# Phenomenology of neutron-antineutron conversion

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Based on S. Gardner and X. Yan,
arXiv:1602.00693, arXiv:1710.09292,
and ongoing work

# Why $n - \bar{n}$ transitions?

- **Two kinds of $\not{B}$ phenomena:**
  - ✻
    - $|\Delta B| = 1$ (& $|B - L| = 0$): $\Lambda_{p \to e^+ \pi^0} \geq 10^{15}$ GeV; [E. Kearns (2013)]
    - $|\Delta B| = 1$ (& $|B - L| = 2$): $\Lambda_{n \to e^- \pi^+} \geq 10^{10}$ GeV; [S. Seidel et al. (1988)]

    [S. Weinberg (1980), H. A. Weldon and A. Zee (1980).]
  - ✻ $|\Delta B| = 2$: $\Lambda_{n\bar{n}} \geq 10^{5.5}$ GeV. Note $(|\Delta B| = 2) \neq (|\Delta B| = 1)^2$;
    $\Rightarrow$ **a relatively low scale of new physics.**
- **Three possible $n - \bar{n}$ transitions.**
  - ✓ $n - \bar{n}$ **oscillation:** neutrons spontaneously transform into antineutrons. It is sensitive to the energy difference between neutrons and antineutrons. Great efforts to deal with environmental effects, such as magnetic fields and matter.
  - ✓ **Dinucleon decay.** Background induced by atmospheric neutrinos can be a problem.
  - ✻ $n - \bar{n}$ **conversion:** A change of a neutron into an antineutron is realized through the interaction with an external source.

**The fermion anticommutation relation and CPT phase constraints leave 3 non-trivial lowest mass dimension operators:** [S. Gardner and X. Yan, (2015)]

- $n^\top C n +$ h.c.. Its search is very sensitive to environmental effects;
- $n^\top C \gamma^5 n +$ h.c., does not contribute to $n\bar{n}$ oscillation; [S. Gardner and E. Jafari (2015), Berezhiani and Vainshtein, (2015) and Fujikawa and Tureanu, (2015)]
- $n^\top C \gamma^\mu \gamma^5 n \partial^\nu F_{\nu\mu} +$ h.c., [Berezhiani and Vainshtein (2015)]

The external source, $j_\mu = \partial^\nu F_{\nu\mu}$, technically can represent any gauge invariant currents.

Since this is a scattering process, the energy degeneracy of the initial and final particles is no longer required, so that $n - \bar{n}$ transition with no sensitivity to environmental effects is possible.

# $n - \bar{n}$ conversion operator

**Exploration of $n - \bar{n}$ oscillations in the basis $(|n+\rangle, |\bar{n}+\rangle, |n-\rangle, \bar{n}-\rangle)$ has been firstly suggested.** [S. Gardner and E. Jafari (2015)]

**Now assuming that momentum transfer is trivially small, in the $\mathbf{P} = 0$ limit, the mass matrix for $n - \bar{n}$ transition in the presence of a magnetic field (whose direction is the spin quantization axis) and an external source $Qej_\mu \equiv \partial^\nu F_{\mu\nu}$ is**

$$M = \begin{pmatrix} m + \omega_0 & \omega_z & 0 & \omega_x - i\omega_y \\ \omega_z & m - \omega_0 & \omega_x - i\omega_y & 0 \\ 0 & \omega_x + i\omega_y & m - \omega_0 & -\omega_z \\ \omega_x + i\omega_y & 0 & -\omega_z & m + \omega_0 \end{pmatrix}$$

where $\omega_0 \equiv -\mu B$, $\boldsymbol{\omega} \equiv \eta \boldsymbol{j}$. **Note $j_0$ does not appear because its matrix element is proportional to $|\mathbf{p}|$, but $\mathbf{p} = 0$.**

# $n - \bar{n}$ conversion operator

**Define $\omega_{xy} = \sqrt{\omega_x^2 + \omega_y^2}$ and calculate the probabilities of a neutron with $s = +$ transforming to $\bar{n}$ in spin $s = +/-$ state respectively,**

$$
\begin{aligned}
\mathcal{P}_{n+\to\bar{n}+} &= \frac{\omega_z^2}{\omega_0^2}\sin^2[t\omega_0]\cos^2[t\omega_{xy}] + \mathcal{O}(\omega_z^3), \\
\mathcal{P}_{n+\to\bar{n}-} &= \sin^2[t\omega_{xy}] - \frac{t\omega_z^2\omega_{xy}}{\omega_0^2}\sin[t\omega_{xy}]\cos[t\omega_{xy}] \\
&\quad -\frac{\omega_z^2}{\omega_0^2}\cos^2[t\omega_0]\sin^2[t\omega_{xy}] + \mathcal{O}(\omega_z^3),
\end{aligned}
$$

where $\omega_i \ll \omega_0$.

**Note that $\mathcal{P}_{n+\to\bar{n}+}$ is still quenched by magnetic field (same spin!).**

**$n - \bar{n}$ oscillation with spin-flip through magnetic field (Rabi formula) has been considered.** [S. Gardner and E. Jafari (2015)]

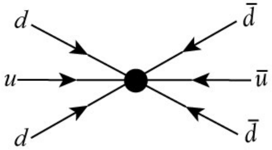
However, it still does not evade quenching. [Berezhiani and Vainshtein (2015), S. Gardner and X. Yan (2016), McKeen and Nelson (2016).]

