron mass Yukawa $\omega = \omega^2 \times 10^{-6}$ electron mass Yukawa y_e $\int \sqrt{P}$ $\int \sqrt{P}$ electron muss fund with *^S±^P [|]*, *[|]*✏*T R*

INT 18-1a, March 8, 2018 2018 \overline{P} be smaller for loop-induced diagrams. As \overline{P} 2010 INT 18-1a, March 8, M. Hor 18 $\frac{8}{2}$ 2018

M. Horoi CMU *|*" *LRz*(*RLz*) ³ *|*, *|*"4*|*, *|*"5*|*

coming from the light left-handed Majorana neutrino coming from the light left-handed Majorana neutrino (Fig. 1b), a long-range part coming from the low-energy (Fig. 1b), a long-range part coming from the low-energy four-fermion charged-current interaction (Fig. 1c), and a short-range part (Fig. 1d). short-range part (Fig. 1d).

We the long-range component of the $0\nu\beta\beta$ diagram as two point-piking refermint the Germi scale, one? exchange a light neutrino. In this case, the Lagrangian exchange a light neutrino. In this case, the Lagrangian can be expressed in terms of effective couplings $[15]$.

$$
\mathcal{L}_6 = \frac{G_F}{\sqrt{20}} \begin{bmatrix} 1 & 0 & 0 \\ \hline \sqrt{20} & 0 & 0 \\ \hline 0 & 0 & 0 \end{bmatrix}
$$

where $J^{\dagger}_{\alpha} = \bar{u} \mathcal{O}_{\alpha} d$ and $\overline{\mathcal{J}_{\beta}} \leftarrow \mathcal{O}_{\beta} \hat{\mu}$ are hadronic and leptonic Lorentz currents, respectively. The definitions of the *O*↵*,* operators are given in Eq. (3) initions of the *O*↵*,* operators are given in Eq. (3) of Ref. $[15]$. The LNV parameters are $\frac{15}{4}$ $\{\epsilon_{V-A}^{V+A},\hspace{0.1cm} \epsilon_{V+A}^{V+A},\hspace{0.1cm} \epsilon_{S+P}^{A} \hspace{0.1cm} \text{if}\hspace{0.1cm} \epsilon_{T B} \}$ and ϵ_{V-A}^{V+A} $\{\epsilon_{V-A}^{V+A}, \ \epsilon_{V+A}^{V+A}, \ \epsilon_{S+P}^{S+P} \ \text{with} \ \alpha = 0 \}$
dicates that the term with $\alpha = 0$. taken out of the sum. $G_F = 1.1663787 \times 10^{-5}$ GeV denotes the Fermi coupling constant. where $J^{\dagger}_{\alpha} = \vec{u} \vec{Q} \vec{d}$ and $\vec{q} \vec{Q}$ \vec{Q} are hadronic dicates that the term with $\alpha = 3$ is explicitly

The $0\nu\beta\beta$ decay amplitude is proportional to the timeordered product of two effective Lagrangians [15]: **444 - 22**

the on-axis values (when only one term in the effective Lagrangian dominates the process) of the LNV parame-Lagrangian dominates the process) of the LNV parameters: ters:

can be expressed in terms of effective couplings [15]:
 $\text{Here, the } \text{Momentum function}$ with $\mathcal{E}_1 = \eta_{0\nu}$ representing the e Here, the *Eⁱ* contain the neutrino physics parameters, Here, the *Eⁱ* contain the neutrino physics parameters, with $\mathcal{E}_1 = \eta_{0\nu}$ representing the exchange of light lefthanded neutrinos corresponding to Fig. 2b, \mathcal{E}_{27} = \mathbb{R} \mathbb{Z} \mathbb{Z} \mathbb{Z} \mathbb{Z} \mathbb{Z} \mathbb{Z} \mathbb{Z} \mathbb{Z} and $\mathbb{Z$ range parameters appearing in Figs. $2c$ and $2e$, and $\mathcal{E}_{\text{S}} = \{ \mathcal{E}_{\text{S}} \in \mathbb{R}^2 \mid \mathcal{E}_{\text{S}} \in \mathbb{R}^2 \}$ denote the short-range parameters at the quark level in-1 denote the short-range parameters at the quark level involved in the diagram of Fig. 2d, 2f, 2g. Following Refs. volved in the diagram of Fig. 2d, 2f, 2g. Following Refs. [13–15, 45], we write *^M*² *ⁱ* as combinations of NME de-[13–15, 45], we write *^M*² *ⁱ* as combinations of NME described in Eqs. $(8, 10, 12, 14, \text{and } 16)$ (see also Eq.(18) in the Appendix for the individial NME) and integrated in the Appendix for the individial NME) and integrated PSF [44] denoted with *G*⁰¹ *G*09. Our values of the PSF [44] denoted with *G*⁰¹ *G*09. Our values of the PSF are presented in Table I. In some cases the inter-PSF are presented in Table I. In some cases the interference terms *E*↵*EM*↵ are small [48] and can be ne-ference terms *E*↵*EM*↵ are small [48] and can be neglected. Considering an on-axis approach when extract-glected. Considering an on-axis approach when extracting the LNV parameters limits, the interference terms are ing the LNV parameters limits, the interference terms are $T[^{76}Ge]/T[^{A}Z]$ corresponds to the set of the with $\varepsilon_1 = \eta_{0\nu}$ representing the

