In Medium Majorana Neutrinos and Double Beta Decay

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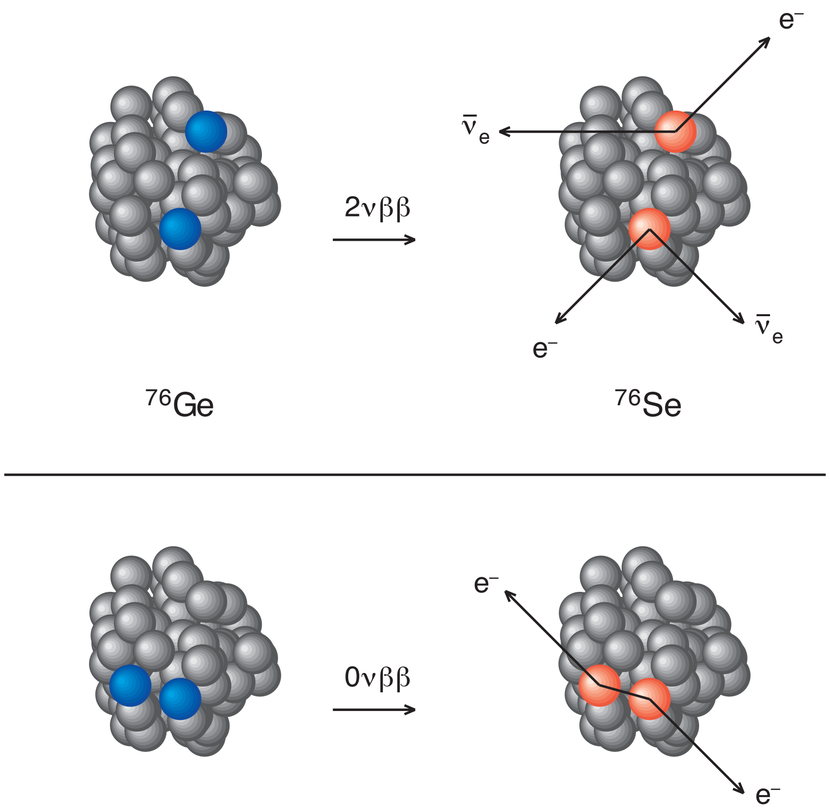
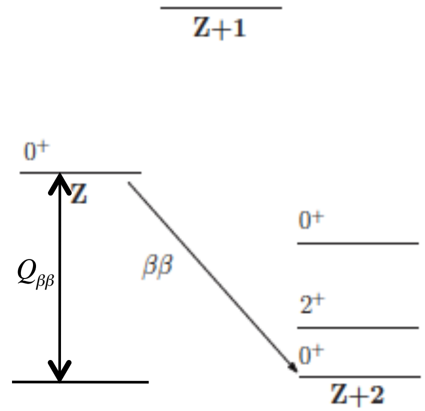
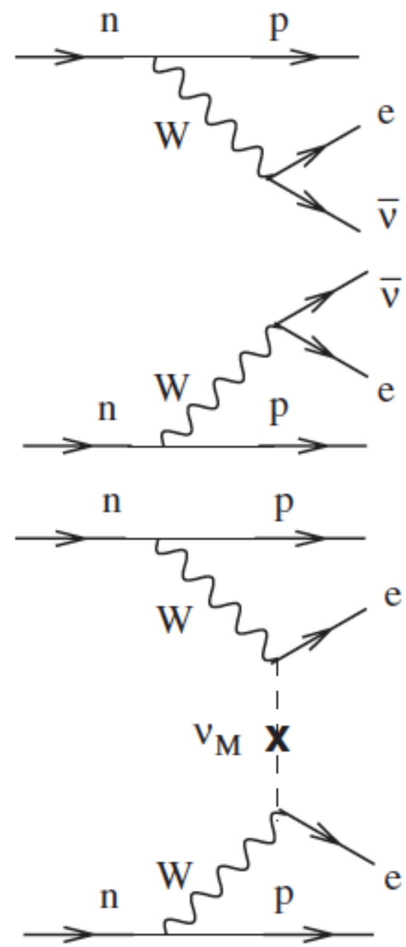
Plan of talk

- In vacuum Majorana neutrinos and double beta decay (DBD)
  - Classical neutrinoless DBD
  - Effective field theory approach
- In medium Majorana neutrino and neutrinoless DBD
  - Neutrino mixing inside atomic nuclei
  - Neutrinoless DBD of atomic nuclei

Classical Double Beta Decay Problem







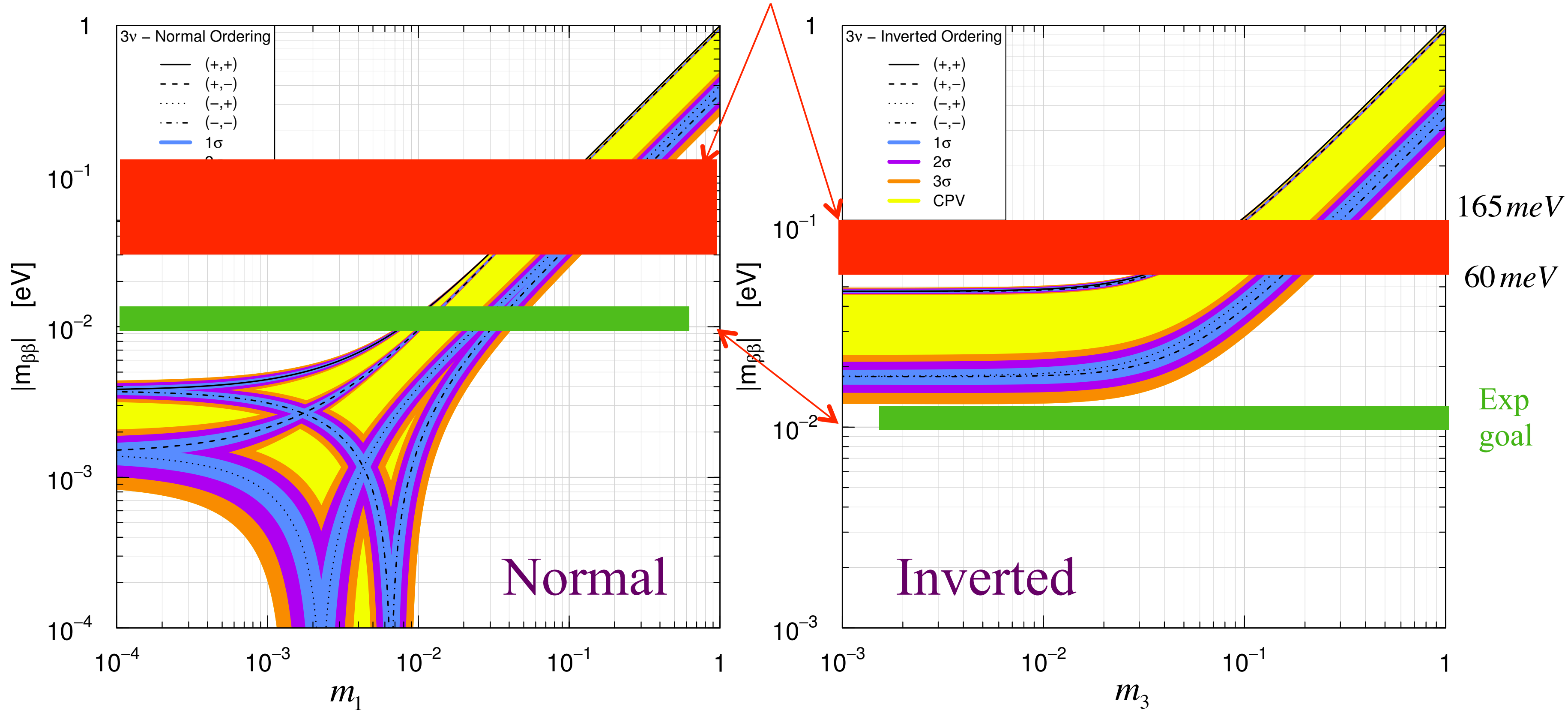
$$\langle m_{\beta\beta} \rangle = \left| \sum_k m_k U_{ek}^2 \right|$$

$$T_{1/2}^{-1}(0\nu) = G^{0\nu}(Q_{\beta\beta})\left[M^{0\nu}(0^+)\right]^2 \left(\frac{< m_{\beta\beta} >}{m_e} \right)^2$$

Neutrino ββ effective mass

arxiv:1507.08204

KamLAND – Zen, PRL 117, 082503 (2016) : ^{136}Xe



$$\left|m_{\beta\beta}\right| = \left|\sum_{k=1}^{3} m_k U_{ek}^2\right| = \left|c_{12}^2 c_{13}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3}\right|$$

$$\phi_2 = \alpha_2 - \alpha_1 \qquad \phi_3 = -\alpha_1 - 2\delta$$

$$\Leftarrow T_{1/2}^{-1}(0\nu) = G^{0\nu}(Q_{\beta\beta})\left[M^{0\nu}(0^+)\right]^2(\eta_{0\nu})^2$$

$$\eta_{0\nu} = \frac{\left|m_{\beta\beta}\right|}{m_e}$$

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}|\nu_i\rangle$$
$$|\nu_i\rangle = \sum_\alpha U_{\alpha i}|\nu_\alpha\rangle$$

Neutrino Masses

PMNS – matrix

$$U = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \begin{bmatrix} e^{i\alpha 1/2} & 0 & 0 \\ 0 & e^{i\alpha 2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$c_{12} \equiv \cos\theta_{12},\ s_{12} = \sin\theta_{12},\ etc$$

- Tritium decay:

$$^3H \rightarrow {}^3He + e^- + \bar{\nu}_e$$

$$m_{\nu_e} = \sqrt{\sum_i |U_{ei}|^2 m_i^2} < 2.2eV\ (Mainz\ \exp.)$$

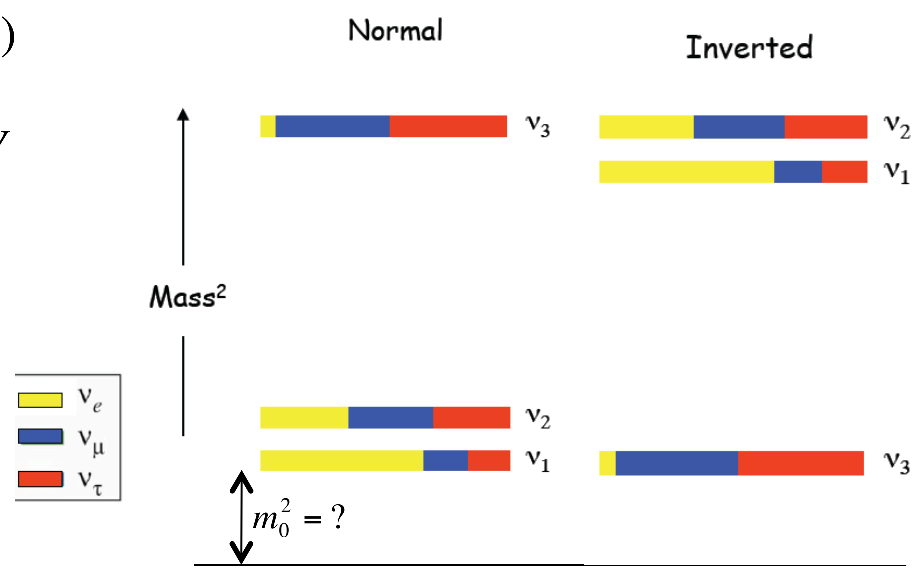
$$KATRIN\ (to\ take\ data):\ goal\ m_{\nu_e} < 0.3eV$$

