Realistic shell-model calculations for double-beta decay based on chiral three-body forces

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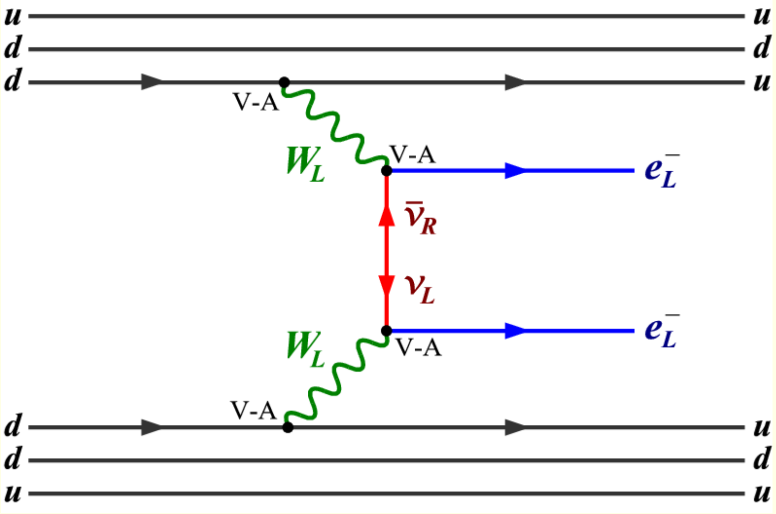
Outline

Double-beta decay study

- **Neutrinoless double-β decay** and nuclear matrix elements
- Theoretical framework: **Realistic shell model**
- Testing the theoretical framework for ^{76}**Ge**, ^{82}**Se**, ^{130}**Te**, and ^{136}**Xe** (Gamow-Teller strength, two-neutrino double-β decay)
- **Renormalization** of the Gamow-Teller operator

Neutrinoless double β-decay

The inverse of the $0\nu\beta\beta$-decay half-life is proportional to the squared nuclear matrix element (NME).
This evidences the relevance to calculate the NME



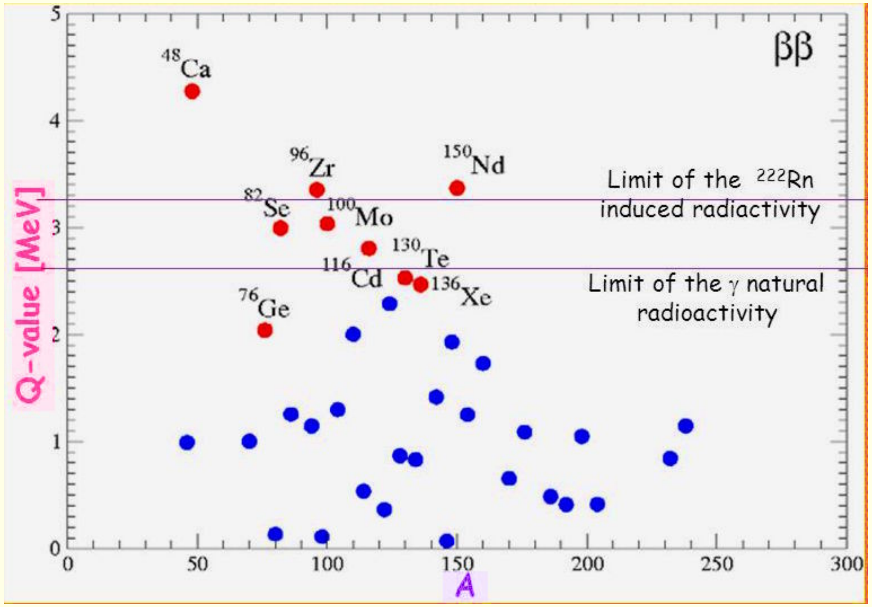
$$\left[T_{1/2}^{0\nu}\right]^{-1} = G^{0\nu}\left|M^{0\nu}\right|^2 \langle m_\nu \rangle^2 \ ,$$

- $G^{0\nu}$ is the so-called phase-space factor, obtained by integrating over she single electron energies and angles, and summing over the final-state spins;
- $\langle m_\nu \rangle = \mid \sum_k m_k U_{ek}^2 \mid$ effective mass of the Majorana neutrino, U_{ek} being the lepton mixing matrix.

Detection of $0\nu\beta\beta$-decay

It is necessary to locate the nuclei that are the best candidates to detect the $0\nu\beta\beta$-decay

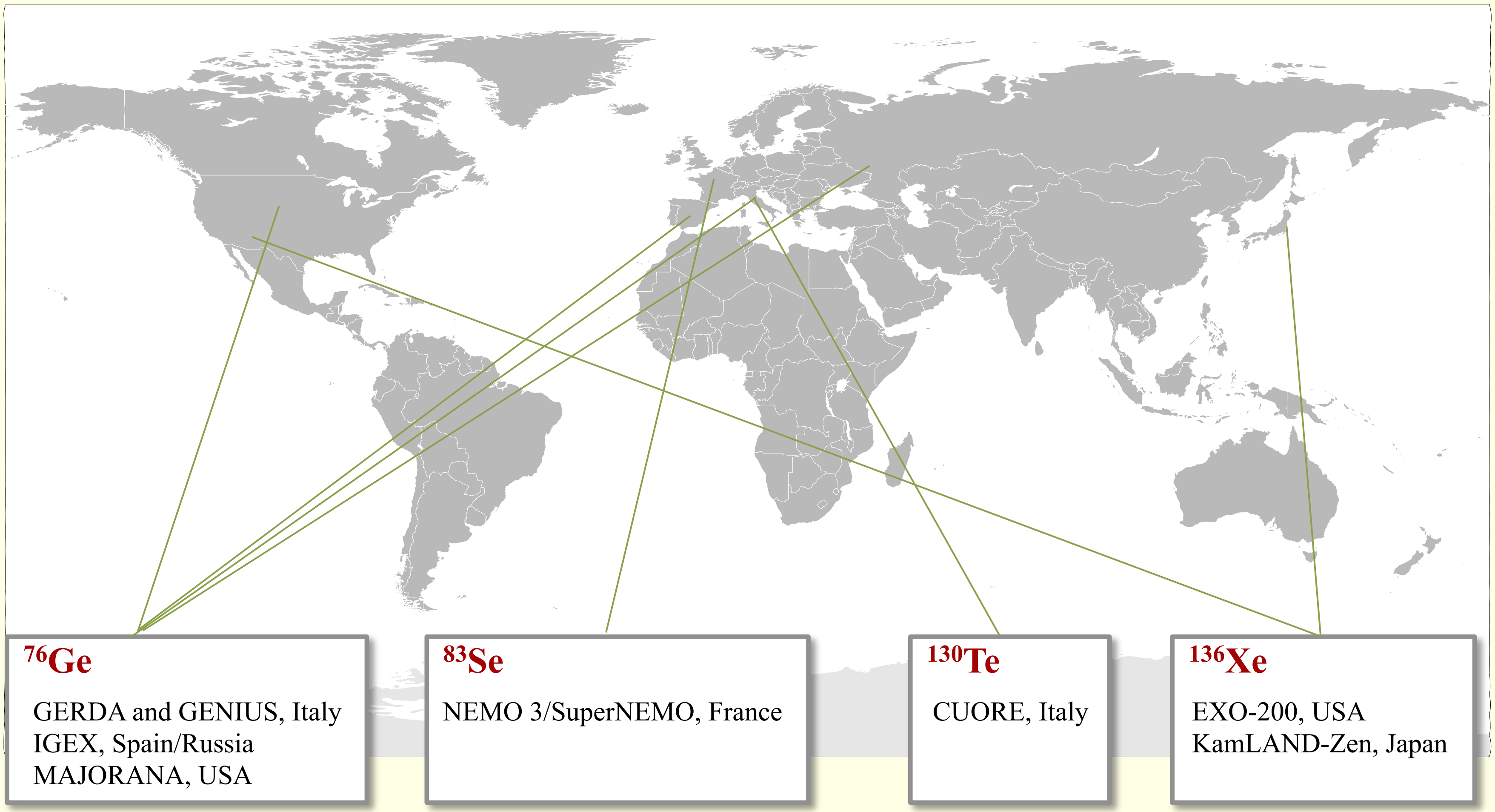
- The main factors to be taken into account are:
  - the Q-value of the reaction;
  - the phase-space factor $G^{0\nu}$;
  - the isotopic abundance



- First group: ^{76}Ge, ^{130}Te, and ^{136}Xe.
- Second group: ^{82}Se, ^{100}Mo, and ^{116}Cd.
- Third group: ^{48}Ca, ^{96}Zr, and ^{150}Nd.

Detection of $0\nu\beta\beta$-decay

Our aim is to compute the $0\nu\beta\beta$-decay NME for ^{76}Ge, ^{82}Se, ^{130}Te, and ^{136}Xe.



