Theoretical Models Can Predict $n - \bar{n}$ Oscillations at an Observable Rate

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Outline

- Some Background
- General formalism and current experimental limits
- Operator analysis and estimate of matrix elements
- Illustrative model giving $n - \bar{n}$ oscillations at an observable rate
- Conclusion: $n - \bar{n}$ oscillations can occur at rates comparable to current limits, providing motivation for a new experiment with greater sensitivity.

Some Background

Idea that an electrically neutral fermion might be its own antiparticle (Majorana, 1937).

Later, idea of a conserved global baryon no.

Producing the observed baryon asymmetry in the universe requires interactions that violate baryon number, B (as well as CP violation and deviation from thermal equilibrium) (Sakharov, 1967).

Suggestion of $n - \bar{n}$ transitions as a mechanism involved in generating baryon asymmetry in the universe (Kuzmin, 1970).

Since (anti)quarks and (anti)leptons are placed in same representations in grand unified theories (GUT's), the violation of B and L is natural in these theories. Besides proton decay, $n - \bar{n}$ oscillations can occur (Glashow, 1979; Marshak and Mohapatra, 1980; Kuo and Love, 1980).

Calculation of matrix elements for $n - \bar{n}$ transitions (Rao and Shrock, 1982).

Although the Standard Model (SM) conserves B perturbatively, SU(2) instantons produce nonperturbative violation B and L, while conserving $B-L$ ('t Hooft, 1976). This is negligible (exponentially small) at zero temperature, T but is important at T of order the electroweak scale (Kuzmin, Rubakov, Shaposhnikov, 1985).

Current interest in various models of baryogenesis.

B and L are global symmetries in the original SM. Neutrino masses and lepton mixing are confirmed physics beyond the SM; the most natural mechanism to explain light neutrino masses is the seesaw mechanism, which involves a combination of Dirac mass terms $\bar{\nu}_{iL} M_{ij}^{(D)} \nu_{j,R} + h.c.$ and Majorana mass terms $\nu_{i,R}^T C M_{ij}^{(R)} \nu_{j,R} + h.c.$; the Majorana terms break L, as $\Delta L = 2$ operators.

In addition to the conventional GUT-scale seesaw, there are also low-scale seesaw mechanisms, e.g., T. Appelquist and R. Shrock, Phys. Lett. B548, 204 (2002); Phys. Rev. Lett. 90, 201801 (2003).

The occurrence of $\Delta L = 2$ operators, possibly at a low-scale, in neutrino mass models gives further motivation to explore the possibility that there might also be $\Delta B = 2$ operators at scales well below a GUT scale.

General Formalism and Experimental Limits

$n - \bar{n}$ Oscillations in Field-Free Vacuum:

CPT: $\langle n|H_{eff}|n\rangle = \langle \bar{n}|H_{eff}|\bar{n}\rangle = m_n - i\lambda_n/2$, where H_{eff} denotes relevant Hamiltonian and $\lambda_n^{-1} = \tau_n = 0.88 \times 10^3$ sec. H_{eff} may also mediate $n \leftrightarrow \bar{n}$ transitions: $\langle \bar{n}|H_{eff}|n\rangle \equiv \delta m$. Consider the matrix in $(n, \bar{n})$ basis:

$$\mathcal{M} = \begin{pmatrix} m_n - i\lambda_n/2 & \delta m \\ \delta m & m_n - i\lambda_n/2 \end{pmatrix}$$

Diagonalizing $\mathcal{M}$ yields mass eigenstates $|n_\pm\rangle = (|n\rangle \pm |\bar{n}\rangle)/\sqrt{2}$

with mass eigenvalues $m_\pm = (m_n \pm \delta m) - i\lambda_n/2$.

So if start with pure $|n\rangle$ state at $t = 0$, then there is a finite probability P for it to be an $|\bar{n}\rangle$ at $t \neq 0$:

$$P(n(t) = \bar{n}) = |\langle \bar{n}|n(t)\rangle|^2 = [\sin^2(t/\tau_{n\bar{n}})]e^{-\lambda_n t}$$

where $\tau_{n\bar{n}} = \hbar/|\delta m| \equiv 1/|\delta m|$. Current limits: $\tau_{n\bar{n}} \gtrsim 10^8$ sec, i.e. (with $\hbar = 0.66 \times 10^{-21}$ MeV-s) $|\delta m| \lesssim 10^{-29}$ MeV), so $\tau_{n\bar{n}} >> \tau_n$.