- Cosmology: CMB power spectrum, BAO, etc,

$$\sum_{i=1}^{3} m_i < 0.23eV$$

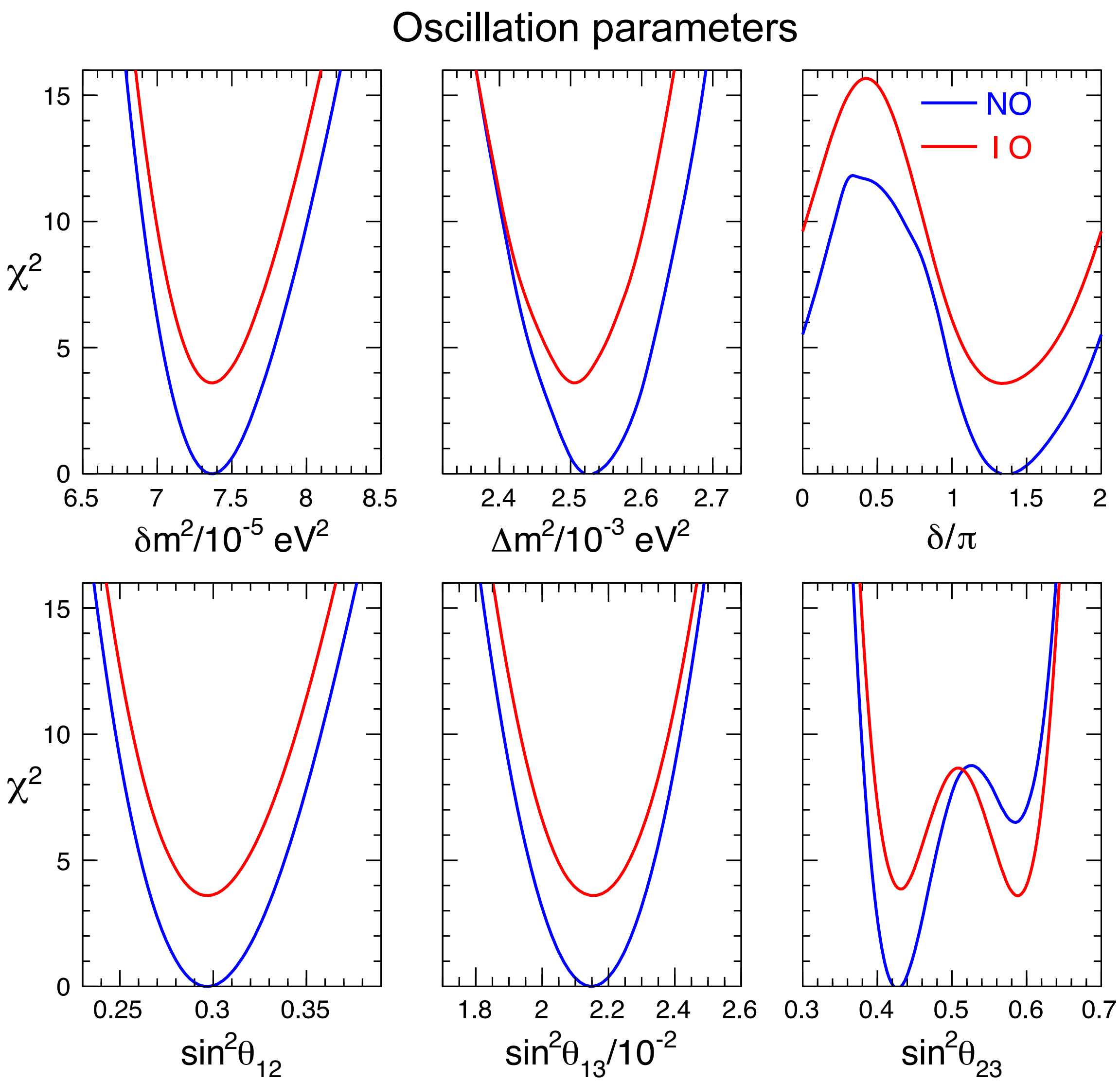
$$Goal: 0.01eV\ (5-10\ y)$$

$$\Delta m^2_{21} \approx 7.5\times 10^{-5}\ eV^2\,(solar)$$
$$|\Delta m^2_{32}| \approx 2.4\times 10^{-3}\ eV^2\,(atmospheric)$$



Two neutrino mass hierarchies

Neutrino oscillations parameters



Bari group:

arxiv.org/1703.04471

$(\Delta\chi^2_{\text{IO-NO}})^{1/2} = 2$

Normal ordering favored at 2σ

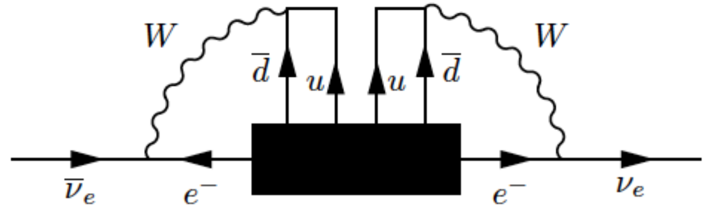
The Black Box Theorems

Black box I (electron neutrino)

J. Schechter and J.W.F Valle, PRD 25, 2951 (1982)

E. Takasugi, PLB 149, 372 (1984)

J.F. Nieves, PLB 145, 375 (1984)



$0\nu\beta\beta$ observed at some level ⇔

(i) Lepton number conservation is violated by 2 units.

(ii) Electron neutrinos are Majorana fermions (with m > 0).

However:

M. Duerr et al, JHEP 06 (2011) 91

$$\left(\delta m_{\nu_e}\right)_{BB} \sim 10^{-24}\, eV << \sqrt{\left|\Delta m_{32}^2\right|} \approx 0.05\, eV$$

Black box II (all flavors + oscillations)

M. Hirsch, S. Kovalenko, I. Schmidt, PLB 646, 106 (2006)

$0\nu\beta\beta$ observed at some level ⇔

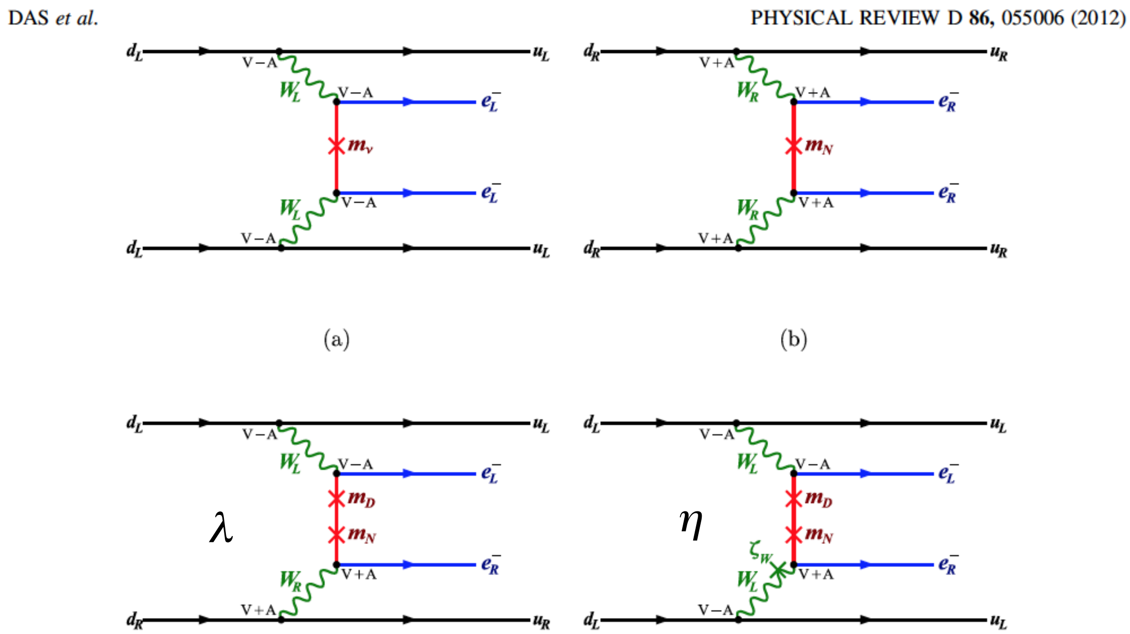
(i) Lepton number conservation is violated by 2 units.

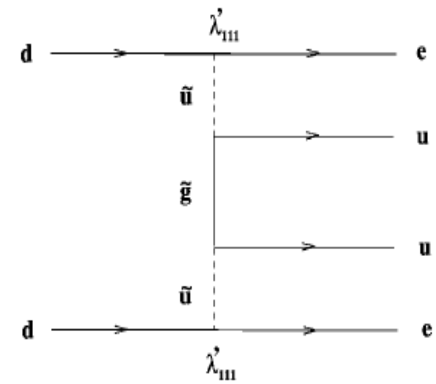
(ii) Neutrinos are Majorana fermions.

Regardless of the dominant $0\nu\beta\beta$ mechanism!

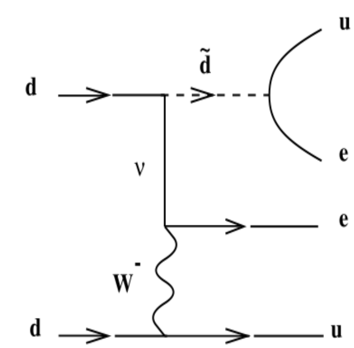
$$(iii)\quad \left\langle m_{\beta\beta} \right\rangle = \left| \sum_{k=1}^{3} m_k U_{ek}^2 \right| = \left| c_{12}^2 c_{13}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3} \right| > 0$$

