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Neutron Oscillations: Appearance, Disappearance, and Baryogenesis

Free n-nbar oscillations can be free of damping and decoherence effects

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Neutron oscillations in gas

$$n \to \bar{n} \quad \text{and} \quad n \leftrightarrow n'$$

- There are similarities and differences
- What is effect of residual gas pressure?
- What should be the level of vacuum?
- $n \to \bar{n}$: both components are in the same gas
- $n \to n'$: components can be in different gas
- Mirror gas pressure (if any) can not be controlled
- In mag. field $\mu_n = -\mu_{\bar{n}}$; $\mu_n = \mu_{n'}$, but $B =\neq B'$

Neutron in a medium (gas)

Hamiltonian of a neutron in an environment

$$(\hbar = c = 1)$$

$$\hat{H} = m - i\frac{\Gamma}{2} + \frac{p^2}{2m} + \mu(\vec{\sigma} \cdot \vec{B}) + V_{opt} +$$

$$+ \text{scattering on residual gas}$$

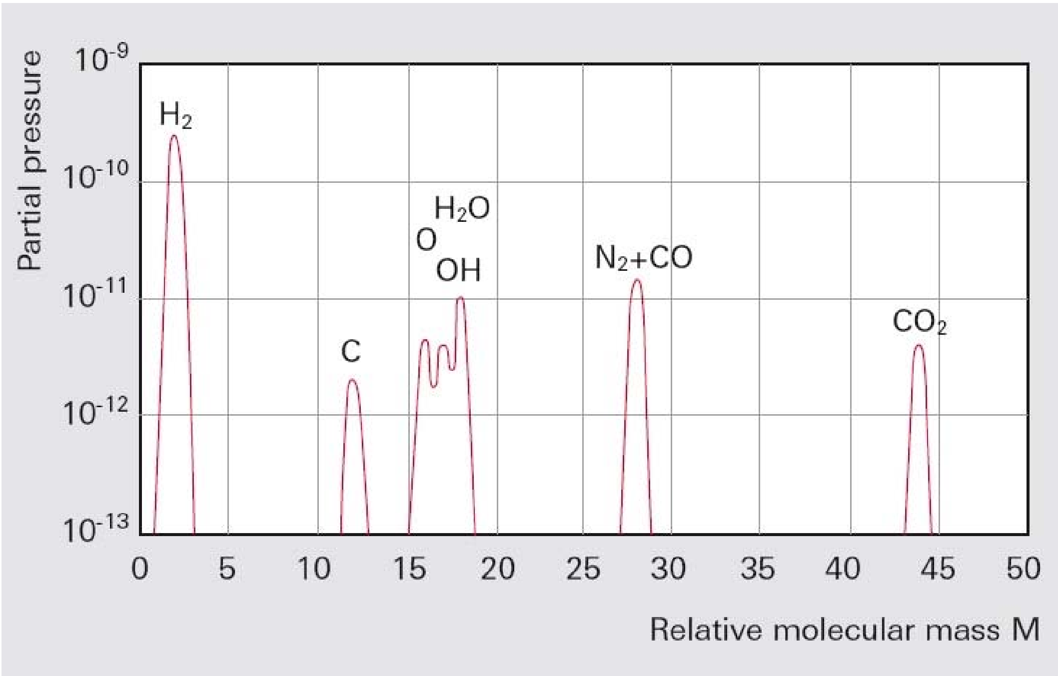
V_{opt} - average nuclear potential of media seen by the neutron

Vacuum gas composition

https://www.pfeiffer-vacuum.com/en/know-how/introduction-to-vacuum-technology/influences-in-real-vacuum-systems/residual-gas-composition/

When working in ultra-high vacuum, it can be important to know the composition of the residual gas before starting vacuum processes or in order to monitor and control processes. The percentages of water (*m/e* = 18) and its fragment OH (*m/e* = 17) will be large in the case of vacuum chambers that are not clean or well baked. Leaks can be identified by the peaks of nitrogen (*m/e* = 28) and oxygen (*m/e* = 32) in the ratio N_2/O_2 of approx. 4 to 1.

Hydrogen (*m/e* = 2), water (*m/e* = 17 and 18), carbon monoxide (*m/e* = 28) and carbon dioxide (*m/e*= 44) will be found in well-baked chambers. No hydrocarbons will be found when using turbomolecular pumps. They are very effectively kept out of the chamber due to the high molecular masses and the resulting high compression ratios. A typical residual gas spectrum for a clean vessel evacuated by a turbomolecular pump is shown in Figure below.



Typical residual gas spectrum of a vessel evacuated by a turbomolecular pump

Mostly H_2

Index of Refraction of the Media.

When neutron with momentum k enters a medium its momentum is modified as $k' = nk$. The n is index of refraction of the media.

$$n = \frac{k'}{k} = \frac{\sqrt{2m(E - Vopt)}}{\sqrt{2mE}} = \sqrt{1 - \frac{V_{opt}}{E}} \approx 1 - \frac{1}{2}\frac{V_{opt}}{E}$$

Modification of the phase induces a modification of the wave velocity is descried by the <u>real part</u> of the index of refraction, while the modification of the amplitude is described by its <u>imaginary part</u>.

V_{opt} - Optical Potential

$$\mathrm{V_{opt}}(k_{rel}) = -2\pi\hbar^2 \frac{\nu_{gas}}{m_*}\langle f_0(k_{rel})\rangle = V - iW$$

Where ν_{gas} - number density of the gas;

m_* - reduced mass;

$f_0(k_{rel})$ - scattering amplitude at angle 0

$k = p/\hbar$ - wave number of neutron

$\mathrm{V_{opt}}$ is part of Hamiltonian

$$\mathrm{V_{opt}}(p_{rel}) = -2\pi\hbar^2 \frac{\nu_{gas}}{m_*} \langle f_0(k_{rel}) \rangle = V - iW$$