coming from the light left-handed Majorana neutrino coming from the light left-handed Majorana neutrino (Fig. 1b), a long-range part coming from the low-energy (Fig. 1b), a long-range part coming from the low-energy four-fermion charged-current interaction (Fig. 1c), and a four-fermion charged-current interaction (Fig. 1c), and a short-range part (Fig. 1d). short-range part (Fig. 1d).

 C^{E} We treat the long-range component of the $0\nu\beta\beta$ diagram **as two coupling domes nance Fermi scale, which one** exchange a light neutrino. In this case, the Lagrangian exchange a light neutrino. In this case, the Lagrangian can be expressed in terms of effective couplings [15]: can be expressed in terms of effective couplings $[15]$:

$$
\mathcal{L}_6 = \frac{G_E}{40}
$$

where $J^{\dagger}_{\alpha} = \overline{\tilde{u}\mathcal{O}_{\alpha}d}$ and $\overline{\mathcal{O}_{\beta}d}$ where hadronic and leptonic Lorentz currents, respectively. The def-8 and leptonic Lorentz currents, respectively. The definitions of the *O*↵*,* operators are given in Eq. (3) initions of the *O*↵*,* operators are given in Eq. (3) of Ref. $[15]$. The LNV parameters are ϵ $\{ \epsilon_{V-A}^{V+A}, \ \ \epsilon_{V+A}^{V+A}, \ \ \epsilon_{V+A}^{V+A}, \ \ \epsilon_{V+A}^{S,B,E,C}$ dicates that the term with $\alpha = \beta$ is explicitly taken out of the sum. *G_{<i>F*} = 1663787 × 105 GeV² denotes the Fermi coupling constant. where $J^{\dagger}_{\alpha} = \bar{u}Q_{\alpha}d^{\dagger}$ and \bar{z} are hadronic $\{e^{V+A}, e^{V+A}, e^{S+A}, e^{S+B}e^{S+B}, e^{S+B}, e^{S+$ dicates that the **derm** with $\alpha = \beta - A$ is explicitly

The $0\nu\beta\beta$ decay amplitude is proportional to the timeordered product of two effective Lagrangians [15]:

the on-axis values (when only one term in the effective Lagrangian dominates the process) of the LNV parameters: ters:

$$
\left[\mathbf{f}_{1/2/2}^{0}\right]^{-\frac{1}{2}}\hspace{-0.5cm}=\hspace{-0.5cm}\underline{\mathbf{g}}_{A}\hspace{-0.5cm}\left[\sum_{i}\hspace{-0.5cm} \delta_{i}\hspace{-0.5cm}\underline{\mathbf{g}}_{i}^{2}\hspace{-0.5cm}\right]^{2}\hspace{-0.5cm}\mathcal{M}^{2+}\hspace{-0.5cm}+\hspace{-0.5cm}\mathrm{Re}\Bigl[\sum_{i\neq j}\hspace{-0.5cm}\delta_{i}\hspace{-0.5cm}\delta_{j}\hspace{-0.5cm}\mathcal{M}_{i}\hspace{-0.5cm}\Biggr]\hspace{-0.5cm}\Biggr]\hspace{-0.5cm}\Biggl].\hspace{-0.5cm}\Biggl[\hspace{-0.5cm}\sum_{i\neq j}\hspace{-0.5cm}\delta_{i}\hspace{-0.5cm}\delta_{j}\hspace{-0.5cm}\mathcal{M}_{i}\hspace{-0.5cm}\Biggr]
$$