Other models: Left-Right symmetric model and SUSY R-parity violation





Gluino exchange



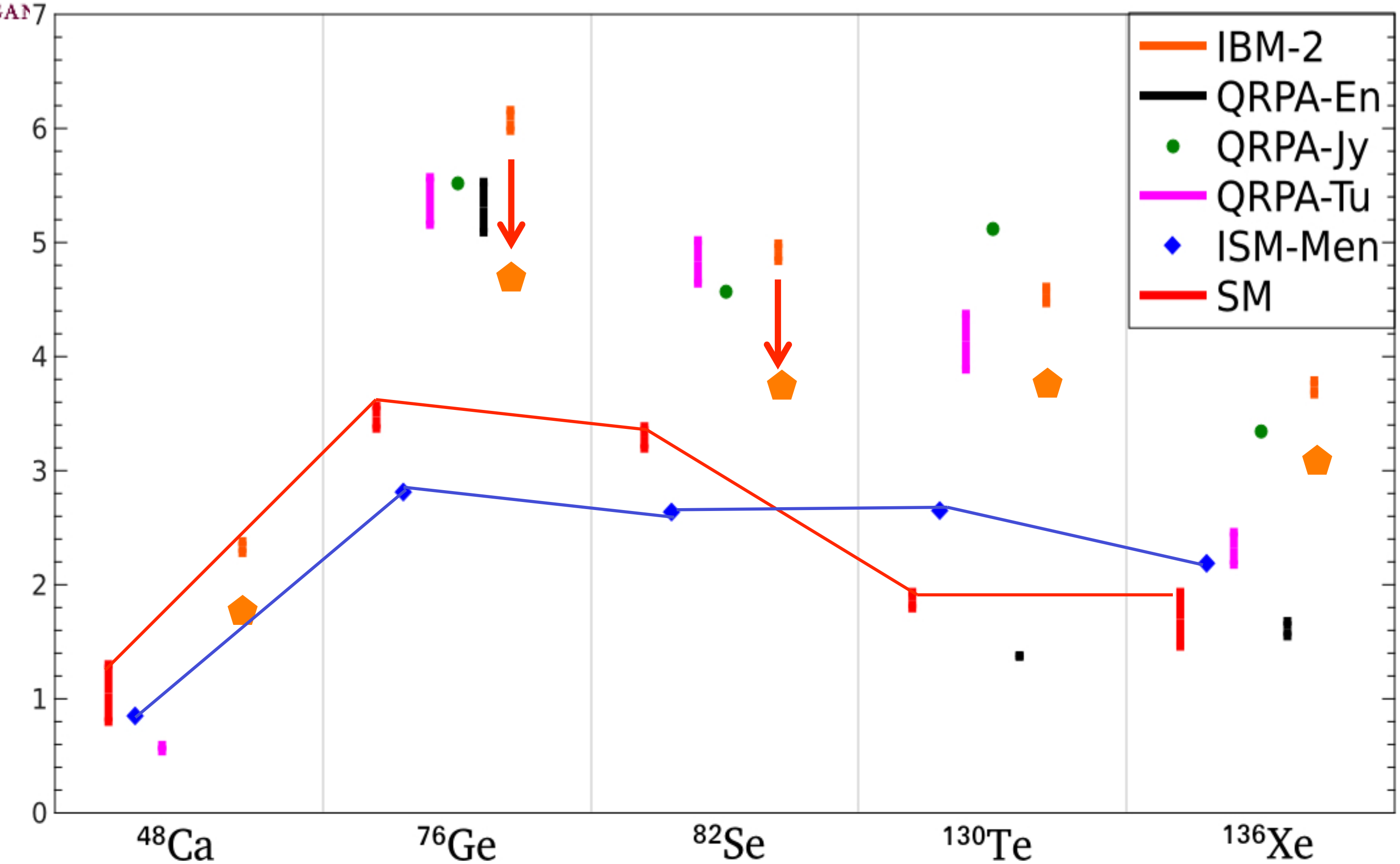
Squark exchange

$$\left[T^{0\nu}_{1/2}\right]^{-1} = G_{01} g_A^4 \left| \eta_{0\nu} M_{0\nu} + \left(\eta^L_{N_R} + \eta^R_{N_R}\right) M_{0N} + \eta_{\tilde q} M_{\tilde q} + \eta_{\lambda'} M_{\lambda'} + \eta_\lambda X_\lambda + \eta_\eta X_\eta \right|^2 .$$

(e)

M. Horoi, A. Neacsu, PRD 93, 113014 (2016)

NME for the light-neutrino exchange mechanism



IBA-2 J. Barea, J. Kotila, and F. Iachello, Phys. Rev. C **87**, 014315 (2013). → IBM-2 PRC **91**, 034304 (2015)

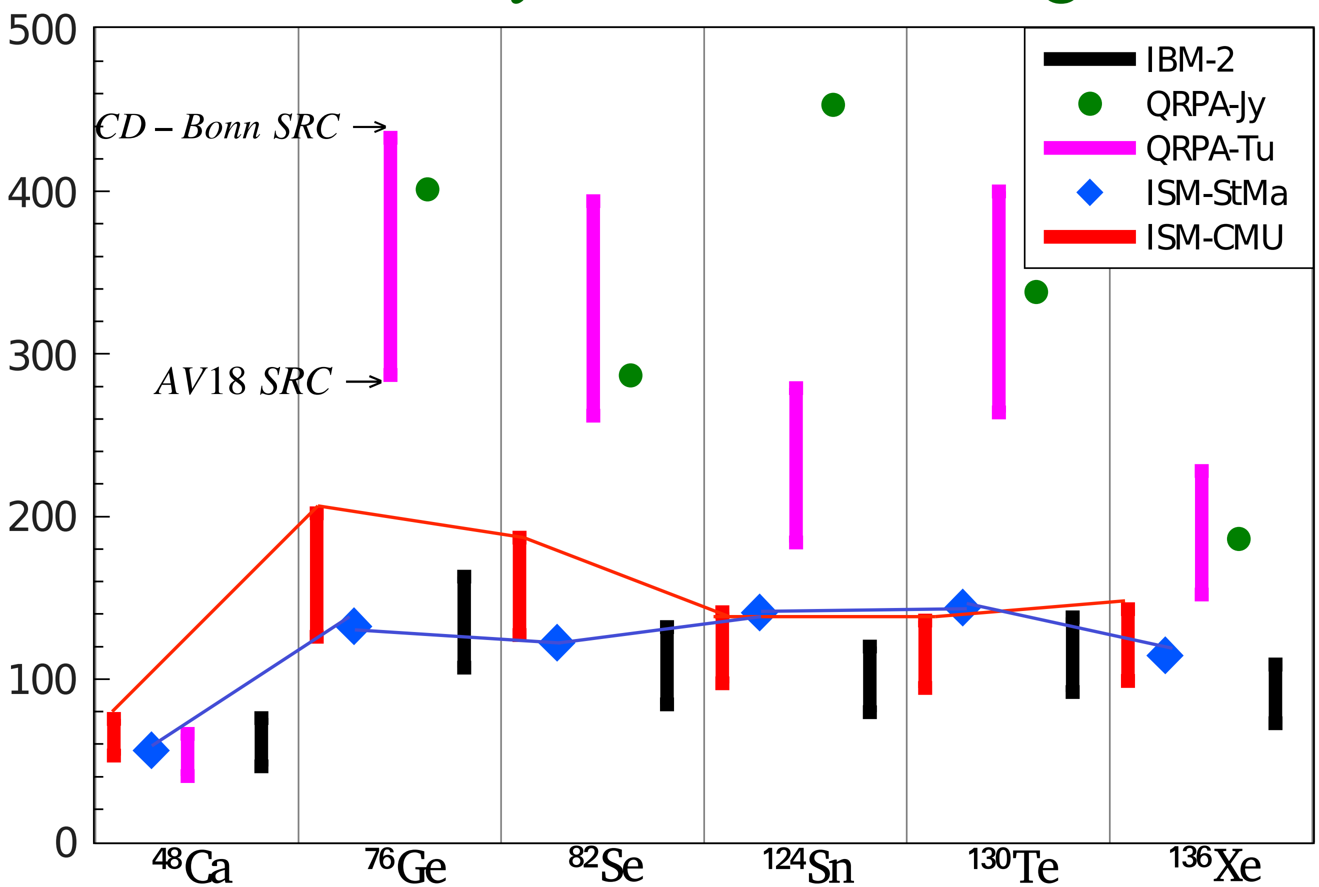
QRPA-En M. T. Mustonen and J. Engel, Phys. Rev. C **87**, 064302 (2013).

QRPA-Jy J. Suhonen, O. Civitarese, Phys. NPA **847** 207–232 (2010).

QRPA-Tu A. Faessler, M. Gonzalez, S. Kovalenko, and F. Simkovic, arXiv:**1408.6077**

ISM-Men J. Menéndez, A. Poves, E. Caurier, F. Nowacki, NPA **818** 139–151 (2009).
SM M. Horoi et. al. PRC **88**, 064312 (2013), PRC **89**, 045502 (2014), PRC **89**, 054304 (2014), PRC **90**, 051301(R) (2014), PRC **91**, 024309 (2015), PRL **110**, 222502 (2013), PRL **113**, 262501(2014).

Heavy neutrino-exchange NME M_{0N}



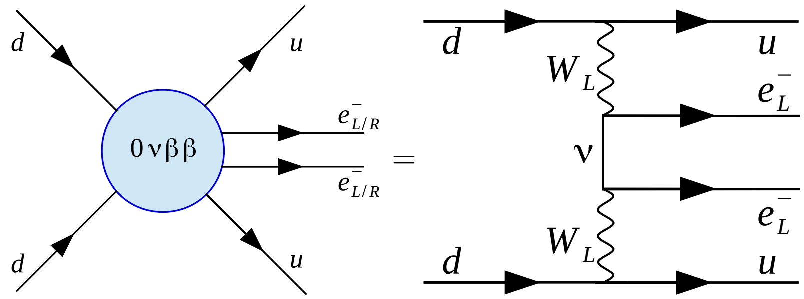
IBA-2 J. Barea, J. Kotila, and F. Iachello, Phys. Rev. C **87**, 014315 (2013).

QRPA-Tu A. Faessler, M. Gonzalez, S. Kovalenko, and F. Simkovic, arXiv:**1408.6077.**

QRPA-Jy J. Hivarynen and J. Suhonen, PRC 91, 024613 (2015), **ISM-StMa** J. Menendez, private communication.

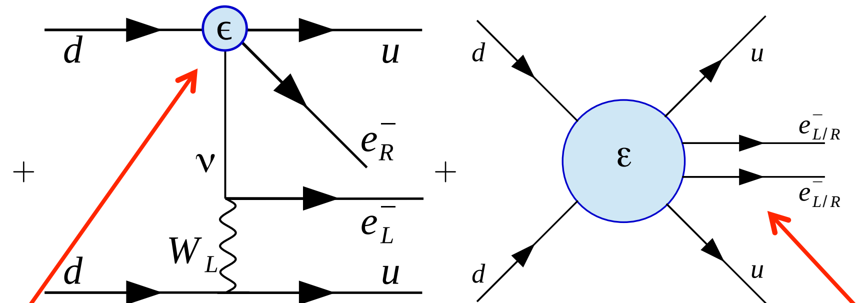
ISM-CMU M. Horoi et. al. PRC **88**, 064312 (2013), PRC **90**, PRC **89**, 054304 (2014), PRC **91**, 024309 (2015), PRL **110**, 222502 (2013).

Effective field theory approach



(a) The generic $0\nu\beta\beta$ decay diagram at the quark-level.

(b) Light left-handed neutrino exchange diagram.



(c) The long-range part of the $0\nu\beta\beta$ diagram.

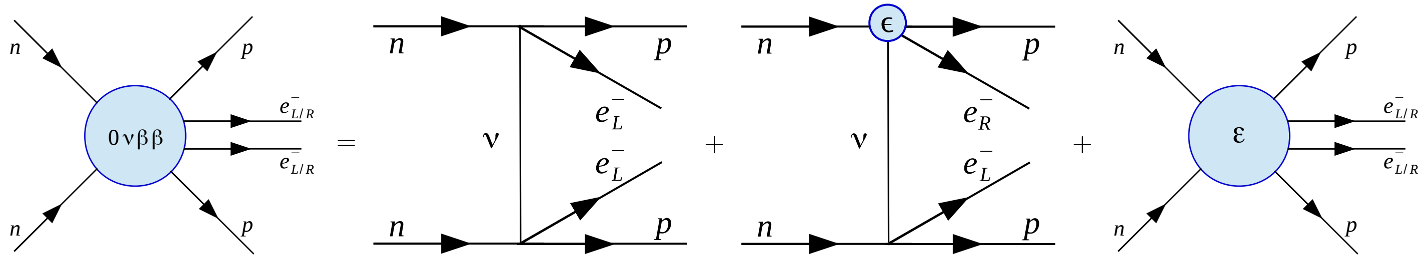
(d) The short-range part of the $0\nu\beta\beta$ diagram.

arxiv:1706.05391

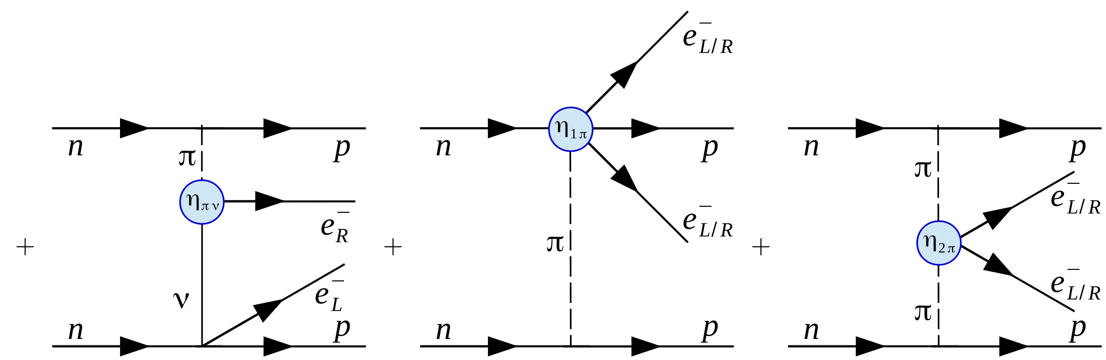
$$\mathcal{L}_6 = \frac{G_F}{\sqrt{2}}\left[j^{\mu}_{V-A} J^{\dagger}_{V-A,\mu} + \sum_{\alpha,\beta}^{*} \epsilon^{\beta}_{\alpha} j_{\beta} J^{\dagger}_{\alpha} \right]$$

$$\mathcal{L}_9 = \frac{G_F^2}{2m_p}\Big[\varepsilon_1 JJj + \varepsilon_2 J^{\mu\nu} J_{\mu\nu} j + \varepsilon_3 J^{\mu} J_{\mu} j + \varepsilon_4 J^{\mu} J_{\mu\nu} j^{\nu} + \varepsilon_5 J^{\mu} J j_{\mu}\Big],$$

Effective field theory after hadronization



(a) The generic $0\nu\beta\beta$ decay process nucleon-level diagram.
(b) Light left-handed neutrino exchange diagram.
(c) The The nucleon-nucleon long-range $\mathcal{L}_6$ mode.
(d) The nucleon-nucleon short-range part of the $0\nu\beta\beta$ diagram.



