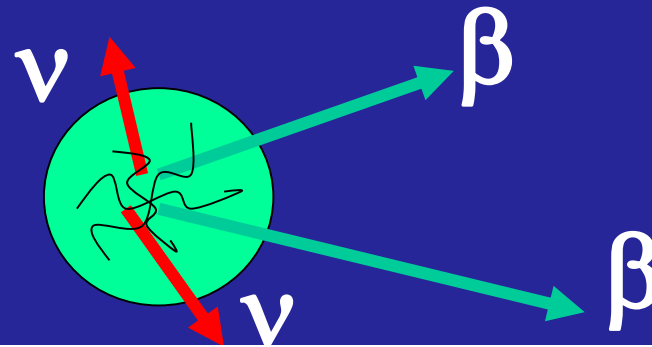


## Novel approaches to the nuclear physics of $\beta\beta$ -decay: chargex reactions, mass-measurements, $\mu$ -capture



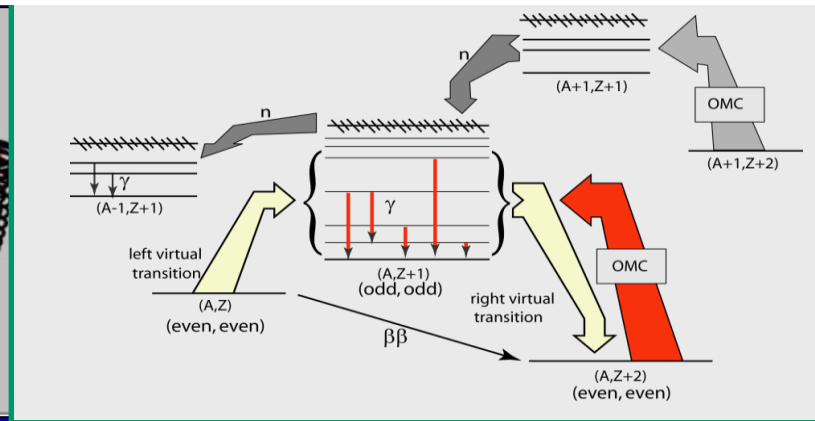
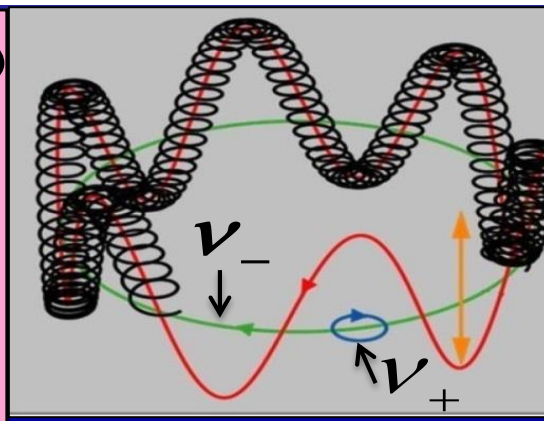
INT- 2018

**Gentle Touch:**

- $q_{tr} = 0$
- $\Delta I = 0$
- $0 \hbar\omega$  excitation

GT ?

A graph showing a red curve that starts at a high value and decays towards zero as energy increases. The x-axis is labeled with 5 and 10.



# Where do we stand in $\beta\beta$ decay when putting together the pieces of the puzzle?

## 1. General features

## 2. Chargex-reactions ( $^3\text{He}, t$ ) & ( $d, ^2\text{He}$ )

- perfect for  $2\nu\beta\beta$  NME's

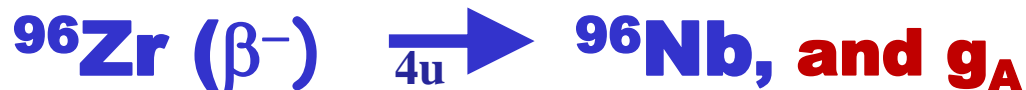
## 3. Chargex-reactions

- limited for  $0\nu\beta\beta$  NME's  
(here:  $2^-$  states and nuclear wave function)

## 4. Mass measurements

- $0\nu\beta\beta$  NME
- $^{96}\text{Zr}$  is a „golden“ case

(PRL116, Feb-2016)



## 5. Muon capture projects starting (MuSIC)

- a high- $q$  transfer phenomenon !!  
gives handle on  **$g_A$  quenching**



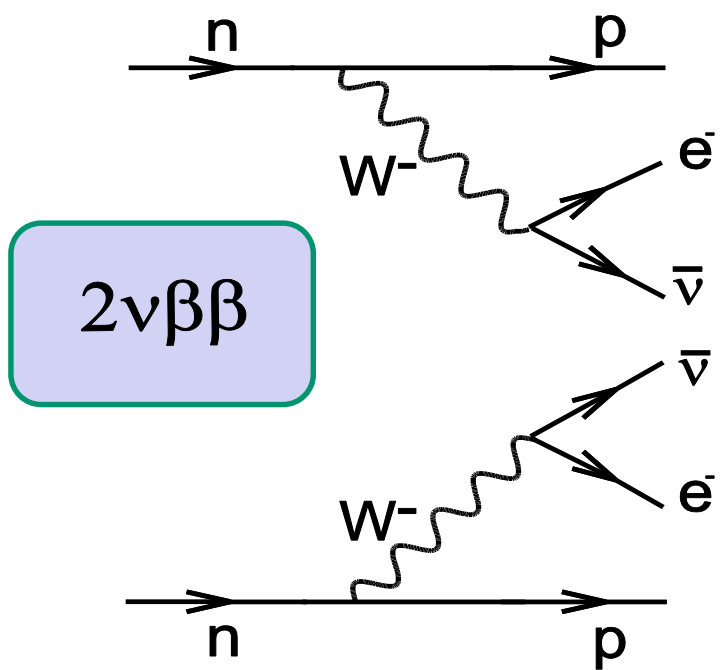
-1-

General features

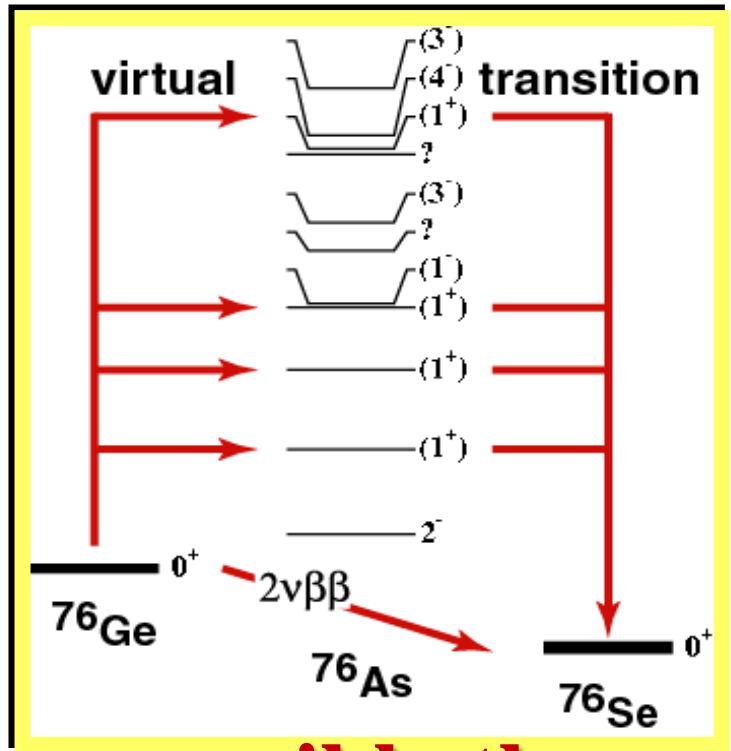
( $2\nu\beta\beta$  /  $0\nu\beta\beta$  decay)

$$\Gamma_{(\beta^-\beta^-)}^{2\nu} = \frac{C}{8\pi^7} \left( \frac{G_F g_A}{\sqrt{2}} \cos(\Theta_C) \right)^4 \left| M_{\text{DGT}}^{(2\nu)} \right|^2 F_{(-)}^2 f(Q)$$

$$= G^{2\nu}(Q, Z) g_A^4 \left| M_{\text{DGT}}^{(2\nu)} \right|^2$$



$$q_{\text{tr}} \sim 0.01 \text{ fm}^{-1}$$



**accessible thru  
charge-ex reaction**

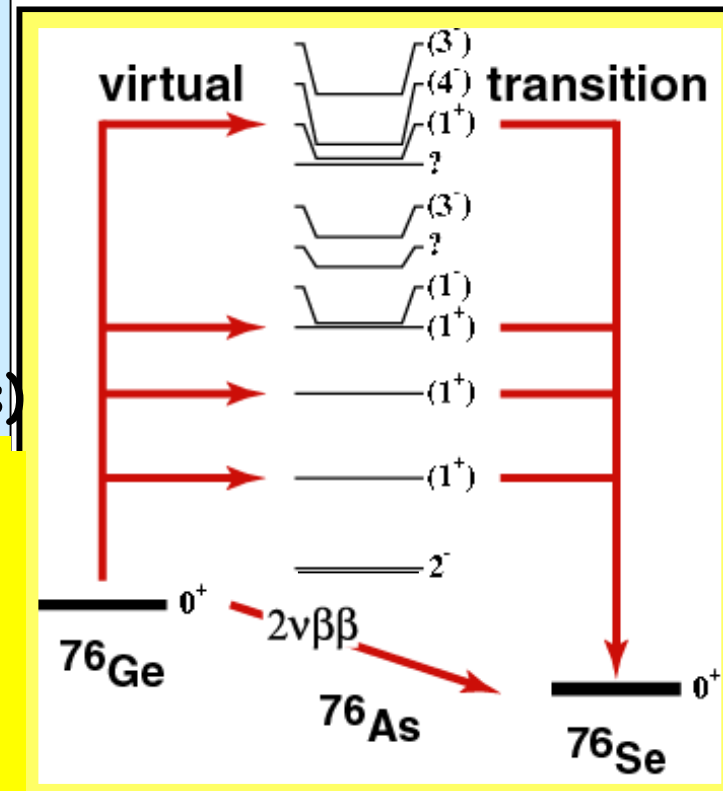
$$M_{\text{DGT}}^{(2n)} = \mathop{\text{a}}_m \frac{\langle \mathbf{0}_{g.s.}^{(f)} | \mathop{\text{a}}_k \mathbf{s}_k \mathbf{t}_{\bar{k}} | \mathbf{1}_m^+ \rangle \langle \mathbf{1}_m^+ | \mathop{\text{a}}_k \mathbf{s}_k \mathbf{t}_{\bar{k}} | \mathbf{0}_{g.s.}^{(i)} \rangle}{\frac{1}{2} Q_{\text{bb}}(\mathbf{0}_{g.s.}^{(f)}) + E(\mathbf{1}_m^+) - E_0}$$

$$= \mathop{\text{a}}_m \frac{M_m(GT^+) M_m(GT^-)}{E_m}$$

to remember:

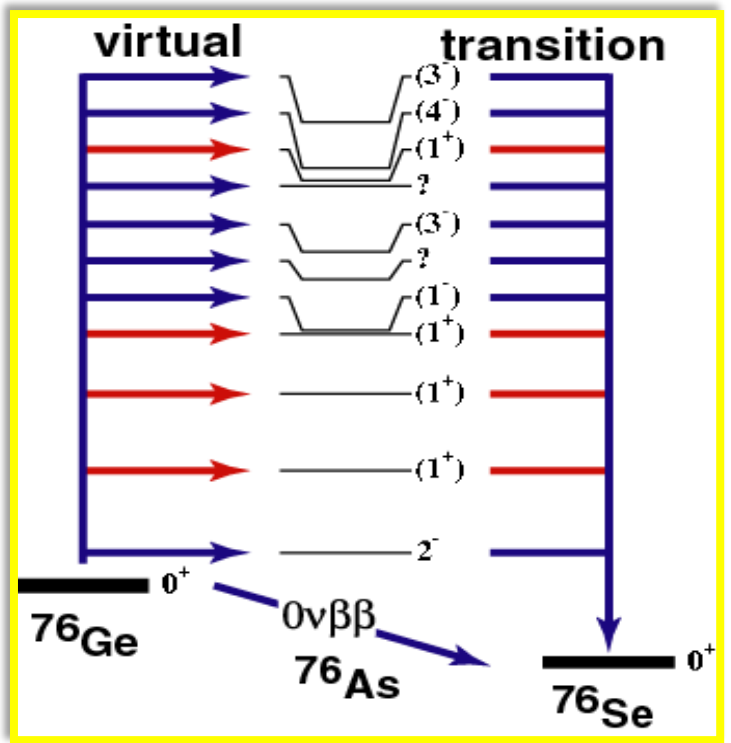
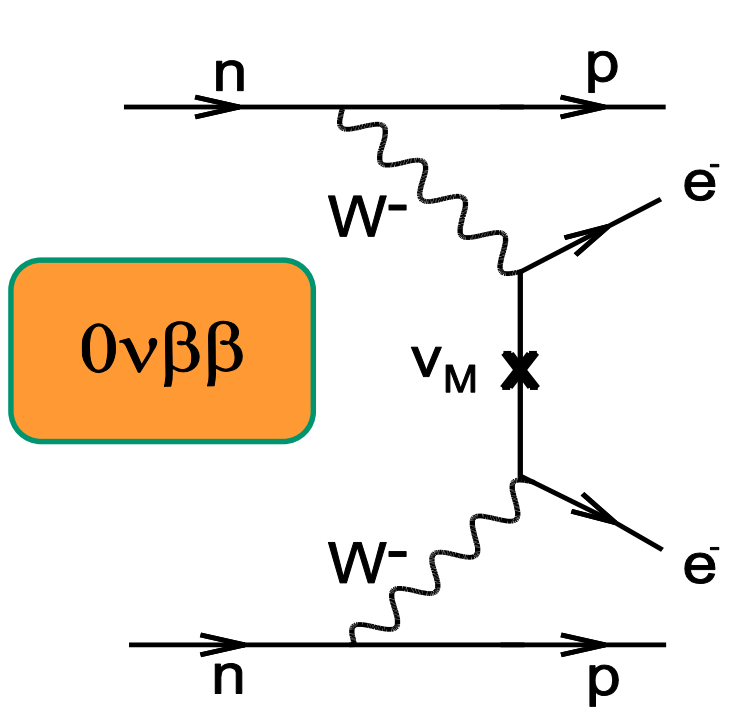
1. 2 sequential & „allowed“  $\beta^-$ -decays of „Gamow-Teller“ type
2. „1, 2, 3, ... forbidden“ decays negligible
3. Fermi-transitions do not contribute (because of different isospin-multiplets)