General Formalism for $n - \bar{n}$ Oscillations

In the $(n, \bar{n})$ basis, write

$$\mathcal{M} = \begin{pmatrix} M_{11} & \delta m \\ \delta m & M_{22} \end{pmatrix}$$

Diagonalization yields mass eigenstates

$$\begin{pmatrix} |n_1\rangle \\ |n_2\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |n\rangle \\ |\bar{n}\rangle \end{pmatrix}$$

where

$$\tan(2\theta) = \frac{2\delta m}{\Delta M}$$

and $\Delta M = M_{11} - M_{22}$. The energy eigenvalues are

$$E_{1,2} = \frac{1}{2}\left[M_{11} + M_{22} \pm \sqrt{(\Delta M)^2 + 4(\delta m)^2} \right]$$

Let

$$\Delta E = E_1 - E_2 = \sqrt{(\Delta M)^2 + 4(\delta m)^2}$$

Transition probability:

$$P(n(t) \to \bar{n}) = |\langle \bar{n} | n(t) \rangle|^2 = \sin^2(2\theta)\ \sin^2[(\Delta E)t/2]\, e^{-\lambda_n t}$$

$$= \left[\frac{(\delta m)^2}{(\Delta M/2)^2 + (\delta m)^2} \right] \sin^2 \left[\sqrt{(\Delta M/2)^2 + (\delta m)^2}\ t \right] e^{-\lambda_n t}$$

If $\sqrt{(\Delta M/2)^2 + (\delta m)^2}\ t << 1$, then by expanding the sin, the quantity $(\Delta M/2)^2 + (\delta m)^2$ cancels out, so

$$P(n(t) \to \bar{n}) \simeq [(\delta m)t]^2\, e^{-\lambda_n t} = (t/\tau_{n\bar{n}})^2\, e^{-\lambda_n t}$$

Most sensitive reactor $n-\bar{n}$ exp. done with ILL High Flux Reactor (HFR) at Grenoble (Baldo-Ceolin, Fidecaro,.., 1985-1994; M. Baldo-Ceolin et al., Z. Phys. C63, 409 (1994))) with neutrons cooled to liquid D_2 temp., kinetic energy $E \simeq 2 \times 10^{-3}$ eV, typical velocity $v \simeq 700$ m/s, $L \simeq 76$ m, $t \simeq 0.11$ sec., $\phi \simeq 1.25 \times 10^{11}$ n/s; set limit

$$\tau_{n\bar{n}} \ge 0.86 \times 10^8 \text{ sec} \quad (90\ \%\ CL)\ , i.e.,$$

$$|\delta m| = \frac{\hbar}{\tau_{n\bar{n}}} = \frac{0.66 \times 10^{-21}\ \text{MeV} - \text{sec}}{\tau_{n\bar{n}}} \le 0.77 \times 10^{-29}\ \text{MeV}$$

In general,

$$|\delta m| = (0.66 \times 10^{-29}\ \text{MeV})\left(\frac{10^8\ \text{sec}}{\tau_{n\bar{n}}}\right)$$

Promising prospects for improvements of this old limit for free neutron propagation with exp. at European Spallation Source, ESS

$n-\bar{n}$ Oscillations in Matter:

For $n-\bar{n}$ oscillations involving a neutron bound in a nucleus, consider

$$\mathcal{M} = \begin{pmatrix} m_{n,eff.} & \delta m \\ \delta m & m_{\bar{n},eff.} \end{pmatrix}$$

with

$$m_{n,eff} = m_n + V_n \ , \quad m_{\bar{n},eff.} = m_n + V_{\bar{n}}$$

where the nuclear potential V_n is real, $V_n = V_{nR}$, but $V_{\bar{n}}$ has an imaginary part representing the $\bar{n}N$ annihilation: $V_{\bar{n}} = V_{\bar{n}R} - iV_{\bar{n}I}$ with $V_{nR}, \ V_{\bar{n}R}, \ V_{\bar{n}I} \sim O(100)$ MeV (Dover, Gal, Richard; Friedman, Gal...). Mixing is thus strongly suppressed; $\tan(2\theta)$ is determined by

$$\frac{2\delta m}{|m_{n,eff.} - m_{\bar{n},eff.}|} = \frac{2\delta m}{\sqrt{(V_{nR} - V_{\bar{n}R})^2 + V_{\bar{n}I}^2}} << 1$$

Using the reactor exp. bound on $|\delta m|$, this gives $|\theta| \lesssim 10^{-31}$. This suppression in mixing is compensated for by the large number of nucleons in a nucleon decay detector such as Soudan-2 or SuperKamiokande e.g., $\sim 10^{33}$ n's in SuperK.