**Dimension analysis of the $j_\mu$ operator shows that**

$$\frac{\eta}{2}(n^T C\gamma^\mu\gamma^5 n j_\mu + \text{h.c.})$$

with $[\eta] = -2$. **Naively expect additional suppression of $\Lambda^3_{BSM}$ compared with $n - \bar{n}$ oscillation?**
**We want to evaluate the mass scale of this suppression.**

**Note that quarks are charged under QED and QCD. However, $j_\mu$ in QCD is not gauge invariant. We explore the connection through QED.**




Quark-level $n - \bar{n}$ oscillation:
$\Lambda_{QCD} \ll \Lambda \ll \Lambda_{BSM}$

# 6-fermion $n - \bar{n}$ oscillation operators

**There are 14 independent operators if $U(1)_{em}$ and $SU(3)_{color}$ symmetries are considered**

[Rao and Shrock (1982), W. E. Caswell et al (1983)]

$$\begin{aligned}
(O_1)_{\chi_1\chi_2\chi_3} &= [u_{\chi_1}^{\top\alpha} C u_{\chi_1}^{\beta}][d_{\chi_2}^{\top\gamma} C d_{\chi_2}^{\delta}][d_{\chi_3}^{\top\rho} C d_{\chi_3}^{\sigma}](T_s)_{\alpha\beta\gamma\delta\rho\sigma}, \\
(O_2)_{\chi_1\chi_2\chi_3} &= [u_{\chi_1}^{\top\alpha} C d_{\chi_1}^{\beta}][u_{\chi_2}^{\top\gamma} C d_{\chi_2}^{\delta}][d_{\chi_3}^{\top\rho} C d_{\chi_3}^{\sigma}](T_s)_{\alpha\beta\gamma\delta\rho\sigma}, \\
(O_3)_{\chi_1\chi_2\chi_3} &= [u_{\chi_1}^{\top\alpha} C d_{\chi_1}^{\beta}][u_{\chi_2}^{\top\gamma} C d_{\chi_2}^{\delta}][d_{\chi_3}^{\top\rho} C d_{\chi_3}^{\sigma}](T_a)_{\alpha\beta\gamma\delta\rho\sigma},
\end{aligned}$$

with $(T_s)_{\alpha\beta\gamma\delta\rho\sigma} = \epsilon_{\rho\alpha\gamma}\epsilon_{\sigma\beta\delta} + \epsilon_{\sigma\alpha\gamma}\epsilon_{\rho\beta\delta} + \epsilon_{\rho\beta\gamma}\epsilon_{\sigma\alpha\delta} + \epsilon_{\sigma\beta\gamma}\epsilon_{\rho\alpha\delta}$
and $(T_a)_{\alpha\beta\gamma\delta\rho\sigma} = \epsilon_{\rho\alpha\beta}\epsilon_{\sigma\gamma\delta} + \epsilon_{\sigma\alpha\beta}\epsilon_{\rho\gamma\delta}$.

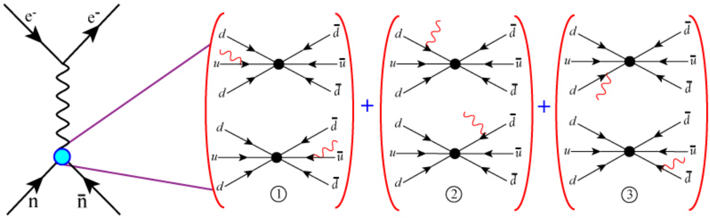
**The number of independent operators can be reduced to 4, if we demand that they are invariant under $\mathbf{SU(2)_L \times U(1)_Y}$.**

[Rao and Shrock (1982), M. Buchoff et al (2012)]

These are $(O_1)_{RRR}$, $(O_2)_{RRR}$, $2(O_3)_{LRR}$, $4(O_3)_{LLR}$.

# EM dressing

The EM interaction with the quark-level $n - \bar{n}$ oscillation operator $\mathcal{O}_1$:




**Consider a process $q^\rho(p) + \gamma(k) \to \bar{q}^\delta(p')$, where $\rho$ and $\delta$ are flavor indices.**

The pertinent terms in interaction Hamiltonian are

$$H_{n\bar{n}} = \frac{\delta_q}{2} \sum_{\chi_1} \int d^3x\, (\psi^{\rho\,T}_{\chi_1} C \psi^\delta_{\chi_1} + h.c.),\ H^\rho = Q_\rho e \sum_{\chi_2} \int d^3x\, \bar{\psi}^\rho_{\chi_2} \not{A} \psi^\rho_{\chi_2}, \text{ and}$$

$$H^\delta = Q_\delta e \sum_{\chi_3} \int d^3x\, \bar{\psi}^\delta_{\chi_3} \not{A} \psi^\delta_{\chi_3}.$$

**We compute the amplitude**

$$\langle \bar{q}^\delta(p') | \mathcal{T}[(-iH_{n\bar{n}})(-iH^\rho - iH^\delta)] | q^\rho(p)\gamma(k)\rangle,$$

**where $\mathcal{T}$ is the time-ordering operator.**

# EM dressing

**We find**

$$-\frac{\delta_q}{2} emi \sum_{\chi} \Big[ \bar{u}^{\delta}(\boldsymbol{p'}, s') \not{\epsilon}(k) u^{\rho}(\boldsymbol{p}, s) \Big( \frac{Q_\rho}{p'^2 - m^2} - \frac{Q_\delta}{p^2 - m^2} \Big)$$

$$+ \chi \bar{u}^{\delta}(\boldsymbol{p'}, s') \not{\epsilon}(k) \gamma^5 u^{\rho}(\boldsymbol{p}, s) \Big( \frac{Q_\rho}{p'^2 - m^2} + \frac{Q_\delta}{p^2 - m^2} \Big) \Big] (2\pi)^4 \delta^4(p' - p - k),$$

