Neutron-Antineutron Oscillations: Appearance, Disappearance, and Baryogenesis

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Neutron-Antineutron Oscillations and Discrete Symmetries

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C, P and T Symmetries In $|\Delta\mathcal{B}| = 2$ Transitions

In our 2015 text Zurab Berezhiani, AV, arXiv:1506.05096
we noted that the parity $\mathbf{P}$, defined in such a way that $\mathbf{P}^2 = 1$, is broken in n-nbar transition as well as $\mathbf{CP}$.
We took away our claim of CP breaking in September of the same 2015 when we presented at a similar workshop here a modified definition of parity $\mathbf{P}_z$, such that $\mathbf{P}_z^2 = -1$.
With this modification all discrete symmetries are preserved in n-nbar transition.

This modification was later discussed by S. Gardner and X. Yan,
I will present some details of an interesting history of the subject which goes back to Majorana and Racah's papers of 1937.

Dirac Lagrangian for neutron

$$\mathcal{L}_D = i\bar{n}\gamma^\mu \partial_\mu n - m\,\bar{n}n$$

describes free neutron and antineutron and preserves the baryon charge, $\mathcal{B} = 1$ for n, $\mathcal{B} = -1$ for $\bar{n}$.
Continuous $\mathrm{U}(1)_\mathcal{B}$ symmetry:

$$n \to \mathrm{e}^{i\alpha} n, \quad \bar{n} \to \mathrm{e}^{-i\alpha}\bar{n}$$

Another term $-im'\bar{n}\gamma_5 n$ consistent with $\mathcal{B}$ conservation can be rotated away by the chiral rotation, $n \to \mathrm{e}^{i\beta\gamma_5} n$.
Four degenerate states: two spin doublets differ by $\mathcal{B}$.

How does baryon number non-conservation shows up?
At the level of free particles it could be only bilinear $|\Delta\mathcal{B}| = 2$ mass terms:

$$C = i\gamma^2\gamma^0$$

$$n^T C n\,, \quad n^T C\gamma_5 n\,, \quad \bar{n}C\bar{n}^T, \quad \bar{n}C\gamma_5\bar{n}^T$$

All these bilinear in fields, presenting the most generic Lorentz invariant modifications, are reduced by field redefinitions to the only one term, breaking baryon charge by two units,

$$\Delta\mathcal{L}_{\not{B}} = -\frac{1}{2}\,\epsilon\left[n^T C n + \bar{n} C \bar{n}^T\right] \qquad C = i\gamma^2\gamma^0$$

where ϵ is a real positive parameter. Redefinitions are thanks to U(2) symmetry of the kinetic term $i\bar{n}\gamma^\mu\partial_\mu n$.

What is the status of $\mathbf{C}$, $\mathbf{P}$ and $\mathbf{T}$ discrete symmetries? Let us start with the charge conjugation $\mathbf{C}$:

$$\mathbf{C}: \quad n \longleftrightarrow n^c = C\bar{n}^T$$

Kind of Z_2 symmetry, $\mathbf{C}^2 = 1$. Most simple in the Majorana representation

$$n^c = n^*\,.$$

Lagrangians can be rewritten as

$$\mathcal{L}_D = \frac{i}{2}\left[\bar{n}\gamma^\mu\partial_\mu n + \overline{n^c}\gamma^\mu\partial_\mu n^c\right] - \frac{m}{2}\left[\bar{n}n + \overline{n^c}\,n^c\right],$$

$$\Delta\mathcal{L}_{\not{B}} = -\frac{1}{2}\,\epsilon\left[\overline{n^c}\,n + \bar{n}\,n^c\right],$$

what makes C-invariance explicit.

Lagrangians are diagonalized in terms of Majorana fields $n_{1,2}$

$$n_1 = \frac{n \pm n^c}{\sqrt{2}}, \qquad \mathrm{C}n_{1,2} = \pm\, n_{1,2}.$$

$$\mathcal{L}_D = \frac{1}{2}\sum_{k=1,2}\left[\bar{n}_k\gamma^\mu\partial_\mu n_k - m\,\bar{n}_k n_k\right],$$

$$\Delta\mathcal{L}_{\not{B}} = -\frac{1}{2}\,\epsilon\left[\bar{n}_1\,n_1 - \bar{n}_2\,n_2\right].$$

Splitting into two Majorana spin doublets with masses

$$M_1 = m + \epsilon \qquad M_2 = m - \epsilon.$$

The parity transformation $\mathbf{P}$ involves, besides reflection of space coordinates, the substitution

$$\mathbf{P}: \qquad n \to \gamma^0 n\,, \qquad n^c \to -\gamma^0 n^c\,.$$

We use $\gamma^0 C \gamma^0 = -C$. The opposite signs reflect the opposite parities of fermion and antifermion C.N. Yang '50 V.B. Berestetsky '51

The definition satisfies $\mathbf{P}^2 = \mathbf{1}$ so eigenvalues of $\mathbf{P}$ are $\pm\mathbf{1}$, opposite parities for fermion and antifermion. Different parities of neutron and antineutron implies that their mixing breaks $\mathbf{P}$ parity. Indeed, $\mathbf{P}$-transformation changes $\Delta\mathcal{L}_{\not B}$ to $(-\Delta\mathcal{L}_{\not B})$. With $\mathbf{C}$- invariance it implies that $\Delta\mathcal{L}_{\not B}$ is also $\mathbf{CP}$ odd.