(e) The pion-neutrino long-range diagram.
(f) The one-pion long-range diagram.
(g) The two-pion long-range diagram.

$$\left[T^{0\nu}_{1/2}\right]^{-1} = g_A^4 \left[\sum_i |\mathcal{E}_i|^2 \mathcal{M}_i^2 + \mathrm{Re}\left[\sum_{i \neq j} \mathcal{E}_i \mathcal{E}_j \mathcal{M}_{ij} \right] \right]$$

$$\mathcal{E}_{2-7} = \{\epsilon^{V+A}_{V-A},\ \epsilon^{V+A}_{V+A},\ \epsilon^{S+P}_{S\pm P},\ \epsilon^{TR}_{TL},\ \epsilon^{TR}_{TR},\ \eta_{\pi\nu}\}$$

$$\mathcal{E}_{8-15} = \{\varepsilon_1,\ \varepsilon_2,\ \varepsilon_3^{LLz(RRz)},\ \varepsilon_3^{LRz(RLz)},\ \varepsilon_4,\ \varepsilon_6,\ \eta_{1\pi},\ \eta_{2\pi}\}$$

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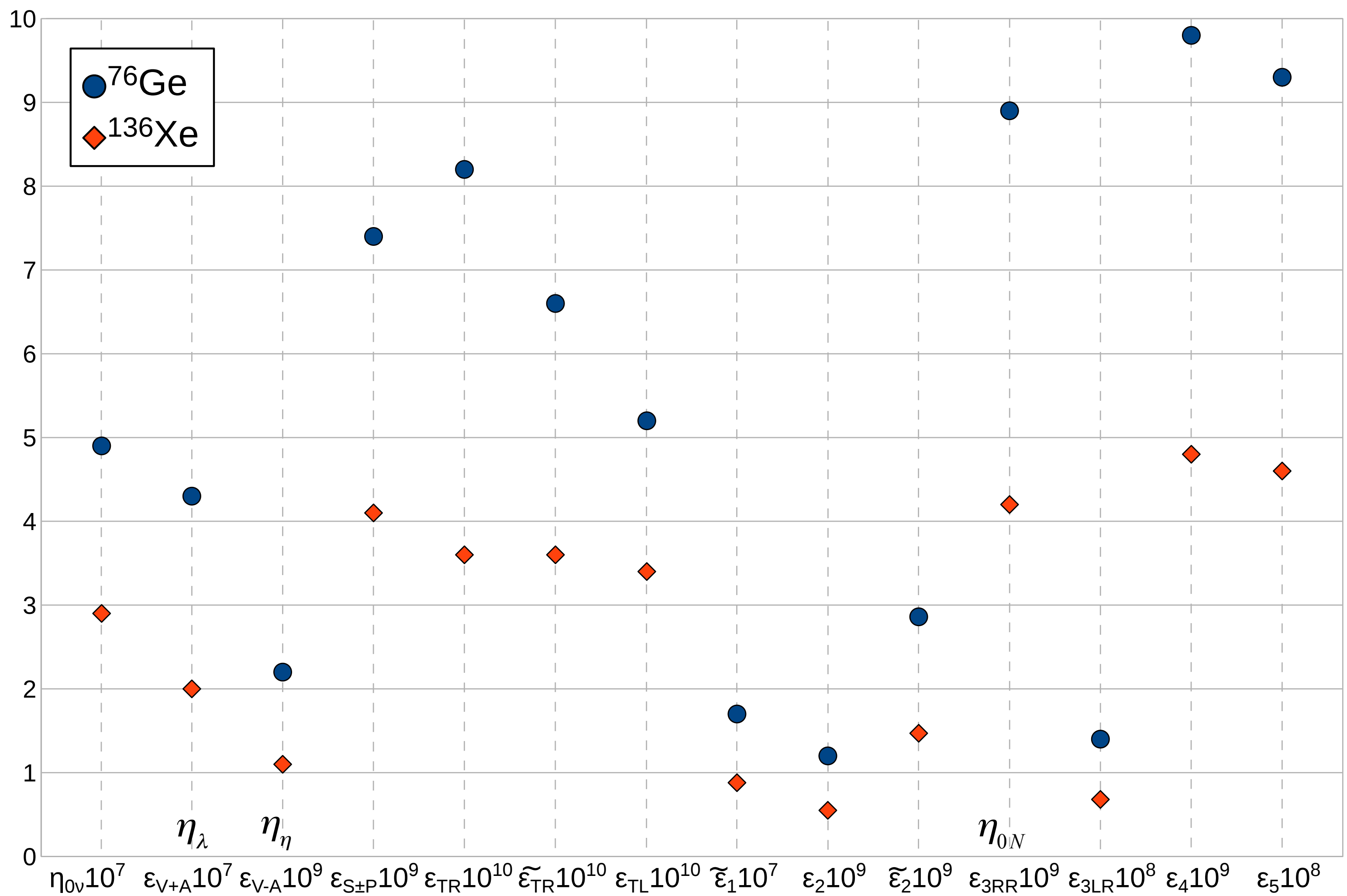
One coupling dominance

$$\left[T^{0\nu}_{1/2}\right]^{-1} = g_A^4 \left[\sum_i |\mathcal{E}_i|^2 \mathcal{M}_i^2 + \mathrm{Re}\left[\sum_{i\neq j} \mathcal{E} \mathcal{M}_{ij}\right]\right]$$

$$\mathcal{E}_{2-7} = \{\epsilon^{V+A}_{V-A},\ \epsilon^{V+A}_{V+A},\ \epsilon^{S+P}_{S\pm P},\ \epsilon^{TR}_{TL},\ \epsilon^{\tilde{TR}}_{TR},\ \eta_{\pi\nu}\} \qquad \mathcal{E}_{8-15} = \{\varepsilon_1,\ \varepsilon_2,\ \varepsilon_3^{LLz(RRz)},\ \varepsilon_3^{LRz(RLz)},\ \varepsilon_4,\ \varepsilon_6,\ \eta_{1\pi},\ \eta_{2\pi}\}$$



$$T^{0\nu}_{1/2}(^{76}Ge) > 5.3\times10^{25}\ years \qquad T^{0\nu}_{1/2}(^{136}Xe) > 1.1\times10^{26}\ years$$

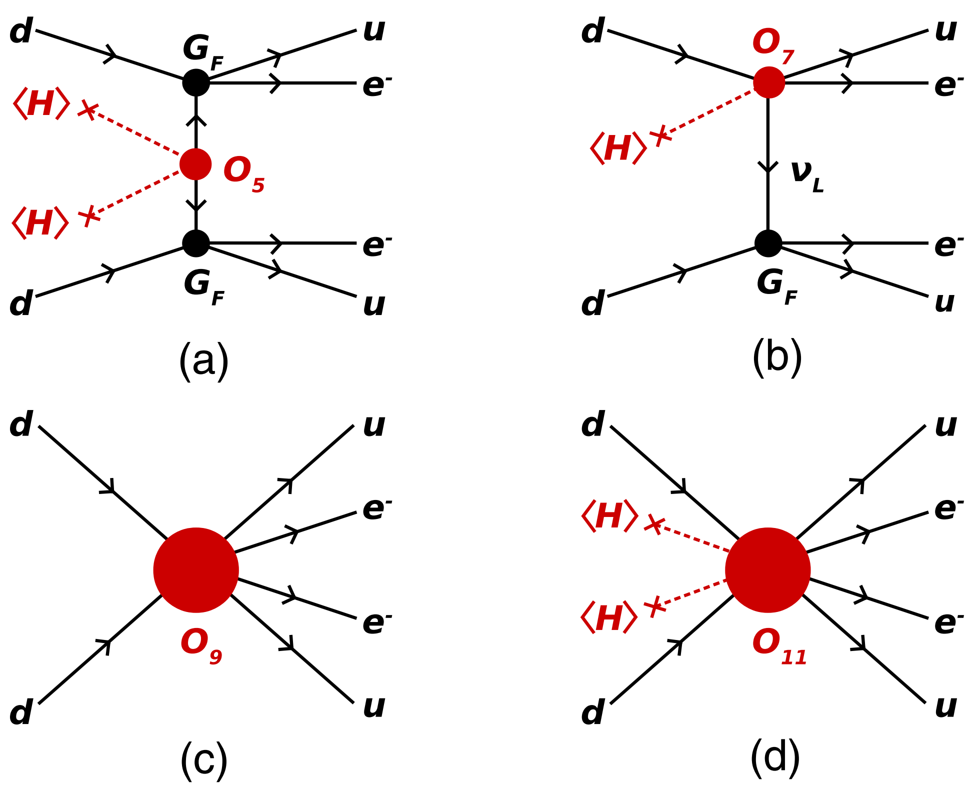
SciDAC Scientific Discovery through Advanced Computing

Consequences: - scales for new physics

- baryogenesis via leptogenesis

PHYSICAL REVIEW D **92,** 036005 (2015)

$$\mathcal{L}_D = \frac{g}{\Lambda_D^{D-4}}\mathcal{O}_D$$



$$m_e\bar{\epsilon}_5 = \frac{g^2v^2}{\Lambda_5}, \qquad \frac{G_F\bar{\epsilon}_7}{\sqrt{2}} = \frac{g^3v}{2\Lambda_7^3},$$

$$\frac{G_F^2\bar{\epsilon}_9}{2m_p} = \frac{g^4}{\Lambda_9^5}, \qquad \frac{G_F^2\bar{\epsilon}_{11}}{2m_p} = \frac{g^6v^2}{\Lambda_{11}^7}$$

$g \approx 1 \quad v = 174\ GeV$ (*Higgs expectation value*)