Here, the *E*_i**f** contain the neutring physics in argumeters $\text{Tr}\left[\text{FPE}^{\text{seed}}_{\text{C}}\right]$ and $\text{FPE}^{\text{S}}_{\text{C}}$ and $\text{FPE}^{\text{$ handed neutrinos corresponding to Fig. 2b, *E*²⁷ = handed neutrinos corresponding to Fig. 2b, *E*²⁷ = $\mathcal{L}(\mathcal{L}(\mathcal{L}))$, we can also the contribution of $\mathcal{L}(\mathcal{L}(\mathcal{L}))$. The contribution of $\mathcal{L}(\mathcal{L}(\mathcal{L}))$ range parameters appearing in Figs. 2c and 2e, and range parameters appearing in Figs. 2c and 2e, and $\mathcal{E}_{\mathcal{S}} = \{ \mathcal{E}_{\mathcal{S}} \in \mathcal{E}_{\mathcal{S}} : \mathcal{E}_{\mathcal{S}} \in \mathcal{E}_{\mathcal{S}} \}$ denote the short-range parameters at the quark level in-denote the short-range parameters at the quark level involved in the diagram of Fig. 2d, 2f, 2g. Following Refs. volved in the diagram of Fig. 2d, 2f, 2g. Following Refs. [13–15, 45], we write *^M*² *ⁱ* as combinations of NME de-[13–15, 45], we write *^M*² *ⁱ* as combinations of NME described in Eqs. $(8, 10, 12, 14,$ and $16)$ (see also Eq.(18) in the Appendix for the individial NME) and integrated in the Appendix for the individial NME) and integrated PSF [44] denoted with *G*⁰¹ *G*09. Our values of the PSF [44] denoted with *G*⁰¹ *G*09. Our values of the PSF are presented in Table I. In some cases the inter-PSF are presented in Table I. In some cases the interference terms *E*↵*EM*↵ are small [48] and can be ne-ference terms *E*↵*EM*↵ are small [48] and can be neglected. Considering an on-axis approach when extract-glected. Considering an on-axis approach when extracting the LNV parameters limits, the interference terms are ing the LNV parameters limits, the interference terms are $\text{Tr}[\text{KZ}]$ $\text{Tr}[\text{KZ$

Neutrinos in atomic nuclei

Atomic nucleus is a high electron density medium:

Consider 2 electrons in the lowest s-orbital of an Hydrogen-like atom

Electron density inside nucleus:
$$
N_e \approx \frac{2}{\pi} \left(\frac{Z}{a_B}\right)^3
$$

Equivalent matter density: $\rho = m_N N_e = 1.67 \times 10^6 \frac{2}{\pi}$ *Z* 53 $\sqrt{2}$ ⎝ $\left(\frac{Z}{52}\right)$ ⎠ $\overline{}$ 3 \int *in* g / cm^2 >> ρ_{Sun}

$$
\rho_{\text{Suncore}} \approx 150 \text{ g}/\text{cm}^3
$$

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$$
|v_e\rangle = \cos\theta |v_1\rangle + \sin\theta |v_2\rangle
$$

Low energy

$$
P_{v_e}(t) = |\langle v_e | v_e(t) \rangle|^2 = 1 - \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 x}{4E}\right)
$$

$$
\frac{1}{\sqrt{2}} \sin^2 2\theta = 0.56 > 0.5
$$

High energy $|v_e(t)\rangle = \sin\theta e^{-iEt}$ $|v_2\rangle$ $P_{v_e}(t) = |\langle v_e(0) | v_e(t) \rangle|^2$ $= \sin^2 \theta = 0.32 < 0.5$

$$
|v_e\rangle = \sum_{i=1}^{N(3)} U_{ei}^* |v_i\rangle
$$

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Figure 8. Solar v_e survival probability as a function of energy. The colour lines correspond to experimental data. The grey line corresponds to the MSW-LMA solution for solar neutrino oscillation. (From [29]).

