Nuclear Matrix Elements for Double Beta Decay BAB, Mihai Horoi and Dong-Liang Fang

 $v =$ light neutrino *N* = heavy neutrino

neutrino mass properties $\left[T^{0\nu}_{1/2}\right]^{-1} = G^{0\nu}\left(\left|M^{0\nu}_\nu\right|^2\left|\eta_\nu\right|^2+\left|M^{0\nu}_N\right|^2\left|\eta_N\right|^2\right)$ nuclear matrix elements M=NME phase space - contains the weak

IBA-2J. Barea, J. Kotila, and F. Iachello, Phys. Rev. C **87**, 014315 (2013).

QRPA-En M. T. Mustonen and J. Engel, Phys. Rev. C **87**, 064302 (2013).

QRPA-Jy J. Suhonen, O. Civitarese, Phys. NPA **847** 207–232 (2010).

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SM M. Horoi et. al. PRC **88**, 064312 (2013), PRC **89**, 045502 (2014), PRC **90**, PRC **89**, 054304 (2014), in preparation, PRL **110**, 222502 (2013).

What are the Nuclear Matrix Elements for the $\beta\beta$ Decay of ⁷⁶Ge?

Results and conclusions

- 1) All of the current results are incomplete in some respect
- 2) Hamiltonians in a model space that reproduce the low-lying spectra for A=76 are a necessary starting point
- 3) But this is not enough for double-beta
- $O_{GT} \approx \tau_{1} \tau_{2} (\sigma_1 \cdot \sigma_2) f(r)$ 4) The most important two-body operators are: $f(r) = 1$ for 2ν
	- $f(r) = a/r$ for light 0ν

 $f(r) = b \,\delta(r)$ for heavy 0ν

 $M_{GT} = \langle f | O_{GT} | i \rangle$ where

all have a different type of renormalization

horizontal truncation

configuration interaction (CI) method can include the complete set of configurations for jj44

jj44 means protons and neutrons in the four orbitals $0f_{5/2}$, $1p_{3/2}$, $1p_{1/2}$ and $0g_{9/2}$

1,358,882,246 m-states for 76Se

IBM also evaluated in jj44 but truncated to a s-d boson space

56Ni

Alex Brown, INT, March 28, 2018

PHYSICAL REVIEW C 80, 064323 (2009)

New effective interaction for f_5pg_9 -shell nuclei

M. Honma,¹ T. Otsuka,^{2,3,4} T. Mizusaki,⁵ and M. Hjorth-Jensen⁶

PHYSICAL REVIEW C 95, 014327 (2017)

Nuclear structure of ⁷⁶Ge from inelastic neutron scattering measurements and shell model calculations

S. Mukhopadhyay,^{1,2,*} B. P. Crider,¹ B. A. Brown,^{3,4} S. F. Ashley,^{1,2} A. Chakraborty,^{1,2,†} A. Kumar,^{1,2} M. T. McEllistrem,¹ E. E. Peters,² F. M. Prados-Estévez,^{1,2} and S. W. Yates^{1,2}

> "complete" spectrum up to about 3.5 MeV. Not only many new levels but also removal of many incorrect levels

Alex Brown, October 27,, 2017 DNP

IBM parameters for 76Ge: Duval et al., PLB 134B, 297 (1983)

one nucleon transfer – orbital occupations

PRL 100, 112501 (2008)

PHYSICAL REVIEW LETTERS

week ending
21 MARCH 2008

Nuclear Structure Relevant to Neutrinoless Double β Decay: ⁷⁶Ge and ⁷⁶Se

J. P. Schiffer,^{1,*} S. J. Freeman,² J. A. Clark,³ C. Deibel,³ C. R. Fitzpatrick,² S. Gros,¹ A. Heinz,³ D. Hirata,^{4,5} C. L. Jiang,¹ B. P. Kay,² A. Parikh,³ P. D. Parker,³ K. E. Rehm,¹ A. C. C. Villa

neutron proton

VOLUME 18, NUMBER 6

 74 Ge(t, p)⁷⁶Ge reaction

S. Mordechai,* H. T. Fortune,[†] R. Middleton, and G. Stephans

VSC

Absolute calculation of two nucleon transfer?