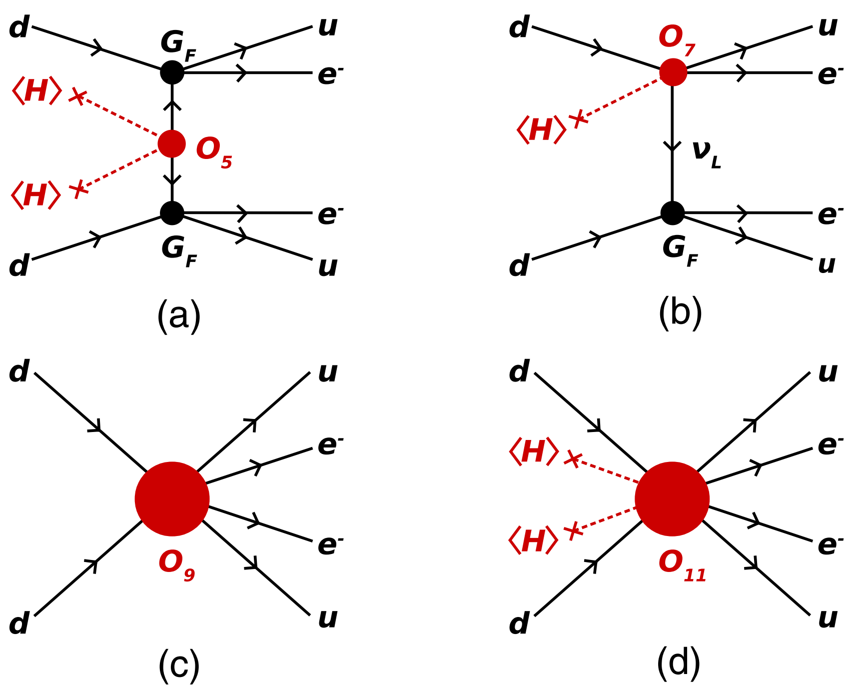
$\mathcal{O}_D$	$\bar{\epsilon}_D$	$\Lambda_D\,(GeV)$
$\mathcal{O}_5$	2.8×10^{-7}	2.12×10^{14}
$\mathcal{O}_7$	2.0×10^{-7}	3.75×10^{4}
$\mathcal{O}_9$	1.5×10^{-7}	2.48×10^{3}
$\mathcal{O}_{11}$	1.5×10^{-7}	1.16×10^{3}

Consequences: - scales for new physics

- baryogenesis via leptogenesis

PHYSICAL REVIEW D **92,** 036005 (2015)

$$\mathcal{L}_D = \frac{g}{(\Lambda_D)^{D-4}}\mathcal{O}_D$$



$$m_e\bar{\epsilon}_5 = \frac{g^2(yv)^2}{\Lambda_5}, \qquad \frac{G_F\bar{\epsilon}_7}{\sqrt{2}} = \frac{g^3(yv)}{2(\Lambda_7)^3},$$

$$\frac{G_F^2\bar{\epsilon}_9}{2m_p} = \frac{g^4}{(\Lambda_9)^5}, \qquad \frac{G_F^2\bar{\epsilon}_{11}}{2m_p} = \frac{g^6(yv)^2}{(\Lambda_{11})^7}$$

TABLE VIII. The BSM effective scale (in GeV) for different dimension-D operators at the present ^{136}Xe half-life limit (Λ_D^0) and for $T_{1/2} \approx 1.1 \times 10^{28}$ years (Λ_D).

$\mathcal{O}_D$	$\bar{\epsilon}_D$	$\Lambda_D^0(y=1)$	$\Lambda_D^0(y=y_e)$	$\Lambda_D(y=y_e)$
$\mathcal{O}_5$	$2.8 \cdot 10^{-7}$	$2.12 \cdot 10^{14}$	1904	19044
$\mathcal{O}_7$	$2.0 \cdot 10^{-7}$	$3.75 \cdot 10^{4}$	541	1165
$\mathcal{O}_9$	$1.5 \cdot 10^{-7}$	$2.47 \cdot 10^{3}$	2470	3915
$\mathcal{O}_{11}$	$1.5 \cdot 10^{-7}$	$1.16 \cdot 10^{3}$	31	43

$$\eta_N \propto \frac{1}{m_{W_R}^4 m_N}$$

$g \approx 1 \quad v = 174\ GeV \quad y_e = 3\times 10^{-6}$ *electron mass Yukawa*

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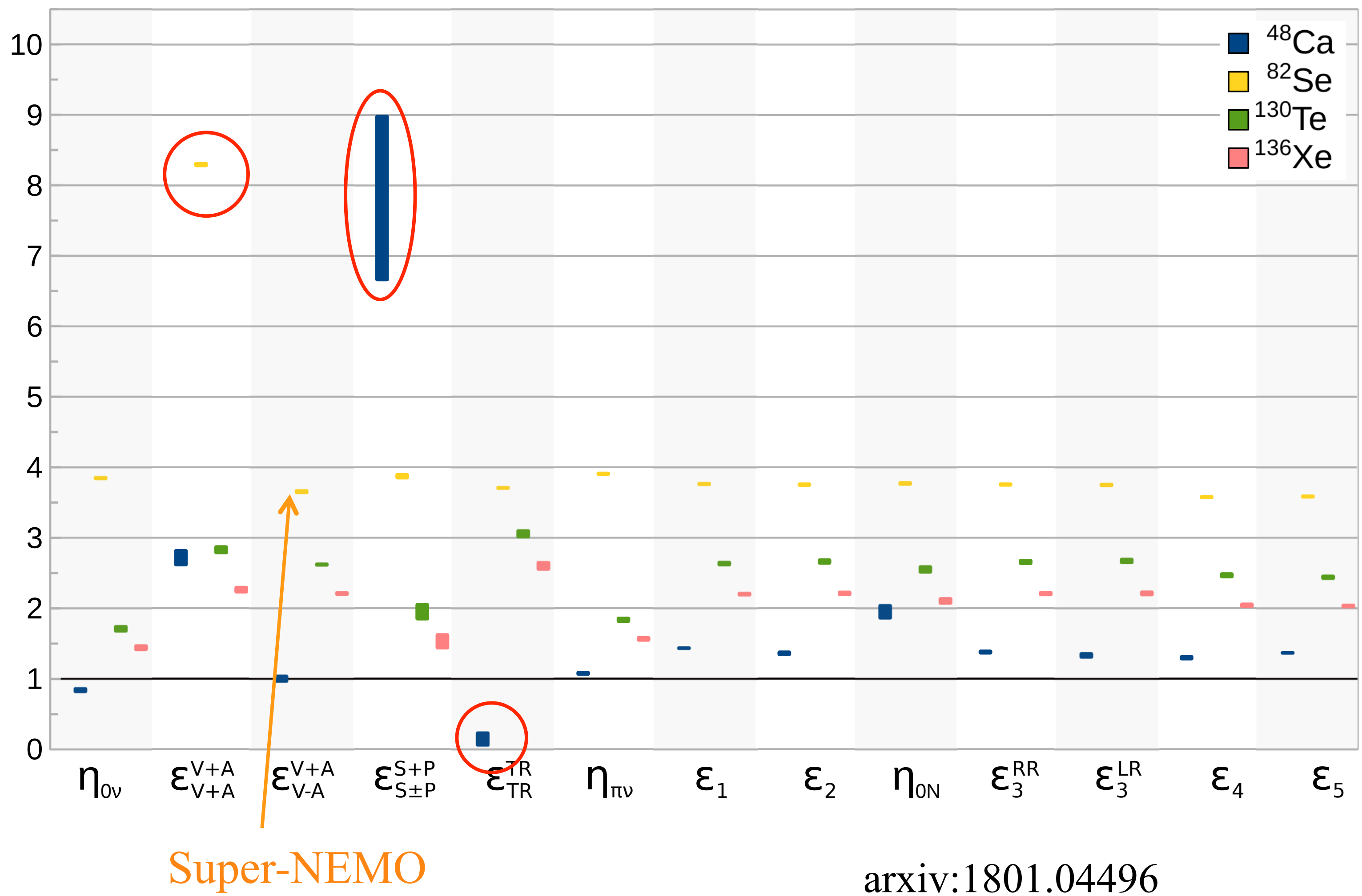
One coupling dominance: which one?

$$\left[T^{0\nu}_{1/2}\right]^{-1} = g_A^4 \left[\sum_i |\mathcal{E}_i|^2 \mathcal{M}_i^2 + \mathrm{Re}\left[\sum_{i\neq j} \mathcal{E}_i \mathcal{E}_j^* \mathcal{M}_{ij}\right]\right]$$

$T[^{76}Ge]/T[^{A}Z]$

CMU Hamiltonians



arxiv:1801.04496

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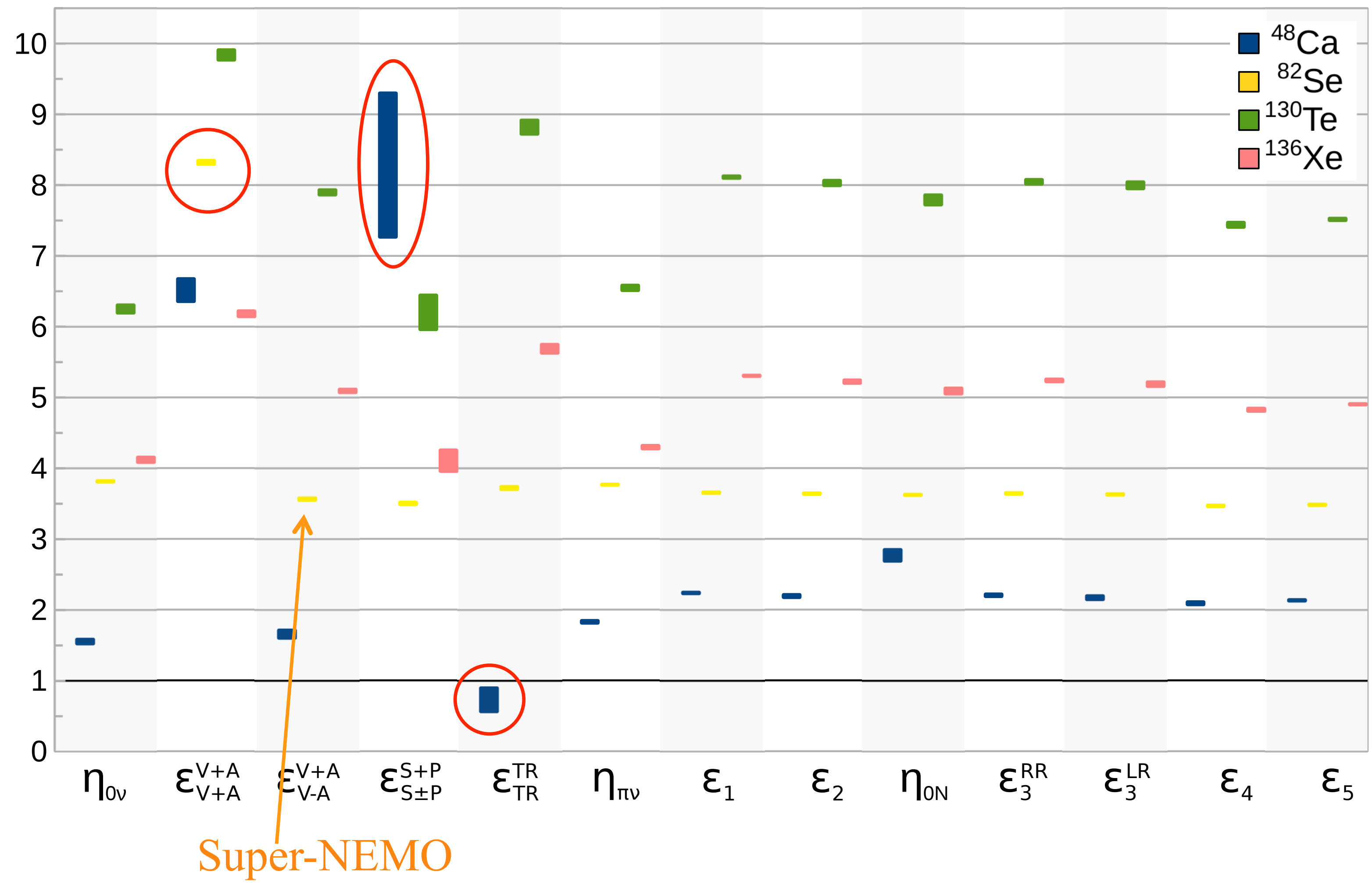
One coupling dominance: which one?