**where $\epsilon$ is the polarization vector of photon. Noting $p^2 = p'^2$, we extract the effective operators associated with the quark-antiquark-photon vertex**

$$-\frac{m\delta_q e}{p^2 - m^2} (Q_\rho \psi_{-\chi}^{\delta T} C\gamma^\mu \psi_{\chi}^{\rho} - Q_\delta \psi_{\chi}^{\delta T} C\gamma^\mu \psi_{-\chi}^{\rho}).$$

**Note that only the $C\gamma^\mu\gamma^5$ Lorentz structure would survive if $\rho = \delta$. Also $\chi$ comes from the EM interaction part. We can recast it as**

$$-\frac{m\delta_q e}{p^2 - m^2} (Q_\rho \psi_{-\chi}^{\delta T} C\gamma^\mu \gamma^5 \psi_{\chi}^{\rho} + Q_\delta \psi_{\chi}^{\delta T} C\gamma^\mu \gamma^5 \psi_{-\chi}^{\rho}).$$

# Effective $n-\bar{n}$ conversion operator

**Following the same procedure, we find the effective $n-\bar{n}$ conversion operator associated with $\mathcal{O}_1$:**

$$
\begin{aligned}
(\tilde{\mathcal{O}}_1)^{\chi}_{\chi_1\chi_2\chi_3} \quad = \quad & (\delta_1)_{\chi_1\chi_2\chi_3}\frac{em}{3(p^2-m^2)}\frac{Qej_\mu}{q^2} \\
& \Big[-2[u^{\alpha\,T}_{-\chi}C\gamma^\mu\gamma^5 u^\beta_\chi + u^{\alpha\,T}_{\chi}C\gamma^\mu\gamma^5 u^\beta_{-\chi}][d^{\gamma\,T}_{\chi_2}Cd^\delta_{\chi_2}][d^{\rho\,T}_{\chi_3}Cd^\sigma_{\chi_3}] \\
& [u^{\alpha\,T}_{\chi_1}Cu^\beta_{\chi_1}][d^{\gamma\,T}_{-\chi}C\gamma^\mu\gamma^5 d^\delta_\chi + d^{\gamma\,T}_{\chi}C\gamma^\mu\gamma^5 d^\delta_{-\chi}][d^{\rho\,T}_{\chi_3}Cd^\sigma_{\chi_3}] \\
& [u^{\alpha\,T}_{\chi_1}Cu^\beta_{\chi_1}][d^{\gamma\,T}_{\chi_2}Cd^\delta_{\chi_2}][d^{\rho\,T}_{-\chi}C\gamma^\mu\gamma^5 d^\sigma_\chi + d^{\rho\,T}_{\chi}C\gamma^\mu\gamma^5 d^\sigma_{-\chi}]\Big](T_s)_{\alpha\beta\gamma\delta\rho\sigma}.
\end{aligned}
$$

**The effective conversion operators associated with $\mathcal{O}_2$ and $\mathcal{O}_3$ can be found in the same way.**

# Matching from the quark to hadron level

**In quark level, the respective effective Lagrangian that mediates $n-\bar{n}$ oscillation and conversion are**

$$\mathcal{L}_q \supset \sum_{i,\chi_1,\chi_2,\chi_3} (\delta_i)_{\chi_1,\chi_2,\chi_3} (\mathcal{O}_i)_{\chi_1,\chi_2,\chi_3} + h.c.,$$

$$\mathcal{L}_q^{\text{conv}} \supset \sum_{\chi} \sum_{i,\chi_1,\chi_2,\chi_3} (\eta_i)^{\chi}_{\chi_1,\chi_2,\chi_3} (\tilde{\mathcal{O}}_i)^{\chi}_{\chi_1,\chi_2,\chi_3} + h.c..$$

**In neutron level, the respective effective Lagrangian are**

$$\mathcal{L}_n \supset -\frac{\delta}{2}(n^T C n + h.c.), \quad \mathcal{L}_n^{\text{con}} \supset -\frac{\eta}{2}(n^T C\gamma^{\mu}\gamma^5 n j_{\mu} + h.c.).$$

**We related the low-energy constants of the effective Lagrangian (oscillation and conversion) to those in quark level by the following matching condition:**

$$\langle \bar{n} | \int d^3\mathbf{r} \mathcal{L}_n(\mathbf{r}) | n \rangle = \langle \bar{n}_q | \int d^3\mathbf{r} \mathcal{L}_q(\mathbf{r}) | n_q \rangle$$

# Matrix elements in MIT bag model

## Matrix element of quark-level $n - \bar{n}$ oscillation operators

| $\langle O_1 \rangle_{RRR}$ | $\langle O_1 \rangle_{LLR}$ | $\langle O_1 \rangle_{RLL}$ | $\langle O_2 \rangle_{RRR}$ | $\langle O_2 \rangle_{LLR}$ | $\langle O_2 \rangle_{RLL}$ | $\langle O_3 \rangle_{RRR}$ | $\langle O_3 \rangle_{LRR}$ | $\langle O_3 \rangle_{LLR}$ |
|---|---|---|---|---|---|---|---|---|
| -5.33 | -4.17 | -0.666 | 1.33 | 1.92 | 0.167 | 2.22 | -2.72 | 2.03 |

