Implications of the mixed (B-L)-gravity anomaly

Archil Kobakhidze
University of Sydney

based on work in progress
with Suntharan Arunasalam

INT Workshop Neutron-Antineutron Oscillations: Appearance, Disappearance, and Baryogenesis

October 23-27, 2017

Motivation

- $\cancel{B}$, $\cancel{C}$, $\cancel{CP}$ (in out-of-equilibrium processes) => dynamical generation of matter-antimatter asymmetry (Sakharov 67')

- Standard Model: $\cancel{C}$, $\cancel{P}$, $\cancel{CP}$ in perturbative weak interactions (the CKM picture)

  $\cancel{B}$ & $\cancel{L}$ in non-perturbative weak processes (anomaly) (Kuzmin, Rubakov, Shaposhnikov 85')

However:
- Hard to get out of equilibrium
- If earlier $B \neq 0$ (but $B-L=0$) baryon number violation by shpalerons => $B=0$

Motivation

- Standard Model + gravity: $\cancel{L}$ (and hence $\cancel{B-L}$) in non-perturbative gravity mediated processes (mixed L-gravity anomaly)

If so,

- Gravitational origin of ν masses? (Dvali, Funcke 16')
- Majorana n mass? (neutron-antineutron oscillations)
- New CP-violation phase?
- Gravity-assisted baryogenesis?

Outline

- Quantum B & L anomalies
- Electroweak instantons
- Gravitational instantons
- Gravitational + $U(1)_Y$ instantons
- Outlook & conclusion

Anomalies in the Standard Model

$$\underbrace{SU(3)\times SU(2)_w \times U(1)_Y}_{\text{gauged}} \times \underbrace{U(1)_B \times U(1)_L}_{\text{global, anomalous}}$$

$$3\times \left\{ \begin{array}{ll} Q_L \sim (3,2,1/6,1/3,0), & d_R \sim (3,1,-1/3,1/3,0) \\ & u_R \sim (3,1,+2/3,1/3\ 0) \\ L_L \sim (1,2,-1/2,0\ 1), & e_R \sim (1,1,-1,0,1) \end{array} \right.$$

Mixed global-gauge anomalies:

$$A_{B2}\left(U(1)_B\, SU(2)_w^2\right) = \left(\frac{1}{3}\,\frac{1}{2}\cdot 3\right)\cdot 3 = \underline{\frac{3}{2}}$$

$$A_{B1}\left(U(1)_B\, U(1)_Y^2\right) = \left(\frac{1}{3}\right)\cdot\left(\frac{1}{6}\right)^2\cdot 6 + (-1/3)\left(\frac{1}{3}\right)^2\cdot 3 + (-1/3)\cdot\left(\frac{2}{3}\right)^2\cdot 3$$

$$= -\underline{\frac{3}{2}}$$

$$A_{L2}\left(U(1)_L\, SU(2)_w^2\right) = \underline{-}\ A_{L1}\left(U(1)_L\, U(1)_Y^2\right) = \underline{\frac{3}{2}}$$

Anomalies in the Standard Model

$$\partial_\mu J^\mu_B = \frac{A_{B2}\, g^2}{16\pi^2} W^a_{\mu\nu} \tilde{W}^{a\mu\nu} + \frac{A_{B1}\, g'^2}{16\pi^2} B_{\mu\nu} \tilde{B}^{\mu\nu}$$

$$\partial_\mu J^\mu_L = \frac{A_{L2}\, g^2}{16\pi^2} W^a_{\mu\nu} \tilde{W}^{a\mu\nu} + \frac{A_{L1}\, g'^2}{16\pi^2} B_{\mu\nu} \tilde{B}^{\mu\nu}$$

$$\Downarrow$$

$$\underline{\partial_\mu J^\mu_{B-L}} \left(\equiv J^\mu_B - J^\mu_L\right) = 0$$

$$\partial_\mu J^\mu_{B+L} = \frac{3 g^2}{16\pi^2} W^a_{\mu\nu} \tilde{W}^{a\mu\nu} - \frac{3 g'^2}{16\pi^2} B_{\mu\nu} \tilde{B}^{\mu\nu}$$

Does this really imply violation of $(B+L)$ $\cancel{\#}$?