 J ournal of Physics: Conference Series **593** (2015) 012007 \mathcal{S} . As said in section 3.6, it is known that solar \mathcal{S} (2000) \mathcal{S} . It is known that solar \mathcal{S}

Neutrino Mixing (MSW effect)

Neutrinos in matter interact with:

- Electrons via charged current
- Any fermion via neutral current

 $V_e = \pm \sqrt{2} G_F N_e$ (*N_e*: *electron density*) + (−) *neutrino* (*antineutrino*)

$$
V_N \t\t P \approx E
$$

$$
\begin{aligned}\n|v_e &>= \cos \theta_m \left| v_1 \right. > + \sin \theta_m \left| v_2 \right. > \\
|v_x &>= - \sin \theta_m \left| v_1 \right. > + \cos \theta_m \left| v_2 \right. > \\
|v_x &>= - \sin \theta_m \left| v_1 \right. > + \cos \theta_m \left| v_2 \right. > \\
\end{aligned}
$$

$$
2P\Delta E_m = 2P(E_{2m} - E_{1m}) = \Delta \mu^2 = \sqrt{\left(\Delta m^2 \cos 2\theta - 2PV_e\right)^2 + \left(\Delta m^2 \sin 2\theta\right)^2}
$$

$$
\cos 2\theta_m = \frac{\Delta m^2 \cos 2\theta - 2PV_e}{\sqrt{\left(\Delta m^2 \cos 2\theta - 2PV_e\right)^2 + \left(\Delta m^2 \sin 2\theta\right)^2}} \xrightarrow[0 \le 2PV_e > \Delta m^2]{} - 1 \implies \theta_m = \frac{\pi}{2}
$$

 $|v_{em}\rangle = 0 |v_1\rangle + 1 |v_2\rangle$ \rightarrow *adiabatic transport* $\rightarrow P_{v_e}(t) = |v_e| |v(t)\rangle^2 = |v_e| |v_{em}\rangle^2 = \sin^2 \theta$

NO oscillations! Just mixing!

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Gonzales-Garcia & Nir, RMP 75, 345 (2003) \overline{a} rcia & Nir, RMP 75, 345 ($\overline{20}$

$$
\begin{pmatrix} \nu_e \\ \nu_X \end{pmatrix} = U(\theta_m) \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix} = \begin{pmatrix} \cos \theta_m & \sin \theta_m \\ -\sin \theta_m & \cos \theta_m \end{pmatrix} \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix}
$$

$$
P_e = \left| \nu_e^m \right|^2 \qquad P_X = \left| \nu_X^m \right|^2
$$

$$
\begin{pmatrix} \dot{\nu}_1^m \\ \dot{\nu}_2^m \end{pmatrix} = \begin{pmatrix} i\Delta(t) & -4E\dot{\theta}_m(t) \\ 4E\dot{\theta}_m(t) & -i\Delta(t) \end{pmatrix} \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix} \qquad \dot{\theta}_m
$$

Adiabatic evolution: *off* − *diagonal terms neglijable*

 p ^{*e*} and *nergies*: matter effects neglijable *Peris next*
co $diabatic$ *evolution* Since the resonance point is not crossed, cos 2"*^m* has the s : matter effects + nonadiabatic evolu α (1) *small energies*: *matter effects neglijable* (2) *lage energies*: *matter effects* + *adiabatic evolution* (3) *lager energies*: *matter effects* + *nonadiabatic evolution*

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the previous section, for small mixture in vacuum, for small mixing and M Horoi CMII m at m and m $\sum_{i=1}^{n}$ $U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$ $\Big)$

Neutrinos in atomic nuclei

Atomic nucleus is a high electron density medium:

Consider 2 electrons in the lowest s-orbital of an Hydrogen-like atom

Equivalent matter density: $\rho(t)$ = 1.67 \times 10⁶ $\frac{2}{\epsilon}$ π *Z* 53 $\sqrt{2}$ ⎝ $\left(\frac{Z}{52}\right)$ ⎠ $\overline{}$ 3 *e*[−]2*tZ*/53 [*in g* / *cm* 2 , *t in pm*]

 $V_e(t)$ [*in* eV] = 7.6 × 10¹⁴ $\rho(t)$ [*in g* / *cm*³]

The typical requirement is:

$$
\lambda \ll \left| \frac{V}{\partial V / \partial t} \right| \iff 2\pi \frac{\hbar c}{p} \ll \frac{53}{2Z} 1000 \quad [\text{in fm}]
$$

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Partial summary

- One can consider the neutrino mixing in atomic nuclei.
- The analysis of neutrino mixing in the Sun and atomic nuclei leads to results backed up by phenomenology.
- These results seem simple and natural, but the road to them is complex!