Lots of experimental data in this region

Is the cross section obtained with jj44 a factor of two smaller than experiment?

A collaboration with reaction theory is needed to answer this.

two nucleon transfer

PHYSICAL REVIEW C 75, 051301(R) (2007)

Pair correlations in nuclei involved in neutrinoless double β decay: ⁷⁶Ge and ⁷⁶Se

S. J. Freeman,¹ J. P. Schiffer,^{2,*} A. C. C. Villari,³ J. A. Clark,⁴ C. Deibel,⁴ S. Gros,² A. Heinz,⁴ D. Hirata,^{3,5} C. L. Jiang,² B. P. Kay,¹ A. Parikh,⁴ P. D. Parker,⁴ J. Qian,⁴ K. E. Rehm,² X.

S
NSCI

$$
^{76}Ge
$$
 E2 map

Width of red line is proportional to the B(E2)

76 Ge

FIG. 7. Partial level scheme of 76 Ge from shell model calculations $[(a)$ and $(c)]$ and experiment (b) . The thicknesses of the solid arrows are proportional to the $B(E2)s$. Dashed arrows indicate that the level lifetime was not determined and the $B(E2)s$ are only approximate.

model space

vertical expansion

particle-hole configurations for all orbitals

1) QRPA in

- a) jj44 = $(0f_{5/2}, 1p_{3/2}, 1p_{1/2}, 0g_{9/2})$ b) fpg = $0f_{7/2}$, $(0f_{5/2}, 1p_{3/2}, 1p_{1/2}, 0g_{9/2}) 0g_{7/2}$ c) 21 orbits
- 2) Many-body perturbation theory (MBPT) to include 2 particle-2 hole (2p-2h) excitations to high excitation.
- 3) Δ particle admixtures and mesonic exchange currents (MEC)

Operator and TBME for 0v double beta

$$
M_{\alpha}^{0\nu} = \sum_{\kappa} \sum_{1234} \langle 13| \mathcal{O}_{\alpha} |24\rangle \langle f|\hat{c}_3^{\dagger} \hat{c}_4|\kappa\rangle \langle \kappa|\hat{c}_1^{\dagger} \hat{c}_2|i\rangle \qquad \alpha = GT
$$

 $\mathcal{O}_{GT} = \tau_{1} - \tau_{2} - (\sigma_1 \cdot \sigma_2) H_{GT}(r, E_K)$

$$
H_{\alpha}(r, E_{\kappa}) = \frac{2R}{\pi} \int_0^{\infty} \frac{f_{\alpha}(qr)h_{\alpha}(q^2)q \,dq}{q + E_{\kappa} - (E_i + E_f)/2}
$$
 Plus short range
correlations (SRC)

$$
f_{GT,F}(qr) = j_0(qr)
$$

\n
$$
h_{GT}(q^2) = \left[1 - \frac{2}{3} \frac{q^2}{q^2 + m_\pi^2} + \frac{1}{3} \left(\frac{q^2}{q^2 + m_\pi^2}\right)^2\right] + \frac{2}{3} \frac{g_M^2(q^2)}{g_A^2} \frac{q^2}{4m_p^2}
$$

\n
$$
[E_\kappa - (E_i + E_f)/2] \to \langle E \rangle
$$

\n
$$
\sum_{\kappa} \langle f|\hat{c}_3^\dagger \hat{c}_4|\kappa \rangle \langle \kappa|\hat{c}_1^\dagger \hat{c}_2|i \rangle = \langle f|\hat{c}_3^\dagger \hat{c}_4 \hat{c}_1^\dagger \hat{c}_2|i \rangle
$$

$$
M_{\alpha}^{0\nu} = \sum_{1234} \langle 13| \mathcal{O}_{\alpha} |24 \rangle \langle f|\hat{c}_3^{\dagger} \hat{c}_4 \hat{c}_1^{\dagger} \hat{c}_2 |i \rangle
$$