Calculation of NME

The NME is given by

$$M^{0\nu} = M^{0\nu}_{GT} - \left(\frac{g_V}{g_A}\right)^2 M^{0\nu}_{F} - M^{0\nu}_{T} \quad ,$$

The matrix elements $M^{0\nu}_{\alpha}$ are defined, for a SM calculation, as follows:

$$M^{0\nu}_{\alpha} = \sum_{j_p j_{p'} j_n j_{n'} J_\pi} TBTD\,(j_p j_{p'}, j_n j_{n'}; J_\pi)\,\langle j_p j_{p'}; J^\pi T \mid \tau_1^- \tau_2^- O^{\alpha}_{12} \mid j_n j_{n'}; J^\pi T \rangle \quad ,$$

with $\alpha = (GT,\ F,\ T)$.
The $TBTD$ are the two-body transition-density matrix elements, and the Gamow-Teller (GT), Fermi (F), and tensor (T) operators as

$$\begin{aligned} O^{GT}_{12} &= \vec{\sigma}_1 \cdot \vec{\sigma}_2 H_{GT}(r) \quad , \\ O^{F}_{12} &= H_F(r) \quad , \\ O^{T}_{12} &= [3\,(\vec{\sigma}_1 \cdot \hat{r})\,(\vec{\sigma}_1 \cdot \hat{r}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2]\, H_T(r) \quad . \end{aligned}$$

These operators should be regularized consistently with the NN potential

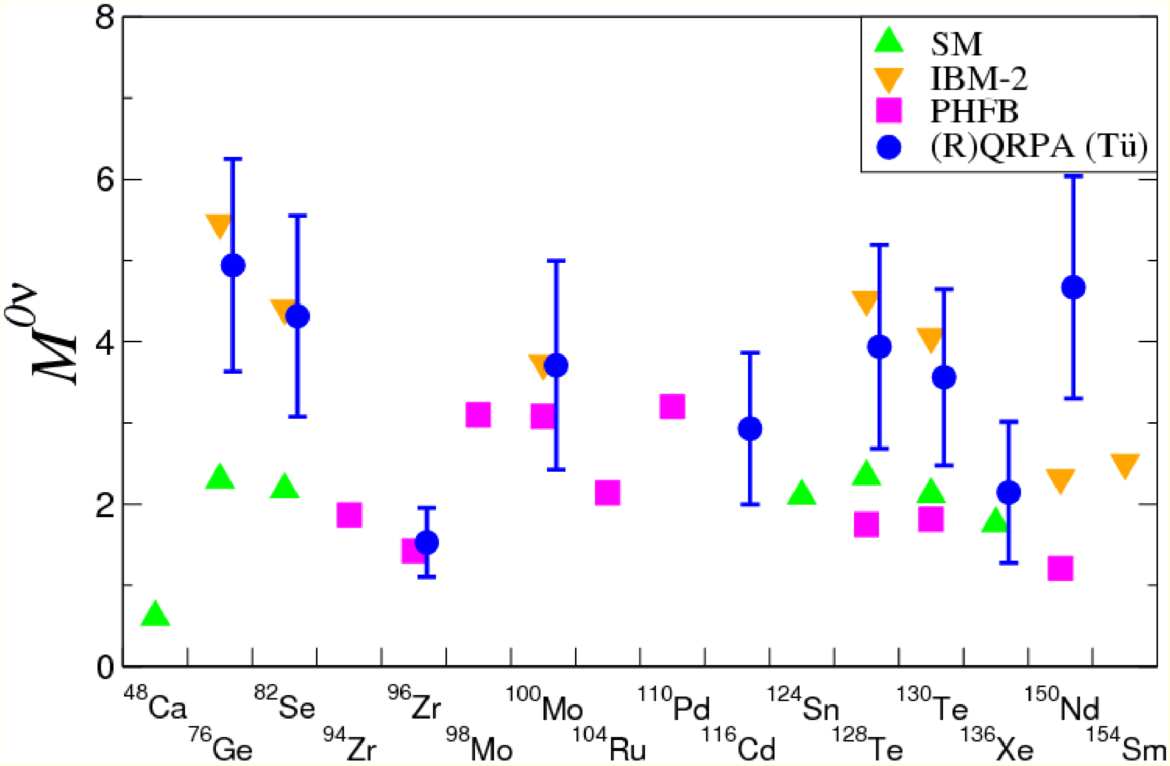
Calculation of NME

To describe the nuclear properties detected in the experiments, one needs to resort to nuclear structure models.

- Every model is characterized by a certain number of parameters.
- The calculated value of the NME may depend upon the chosen nuclear structure model.

All models may present advantages and/or shortcomings to calculate the NME

Nuclear structure models for NME



F. Iachello, Majorana Lectures 2016.

- The spread of nuclear structure calculations evidences inconsistencies among results obtained with different models

The spread of theoretical NME depends on the calculated wave function (model dependence), but also on the choice of the effective values of the axial and vector coupling constants g_A, g_V

The quenching of g_A, g_V

There are different arguments to employ effective values of g_A, g_V.
They are necessary to take into account:

- the degrees of freedom that have been not taken explicitly into account because of the truncation of the Hilbert space;
- the corrections to the free values of g_A, g_V due to meson exchange currents;
- the short-range correlations excluded to soften the *NN* force, in calculations starting from realistic potentials.

Renormalization of g_A, g_V

The standard approach is to renormalize g_A by reproducing GT data (single- and double β-decay with neutrinos), but two main issues arises:

- $2\nu\beta\beta$- and $0\nu\beta\beta$-decay operators are quite different,
- GT data may provide informations about g_A^{eff}, but not for g_V^{eff}.

- The derivation of the shell-model hamiltonian, starting from a realistic nuclear potential V_{NN} and using the many-body theory, may provide a reliable approach to the study of the $0\nu\beta\beta$ decay
- The model space may be “shaped” according to the computational needs of the diagonalization of the shell-model hamiltonian
- In such a case, the effects of the neglected degrees of freedom are taken into account by the effective hamiltonian H_{eff} and the effective transition operators O_{eff} via the many-body theory

Workflow for realistic shell model calculations

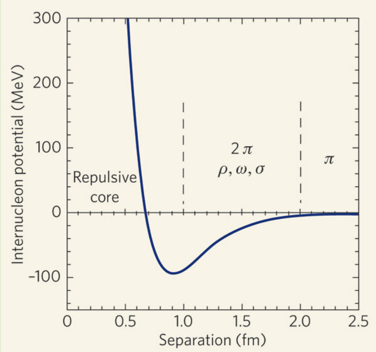
1. Choose a realistic *NN* potential (*NNN*)
2. Renormalize its short range correlations
3. Determine the model space better tailored to study the system under investigation
4. Derive the effective shell-model hamiltonian and consistently effective transition operators, expanding it up to third order in the many-body perturbation theory.
5. Calculate the physical observables (energies, e.m. transition probabilities, ...), using only theoretical SP energies, two-body matrix elements, and effective operators.

Realistic nucleon-nucleon potential

Several realistic potentials $\chi^2/datum \simeq 1$:
CD-Bonn, Argonne V18, Nijmegen, ...

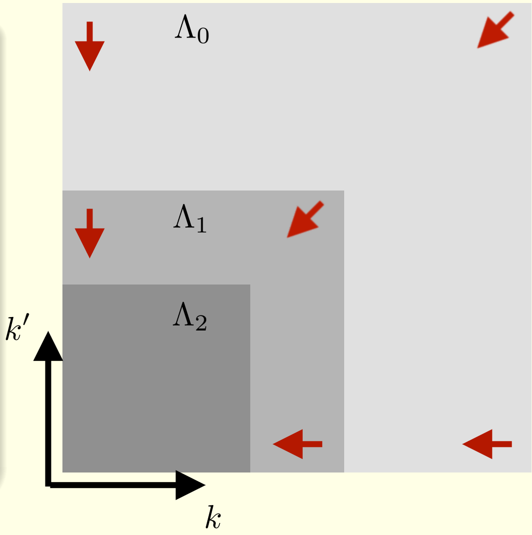
Our choice

Strong short-range repulsion



How to handle the short-range repulsion ?

- Brueckner *G* matrix
- EFT inspired approaches
  - $V_{\rm low-k}$, our choice
  - SRG
  - chiral potentials



Three-body matrix elements (3BMEs)

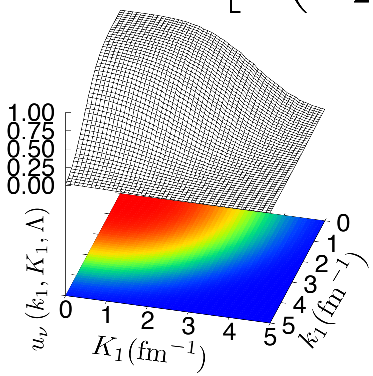
Our approach

- We have developed **our own code** for 3BMEs.
- CM separation, antisymmetrization, and calculations for c_E and c_D MEs have been performed based on previous techniques.

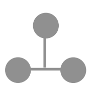
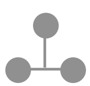
P. Navrátil *et al.*, Phys. Rev. C **61**, 044001 (2000).
E. Epelbaum *et al.*, Phys. Rev. C **66**, 064001 (2002).
P. Navrátil, Few-Body Syst. **41**, 117 (2007).