Can be determined via charge-exchange reactions in the (n,p) and (p,n) direction ( e.g. (d, $^2\text{He}$ ) or ( $^3\text{He}$ ,t) )



$$G_{(b^- b^-)}^{0n} = G^{0n}(Q,Z) g_A^4 \left| M_{DGT}^{(0n)} \right|^2 \frac{g_V^2}{g_A^2} \frac{Q^2}{0} M_{DF}^{(0n)} \left| m_{n_e} \right|^2$$

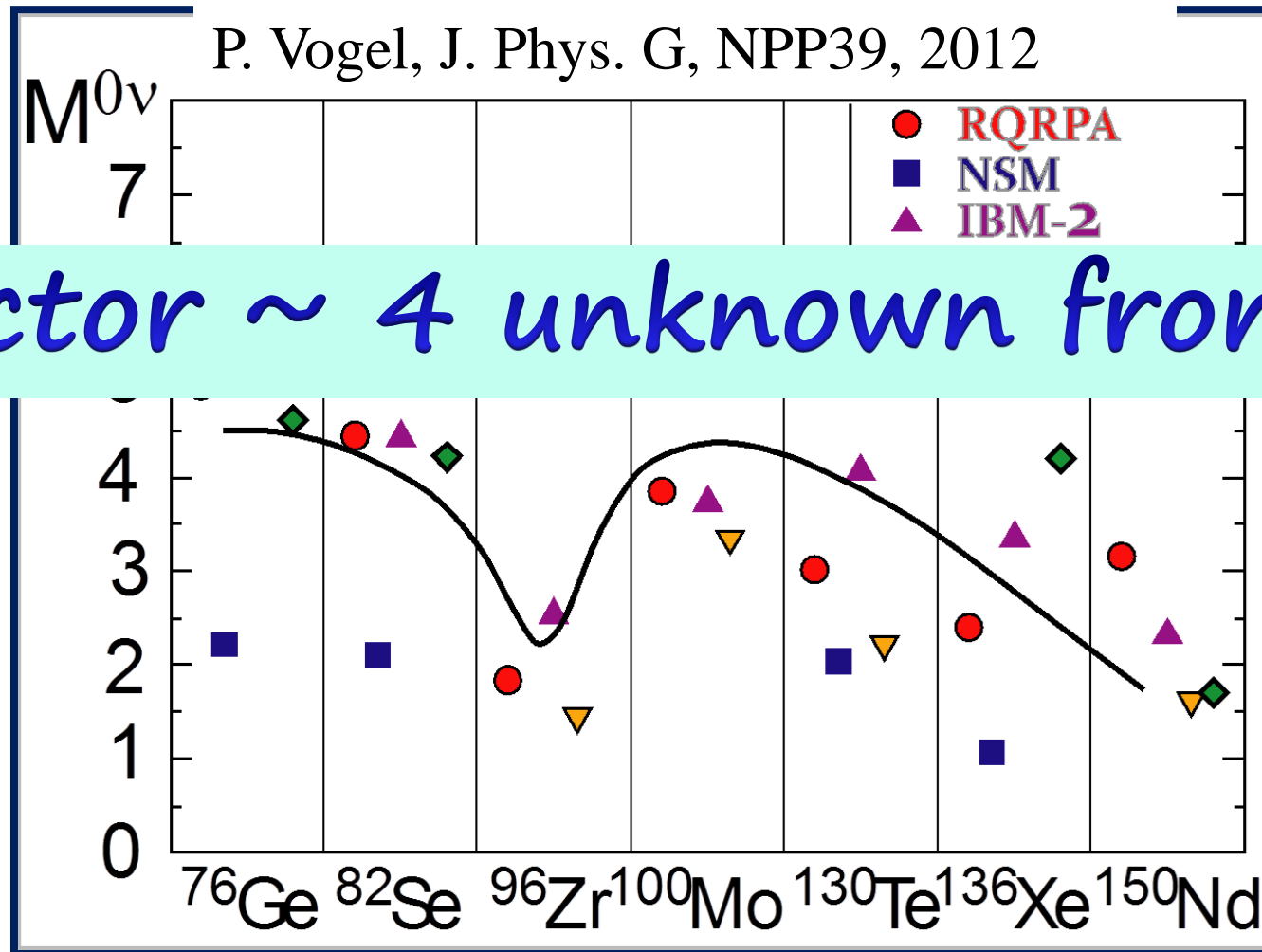
Majorana- $\nu$  !



$q_{tr} \sim 0.5 \text{ fm}^{-1} !!$

**NOT accessible thru charge-ex reaction**

# The situation of the Nuclear Matrix Elements for neutrinoless $\beta\beta$ decay



+ factor  $\sim 4$  unknown from  $g_A^4$ .

-2-

Charge-exchange reactions

GT-part

( $2\nu\beta\beta$  decay)

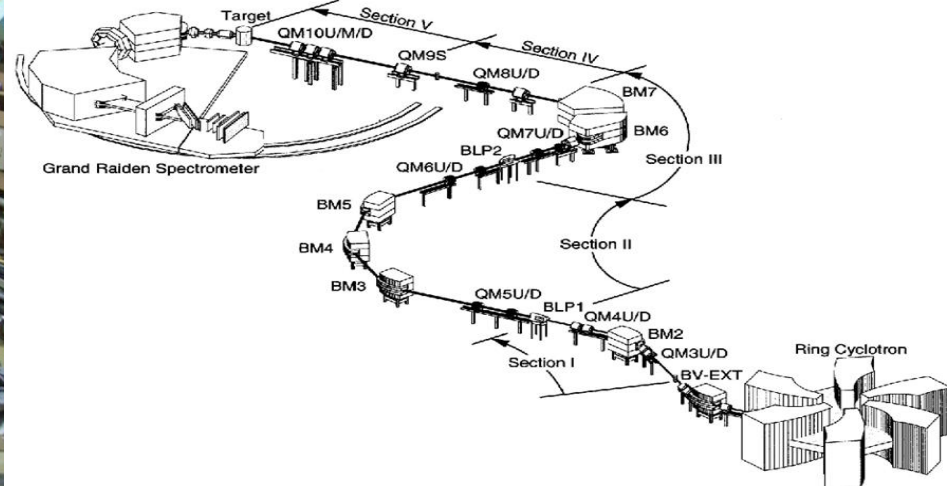


# Charge-exchange reactions

## Grand Raiden Magnetic Spectrometer



$\Delta E/E \sim 5 \times 10^{-5}$   $\sim 25$  keV  
at 420 MeV ( $^3\text{He}$ )

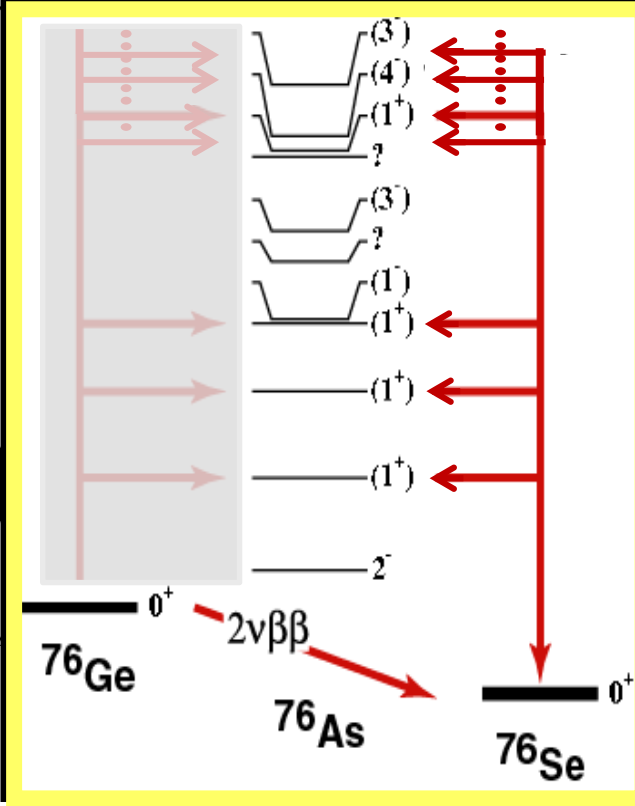
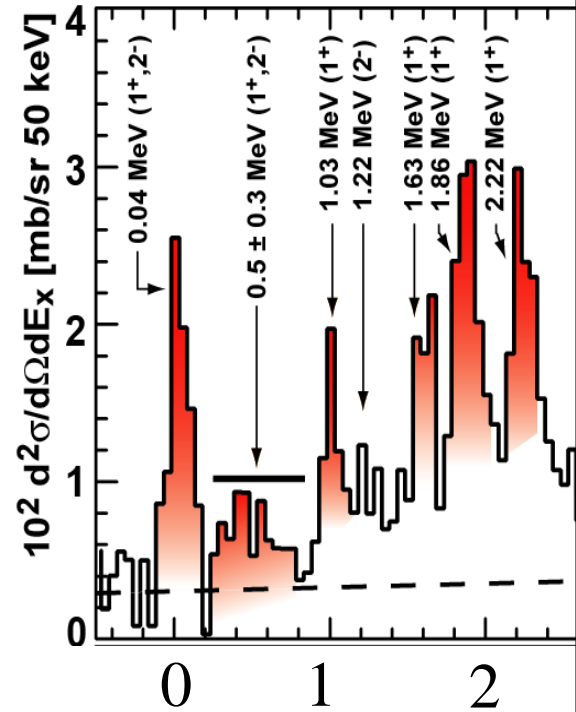


**$^{76}\text{Ge}$**

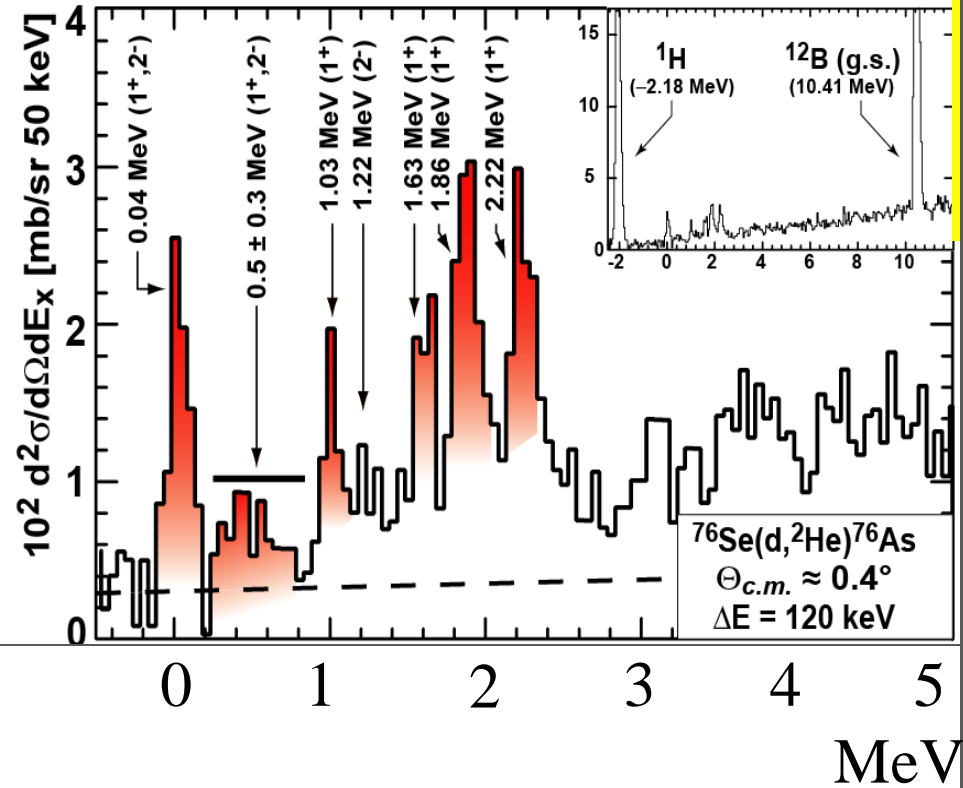
**$N-Z=10$**

**Resolution is the key !!!**





the other leg  
 ( $\text{BGT}^+$ ):  
 $^{76}\text{Se}(d, ^2\text{He})^{76}\text{As}$   
 ( $\Delta E = 120 \text{ keV}$ )

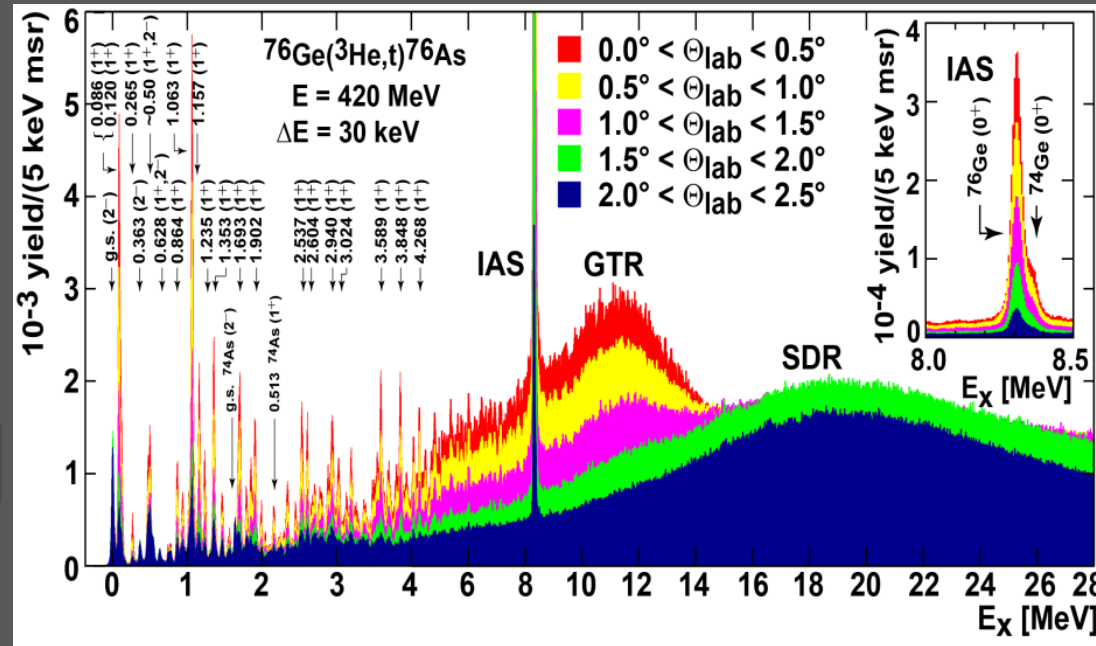


the other leg  
 (BGT<sup>+</sup>):  
 $^{76}\text{Se}(d, ^2\text{He})^{76}\text{As}$   
 ( $\Delta E = 120 \text{ keV}$ )