This CP-oddness, however, does not translates immediately into observable CP-breaking effects. To get them one needs an interference of amplitudes provided by interaction.

This subtlety is discussed in textbooks, see e.g.
V. B. Berestetsky, E. M. Lifshitz and L. P. Pitaevsky,
Let's remind it.

When $\mathcal{B}$ is conserved there is there is no transition between sectors with different $\mathcal{B}$. One can combine $\mathbf{P}$ with a $\mathrm{U}(1)_{\mathcal{B}}$ phase rotation and define $\mathbf{P}_\alpha$

$$\mathbf{P}_\alpha = \mathbf{P}\,\mathrm{e}^{i\mathcal{B}\alpha}: \quad n \to \mathrm{e}^{i\alpha}\gamma^0 n\,, \quad n^c \to -\mathrm{e}^{-i\alpha}\gamma^0 n^c$$

Of coarse, then $\mathbf{P}_\alpha^2 = \mathrm{e}^{2i\mathcal{B}\alpha} \neq 1$ but the phase is unobservable while $\mathcal{B}$ is is conserved.

When $\mathcal{B}$ is its not conserved the only remnant of $\mathrm{U}(1)_{\mathcal{B}}$ rotations is Z_2 symmetry, $n \to -n$. It means that we can consider a different parity definition P_z, such that $\mathrm{P}_z^2 = -1$.

Thus, choosing $\alpha = \pi/2$ we come to

$$\mathrm{P}_z = \mathrm{P}e^{iB\pi/2}: \qquad n \to i\gamma^0 n\,, \qquad n^c \to i\gamma^0 n^c\,.$$

Moreover, in case of Majorana fermions it is the only possible choice. Indeed, in Majorana representation where

$$\gamma^0 = \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix}$$

only $i\gamma^0$ preserves reality of the Majorana spinor.

This was derived by Giulio Racah in 1937.

Now P_z parities of n and $\bar{n}$ are the same $\cdot i$, so their mixing does not break the P_z parity. It means that all discrete symmetries, C, P_z and T are preserved by $\Delta\mathcal{L}_{\not B}$.

A few comments. First, preservation of T follows from CPT theorem provided by Lorentz invariance and locality. Second, it is amusing that the same parity for n and n^c equal to $\cdot i$ is consistent with the notion of the opposite parities for fermion and antifermion: one should compare $\mathrm{P}_z(n)$ with $[\mathrm{P}_z(n^c)]^*$. Third, P_z commutes with C , i.e., $\mathrm{CP}_z = \mathrm{P}_z\mathrm{C}$, in contrast with P which anticommutes, $\mathrm{CP} = -\mathrm{PC}$.

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Similar effects for neutrino was noted by Wolfenstein '81.

Six-quarks operators: discrete symmetries

New physics beyond the Standard Model, leading to $|\Delta B| = 2$ transitions, induces the effective six-quark interaction,

$$\mathcal{L}\left(\Delta\mathcal{B} = -2\right) = \frac{1}{M^5}\sum c_i \mathcal{O}^i \,,$$
$$\mathcal{O}^i = T^i_{A_1 A_2 A_3 A_4 A_5 A_6} q^{A_1} q^{A_2} q^{A_3} q^{A_4} q^{A_5} q^{A_6} \,,$$

where coefficients T^i account for color, flavor and spinor structures.

In particular, for n-nbar mixing

$$\langle\bar{n}|\, \mathcal{L}\left(\Delta\mathcal{B} = -2\right) |n\rangle = -\frac{1}{2}\,\epsilon\, v^T_{\bar{n}} C\, u_n$$

it lead to an estimate .

$$\epsilon = \frac{1}{\tau_{n\bar{n}}} \sim \frac{\Lambda^6_{\mathrm{QCD}}}{M^5} \,.$$

For u and d quarks of the first generation the full list of operators was determined S. Rao and R. Shrock, W. E. Caswell, J. Milutinovic and G. Senjanovic

$$\mathcal{O}^1_{\chi_1\chi_2\chi_3} = u^{iT}_{\chi_1} C u^j_{\chi_1} d^{kT}_{\chi_2} C d^l_{\chi_2} d^{mT}_{\chi_3} C d^n_{\chi_3} \left[\epsilon_{ikm}\epsilon_{jln} + \epsilon_{ikn}\epsilon_{jlm} + \epsilon_{jkm}\epsilon_{nil} + \epsilon_{jkn}\epsilon_{ilm}\right],$$

$$\mathcal{O}^2_{\chi_1\chi_2\chi_3} = u^{iT}_{\chi_1} C d^j_{\chi_1} u^{kT}_{\chi_2} C d^l_{\chi_2} d^{mT}_{\chi_3} C d^n_{\chi_3} \left[\epsilon_{ikm}\epsilon_{jln} + \epsilon_{ikn}\epsilon_{jlm} + \epsilon_{jkm}\epsilon_{nil} + \epsilon_{jkn}\epsilon_{ilm}\right],$$

$$\mathcal{O}^3_{\chi_1\chi_2\chi_3} = u^{iT}_{\chi_1} C d^j_{\chi_1} u^{kT}_{\chi_2} C d^l_{\chi_2} d^{mT}_{\chi_3} C d^n_{\chi_3} \left[\epsilon_{ijm}\epsilon_{kln} + \epsilon_{ijn}\epsilon_{klm}\right].$$