**Consistent with the published results.** [Rao and Shrock (1982)]

**The matrix elements of quark level $n - \bar{n}$ conversion operators with $\mu = z$ only:**

| | $I_1$ | | | | $I_2$ | | | | | $I_3$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\chi_1\chi_2\chi_3$ | $\chi = R$ | $\chi = L$ | EM | $\chi_1\chi_2\chi_3$ | $\chi = R$ | $\chi = L$ | EM | $\chi_1\chi_2\chi_3$ | $\chi = R$ | $\chi = L$ | EM |
| RRR | 19.8 | 19.8 | 0 | RRR | -4.95 | -4.95 | 0 | RRR | 1.80 | -8.28 | 10.1 |
| RRL | 17.3 | 17.3 | 0 | RRL | -2.00 | -9.02 | 7.02 | RRL | -1.07 | -8.81 | 7.74 |
| RLR | 17.3 | 17.3 | 0 | RLR | -4.09 | -0.586 | -3.50 | RLR | 7.20 | 6.03 | 1.17 |
| RLL | 6.02 | 6.02 | 0 | RLL | -0.586 | -4.09 | 3.50 | RLL | 6.03 | 7.20 | -1.17 |
| LRR | 6.02 | 6.02 | 0 | LRR | -4.09 | -0.586 | -3.50 | LRR | 7.20 | 6.03 | 1.17 |
| LRL | 17.3 | 17.3 | 0 | LRL | -0.586 | -4.09 | 3.50 | LRL | 6.03 | 7.20 | -1.17 |
| LLR | 17.3 | 17.3 | 0 | LLR | -9.02 | -2.00 | -7.02 | LLR | -8.78 | -1.04 | -7.74 |
| LLL | 19.8 | 19.8 | 0 | LLL | -4.95 | -4.95 | 0 | LLL | -8.28 | 1.80 | -10.1 |

**The matrix elements of $\tilde{\mathcal{O}}_1$ vanish because its operator structure is associated with the same flavor.** [S. Gardner. and X. Yan (2016)]

# Relations between coupling parameters

**Assuming $\mathbf{SU}(2)_\mathbf{L} \times \mathbf{U}(1)_\mathbf{Y}$, only the matrix elements of $(\mathcal{O}_3)_{LLR}$ and $(\mathcal{O}_3)_{LLR}$ conversion operators are non-vanishing. We pick $(\mathcal{O}_3)_{LLR}$ as an illustration.**

**Applying the matching condition to $n-\bar{n}$ oscillation gives $\delta = (\delta_3)_{LLR}\langle\mathcal{O}_3\rangle_{LLR}$, and to $n-\bar{n}$ conversion yields**

$$\eta j_z = (\delta_3)_{LLR}((I_3)^{R3}_{LLR} - (I_3)^{L3}_{LLR})\frac{e}{3}\frac{m}{p^2-m^2}\frac{Qej_z}{q^2},$$

$$\Rightarrow \eta = \left(\frac{\delta}{q^2}\right)\left(\frac{m}{p^2-m^2}\right)\left(\frac{Qe^2}{3}\right)\left(\frac{(I_3)^{R3}_{LLR}-(I_3)^{L3}_{LLR}}{\langle\mathcal{O}_3\rangle_{LLR}}\right)$$
$$= \left(\frac{\delta}{q^2}\right)\left(\frac{0.108\ \text{GeV}^{-1}}{0.365^2}\right)\left(\frac{Qe^2}{3}\right)\left(\frac{-7.74}{2.03}\right).$$

**More generally, we replace $\delta$ by $\tilde{\delta}$, with**

$$\tilde{\delta} \equiv \left[\frac{\langle\mathcal{O}_3\rangle_{LLR}}{(I_3)^{R3}_{LLR} - (I_3)^{L3}_{LLR}}\right]\sum^{i=2,3}_{\chi_1,\chi_2,\chi_3}\left[(\delta_i)_{\chi_1,\chi_2,\chi_3}\left((I_i)^{\mathrm{R}\,3}_{\chi_1,\chi_2,\chi_3} - (I_i)^{\mathrm{L}\,3}_{\chi_1,\chi_2,\chi_3}\right)\right]$$

- **The mass scale of the suppression need not come from BSM theory.**
- **If $n-\bar{n}$ oscillation can occur, so can $n-\bar{n}$ conversion. They are complementary.**
- **Not all $n-\bar{n}$ oscillation operators contribute to $n-\bar{n}$ conversion processes.**

# Applications of $n - \bar{n}$ conversion

**Set limit on $\tilde{\delta}$ through $n - \bar{n}$ conversion processes at low energies, mediated by electromagnetically charged particle scattering.**

Two processes are considered:

- $n + Q \to \bar{n} + Q$, **i.e. neutrons scatter with a charged particle (with electric charge Q) target.**
- $Q + n \to Q + \bar{n}$, **i.e. a charged particle scatter with a neutron target.**

**The event rate for a fixed-target experiment is given by**

$$\frac{dR}{dt} = \mathcal{L}\sigma = \phi\rho L\sigma,$$

**where $\mathcal{L}$ denotes luminosity with units $\mathrm{cm}^{-2}\mathrm{s}^{-1}$, $R$ is the number of events, $\phi$ is the flux of incoming particles.**

**Comments about the cross section:**

- ✻ **It increases as $\theta$ goes to zero, we only focus on the forward scattering.**
- ✻ **We estimate the total cross section within a solid angle $\pi * \theta_0^2$.**

# Experimental setups

**The angle $\theta_0$ (in radians):**

- **Neutron scattering process: $\theta_0 \approx 0.003$ , which is based on a setup for a T violation search in neutron transmission experiment.** [J. D. Bowman and V. Gudkov (2014)] **Note there exists an upper limit of $\theta_0 = m_e/m_n$ from the energy-momentum conservation constrain.**
- **Electron beams: $\theta_0 = 3.73 \times 10^{-5}$, which is determined by the uncertainty principle vs. the Coulomb interaction.** [R. C. Fernow (1986)]