# a surprise:

low-E part of  
NME makes up  
~100% of total  
 $2\nu\beta\beta$ -ME

(given by  $T_{1/2}^{2\nu\beta\beta} = 1.4 \times 10^{21}\text{yr}$ )



no need for

Giant resonance contribution

# Nuclear matrix elements and deformation

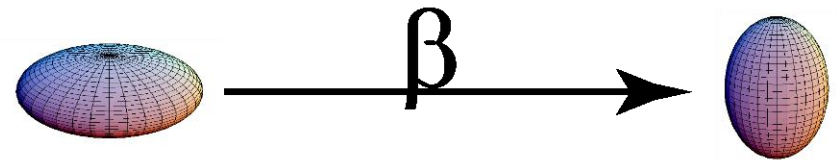
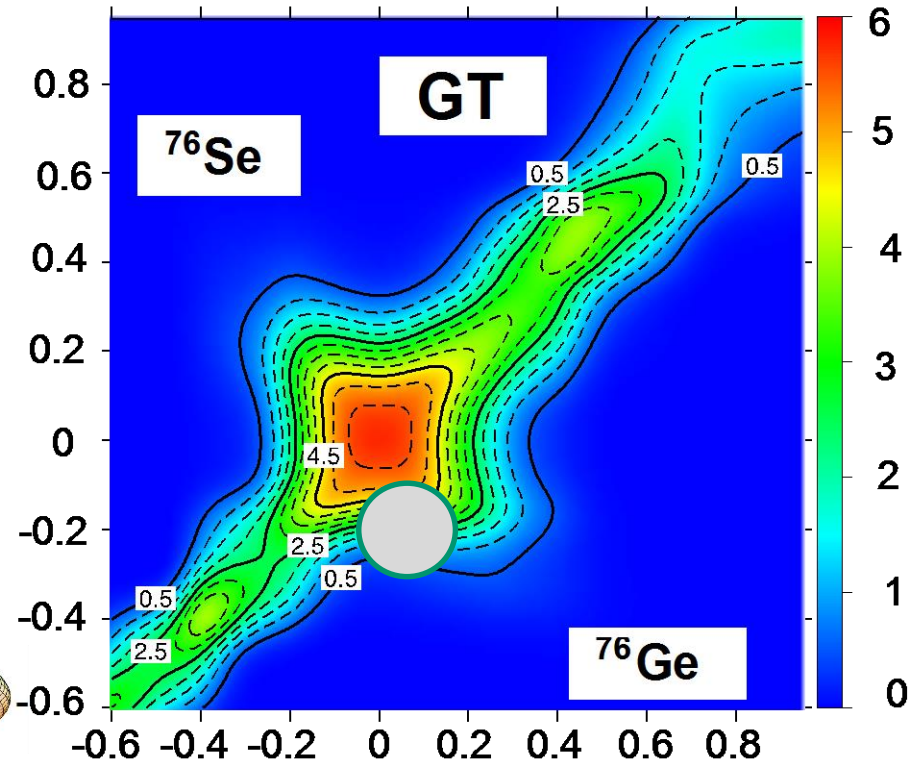
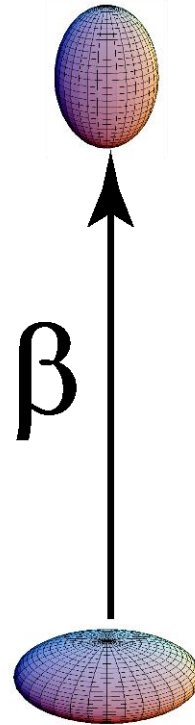
$${}^{76}\text{Ge}: \beta \sim +0.1$$

$${}^{76}\text{Se}: \beta \sim -0.2$$

reduction of the NME  
due to deformation is  
theoretically confirmed

**but**

expm'ly it seems to manifests  
itself (in  $2\nu\beta\beta$  decay) by a lack  
of correlation between the two  
different  $B(\text{GT})$  „legs“, rather  
than a reduction of individual  
strength



From: T. R. Rodriguez, et al, PRL105 (2010)

# 100 Mo

N-Z=16

$$T_{1/2}^{2\nu\beta\beta} = 6.9 \times 10^{18} \text{ yr}$$

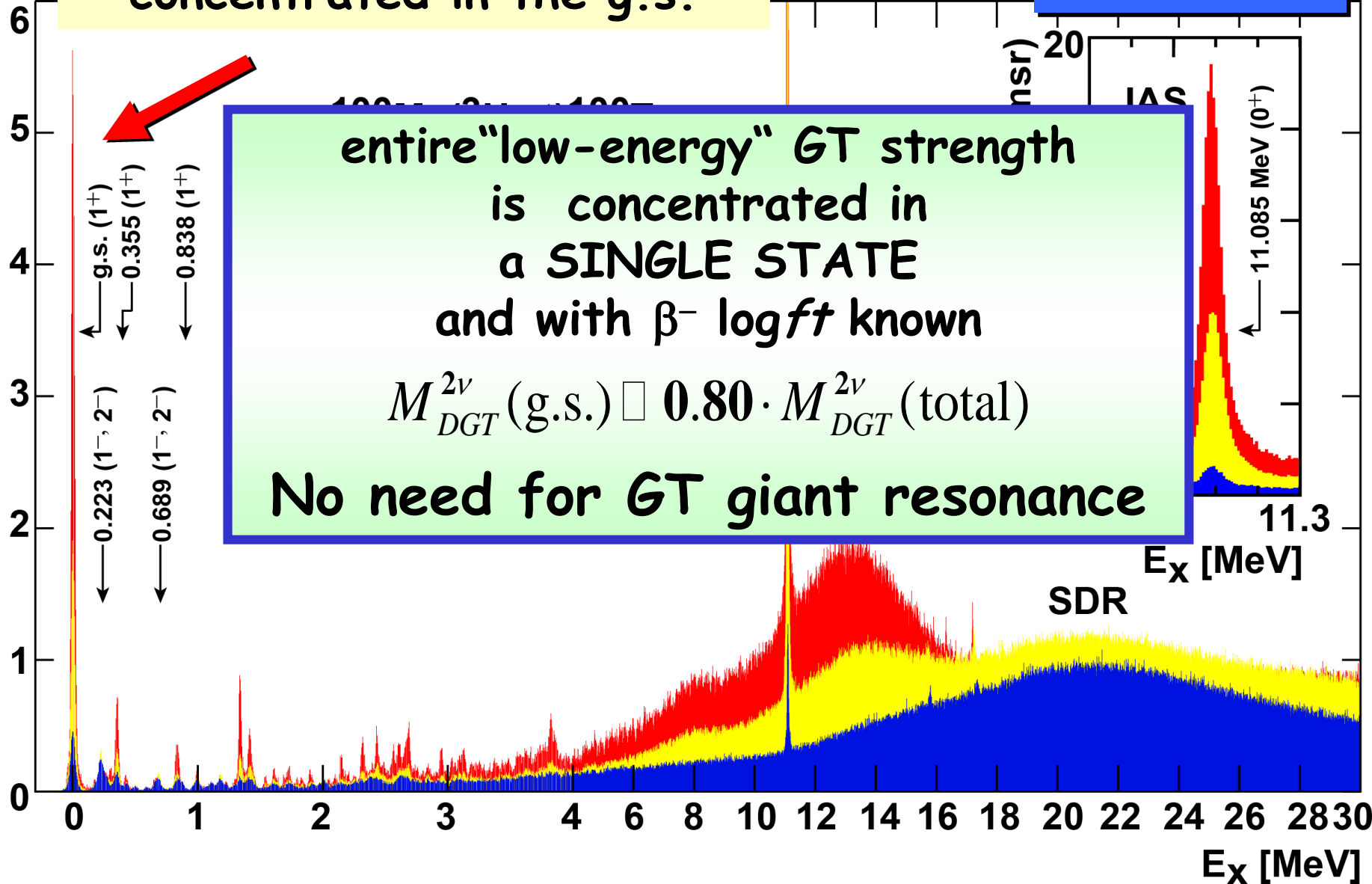
has the largest NME  
(NME ~ 0.24/MeV)



HERE: almost the entire low-E GT strength is concentrated in the g.s.

100Mo

10<sup>-3</sup> yield/(5 keV msr)



# $^{136}\text{Xe}$

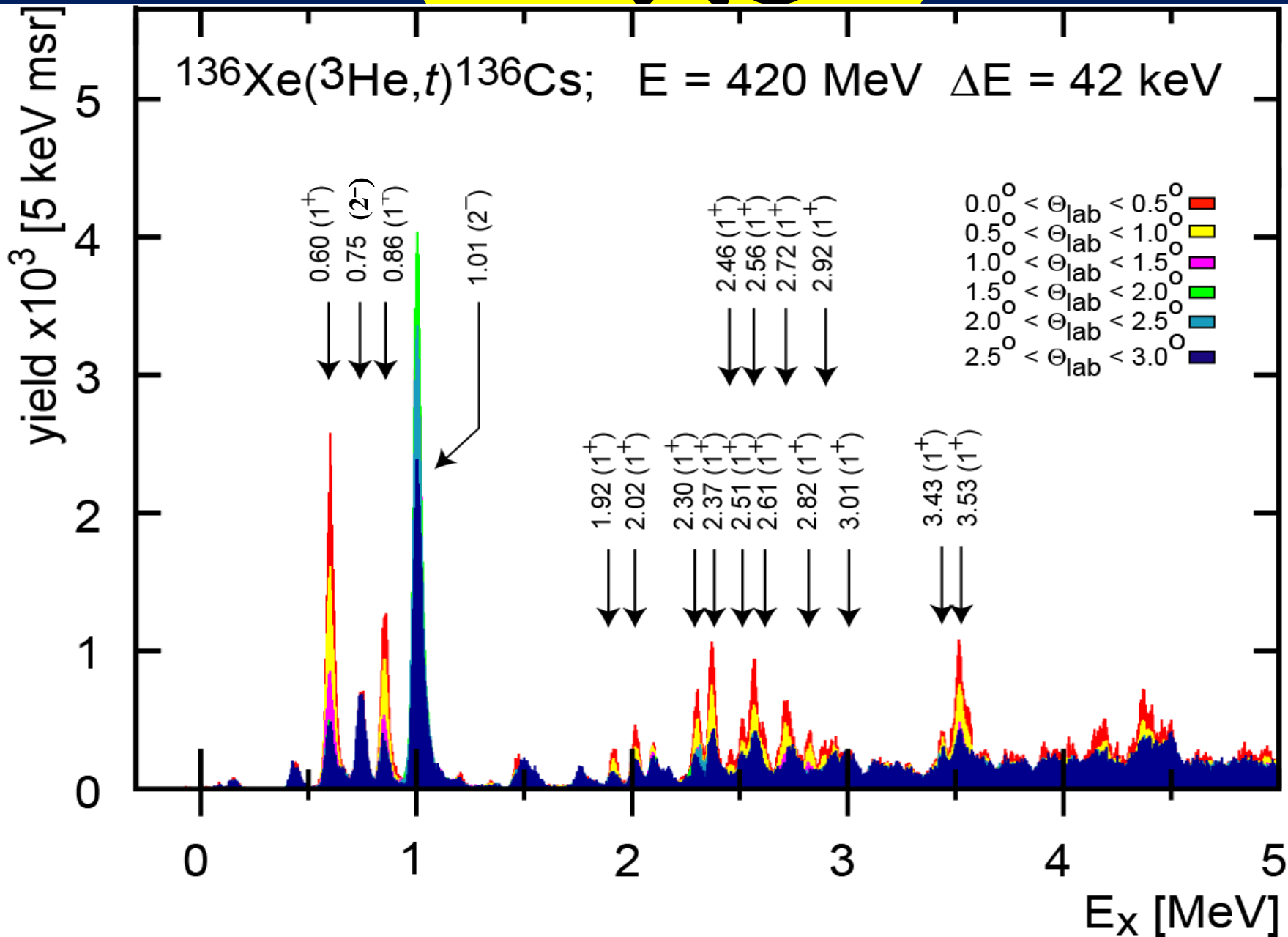
$$T_{1/2}^{2\nu\beta\beta} = 2.2 \times 10^{21} \text{ yr}$$

has the smallest NME

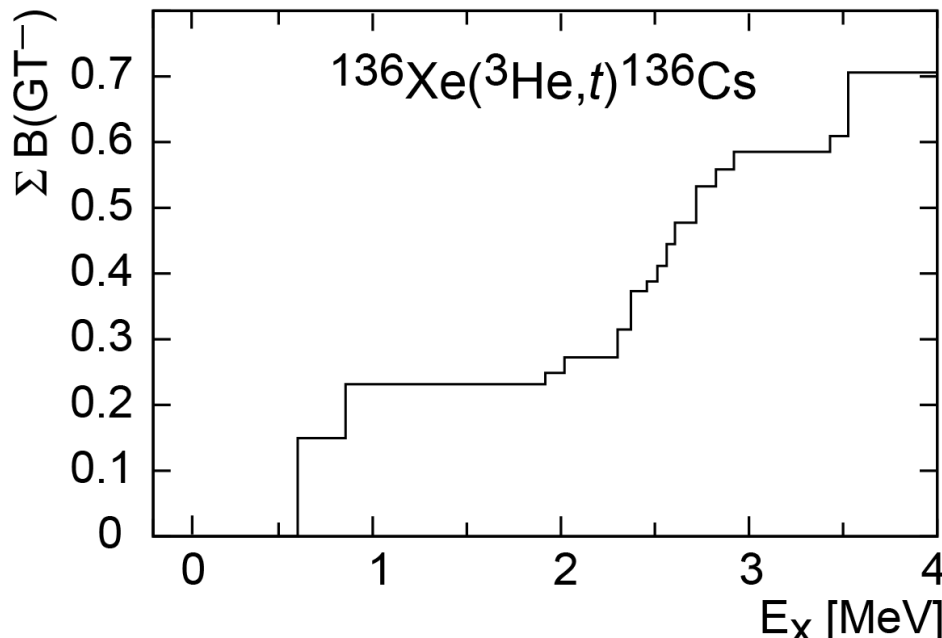
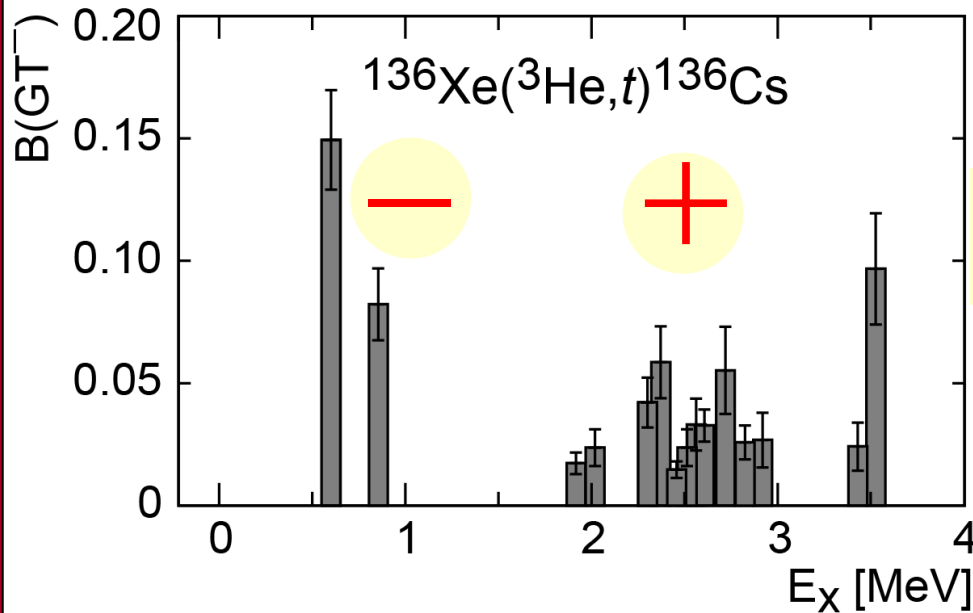
question: why so stable !!!