Here χ_i stand for L or R quark chirality. Accounting for relations

$$\mathcal{O}^1_{\chi LR} = \mathcal{O}^1_{\chi RL}, \quad \mathcal{O}^{2,3}_{LR\chi} = \mathcal{O}^{2,3}_{RL\chi},$$

$$\mathcal{O}^2_{\chi\chi\chi'} - \mathcal{O}^1_{\chi\chi\chi'} = 3\mathcal{O}^3_{\chi\chi\chi'},$$

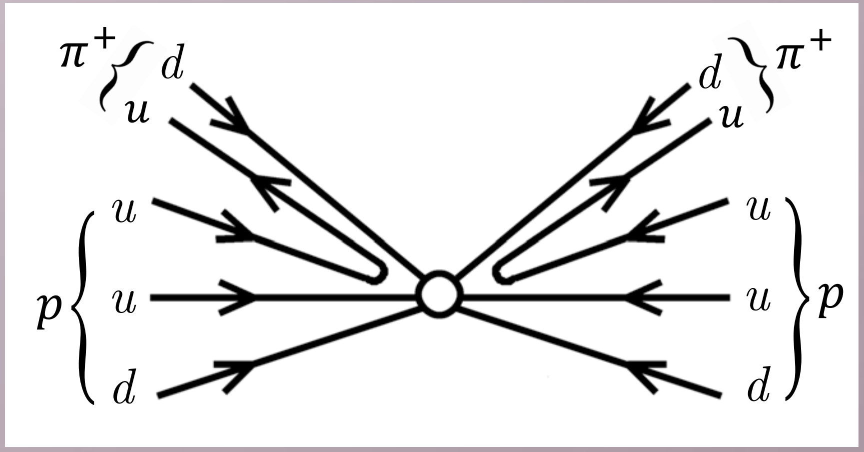
we deal with 14 operators for $\Delta\mathcal{B} = -2$ transitions.

Only combinations of operators which are $\mathbf{P}_z$ even contributes to n-nbar mixing. The $\mathbf{P}_z$ reflection interchanges L and R chiralities χ_i in the operators $O^i_{\chi_1\chi_2\chi_3}$.
Thus, only 7 combinations

$$O^i_{\chi_1\chi_2\chi_3} + L \leftrightarrow R$$

of 14 operators contribute to n-nbar mixing.

What about remaining $\mathbf{P}_z$ odd combinations $(O^i_{\chi_1\chi_2\chi_3} - L \leftrightarrow R)$? Although they do not contribute to n-nbar mixing their effect shows up in instability of nuclei. This source of instability is due to two nucleon annihilation into pions.



The charge conjugation $\mathbf{C}$ transforms operators $O^i_{\chi_1\chi_2\chi_3}$ into Hermitian conjugated $[O^i_{\chi_1\chi_2\chi_3}]^\dagger$. So, we have 14 C-even operators, $O^i_{\chi_1\chi_2\chi_3} + \mathbf{H.c.}$, and 14 C-odd ones, $O^i_{\chi_1\chi_2\chi_3} - \mathbf{H.c.}$

In total,we break all 28 operators in four sevens with different $\mathbf{P}_z$, $\mathbf{C}$ and $\mathbf{CP}_z$ features,

$$\left[O^i_{\chi_1\chi_2\chi_3} + L \leftrightarrow R\right] + \mathbf{H.c.}, \quad \mathbf{P}_z = +, \ \mathbf{C} = +, \ \mathbf{CP}_z = +$$

$$\left[O^i_{\chi_1\chi_2\chi_3} + L \leftrightarrow R\right] - \mathbf{H.c.}, \quad \mathbf{P}_z = +, \ \mathbf{C} = -, \ \mathbf{CP}_z = -$$

$$\left[O^i_{\chi_1\chi_2\chi_3} - L \leftrightarrow R\right] + \mathbf{H.c.}, \quad \mathbf{P}_z = -, \ \mathbf{C} = +, \ \mathbf{CP}_z = -$$

$$\left[O^i_{\chi_1\chi_2\chi_3} - L \leftrightarrow R\right] - \mathbf{H.c.}, \quad \mathbf{P}_z = -, \ \mathbf{C} = -, \ \mathbf{CP}_z = +$$

Only the first seven which are both $\mathbf{P}_z$ and $\mathbf{C}$ even contribute to n-nbar mixing.