Regulator (non-local form)

$$u_\nu\left(k_1, K_1, \Lambda\right) = \exp\left[-\left(\frac{k_1^2+K_1^2}{2\Lambda^2}\right)^\nu\right]$$



New (brute-force) formalism for two-pion exchange term

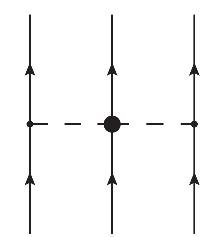
$\langle$  $\left| W_{3N}^{(2\pi;c_1)} \right|$  $\rangle$

$= \sum\sum\sum$ (coeff.) $\ni$ **c_1, ten 3*j*, eight 6*j*, five 9*j* symbols, etc.**

19 summations

$$\times \iiiint dk_1 dK_1 dk_1' dK_1' k_1^{2-\lambda_b-\lambda_b'+\lambda_3-\lambda_3'} k_1'^{\lambda_b'+\lambda_3'+1} K_1^{2-\lambda_c+\lambda_b-\lambda_b''+\lambda_3-\lambda_3''} K_1'^{\lambda_c+\lambda_b''+\lambda_3''+1}$$

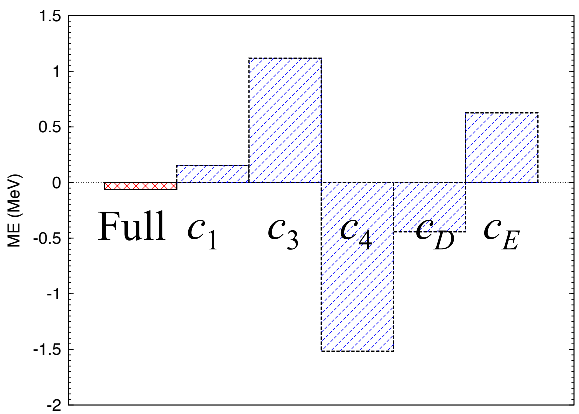
$$\times f_{\lambda_1\lambda_2\lambda_3}(k_1, k_1', K_1, K_1') P_{n_{12}l_{12}}(k_1)\, P_{n_{12}'l_{12}'}(k_1')\, P_{nl}(K_1)\, P_{n'l'}(K_1')\, u_\nu\left(k_1, K_1, \Lambda\right) u_\nu\left(k_1', K_1', \Lambda\right)$$



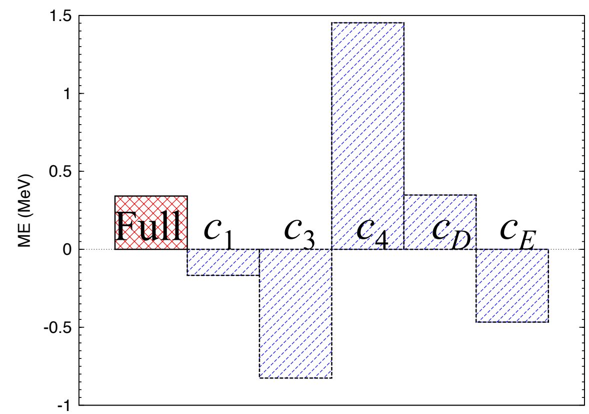
A few examples of 3BMEs (p-shell) $\,_A\left\langle \left[\left[a'b' \right]_{J'_{12}T'_{12}} c' \right]_{JT} \middle| V_{3N} \middle| \left[\left[ab \right]_{J_{12}T_{12}} c \right]_{JT} \right\rangle_A$

- The c_4 MEs play largest contribution, almost universally.

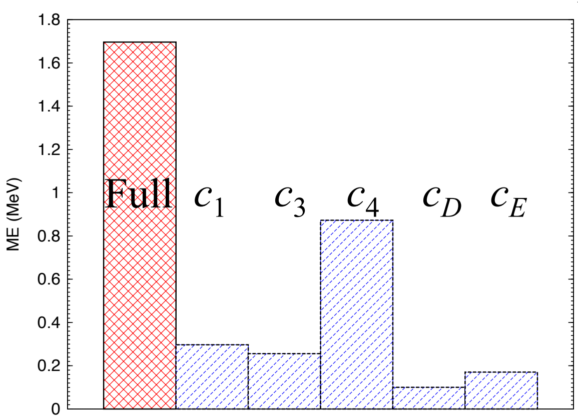
$_A\left\langle \left[\left[s_{\frac{1}{2}} s_{\frac{1}{2}} \right]_{01} p_{\frac{3}{2}} \right]_{\frac{3}{2}\frac{3}{2}} \middle| V_{3N} \middle| \left[\left[s_{\frac{1}{2}} s_{\frac{1}{2}} \right]_{01} p_{\frac{3}{2}} \right]_{\frac{3}{2}\frac{3}{2}} \right\rangle_A$



$_A\left\langle \left[\left[s_{\frac{1}{2}} s_{\frac{1}{2}} \right]_{01} p_{\frac{3}{2}} \right]_{\frac{3}{2}\frac{3}{2}} \middle| V_{3N} \middle| \left[\left[s_{\frac{1}{2}} s_{\frac{1}{2}} \right]_{10} p_{\frac{3}{2}} \right]_{\frac{3}{2}\frac{3}{2}} \right\rangle_A$



$_A\left\langle \left[\left[s_{\frac{1}{2}} p_{\frac{3}{2}} \right]_{11} s_{\frac{1}{2}} \right]_{\frac{1}{2}\frac{1}{2}} \middle| V_{3N} \middle| \left[\left[s_{\frac{1}{2}} s_{\frac{1}{2}} \right]_{10} p_{\frac{3}{2}} \right]_{\frac{1}{2}\frac{1}{2}} \right\rangle_A$



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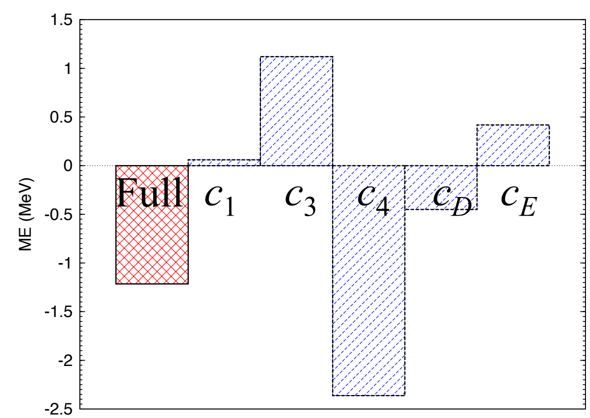

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- The c_i MEs play largest contribution, almost universally

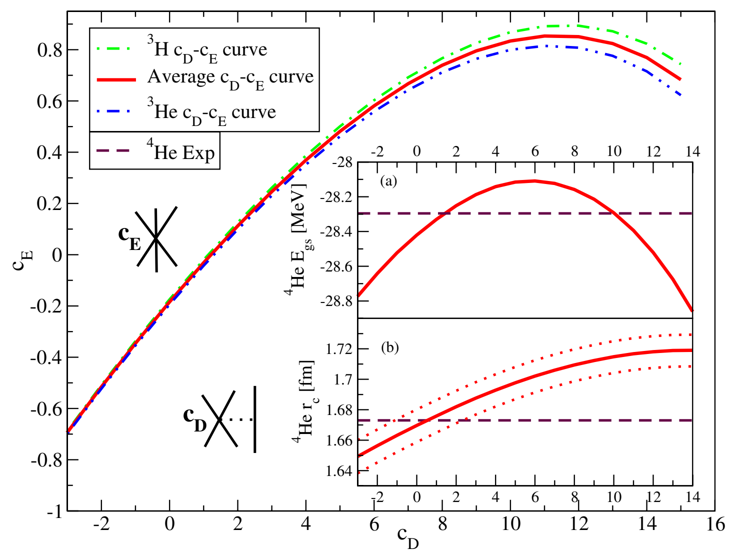
- LECs

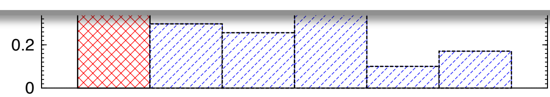
$$c_1 = -0.81 \text{ GeV}^{-1}, \ c_3 = -3.20 \text{ GeV}^{-1}, \ c_4 = 5.40 \text{ GeV}^{-1}$$

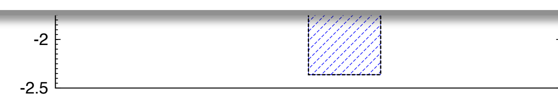
D. R. Entem and R. Machleidt, Phys. Rev. C **68**, 041001(R) (2003).

$$c_D = -1.0, \ c_E = -0.34$$

P. Navrátil *et al.*, Phys. Rev. Lett. **99**, 042501 (2007).







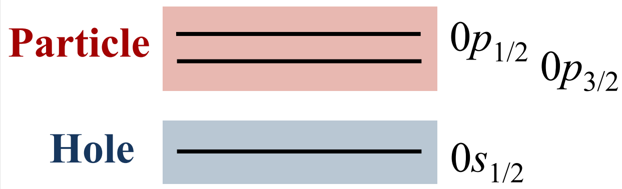
No empirical input

Interactions and LECs

2NF: Chiral EFT N³LO, **3NF**: Chiral EFT N²LO

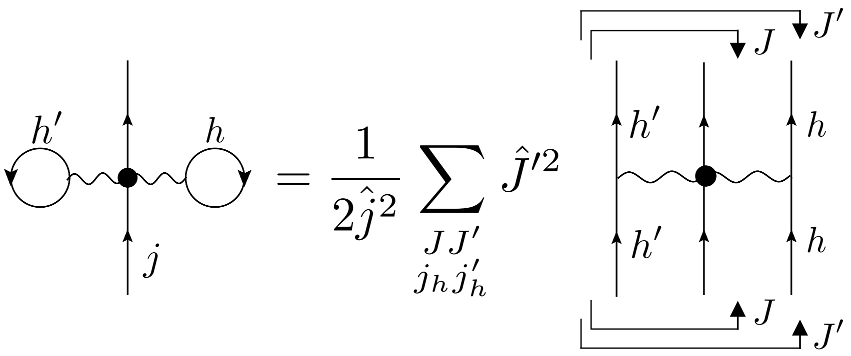
D. R. Entem and R. Machleidt, Phys. Rev. C **68**, 041001(R) (2003).
P. Navrátil *et al*., Phys. Rev. Lett. **99**, 042501 (2007).

