Two-Body Currents and ββ Decay

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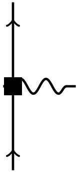
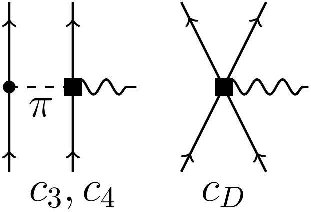
Weak Current in Chiral EFT

Leading order:



Three orders down:



In Coordinate Space (Don't Ask…)

$$\hat{\boldsymbol{J}}_{2b}(\boldsymbol{x}) = \sum_{k<l}^{A} \boldsymbol{J}_{kl}(\boldsymbol{x})$$

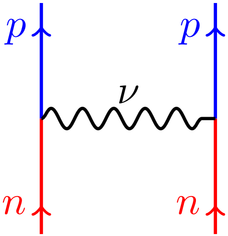
$$\begin{aligned}
\boldsymbol{J}_{kl}(\boldsymbol{x}) = & \frac{g_A}{8m_N f_\pi^2}\left\{\hat{\boldsymbol{p}}_k,\ \boldsymbol{\tau}_\times^a \boldsymbol{\sigma}_l \cdot \hat{\boldsymbol{r}} \frac{Y_1}{r}\right\}\delta(\boldsymbol{x}-\boldsymbol{r}_k) + \frac{i g_A}{4m_N f_\pi^2}\boldsymbol{\tau}_\times^a \boldsymbol{\sigma}_l \cdot \hat{\boldsymbol{r}} \frac{Y_1}{r} \nabla_{\boldsymbol{x}} \delta(\boldsymbol{x}-\boldsymbol{r}_k) \\
& + \frac{i g_A}{8m_N f_\pi^2}\boldsymbol{\tau}_\times^a \left[\boldsymbol{\sigma}_l \cdot \hat{\boldsymbol{r}}\hat{\boldsymbol{r}} m_\pi^2 \left(1+\frac{2}{m_\pi r}+\frac{2}{m_\pi^2 r^2}\right)Y_0 + \frac{\boldsymbol{\sigma}_l \cdot \hat{\boldsymbol{r}}\hat{\boldsymbol{r}}}{r^2}Y_1 - \frac{\boldsymbol{\sigma}_l}{r^2}Y_1\right]\delta(\boldsymbol{x}-\boldsymbol{r}_k) \\
& - \frac{c_3 g_A}{m_N f_\pi^2}\boldsymbol{\tau}_l^a \left[m_\pi^2\left(\left(\boldsymbol{\sigma}_l \cdot \hat{\boldsymbol{r}}\hat{\boldsymbol{r}} - \frac{\boldsymbol{\sigma}_l}{3}\right)Y_2 + \frac{\boldsymbol{\sigma}_l}{3}Y_0\right) - \frac{\boldsymbol{\sigma}_l}{3}\delta(\boldsymbol{r})\right]\delta(\boldsymbol{x}-\boldsymbol{r}_k) \\
& - \frac{g_A}{2m_N f_\pi^2}\left(c_4+\frac{1}{4}\right)\boldsymbol{\tau}_\times^a \left[m_\pi^2\left(\left(\boldsymbol{\sigma}_k \times \hat{\boldsymbol{r}}\boldsymbol{\sigma}_l \cdot \hat{\boldsymbol{r}} - \frac{\boldsymbol{\sigma}_\times}{3}\right)Y_2 + \frac{\boldsymbol{\sigma}_\times}{3}Y_0\right) - \frac{\boldsymbol{\sigma}_\times}{3}\delta(\boldsymbol{r})\right]\delta(\boldsymbol{x}-\boldsymbol{r}_k) \\
& - \frac{g_A}{2m_N f_\pi^2}\left(\frac{1+c_6}{4}\right)\boldsymbol{\tau}_\times^a \boldsymbol{\sigma}_k \times \nabla_{\boldsymbol{x}} \boldsymbol{\sigma}_l \cdot \hat{\boldsymbol{r}} \frac{Y_1}{r}\delta(\boldsymbol{x}-\boldsymbol{r}_k) \\