Eigenvalues:

$$m_{1,2} = \frac{1}{2}\Big[m_{n,eff.} + m_{\bar{n},eff.} \pm \sqrt{(m_{n,eff.} - m_{\bar{n},eff.})^2 + 4(\delta m)^2}\ \Big]$$

Expanding m_1 for the mostly n mass eigenstate $|n_1\rangle \simeq |n\rangle$,

$$m_1 \simeq m_n + V_n - i\frac{(\delta m)^2\, V_{\bar{n}I}}{(V_{nR} - V_{\bar{n}R})^2 + V_{\bar{n}I}^2}$$

Imaginary part leads to matter instability via $\bar{n}n,\ \bar{n}p \to \pi$'s, with rate

$$\Gamma_m = \frac{1}{\tau_m} = \frac{2(\delta m)^2|V_{\bar{n}I}|}{(V_{nR} - V_{\bar{n}R})^2 + V_{\bar{n}I}^2}$$

So $\tau_m \propto (\delta m)^{-2} = \tau_{n\bar{n}}^2$. Writing $\tau_m = R\,\tau_{n\bar{n}}^2$, one has $R \sim O(100)$ MeV, dependent on nucleus, equivalently, $R \sim 10^{23}$ s^{-1}; e.g. calculations give $R \sim 1 \times 10^{23}$ s^{-1} for Fe, $R \sim 0.5 \times 10^{23}$ s^{-1} for Oxygen.

After earlier limits from other detectors, Soudan-2 and SuperK have reported matter instability limits:

Soudan-2: $\tau_m > 0.72 \times 10^{32}$ yr (90 % CL) [Chung et al., PRD 66, 032004 (2002)]

From $\tau_{n\bar{n}} = \sqrt{\frac{\tau_m}{R}}$ with $R \simeq 1 \times 10^{23}$ s^{-1} for Fe, this gives $\tau_{n\bar{n}} \gtrsim 1.5 \times 10^8$ s.

SuperK [Abe et al., PRL 91, 072006 (2015)]: $\tau_m > 1.9 \times 10^{32}$ yr (90 % CL); with $R \simeq 0.5 \times 10^{23}$ s^{-1}, this yields $\tau_{n\bar{n}} \gtrsim 2.7 \times 10^8$ s

Weaker limit $\tau_m > 1.5 \times 10^{31}$ yr (90 % CL) from SNO using deuterium, yielding $\tau_{n\bar{n}} \gtrsim 1.3 \times 10^8$ s [SNO Collab., Aharmim et al., arXiv:1705.00696].

Future ESS quasi-free neutron propagation exp. could improve substantially on existing limits; a limit of $\tau_{n\bar{n}} > 10^9$ s is equivalent to

$$\tau_m > (1.6 \times 10^{33} \text{ yr})\left(\frac{\tau_{n\bar{n}}}{10^9 \text{ s}}\right)^2\left(\frac{R}{0.5 \times 10^{23} \text{ s}^{-1}}\right)$$

Operator Analysis and Estimate of Matrix Elements

At the quark level $n \to \bar{n}$ is $(udd) \to (u^c d^c d^c)$. This is mediated by 6-quark operators $\mathcal{O}_i$, so the effective Hamiltonian is

$$\mathcal{H}_{eff} = \sum_i c_i \mathcal{O}_i$$

For d-dimensional spacetime the dimension of a fermion field ψ in mass units is $d_\psi = (d-1)/2$, so dimension $d_{\mathcal{O}_i} = 6 d_\psi = 3(d-1)$ and $d_{c_i} = d - d_{\mathcal{O}_i} = 3 - 2d$. For $d = 4$, $d_\psi = 3/2$, $d_{\mathcal{O}_i} = 9$, and $d_{c_i} = -5$.

With $H_{eff} = \int d^3x \, \mathcal{H}_{eff}$, the transition amplitude is

$$\delta m = \langle \bar{n} | H_{eff} | n \rangle = \frac{1}{M_X^5} \sum_i a_i \langle \bar{n} | \mathcal{O}_i | n \rangle$$

where $c_i = a_i / M_X^5$, and M_X is an effective mass scale characterizing the interaction(s) responsible for the $n - \bar{n}$ transition.

This effective mass scale M_X can arise as a function of several particle mass scales and dimensionless couplings.