^L(*x*) = ✓ ⁰

Neutrinoless double beta decay in vacuum 1 derivative interesting the original extending the Neutrinoless double beta decay in \mathbb{R}^n ϵ derivation of the decay have the complete assume that the complete μ

$$
A_{0\beta\beta} \propto NP = \langle 0| T \left[\psi_{eL}(x_1) \psi_{eL}^T(x_2) \right] | 0 \rangle
$$

$$
\psi_e(x) = \sum_{a=1}^{N(3)} U_{ea} \psi_a(x)
$$

$$
NP = \sum_{a=1}^{3} U_{ea}^{2} \langle 0| T [\psi_{aL}(x_{1}) \psi_{aL}^{T}(x_{2})] |0\rangle
$$

= $\sum_{a=1}^{3} U_{ea}^{2} [-i \int \frac{d^{4}p}{(2\pi)^{4}} \frac{m_{a}e^{-ip(x_{1}-x_{2})}}{p^{2}-m_{a}^{2}+i\epsilon} P_{L}C]$

$$
P_{L} = \frac{1}{2} (1-\gamma^{5}) \qquad \hat{\psi}(x) = C \psi^{*}(x)
$$

fields necessary to further process the electron current. nt, and one finally gets: **P**_{*L*}^C product is further used to process the electron current and $T_{\rm L}$ C product is further used to process the electron current, as \overline{D} and decay is for the special formula formula for the decay standard formula for the \overline{D} $P_L C$ product is further used to process the electron current, and one finally gets:

2018

Neutrino Fields

Dirac equation: states vs fields

$$
(i\gamma^{0}\partial_{0} + i\gamma^{i}\partial_{i} - m)\psi(x) = 0
$$
\n
$$
\gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2g^{\mu\nu} \qquad \gamma^{5} = i\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3}
$$
\n
$$
i\frac{\partial\psi(x)}{\partial t} = (-i\gamma^{0}\gamma^{i}\partial_{i} + \gamma^{0}m)\psi(x)
$$
\n
$$
\gamma^{\dagger}_{\mu} = \gamma_{0}\gamma_{\mu}\gamma_{0} \qquad \Rightarrow \qquad \gamma^{\dagger}_{0} = \gamma_{0} \qquad \gamma^{\dagger}_{i} = -\gamma_{i}
$$
\n
$$
\psi(x) = \int \frac{d^{3}p}{(2\pi)^{3/2}2E_{p}} \Big[\Big(u^{(+)}(\vec{p})a(\vec{p},+) + u^{(-)}(\vec{p})a(\vec{p},-) \Big) e^{-ipx} + \Big(v^{(+)}(\vec{p})b^{\dagger}(\vec{p},+) + v^{(-)}(\vec{p})b^{\dagger}(\vec{p},-) \Big) e^{ipx} \Big]
$$
\n
$$
\Big[a(\vec{p},h), a^{\dagger}(\vec{p}',h') \Big]_{+} = \delta_{hh} \delta(\vec{p} - \vec{p}') \qquad \Big[a(\vec{p},h), a(\vec{p}',h') \Big]_{+} = \Big[a^{\dagger}(\vec{p},h), a^{\dagger}(\vec{p}',h') \Big]_{+} = 0
$$
\nSame for b, b[†]

In addition one needs a vaccum state : $|0>$ \Rightarrow $|\psi\rangle = \psi(x)|0\rangle$

P. Mannheim, PRD **37**, 1935 (1988): used *a(p, -)* piece of the field to justify Wolfenstein's Eqs. provided that neutrinos, Dirac or Majorana, are ultra relativistic!

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PHYSICAL REVIEW 0 VOLUME 45, NUMBER ⁵ (2.15) For relativistic neutrinos, for which m, «P there are ^N solutions for j =1, . . . , ^N corresponding to \overline{R} and \overline{R} are ceived manuscript received manuscript received \overline{R} or \overline $\frac{1}{\sqrt{N}}$

Majoron decay of neutrinos in matter fully conserved and helicit in the matter matter in decays of neutrinos in matter Majoron decay of neutrinos in matter \mathbf{R} t extrins ϵ in matter.