$$
O_{GT} \approx \tau_{1} - \tau_{2} - (\sigma_1 \cdot \sigma_2) \, a/r
$$

Alex Brown, INT, March 28, 2018

$$
M^{2\nu} = M_{GT}^{2\nu} - \left(\frac{g_V}{g_A}\right)^2 M_F^{2\nu}
$$

$$
M^{0\nu} = M_{GT}^{0\nu} - \left(\frac{g_V}{g_A}\right)^2 M_F^{0\nu} + M_T^{0\nu}
$$
Gamow-Teller Fermi tensor

 $M_F^{2\nu} \approx 0$ due to isospin conservation. This was recently corrected in QRPA (2013) and IBM (2015) and this also reduced $M_F^{0\nu}$ in these models.

tensor term is small

Thus, based on an average of all recent models we use $M = R_{GT} M_{GT}$ with $M^{(2\nu)}=M^{(2\nu)}_{GT}$
 $M^{(0\nu \text{ light})}=1.12(7)\,M^{(0\nu \text{ light})}_{GT}$
 $M^{(0\nu \text{ heavy})}=1.13(13)\,M^{(0\nu \text{ heavy})}_{GT}$

GT operators $M_{GT} = \langle f | O_{GT} | i \rangle$ where
 $O_{GT} \approx \tau_{1-\tau_{2-}} (\sigma_1 \cdot \sigma_2) f(r)$
 $f(r) = 1$ for 2ν $f(r) = a/r$ for light 0ν
 $f(r) = b \delta(r)$ for heavy 0ν

Nuclear Structure Aspects of Neutrinoless Double- β Decay

B. A. Brown,¹ M. Horoi,² and R. A. Sen'kov^{2,3}

heavy $0\nu\beta\beta$ decay

CI conclusion: The J_{pp} =0⁺ term is partly cancelled by $J_{pp} = 2^+$

CI conclusion: The J_{pp} =0⁺ term is reduced by a factor of two from higher J_{pp} .

Observation (old): CI theory for 2v decay is a factor of 2.5 too large

We will write

 $M = R_V M(CI)$ where $R_V = 0.4$

Where R_{V} is a correction factor that takes into account the "vertical expansion" of the model space (beyond jj44)

β decay

We have known for 30 years that the experimental B(GT) strengths observed in experiment are systematically smaller than that obtained in $\,$ sd and $\rm pf$ shell-model calculations by a factor of $\,$ R=0.5 to 0.6. $\,$ Compilations:

ADNDT 33, 345 (1985) PRC 53, R2602 (1996).

pf shell model with R=0.55

In this case the Ikeda sum rule $3(N-Z) = 6$

β decay: some recent examples

PHYSICAL REVIEW C 91, 064316 (2015)

High-resolution study of Gamow-Teller excitations in the ⁴²Ca(³He,t)⁴²Sc reaction and the observation of a "low-energy super-Gamow-Teller state"

Y. Fujita, ^{1,2,*} H. Fujita, ^{1,2} T. Adachi,¹ G. Susoy, ^{1,3} A. Algora, ^{4,5} C. L. Bai,⁶ G. Colò,⁷ M. Csatlós,⁵ J. M. Deaven, ^{8,9,10} E. Estevez-Aguado,⁴ C. J. Guess,^{8,9,10,†} J. Gulyás,⁵ K. Hatanaka,¹ K. Hirota,¹ M. Honma,¹¹ D. Ishikawa,¹ A. Krasznahorkay,⁵ H. Matsubara, ^{1,‡} R. Meharchand, ^{8,9,10,§} F. Molina,^{4,||} H. Nakada, ¹² H. Okamura, ^{1,¶} H. J. Ong, ¹ T. Otsuka, ¹³ G. Perdikakis, ^{8,14}
B. Rubio,⁴ H. Sagawa, ^{15,16} P. Sarriguren, ¹⁷ C. Scholl, ^{18,#} Y. J. H. Thies,²¹ K. Yoshida,²² R. G. T. Zegers,^{8,9,10} and J. Zenihiro^{1,**}

Alex Brown, INT, March 28, 2018

 $B(GT-) - B(GT+) = 3(N-Z)$ sum rule is always satisfied but we must go up to about 50 MeV.