Electroweak instantons

Note: $$W^a_{\mu\nu}\tilde{W}^{a\mu\nu} = \partial_\mu K^\mu \,, \quad K^\mu = \epsilon^{\mu\nu\alpha\beta}\left[W^a_{\nu\alpha} W^a_\beta - \frac{1}{3} g \epsilon_{abc} W^a_\nu W^b_\alpha W^c_\beta \right]$$

$$B_{\mu\nu}\tilde{B}^{\mu\nu} = \partial_\mu \varkappa^\mu \,, \quad \varkappa^\mu = \epsilon^{\mu\nu\alpha\beta} B_{\nu\alpha} B_\beta$$

not gauge invariant

Integrate $(B+L)$ anomaly equation:

$$\Delta(B+L) = 3\underbrace{\int d^4x \, \frac{g^2}{16\pi^2} \partial_\mu K^\mu}_{\text{Chern-Pontryagin index } P_{SU(2)}} - 3\underbrace{\int d^4x \, \frac{g'^2}{16\pi^2} B_{\mu\nu}\tilde{B}^{\mu\nu}}_{P_{U(1)} = 0}$$

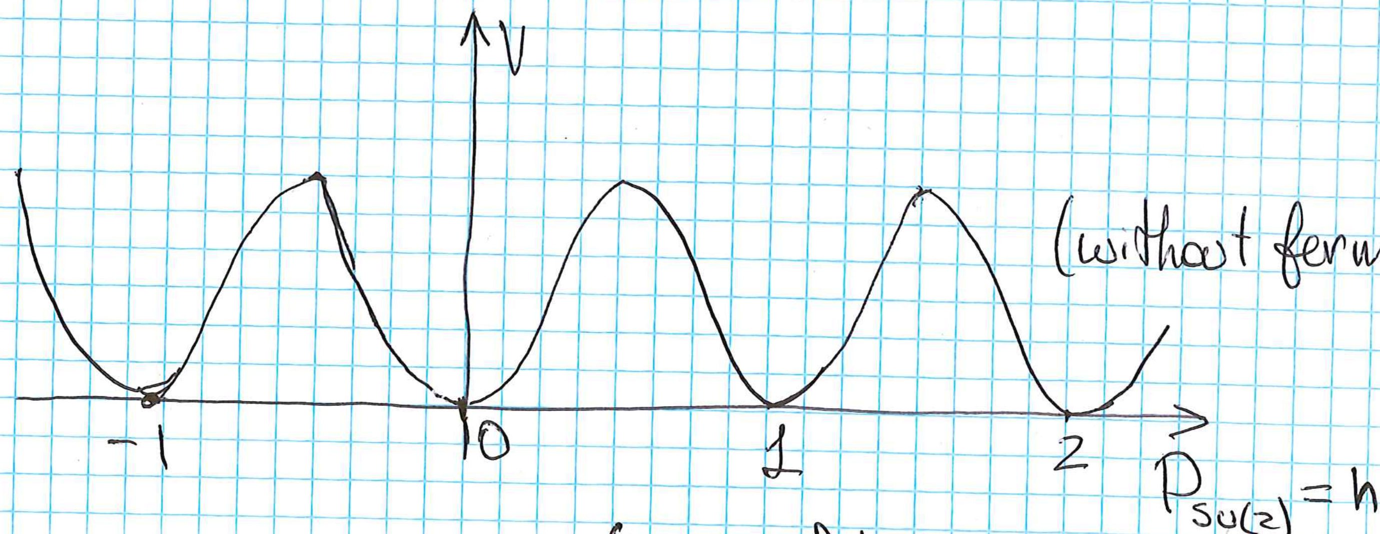
$$\pi_3(S^3) = \mathbb{Z}$$

Electroweak instantons

$$\Delta(B+L) = \int_{\partial V \equiv S^3} dx_\mu K^\mu \Big|_{W_\mu^a \xrightarrow{|x|\to\infty} \frac{i}{g} U \partial_\mu U^+ \quad U \in SU(2)_w}$$

$$\parallel$$

$$n = n_L - n_R \quad \text{(Atiyah, Singer 63')}$$



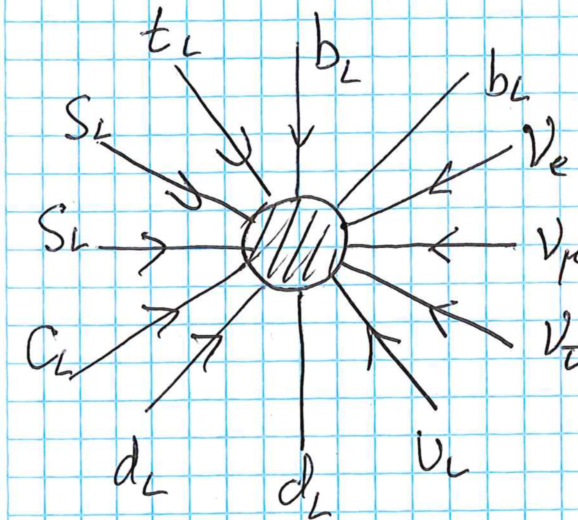
(without fermions)