$$\left[T_{1/2}^{0\nu}\right]^{-1} = g_A^4 \left[\sum_i |\mathcal{E}_i|^2 \mathcal{M}_i^2 + \mathrm{Re}\left[\sum_{i\neq j} \mathcal{E}_i \mathcal{E}_j \mathcal{M}_{ij}\right]\right]$$

$T[^{76}Ge]/T[^{A}Z]$

Strasbourg-Madrid Hamiltonians



Neutrinos in atomic nuclei

Atomic nucleus is a high electron density medium:

Consider 2 electrons in the lowest s-orbital of an Hydrogen-like atom

$$\textit{Electron density inside nucleus}: N_e \approx \frac{2}{\pi}\left(\frac{Z}{a_B}\right)^3$$

$$\textit{Equivalent matter density}: \rho = m_N N_e = 1.67\times 10^6 \frac{2}{\pi}\left(\frac{Z}{53}\right)^3 \; in\; g/cm^2 >> \rho_{Sun}$$

$$\rho_{Sun\,core} \approx 150\; g/cm^3$$

Solar Neutrinos Survival Probability

$$|\nu_e\rangle = \cos\theta |\nu_1\rangle + \sin\theta |\nu_2\rangle$$

Low energy

$$P_{\nu_e}(t) = |\langle \nu_e | \nu_e(t)\rangle|^2 = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 x}{4E}\right)$$

$$\xrightarrow[x\to\infty]{} 1 - \frac{1}{2}\sin^2 2\theta = 0.56 > 0.5$$

High energy

$$|\nu_e(t)\rangle = \sin\theta\, e^{-iEt} |\nu_2\rangle$$

$$P_{\nu_e}(t) = |\langle \nu_e(0) | \nu_e(t)\rangle|^2$$

$$= \sin^2\theta = 0.32 < 0.5$$

$$|\nu_e\rangle = \sum_{i=1}^{N(3)} U^*_{ei} |\nu_i\rangle$$

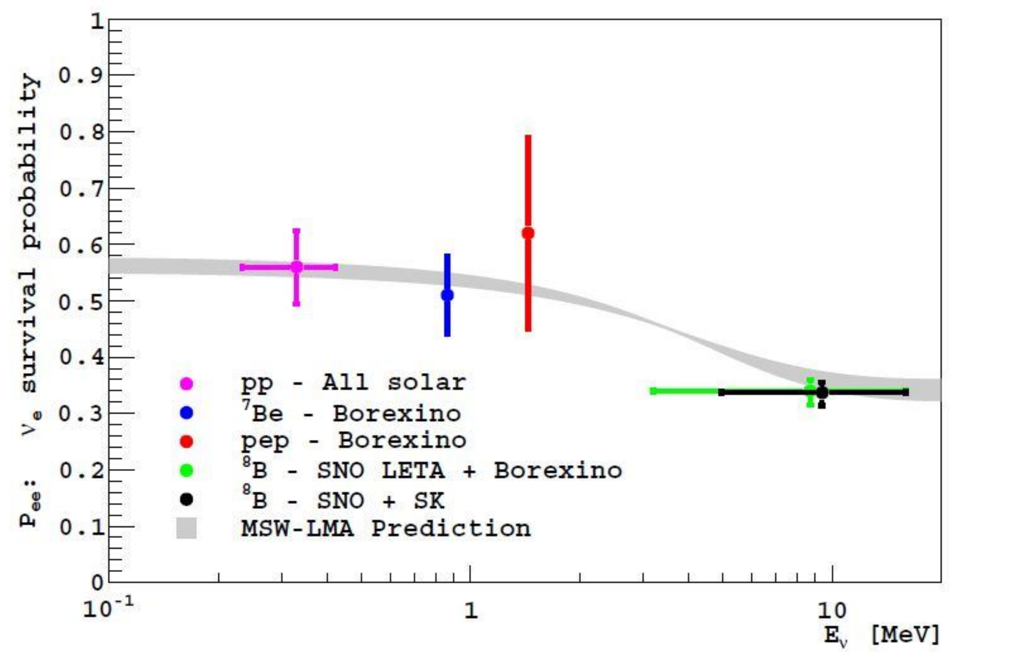


Figure 8. Solar ν_e survival probability as a function of energy. The colour lines correspond to experimental data. The grey line corresponds to the MSW-LMA solution for solar neutrino oscillation. (From [29]).

Journal of Physics: Conference Series **593** (2015) 012007

Neutrino Mixing (MSW effect)

Neutrinos in matter interact with:

- Electrons via charged current
- Any fermion via neutral current

$$V_e = \pm\sqrt{2}G_F N_e \quad (N_e : \textit{electron density})$$
$$+(-)\ \textit{neutrino (antineutrino)}$$

V_N $\qquad P \approx E$

$$|\nu_e\rangle = \cos\theta_m |\nu_1\rangle + \sin\theta_m |\nu_2\rangle$$
$$|\nu_X\rangle = -\sin\theta_m |\nu_1\rangle + \cos\theta_m |\nu_2\rangle$$

$$\leftarrow i\frac{d}{dt}\begin{pmatrix}|\nu_e\rangle \\ |\nu_X\rangle\end{pmatrix} = \left[U^\dagger \begin{pmatrix} P + m_1^2/2P & 0 \\ 0 & P + m_2^2/2P \end{pmatrix} U + \begin{pmatrix} V_e + V_N & 0 \\ 0 & V_N \end{pmatrix}\right]\begin{pmatrix}|\nu_e\rangle \\ |\nu_X\rangle\end{pmatrix}$$

$$2P\Delta E_m \equiv 2P\left(E_{2m} - E_{1m}\right) = \Delta\mu^2 = \sqrt{\left(\Delta m^2 \cos 2\theta - 2PV_e\right)^2 + \left(\Delta m^2 \sin 2\theta\right)^2}$$

$$\cos 2\theta_m = \frac{\Delta m^2 \cos 2\theta - 2PV_e}{\sqrt{\left(\Delta m^2 \cos 2\theta - 2PV_e\right)^2 + \left(\Delta m^2 \sin 2\theta\right)}} \xrightarrow[0<2PV_e>>\Delta m^2]{} -1 \quad \Rightarrow \theta_m = \frac{\pi}{2}$$

$$|\nu_{em}\rangle = 0|\nu_1\rangle + 1|\nu_2\rangle \quad \rightarrow \textit{adiabatic transport} \rightarrow P_{\nu_e}(t) = |\langle\nu_e|\nu(t)\rangle|^2 = |\langle\nu_e|\nu_{em}\rangle|^2 = \sin^2\theta$$

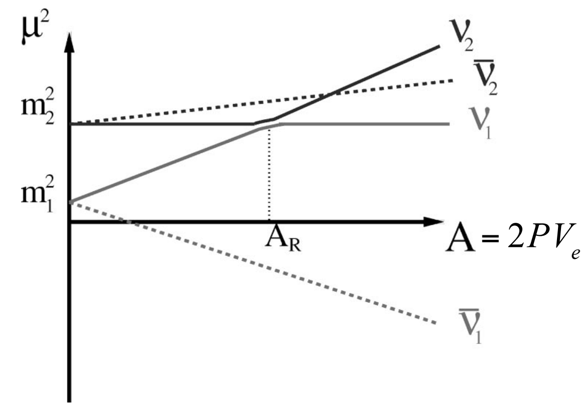
NO oscillations! Just mixing!

Neutrino mixing

Gonzales-Garcia & Nir, RMP 75, 345 (2003)

$$U=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$$

$$\begin{pmatrix}\nu_e\\ \nu_X\end{pmatrix}=U(\theta_m)\begin{pmatrix}\nu_1^m\\ \nu_2^m\end{pmatrix}=\begin{pmatrix}\cos\theta_m & \sin\theta_m\\ -\sin\theta_m & \cos\theta_m\end{pmatrix}\begin{pmatrix}\nu_1^m\\ \nu_2^m\end{pmatrix}$$



$$P_e=\left|\nu_e^m\right|^2 \qquad P_X=\left|\nu_X^m\right|^2$$

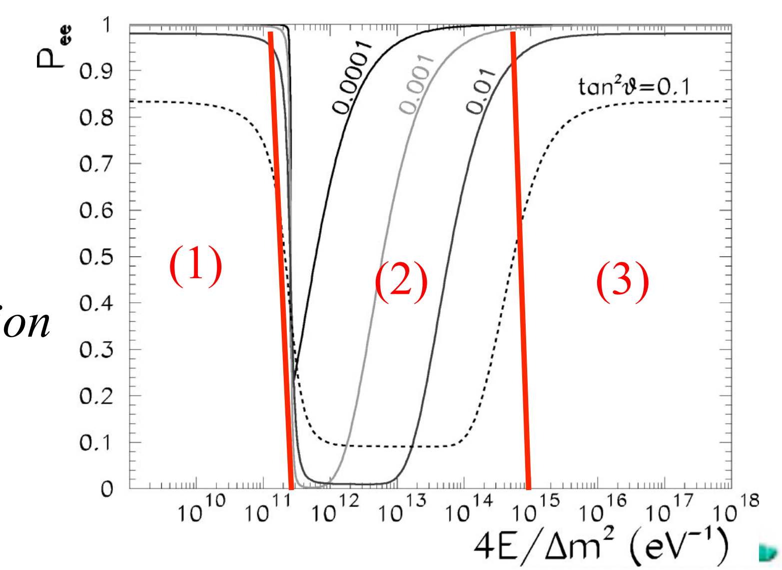
$$\begin{pmatrix}\dot{\nu}_1^m\\ \dot{\nu}_2^m\end{pmatrix}=\begin{pmatrix}i\Delta(t) & -4E\dot{\theta}_m(t)\\ 4E\dot{\theta}_m(t) & -i\Delta(t)\end{pmatrix}\begin{pmatrix}\nu_1^m\\ \nu_2^m\end{pmatrix} \qquad \dot{\theta}_m \propto \dot{V}_e$$

Adiabatic evolution: off – diagonal terms neglijable

(1) *small energies: matter effects neglijable*

(2) *lage energies: matter effects + adiabatic evolution*

(3) *lager energies: matter effects + nonadiabatic evolution*



Neutrinos in atomic nuclei

Atomic nucleus is a high electron density medium:

Consider 2 electrons in the lowest s-orbital of an Hydrogen-like atom

$$Equivalent\ matter\ density:\ \rho(t) = 1.67 \times 10^6 \frac{2}{\pi}\left(\frac{Z}{53}\right)^3 e^{-2tZ/53}\ [in\ g/cm^2,\ t\ in\ pm]$$

$$V_e(t)\,[in\ eV] = 7.6 \times 10^{14}\,\rho(t)\,[in\ g/cm^3]$$

The typical requirement is:

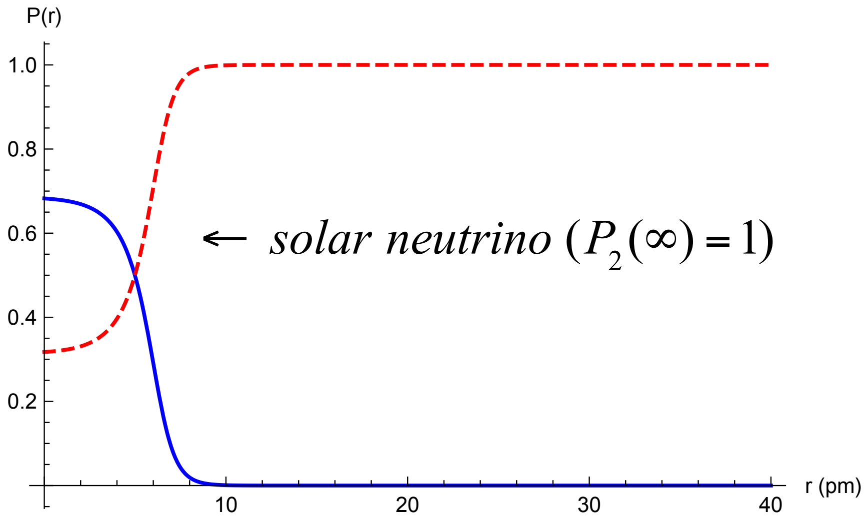
$$\lambda << \left|\frac{V}{\partial V/\partial t}\right| \Leftrightarrow 2\pi\frac{\hbar c}{p} << \frac{53}{2Z}1000\ \ [in\ \ fm]$$