The two-neutrino double-beta only depends on the low-lying strength (up to about 10 MeV) that is reduced (quenched) due to configuration mixing

 β decay – where does the GT strength go?

example of ¹⁷F with the Ikeda sum rule $S = B(GT-) - B(GT+) = 3(Z-N) = 3$

β decay: recent example

Physics Letters B 737 (2014) 383-387

Sizeable beta-strength in $31Ar$ ($\beta 3p$) decay

G.T. Koldste^a, B. Blank^b, M.J.G. Borge^c, J.A. Briz^c, M. Carmona-Gallardo^c, L.M. Fraile^d, H.O.U. Fynbo^{a,*}, J. Giovinazzo^b, J.G. Johansen^{a,1}, A. Jokinen^e, B. Jonson^f, T. Kurturkian-Nieto^b, T. Nilsson A. Saastamoinen^{e,2}, O. Tengblad^c, J.-C. Thomas^h, J. Van de Walleⁱ

 β decay for sd and pf shell

 $B_{\text{exp}} = 0.60 \text{ B}_{\text{CI}} = (1 - 0.223)^2 \text{ B}_{\text{CI}}$

Explained (1980's) using MBPT as being due to 2p-2h admixtures up to high excitations, and mesonic exchange currents (MEC) including the Δ isobar:

Arima, Shimizu, Bentz and Hyuga (for sd shell) (1987): $\rm B_{ASBH}$ = 0.65 $\rm B_{CI}$ = (1 - 0.220 + 0.025)² $\rm B_{CI}$

Towner and Khanna (for sd shell) (1987): $\rm B_{TK}$ = 0.65 $\rm B_{CI}$ = (1 - 0.175 - 0.020)² $\rm B_{CI}$

I. S. Towner, Phys. Rep. 155, 264 (1987).

A. Arima, K. Schimizu, W. Bentz and H. Hyuga, Adv. Nucl. Phys. 18, 1 (1987).

B. A. Brown and B. H. Wildenthal, Nucl. Phys. A474, 290 (1987).

$2\nu\beta\beta$ decay

Conclusion I: from QRPA Inclusion of both spin-orbit pairs reduces the 2 NME by about a factor of about 0.5

Conclusion II: 2p-2h beyond QRPA reduces the low-lying GT strength further by a factor of about 0.6

 III – the total reduction is thus (0.5) x $(0.6) = 0.30$

Compared to 0.40 from exp

heavy $0\nu\beta\beta$ decay

Experiment is not known

we must predict this

Within jj44 all model are rather consistent.

Should CI should be reduced by a factor of 0.4?

(equivalent to reducing $\rm g_A$ from 1.27 to about 0.8)

heavy $0\nu\beta\beta$ decay

QRPA conclusion: Vertical expansion enhances the heavy 0 NME by about a factor of 2.

The short-range pairing interaction causes an odd-even oscillations in the separation energies. This can be used to extract a vertical correction to the jj44 model space for a similar operator of about $R_V = 1.65(25)$

Particle transfer cross section calculated within the jj44 model space should under-predict the data and must be corrected for the pairing from other orbitals using MBPT.

Corrections to the calculated observables depend on the observable and the model space. Examples in the sd shell from the 1980's

exp/theory (average)

All of these have been understood in terms of MBPT and the vertical expansion

We write $\rm\ M_{GT}$ = $\rm R_{V}\ M_{GT\text{-}CI}$ where $\rm R_{V}$ is the vertical expansion correction factor $2v$ v $R_V = 0.40$ f(r) = 1 empirical value consistent with other data 0v heavy $R_V = 1.65(25)$ f(r) = b $\delta(r)$ pairing enhancement from other data 0v light $R_V = ?$ f(r) = a/r

light $0 \vee \beta \beta$ decay $f(r) = a/r$

light $0 \vee \beta \beta$ decay $f(r) = a/r$

Effects of vertical expansion within QRPA are small. This due to competition between ph (1^+) reduction and pp (0^{+}) enhancement.