Model space $\hbar\omega = 19$ MeV

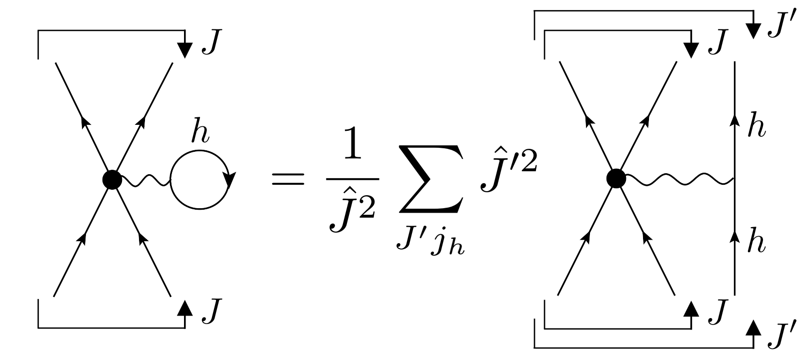


2 valence nucleons with ⁴He core

Normal-ordered SPE (1st order)



Normal-ordered 2BME (1st order)



Many-body perturbation theory

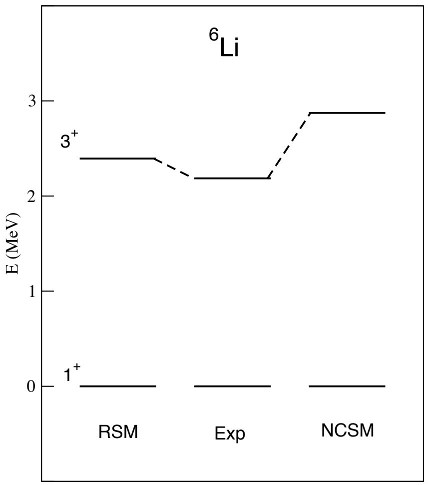
2NF: Up to the 3rd-order of the folded-diagram expansion
3NF: Up to the 1st-order, at this moment

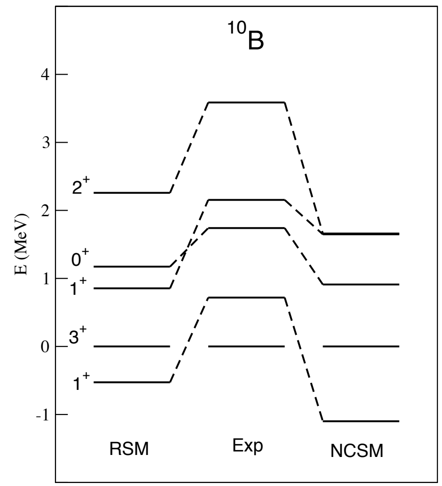
Renormalization

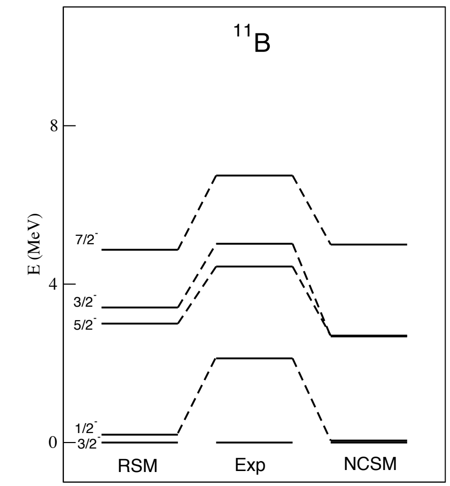
Our realistic forces are **NOT** renormalized.

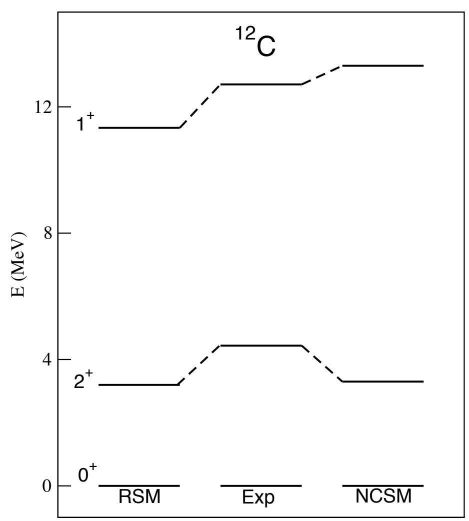
2NF only

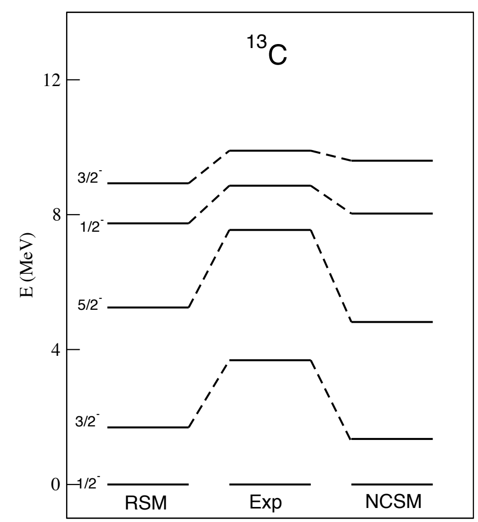
- Comparison with no-core shell model (NCSM) P. Navrátil *et al*., Phys. Rev. Lett. **99**, 042501 (2007).





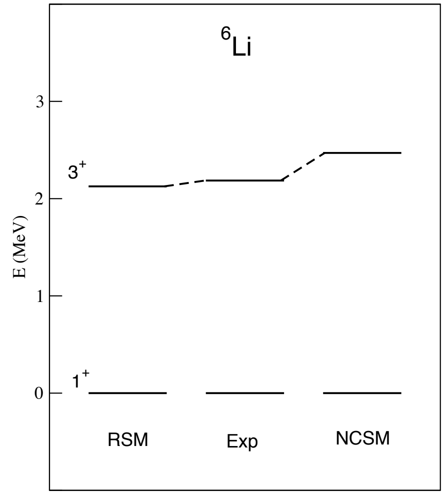


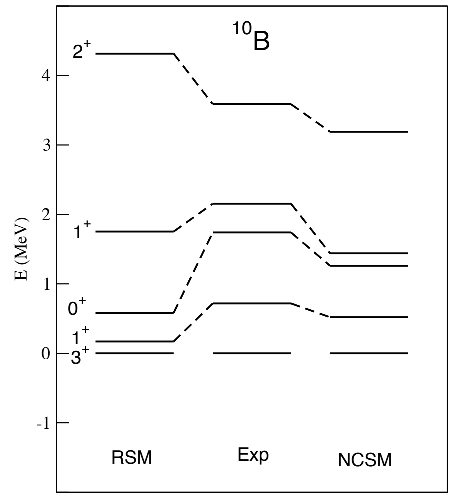


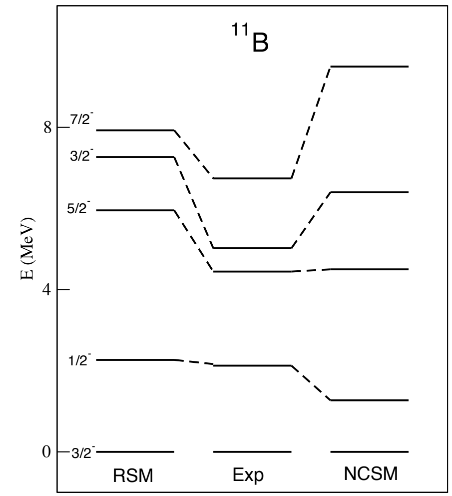


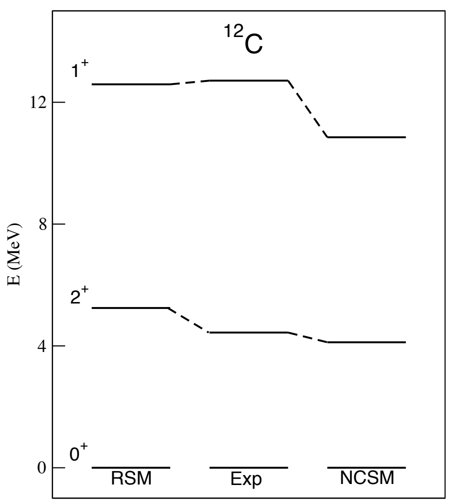
2NF + 3NF (very preliminary)

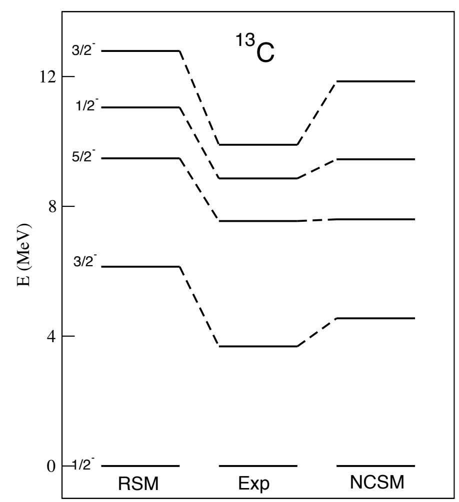
- Comparison with no-core shell model (NCSM) P. Navrátil *et al.*, Phys. Rev. Lett. **99**, 042501 (2007).