(lives 300 times longer than  $^{100}\text{Mo}$ )

# $^{136}\text{Xe}$



# What's the size of the NME?



$$T_{1/2}^{2n} = 2.2 \times 10^{21} \text{ yr}$$

$$M_{\text{DGT}}^{(2n)} \square 0.019 \text{ MeV}^{-1}$$

all signs positive  $\rightarrow$

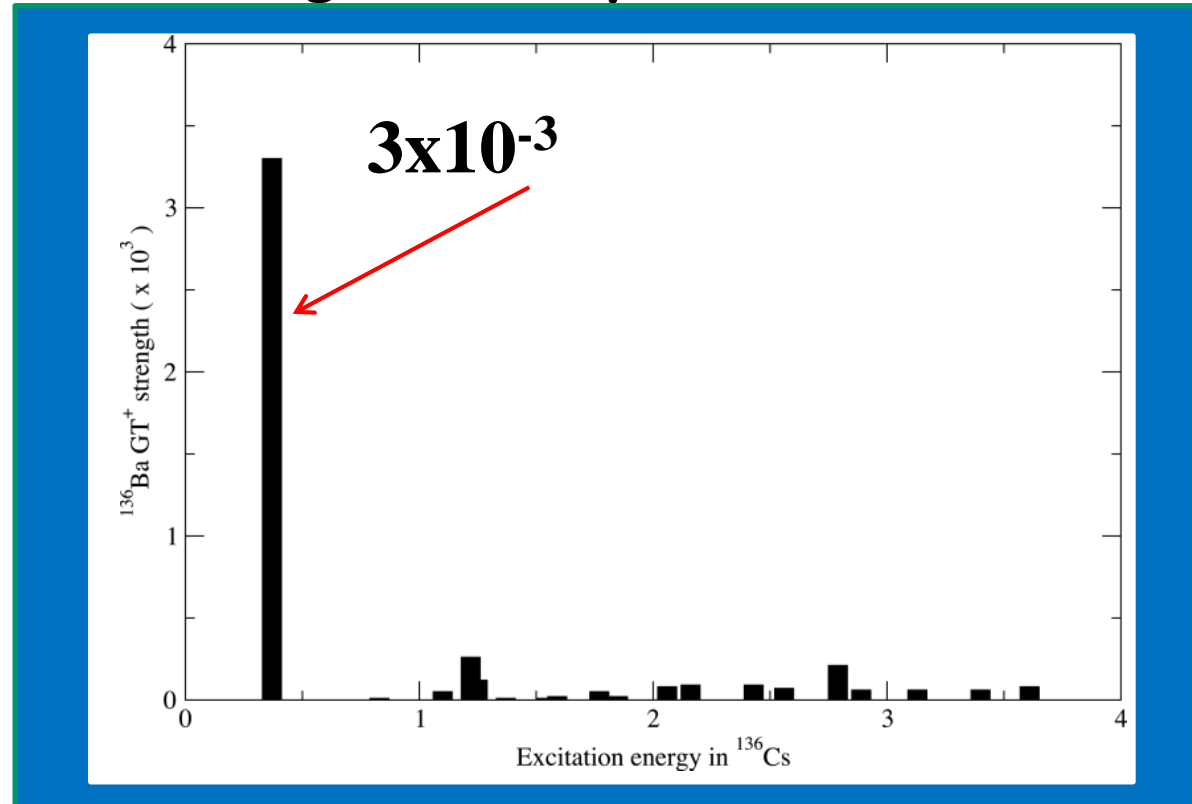
$$B_m(GT^+) \gg 10^{-2} \times B_m(GT^-)$$

$$B_m(GT^+) \gg 10^{-3} \text{ !!!!}$$

**A. Poves** (simultaneous to our publication):

there is no  $B(GT^+)$  strength, except for lowest  $1^+$  state

Recall:  
 $^{136}\text{Xe}$  is almost  
doubly magic!!



Shell model provides conclusive explanation for the deemed „pathologically“ long half-life of  $^{136}\text{Xe}$ .

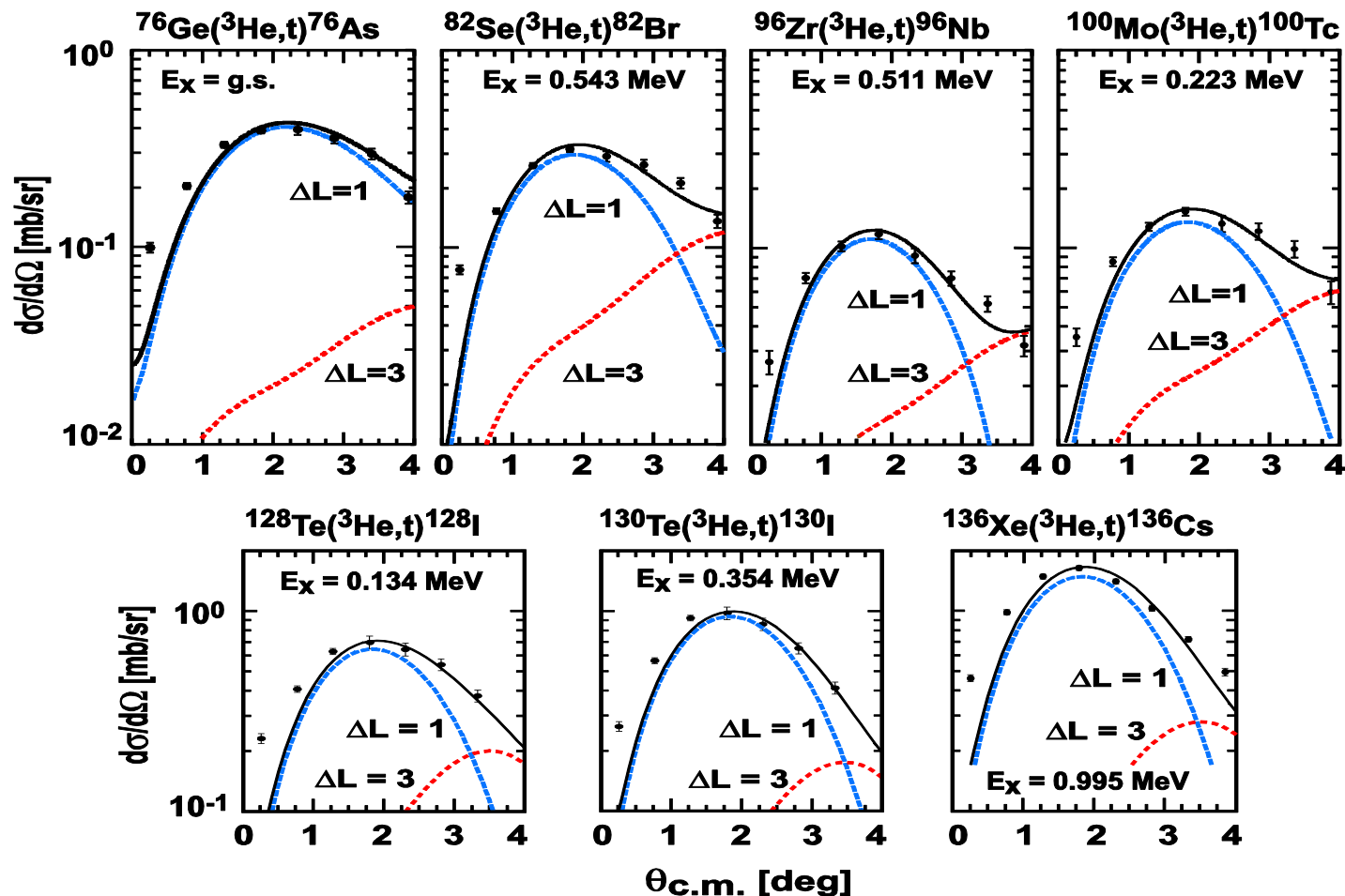
Expt'l test:  $^{136}\text{Ba}(d, ^2\text{He})^{136}\text{Cs}$

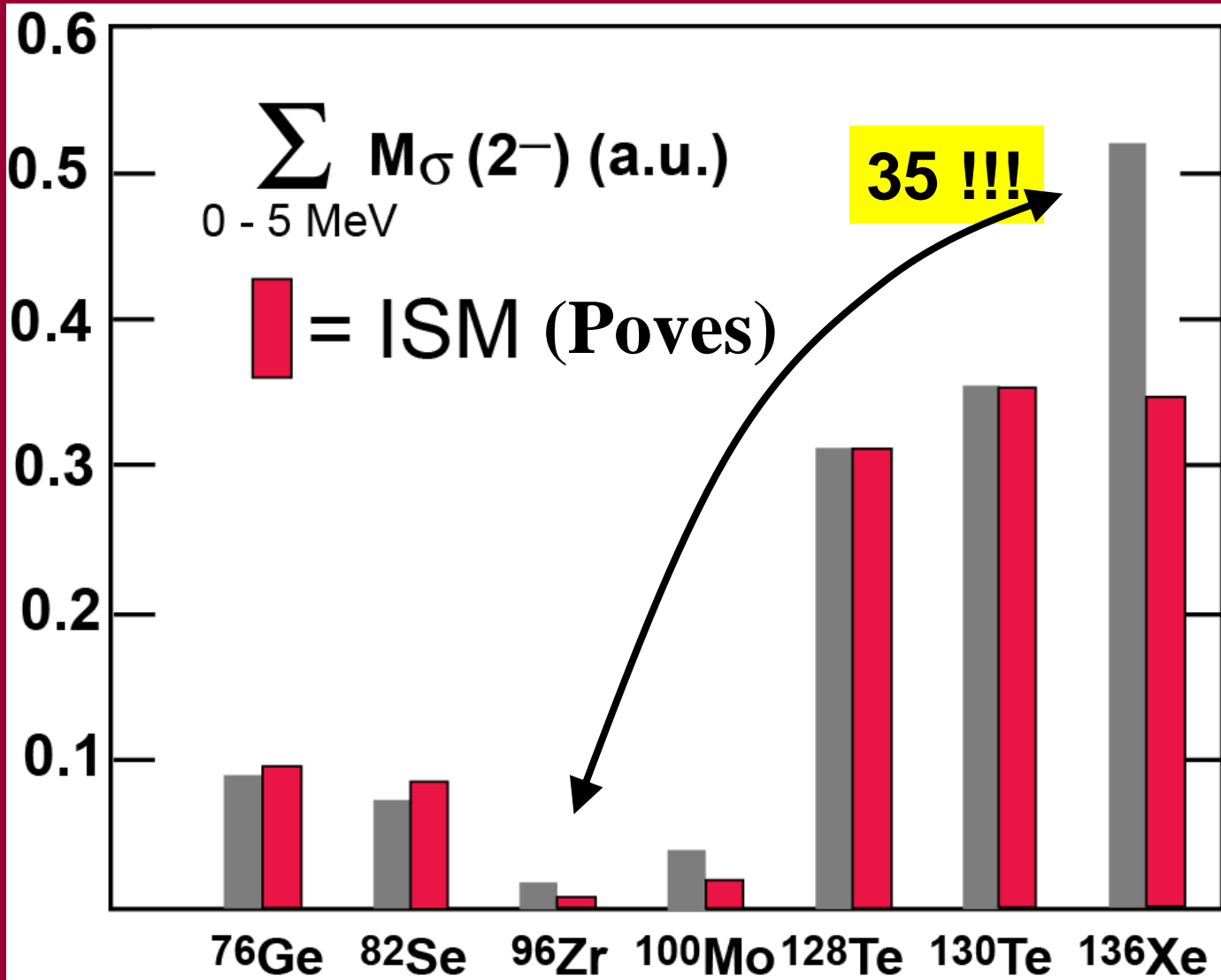
-3-

Charge-exchange reactions  
spin-dipole part  
( $0\nu\beta\beta$  decay)