$$ReV_{opt} = 2\pi\hbar^2 b \frac{\nu_{gas}}{m_*} = V$$

where $b = -Ref_{\theta=0}$ $- \, coherent$ scattering length

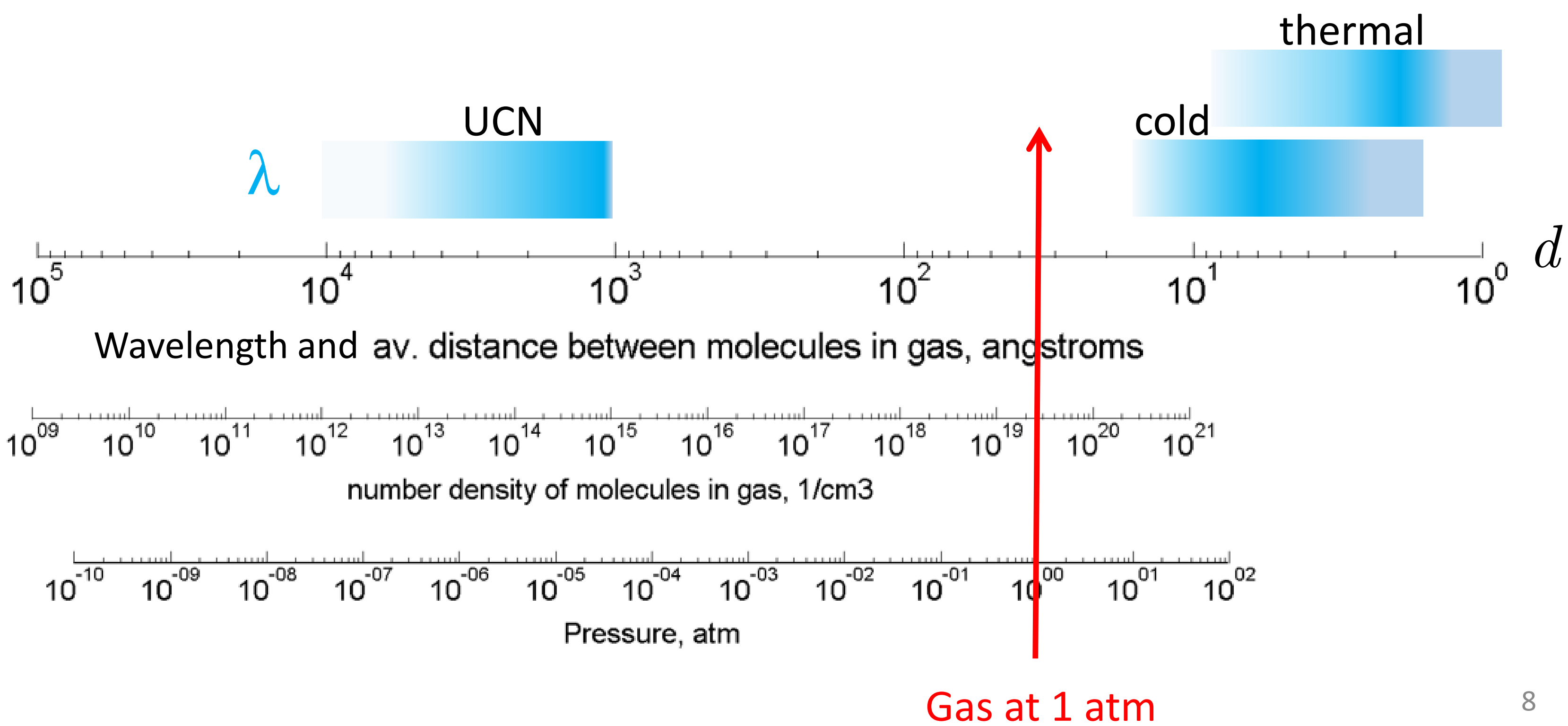
For s-wave elastic scattering: $f_{\theta=0} = -b(1 - ik_{rel}b)$

More generally: $\mathrm{I}mf_{\theta=0} = \frac{k}{4\pi}\sigma_{tot}$ optical theorem

$$ImV_{opt} = \frac{\hbar\nu_{gas}(\sigma_{tot}.\upsilon\,)m_n}{2m_*} = W$$

Neutron wavelength and intermolecular distances

Question: is $\lambda > d$ a necessary condition for defining the refraction index of the media?



About Fermi potential

or refractive index for the neutron in the media

Wavenumber k is a vector: $\vec{k}\ (k_x, k_y, k_z)$ $\quad \left|\vec{k}\right| = k = \frac{2\pi}{\lambda}\ ;\ \lambda = \frac{2\pi}{k}$

that is determining a wavepacket of particle along beam axis z.
For neutron moving along z the k_x and k_y are $\simeq 0$.
That means that in transversal direction wavepackets are not localized and neutron sees other atoms. Thus, the average Fermi potential is defined.

Description of matter effects of oscillating system

- L. D. Landau and E. M. Lifshitz, Quantum Mechanics, Course Theoretical Physics v. 3
- R. P. Feynman, Statistical Mechanics, A Set Of Lectures (Advanced Books Classics)
- https://en.wikipedia.org/wiki/Density_matrix

$$|n> = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \quad |n'> = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Mixed state: $\begin{pmatrix} n \\ n' \end{pmatrix} = a \begin{pmatrix} 1 \\ 0 \end{pmatrix} + b \begin{pmatrix} 0 \\ 1 \end{pmatrix}$; $\quad |a|^2 + |b|^2 = 1$

$$\hat{\rho} = \begin{pmatrix} aa^* & ab^* \\ ba^* & bb^* \end{pmatrix} = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}$$ – density matrix

$$\frac{\partial}{\partial t}\hat{\rho} = -i\left[\hat{H} \cdot \hat{\rho}\right] = -i\hat{H}\hat{\rho} + i\hat{\rho}\hat{H}^\dagger$$

Liouville–von Neumann equation for density matrix evolution

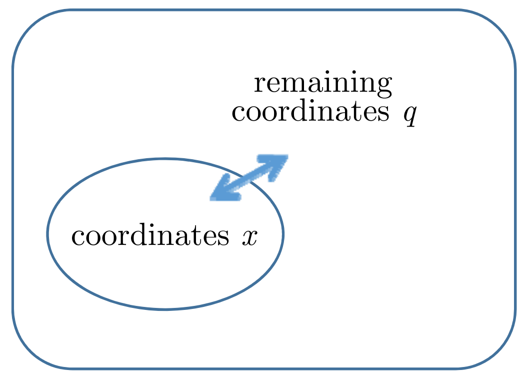
$$\hat{\rho}(t = 0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

For Hermitian, unitary Hamiltonians the Liouville–von Neumann equation is equivalent to Schrödinger equation. The concept of density matrix ρ was introduced to describe the behavior of the system with coordinates x averaged over external coordinates q of the environment:

In Landau v. 3:

$$\bar{f} = \iint \Psi^*(q,x)\hat{f}\,\Psi(q,x)dqdx$$

$$\rho(x,x') \doteq \int \Psi(q,x)\Psi^*(q,x')dq$$



including decaying systems interacting with the environment via absorption.

In our case the environment is some gas in the neutron flight path ("vacuum") and "system with coordinates x" is oscillating $n \rightarrow n'$

Let's consider simple Hamiltonian with $E \neq E'$:

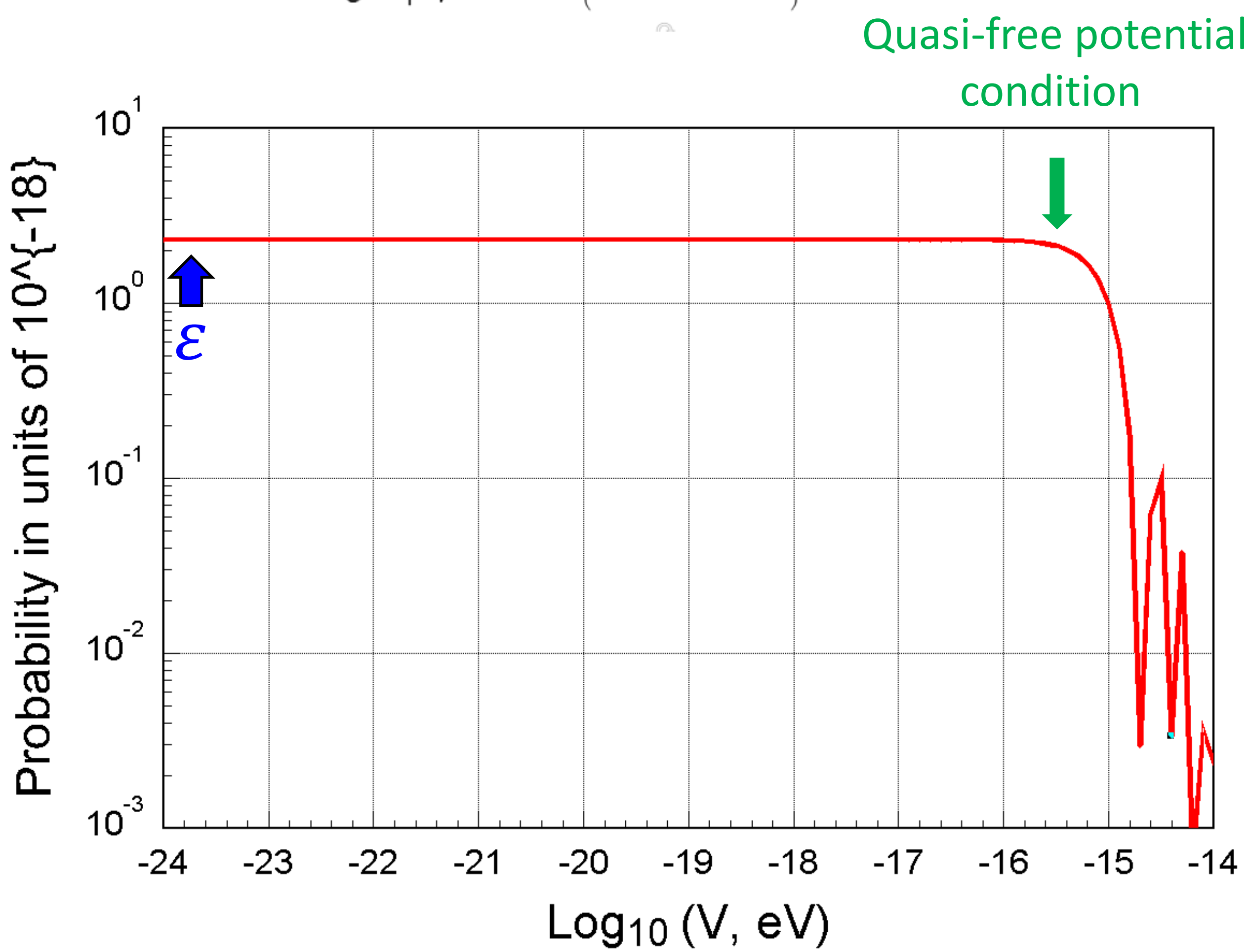
$$\hat{H} = \begin{pmatrix} E & \epsilon \\ \epsilon & E' \end{pmatrix} \qquad E_{1,2} = \underbrace{\frac{E+E'}{2}}_{\text{Average energy level of system } E_0} \pm \underbrace{\sqrt{\left(\frac{E-E'}{2}\right)^2 + \epsilon^2}}_{\omega}$$

If all parameters are real (if H is Hermitian) the oscillating system can be equivalently solved either with time dependent Schrödinger equation or with density matrix. Real part of the energy (potential energy) similar for both components doesn't change oscillation dynamics: $E_0 = (E + E')/2$. Define difference as $2V = |E - E'|$.

$$P_{n'}(t) = \frac{\epsilon^2}{\omega^2} \cdot sin^2 \omega t; \qquad \omega = \sqrt{V^2 + \epsilon^2}$$

The increase of frequency here suppresses the appearance probability.

$$P_{n\to\bar{n}}(t)=\frac{\varepsilon^2}{\varepsilon^2+V^2}\cdot\sin^2\left(\frac{\sqrt{\varepsilon^2+V^2}}{\hbar}\cdot t\right)$$



Quasi-free potential condition doesn’t depend on ε in the wide range of the latter. However, it depends on the flight time t in experiment.

Hamiltonian for the oscillating neutron (no polarization) moving through the gas (no mag. field)

$$\hat{H} = \begin{pmatrix} E + V - iW - \cancel{i\frac{\gamma}{2}} & \epsilon \\ \epsilon & E + V' - iW' - \cancel{i\frac{\gamma}{2}} \end{pmatrix}$$

we should expect for frequency:

Proper oscillation

Real potential diff. increases frequency

$$\boxed{\omega^2 = \epsilon^2 - \xi^2 + V^2 + 2i\xi V}$$

Frequency of the system

Absorption damping

$$\xi = \frac{W - W'}{2}$$

For even more general case of the complex potential, decay, and including magnetic field and spin:

$$\hat{H} = \begin{pmatrix} -i\frac{\gamma}{2} + V - iW + \mu B & 0 & \varepsilon\cos\left(\frac{\beta}{2}\right) & -\varepsilon\sin\left(\frac{\beta}{2}\right) \\ 0 & -i\frac{\gamma}{2} + V - iW - \mu B & \varepsilon\sin\left(\frac{\beta}{2}\right) & \varepsilon\cos\left(\frac{\beta}{2}\right) \\ \varepsilon\cos\left(\frac{\beta}{2}\right) & \varepsilon\sin\left(\frac{\beta}{2}\right) & -i\frac{\gamma}{2} + V' - iW' + \mu' B' & 0 \\ -\varepsilon\sin\left(\frac{\beta}{2}\right) & \varepsilon\cos\left(\frac{\beta}{2}\right) & 0 & -i\frac{\gamma}{2} + V' - iW' - \mu' B' \end{pmatrix}$$

$$\hat{\rho} = \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{21} & \rho_{22} & \rho_{23} & \rho_{24} \\ \rho_{31} & \rho_{32} & \rho_{33} & \rho_{34} \\ \rho_{41} & \rho_{42} & \rho_{43} & \rho_{44} \end{pmatrix} \qquad \hat{\rho}(t=0) = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