$n = \pm 1$ (Belavin, Polyakov, Schwartz, Tyupkin 75')

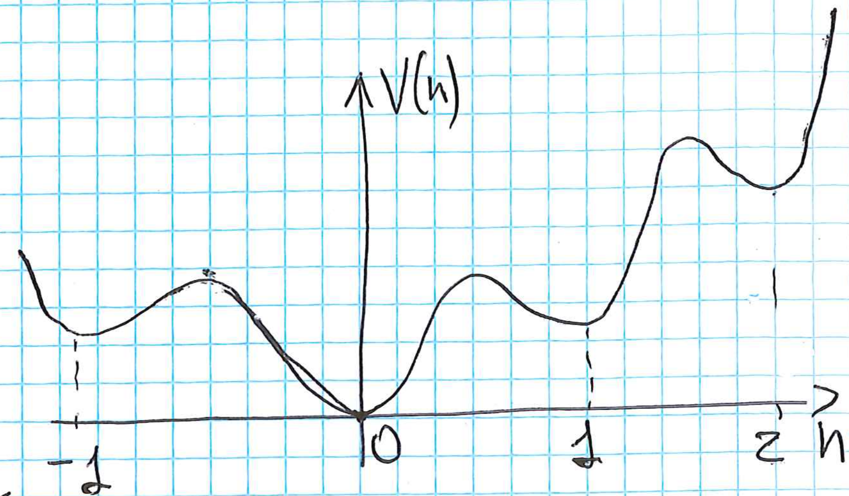
$$W_{\mu\nu}^a = \pm \tilde{W}_{\mu\nu}^a \,, \quad S_E^1 = \frac{8\pi^2}{g^2}$$

Electroweak instanton

In the presence of fermions:



$$\sim e^{-\frac{8\pi^2}{g^2(v)}} \simeq 10^{-54\pi}$$

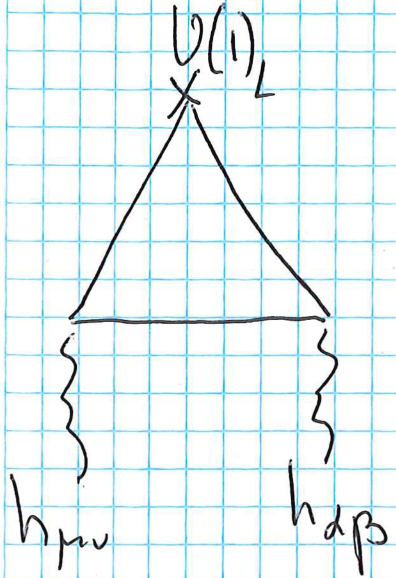


$\Delta B = \Delta L = 3$ process

- Zero modes (Anselm, Johansen 94') $\Rightarrow$ $_\theta\langle 0 | 0 \rangle_\theta \sim \det(i\not{D} - m) = 0$

Coherence of θ-vacua is broken, no CP-violating θ_{EW} (UV completion, Shifman, Vainshtein 17')

Anomalies in the SM + gravity



$$\mathcal{A}_{LG} = 3\,(2\cdot 1 - 1) = 3 \quad \text{(no right-handed } \nu\text{'s)}$$

$$\nabla_\mu J^\mu_L = -\frac{3}{192\pi^2} R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma}$$

$$\tilde{R}^{\mu\nu\rho\sigma} = \frac{1}{2\sqrt{g}}\,\epsilon^{\mu\nu\alpha\beta} R_{\alpha\beta}{}^{\rho\sigma}$$

$$\Downarrow$$

$$\nabla_\mu J^\mu_{B-L} \neq 0$$

Gravitational instantons?