**Experiment setups we used:**

- Slow neutron beams: $\phi = 1.7 \times 10^{11}$ s$^{-1}$ and $|\mathbf{p_n}| \simeq 2$ keV, [Baldo-Ceolin et al. (ILL) (1994)] Fast neutron beams: $\phi = 5 \times 10^{8}$ s$^{-1}$ with $|\mathbf{p_n}| \simeq 0.447$ GeV.
- Electron beams in the DarkLight experiment: $\phi = 2 \times 10^{17}$ s$^{-1}$ and $|\mathbf{p_e}| \simeq 100$ MeV. [J. Balewski et al. (2014)]
- Density of the liquid deuterium at 19K: $\rho = 5 \times 10^{22}$ cm$^{-3}$. [Clusius, K., and Bartholome E (1935)]
- Density of the solid oxygen at 24K: $\rho = 5.76 \times 10^{22}$ cm$^{-3}$. [H. M. Roder (1978)]

# Set limits on $|\delta|$

**If we run the experiment for one year without observing one single event, i.e.,**

$$R = \phi L T \rho \sigma < 1,$$

**we can set a limit**

$$|\tilde{\delta}| < 1 \times 10^{-19} \left(\frac{|\boldsymbol{p_n}|}{2\ \text{KeV}}\right) \sqrt{\frac{1\ \text{yr}}{\text{t}}} \sqrt{\frac{1.7 \times 10^{11}\ \text{s}^{-1}}{\phi}} \sqrt{\frac{1\ \text{m}}{L}} \sqrt{\frac{5.76 \times 10^{22}\ \text{cm}^{-3}}{\rho}} \text{GeV}.$$

- $\mathbf{n} + \mathbf{d} \to \bar{\mathbf{n}} + \mathbf{d}$: $|\tilde{\delta}| \lesssim 3 \times 10^{-19}$ GeV for $|\boldsymbol{p_n}| = 2$ keV neutron beams, $|\delta| < 2 \times 10^{-11}$ GeV for $|\boldsymbol{p_n}| = 0.447$ GeV neutron beams;
- $\mathbf{n} + \mathbf{O} \to \bar{\mathbf{n}} + \mathbf{O}$: $|\tilde{\delta}| < 1.253 \times 10^{-19}$ GeV for $|\boldsymbol{p_n}| = 2$ keV neutron beams, $|\delta| < 7 \times 10^{-12}$ GeV for $|\boldsymbol{p_n}| = 0.447$ GeV neutron beams;
- $\mathbf{e} + \mathbf{n} \to \mathbf{e} + \bar{\mathbf{n}}$: $|\tilde{\delta}| < 1.784 \times 10^{-15}$ GeV for $|\boldsymbol{p_n}| = 100$ MeV electron beams;

**Cold neutrons scattering with a deuterium target or a solid oxygen target seems promising.**

# Summary and Outlook

- Through $n - \bar{n}$ conversion operators, we argue that it is possible to realize a $n - \bar{n}$ transition process with no sensitivity to the environmental effects.
- Phase constraints associated with discrete symmetry transformations show that only one additional $n - \bar{n}$ transition operator is left.
- Due to the connection between $n - \bar{n}$ oscillation and $n - \bar{n}$ conversion, we can determine the low energy "constant" of this operator through EM interaction and find that the additional mass scale of suppression needs not come from BSM physics.
- This operator offers us an opportunity to realize $n - \bar{n}$ transition through scattering experiments. We explore various limits on $n - \bar{n}$ oscillation through three promising $n - \bar{n}$ conversion proposals.

# Backup slides

# CPT transformation of $B - L$ violating operators

**For a general four-component fermion field:**

$$\begin{aligned} \mathbf{C}\psi(x)\mathbf{C}^{-1} &= \eta_c C\gamma^0\psi^*(x) = \eta_c i\gamma^2\psi^*(x) \equiv \eta_c\psi^c(x)\,, \\ \mathbf{P}\psi(t,\mathbf{x})\mathbf{P}^{-1} &= \eta_p\gamma^0\psi(t,-\mathbf{x})\,, \\ \mathbf{T}\psi(t,\mathbf{x})\mathbf{T}^{-1} &= \eta_t\gamma^1\gamma^3\psi(-t,\mathbf{x})\,. \end{aligned}$$

Thus $\mathbf{C}^2\psi(x)\mathbf{C}^{-2} = \psi(x)$, $\mathbf{T}^2\psi(x)\mathbf{T}^{-2} = -\psi(x)$ and $\mathbf{P}^2\psi(x)\mathbf{P}^{-2} = \eta_p^2\psi(x)$.

Note: **In normal case: $\mathcal{L} \ni \bar{\psi}\Gamma\psi$, $\Gamma$ is some product of gamma matrices.**

**Under discrete symmetry transformations:**
$\bar{\psi}\Gamma\psi \Rightarrow \pm|\eta|^2\bar{\psi}\Gamma\psi = \pm\bar{\psi}\Gamma\psi$, **therefore, phases do not matter.**

**IN THIS CASE...**

# CPT transformation of $B - L$ violating operators

$$\mathcal{O}_1 = \psi^T C \psi + \text{h.c.} \quad \overset{\text{CPT}}{\Longrightarrow} -(\eta_c \eta_p \eta_t)^2 \,,$$

$$\mathcal{O}_2 = \psi^T C \gamma_5 \psi + \text{h.c.} \quad \overset{\text{CPT}}{\Longrightarrow} -(\eta_c \eta_p \eta_t)^2 \,,$$

$$\mathcal{O}_3 = \psi^T C \gamma^\mu \psi \, \partial^\nu F_{\mu\nu} + \text{h.c.} \quad \overset{\text{CPT}}{\Longrightarrow} +(\eta_c \eta_p \eta_t)^2 \,,$$

$$\mathcal{O}_4 = \psi^T C \gamma^\mu \gamma_5 \psi \, \partial^\nu F_{\mu\nu} + \text{h.c.} \quad \overset{\text{CPT}}{\Longrightarrow} -(\eta_c \eta_p \eta_t)^2 \,,$$

$$\mathcal{O}_5 = \psi^T C \sigma_{\mu\nu} \psi \, F^{\mu\nu} + \text{h.c.} \quad \overset{\text{CPT}}{\Longrightarrow} +(\eta_c \eta_p \eta_t)^2 \,,$$

$$\mathcal{O}_6 = \psi^T C \sigma_{\mu\nu} \gamma_5 \psi \, F^{\mu\nu} + \text{h.c.} \quad \overset{\text{CPT}}{\Longrightarrow} +(\eta_c \eta_p \eta_t)^2 \,.$$

**Note:** The operators do not transform under CPT with definite sign.

If $\eta_c = \eta_p = \eta_t = 1$, all CPT even operators vanish identically due to the anticommutation relations of fermion fields.