For example, one type of Feynman diagram contributing to $n - \bar{n}$ transition amplitude involves ingoing uud quarks and outgoing $u^c d^c d^c$ quarks with Higgs coupling to these, each transforming $q \longrightarrow q^c$. Denote these Higgs couplings to the quarks generically as y_i and the Higgs masses as m_{H_i}. The three Higgs propagators meet at a triple-Higgs vertex, denoted g_H. Resultant coefficient

$$c_i = \frac{a_i}{M_X^5} = \frac{y_1\, y_2\, y_3\, g_H}{m_{H_1}^2 m_{H_2}^2 m_{H_3}^2}$$

The matrix element $\langle \bar{n} | \mathcal{O}_\rangle | n \rangle \sim \Lambda_{QCD}^6$, where $\Lambda_{QCD} \simeq 200$ MeV. Then

$$|\delta m| = \frac{1}{\tau_{n\bar{n}}} \sim \frac{\Lambda_{QCD}^6}{M_X^5}$$

Numerically,

$$\tau_{n\bar{n}} = (2 \times 10^8 \text{ s}) \left(\frac{M_X}{4 \times 10^5 \text{ GeV}} \right)^5 \left(\frac{3 \times 10^{-5} \text{ GeV}^6}{|\langle \bar{n} | \sum_i a_i \mathcal{O}_i | n \rangle|} \right)$$

Operators $\mathcal{O}_i$ must be color singlets and, for M_X larger than the electroweak symmetry breaking scale, also $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$-singlets. Relevant SU(3)$_c$ and SU(2)$_L$ contractions:

$$[\mathbf{3} \times \mathbf{3}]_a \times [\mathbf{3} \times \mathbf{3}]_a \times [\mathbf{3} \times \mathbf{3}]_s \to [\bar{\mathbf{3}} \times \bar{\mathbf{3}}]_s \times \mathbf{6} \to \bar{\mathbf{6}} \times \mathbf{6} \to \mathbf{1}$$

$$[\mathbf{3} \times \mathbf{3}]_s \times [\mathbf{3} \times \mathbf{3}]_s \times [\mathbf{3} \times \mathbf{3}]_s \to [\mathbf{6} \times \mathbf{6}]_s \times \mathbf{6} \to \bar{\mathbf{6}} \times \mathbf{6} \to \mathbf{1}$$

The color tensors that perform these contractions are

$$(T_s)_{\alpha\beta\gamma\delta\rho\sigma} = \epsilon_{\rho\alpha\gamma}\epsilon_{\sigma\beta\delta} + \epsilon_{\sigma\alpha\gamma}\epsilon_{\rho\beta\delta} + \epsilon_{\rho\beta\gamma}\epsilon_{\sigma\alpha\delta} + \epsilon_{\sigma\beta\gamma}\epsilon_{\rho\alpha\delta}$$

$$(T_a)_{\alpha\beta\gamma\delta\rho\sigma} = \epsilon_{\rho\alpha\beta}\epsilon_{\sigma\gamma\delta} + \epsilon_{\sigma\alpha\beta}\epsilon_{\rho\gamma\delta}$$

$(T_s)_{\alpha\beta\gamma\delta\rho\sigma}$ is symmetric in the indices $(\alpha\beta)$, $(\gamma\delta)$, $(\rho\sigma)$.

$(T_a)_{\alpha\beta\gamma\delta\rho\sigma}$ is antisymmetric in $(\alpha\beta)$ and $(\gamma\delta)$ and symmetric in $(\rho\sigma)$.

Types of quark bilinears involve SU(2) singlets such as (where $C = i\gamma_2\gamma_0$)

$$[u_R^{\alpha T} C u_R^{\beta}], \quad [d_R^{\gamma T} C d_R^{\delta}], \quad [u_R^{\alpha T} C d_R^{\beta}]$$

and SU(2) doublets $Q_L^{i\alpha} = \binom{u^\alpha}{d^\alpha}_L$ and $Q_L^{i\beta}$ in bilinears $[Q_L^{i\alpha T} C Q_L^{j\beta}]$, etc., with appropriate SU(2) ϵ_{ij} tensors to yield SU(2)$_L$-invariant operator products. Operators:

$$\mathcal{O}_1 = [u_R^{\alpha T} C u_R^{\beta}][d_R^{\gamma T} C d_R^{\delta}][d_R^{\rho T} C d_R^{\sigma}](T_s)_{\alpha\beta\gamma\delta\rho\sigma}$$

$$\mathcal{O}_2 = [u_R^{\alpha T} C d_R^{\beta}][u_R^{\gamma T} C d_R^{\delta}][d_R^{\rho T} C d_R^{\sigma}](T_s)_{\alpha\beta\gamma\delta\rho\sigma}$$

$$\mathcal{O}_3 = [Q_L^{i\alpha T} C Q_L^{j\beta}][u_R^{\gamma T} C d_R^{\delta}][d_R^{\rho T} C d_R^{\sigma}]\epsilon_{ij}(T_a)_{\alpha\beta\gamma\delta\rho\sigma}$$

$$\mathcal{O}_4 = [Q_L^{i\alpha T} C Q_L^{j\beta}][Q_L^{k\gamma T} C Q_L^{m\delta}][d_R^{\rho T} C d_R^{\sigma}]\epsilon_{ij}\epsilon_{km}(T_a)_{\alpha\beta\gamma\delta\rho\sigma}$$

A given theory determines the coefficients c_i and the effective mass scale M_X. Then one needs to calculate the matrix elements $\langle \bar{n}|\mathcal{O}_i|n\rangle$ to predict δm and thus the resultant $n - \bar{n}$ rate.