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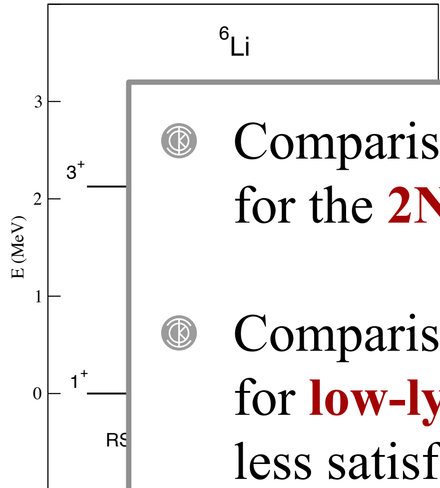
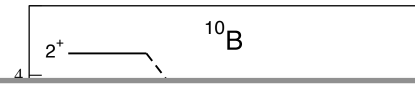
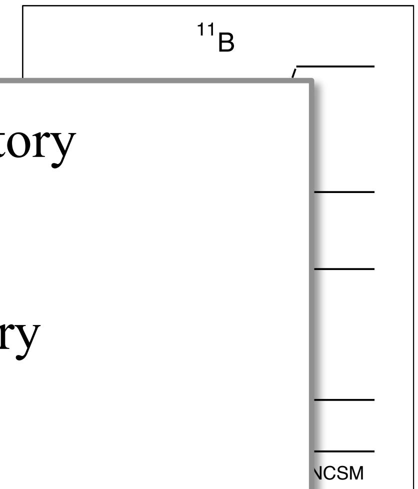
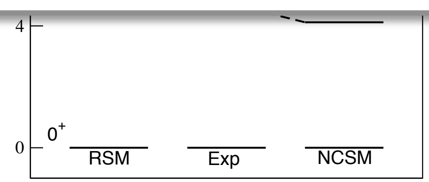
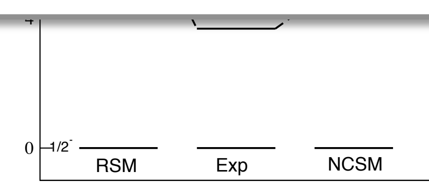






- Comparison between RSM and NCSM are satisfactory for the **2NF** only.
- Comparison between SM and NCSM are satisfactory for **low-lying spectra** when including **3NF**, less satisfactory for **higher-energy states**.
- RSM calculations with three-body forces are **very preliminary**, there are more correlations to be considered:
  → Include contributions beyond the first order.
  → Include the many-body correlations induced by the 3NF.





We start from the many-body hamiltonian H defined in the full Hilbert space:

$$H = H_0 + H_1 = \sum_{i=1}^{A} (T_i + U_i) + \sum_{i<j} (V_{ij}^{NN} - U_i)$$

$$\left(\begin{array}{c|c} PHP & PHQ \\ \hline QHP & QHQ \end{array} \right) \quad \begin{array}{c} \mathcal{H} = X^{-1} H X \\ \Longrightarrow \\ Q\mathcal{H}P = 0 \end{array} \quad \left(\begin{array}{c|c} P\mathcal{H}P & P\mathcal{H}Q \\ \hline 0 & Q\mathcal{H}Q \end{array} \right)$$

$$H_{\text{eff}} = P\mathcal{H}P$$

Suzuki & Lee $\Rightarrow X = e^{\omega}$ with $\omega = \left(\begin{array}{c|c} 0 & 0 \\ \hline Q\omega P & 0 \end{array} \right)$

$$H_1^{\text{eff}}(\omega) = PH_1P + PH_1Q \frac{1}{\epsilon - QHQ} QH_1P -$$

$$- PH_1Q \frac{1}{\epsilon - QHQ} \omega H_1^{\text{eff}}(\omega)$$

Folded-diagram expansion

This recursive equation for H_{eff} may be solved using iterative techniques (Krenciglowa-Kuo, Lee-Suzuki, ...)

$$H_{\mathrm{eff}} = \hat{Q} - \hat{Q}' \int \hat{Q} + \hat{Q}' \int \hat{Q} \int \hat{Q} - \hat{Q}' \int \hat{Q} \int \hat{Q} \int \hat{Q} \cdots ,$$

$\hat{Q}$-box vertex function

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P$$

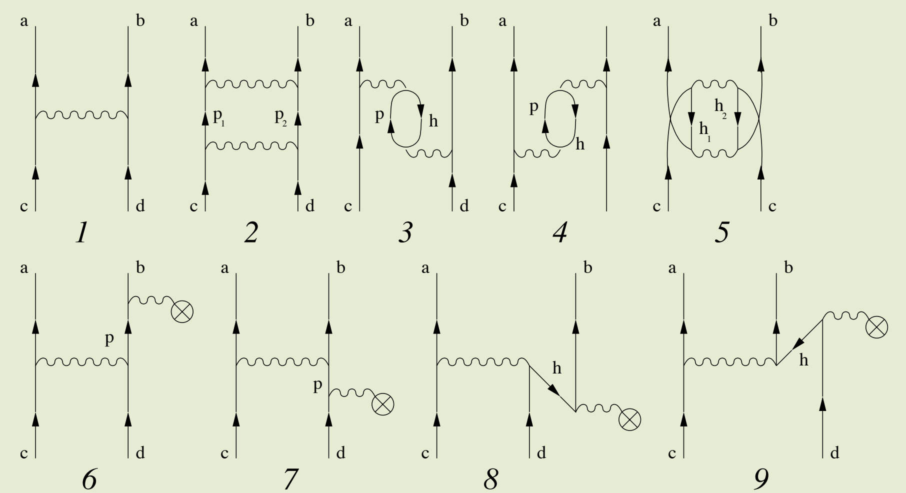
Perturbative approach to H_{eff}

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P$$

Exact calculation of the $\hat{Q}$-box is computationally prohibitive for many-body system $\Rightarrow$ we perform a perturbative expansion

$$\frac{1}{\epsilon - QHQ} = \sum_{n=0}^{\infty} \frac{(QH_1Q)^n}{(\epsilon - QH_0Q)^{n+1}}$$

The diagrammatic expansion of the $\hat{Q}$-box



Consistently, any shell-model effective operator may be calculated

It has been demonstrated that, for any bare operator Θ, a non-Hermitian effective operator $\Theta_{\rm eff}$ can be written in the following form:

$$\Theta_{\rm eff} = (P + \hat{Q}_1 + \hat{Q}_1\hat{Q}_1 + \hat{Q}_2\hat{Q} + \hat{Q}\hat{Q}_2 + \cdots)(\chi_0 + \\ + \chi_1 + \chi_2 + \cdots) ,$$

where

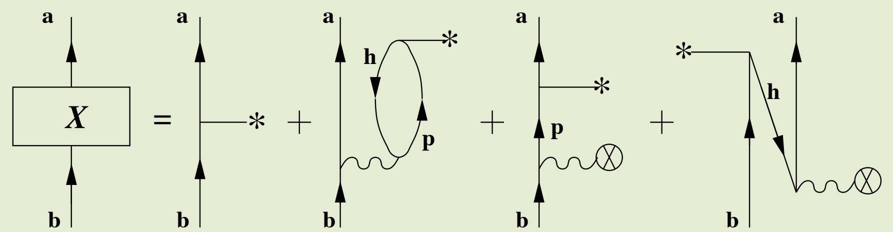
$$\hat{Q}_m = \frac{1}{m!} \frac{d^m \hat{Q}(\epsilon)}{d\epsilon^m} \Big|_{\epsilon=\epsilon_0} ,$$

ϵ_0 being the model-space eigenvalue of the unperturbed hamiltonian H_0

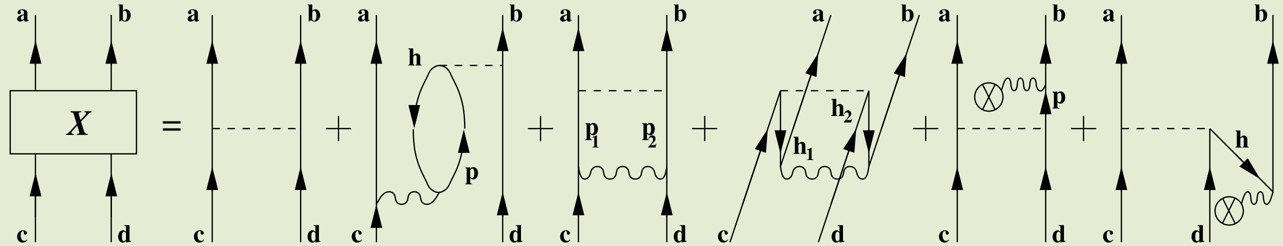
K. Suzuki and R. Okamoto, Prog. Theor. Phys. **93** *, 905 (1995)*

We arrest the χ series at the leading term χ_0, and then expand it perturbatively:

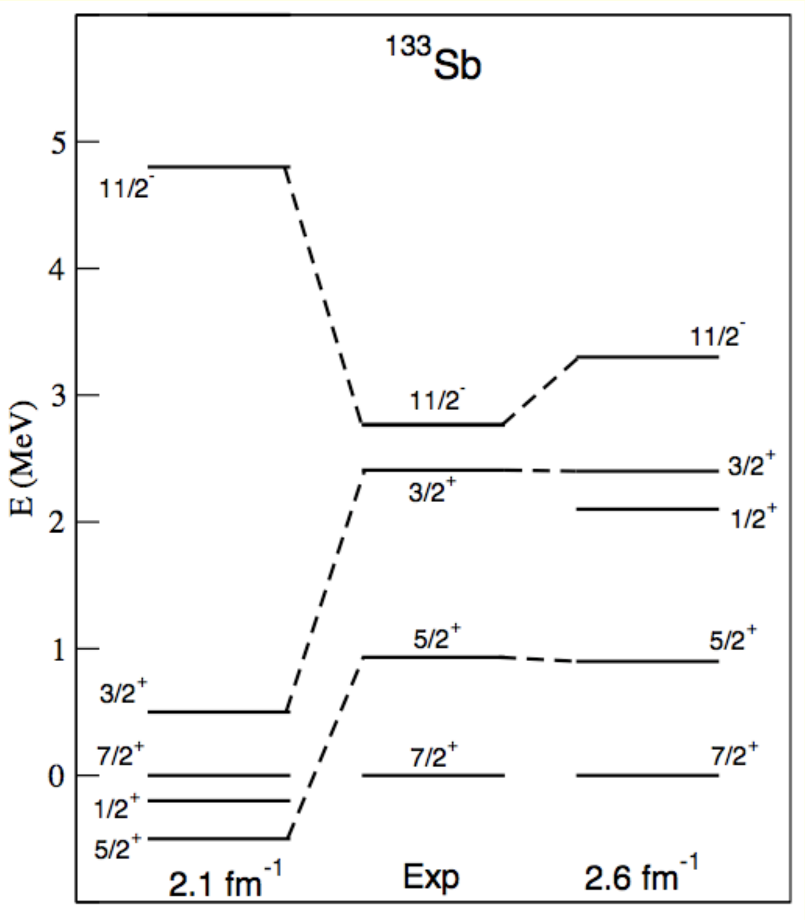
One-body operator

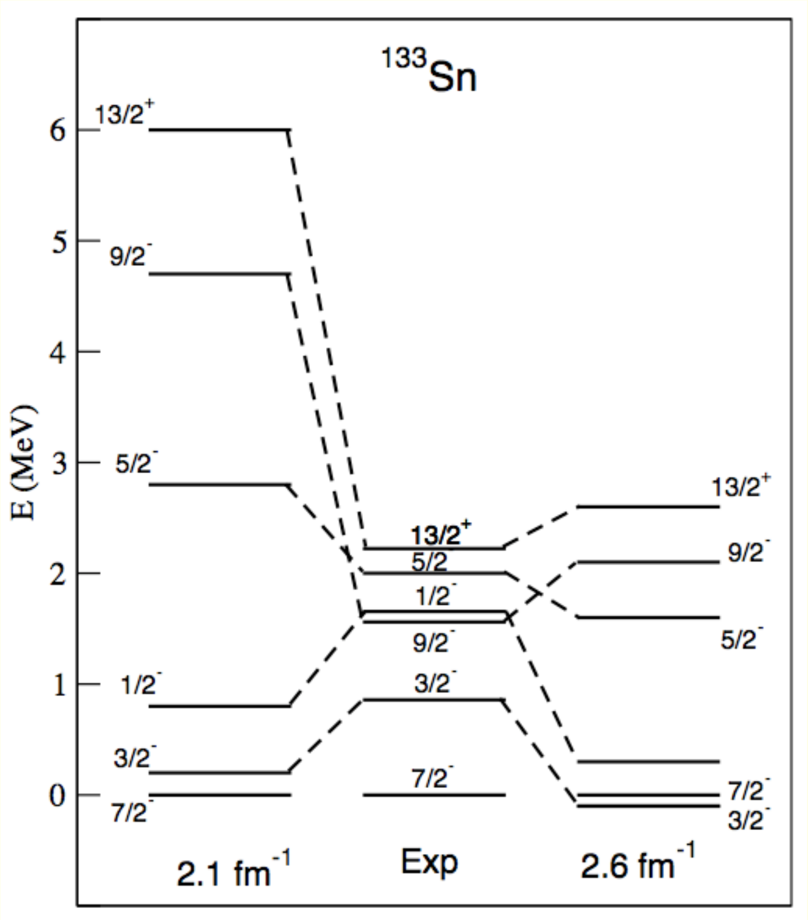


Two-body operator



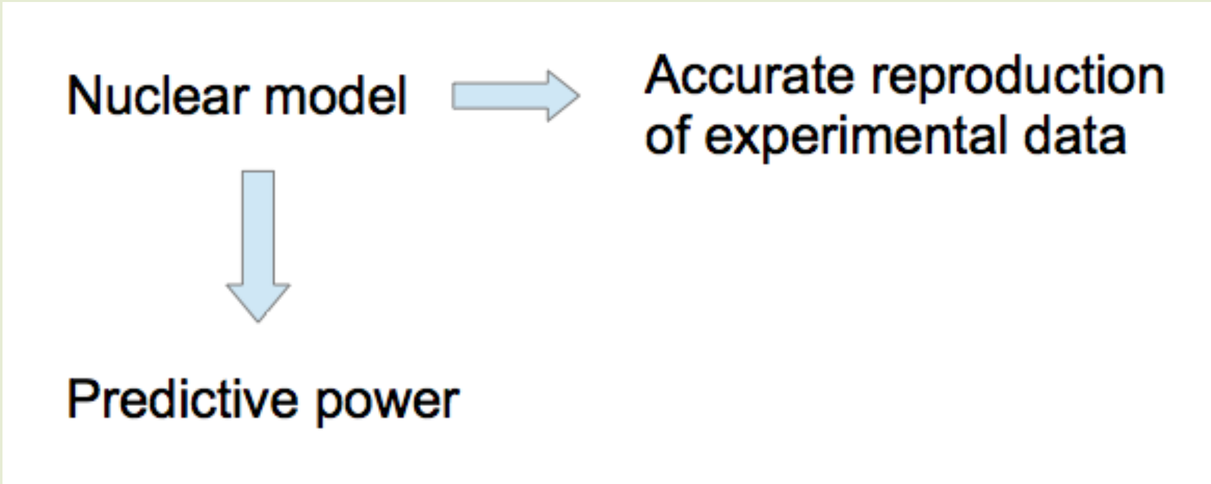
Choice of cutoff $\Lambda = 2.6$ fm^{-1}





L. Coraggio, A. Gargano, and N. Itaco, JPS Conf. Proc. ***6****, 020046 (2015)*

Nuclear models and predictive power



Realistic SM calculations for ^{76}Ge, ^{82}Se, ^{130}Te, and ^{136}Xe

$\Downarrow$

Check this approach calculating observables related to the GT strengths and $2\nu\beta\beta$ decay and compare the results with data.

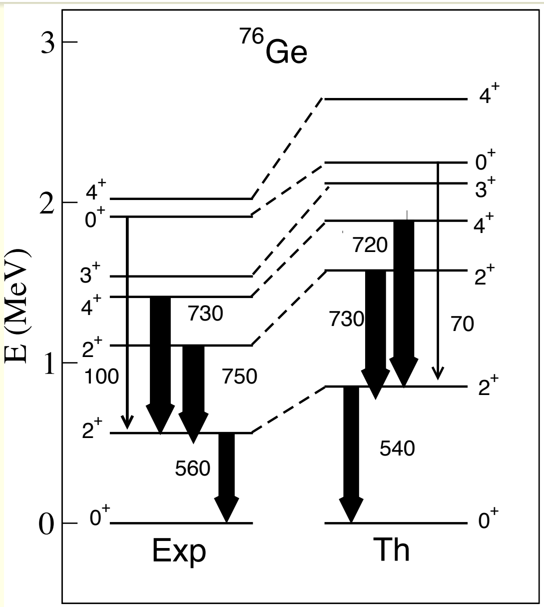
$$\left[T_{1/2}^{2\nu}\right]^{-1} = G^{2\nu}\left|M_{2\nu}^{\mathrm{GT}}\right|^2$$

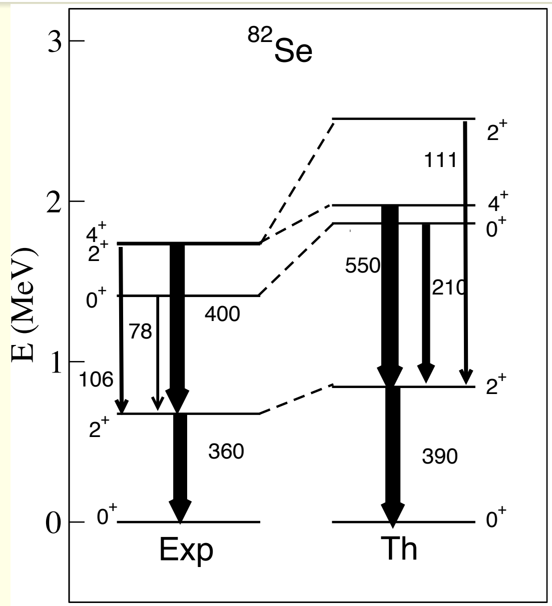
Shell-model calculations

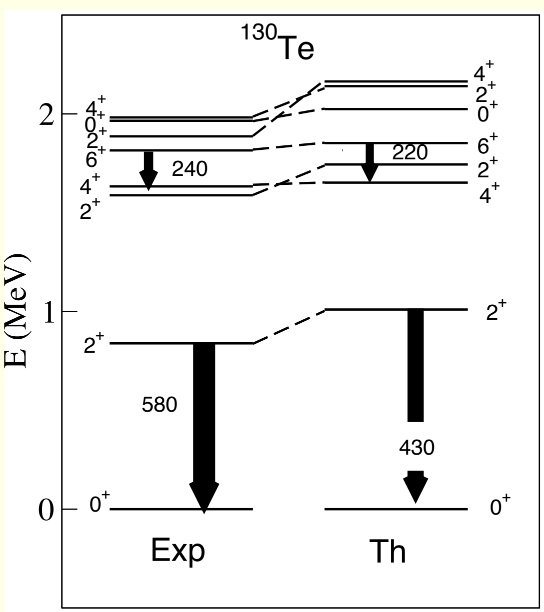
- ^{76}Ge, ^{82}Se: four proton and neutron orbitals outside double-closed ^{56}Ni
  $0f_{5/2}, 1p_{3/2}, 1p_{1/2}, 0g_{9/2}$
- ^{130}Te, ^{136}Xe: five proton and neutron orbitals outside double-closed ^{100}Sn *
  $0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2}, 0h_{11/2}$