# Majorana phase constraints

The plane-wave expansion of a general Majorana field $\psi_m$ is

$$\psi_m(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}\sqrt{2E}} \sum_s \left\{ f(\mathbf{p}, s)u(\mathbf{p}, s)e^{-ip\cdot x} + \lambda f^\dagger(\mathbf{p}, s)v(\mathbf{p}, s)e^{ip\cdot x} \right\} .$$

where $\lambda$ is the creation phase factor and can be chosen arbitrarily. Now applying C transformation and Majorana condition,

$$i\gamma^2\psi_m^*(x) = \lambda^*\psi_m(x),$$

yields

$$\mathbf{C}\psi_m(x)\mathbf{C}^{-1} \quad = \quad \eta_c\lambda^*\psi_m(x),$$

i.e. $\mathbf{C}f(\mathbf{p}, s)\mathbf{C}^{-1} = \eta_c\lambda^* f(\mathbf{p}, s)$ and $\mathbf{C}f^\dagger(\mathbf{p}, s)\mathbf{C}^{-1} = \eta_c\lambda^* f^\dagger(\mathbf{p}, s)$.
**Since C is a unitary operator, Hermitian conjugate shows $\eta_c^*\lambda$ is real.**

# Majorana phase constraints

Under **CP**, we find $\eta_p^*\eta_c^*\lambda$ must be imaginary, or $\eta_p^*$ must be imaginary.
Under **T**, we have $\eta_t\lambda$ must be real.
Under **CPT**, we have

$$\mathbf{CPT}\psi_m(x)(\mathbf{CPT})^{-1} = -\eta_c\eta_p\eta_t\gamma^5\psi_m^*(-x)\,,$$

or

$$\begin{aligned}\mathbf{CPT}f(\mathbf{p},s)(\mathbf{CPT})^{-1} &= s\lambda^*\eta_c\eta_p\eta_t f(\mathbf{p},-s)\,,\\ \mathbf{CPT}f^\dagger(\mathbf{p},s)(\mathbf{CPT})^{-1} &= -s\lambda\eta_c\eta_p\eta_t f^\dagger(\mathbf{p},-s)\,.\end{aligned}$$

Notice **CPT** is **antiunitary** and define $\mathbf{CPT} = KU_{cpt}$, where $U_{cpt}$ denotes a unitarity operator. We find $\eta_c\eta_p\eta_t$ is pure imaginary!.

- **C**: $\eta_c^*\lambda$ is real;
- **CP**: $\eta_p^*\eta_c^*\lambda$ is imaginary or $\eta_p^*$ is imaginary;
- **T**: $\eta_t\lambda$ is real;
- **CPT**: $\eta_c\eta_p\eta_t$ is imaginary. $\Rightarrow \eta_c\eta_t$ is real.

**Notice order does not matter and no constraint for** $\eta_c\eta_p$.

# Phase constraints for Dirac field in B-L violation theories

The plane-wave expansion of a Dirac field $\psi(x)$ is given by

$$\psi(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}\sqrt{2E}} \sum_{s=\pm} \{b(\mathbf{p},s)u(\mathbf{p},s)e^{-ip\cdot x} + d^\dagger(\mathbf{p},s)v(\mathbf{p},s)e^{ip\cdot x}\}$$

Construct a Majorana field from Dirac fields:

$$\psi_{m\pm}(x) = \frac{1}{\sqrt{2}}(\psi(x) \pm \mathbf{C}\psi(x)\mathbf{C}^\dagger)$$

then plane-wave expansion is

$$\psi_{m\pm}(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}\sqrt{2E}} \sum_{s} \left\{w_\pm(\mathbf{p},s)u(\mathbf{p},s)e^{-ip\cdot x} \pm \eta_c w_\pm^\dagger(\mathbf{p},s)v(\mathbf{p},s)e^{ip\cdot x}\right\}.$$

where $w_{m\pm}(\mathbf{p},s) \equiv \frac{1}{\sqrt{2}}[b(\mathbf{p},s) \pm \eta_c d(\mathbf{p},s)]$ and $\lambda = \pm\eta_c$. We find the **same** phase constraints for Dirac fields as Majorana fields.