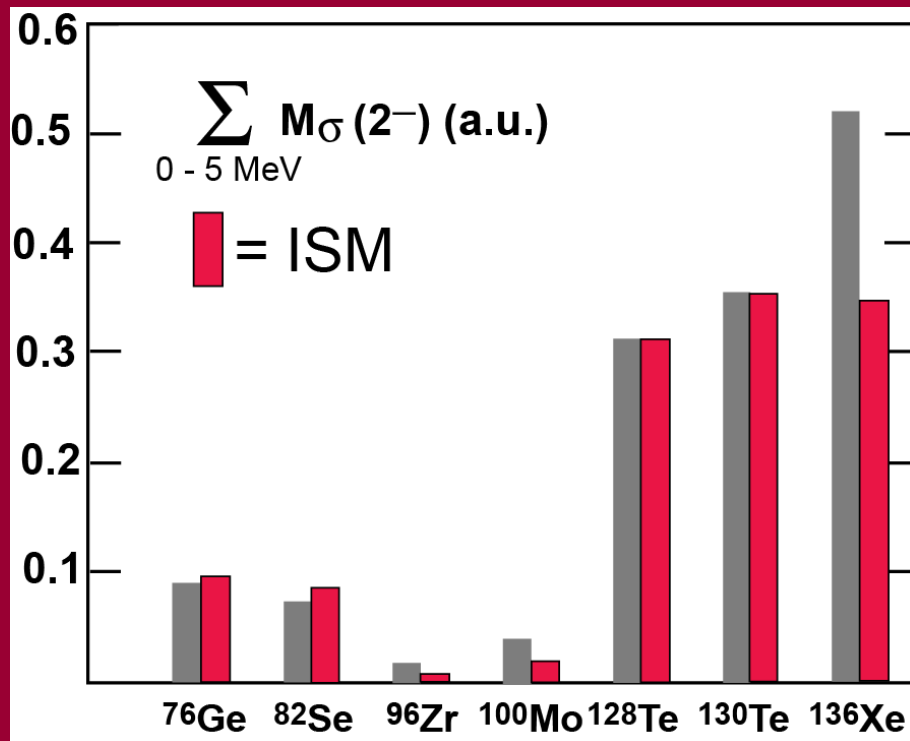
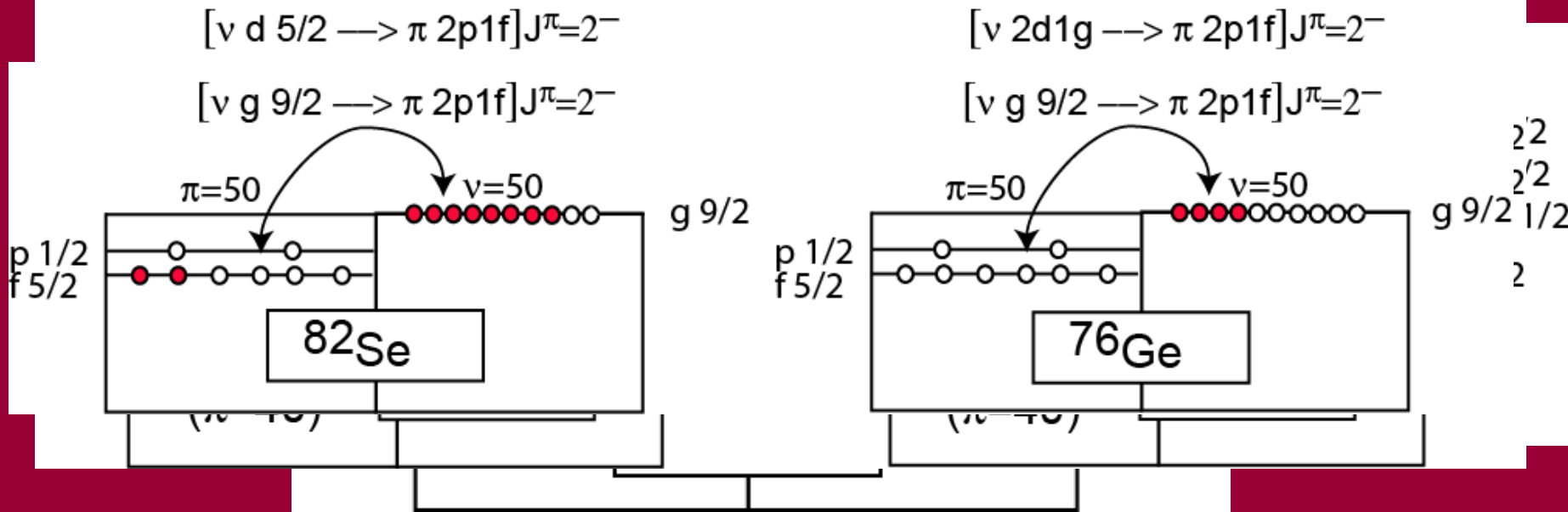
# Charge-exchange reaction towards the $0\nu\beta\beta$ NME's

Here:  $2^-$  states via chargex reactions



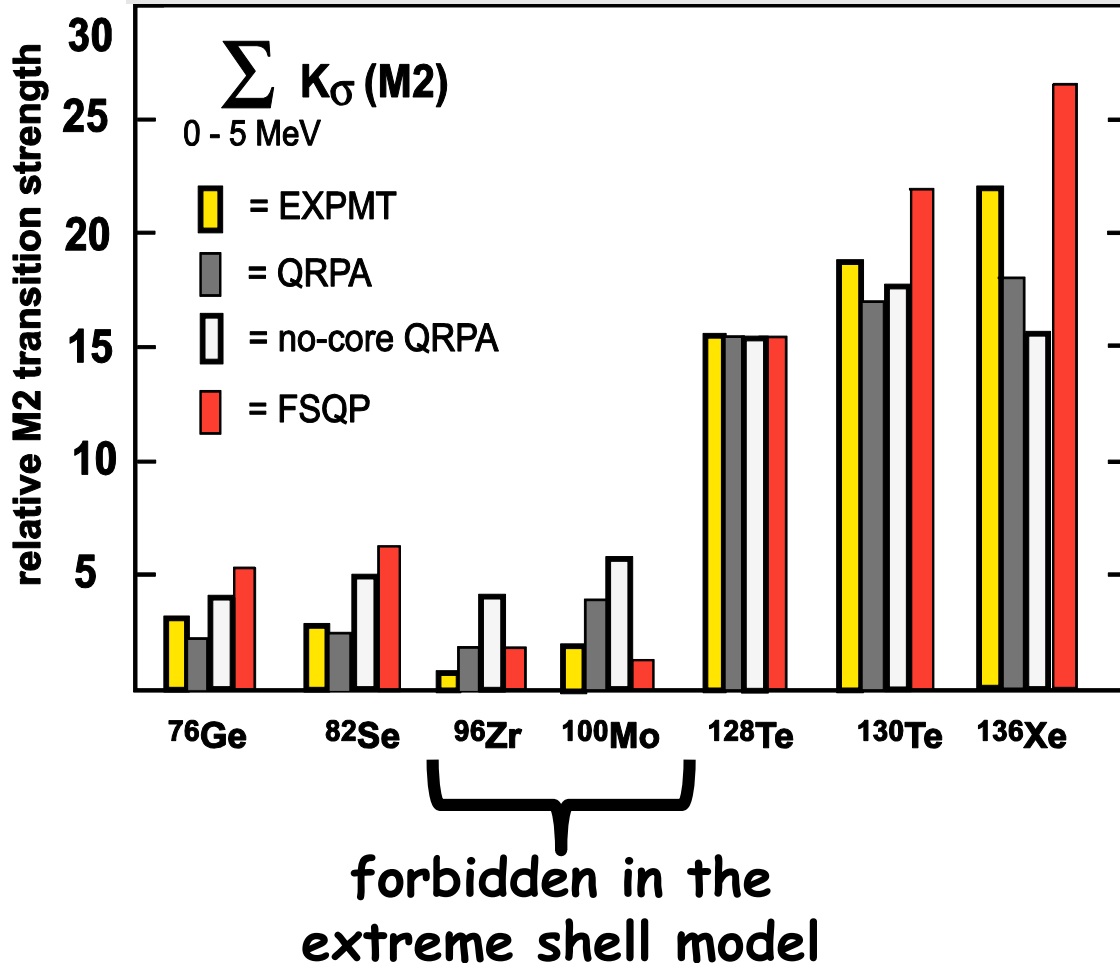






# Low-energy spin-dipole (2-) strength to test nuclear wave function for $0\nu\beta\beta$ decay NME's

$$\left. \frac{d\sigma^{\text{SD}}}{d\Omega} \right|_{q_{\text{max}}} = \left[ \frac{\mu}{\pi\hbar^2} \right]^2 \frac{k_f}{k_i} N_D^{\sigma\tau} \left| \frac{J_{\sigma\tau}^{q_{\text{max}}}}{r_0 A^{1/3}} \right|^2 K_{\sigma}(M2).$$



## MODELS

### QRPA

→ reasonable description

### no-core QRPA

→ washes out shell structures

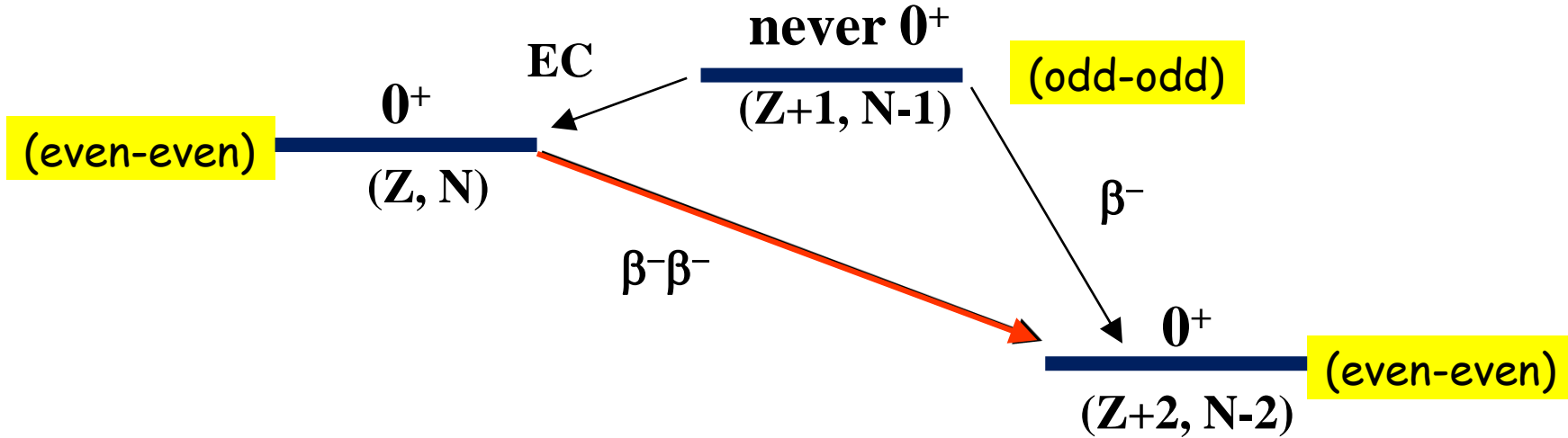
### FSQP

→ semi-microscopic model,  
excellent description of data

-4-

Mass measurements  
and  $0\nu\beta\beta$  NMEs  
 $^{96}\text{Zr}$

# $\beta\text{-}\beta\text{-}$ decay



1.  $^{48}\text{Ca}$

2.  $^{150}\text{Nd}$

3.  $^{96}\text{Zr}$

1.  $^{100}\text{Mo}$

2.  $^{82}\text{Se}$

3.  $^{116}\text{Cd}$

7.  $^{130}\text{Te}$

8.  $^{136}\text{Xe}$

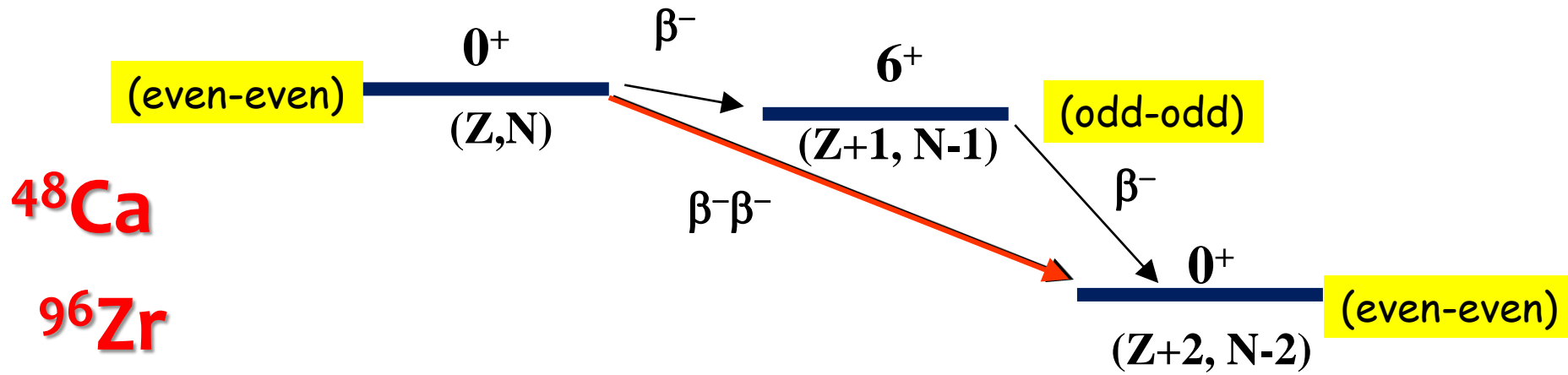
9.  $^{124}\text{Sn}$

10.  $^{76}\text{Ge}$

11.  $^{110}\text{Pd}$

The  $\beta\text{-}\beta\text{-}$  decay candidates with highest Q-value

# $\beta\text{-}\beta\text{-}$ decay



1.  $^{48}\text{Ca}$

2.  $^{150}\text{Nd}$

3.  $^{96}\text{Zr}$

1.  $^{100}\text{Mo}$

2.  $^{82}\text{Se}$

3.  $^{116}\text{Cd}$

7.  $^{130}\text{Te}$

8.  $^{136}\text{Xe}$

9.  $^{124}\text{Sn}$

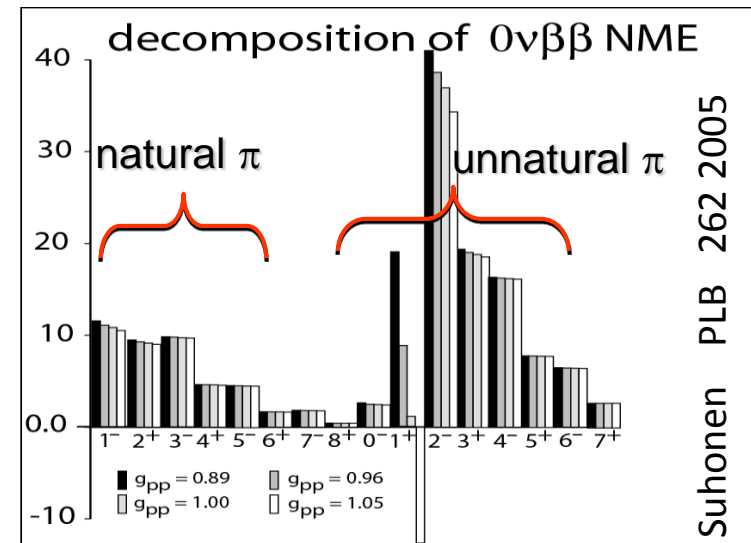
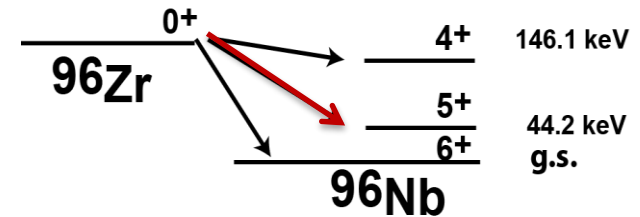
10.  $^{76}\text{Ge}$