& + \frac{g_A}{8m_N f_\pi^2}\left\{\hat{\boldsymbol{p}}_l,\ \boldsymbol{\tau}_\times^a \boldsymbol{\sigma}_k \cdot \hat{\boldsymbol{r}} \frac{Y_1}{r}\right\}\delta(\boldsymbol{x}-\boldsymbol{r}_l) + i\frac{g_A}{4m_N f_\pi^2}\boldsymbol{\tau}_\times^a \boldsymbol{\sigma}_k \cdot \hat{\boldsymbol{r}} \frac{Y_1}{r} \nabla_{\boldsymbol{x}} \delta(\boldsymbol{x}-\boldsymbol{r}_l) \\
& - \frac{i g_A}{8m_N f_\pi^2}\boldsymbol{\tau}_\times^a \left[\boldsymbol{\sigma}_k \cdot \hat{\boldsymbol{r}}\hat{\boldsymbol{r}} m_\pi^2 \left(1+\frac{2}{m_\pi r}+\frac{2}{m_\pi^2 r^2}\right)Y_0 + \frac{\boldsymbol{\sigma}_k \cdot \hat{\boldsymbol{r}}}{r^2}\hat{\boldsymbol{r}}Y_1 - \frac{\boldsymbol{\sigma}_k}{r^2}Y_1\right]\delta(\boldsymbol{x}-\boldsymbol{r}_l) \\
& - \frac{g_A c_3}{m_N f_\pi^2}\boldsymbol{\tau}_k^a \left[m_\pi^2\left(\left(\boldsymbol{\sigma}_k \cdot \hat{\boldsymbol{r}}\hat{\boldsymbol{r}} - \frac{\boldsymbol{\sigma}_k}{3}\right)Y_2 + \frac{\boldsymbol{\sigma}_k}{3}Y_0\right) - \frac{\boldsymbol{\sigma}_k}{3}\delta(\boldsymbol{r})\right]\delta(\boldsymbol{x}-\boldsymbol{r}_l) \\
& + \frac{g_A}{2m_N f_\pi^2}\left(c_4+\frac{1}{4}\right)\boldsymbol{\tau}_\times^a \left[m_\pi^2\left(\left(\boldsymbol{\sigma}_l \times \hat{\boldsymbol{r}}\boldsymbol{\sigma}_k \cdot \hat{\boldsymbol{r}} + \frac{\boldsymbol{\sigma}_\times}{3}\right)Y_2 - \frac{\boldsymbol{\sigma}_\times}{3}Y_0\right) + \frac{\boldsymbol{\sigma}_\times}{3}\delta(\boldsymbol{r})\right]\delta(\boldsymbol{x}-\boldsymbol{r}_l) \\
& - \frac{g_A}{2m_N f_\pi^2}\left[\left(\frac{1+c_6}{4}\right)\boldsymbol{\tau}_\times^a \boldsymbol{\sigma}_l \times \nabla_{\boldsymbol{x}} \boldsymbol{\sigma}_k \cdot \hat{\boldsymbol{r}} \frac{Y_1}{r}\right]\delta(\boldsymbol{x}-\boldsymbol{r}_l) \\
& + \frac{g_A}{m_N f_\pi^2}\left[\hat{d}_1(\boldsymbol{\tau}_k^a \boldsymbol{\sigma}_k + \boldsymbol{\tau}_l^a \boldsymbol{\sigma}_l) + \hat{d}_2 \boldsymbol{\tau}_\times^a \boldsymbol{\sigma}_\times\right]\delta(\boldsymbol{r})\delta(\boldsymbol{x}-\boldsymbol{r}_k)
\end{aligned}$$

$0\nu\beta\beta$ Decay

Use closure approximation:

$$\hat{O} \propto \int \frac{J^{+}(\vec{q})J^{+}(-\vec{q})}{q(q+\bar{E})}\, d^3q$$