Calculation of these matrix elements $\langle \bar{n}|\mathcal{O}_i|n\rangle$ was performed using the MIT bag model (Rao and Shrock, Phys. Lett. B 116, 239 (1982); further results taking into account multigenerational contributions and varying MIT bag parameters in Rao and Shrock, Nucl. Phys. B 232, 143 (1984).

Recall that the MIT bag model is a phenomenological model of hadrons (DeGrand, Jaffe, Johnson, Kiskis, PRD 12, 2060 (1975)). For ground-state hadrons, quarks with effective mass m_q are confined inside a spherical cavity, and the Dirac equation is solved to obtain their eigenfunctions and and energy eigenvalues. Taking $m_q = 0$ for u and d quarks, this yields the quark energy $E_q = N_q\kappa/R$, where $\kappa = 2.04$ and $N_q = 3$ for baryons (and $N_q = 2$ for $q\bar{q}$ mesons).

The second main contribution to the hadron energy is from a gluonic energy $E_g = BV$, where B is the gluonic energy density and $V = (4\pi/3)R^3$ is the volume.

So in the simplest bag model, the hadron mass is

$$m_{hadron} = \frac{N_q\kappa}{R} + \frac{4\pi BR^3}{3}$$

One then minimizes this mass-energy, setting $dm_{hadron}/dR = 0$ with $d^2m_{hadron}/dR^2 > 0$ and solves for the radius R, obtaining

$$R = \left(\frac{N_f \kappa}{4\pi B}\right)^{1/4}$$

Substituting R back into m_{hadron}, one gets

$$m_{baryon} = 4(4\pi/3)^{1/4}\kappa^{3/4}B^{1/4} = 9.77B^{1/4}$$

and

$$\frac{m_{baryon}}{m_{meson}} = (3/2)^{3/4} = 1.36$$

Compare $m_N/m_\rho = 1.2$ and $m_\Delta/m_\rho = 1.6$. In the full bag model there are several additional terms contributing to m_{hadron}, including a color hyperfine interaction, etc., giving reasonable fit to ground-state hadron masses. MIT fit A with $m_{u,d}$ taken to be negligibly small yields $B^{1/4} = 145$ MeV. MIT fit B with effective constituent $m_{u,d} = 108$ MeV yields $B^{1/4} = 125$ MeV.

The $n - \bar{n}$ matrix element calculations involve integrals over sixth-power polynomials of spherical Bessel functions from the six quarks in the transition operator. Illustrative results for the two MIT fits: properties:

$$\langle \mathcal{O}_1 \rangle = -6.6 \times 10^{-5}\ \mathrm{GeV}^6 \quad (\mathrm{Fit\ A})$$

$$\langle \mathcal{O}_1 \rangle = -5.3 \times 10^{-5}\ \mathrm{GeV}^6 \quad (\mathrm{Fit\ B})$$

$$\langle \mathcal{O}_2 \rangle = 1.6 \times 10^{-5}\ \mathrm{GeV}^6 \quad (\mathrm{Fit\ A})$$

$$\langle \mathcal{O}_2 \rangle = 1.3 \times 10^{-5}\ \mathrm{GeV}^6 \quad (\mathrm{Fit\ B})$$

Similarly for others; details in our papers. In general, MIT bag calc. gives

$$|\langle \bar{n}|\mathcal{O}_i|n\rangle| \sim\ 10^{-4} - O(10^{-5})\ \mathrm{GeV}^6 \simeq (200\ \mathrm{MeV})^6 \simeq \Lambda_{QCD}^6$$

These will be sufficient for our estimates here. Preliminary lattice calcs. by Buchoff, Syritsyn, Wagman; eventually, lattice calcs. should determine the matrix elements.

$n - \bar{n}$ Oscillations in an Extra-Dimensional Model

We provide an illustrative model in which $n - \bar{n}$ oscillations can occur observable level (Nussinov and Shrock, Phys. Rev. Lett. 88, 171601 (2002)), while proton decay is strongly suppressed. This model involves extra (spatial) dimensions, associated BSM physics.

We focus on theories where SM fields can propagate in the extra dimensions and the wavefunctions of SM fermions have strong localization (with Gaussian profiles) in this extra-dimensional space. Effective size of extra dimension(s) is L; $\Lambda_L = L^{-1}$ can be ~ 100 TeV, $<< M_{Pl}$.