IBM and QRPA are too large within jj44. Due to lack of cancellation from $J_{pp} = 2^+$ contributions in those models.

light $0\nu\beta\beta$ decay

PHYSICAL REVIEW C 87, 064315 (2013)

Effective double- β -decay operator for ⁷⁶Ge and ⁸²Se

Jonathan Engel[†] Jason D. H

Used 2p-2h MBPT:

 0ν NME for 76 Ge was increased by about 20%

So we use $M_{GT} = R_V M_{GT-CI}$ with $R_V = 1.20(20)$

Short range correlations – repulsive part of NN potential pushes out the relative wave function at short distance and reduced the radial matrix elements for $1/r$ and $\delta(r)$

Several options exits – I started with the CD-Bonn – the weakest.

Others are AV18 (the strongest) and UCOM (in between).

So for now I take the mean with the error coming from the difference.

Heavy neutrino – NME reduced by $R_S = 0.80(20)$ Light neutrino – NME reduced by $R_S = 0.97(3)$

Final results for ⁷⁶Ge starting with CI in jj44 called $\rm M_{CI}$

2v
$$
M_{exp} = 0.140(5)
$$

\n
$$
= (R_V) (R_S) (R_{GT}) M_{CI}
$$
\n
$$
= (0.40) (1) (1) 0.31(3)
$$
\n
$$
= (1 - 0.37)^2 0.31(3)
$$
\n
$$
= (1 - 0.10 - 0.27 - 0.02)^2 0.31(3)
$$
\n2p-2h 1p-1h MEC
\n(number are a guess – need to be confirmed)

Results for 76 Ge starting with CI in jj44 called M_{CI-GT}

Ov heavy neutrino

 $\rm M = \rm \quad R_{V} \rm \qquad \quad R_{S} \rm \qquad \quad \quad R_{GT} \qquad M_{CI\text{-}GT}$ $= 1.65(25)$ 0.80(20) 1.13(13) 155(10) $= 232(80)$

Results for 76 Ge starting with CI in jj44 called M_{CI-GT}

0v light neutrino

 $\rm M = \rm \quad R_{V} \rm \qquad \quad R_{S} \rm \qquad \quad R_{GT} \quad \quad M_{CI-GT}$ $= 1.20(20)$ 0.97(3) 1.12(7) 3.0(3) $= 3.9(8)$

IBA-2J. Barea, J. Kotila, and F. Iachello, Phys. Rev. C **87**, 014315 (2013).

QRPA-En M. T. Mustonen and J. Engel, Phys. Rev. C **87**, 064302 (2013).

QRPA-Jy J. Suhonen, O. Civitarese, Phys. NPA **847** 207–232 (2010).

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PHYSICAL REVIEW LETTERS 120, 132503 (2018)

Editors' Suggestion

Featured in Physics

Improved Limit on Neutrinoless Double- β Decay of ⁷⁶Ge from GERDA Phase II

TABLE II. Comparison of lower half-life limits $T_{1/2}^{0\nu}$ (90% C.L.) and corresponding upper Majorana neutrino mass $m_{\beta\beta}$ limits of different $0\nu\beta\beta$ experiments. The experiments, the isotopes, and the isotopic m of nuclear matrix elements (NME) [15–22] are given in column 4. The lower half-life sensitivities and limits are shown in columns 5 and 7, respectively. The corresponding upper limits for $m_{\beta\beta}$ derived with the NME are shown in columns 6 and 8.