Gravitational instantons: generalities

- In quantum gravity imitate instanton effects by summing over manifolds $(M, g_{\mu\nu})$ with arbitrary topology in the Euclidean path integral

  Problem: $S_G \lesseqgtr 0$

- Positive action theorem (Schoen, Yau 79', Witten 81')

  For $R = 0$, (a) AE, $S_G \geq 0$; $S_G = 0 \Rightarrow$ flat

  (b) ALE, $S_G \geq 0$; $S_G = 0 \Rightarrow$ self-dual configurations

$$\Downarrow$$

$$S_G^{\text{inst}} = 0 \, !$$

Gravitational instantons: generalities

For $R=0$ AE/ALE no zero-fermion modes:

$$\not\nabla \psi = 0 \, , \quad \not\nabla = \gamma^\mu \nabla_\mu = \gamma^\mu \left(\partial_\mu + \frac{1}{4} \omega_\mu^{\ ab} \sigma_{ab} \right)$$

$$\not\nabla \not\nabla = \nabla^2 + \frac{1}{8} \underset{\substack{\parallel \\ 0}}{R_{\mu\nu ab}} \sigma^{\mu\nu} \sigma^{ab} = \nabla^2$$

$$\underset{\substack{/ \\ \text{positive definite}}}{\nabla^2} \psi = 0 \quad \Rightarrow \quad \psi = 0$$

Such instantons would not induce $\not L$

Eguchi-Hanson instanton

(Eguchi, Hanson 78'; Gibbons, Hawking, Perry 78')

$$R_{\mu\nu}{}^{ab} = \partial_\mu \omega_\nu{}^{ab} - \partial_\nu \omega_\mu{}^{ab} + \omega_\mu{}^{ac} \omega_\nu{}^{cb} - \omega_\nu{}^{ac} \omega_\mu{}^{cb}$$

$$T_{\mu\nu}{}^{a} = \partial_\mu e_\nu^a - \partial_\nu e_\mu^a + \omega_\mu{}^{ab} e_{\nu b} - \omega_\nu{}^{ab} e_{\mu b} \equiv 0 \quad \text{(torsion free)}$$

$$g_{\mu\nu} = \delta_{ab}\, e_\mu^a e_\nu^b \quad \text{(metric formulation)}$$

We seek for (anti) self-dual solutions:

$$\omega_\mu{}^{ab} = \pm \tilde{\omega}_\mu{}^{ab} = \pm \frac{1}{2} \epsilon^{abcd} \omega_{\mu\, cd}$$

$$R_{\mu\nu}{}^{ab} = \pm \tilde{R}_{\mu\nu}{}^{ab} = \pm \frac{1}{2} \epsilon^{abcd} R_{\mu\nu cd}$$

Eguchi-Hanson instanton

Coordinates $(r, \theta, \varphi, \psi)$

flat spacetime: $ds^2 = dr^2 + r^2(\sigma_x^2 + \sigma_y^2 + \sigma_z^2)$

$$\sigma_x = \frac{1}{2}(\sin\psi \, d\theta - \sin\theta \cos\psi \, d\phi)$$

$$\sigma_y = \frac{1}{2}(-\cos\psi \, d\theta - \sin\theta \sin\psi \, d\phi)$$

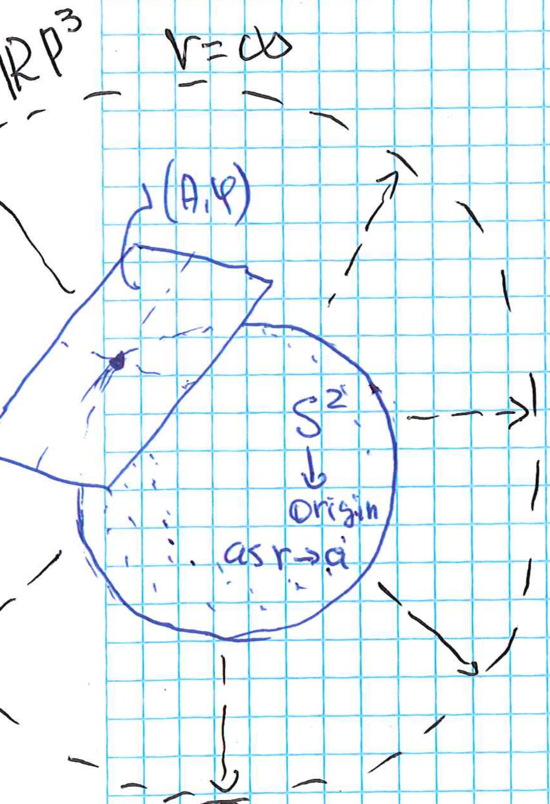
$$\sigma_z = \frac{1}{2}(d\psi + \cos\theta \, d\phi)$$

$$0 \leq \theta < \pi, \quad 0 \leq \phi < 2\pi, \quad 0 \leq \psi \leq \underline{4\pi}$$