C. Giunti, C. W. Kim, and U. W. Lee C. Giunti, C. W. Kim, and U. W. Lee
Department of Physics and Astronomy, The Johns Hopkins University, Baltimore, Maryland 21218 ^N —iE'"'f+~ ^x iE'"'~ —I ^x C. Giunti, C. W. Kim, and U. W. Lee
Department of Physics and Astronomy, The Johns Hopkins University, Baltimore, Maryland 21218 Departi

W. P. Lam model-independent approach, can be approach, can be applied to any number of neutrino generations and to the decay number of neutrino generations and to the decay number of neutrino generations and to the decay number of n

Institute ofPhysics, Academia Sinica, Taipei, Taiwan 11529,Republic of China w. P. Lam
with production of any massless pseudoscalar boson. In particular, we discuss the two-generation case with production of any massless pseudoscalar boson. In particular, we discuss the two-generation case
and show that *in matter* the helicity-flipping decays are dominant over the helicity-conserving decays. flipping and helicity-conserving α of neutrinos propagating in dense media. We discuss α The implications of the Majoron decay for the neutrinos from astrophysical objects are also briefly disrne m The implications of the Majoron decay for the ne
cussed. \sim documentations. The values of the and show that *in matter* the hencity-inpping decays are dominant over the hencity-conserving decays.
The implications of the Majoron decay for the neutrinos from astrophysical objects are also briefly dissino improducino et the majorem decay for the heath mentry-conserving accays.
objects are also briefly disand show that *in matter* the helicity-flipping decays are dominant over the helicity-conserving decays.
The implications of the Majoron decay for the neutrinos from astrophysical objects are also briefly dissquared in matter. Since to lowest order eration case d destruction and creation operators obeying the canonical theorem d α and α is α $w = c$ = c = c + c + n , n are the energy eigenthe interpretation of the interpretation of the operation of the second and and a second and and a second and a $\overline{\mathbf{a}}$ and particular, we discuss the two-generation can

$$
Weyl: \ \gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad P_L C = \begin{pmatrix} 0 & 0 \\ 0 & i\sigma^2 \end{pmatrix} \qquad \psi(x) = \begin{pmatrix} -i\sigma^2 \Phi^*(x) \\ \Phi(x) \end{pmatrix} \Rightarrow \psi_L (x) = \begin{pmatrix} 0 \\ \Phi(x) \end{pmatrix}
$$

$$
\Phi_{a}^{M}(x) = \int \frac{d\mathbf{p}}{\sqrt{(2\pi)^{3}}} \sum_{j=1}^{N} \sum_{h=\pm 1} [\alpha_{aj}^{(h)}(P)w(\mathbf{p}, h)a_{j}(\mathbf{p}, h)e^{-iE_{j}^{(h)}t + ip\cdot x} - h\beta_{aj}^{(h)^{*}}(P)w(\mathbf{p}, -h)a_{j}^{\dagger}(\mathbf{p}, h)e^{iE_{j}^{(h)}t - ip\cdot x}]
$$

$$
\beta_{aj}^{(-)}(P) = \frac{m_a}{2P} \alpha_{aj}^{(-)}(P) \qquad \frac{m_j^{(-)}^2 - m_a^2}{2P} \alpha_{aj}^{(-)}(P) - \sum_{b=1}^N V_{ab}^M \alpha_{bj}^{(-)}(P) = 0
$$

$$
\alpha_{aj}^{(+)}(P) = \frac{m_a}{2P} \beta_{aj}^{(+)}(P) \qquad m^{(+)^2} - m^2 \qquad N
$$

$$
\alpha_{aj}^{(+)}(P) = \frac{m_a}{2P} \beta_{aj}^{(+)}(P) \qquad \frac{m_j^{(+)}^2 - m_a^2}{2P} \beta_{aj}^{(+)}(P) + \sum_{b=1}^N V_{ab}^M{}^* \beta_{bj}^{(+)}(P) = 0
$$

$$
E_j^{(h)} = P + \frac{m_j^{(h)^2}}{2P}, \quad j = 1, ..., N
$$

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INT 18-1a, March 8, M. Hord a, M. Hord and a strategies of the masses of the effective masses of the e 2018 \mathcal{S} . We derive the spinorial wave the spinori 2018 $\mathbf I$ 2018 2018 $\text{INT } 18-1a$, March 8, M. F 2018