System of 16 coupled diff. eqs can be handled by Mathematica

4x4 System of ODEs

$$
\begin{pmatrix}
\dot{\rho}_{11} \\ \dot{\rho}_{12} \\ \dot{\rho}_{13} \\ \dot{\rho}_{14} \\ \dot{\rho}_{21} \\ \dot{\rho}_{22} \\ \dot{\rho}_{23} \\ \dot{\rho}_{24} \\ \dot{\rho}_{31} \\ \dot{\rho}_{32} \\ \dot{\rho}_{33} \\ \dot{\rho}_{34} \\ \dot{\rho}_{41} \\ \dot{\rho}_{42} \\ \dot{\rho}_{43} \\ \dot{\rho}_{44}
\end{pmatrix}
=
\begin{pmatrix}
\rho 11\,(-\gamma - v\,v\,\sigma) + i\,\varepsilon\,\rho 13\cos\left(\frac{\beta}{2}\right) - i\,\varepsilon\,\rho 14\sin\left(\frac{\beta}{2}\right) - i\,\varepsilon\,\rho 31\cos\left(\frac{\beta}{2}\right) + i\,\varepsilon\,\rho 41\sin\left(\frac{\beta}{2}\right) \\
\rho 12\,(-\gamma - v\,v\,\sigma - 2\,i\,B\,\mu) + i\,\varepsilon\,\rho 13\sin\left(\frac{\beta}{2}\right) + i\,\varepsilon\,\rho 14\cos\left(\frac{\beta}{2}\right) - i\,\varepsilon\,\rho 32\cos\left(\frac{\beta}{2}\right) + i\,\varepsilon\,\rho 42\sin\left(\frac{\beta}{2}\right) \\
\rho 13\left(-\gamma - \frac{v\,v\,\sigma}{2} - \frac{1}{2}v'\,v'\,\sigma' - i\,B\,\mu + i\,\mu\,B' + \frac{2\,i\,\pi\,v\,v\,\mathrm{Re}(f)}{k} - \frac{2\,i\,\pi\,\mathrm{Re}(f')\,v'\,v'}{k}\right) + i\,\varepsilon\,\rho 11\cos\left(\frac{\beta}{2}\right) + i\,\varepsilon\,\rho 12\sin\left(\frac{\beta}{2}\right) - i\,\varepsilon\,\rho 33\cos\left(\frac{\beta}{2}\right) + i\,\varepsilon\,\rho 43\sin\left(\frac{\beta}{2}\right) \\
\rho 14\left(-\gamma - \frac{v\,v\,\sigma}{2} - \frac{1}{2}v'\,v'\,\sigma' - i\,B\,\mu - i\,\mu\,B' + \frac{2\,i\,\pi\,v\,v\,\mathrm{Re}(f)}{k} - \frac{2\,i\,\pi\,\mathrm{Re}(f')\,v'\,v'}{k}\right) - i\,\varepsilon\,\rho 11\sin\left(\frac{\beta}{2}\right) + i\,\varepsilon\,\rho 12\cos\left(\frac{\beta}{2}\right) - i\,\varepsilon\,\rho 34\cos\left(\frac{\beta}{2}\right) + i\,\varepsilon\,\rho 44\sin\left(\frac{\beta}{2}\right) \\
\rho 21\,(-\gamma - v\,v\,\sigma) + i\,\varepsilon\,\rho 23\cos\left(\frac{\beta}{2}\right) - i\,\varepsilon\,\rho 41\cos\left(\frac{\beta}{2}\right) - i\,\varepsilon\,\rho 24\sin\left(\frac{\beta}{2}\right) - i\,\varepsilon\,\rho 31\sin\left(\frac{\beta}{2}\right) \\
\rho 22\,(-\gamma - v\,v\,\sigma + 2\,i\,B\,\mu) + i\,\varepsilon\,\rho 23\sin\left(\frac{\beta}{2}\right) + i\,\varepsilon\,\rho 24\cos\left(\frac{\beta}{2}\right) - i\,\varepsilon\,\rho 32\sin\left(\frac{\beta}{2}\right) - i\,\varepsilon\,\rho 42\cos\left(\frac{\beta}{2}\right) \\
\rho 23\left(-\gamma - \frac{v\,v\,\sigma}{2} - \frac{1}{2}v'\,v'\,\sigma' + i\,B\,\mu + i\,\mu\,B' + \frac{2\,i\,\pi\,v\,v\,\mathrm{Re}(f)}{k} - \frac{2\,i\,\pi\,\mathrm{Re}(f')\,v'\,v'}{k}\right) + i\,\varepsilon\,\rho 21\cos\left(\frac{\beta}{2}\right) + i\,\varepsilon\,\rho 22\sin\left(\frac{\beta}{2}\right) - i\,\varepsilon\,\rho 33\sin\left(\frac{\beta}{2}\right) - i\,\varepsilon\,\rho 43\cos\left(\frac{\beta}{2}\right) \\
\rho 24\left(-\gamma - \frac{v\,v\,\sigma}{2} - \frac{1}{2}v'\,v'\,\sigma' + i\,B\,\mu - i\,\mu\,B' + \frac{2\,i\,\pi\,v\,v\,\mathrm{Re}(f)}{k} - \frac{2\,i\,\pi\,\mathrm{Re}(f')\,v'\,v'}{k}\right) - i\,\varepsilon\,\rho 21\sin\left(\frac{\beta}{2}\right) + i\,\varepsilon\,\rho 22\cos\left(\frac{\beta}{2}\right) - i\,\varepsilon\,\rho 34\sin\left(\frac{\beta}{2}\right) - i\,\varepsilon\,\rho 44\cos\left(\frac{\beta}{2}\right) \\
\rho 31\left(-\gamma - \frac{v\,v\,\sigma}{2} - \frac{1}{2}v'\,v'\,\sigma' + i\,B\,\mu - i\,\mu\,B' - \frac{2\,i\,\pi\,v\,v\,\mathrm{Re}(f)}{k} + \frac{2\,i\,\pi\,\mathrm{Re}(f')\,v'\,v'}{k}\right) - i\,\varepsilon\,\rho 11\cos\left(\frac{\beta}{2}\right) - i\,\varepsilon\,\rho 21\sin\left(\frac{\beta}{2}\right) + i\,\varepsilon\,\rho 33\cos\left(\frac{\beta}{2}\right) - i\,\varepsilon\,\rho 34\sin\left(\frac{\beta}{2}\right) \\
\rho 32\left(-\gamma - \frac{v\,v\,\sigma}{2} - \frac{1}{2}v'\,v'\,\sigma' - i\,B\,\mu - i\,\mu\,B' - \frac{2\,i\,\pi\,v\,v\,\mathrm{Re}(f)}{k} + \frac{2\,i\,\pi\,\mathrm{Re}(f')\,v'\,v'}{k}\right) - i\,\varepsilon\,\rho 12\cos\left(\frac{\beta}{2}\right) - i\,\varepsilon\,\rho 22\sin\left(\frac{\beta}{2}\right) + i\,\varepsilon\,\rho 33\sin\left(\frac{\beta}{2}\right) + i\,\varepsilon\,\rho 34\cos\left(\frac{\beta}{2}\right) \\
\rho 33\,(-\gamma - v'\,v'\,\sigma') - i\,\varepsilon\,\rho 13\cos\left(\frac{\beta}{2}\right) - i\,\varepsilon\,\rho 23\sin\left(\frac{\beta}{2}\right) + i\,\varepsilon\,\rho 31\cos\left(\frac{\beta}{2}\right) + i\,\varepsilon\,\rho 32\sin\left(\frac{\beta}{2}\right) \\
\rho 34\,(-\gamma - v'\,v'\,\sigma' - 2\,i\,\mu\,B') - i\,\varepsilon\,\rho 14\cos\left(\frac{\beta}{2}\right) - i\,\varepsilon\,\rho 24\sin\left(\frac{\beta}{2}\right) - i\,\varepsilon\,\rho 31\sin\left(\frac{\beta}{2}\right) + i\,\varepsilon\,\rho 32\cos\left(\frac{\beta}{2}\right) \\
\rho 41\left(-\gamma - \frac{v\,v\,\sigma}{2} - \frac{1}{2}v'\,v'\,\sigma' + i\,B\,\mu + i\,\mu\,B' - \frac{2\,i\,\pi\,v\,v\,\mathrm{Re}(f)}{k} + \frac{2\,i\,\pi\,\mathrm{Re}(f')\,v'\,v'}{k}\right) + i\,\varepsilon\,\rho 11\sin\left(\frac{\beta}{2}\right) - i\,\varepsilon\,\rho 21\cos\left(\frac{\beta}{2}\right) + i\,\varepsilon\,\rho 43\cos\left(\frac{\beta}{2}\right) - i\,\varepsilon\,\rho 44\sin\left(\frac{\beta}{2}\right) \\
\rho 42\left(-\gamma - \frac{v\,v\,\sigma}{2} - \frac{1}{2}v'\,v'\,\sigma' - i\,B\,\mu + i\,\mu\,B' - \frac{2\,i\,\pi\,v\,v\,\mathrm{Re}(f)}{k} + \frac{2\,i\,\pi\,\mathrm{Re}(f')\,v'\,v'}{k}\right) + i\,\varepsilon\,\rho 12\sin\left(\frac{\beta}{2}\right) - i\,\varepsilon\,\rho 22\cos\left(\frac{\beta}{2}\right) + i\,\varepsilon\,\rho 43\sin\left(\frac{\beta}{2}\right) + i\,\varepsilon\,\rho 44\cos\left(\frac{\beta}{2}\right) \\
\rho 43\,(-\gamma - v'\,v'\,\sigma' + 2\,i\,\mu\,B') + i\,\varepsilon\,\rho 13\sin\left(\frac{\beta}{2}\right) - i\,\varepsilon\,\rho 23\cos\left(\frac{\beta}{2}\right) + i\,\varepsilon\,\rho 41\cos\left(\frac{\beta}{2}\right) + i\,\varepsilon\,\rho 42\sin\left(\frac{\beta}{2}\right) \\
\rho 44\,(-\gamma - v'\,v'\,\sigma') + i\,\varepsilon\,\rho 14\sin\left(\frac{\beta}{2}\right) - i\,\varepsilon\,\rho 24\cos\left(\frac{\beta}{2}\right) - i\,\varepsilon\,\rho 41\sin\left(\frac{\beta}{2}\right) + i\,\varepsilon\,\rho 42\cos\left(\frac{\beta}{2}\right)
\end{pmatrix}
$$