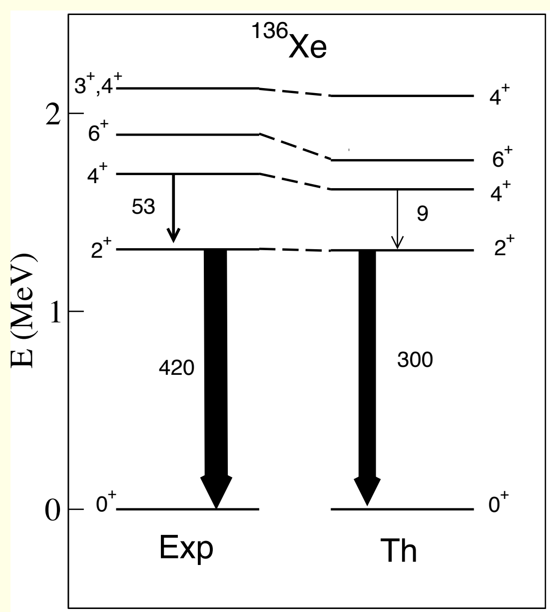
* *L. Coraggio, L. De Angelis, T. Fukui, A. Gargano, and N. Itaco, Phys. Rev. C* **95**, *064324 (2017)*

Spectroscopic properties

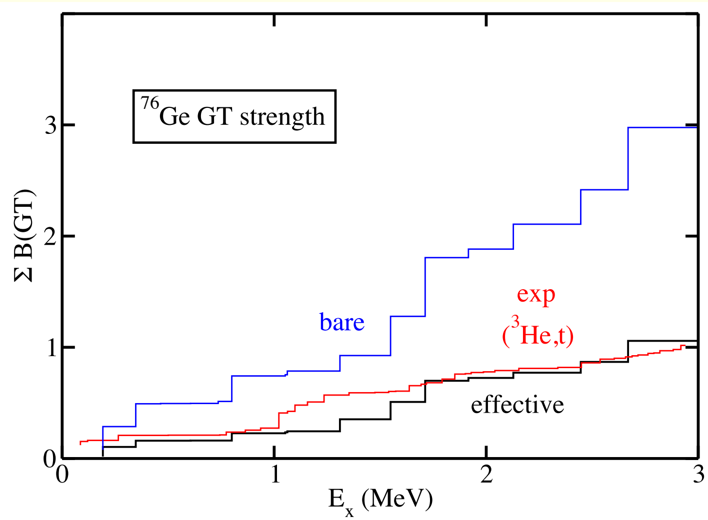


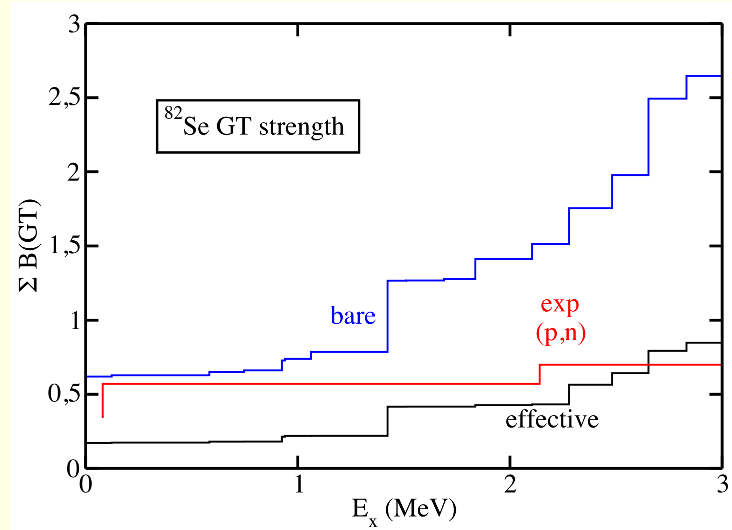


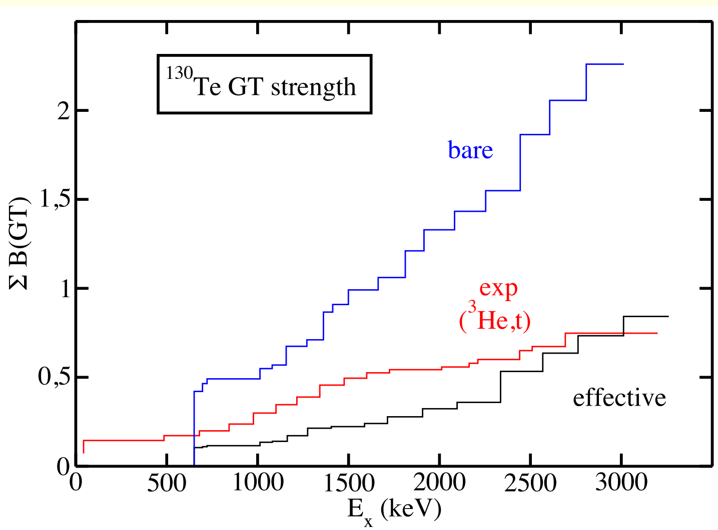


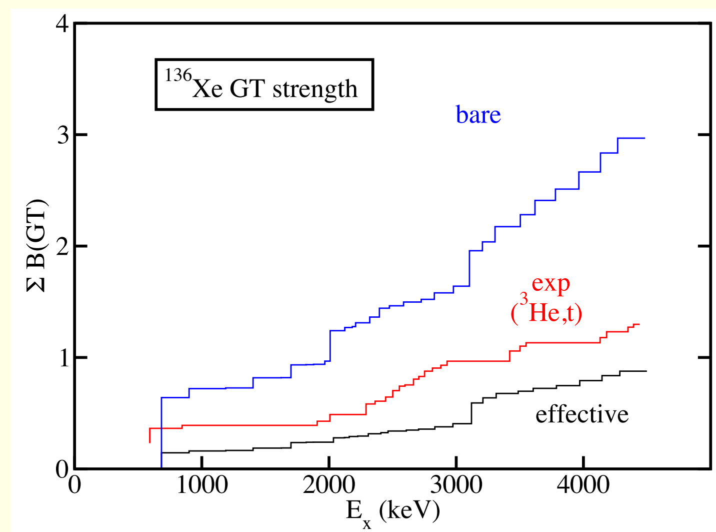


GT$^-$ running sums

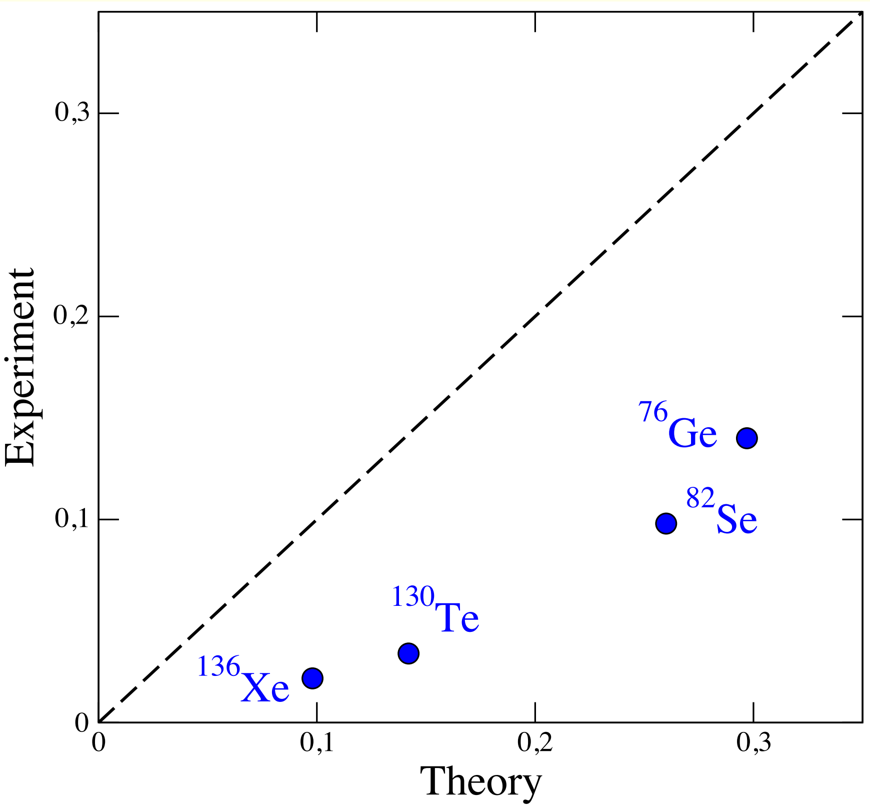








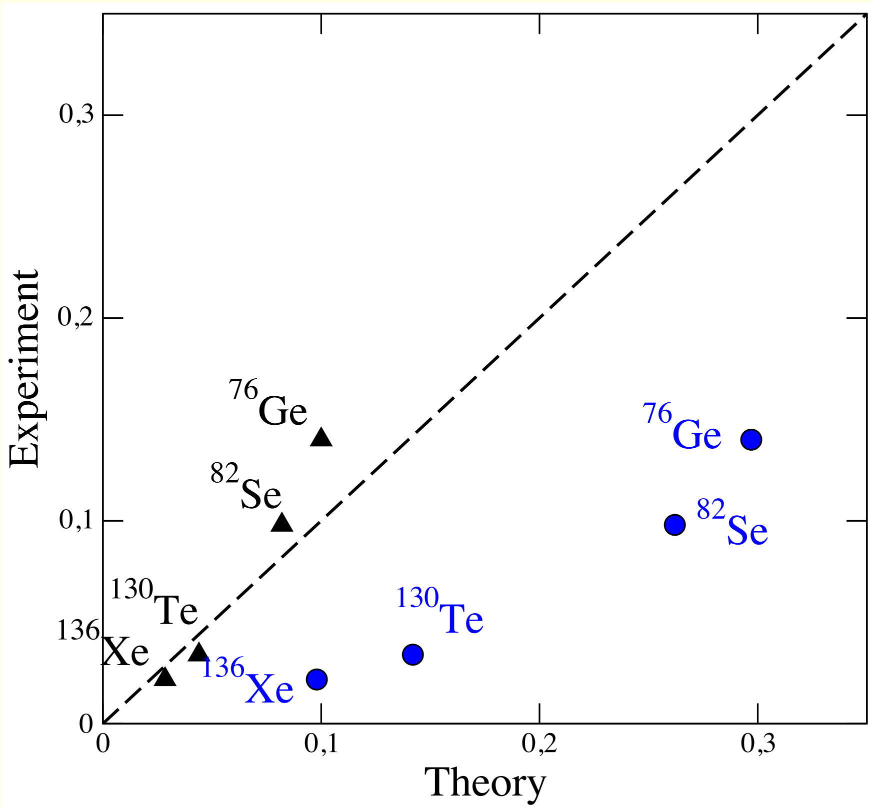
$2\nu\beta\beta$ NME



Blue dots: bare GT operator

Decay	Expt.	Bare
$^{76}Ge \rightarrow ^{76}Se$	0.140 ± 0.005	0.297
$^{82}Se \rightarrow ^{82}Kr$	0.098 ± 0.004	0.262
$^{130}Te \rightarrow ^{130}Xe$	0.034 ± 0.003	0.142
$^{136}Xe \rightarrow ^{136}Ba$	0.0218 ± 0.0003	0.0975

$2\nu\beta\beta$ NME



Blue dots: bare GT operator
Black triangles: effective GT operator

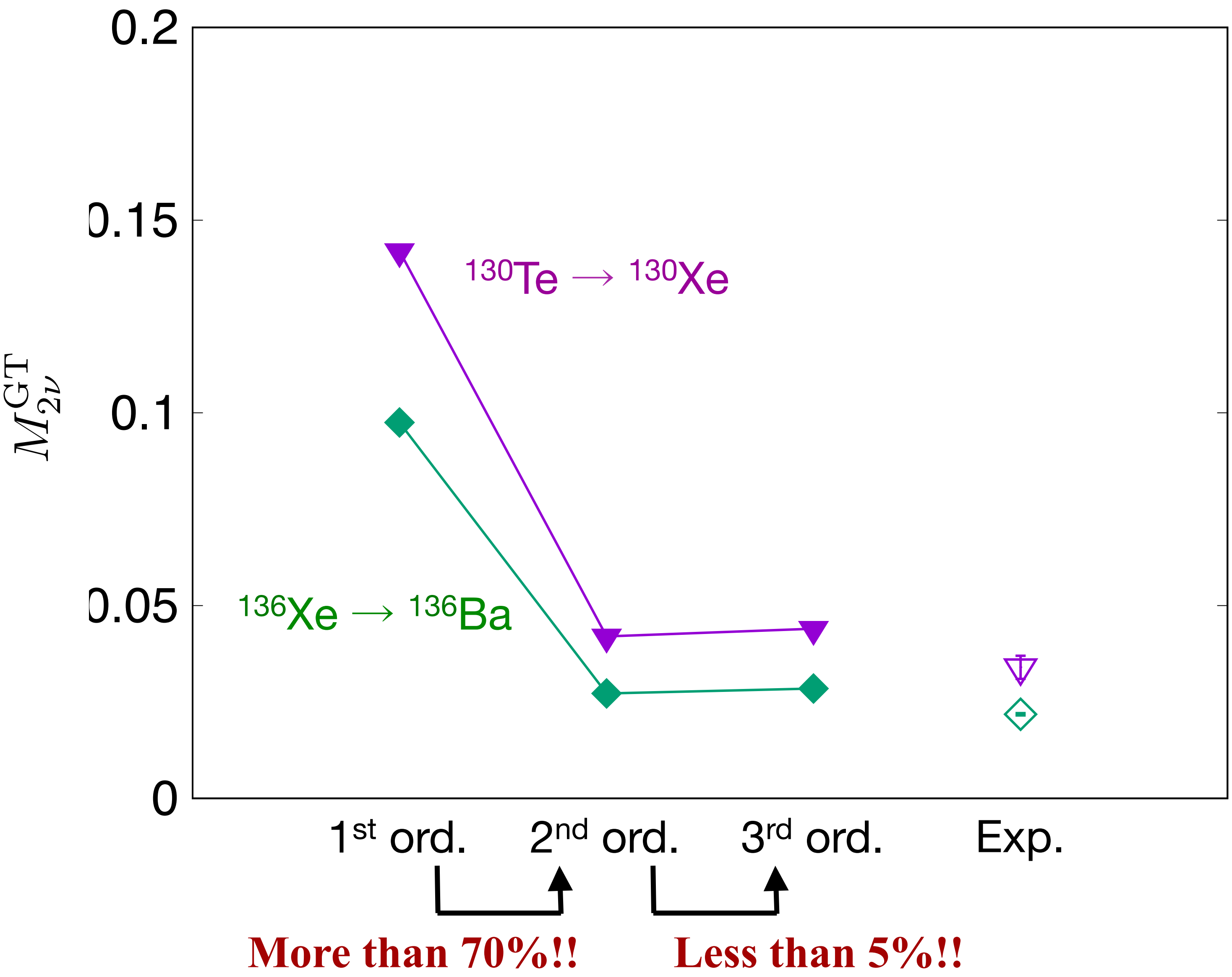
Decay	Expt.	Eff.
$^{76}Ge \rightarrow ^{76}Se$	0.140 ± 0.005	0.100
$^{82}Se \rightarrow ^{82}Kr$	0.098 ± 0.004	0.082
$^{130}Te \rightarrow ^{130}Xe$	0.034 ± 0.003	0.044
$^{136}Xe \rightarrow ^{136}Ba$	0.0218 ± 0.0003	0.0285

"Quenching"

Matrix elements of the neutron-proton effective GT$^-$ operator