11.  $^{110}\text{Pd}$

The  $\beta\text{-}\beta\text{-}$  decay candidates with highest Q-value

# Idea

- measure **Q-value** for  $^{96}\text{Zr} \rightarrow ^{96}\text{Nb}$  **single  $\beta$ -decay** by precision mass measurement
- measure the **single  $\beta$ -decay** rate
- $\rightarrow$  ft-value
- determine the  $^{96}\text{Zr}$  **4-fold forbidden  $\beta$ -decay NME** and confront with theory
- confront with same theories aimed at calculating  **$0\nu\beta\beta$ -decay NME for the same nucleus!!**



# Competition between $\beta$ & $\beta\beta$ decay of $^{96}\text{Zr}$

two conflicting half-lives:

NEMO-3:  $T_{1/2}^{2\nu\beta\beta} = (2.3 \pm 0.2) \times 10^{19} \text{ y}$   
 geo-chem:  $T_{1/2}^{\beta} = (0.94 \pm 0.32) \times 10^{19} \text{ y}$  1

can this difference be reconciled ?  
 yes, if single  $\beta$  competes with  $\beta\beta$  decay

$$(T_{1/2})^{-1} = (T_{1/2}^{2\nu\beta\beta})^{-1} + (T_{1/2}^{\beta})^{-1}$$

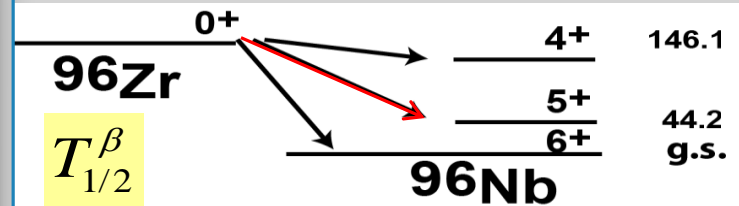
expected  $T_{1/2}^{\beta} = (1.6 \pm 0.9) \times 10^{19} \text{ y}$

experiment  $T_{1/2}^{\beta} > 2.6 \times 10^{19} \text{ y}$  2

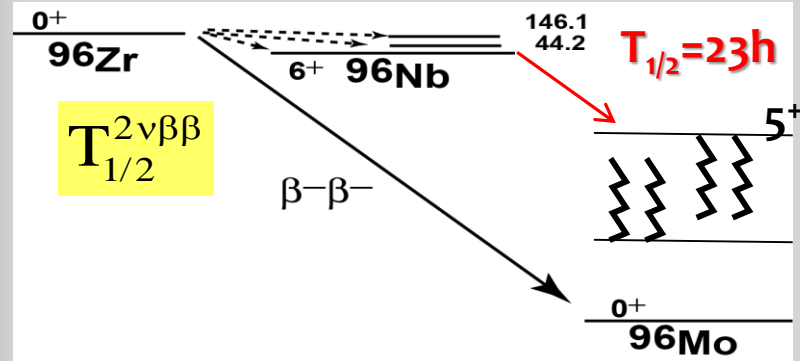
pred. (QRPA)  $T_{1/2}^{\beta} = 24 \times 10^{19} \text{ y}$  3

**BUT**

$$(T_{1/2}^{\beta})^{-1} \propto 0(Q^{13}) g_A^2 \langle M_{\beta}^{4u} \rangle^2$$

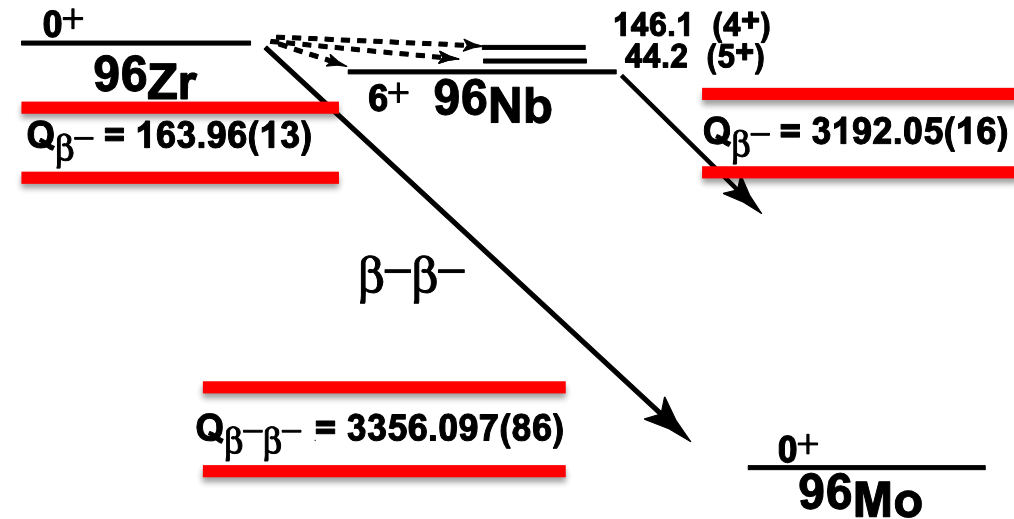


$0^+ \rightarrow 6^+$  6-fold non-unique (unobservably long)  
 $0^+ \rightarrow 5^+$  4-fold unique (possible)  
 $0^+ \rightarrow 4^+$  4-fold non-unique (no phase space)



**Q-value**  $\rightarrow M_{\beta}^{4u} \rightarrow (T_{1/2}^{0\nu\beta\beta})^{-1} \propto Q^5 |M_{\beta\beta}^{0\nu}|^2 \langle m_{\beta\beta} \rangle^2$

# Results

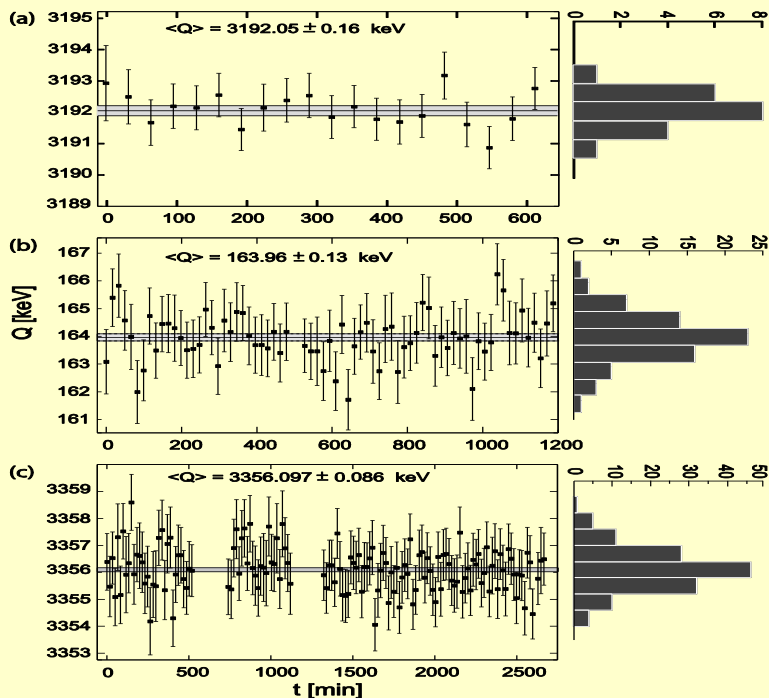


## 96Zr

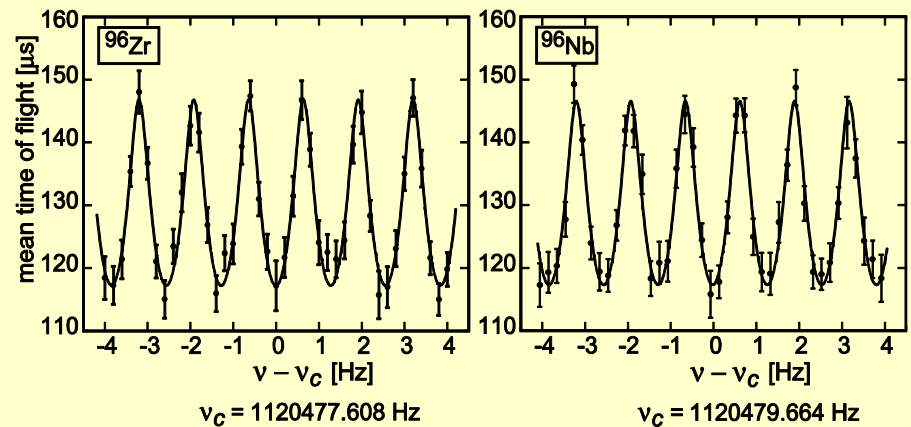
$$Q_{\beta\beta} = 3356.097 \pm 0.086 \text{ keV}$$

7.1 keV higher than AME2012

$$Q_{\beta} = 163.96 \pm 0.13 \text{ keV} !!$$

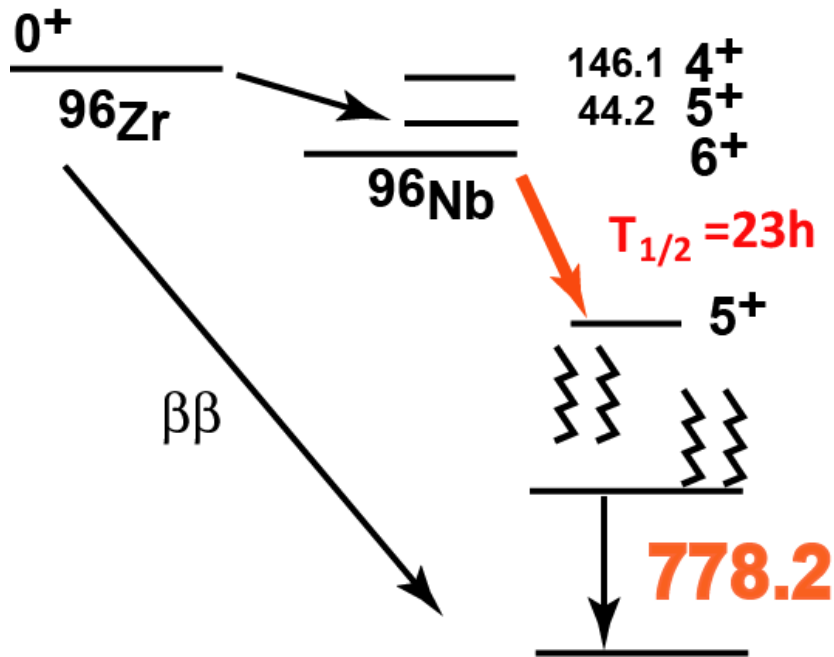


## Ramsey excitation





# Next: need $T_{1/2}$ of single $\beta$ decay



$$T_{1/2}(\text{QRPA}) = \frac{24}{g_A^2} \times 10^{19} \text{ yr}$$

$$T_{1/2}(\text{SM}) = \frac{11}{g_A^2} \times 10^{19} \text{ yr}$$

$$T_{1/2}(\text{exp}) > 2.3 \times 10^{19} \text{ yr}$$

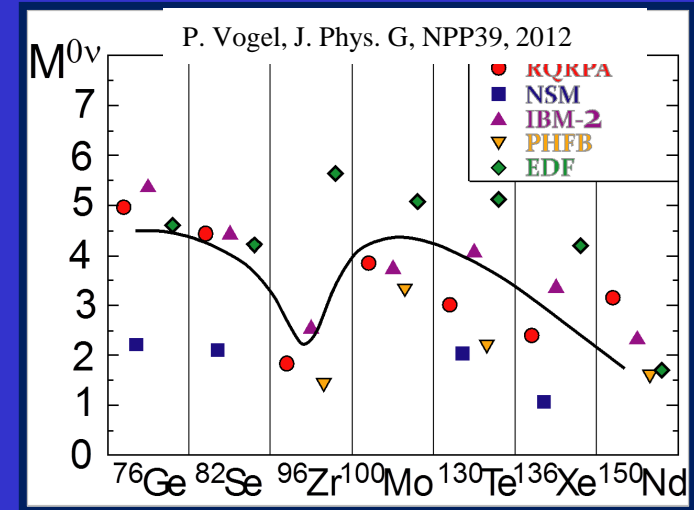
## Important side effect:

single  $\beta$  decay depends on  $g_A^2$   
 $2\nu/0\nu\beta\beta$  decay depends on  $g_A^4$

A measurements of single  $\beta$  decay gives expmtl handle on the quenching of  $g_A$

# $0\nu\beta\beta$ NMEs

1. what about getting the  $0\nu\beta\beta$  NMEs ?
2. what about how to test the models ?
3. what are the expmtl tools?



investigate „higher-order forbidden“ matrix elements of  $\sigma\tau$ - type  
( $0^+ \rightarrow 2^-$ ;  $0^+ \rightarrow 3^+$ ; ..... )  $\rightarrow$  limited possibilities, not promising

are there nuclei, where  $\beta$  and  $\beta\beta$  decay are in competition ?

$\rightarrow$  YES!  $^{48}\text{Ca}$  and  $^{96}\text{Zr}$

can muon-nuclear physics help?