Thi description by Liouville–von Neumann equation is still not complete

$$\frac{\partial}{\partial t}\hat{\rho} = -i\left[\hat{H}\cdot\hat{\rho}\right] = -i\hat{H}\hat{\rho} + i\hat{\rho}\hat{H}^{\dagger}$$ – Liouville–von Neumann equation

Direct interaction with environment via incoherent elastic collisions is not included. This will lead to decoherence.

“Most general evolution of probabilities satisfies an equation of a class known as Lindblad equations.”

The New York Review of Books

The Trouble with Quantum Mechanics

Steven Weinberg JANUARY 19, 2017 ISSUE

http://www.nybooks.com/articles/2017/01/19/trouble-with-quantum-mechanics/

Lindblad Equation

https://en.wikipedia.org/wiki/Lindblad_equation

The Lindblad master equation for an N-dimensional system's reduced density matrix ρ can be written:

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] + \sum_{n,m=1}^{N^2-1} h_{n,m}\left(L_n\rho L_m^\dagger - \frac{1}{2}\left(\rho L_m^\dagger L_n + L_m^\dagger L_n \rho\right)\right)$$

This equation includes in general the loss of coherence of oscillating system to environment.

Loss of coherence is the reset of the oscillation phase between two components.
At this moment oscillating system collapses (with some probability) into one of its pure states and continue motion with this boundary conditions. It is “measurement” of the system by the environment; system remains in the oscillating state with reset boundary conditions.

Example of decoherence of two-level system in old classical paper

PHYSICAL REVIEW VOLUME 123, NUMBER 4 AUGUST 15, 1961

Conversion of Muonium into Antimuonium*

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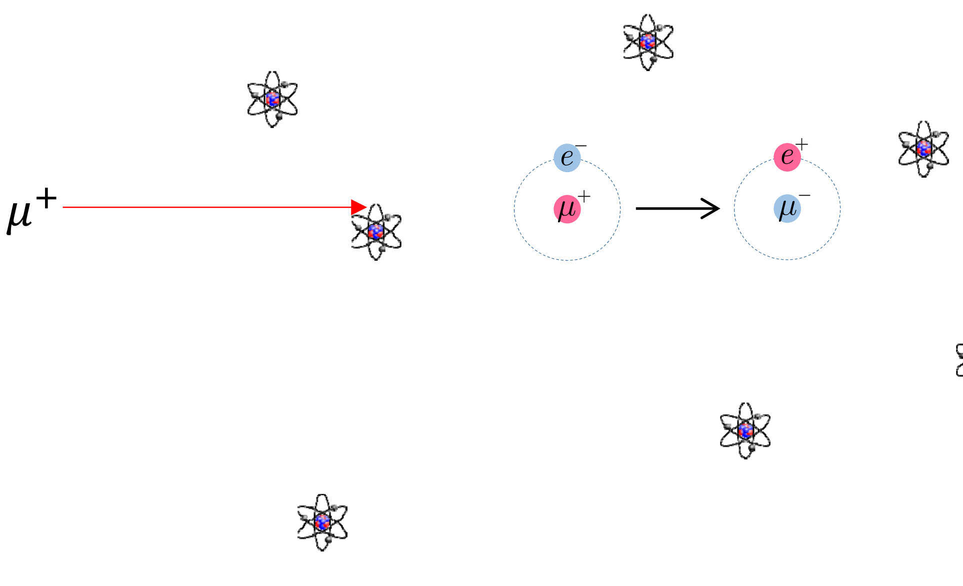
(Received April 4, 1961)

A detailed analysis is made of the possible conversion of muonium into antimuonium in various environments. An assumed $\bar{\mu}e\bar{\mu}e$ weak interaction of the usual form and strength gives a probability of 2.5×10^{-5} in vacuum, even in the presence of reasonable external electric fields. In a solid the probability is less by at least 10, and probably 20, orders of magnitude. In an inert gas the probability is roughly to be divided by the numbers of collisions during a muon lifetime, and hence is quite small unless the pressure at room temperature is less than about 10^{-4} atm. Lowering the temperature does not help. A possible experiment is suggested.



<u>Measurement</u> of the system (the effect):
μ^- decay with emission of Michel's e^-;
or inelastic collision with molecules
leading again to the decay of the μ^-

Here the evolution equation for ρ has resulted into simple form of Lindblad equation

Muonium in vacuum

… and the probability that the muon decays as μ^- rather than μ^+ is

$$P(\bar{M})=\int_0^\infty \lambda e^{-\lambda t}|\langle \bar{M}|\Psi(t)\rangle|^2=\frac{|\delta|^2}{2(|\delta|^2+\Delta^2+\lambda^2)}, \quad (6)$$

where $\lambda=3\times10^{-10}$ ev is the muon decay rate.

In the absence of external fields,

$$P(\bar{M})\simeq(|\delta|^2/2\lambda^2)=2.5\times10^{-5} \quad (7)$$

IV. MUONIUM IN GASES

In treating the $M \rightarrow \bar{M}$ process in a gas we shall assume that the muonium system is scattered incoherently by the gas molecules, except of course for the coherent forward scattering responsible for the index of refraction. However, we do not want to assume that in general the muonium simply moves classically from molecule to molecule. In this situation it seems essential to use a statistical matrix treatment.

Suppose that we refer to the sequence of elastic scatterings up to time t as a "history" H, with probability $P(H)$. [The sum of the $P(H)$ is the probability that a decay or inelastic collision has not yet occurred, and hence vanishes as $t \to \infty$.] Each history gives rise to a 2-dimensional state vector $u(H)$ with components $u_1 = \langle M | \Psi \rangle$ and $u_2 = \langle \bar{M} | \Psi \rangle$, normalized in the sense that

$$\|u\|^2 = |u_1|^2 + |u_2|^2 = 1. \tag{12}$$

The statistical matrix $\rho(t)$ is defined as

$$\rho(t) = \sum_H P(H) u(H) u^\dagger(H). \tag{13}$$

(1)

At time $t' = t + dt$, the history H' might consist of either:

(i) A history H followed by elastic scattering through an angle θ, giving

$$u(H') = F(\theta) u(H) / \|F(\theta) u(H)\|, \tag{14}$$

$$P(H') = \|F(\theta) u(H)\|^2 P(H) n v dt, \tag{15}$$

where $F(\theta)$ is the matrix

$$F(\theta) = \begin{pmatrix} f(\theta) & 0 \\ 0 & \bar{f}(\theta) \end{pmatrix}, \tag{16}$$

and n is the number density of gas molecules, v is the muonium velocity (assumed fixed), and f and $\bar{f}$ are the M and $\bar{M}$ elastic scattering amplitudes.