		Isotope	M_i (kg)	NME	Sensitivity		Limit	
Experiment					(10^{25} yr) $T^{0\nu}_{1/2}$	$m_{\beta\beta}$ (eV)	$T_{1/2}^{0\nu}$ (10 ²⁵) yr)	$m_{\beta\beta}$ (eV)
GERDA		76 Ge	31	$2.8 - 6.1$	5.8	$0.14 - 0.30$	8.0	$0.12 - 0.26$
Majorana	13]	76 Ge	26	$2.8 - 6.1$	2.1	$0.23 - 0.51$	1.9	$0.24 - 0.52$
KamLAND-Zen	[24]	136Xe	343	$1.6 - 4.8$	5.6	$0.07 - 0.22$	10.7	$0.05 - 0.16$
EXO	[25, 26]	136Xe	161	$1.6 - 4.8$	1.9	$0.13 - 0.37$	1.1	$0.17 - 0.49$
CUORE	[27, 28]	130 Te	206	$1.4 - 6.4$	0.7	$0.16 - 0.73$	1.5	$0.11 - 0.50$

IBA-2 J. Barea, J. Kotila, and F. Iachello, Phys. Rev. C **87**, 014315 (2013). **QRPA-Tu** A. Faessler, M. Gonzalez, S. Kovalenko, and F. Simkovic, arXiv:**1408.6077 SM** M. Horoi et. al. PRC **88**, 064312 (2013), PRC **90**, PRC **89**, 054304 (2014), in preparation, PRL **110**, 222502 (2013).

IBA-2 J. Barea, J. Kotila, and F. Iachello, Phys. Rev. C **87**, 014315 (2013). **QRPA-Tu** A. Faessler, M. Gonzalez, S. Kovalenko, and F. Simkovic, arXiv:**1408.6077 SM** M. Horoi et. al. PRC **88**, 064312 (2013), PRC **90**, PRC **89**, 054304 (2014), in preparation, PRL **110**, 222502 (2013).

Next

- 1) Check the jj44 CI Hamiltonian for the A=76 region by making more comparisons to known spectroscopy including two nucleon transfer and other transfer reactions.
- 2) Understand the horizontal vertical division with ab-initio and no core calculations for lighter nuclei.
- 3) Check and improve the corrections beyond jj44 with MBPT and other methods like coupled cluster (CC) for all relevant operators including B(GT).
- 4) Reduce the error on the short range correlations using for example the techniques being developed by Bogner et al for in-medium similarity renormalization group (IM-SRG)
- 5) Make a similar evaluation for other cases of interest.

Collaborators:

Dong-Liang Fang Jilin University, Changchun, China Mihai Horoi Central Michigan University Bill Rae Garsington, England

Input from:

Jose Barea (Chile) Mark Caprio (Notre Dame) Alfredos Poves (Madrid) Fedor Šimkovic (Dubna)

Germanium detectors – detector is its own source of decay ${\rm T_{1/2}}$ > 3 x 10²⁵ yr (90% C.L.) $\rm{m_{v}>10.30~eV^{-1}}$

GERDA

The GERmanium Detector Array for the search of neutrinoless double beta decay in Ge-76 at the Laboratori Nazionali del Gran Sasso (LNGS)

Shaded areas shows what we know from neutrino oscillations

- •Covering the inverted hierarchy region ($\langle m \rangle \sim 15$ meV) requires sensitivity to half-lives of $10^{27} - 10^{28}$ years.
- •Corresponds to one decay per year for a tonne of material

Shaded areas show what is allowed from neutrino oscillations

Fig. 10. Second-order core-polarisation graphs that give a correction to the magnetic moment of a closed-LS-shell-plus-one nucleus. Graphs (a), (b), (c) involve 2p-1h intermediate states, graphs (d), (e), (f) and (g) involve 3p-2h intermediate states. Graphs (c) and (g) are 'folded' graphs that correct (to second order) the normalization of the single-particle state.

Fig. 13. Meson-exchange graphs of pion range: (a) nucleon intermediate states, (b) antinucleon intermediate states, pair graph, (c) pionic-current graph, (d) isobar intermediate states, and (e) heavy-meson current graph.