$\rightarrow$  YES  $\mu$ -capture a potentially powerful tool

# Muon capture and $0\nu\beta\beta$ NMEs

$^{56}\text{Fe}$

$^{32}\text{S}$

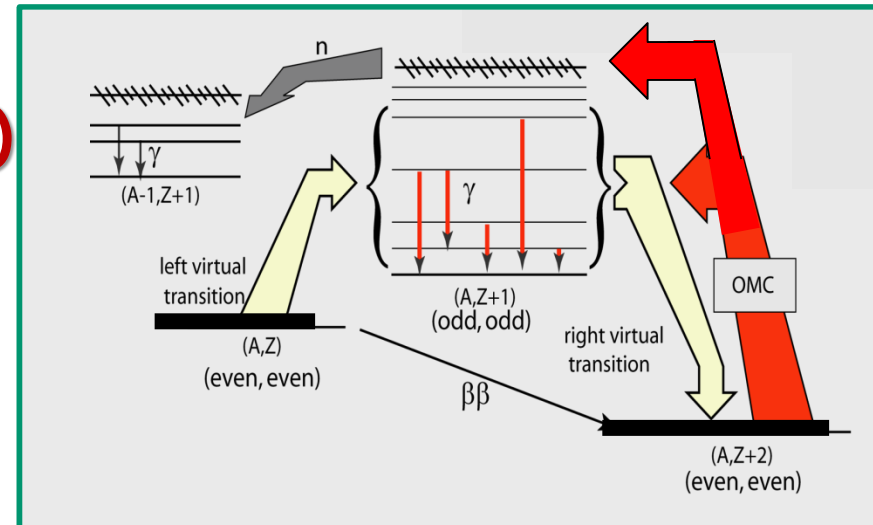
$^{24}\text{Mg}$

# Motivations

- $\mu$ -cap features momentum transfers similar to  $0\nu\beta\beta$  decay ( $q_{tr} \sim 0.5\text{fm}^{-1} \sim 100\text{MeV}/c$ )
- $\mu$ -cap processes to  $1^+$  states in  $A(\mu^-, \nu)B$  may be compared with charge-ex reactions of (n,p) type.
- $\mu$ -cap may give access to  $g_A$  quenching issue

## However

- only the On-channel ( $\sim 10\%$ ) is relevant for  $0\nu\beta\beta$  decay
- level scheme of final odd-odd nucleus is extremely poorly known



# The muon capture and $g_A$ in weak decays

## Title

Exclusive  $\mu$ -capture on  $^{24}\text{Mg}$ ,  $^{32}\text{S}$  and  $^{56}\text{Fe}$  populating low-lying  $1^+$  states to probe the weak axial current at high momentum transfer

M. Alanssari,<sup>1</sup> I. H. Hashim,<sup>2</sup> L. Jokiniemi,<sup>3</sup> H. Ejiri,<sup>4</sup>  
E. Ideguchi,<sup>4</sup> A. Sato,<sup>4</sup> J. Suhonen,<sup>3</sup> and D. Frekers<sup>1</sup>

<sup>1</sup>Institut für Kernphysik, Westfälische Wilhelms-Universität, D-48149 Münster, Germany

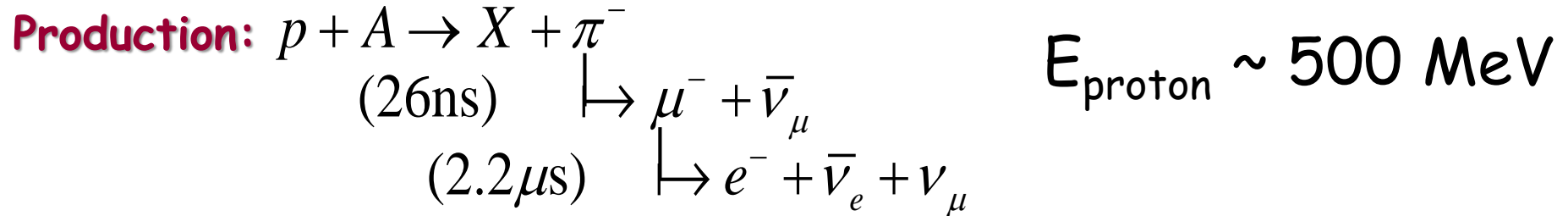
<sup>2</sup>Universiti Teknologi Malaysia, 81310 Johor Bahru, Malaysia

<sup>3</sup>University of Jyväskylä, Department of Physics, FI-40014, Finland

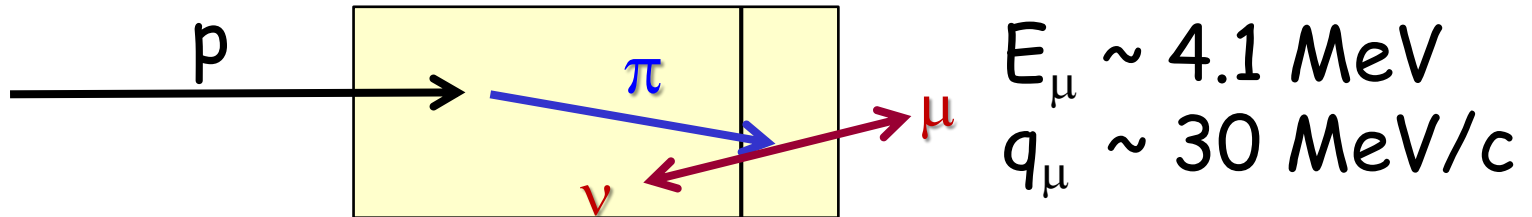
<sup>4</sup>Research Center for Nuclear Physics, Osaka University, Ibaraki, Osaka 567-0047, Japan

# The amazing muon

There has not been any other elementary particle so „successful“ in advancing our knowledge in so many different areas of physics.



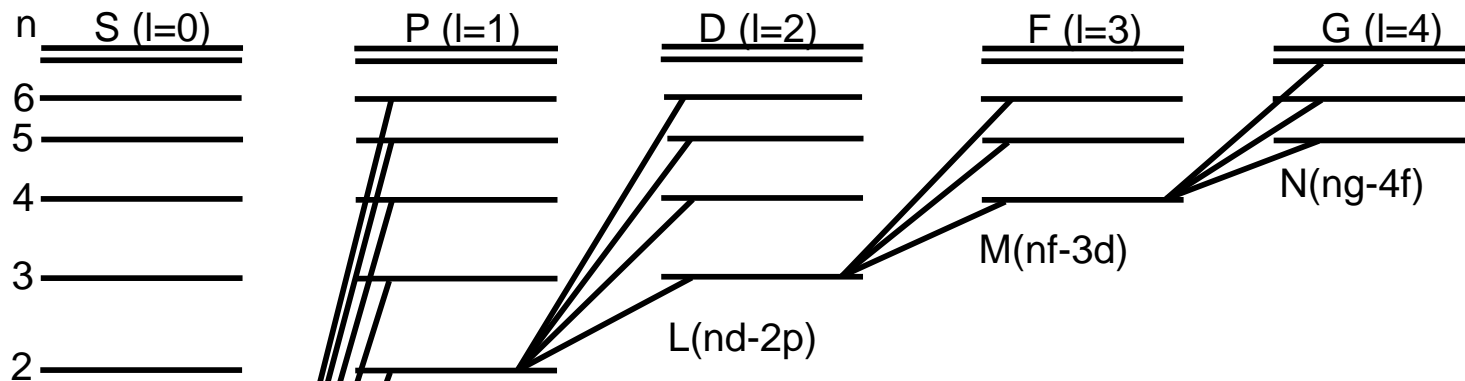
**Surface muons:** produced from stopped (usually negative) pions at end of target



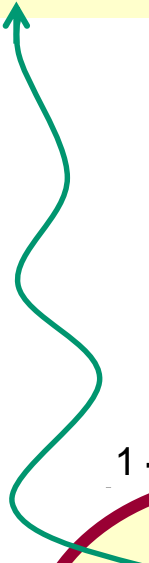
**Life-time:**  $\Gamma_\mu = \frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3} (1 - \varepsilon) = (2,196981(2) \mu\text{sec})^{-1} \quad (\varepsilon \approx 10^{-3})$

$G_F = 1,16637(2) \cdot 10^{-5} \text{ GeV}^{-2} \quad m_\mu = 105.6583745(24) \text{ MeV}$

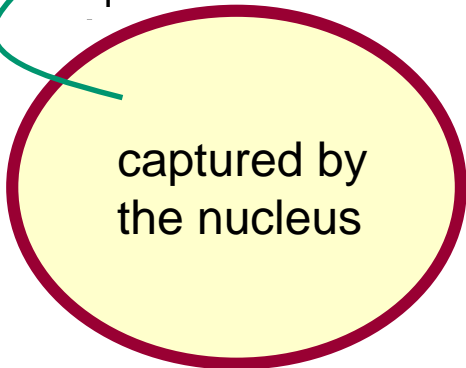
**Muonium:**  $\mu^+ + e^-$  an exotic hydrogen  $I \sim 13.6 \text{ eV}$



neutrino



K(np-1s)



captured by the nucleus

prompt Lyman  $\alpha$ -series of atomic  $\mu$ -capture followed by delayed nucl. capture

$$\lambda_{cap} = \lambda_{total} - Q\lambda_{decay}$$

Huff-factor

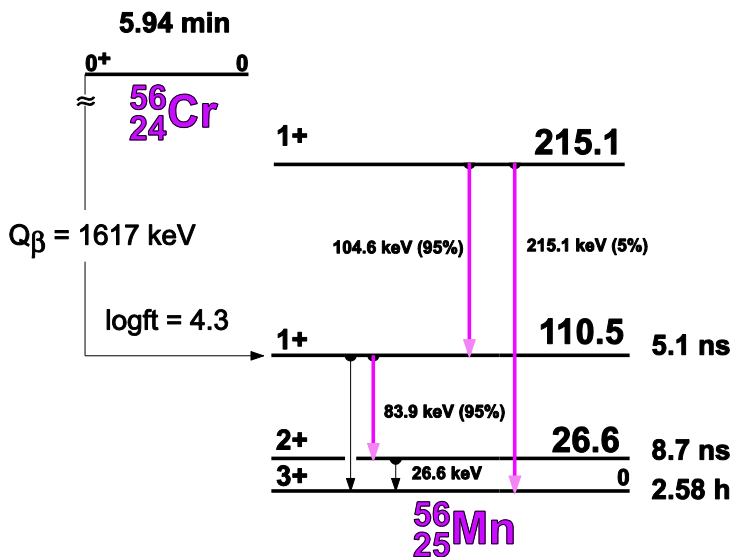
$$\lambda_{decay} = (2.2\mu s)^{-1} = 4.54 \cdot 10^5 s^{-1}$$

$$\lambda_{total} \sim (10 - 1000 ns)^{-1} \sim 10^8 - 10^6 s^{-1}$$

$$Q \sim 0.9 - 1.0$$

- the neutrino takes most of the energy
- $E_x(\text{nucl}) < 10 - 20 \text{ MeV}$

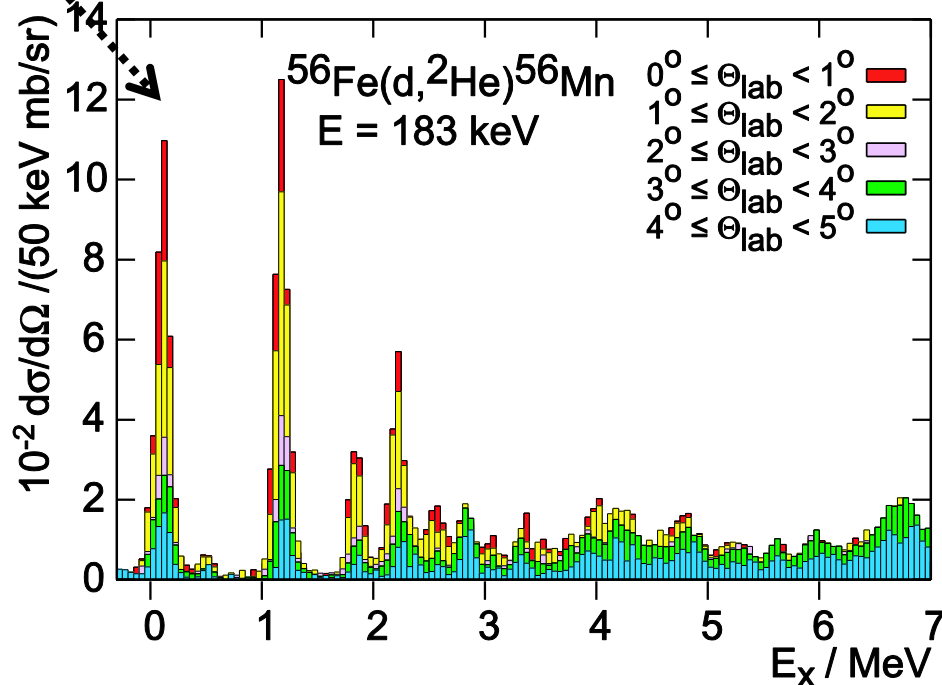
# The issue of $g_A$ queching



$^{56}\text{Fe}(\mu, \nu_\mu)^{56}\text{Mn}, (0 \text{ n})$

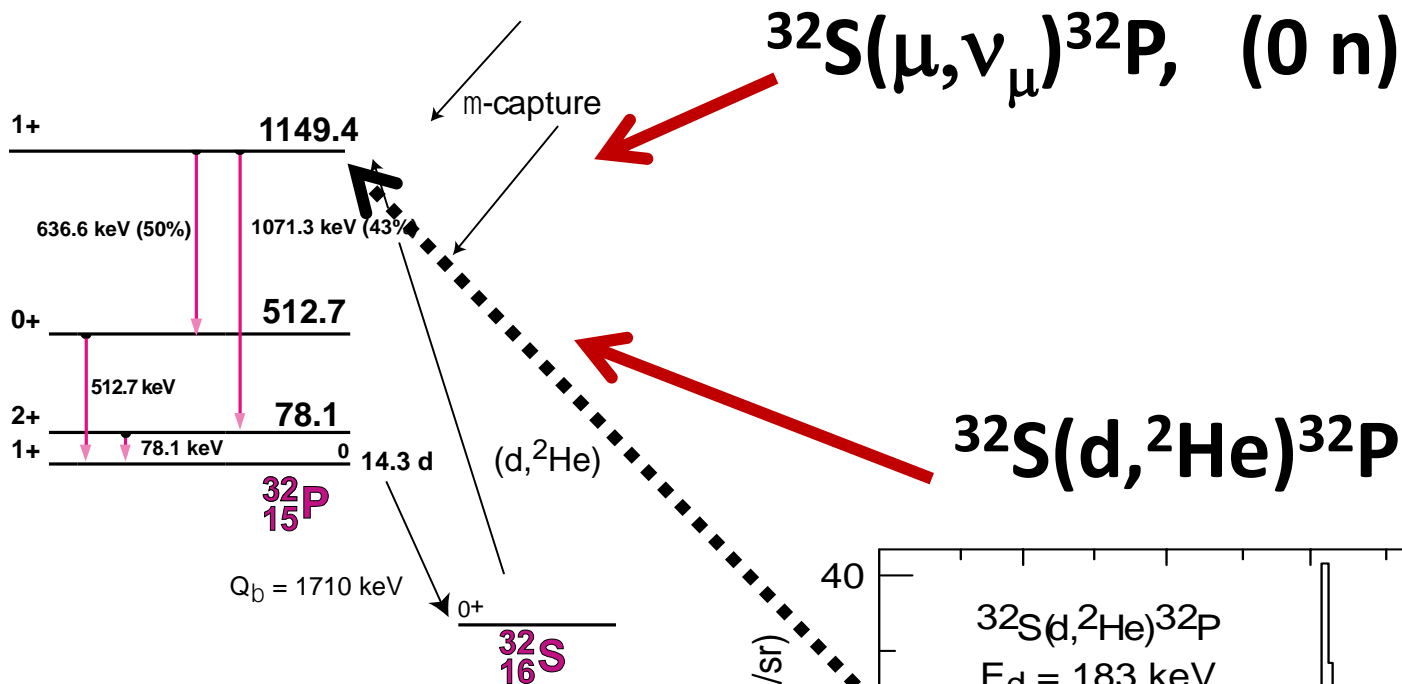
$^{56}\text{Fe}(d, ^2\text{He})^{56}\text{Mn}$

