INT Workshop "$N-\bar{N}$..."

CONVERSION OF N into $\bar{N}$ WITHOUT OSCILLATIONS

Boris Kerbikov, ITEP, Lebedev, MIPT

THE KEY MESSAGE (arXiv: 1704.07117 v:3)

In vacuum $|\psi_{\bar{n}}(t)|^2 = \frac{4\varepsilon^2}{\Lambda^2} e^{-\Gamma_\beta t} \sin^2 \frac{\Lambda t}{2}$

$\Lambda^2 = 4\varepsilon^2 + d^2$, $d = |2\vec{\mu}_n \vec{B}|$, $|\psi_{\bar{n}}(t)|^2 = 1$ at $t \cong 10^8$ s for $B = 0$.

In an environment (trap, nuclei, gas with $n \gtrsim 10^{+6}\,\mathrm{cm}^{-3}$)

$$|\psi_{\bar{n}}(t)|^2 = \frac{4\varepsilon^2}{\Omega^2} e^{-(\lambda/2 + \Gamma_\beta)t} \sinh^2 \frac{\Omega t}{2}, \quad \Omega^2 = \frac{\lambda^2}{4} - 4\varepsilon^2, \quad B = 0$$

$$\lambda = \left\{ \begin{array}{ll} 2/\tau \simeq 20\,s^{-1} & \text{trap} \\ \Gamma_a \simeq 100\,\mathrm{MeV} \simeq 10^{23} s^{-1} & \text{nuclei} \\ n v \sigma_a \simeq 0.75 \cdot 10^{-4} s^{-1} & \text{ILL experiment } (n \simeq 0.5 \cdot 10^{11} \mathrm{cm}^{-3}) \end{array} \right\} \Rightarrow \Omega^2 > 0 \Rightarrow \sinh^2$$

λ – w.f. reduction by the environment

(LIGO $P = 10^{-9}$ torr vs $1.5 \cdot 10^{-6}$ torr ILL)

$|\psi_{\bar{n}}(t)|^2 = 1$ NEVER even for $\Gamma_\beta = 0$ and LIGO vacuum and $t \to \infty$

$$|\Psi_n(t)|^2 = \frac{4\delta^2}{\Omega^2} \exp\left(-\left(\frac{\lambda}{2}+\Gamma_\beta\right)t\right) \sinh^2 \frac{\Omega t}{2}$$

Asymptotics for long ($t \gtrsim \lambda^{-1}$) and short ($t \ll \lambda^{-1}$) times

- Nuclei at $t \gtrsim 1/\Gamma_a \simeq 10^{-23}$ s "long" time

$$|\Psi_{\bar{n}}(t)|^2 \simeq \frac{4\delta^2}{\Gamma_a^2} \exp\left(-\frac{4\delta^2}{\Gamma_a} t\right) \Rightarrow \frac{|\Psi_{\bar{n}}(t)|^2}{|\Psi_n(t)|^2} \simeq \frac{4\delta^2}{\Gamma_a^2} \simeq 10^{-62} \text{ at all times}$$

Disappearance lifetime A. Gal 2000

$$T = \Gamma_a / 4\delta^2 \gtrsim 10^{32}\, yr.$$

- Gas at $t \lesssim 10^4$ s (with $\Gamma_\beta = 0$)

$$|\Psi_{\bar{n}}(t)|^2 \simeq \frac{4\delta^2}{\lambda^2} \exp\left(-\frac{4\delta^2}{\lambda} t\right) \simeq (ILL) \simeq 10^{-7} \exp\left(-\frac{t(yr)}{3000}\right).$$

• What happens in gas (III) at short times?

$t \ll \lambda^{-1} \simeq 10^4 s$

$$|\psi_{\bar{n}}(t)|^2 \simeq \varepsilon^2 t^2 - \tfrac{1}{2}\varepsilon^2 \lambda t^3 + \dots \quad (\lambda t \gtrsim 1 \text{ for } t \lesssim 10^4 s)$$