(2)

(ii) A history H followed by no decay or collisions, except for the unavoidable coherent forward scattering. This gives

$$u(H')=(1+A\,dt)u(H)/\|(1+A\,dt)u(H)\|, \qquad (17)$$

$$P(H')=1-u(H)^{\dagger}Bu(H)dt, \qquad (18)$$

where

$$A=\begin{bmatrix} \frac{2\pi inv}{k}f(0)-iE_0-\frac{\lambda}{2} & -i\frac{\delta}{2} \\ -i\frac{\delta^*}{2} & \frac{2\pi inv}{k}\bar{f}(0)-i\bar{E}_0-\frac{\lambda}{2} \end{bmatrix}, \qquad (19)$$

$$B=\begin{bmatrix} \omega_c+\lambda & 0 \\ 0 & \bar{\omega}_c+\lambda \end{bmatrix}. \qquad (20)$$

Here E_0 and $\bar{E}_0$ are the M and $\bar{M}$ energies between collisions, $k\cong m_\mu v$, and ω_c and $\bar{\omega}_c$ are the total (elastic plus inelastic) M and $\bar{M}$ collision rates. It follows from the optical theorem that $A+A^{\dagger}=-B$, so that

$$P(H')=\|(1+A\,dt)u(H)\|^2. \qquad (21)$$

Now at time t' the statistical matrix is

$$\rho(t') = \sum_{H'} P(H')u(H')u(H')^\dagger$$

$$= \sum_H P_H \left\{ nvdt \int F(\theta)u(H)u(H)^\dagger F^\dagger(\theta)d\Omega \right.$$

$$\left. + (1+Adt)u(H)u(H)^\dagger(1+Adt)^\dagger \right\}$$

(1)

$$= hvdt \int F(\theta)\rho(t)F^\dagger(\theta)d\Omega$$

(2)

$$+ (1+Adt)\rho(t)(1+Adt)^\dagger, \quad (22)$$

and hence, finally,

$$d\rho/dt = A\rho + \rho A^\dagger + nv \int F(\theta)\rho F^\dagger(\theta)\, d\Omega. \tag{23}$$

We have derived here four coupled linear differential equations for the four components of $\rho(t)$. It might be necessary to go through the straightforward but tedious task of solving them if (as in an experiment with gated counters) it were necessary to know the time dependence of $\rho(t)$. We shall only solve for $P(\bar{M})$.

It looks like the Lindblad “super-operator” term is coming here from the incoherent elastic scattering of the oscillating system on the molecules of environment.

… and produces a decoherence effect on oscillation.

At all reasonable temperatures in a dilute inert gas, ω_I will probably be small compared to λ. It is known experimentally[8] that a good fraction of the muonium formed in argon at room temperature and 50 atm lasts long enough for the μ^+ to decay. (The same is probably not true for $\bar{M}$.) Hence $\omega \simeq \lambda$, so that

$$P(\bar{M}) \simeq |\delta|^2/2\lambda\Lambda_0 = 2.5\times10^{-5}/N, \qquad (38)$$

Probability of oscillation in vacuum. Noted by Kerbikov: $\omega \simeq \lambda$ might be not the case for $n \to \bar{n}$ in gas.

where $N = \Lambda_0/\lambda$ is (for large N) the mean of the number of collisions suffered during a muon lifetime for M and $\bar{M}$. It makes no difference whether the $\bar{M}$ collisions are elastic or inelastic, providing that an $\bar{M}$ breakup is detectable through μ^- decay or absorption. We may estimate N roughly (for $N > 1$) as

$$N \simeq \pi n v R^2/\lambda \simeq \pi(kR)Rn/m_\mu\lambda$$
$$\simeq n(kR)/(3\times10^{15}\ \text{cm}^{-3}), \qquad (39)$$

so at room temperature we need $n \ll 3\times10^{15}$ cm^{-3} to avoid quenching of the $M \to \bar{M}$ process. (In the

Scattering Integral in Feinberg & Weinberg evolution equation

$$d\rho/dt = A\rho + \rho A^{\dagger} + nv \int F(\theta)\rho F^{\dagger}(\theta)\, d\Omega. \qquad (23)$$

$$F(\theta) = \begin{pmatrix} f_1(\theta) & 0 \\ 0 & f_2(\theta) \end{pmatrix}$$

Source of decoherence

$$\int F(\theta)\rho F^{\dagger}(\theta)\, d\Omega = 4\pi \begin{pmatrix} |f_1|^2\rho_{11} & f_1 f_2^{*}\rho_{12} \\ f_1^{*} f_2 \rho_{21} & |f_2|^2\rho_{22} \end{pmatrix}$$

Will be zero if
one of f's is zero

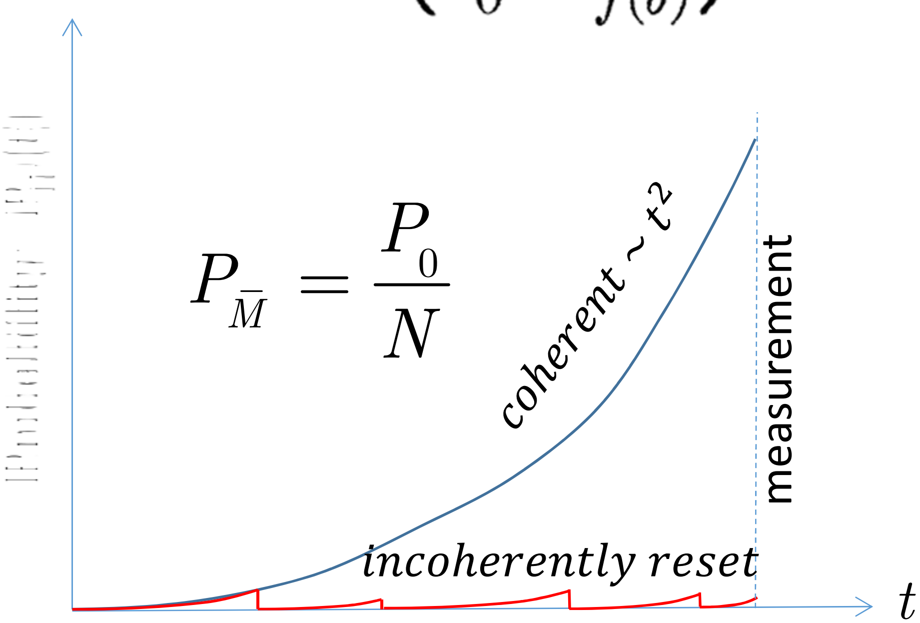
Loss of COHERENCE is due to scattering integral

$$d\rho/dt = A\rho + \rho A^{\dagger} + nv \int F(\theta)\rho F^{\dagger}(\theta)\, d\Omega. \qquad (23)$$

Absorption and decay make here negative contribution

$$F(\theta) = \begin{pmatrix} f(\theta) & 0 \\ 0 & \bar{f}(\theta) \end{pmatrix}$$

$$P_{\bar{M}} = \frac{P_0}{N}$$



In the vacuum the probability ρ_{22} coherently grows as $\sim t^2$ [$sin^2(\omega t)$] . Every incoherent elastic collision resets the oscillating system's phase to zero, but the system continues its motion in the environment contributing (with + sign) to the evolution of ρ until it is being "measured" at some later time. Weinberg's recommendation is to make number of collisions < 1. That is the reason why current muonium oscillation search experiments are being performed in vacuum or with zero pressure.