# Majorana phase constraints

**The plane-wave expansion of a general Majorana field $\psi_m$ is**

$$\psi_m(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}\sqrt{2E}} \sum_s \left\{ f(\mathbf{p},s)u(\mathbf{p},s)e^{-ip\cdot x} + \lambda f^\dagger(\mathbf{p},s)v(\mathbf{p},s)e^{ip\cdot x} \right\} ,$$

where $\lambda$ is the creation phase factor and can be chosen arbitrarily. Now applying **C transformation** and **Majorana condition**,

$$i\gamma^2\psi_m^*(x) = \lambda^*\psi_m(x),$$

yields

$$\mathbf{C}\psi_m(x)\mathbf{C}^{-1} = \eta_c\lambda^*\psi_m(x),$$

i.e. $\mathbf{C}f(\mathbf{p},s)\mathbf{C}^{-1} = \eta_c\lambda^* f(\mathbf{p},s)$ and $\mathbf{C}f^\dagger(\mathbf{p},s)\mathbf{C}^{-1} = \eta_c\lambda^* f^\dagger(\mathbf{p},s)$.
**Since C is a unitary operator, Hermitian conjugate shows $\eta_c^*\lambda$ is real.**

# Majorana phase constraints

Under **CP**, we find $\eta_p^* \eta_c^* \lambda$ must be imaginary, or $\eta_p^*$ must be imaginary.
Under **T**, we have $\eta_t \lambda$ must be real.
Under **CPT**, we have

$$\mathbf{CPT}\psi_m(x)(\mathbf{CPT})^{-1} = -\eta_c \eta_p \eta_t \gamma^5 \psi_m^*(-x)\,,$$

or

$$\begin{aligned} \mathbf{CPT} f(\mathbf{p}, s)(\mathbf{CPT})^{-1} &= s\lambda^* \eta_c \eta_p \eta_t f(\mathbf{p}, -s)\,, \\ \mathbf{CPT} f^\dagger(\mathbf{p}, s)(\mathbf{CPT})^{-1} &= -s\lambda \eta_c \eta_p \eta_t f^\dagger(\mathbf{p}, -s)\,. \end{aligned}$$

Notice **CPT** is **antiunitary** and define $\mathbf{CPT} = KU_{cpt}$, where $U_{cpt}$ denotes a unitarity operator and $K$ denotes complex conjugation. We find $\eta_c \eta_p \eta_t$ is pure imaginary!.

- **C**: $\eta_c^* \lambda$ is real;
- **CP**: $\eta_p^* \eta_c^* \lambda$ is imaginary or $\eta_p^*$ is imaginary;
- **T**: $\eta_t \lambda$ is real;
- **CPT**: $\eta_c \eta_p \eta_t$ is imaginary. $\Rightarrow \eta_c \eta_t$ is real.

**Notice order does not matter and no constraint for $\eta_c \eta_p$ exists.**

# Phase constraints for Dirac field in B-L violation theories

**The plane-wave expansion of a Dirac field $\psi(x)$ is given by**

$$\psi(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}\sqrt{2E}} \sum_{s=\pm} \{ b(\mathbf{p},s)u(\mathbf{p},s)e^{-ip\cdot x} + d^\dagger(\mathbf{p},s)v(\mathbf{p},s)e^{ip\cdot x} \}$$

**Construct a Majorana field from Dirac fields:**

$$\psi_{m\pm}(x) = \frac{1}{\sqrt{2}}(\psi(x) \pm \mathbf{C}\psi(x)\mathbf{C}^\dagger)$$

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$$\psi_{m\pm}(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}\sqrt{2E}} \sum_{s} \left\{ w_\pm(\mathbf{p},s)u(\mathbf{p},s)e^{-ip\cdot x} \pm \eta_c w_\pm^\dagger(\mathbf{p},s)v(\mathbf{p},s)e^{ip\cdot x} \right\}.$$

where $w_{m\pm}(\mathbf{p},s) \equiv \frac{1}{\sqrt{2}}[b(\mathbf{p},s) \pm \eta_c d(\mathbf{p},s)]$ and $\lambda = \pm\eta_c$. **Therefore, we find the same phase constraints for Dirac fields as Majorana fields.**

# Phase constraints are intrinsic

Now,

$$\begin{aligned}
\mathcal{O}_1 &= \psi^T C \psi + \text{h.c.} & \stackrel{\textsf{CPT}}{\Longrightarrow} +1\,,\\
\mathcal{O}_2 &= \psi^T C \gamma_5 \psi + \text{h.c.} & \stackrel{\textsf{CPT}}{\Longrightarrow} +1\,,\\
\mathcal{O}_3 &= \psi^T C \gamma^\mu \psi\, \partial^\nu F_{\mu\nu} + \text{h.c.} & \stackrel{\textsf{CPT}}{\Longrightarrow} -1\,,\\
\mathcal{O}_4 &= \psi^T C \gamma^\mu \gamma_5 \psi\, \partial^\nu F_{\mu\nu} + \text{h.c.} & \stackrel{\textsf{CPT}}{\Longrightarrow} +1\,,\\
\mathcal{O}_5 &= \psi^T C \sigma_{\mu\nu} \psi\, F^{\mu\nu} + \text{h.c.} & \stackrel{\textsf{CPT}}{\Longrightarrow} -1\,,\\
\mathcal{O}_6 &= \psi^T C \sigma_{\mu\nu} \gamma_5 \psi\, F^{\mu\nu} + \text{h.c.} & \stackrel{\textsf{CPT}}{\Longrightarrow} -1\,.
\end{aligned}$$

**Once the anticommuting nature of the fermion fields is taken into account, only CPT even operators survive.**

**A natural way to understand the existence of phase constraints is to consider them as intrinsic properties of discrete symmetry transformations.**

# Implications of the CPT phases

**4×4 effective Hamiltonian framework**

Work in the basis $|n(\mathbf{p},+)\rangle, |\bar{n}(\mathbf{p},+)\rangle, n(\mathbf{p},-)\rangle, |\bar{n}(\mathbf{p},-)\rangle$.
[SG and Jafari (2015)]

**Spin-dependent SM effects involving transverse magnetic fields could realize $n-\bar{n}$ transitions in which the particle spin flips without magnetic quenching.**

However, it is sensitive to the CPT phase constraint.