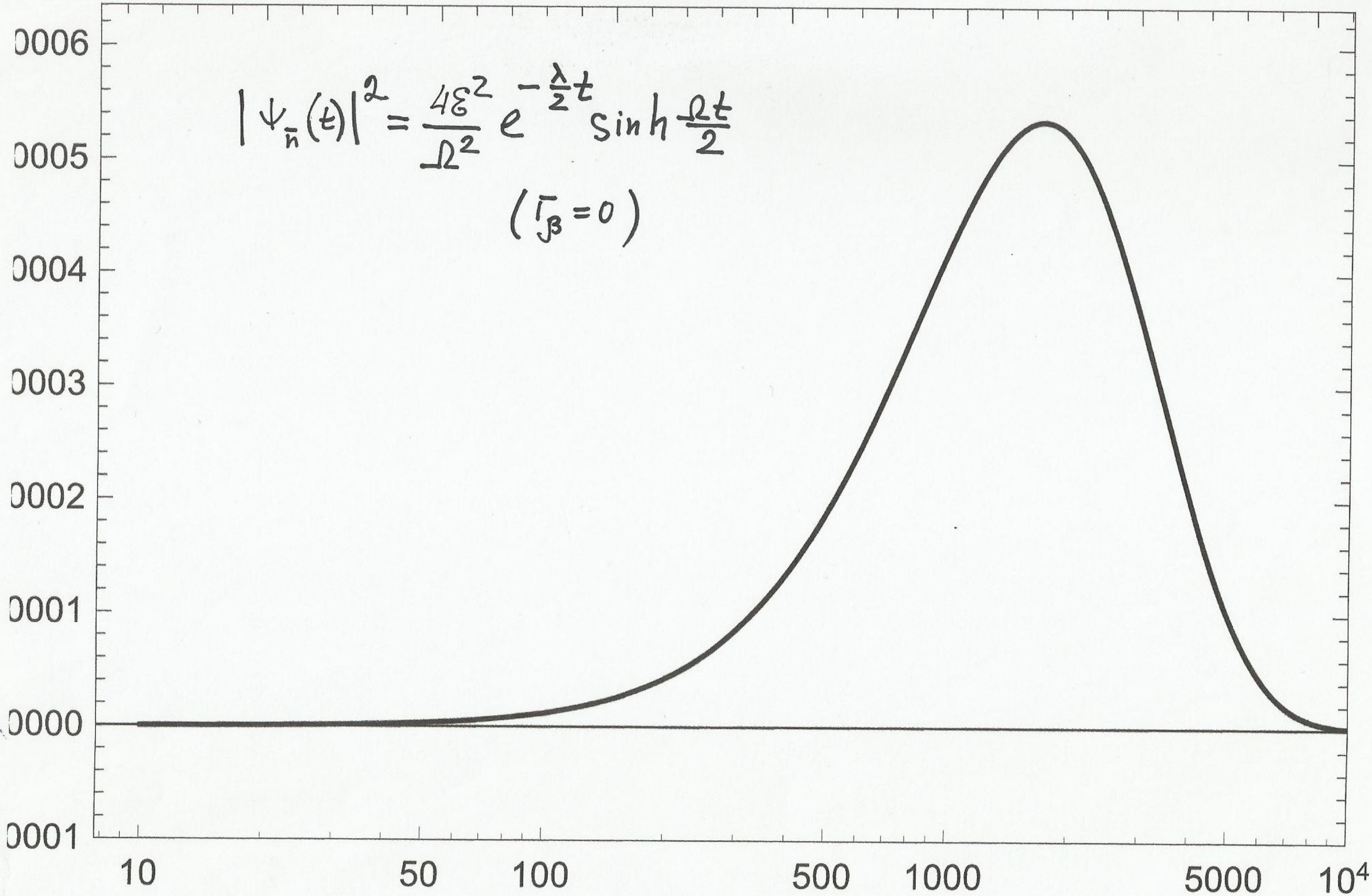
$$R_z(t) = |\psi_n(t)|^2 - |\psi_{\bar{n}}(t)|^2 = 1 - 2\varepsilon^2 t^2 + \dots \quad \left(\varepsilon^2 t^2 \simeq 10^{-18} \text{ for } t = 0.1 s\right)$$

Important point: deviation of $R_z(t)$ from one starts quadratically, "flat top" at $t=0$ →

$$R_z(t) = \left[1 - 2\varepsilon^2\left(\frac{T}{N}\right)^2\right]^N \to 1, \quad N \to \infty$$

Turing's-Xeno's-Watched Pot Paradox: In the limit of infinitely frequent monitoring the system will become frozen forever. Repeated "measurement" keeps the state from evolving

3a



$|\psi_{\bar{n}}(t)|^2 = \frac{4\varepsilon^2}{\Omega^2} e^{-\frac{\lambda}{2}t} \sinh\frac{\Omega t}{2}$
$(\Gamma_\beta = 0)$
0006
0005
0004
0003
0002
0001
0000
0001
10
50
100
500
1000
5000
10^4