Such models are of interest partly because they can provide a mechanism for obtaining a hierarchy in fermion masses and quark mixing. They show how observable $n - \bar{n}$ oscillations can arise in physics beyond the SM.

In generic models of this type, excessively rapid proton decay, e.g., via $p \longrightarrow e^+\pi^0$, etc., can be avoided by arranging that the wavefunction centers of the u and d quarks are separated far from those of the e and μ. However, this does not guarantee adequate suppression of $n - \bar{n}$ oscillations. We have analyzed this.

Denote usual spacetime coords. as x_ν, $\nu = 0, 1, 2, 3$ and consider ℓ extra compact coordinates, y_λ, so $d = 4 + \ell$. Let SM fermion have the form $\Psi(x, y) = \psi(x)\chi(y)$, where $\chi(y)$ has support for $0 \le y_\lambda \le L$.

Use a low-energy effective field theory approach with an ultraviolet cutoff M_* and consider only lowest relevant mode in the Kaluza-Klein (KK) mode decompositions of each Ψ field.

To get hierarchy in 4D fermion mass matrices, have the fermion wavefunctions $\chi(y)$ localized with Gaussian profiles of half-width $\mu^{-1} << L$ at various points in the higher-dimensional space:

$$\chi_f(y) = Ae^{-\mu^2|y-y_f|^2}$$

where $|y_f| = (\sum_{\lambda=1}^{\ell} y_{f,\lambda}^2)^{1/2}$.

Starting from the Lagrangian in the d-dimensional spacetime, one obtains the resultant low-energy effective field theory in 4D by integrating over the extra ℓ dimension(s). The normalization factor $A = (2/\pi)^{\ell/4}\mu^{\ell/2}$ is included so that after this integration the 4D kinetic term $\bar{\psi}(x)i\partial\!\!\!/\psi(x)$ has canonical normalization.

Denote $\xi = \mu/\Lambda_L$; choice $\xi \sim 30$, i.e., $\mu^{-1} \sim L/30$, yields adequate separation of fermions while fitting in interval $[0, L]$. (Localization method for $\ell = 1$: coupling fermion to scalar field with a kink; similarly for $\ell = 2$.)

A Yukawa interaction in the d-dimensional space with coefficients of order unity and moderate separation of localized wavefunctions yields a strong hierarchy in the effective low-energy 4D Yukawa interaction because the convolution of two of the fermion Gaussian wavefunctions is another Gaussian,

$$\int d^\ell y \bar{\chi}(y_f)\chi(y_{f'}) \sim \int d^\ell y e^{-\mu^2|y-y_f|^2} e^{-\mu^2|y-y_{f'}|^2} \sim e^{-(1/2)\mu^2|y_f-y_{f'}|^2}$$

Have UV cutoff M_* satisfying $M_* > \mu$ for the validity the low-energy effective field theory analysis. Take $\Lambda_L \sim 100$ TeV for adequate suppression of neutral flavor-changing currents; with $\xi = \mu/\Lambda = 30$, this yields $\mu \sim 3 \times 10^3$ TeV.

In d-dimensions, $\mathcal{H}_{eff,4+\ell} = \sum_{i=1}^{4} \kappa_i O_i$, where the operators O_i are comprised of the $(4+\ell)$-dimensional quark fields corresponding to those in $\mathcal{O}_i$ as Ψ corresponds to ψ. Here mass dimension of coefficients $d_{\kappa_i} = 3 - 2d = -(5+2\ell)$. Hence we write $\kappa_i = \eta_i / M_X^{5+2\ell}$ and, with no loss of generality, take $\eta_4 = 1$. Assume $M_X \sim \Lambda_L$.

Now carry out the integrations over y to get, for each i,

$$c_i \mathcal{O}_i(x) = \kappa_i \int d^\ell y O_i(x,y)$$

Consider case $\ell = 2$. Denoting

$$\rho_c \equiv \frac{4\mu^4}{3\pi^2 M_X^9}$$

we find

$$c_i = \rho_c \eta_i \exp\left[-(4/3)\mu^2 |y_{u_R} - y_{d_R}|^2\right] \ , i = 1, 2$$

$$c_3 = \rho_c \eta_3 \exp[-(1/6)\mu^2 (2|y_{Q_L} - y_{u_R}|^2 + 6|y_{Q_L} - y_{d_R}|^2 + 3|y_{u_R} - y_{d_R}|^2)]$$

$$c_4 = \rho_c \exp\left[-(4/3)\mu^2 |y_{Q_L} - y_{d_R}|^2\right]$$

Fit to values of quark masses and CKM mixing for $\ell = 2$ gives

$$|y_{Q_L} - y_{u_R}| = |y_{Q_L} - y_{d_R}| \simeq 5\mu^{-1}$$

$$|y_{u_R} - y_{d_R}| \simeq 7\mu^{-1}$$

Can also include corrections due to Coulombic gauge interactions between fermions, analyzed in Nussinov and Shrock, Phys. Lett. B 526, 137 (2002).