**Example:**  
 Compare transition strength in  $\mu$ -cap and  $(d, ^2\text{He})$  charge-ex

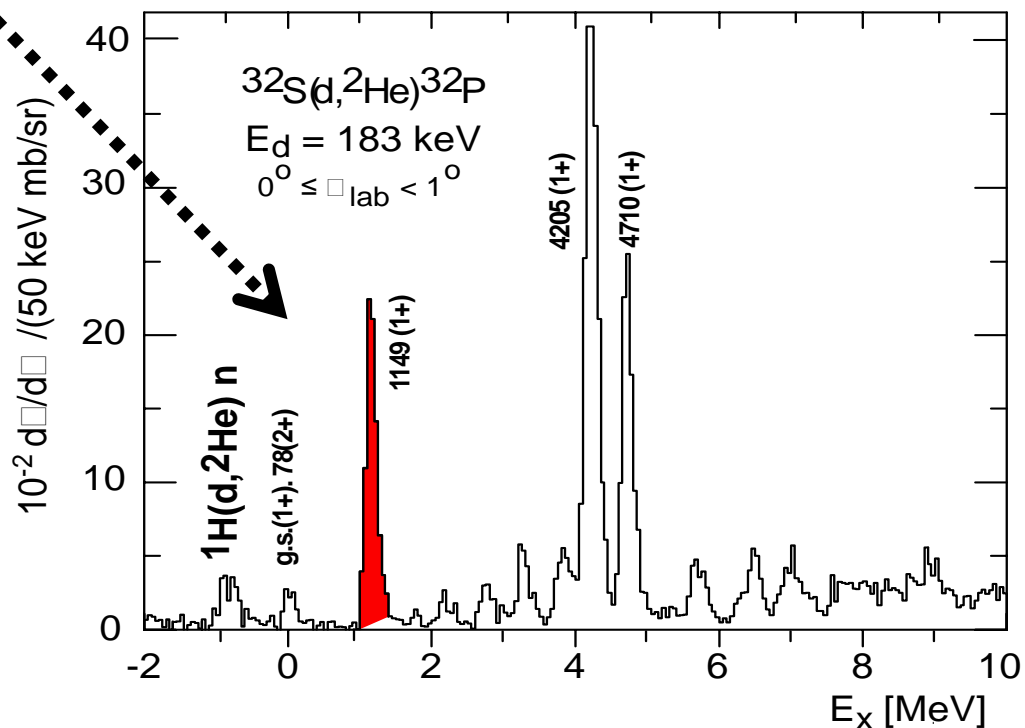




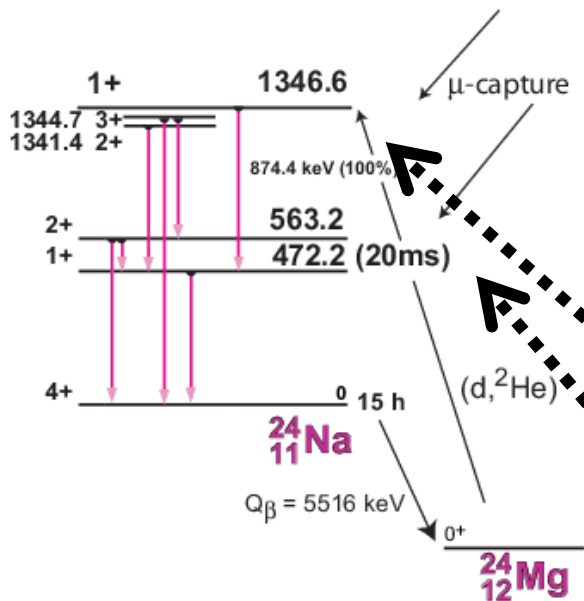
# The issue of $g_A$ queching



**Example:**  
 Compare transition strength in  $\mu$ -cap and  $(d, ^2\text{He})$  charge-ex



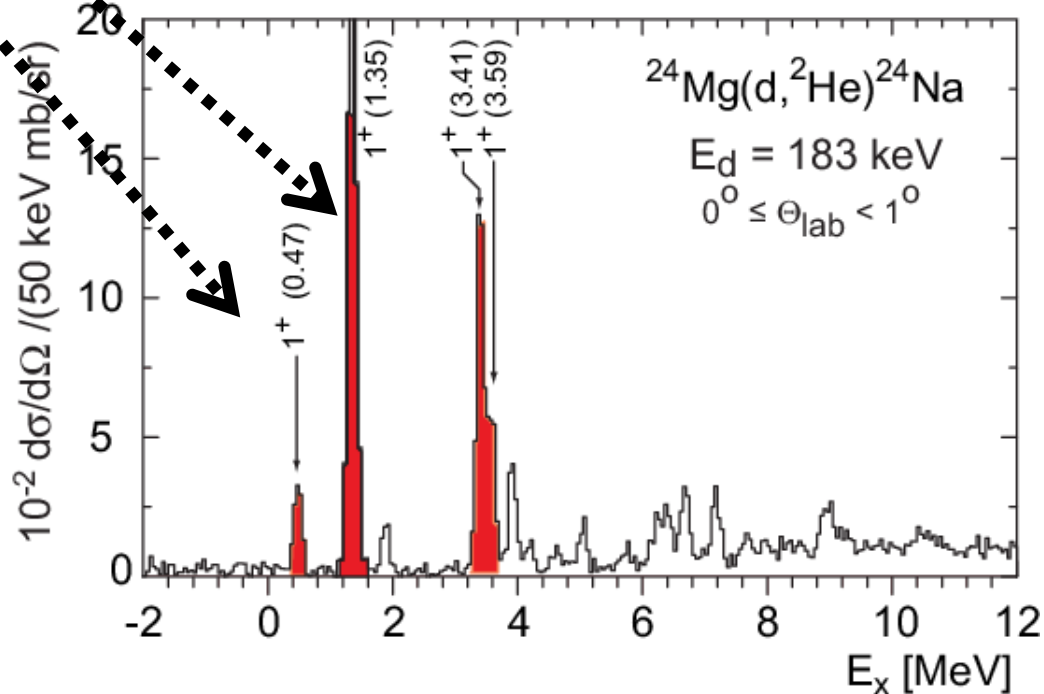
# The issue of $g_A$ queching



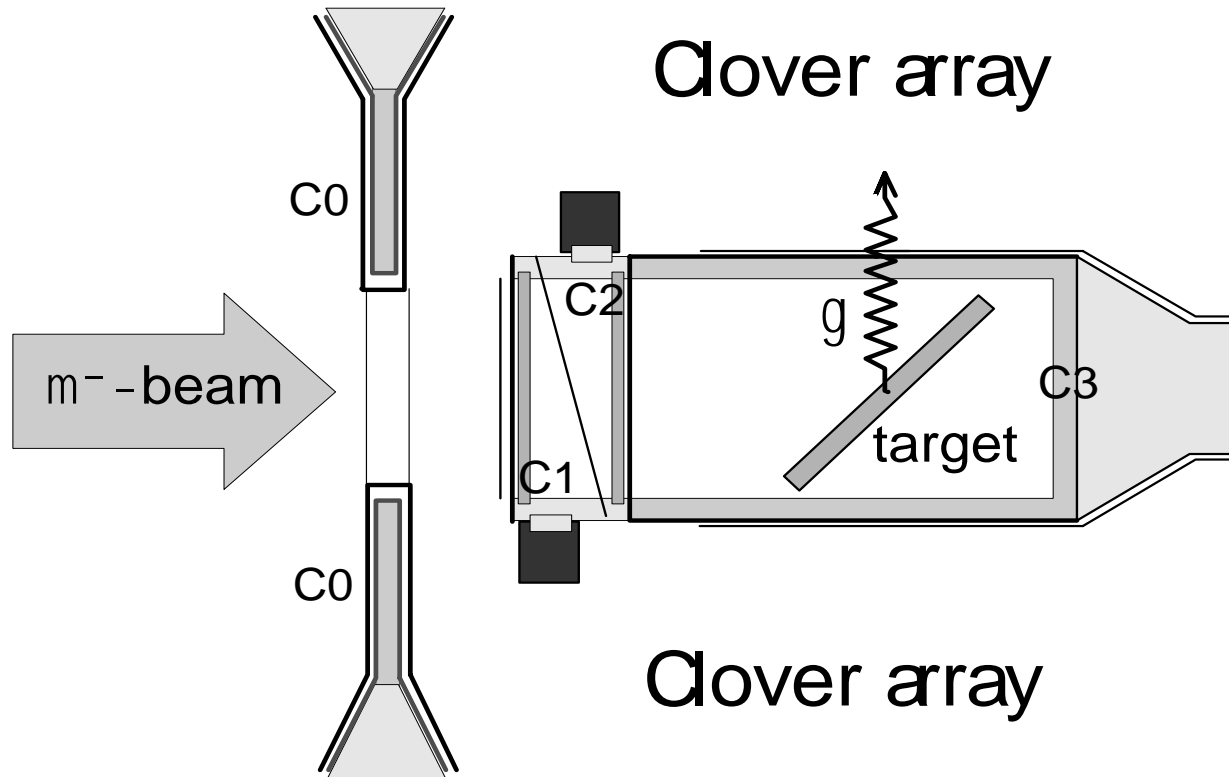
$^{24}\text{Mg}(\mu, \nu_\mu)^{24}\text{Na}$ , (0 n)

$^{24}\text{Mg}(d, ^2\text{He})^{24}\text{Na}$

**Example:**  
Compare transition strength in  $\mu$ -cap and  $(d, ^2\text{He})$  charge-ex



# Schematics of set-up

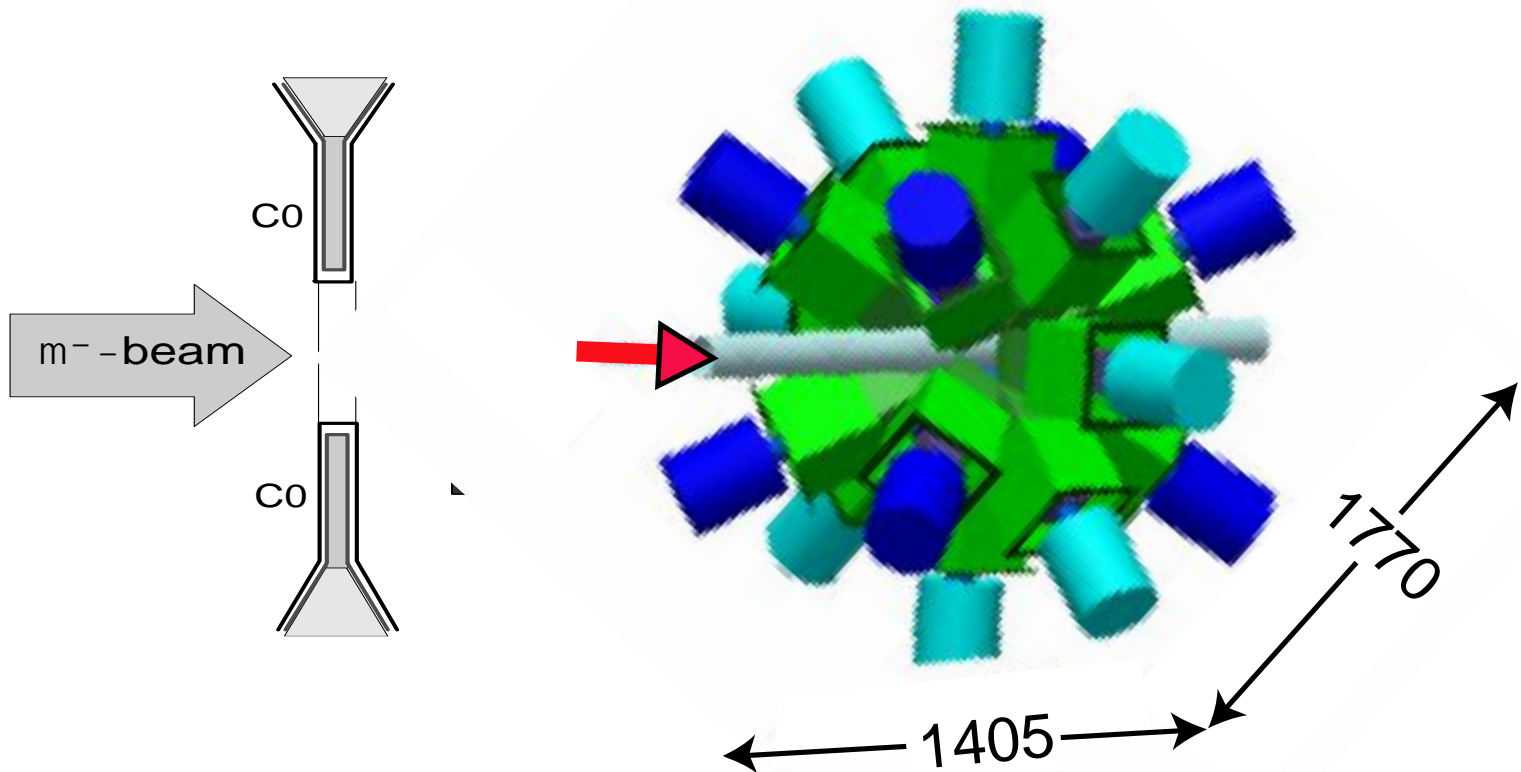


$$\mu_{stop} = \overline{C0} \wedge C1 \wedge C2 \wedge \overline{C3}$$

# of  $\mu$ -stop =  $8 - 25 \times 10^3$  with  $20 - 30$  MeV/c

target can be used for solids and gas

# Schematics of set-up muon beam line facility “MuSIC” + “CAGRA” at RCNP Osaka



CAGRA = Clover Array Gamma RAY spectrometer

MuSIC = MUon Science Innovative muon beam Channel

# The issue of $g_A$ quenching

**But: Hold your horses !!!  
things are a bit more complicated**

there is a pseudo-scalar coupling effective in  $\mu$ -capture with a constant  $g_p$  ( also badly known !!)

**What is this ????**

Inside the nucleus the muon can decay back into a virtual pion (lots of energy available!!), and the pion generates a final state imprinting it with the parity of the pion. ( $P(\pi) = -1$ )

**The effect depends on how many protons there are.**

**$^{24}\text{Mg} \rightarrow 12$  protons**

**$^{32}\text{S} \rightarrow 16$  protons**

**$^{56}\text{Fe} \rightarrow 26$  protons**

# Conclusion

- **Charge-ex reactions:**
  - useful tool for  $2\nu\beta\beta$  decay NME's.
- **Spin-dipole excitation via charge-ex:**
  - used for first time, low-E spin-dipole strength mirrors ground-state properties
- **Precision mass measurement:**
  - $^{96}\text{Zr}$  is a golden case for testing  $0\nu$ -NME's and getting experimental handle on  $g_A$
- **$\mu$ -cap:**
  - maybe the only viable tool to study weak response at high momentum transfer and to fix the  $g_A$  problem by comparing with  $(d, ^2\text{He})$