• PRELIMINARY CONCLUSIONS

The time evolution of $|\Psi_n(t)|^2$ and $|\Psi_{\bar{n}}(t)|^2$ is determined by the interplay of:

ε-mixing and λ-the measure of the w.f. "reduction" by the environment

$$\Omega^2 = \frac{\lambda^2}{4} - 4\varepsilon^2$$

(i) free space $\lambda = 0$, $\sin^2$ law

(ii) $\Omega^2 < 0$ – underdamped, slightly or strongly damped oscillations

(iii) $\Omega^2 = 0$ – critical damping, fine tuning of ε and λ

(iv) $\Omega^2 > 0$ – overdamping, $\sinh^2$ law; the system never turn from $(|\Psi_n|^2 = 1, |\Psi_{\bar{n}}|^2 = 0)$ to $(|\Psi_n|^2 = 0, |\Psi_{\bar{n}}|^2 = 1)$.

Any conceivable experiment (including LIGO vacuum) belongs to (iv) category

• Why the density matrix? What's wrong with the Schrodinger w.f.?

- W.F. – propagation of a beam of particles, whether it be in a vacuum, or in a medium. Refractive index, etc.
- Another class of problems – evolution of a statistical ensemble simultaneously mixing and interacting in a medium. Schrodinger eq-n needs an unavailable Hamiltonian of a test system + environment
- Density matrix reflects our ignorance of the full system
- A clear-cut example: neutrino ~~[illegible]~~ mixing and scattering in a medium:

$$\alpha|\nu_e\rangle + \beta|\nu_\mu\rangle \xrightarrow[\text{or } \nu + d \to u + e^-]{\text{collision}} \to \begin{cases} \text{linear combination?} \\ \text{incoherent mixture?} \end{cases}$$

Quantum Boltzmann eq-n with anti-commutator $\dot{\rho} = \ldots - \{\Gamma, \rho\}$ – Dolgov 1981, Raffelt, Singl, Stodolsky 1992, Bloch eq-n – Stodolsky 1987

Back to the Beginning — Density Matrix, von Neumann-Liouville, Bloch, Lindblad Equations

Pure state $|\psi\rangle = \psi_1(t)|n\rangle + \psi_2(t)|\bar{n}\rangle$

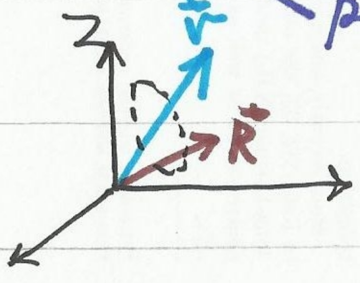
$$\hat{\rho}(t) = \begin{pmatrix} \psi_1\psi_1^* & \psi_1\psi_2^* \\ \psi_2\psi_1^* & \psi_2\psi_2^* \end{pmatrix} = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}$$

$i\frac{d\hat{\rho}}{dt} = [\hat{H}, \hat{\rho}]$ vs. Heisenberg $i\frac{d\hat{O}}{dt} = -[\hat{H}, \hat{O}]$

$\hat{\rho} = \frac{1}{2}(1 + \vec{R}\vec{\sigma})$, $\vec{R} = \begin{pmatrix} \rho_{12}+\rho_{21} \\ -i(\rho_{21}-\rho_{12}) \\ \rho_{11}-\rho_{22} \end{pmatrix}$ — spin (flavor) polarization vector

$= \dot{\vec{R}} = \vec{V} \times \vec{R}$, $\hat{V} = \begin{pmatrix} 2\varepsilon \\ 0 \\ d \end{pmatrix}$, ε-mixing, d-magnetic field, refraction index

precession of Bloch vector around "magnetic field" $\vec{V}$



$\vec{R}$ up $\sim |n\rangle$

$\vec{R}$ down $\sim |\bar{n}\rangle$ $\quad \rho_{22}(t) = \frac{4\varepsilon^2}{\Lambda^2}\sin^2\frac{\Lambda}{2} +$

von Neumann – Bloch – Lindblad

von Neumann $\quad\quad\quad\quad\quad\quad$ Bloch

$$i\frac{d\hat{\rho}}{dt} = [\hat{H}, \hat{\rho}] \longleftrightarrow \mathring{\vec{R}} = \vec{V} \times \vec{R}$$

next :

$$\mathring{\rho}_S(t) = L(\rho_S(t)), \quad \rho_S(t) = tr_R\{\rho_{full}(t)\}$$

$$i\,\mathring{\rho}_S(t) = [H, \rho_S] - \frac{i}{2}\sum_n \left(L_n^+ L_n \rho_S + \rho_S L_n^+ L_n - 2 L_n \rho L_n^+\right)$$