We find c_j for $j = 1, 2, 3$ are $<< c_4$, and hence focus on c_4.

To leading order (neglecting small CKM mixings), $|y_{Q_L} - y_{d_R}|$ is determined by m_d via relation (with $v = 246 \text{ GeV} = 2m_W/g$)

$$m_d = h_d \frac{v}{\sqrt{2}}$$

with

$$h_d = h_{d,0} \exp[-(1/2)\mu^2 |y_{Q_L} - y_{d_R}|^2]$$

where $h_{d,0}$ is the Yukawa coupling in the $(4+\ell)$-dimensional space, so that

$$\exp\left[-(1/2)\mu^2 |y_{Q_L} - y_{d_R}|^2\right] = \frac{2^{1/2} m_d}{h_{d,0} v}$$

Take $h_{d,0} \sim 1$ and $m_d \simeq 10$ MeV; then contribution to δm from $\mathcal{O}_4$ term is

$$\delta m \simeq c_4 \langle \bar{n} | \mathcal{O}_4 | n \rangle \simeq \left(\frac{4\mu^4}{3\pi^2 M_X^9} \right) \left(\frac{2^{1/2} m_d}{v} \right)^{8/3} \langle \bar{n} | \mathcal{O}_4 | n \rangle$$

From MIT bag model calculation we have

$$\langle \bar{n} | \mathcal{O}_4 | n \rangle \simeq 0.9 \times 10^{-4} \text{ GeV}^6$$

Requiring that the resultant $|\delta m|$ be less than the experimental limit $\tau_{n\bar{n}} > 3 \times 10^8$ sec, i.e., $|\delta m| < 2 \times 10^{-30}$ MeV, we obtain the bound

$$M_X \gtrsim (50\ \text{TeV}) \left(\frac{\tau_{n\bar{n}}}{3 \times 10^8\ \text{sec}}\right)^{1/9} \times \left(\frac{\mu}{3 \times 10^3\ \text{TeV}}\right)^{4/9} \left(\frac{|\langle \bar{n}|O_4|n\rangle|}{0.9 \times 10^{-4}\ \text{GeV}^6}\right)^{1/9}$$

Uncertainty in calculation of matrix element $\langle \bar{n}|\mathcal{O}_4|n\rangle$ is relatively unimportant for this bound because of the $1/9$ power in the exponent.

Hence, for relevant values of $M_X \sim 50 - 100$ TeV, $n - \bar{n}$ oscillations might occur at levels that are in accord with the current experiment limit but not too far below this limit.

This model also illustrates how baryon number violation can occur via $n - \bar{n}$ oscillations with strongly suppressed proton decay.

Other models can also predict $n - \bar{n}$ oscillations near to current limits, e.g., Babu, Mohapatra, and Nasri, PRL 97, 131301 (2006), review in $n - \bar{n}$ Collab. (D. Phillips et al.), Phys. Rept. 612, 1 (2016).

A different possibility is that there might be a "mirror" world (Lee, Yang, Okun,Berezhiani, Kamyshkov, Mohapatra, Nussinov,...) with weak coupling to our world, allowing a n to oscillation to the mirror neutron, n'; so n disappearance.

Conclusions

- $n - \bar{n}$ oscillations are an interesting possible manifestation of baryon number violation, of $|\Delta B| = 2$ type, complementary to proton decay. A discovery of $n - \bar{n}$ oscillations would be of profound significance.

- Current lower limit on $\tau_{n\bar{n}}$ for $n - \bar{n}$ oscillations: $\tau_{n\bar{n}} \gtrsim 3 \times 10^8$ s from SuperK.

- We have presented a model that shows how new physics beyond the SM can produce $n - \bar{n}$ oscillations at rates comparable with this current limit. This model also demonstrates that $n - \bar{n}$ oscillations can be the main manifestation of baryon number violation, since proton decay is strongly suppressed.

- The possibility of $n - \bar{n}$ oscillations with $\tau_{n\bar{n}}$ only slightly greater than current lower limit provide a strong motivation for a new experiment to search for $n - \bar{n}$ oscillations with greater sensitivity, e.g., at the ESS.