Loss of coherence in $n - \bar{n}$ transformation?

- In ESS n-nbar experiment with L=200-m vacuum vessel and residual pressure is <10^{-8} atm or < 10^{-3} Pa , vacuum gas H_2 , with total cross section 82 barns for thermal neutrons (overestimate), the probability of elastic collision for the neutron component with gas molecules is $\nu\sigma L \lesssim 10^{-6}$ per flight. Elastic x-section for $\bar{n}$ component is not well known but its x-section is not larger than for np.

- If incoherent elastic scattering will occur to oscillating $n\bar{n}$ system in the beam, the $n\bar{n}$ system will be scattered isotropically in s-wave and will be mostly removed from the beam. So it will not contribute to the evolution of ρ matrix through scattering integral, since the system after scattering can not be measured.

Conclusion: for $n\bar{n}$ oscillation in gas with residual pressure < 10^{-5} $mbar$ the evolution equation has the form :

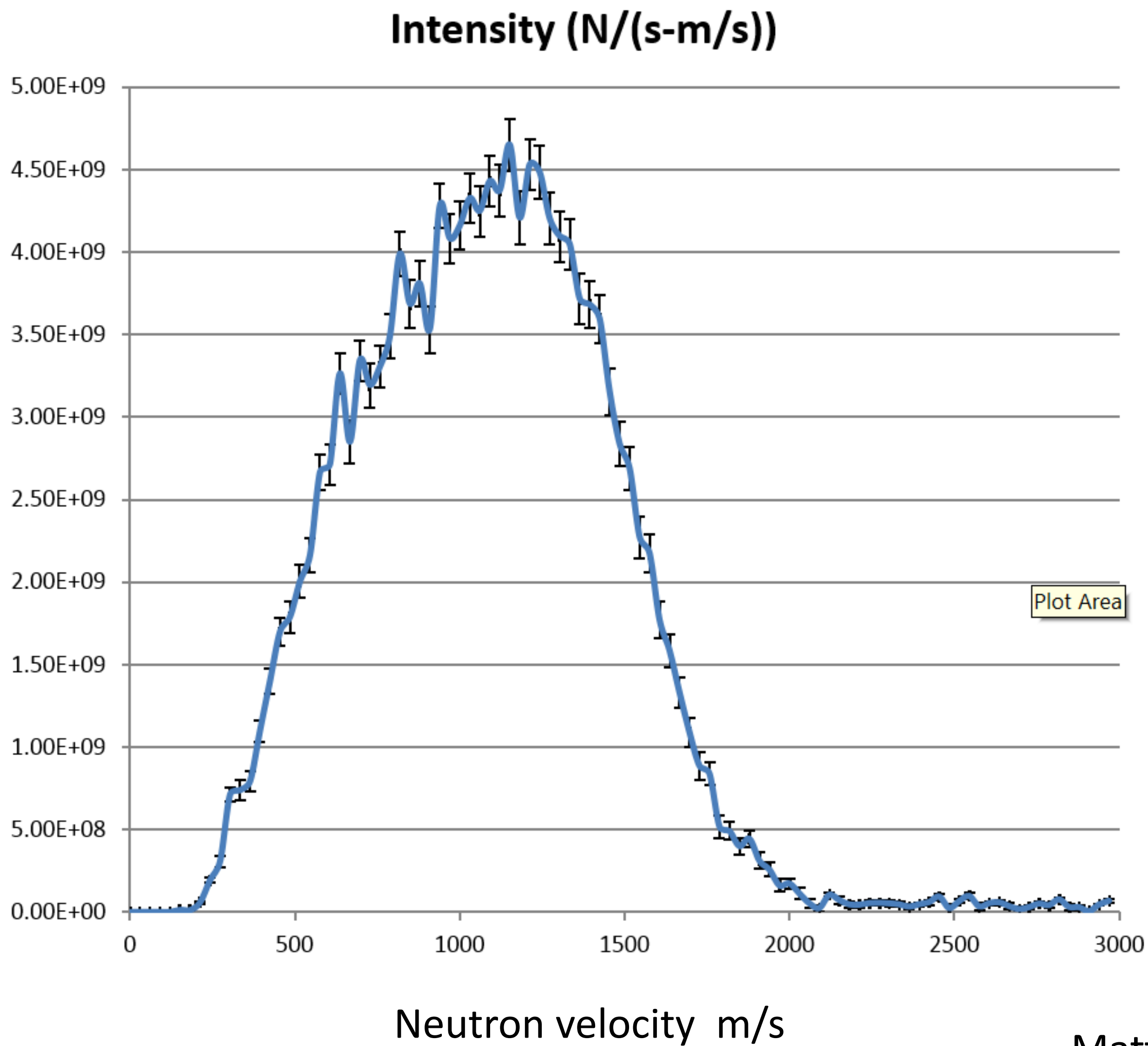
$$\frac{\partial}{\partial t}\hat{\rho} = -i\hat{H}\hat{\rho} + i\hat{\rho}\hat{H}^{\dagger} + O(0)$$

without scattering integral introducing decoherence.

Parameters used in calculation

Variable	Value
Neutron Lifetime (PDG)	880.3 s
$n - \bar{n}$ Oscillation Time	2.4×10^8 s
gas	diatomic hydrogen
mass	2 amu
neutron-gas reduced mass	$\frac{2}{3} \times 939.57$ MeV
gas temperature	293 K
mean gas velocity	from Maxwell-Boltzmann distribution
relative velocity	$\lvert v_n - v_g \rvert$
length of tube	200 m
b_{H_2}	$2 \times -3.74 \times 10^{-13}$ cm
$\sigma_{H_2 coh}$	7.03×10^{-24} cm^2
$\sigma_{H_2 incoh}$	$2 \times 80.26 \times 10^{-24}$ cm^2 **nH_2**
σ_{abs} (at 2200 m/s)	$2 \times 33.26 \times 10^{-26}$ cm^2
$b_{\bar{H}_2}$	$2 \times 0.94 \times 10^{-13}$ cm
$\sigma_{\bar{H}_2 coh}$	4.44×10^{-25} cm^2
$\sigma_{\bar{H}_2 incoh}$	0 ? **$\bar{n}H_2$**
σ_{ann} (at 0 m/s)	$2 \times 44 \times 10^{-27}$ cm^2