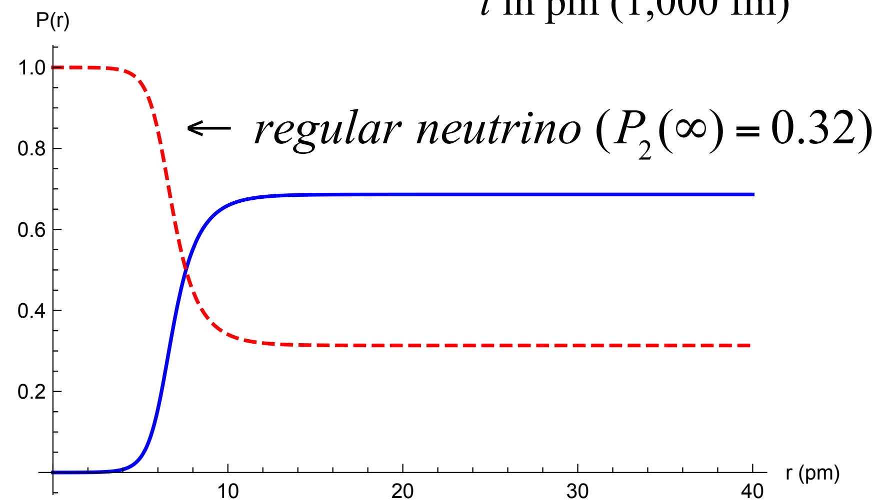
Neutrino mixing in atomic nuclei

$$V_e(in\ eV) = 7.6 \times 10^{-14} \rho(in\ g/cm^3)$$

$$\rho(t) = m_p \times N_e(t) = 1.67 \times 10^6 \frac{2}{\pi} \left(\frac{Z}{53}\right)^3 e^{-2tZ/53}$$

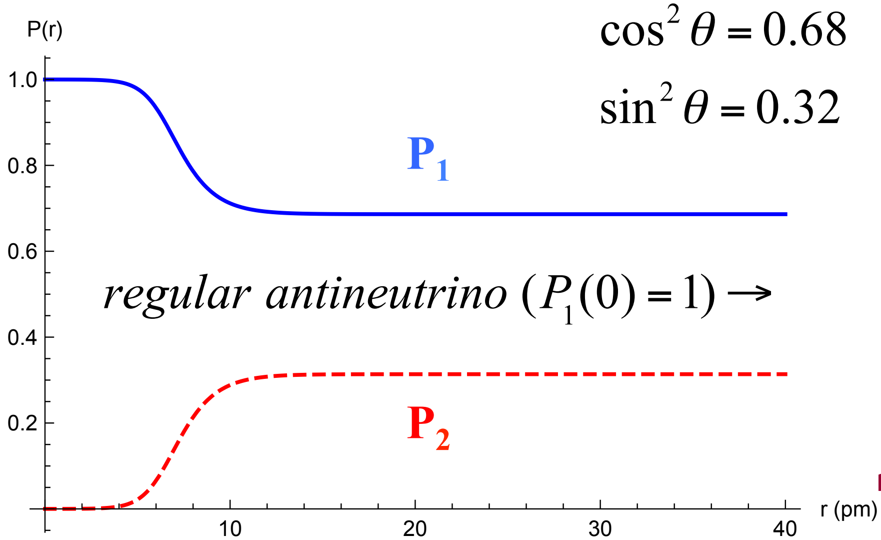
t in pm (1,000 fm)



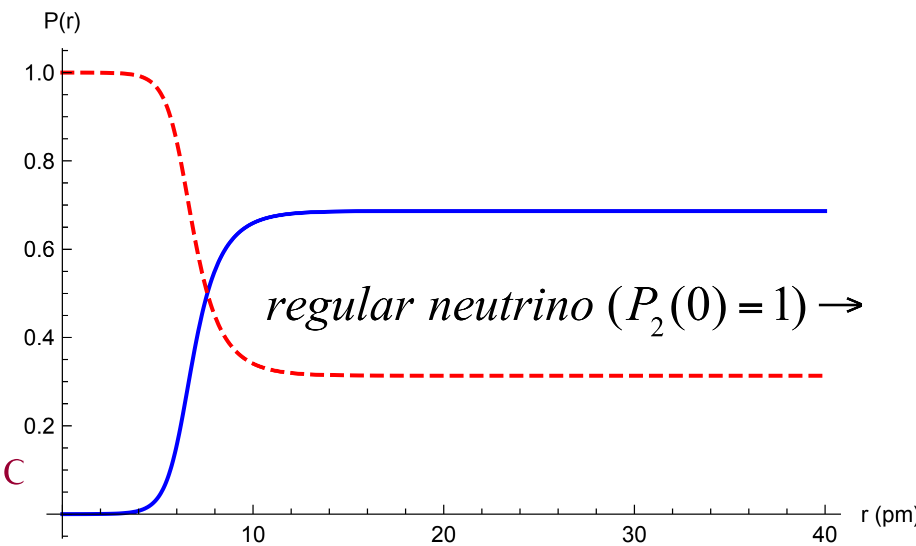


$\cos^2\theta = 0.68$

$\sin^2\theta = 0.32$



roi C



Partial summary

- One can consider the neutrino mixing in atomic nuclei.
- The analysis of neutrino mixing in the Sun and atomic nuclei leads to results backed up by phenomenology.
- These results seem simple and natural, but the road to them is complex!

Neutrinoless double beta decay in vacuum

$$A_{0\beta\beta} \propto NP = \langle 0| T\left[\psi_{eL}(x_1)\psi^T_{eL}(x_2)\right]|0\rangle$$

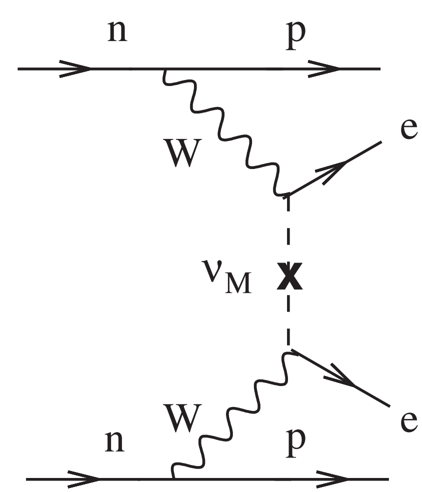
$$\psi_e(x) = \sum_{a=1}^{N(3)} U_{ea}\psi_a(x)$$

$$NP = \sum_{a=1}^{3} U^2_{ea}\langle 0| T\left[\psi_{aL}(x_1)\psi^T_{aL}(x_2)\right]|0\rangle$$
$$= \sum_{a=1}^{3} U^2_{ea}\left[-i\int \frac{d^4p}{(2\pi)^4}\frac{m_a e^{-ip(x_1-x_2)}}{p^2-m_a^2+i\epsilon}P_L\mathcal{C}\right]$$

$$P_L = \frac{1}{2}\left(1-\gamma^5\right) \qquad \hat{\psi}(x) = C\psi^*(x)$$

$P_L C$ product is further used to process the electron current, and one finally gets:

$$\frac{1}{T_{1/2}} = G(Z,Q)\left|M_{0\nu}\right|^2 \left|\sum_{a=1}^{3} U^2_{ea} m_a\right|^2 / m_e^2$$



Neutrino Fields

Dirac equation: states vs fields

$$\left(i\gamma^0\partial_0 + i\gamma^i\partial_i - m\right)\psi(x) = 0 \qquad \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu} \qquad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$$

$$i\frac{\partial\psi(x)}{\partial t} = \left(-i\gamma^0\gamma^i\partial_i + \gamma^0 m\right)\psi(x) \qquad \gamma_\mu^\dagger = \gamma_0\gamma_\mu\gamma_0 \quad \Rightarrow \quad \gamma_0^\dagger = \gamma_0 \quad \gamma_i^\dagger = -\gamma_i$$

$$\psi(x) = \int \frac{d^3p}{(2\pi)^{3/2}2E_p}\left[\left(u^{(+)}(\vec{p})a(\vec{p},+) + u^{(-)}(\vec{p})a(\vec{p},-)\right)e^{-ipx} + \left(v^{(+)}(\vec{p})b^\dagger(\vec{p},+) + v^{(-)}(\vec{p})b^\dagger(\vec{p},-)\right)e^{ipx}\right]$$

$$\left[a(\vec{p},h),\, a^\dagger(\vec{p}\,',h')\right]_+ = \delta_{hh'}\delta(\vec{p}-\vec{p}\,') \qquad \left[a(\vec{p},h),\, a(\vec{p}\,',h')\right]_+ = \left[a^\dagger(\vec{p},h),\, a^\dagger(\vec{p}\,',h')\right]_+ = 0$$

Same for b, $b^\dagger$

$$\textit{In addition one needs a vaccum state}: |0> \quad \Rightarrow \quad |\psi> = \psi^\dagger(x)|0>$$

P. Mannheim, PRD **37**, 1935 (1988): used *a(**p**, -)* piece of the field to justify Wolfenstein's Eqs. provided that neutrinos, Dirac or Majorana, are ultra relativistic!

PHYSICAL REVIEW D VOLUME 45, NUMBER 5 1 MARCH 1992

Majoron decay of neutrinos in matter

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with production of any massless pseudoscalar boson. In particular, we discuss the two-generation case and show that *in matter* the helicity-flipping decays are dominant over the helicity-conserving decays. The implications of the Majoron decay for the neutrinos from astrophysical objects are also briefly discussed.

$$Weyl: \quad \gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad P_L C = \begin{pmatrix} 0 & 0 \\ 0 & i\sigma^2 \end{pmatrix} \qquad \psi(x) = \begin{pmatrix} -i\sigma^2 \Phi^*(x) \\ \Phi(x) \end{pmatrix} \Rightarrow \psi_L(x) = \begin{pmatrix} 0 \\ \Phi(x) \end{pmatrix}$$

$$\Phi_a^M(x) = \int \frac{d\mathbf{p}}{\sqrt{(2\pi)^3}} \sum_{j=1}^{N} \sum_{h=\pm 1} [\alpha_{aj}^{(h)}(P) w(\mathbf{p},h) a_j(\mathbf{p},h) e^{-iE_j^{(h)}t + i\mathbf{p}\cdot\mathbf{x}} - h\beta_{aj}^{(h)*}(P) w(\mathbf{p},-h) a_j^\dagger(\mathbf{p},h) e^{iE_j^{(h)}t - i\mathbf{p}\cdot\mathbf{x}}]$$

$$\beta_{aj}^{(-)}(P) = \frac{m_a}{2P}\alpha_{aj}^{(-)}(P) \qquad \frac{m_j^{(-)2} - m_a^2}{2P}\alpha_{aj}^{(-)}(P) - \sum_{b=1}^{N} V_{ab}^M \alpha_{bj}^{(-)}(P) = 0$$

$$\alpha_{aj}^{(+)}(P) = \frac{m_a}{2P}\beta_{aj}^{(+)}(P) \qquad \frac{m_j^{(+)2} - m_a^2}{2P}\beta_{aj}^{(+)}(P) + \sum_{b=1}^{N} V_{ab}^{M*} \beta_{bj}^{(+)}(P) = 0$$

$$E_j^{(h)} = P + \frac{m_j^{(h)2}}{2P}, \quad j = 1, \ldots, N \qquad \Phi_e^W(x) = \sum_{a=1}^{N(3)} U_{ea} \Phi_a^M(x)$$