Some Backup Slides

$n-\bar{n}$ Oscillations in a Magnetic Field $\vec{B}$:

Relevant to Institut Laue-Langevin (ILL) and planned ESS experiments searching for $n-\bar{n}$ oscillations

n and $\bar{n}$ interact with $\vec{B}$ via magnetic moment $\vec{\mu}=\mu\,\vec{\sigma}$, $\mu_n=-\mu_{\bar{n}}=\kappa\mu_N$, where $\kappa=-1.91$, $\mu_N=e/(2m_N)=3.15\times 10^{-14}$ MeV/Tesla, so

$$\mathcal{M}=\begin{pmatrix} m_n-\vec{\mu}_n\cdot\vec{B}-i\lambda_n/2 & \delta m \\ \delta m & m_n+\vec{\mu}_n\cdot\vec{B}-i\lambda_n/2 \end{pmatrix}$$

So $\Delta M=M_{11}-M_{22}=-2\vec{\mu}_n\cdot\vec{B}$ and diagonalization yields mass eigenstates $|n_1\rangle$, $|n_2\rangle$, with mixing

$$\tan(2\theta)=-\frac{\delta m}{\vec{\mu}_n\cdot\vec{B}}$$

and energy eigenvalues

$$E_{1,2}=m_n\pm\sqrt{(\vec{\mu}_n\cdot\vec{B})^2+(\delta m)^2}\;-i\lambda_n/2$$

ILL experiment reduced $|\vec{B}| = B$ to $B \sim 10^{-4}\ \mathrm{G} = 10^{-8}$ T, so

$$|\mu_n| B = (6.03 \times 10^{-22}\ \mathrm{MeV}) \left(\frac{B}{10^{-8}\ \mathrm{T}} \right)$$

Since $|\delta m| \lesssim 10^{-29}\ \mathrm{MeV} \ \ << |\mu_n| B$ from exp., it follows that
$|\theta| \lesssim 10^{-8} << 1$ and

$$\Delta E = 2\sqrt{(\vec{\mu}_n \cdot \vec{B})^2 + (\delta m)^2} \simeq 2|\vec{\mu}_n \cdot \vec{B}|$$

In a reactor $n - \bar{n}$ experiment, arrange that n's propagate a time t such that

$$|\vec{\mu}_n \cdot \vec{B}| t = 0.92 \left(\frac{B}{10^{-8}\ \mathrm{T}} \right) \left(\frac{t}{1\ \mathrm{sec}} \right) << 1 \quad \text{and } t << \tau_n$$

Then

$$P(n(t) \to \bar{n}) \simeq (2\theta)^2 \left(\frac{\Delta E t}{2} \right)^2 \simeq \left(\frac{\delta m}{\vec{\mu}_n \cdot \vec{B}} \right)^2 \left(\vec{\mu}_n \cdot \vec{B}\, t \right)^2 = [(\delta m)\, t]^2 = (t/\tau_{n\bar{n}})^2$$

So

$$N_{\bar{n}} = P(n(t) = \bar{n})N_n$$

where $N_n = \phi T_{run}$, with $\phi =$ neutron flux, $T_{run} =$ running time. Sensitivity of exp. depends in part on the product

$$N_n \left(\frac{t}{\tau_{n\bar{n}}}\right)^2 = \phi T_{run} \left(\frac{t}{\tau_{n\bar{n}}}\right)^2$$

so, with adequate magnetic shielding, want to maximize t, subject to condition $|\vec{\mu}_n \cdot \vec{B}|t << 1$ (and neutrons not falling too far in gravity to reach detector).

One may further generalize to a theoretical context with possible CPT violation. Then $m_n \neq m_{\bar{n}}$, $\tau_n \neq \tau_{\bar{n}}$, $\mu_n \neq -\mu_{\bar{n}}$. So

$$\Delta M = M_{11} - M_{22} = m_n - m_{\bar{n}} - (\vec{\mu}_n - \vec{\mu}_{\bar{n}}) \cdot \vec{B} - i(\lambda_n - \lambda_{\bar{n}})/2$$

Assume $(|\mu_n| - |\mu_{\bar{n}}|)/|\mu_n| << 1$ and use the fact that $t << \tau_n$ so that the $-i(\lambda_n - \lambda_{\bar{n}})/2$ is not important; then

$$\Delta M \simeq m_n - m_{\bar{n}} - 2\vec{\mu}_n \cdot \vec{B}$$

If one arranged so that $|\vec{\mu}_n \cdot \vec{B}|t << 1$ and if an experiment would observe $n - \bar{n}$ oscillations, then this would set an upper bound $|m_n - m_{\bar{n}}|t < 1$, since a larger $|m_n - m_{\bar{n}}|$ would suppress the mixing and hence the oscillation (Okun, 1984).

Numerically, one might achieve an upper bound $|m_n - m_{\bar{n}}| \lesssim 10^{-23}$ MeV in this way.