$n_a l_a j_a$ $n_b l_b j_b$	3rd order $\mathrm{GT}^-_{\mathrm{eff}}$	quenching	$n_a l_a j_a$ $n_b l_b j_b$	3rd order $\mathrm{GT}^-_{\mathrm{eff}}$	quenching
$0f_{5/2}$ $0f_{5/2}$	-0.977	0.37	$0g_{7/2}$ $0g_{7/2}$	-1.239	0.50
$0f_{5/2}$ $1p_{3/2}$	-0.143		$0g_{7/2}$ $1d_{5/2}$	-0.019	
$1p_{3/2}$ $0f_{5/2}$	0.046		$1d_{5/2}$ $0g_{7/2}$	0.131	
$1p_{3/2}$ $1p_{3/2}$	2.030	0.62	$1d_{5/2}$ $1d_{5/2}$	1.864	0.64
$1p_{3/2}$ $1p_{1/2}$	-1.621	0.55	$1d_{5/2}$ $1d_{3/2}$	-1.891	0.61
$1p_{1/2}$ $1p_{3/2}$	1.713	0.58	$1d_{3/2}$ $1d_{5/2}$	1.794	0.58
$1p_{1/2}$ $1p_{1/2}$	-0.697	0.67	$1d_{3/2}$ $1d_{3/2}$	-1.023	0.66
$0g_{9/2}$ $0g_{9/2}$	3.121	0.70	$1d_{3/2}$ $2s_{1/2}$	-0.093	
			$2s_{1/2}$ $1d_{3/2}$	0.117	
			$2s_{1/2}$ $2s_{1/2}$	1.598	0.65
			$0h_{11/2}$ $0h_{11/2}$	2.597	0.69

Perturbative properties of effective operator



- $2\nu\beta\beta$
  - Role of real three-body forces and two-body currents (present collaboration with Pisa group)
  - Evaluation of the contribution of many-body correlations (blocking effect)
- $0\nu\beta\beta$
  - Derivation of the two-body effective operator
  - SRC calculated consistently with $V_{\mathrm{low-k}}$