 $M.$ HOTOI UNIU

$\begin{array}{ll} \textbf{CENTRAL} \textbf{MICHIGAN} \\ \textbf{university} \end{array}$ Neutrinoless double beta decay of atomic nuclei versions

$$
\Phi_e^W(x) = \int \frac{d^3 p}{(2\pi)^{3/2}} \sum_{a,j} U_{ca} \left[\left(\alpha_a^{(-)} \chi^{(-)}(\vec{p}) a_j(\vec{p}, -) + \frac{m_a}{2\hat{P}} \beta_a^{(+)} \chi^{(+)}(\vec{p}) a_j(\vec{p}, +) \right) e^{-ip \cdot x} \right]
$$
\n
$$
\int \left(\frac{m_a}{2\hat{P}} \alpha_a^{-} \gamma^* \chi^{(+)}(\vec{p}) a_j^{\dagger}(\vec{p}, -) - \beta_{a,j}^{(+)*} \chi^{(-)}(\vec{p}) a_j^{\dagger}(\vec{p}, +) \right) e^{ip \cdot x} \right]
$$
\n
$$
\sum_a U_{ca} \alpha_a^{(-)} = \delta_{j,j_b}
$$
\n
$$
\sum_{a} U_{ca} \beta_a^{(+)*} = \delta_{j,j_b}
$$
\n
$$
j_b - \text{ highest mass eigenstate (3 for NO, 2 for IO)}
$$
\n
$$
j_l - \text{ lowest mass eigenstate (1 for NO, 3 for IO)}
$$
\n
$$
\chi^{(+)}(\vec{p}) = \begin{pmatrix} \cos \frac{\theta}{2} \\ \sin \frac{\theta}{2} e^{i\phi} \end{pmatrix}, \qquad \chi^{(-)}(\vec{p}) = \begin{pmatrix} -\sin \frac{\theta}{2} e^{-i\phi} \\ \cos \frac{\theta}{2} \end{pmatrix}
$$
\n
$$
N = \text{NFT 18-1a, March 8}, \qquad \text{M. Horoi CMU}
$$

inverted ordering). The index of the normal ordering and state 3 for the normal ordering and state 3 μ ordering), and *j^l* is the index of the lowest mass eigenstate (i.e. state 1 for the normal ordering and state 3 for the where *j^h* is the index of the highest mass eigenstate (i.e. state 3 for the normal ordering and state 2 for the inverted

$$
\chi^{(+)}(\vec{p}) = \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}e^{i\phi} \end{pmatrix} , \qquad \chi^{(-)}(\vec{p}) = \begin{pmatrix} -\sin\frac{\theta}{2}e^{-i\phi} \\ \cos\frac{\theta}{2} \end{pmatrix}
$$

INT 18-1a, March 8, 2018 *m^a a,j Uea* ²*^P* ↵()⇤

^Uea ⇥ ⇣

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M. Horoi CMU 28 *M.* Horoi CMU

CENTRAL MICHIGAN Neutrinoless double beta decay of atomic *u d decay* of a domine to the *decay* of a domine nuclei

$$
\Phi_e^W(x) = \int \frac{d^3 p}{(2\pi)^{3/2}} \left[\left(\chi^{(-)}(\vec{p}) a_{j_h}(\vec{p}, -) + \sum_{a,j} U_{ea} \frac{m_a}{2P} \beta_{a}^{(+)} \chi^{(+)}(\vec{p}) a_j(\vec{p}, +) \right) e^{-ip \cdot x} + \left(\sum_{a,j} U_{ea} \frac{m_a}{2P} \alpha_{a}^{(-)*} \chi^{(+)}(\vec{p}) a_j^{\dagger}(\vec{p}, -) - \chi^{(-)}(\vec{p}) a_{j_l}^{\dagger}(\vec{p}, +) \right) e^{ip \cdot x} \right]
$$

 j_h – highest mass eigenstate (3 for NO, 2 for IO)

 j_l – lowest mass eigenstate (1 for NO, 3 for IO) *b j* (+)(~*p*)()*^T* (~*p*)*eip·*(*x*1*x*2)

ative 4-component spinors
M. Horoi CMU Phys.Lett. B336 (1994) 439-445 alternative 4-component spinors
M. Horsi CMU