L – Lindblad operators, Lindblad 1976, Gorini, Kossakovski, Sudarshan 1976

$\hat{L}$ – departure from ordinary quantum mechanics

$$\mathring{\vec{R}} = \vec{V} \times \vec{R} \longrightarrow \mathring{\vec{R}} = \vec{V} \times \vec{R} - D_T \vec{R}_T, \quad \vec{R}_T = \begin{pmatrix} R_x \\ R_y \end{pmatrix}$$

decoherence

• "Stodolsky" Equation

$$\dot{\vec{R}} = \vec{V} \times \vec{R} - D_T \vec{R}_T - \gamma \vec{R}$$

$\gamma = 1/\tau_\beta$

$$V = \begin{pmatrix} 2\varepsilon \\ 0 \\ d + \mathrm{Re}\,\Lambda \end{pmatrix} \quad D_T = \begin{pmatrix} \mathrm{Im}\,\Lambda & 0 \\ 0 & \mathrm{Im}\,\Lambda \end{pmatrix} \quad R_T = \begin{pmatrix} R_x \\ R_y \end{pmatrix}$$

$$R_x = \rho_{12} + \rho_{21}\,, \quad R_y = -i(\rho_{21} - \rho_{12})$$

$$\Lambda = i n \vartheta \frac{\pi}{k^2}\left(1 - S_1^* S_2\right)$$

$\mathrm{Re}\,\Lambda$ – the refraction energy shift, no decoherence

$\mathrm{Im}\,\Lambda$ – reduction rate, **decoherence**

$$\dot{\vec{R}} = \begin{pmatrix} -(\mathrm{Im}\,\Lambda + \gamma) & -(d + \mathrm{Re}\,\Lambda) & 0 \\ (d + \mathrm{Re}\,\Lambda) & -(\mathrm{Im}\,\Lambda + \gamma) & -2\varepsilon \\ 0 & 2\varepsilon & -\gamma \end{pmatrix} \begin{pmatrix} R_x \\ R_y \\ R_z \end{pmatrix}$$

$$\dot{\vec{R}} = \begin{pmatrix} -(\mathrm{Im}\Lambda+\gamma) & -(d+\mathrm{Re}\Lambda) & 0 \\ (d+\mathrm{Re}\Lambda) & -(\mathrm{Im}\Lambda+\gamma) & -2\varepsilon \\ 0 & 2\varepsilon & -\gamma \end{pmatrix} \begin{pmatrix} R_x \\ R_y \\ R_z \end{pmatrix}$$

This is: Bloch, Lindblad, Quantum Boltzmann, as you wish

Next
- (i) take a reliable set of n and $\bar{n}$ amplitudes and solve either numerically, or analytically (1704.00826)
- (ii) simplify the task to reveal the character of the solution

(ii) Let $d = \mathrm{Re}\,\Lambda = \gamma = 0$, $\mathrm{Im}\,\Lambda \equiv \lambda = \begin{cases} 2/\tau & \text{bottle} \\ \Gamma_a & \text{nuclei} \\ n v \sigma_a & \text{gas} \end{cases}$

Then $\ddot{R}_z + \lambda \dot{R}_z + 4\varepsilon^2 R_z = 0$

Oscillator with friction

$\Omega^2 = \frac{\lambda^2}{4} - 4\varepsilon^2 \rightarrow \Omega^2 > 0$ no oscillations

The relaxation time of $\vec{R}$ τ_{relax} $\tau_z = \left[4\varepsilon^2 \frac{\lambda}{\lambda^2+(d+\mathrm{Re}\Lambda)^2}\right]^{-1} \simeq$

$\simeq \frac{\lambda}{4\varepsilon^2} \gg \tau_{n\bar{n}} = 1/\varepsilon$ The $n \to \bar{n}$ swap is inhibited, $\vec{R}$ remains roughly in the original position.

CONCLUSIONS

- The three settings to search for $n \to \bar{n}$:
  - a) in a trap
  - b) in nuclei
  - c) in gas

  May be treated within the same formalism and described by the same equation
- Decoherence washes out oscillations (but does not inhibit conversion)

Thank you for attention