Neutrinoless double beta decay of atomic nuclei

$$\Phi_e^W(x) = \int \frac{d^3p}{(2\pi)^{3/2}} \sum_{a,j} U_{ea} \; \Big[\Big(\alpha_{a\,j}^{(-)} \chi^{(-)}(\vec{p}) a_j(\vec{p},-) + \tfrac{m_a}{2P} \beta_{a\,j}^{(+)} \chi^{(+)}(\vec{p}) a_j(\vec{p},+)\Big) e^{-ip\cdot x}$$

$$+ \Big(\tfrac{m_a}{2P} \alpha_{a\,j}^{(-)*} \chi^{(+)}(\vec{p}) a_j^\dagger(\vec{p},-) - \beta_{a\,j}^{(+)*} \chi^{(-)}(\vec{p}) a_j^\dagger(\vec{p},+)\Big) e^{ip\cdot x}\Big]$$

$$\sum_a U_{ea} \alpha_{a\,j}^{(-)} = \delta_{j,j_h}$$

$$\sum_a U_{ea} \beta_{a\,j}^{(+)*} = \delta_{j,j_l}$$

j_h – highest mass eigenstate (3 for NO, 2 for IO)

j_l – lowest mass eigenstate (1 for NO, 3 for IO)

MSW effect

$$\chi^{(+)}(\vec{p}) = \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}\, e^{i\phi} \end{pmatrix}, \qquad \chi^{(-)}(\vec{p}) = \begin{pmatrix} -\sin\frac{\theta}{2}\, e^{-i\phi} \\ \cos\frac{\theta}{2} \end{pmatrix}$$

Neutrinoless double beta decay of atomic nuclei

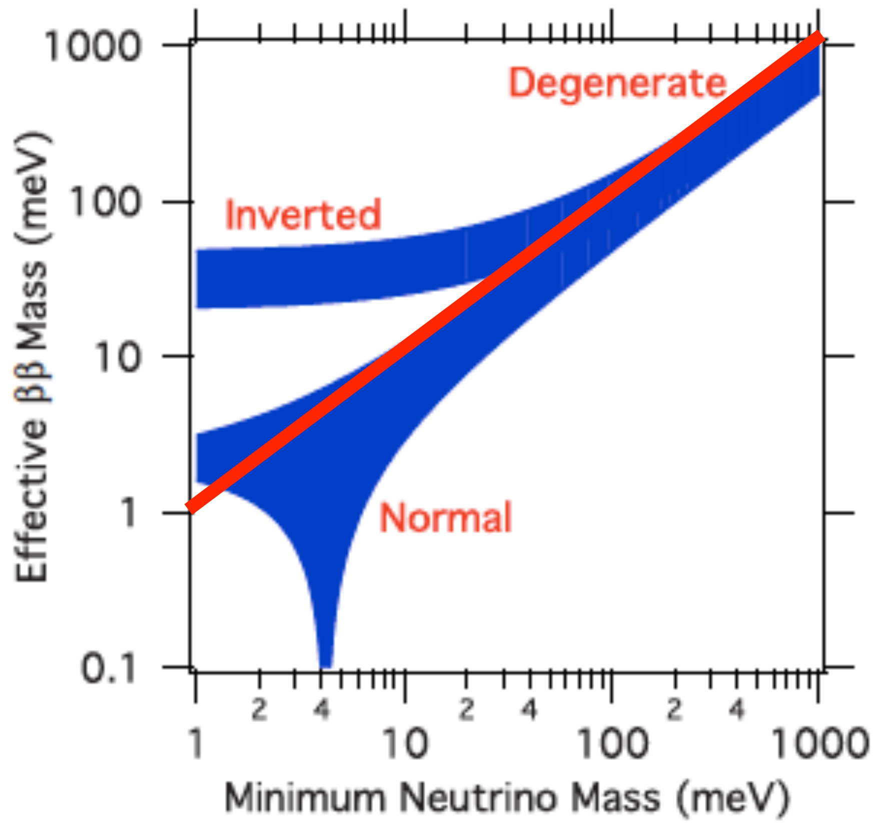
$$\Phi_e^W(x) = \int \frac{d^3p}{(2\pi)^{3/2}} \left[\left(\chi^{(-)}(\vec{p}) a_{j_h}(\vec{p},-) + \sum_{a,j} U_{ea} \frac{m_a}{2P} \beta_{a\ j}^{(+)} \chi^{(+)}(\vec{p}) a_j(\vec{p},+) \right) e^{-ip\cdot x} \right.$$
$$\left. + \left(\sum_{a,j} U_{ea} \frac{m_a}{2P} \alpha_{a\ j}^{(-)*} \chi^{(+)}(\vec{p}) a_j^\dagger(\vec{p},-) - \chi^{(-)}(\vec{p}) a_{j_l}^\dagger(\vec{p},+) \right) e^{ip\cdot x} \right]$$

j_h – highest mass eigenstate (3 for NO, 2 for IO)

j_l – lowest mass eigenstate (1 for NO, 3 for IO)

Phys.Lett. B336 (1994) 439-445 alternative 4-component spinors

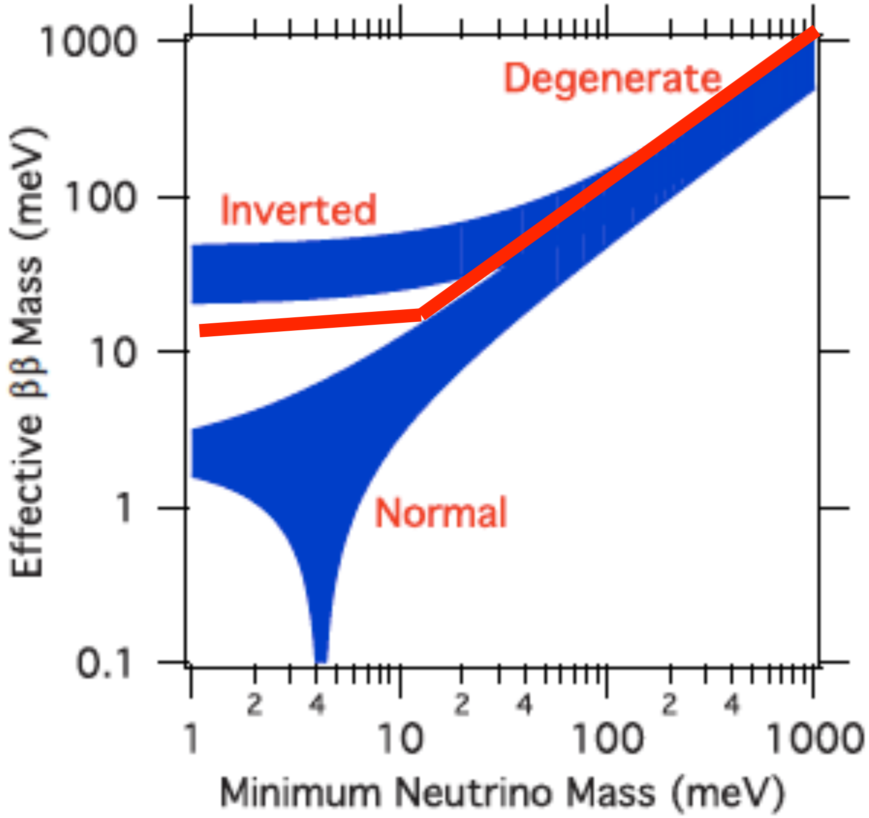
Effective neutrino mass



$$\left|m_{\beta\beta}\right| = \left|U_{e j_l}^2 m_{j_l}\right| \qquad \Leftarrow T_{1/2}^{-1}(0\nu) = G^{0\nu}(Q_{\beta\beta})\left[M^{0\nu}(0^+)\right]^2\left(\eta_{0\nu}\right)^2$$

$$\eta_{0\nu} = \frac{\left|m_{\beta\beta}\right|}{m_e}$$

Effective neutrino mass



$$\left|m_{\beta\beta}\right| = \left|U_{e j_h}^{2} m_{j_h}\right| \qquad \Leftarrow T_{1/2}^{-1}(0\nu) = G^{0\nu}(Q_{\beta\beta})\left[M^{0\nu}(0^{+})\right]^{2}\left(\eta_{0\nu}\right)^{2}$$

$$\eta_{0\nu} = \frac{\left|m_{\beta\beta}\right|}{m_e}$$

Neutrinoless double beta decay of atomic nuclei

$$\Phi_e^W(x) = \int \frac{d^3p}{(2\pi)^{3/2}} \sum_{a,j} U_{ea} \ \Big[\left(\alpha_{a\ j}^{(-)} \chi^{(-)}(\vec{p}) a_j(\vec{p},-) + \tfrac{m_a}{2P} \beta_{a\ j}^{(+)} \chi^{(+)}(\vec{p}) a_j(\vec{p},+) \right) e^{-ip\cdot x}$$
$$+ \left(\tfrac{m_a}{2P} \alpha_{a\ j}^{(-)*} \chi^{(+)}(\vec{p}) a_j^\dagger(\vec{p},-) - \beta_{a\ j}^{(+)*} \chi^{(-)}(\vec{p}) a_j^\dagger(\vec{p},+) \right) e^{ip\cdot x} \Big]$$

$$\sum_{j=1}^{N(3)} \alpha_{a\,j}^{(-)} \alpha_{b\,j}^{(-)*} = \delta_{ab}$$

$$\sum_{j=1}^{N(3)} \beta_{a\,j}^{(-)} \beta_{b\,j}^{(-)*} = \delta_{ab}$$

$$NP_a = \langle 0 | T\left[\psi_{aL}(x_1)\psi_{aL}^T(x_2)\right] |0\rangle = \begin{pmatrix} 0 & 0 \\ 0 & \langle 0| T\left[\Phi_a(x_1)\Phi_a^T(x_2)\right] |0\rangle \end{pmatrix}$$

$t_1 > t_2$

$$\langle 0| \Phi_e^W(x_1) \left(\Phi_e^W(x_2)\right)^T |0\rangle = \sum_a U_{ea}^2 \int \frac{d^3p}{(2\pi)^3} \frac{m_a}{2P} \left[\chi^{(-)}(\vec{p})\chi^{(+)T}(\vec{p}) - \chi^{(+)}(\vec{p})\chi^{(-)T}(\vec{p})\right] e^{-ip\cdot(x_1-x_2)}$$

$$\langle 0| T\left[\Phi_e^W(x_1)\left(\Phi_e^W(x_2)\right)^T\right] |0\rangle = -i \sum_a U_{ea}^2 \int \frac{d^4p}{(2\pi)^4} \frac{m_a e^{-ip(x_1-x_2)}}{p^2 - m_a^2 + i\epsilon} \left(i\sigma^2\right)$$

$$P_L C = \begin{pmatrix} 0 & 0 \\ 0 & i\sigma^2 \end{pmatrix}$$

In atomic nuclei NP = In vacuum NP

Vacuum result stands : $m_{\beta\beta} = \left| \sum_{a=1}^{3} U_{ea}^2 m_a \right|$

Summary

- Neutrinoless DBD, if observed, will represent a big step forward in our understanding of the neutrinos, and of physics beyond the Standard Model.
- Better nuclear matrix elements and effective DBD operators are needed to identify the underlying mechanism(s).
- The effects of the high electron densities in atomic nuclei were investigated and they do not change the neutrino emission or detection, nor the $0\nu\beta\beta$ outcome.
- These results look simple, but the road to them is complex. Other observables (Majoron) may be affected!