Consider $n-\bar{n}$ oscillate in a static $\mathbf{B}_0$ with $\omega_0 \equiv -\mu_n B_0$. Apply a static $\mathbf{B}_1$ suddenly at $t=0$ and define $\omega_1 \equiv -\mu_n B_1$. The Hamiltonian matrix at $t>0$ is

$$\mathcal{H} = \begin{pmatrix} M+\omega_0 & \delta & \omega_1 & 0 \\ \delta & M-\omega_0 & 0 & -\omega_1 \\ \omega_1 & 0 & M-\omega_0 & -\delta\eta_{cpt}^2 \\ 0 & -\omega_1 & -\delta\eta_{cpt}^2 & M+\omega_0 \end{pmatrix},$$

where $\delta$ denotes a $n(+)\to\bar{n}(+)$ transition matrix element.

**The matrix element $\langle \bar{n} | O | n \rangle$ can be calculated in the MIT bag model** [Rao and Shrock (1982)] **or through lattice QCD.** [M. Buchoff et al (2012), S. Syritsyn et al (2015)]

**Table 2:** Preliminarly results for matrix elements of 6-quark operators 2.1 and comparison to the MIT Bag Model results [8]. The first line shows matrix elements for $I = 3_{R,L}$ operators vanishing identically.

| | $Z(\text{lat} \to \overline{MS})$ | $\mathcal{O}^{\overline{MS}(2\text{ GeV})}[10^{-5}\text{GeV}^6]$ | Bag "A" | $\frac{\text{LQCD}}{\text{Bag "A"}}$ | Bag "B" | $\frac{\text{LQCD}}{\text{Bag "B"}}$ |
|---|---|---|---|---|---|---|
| $[(RRR)_3]$ | 0.62(12) | 0 | 0 | – | 0 | – |
| $[(RRR)_1]$ | 0.454(33) | 45.4(5.6) | 8.190 | 5.5 | 6.660 | 6.8 |
| $[R_1(LL)_0]$ | 0.435(26) | 44.0(4.1) | 7.230 | 6.1 | 6.090 | 7.2 |
| $[(RR)_1 L_0]$ | 0.396(31) | -66.6(7.7) | -9.540 | 7.0 | -8.160 | 8.1 |
| $[(RR)_2 L_1]^{(1)}$ | 0.537(52) | -2.12(26) | 1.260 | -1.7 | -0.666 | 3.2 |
| $[(RR)_2 L_1]^{(2)}$ | 0.537(52) | 0.531(64) | -0.314 | -1.7 | 0.167 | 3.2 |
| $[(RR)_2 L_1]^{(3)}$ | 0.537(52) | -1.06(13) | 0.630 | -1.7 | -0.330 | 3.2 |

**We use the MIT bag model to evaluate the matrix elements.**

# B-L violation and theories of self-conjugate fermions

**In 1967, in attempting to rationalize the spectral pattern of the low-lying, light hadrons, Carruthers discovered a class of theories for which the CPT theorem does not hold.** [Carruthers (1967)]

**The pions form a self-conjugate isospin multiplet $(\pi^+, \pi^0, \pi^-)$, but the kaons form pair-conjugate multiplets $(K^+, K^0)$ and $(\bar{K}^0, K^-)$.**

**Carruthers discovered that free theories of self-conjugate bosons with half-integer isospin are nonlocal, that the commutator of two self-conjugate fields with opposite isospin components do not vanish at space-like separations.** [Carruthers (1967)]

**Same conclusion for theories of arbitrary spin.** [Lee (1967), Fleming and Kazes (1967), Jin (1967).]

# B-L violation and theories of self-conjugate fermions

**Failure of weak local communitivity $\Rightarrow$ CPT symmetry is not expected to hold, nor should the CPT theorem of Greenberg apply.**
[Carruthers (1968), Streater and Wightman (2000), Greenberg (2002)]

**The conclusion is it is possible to have self-conjugate theories of I=0, but it is not possible to have self-conjugate theories of I=1/2**

**Note neutron and antineutron are members of pair-conjugate I = 1/2 multiplets. In addition, the quark-level operators that generate $n - \bar{n}$ oscillations would also produce $p - \bar{p}$ oscillations under the isospin transformation $u \leftrightarrow d$. In QCD in the chiral limit, $n - \bar{n}$ oscillations are indistinguishable with $p - \bar{p}$ oscillations. Then if $n - \bar{n}$ oscillation happens, neutron and proton would form a self-conjugate isofermion pair with a half- integer isospin and break weak locality.**

**Therefore, $B - L$ violation is not compatible with QCD in the chiral limit.**

# CPT transformation of $B - L$ violating operators

**For a general four-component fermion field:**

$$\mathbf{C}\psi(x)\mathbf{C}^{-1} = \eta_c C\gamma^0\psi^*(x) = \eta_c i\gamma^2\psi^*(x) \equiv \eta_c\psi^c(x)\,,$$
$$\mathbf{P}\psi(t,\mathbf{x})\mathbf{P}^{-1} = \eta_p\gamma^0\psi(t,-\mathbf{x})\,,$$
$$\mathbf{T}\psi(t,\mathbf{x})\mathbf{T}^{-1} = \eta_t\gamma^1\gamma^3\psi(-t,\mathbf{x})\,.$$

Thus $\mathbf{C}^2\psi(x)\mathbf{C}^{-2} = \psi(x)$, $\mathbf{T}^2\psi(x)\mathbf{T}^{-2} = -\psi(x)$ and $\mathbf{P}^2\psi(x)\mathbf{P}^{-2} = \eta_p^2\psi(x)$.

Note: **In normal case: $\mathcal{L} \ni \bar{\psi}\Gamma\psi$, $\Gamma$ is some product of gamma matrices.**

**Under discrete symmetry transformations:**
$\bar{\psi}\Gamma\psi \Rightarrow \pm|\eta|^2\bar{\psi}\Gamma\psi = \pm\bar{\psi}\Gamma\psi$, **therefore, phases do not matter.**