Eguchi-Hanson instanton

Anti-self-dual solution:

$$\boxed{ds^2 = \frac{dr^2}{1-\left(\frac{a}{r}\right)^4} + r^2\left(\sigma_x^2 + \sigma_y^2 + \left[1-\left(\frac{a}{r}\right)^4\right]\sigma_z^2\right)}$$



a - an arbitrary constant length, instanton size

Apparent singularity at $r=a$

Geodesically complete if $0 \leq \psi < 2\pi$

We 'half' the space, it got a boundary

A $r\to\infty$, $S^3/\mathbb{Z}_2 = \mathbb{R}P^3$

Eguchi-Hanson instanton

$$S_E = \frac{-1}{16\pi G}\int \sqrt{g}\,\cancel{R}\,d^4x - \frac{1}{8\pi}\int_{\partial M(r\to\infty)} K\, d\Sigma$$

$\cancel{R} \to 0$ Ricci flat

$$= \frac{\pi}{8}\left[3r^2 - \frac{a^4}{r^2} - 3r^2\left(1-\left(a/r\right)^4\right)^{1/2}\right]\Bigg|_{r\to\infty}$$

$$\sim \frac{\pi}{16}\,\frac{a^4}{r^2}\Bigg|_{r\to\infty} = 0$$

$$I_{1/2}(\not{\nabla}) = 0 \qquad \text{(index of the Dirac operator)}$$

Gravitational + $U(1)_Y$ instantons

- Consider sequestered SM: $U(1)_Y$, $L_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$, e_R

- Assume $B_{\mu\nu} = \pm \tilde{B}_{\mu\nu} \Rightarrow T^{U(1)_Y}_{\mu\nu} = 0 \Rightarrow$

  $\Rightarrow$ Eguchi-Hanson metric is <u>unaffected</u>

- Anti-self dual configuration:

$$A_r = A_\theta = 0 \,, \quad A_\psi = \frac{qa}{2r^2} \,, \quad A_\phi = \frac{qa}{2r^2} \cos\theta$$

q - hypercharge of the instanton

Gravitational + $U(1)_Y$ instantons

Compute action:

$$S_E = \frac{1}{4g'^2}\int d^4x \sqrt{-g}\, F_{\mu\nu}F^{\mu\nu}$$

$$= \frac{4\pi^2 q^2}{g'^2} = \frac{\pi q^2}{\alpha_Y(a)}$$

- Note: π vs 2π – the effect of half-space.
- Also, $\alpha_Y(a \to 0) \to \infty$ (Landau pole) $\Rightarrow$

  $\Rightarrow$ small instantons give dominant contribution to transition amplitudes
- No flat space analogue.

Gravitational + $U(1)_Y$ instantons

- Spinor structure is supported if q is quantised

$$q \cdot Y = n \in \mathbb{Z}$$

$$q = 2 \quad \text{(the smallest charge)}$$

$$S_E = \frac{4\pi}{\alpha_Y(e_P)} \hat{\simeq} 700 \text{ (in the SM, very sensitive to UV physics)}$$

Numerics will be different in a full model.

Gravitational + $U(1)_Y$ instantons

- θ_{EW} can no longer be rotated away by phase transformations of right-handed fermions

  Extra, unexplored source of CP-violation in the SM (needs explicit analysis of fermion zero modes in the Higgsed regime, work in progress)

- May produce a seed of $(B-L)$ in the early universe $\Rightarrow$ baryogenesis

  Analogue of sphalerons (?)

Conclusion

- Both $\cancel{B-L}$ & $\cancel{B+L}$ are violated in the Standard Model due to the electroweak + gravitational anomalies
- Instanton picture: Hypercharged Eguchi-Hanson instanton
  - Spin structure $\Rightarrow$ hypercharge quantization
  - additional CP violating phase (work in progress)
  - Potential implications in cosmology (work in progress) & elswhere