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Effective neutrino mass

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Effective neutrino mass

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M. Horoi CMU m_e $\frac{81}{31}$

Neutrinoless double beta decay of atomic nuclei *^M ^a* (*x*) = ^Z *^d*³*^p* ihle he ↵() *a j ^w*(~*p,*)*a^j* (~*p,*) + *^m^a* ²*^p* (+) *a j ^w*(~*p,* +)*a^j* (~*p,* +)⌘ *^e* (*x*2) *^T [|]*0ⁱ ⁼ *b j* (+)(~*p*)()*^T* (~*p*)*eip*(*x*1*x*2) $\mathcal{L}_{\text{CENTRAL MICHIGAN}}$ Neutrinoless double beta decay of atomic nuclei (13). It is preferable to calculate the contributions to electron neutrino propagator, Eq. (11), in an electron density

$$
\Phi_e^W(x) = \int \frac{d^3 p}{(2\pi)^{3/2}} \sum_{a,j} U_{ea} \left[\left(\alpha_{a}^{(-)} \chi^{(-)}(\vec{p}) a_j(\vec{p},+) + \frac{m_a}{2P} \beta_{a}^{(+)} \chi^{(+)}(\vec{p}) a_j(\vec{p},+) \right) e^{-ip \cdot x} \right. \\
\left. + \left(\frac{m_a}{2P} \alpha_{a}^{(-)} \chi^{(+)}(\vec{p}) a_j^{\dagger}(\vec{p},-) - \beta_{a}^{(+)} \chi^{(-)}(\vec{p}) a_j^{\dagger}(\vec{p},+) \right) e^{ip \cdot x} \right] \\
\sum_{j=1}^{N(3)} \alpha_{a,j}^{(-)} \alpha_{b,j}^{(-)^*} = \delta_{ab} \\
N P_a = \left\langle 0 \right| T \left[\psi_{aL}(x_1) \psi_{aL}^T(x_2) \right] \left| 0 \right\rangle = \begin{pmatrix} 0 & 0 \\ 0 & \langle 0 \right| T \left[\Phi_a(x_1) \Phi_a^T(x_2) \right] \left| 0 \right\rangle \end{pmatrix}
$$

$$
t_1 > t_2
$$

\n
$$
\langle 0 | \Phi_e^W(x_1) (\Phi_e^W(x_2))^{T} | 0 \rangle = \sum_a U_{ea}^2 \int \frac{d^3 p}{(2\pi)^3} \frac{m_a}{2P} \left[\chi^{(-)}(\vec{p}) \chi^{(+)T}(\vec{p}) - \chi^{(+)}(\vec{p}) \chi^{(-)T}(\vec{p}) \right] e^{-ip \cdot (x_1 - x_2)}
$$

$$
\langle 0|T\left[\Phi_e^W(x_1)\left(\Phi_e^W(x_2)\right)^T\right]|0\rangle = -i\sum_a U_{ea}^2 \int \frac{d^4p}{(2\pi)^4} \frac{m_a e^{-ip(x_1 - x_2)}}{p^2 - m_a^2 + i\epsilon} \left(i\sigma^2\right)
$$

In atomic nuclei NP = In vacuum NP

In atomic nuclei NP = *In vacuum NP iic nuclei NP* = *In vacuum NF In atomic nuclei NP* = *In vacuum NP*

$$
Vacuum result stands: \ m_{\beta\beta} = \left| \sum_{a=1}^{3} U_{ea}^{2} m_{a} \right|
$$

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0 $i\sigma^2$

⎠ $\overline{\mathcal{L}}$

 $f_L C = \begin{pmatrix} 0 & i\sigma^2 \end{pmatrix}$

⎝

Summary

- Neutrinoless DBD, if observed, will represent a big step forward in our understanding of the neutrinos, and of physics beyond the Standard Model.
- Better nuclear matrix elements and effective DBD operators are needed to identify the underlying mechanism(s).
- The effects of the high electron densities in atomic nuclei were investigated and they do not change the neutrino emission or detection, nor the 0*v*ββ outcome.
- These results look simple, but the road to them is complex. Other observables (Majoron) may be affected!