Leading diagram:



Product of Currents

In first quantization, let

$$\sum_i \hat{O}_i^{1b} = \text{1-body operator in } J^+$$

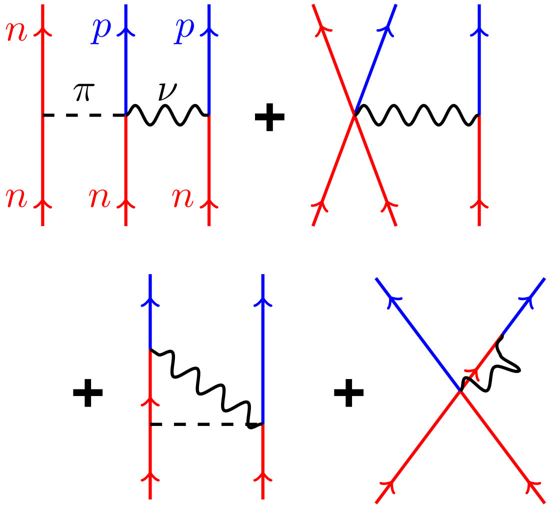
$$\sum_{ij} \hat{O}_{ij}^{2b} = \text{2-body operator in } J^+$$

$$J^+(\vec{q})J^+(-\vec{q}) = \sum_{ij} \hat{O}_i^{1b}\hat{O}_j^{1b} + \overbrace{\sum_{ijk}\left(\hat{O}_{ij}^{2b}\hat{O}_k^{1b} + \hat{O}_i^{1b}\hat{O}_{jk}^{2b}\right)}^{\text{3-body op.}} + \text{4-body}$$

$$+ \underbrace{\sum_{ij}\left(\hat{O}_{ij}^{2b}[\hat{O}_i^{1b} + \hat{O}_j^{1b}] + [\hat{O}_i^{1b} + \hat{O}_j^{1b}]\hat{O}_{ij}^{2b}\right)}_{\text{2-body op.}}$$

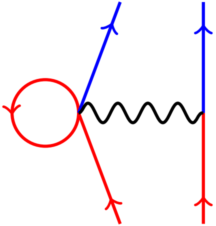
Two-Body Currents in $0\nu\beta\beta$ Decay

Diagrams for these contributions:



Javier's Famous Nuclear Matter Work

Normal-ordering of two-body current, to get effective one-body current. Corresponds to:



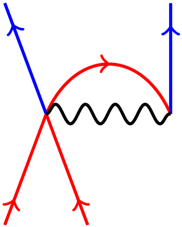
+ . . .

$$g_A \longrightarrow g_A - g_A \frac{\rho}{F_\pi^2} \left[\frac{c_d}{g_A \Lambda} + \frac{2c_3}{3} \frac{q^2}{q^2 + 4m_\pi^2} + I(\rho, P) \left(\frac{2c_4 - 3_3}{3} + \frac{1}{6m} \right) \right]$$

$I(\rho, P)$ is a complicated function of ρ and P that depends weakly on P an is about 2/3 at nuclear density.

Other Contributions

L.J. Wang has done calculation with approximate ^{76}Ge wave function in *fp* shell, inert core underneath. When evaluating contribution of nucleons in core, he finds that in addition a different sum over orbitals contributes:



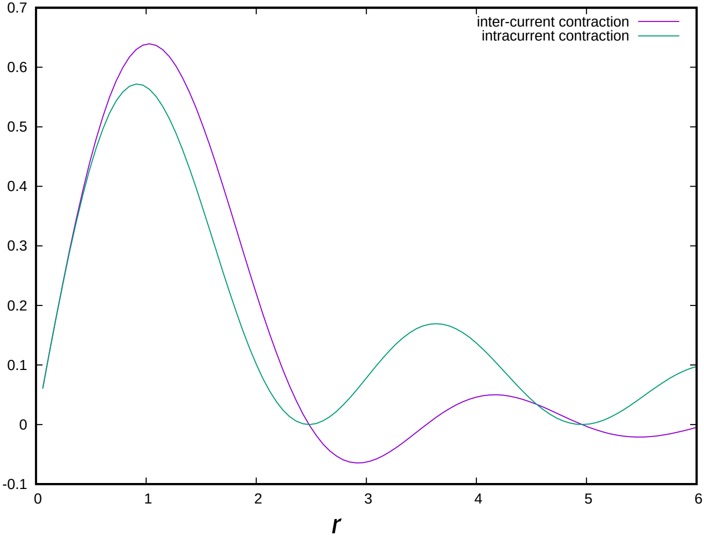
Comes from one-body operator acting, then two-body operator:

$$\sum_{i<F} \langle F| \; p_b^\dagger \underbrace{n_i^\dagger n_a n_b \quad p_a^\dagger}_{} n_i \; |I\rangle$$

Can also view as exchange piece of the three-body-operator that comes form multiplying two- and one-body operators.

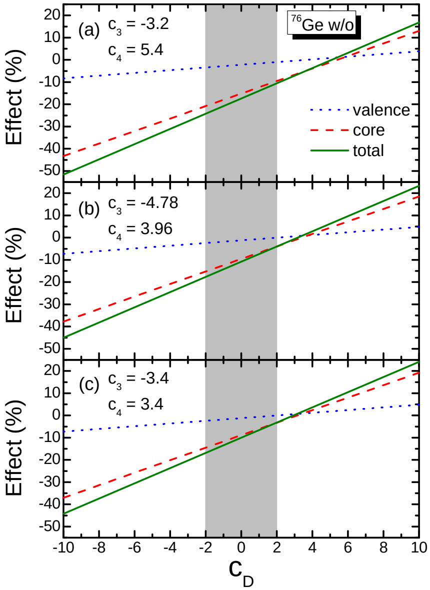
The Two Contractions in Symmetric Nuclear Matter

Contribution to matrix element vs. distance between two-decay vertices.

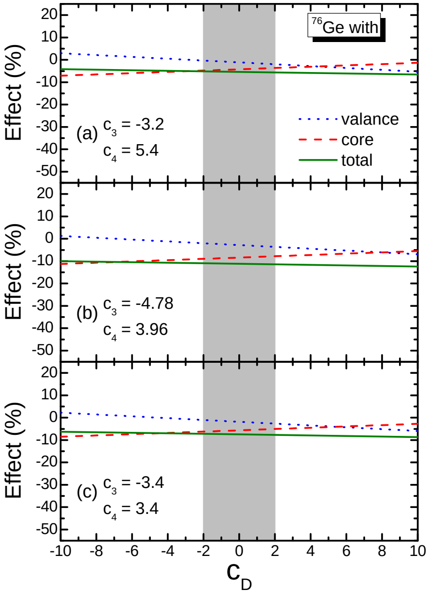


Longjun Result:

Javier contraction only

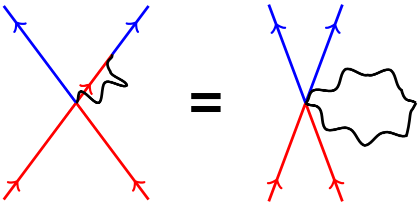


All contractions



Counterterms?

The two-body operator represented by this diagram:



contains a divergent loop integral. The Los Alamos EFT people say that there should be a counter term (with an unknown coefficient) to compensate. There may be other such terms as well; I haven't really looked yet.

Counterterms?

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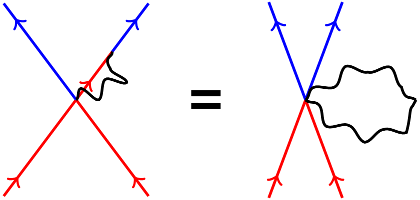

contains a divergent loop integral. The Los Alamos EFT people say that there should be a counter term (with an unknown coefficient) to compensate. There may be other such terms as well; I haven't really looked yet.

This is not great news, IMO.

So, to Sum Up…

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