Neutron velocity spectrum for ESS beam arriving to n-nbar target



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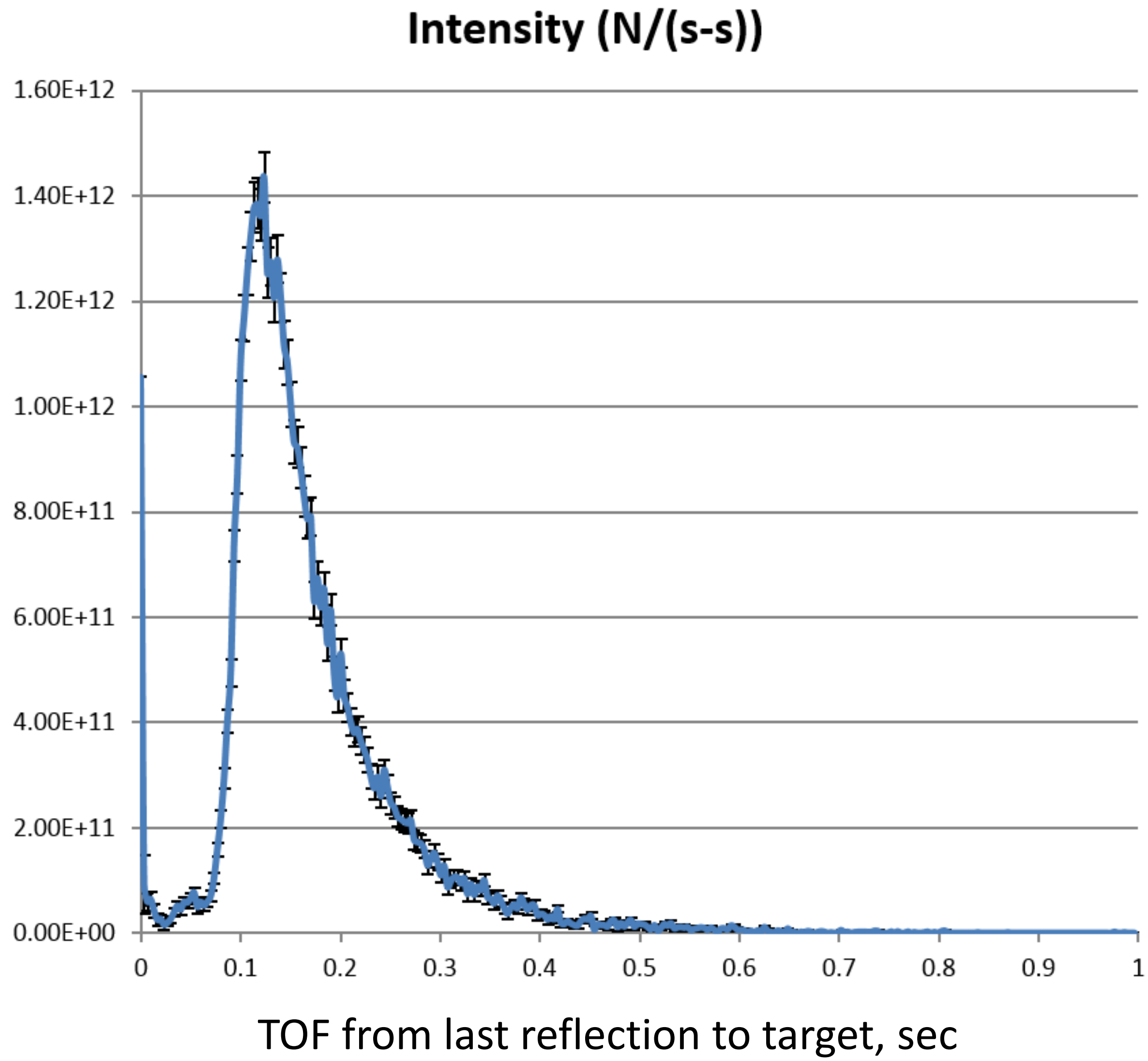
Neutron TOF spectrum for ESS beam In n-nbar layout



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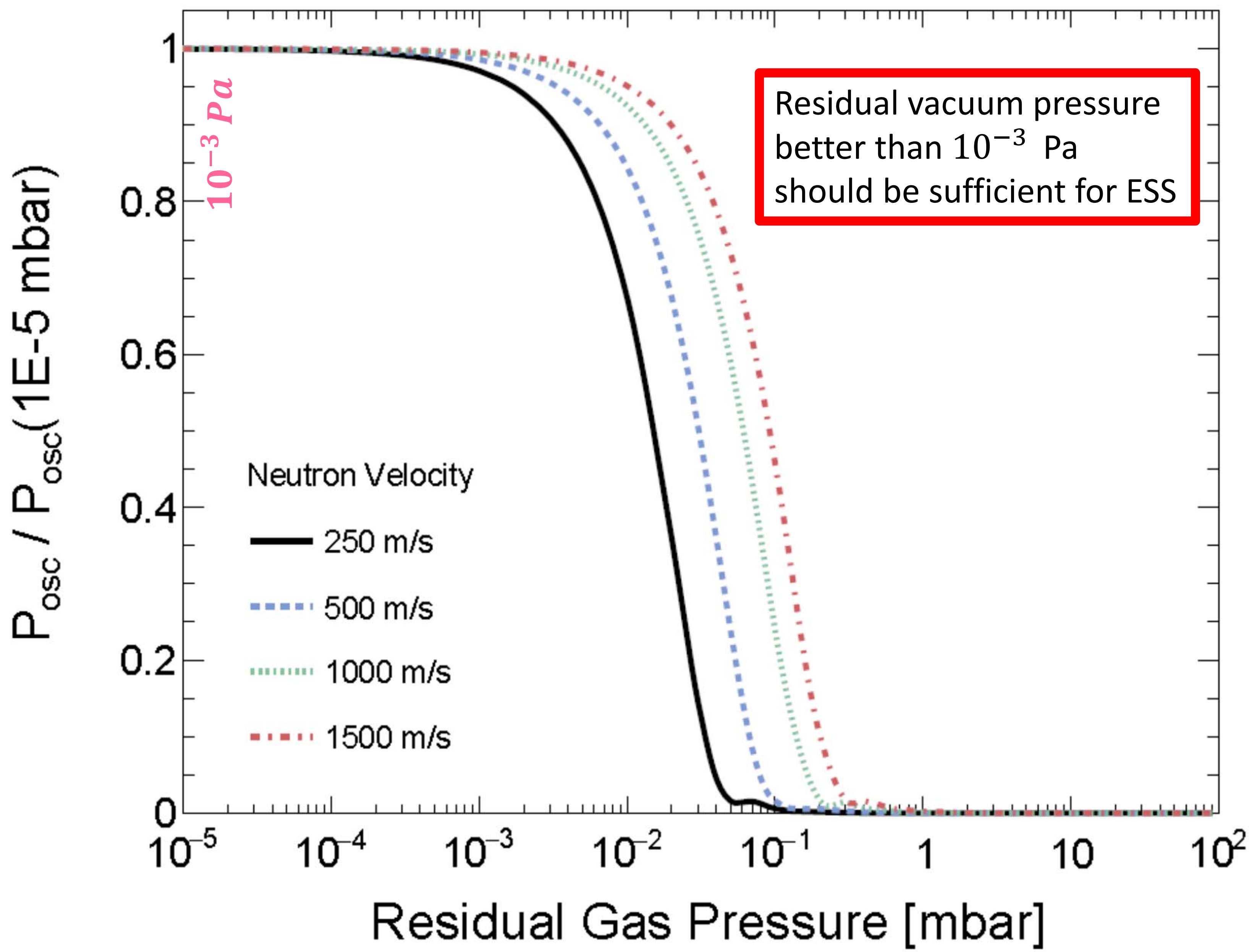
Requirement for $n\bar{n}$ residual gas pressure (assuming hydrogen) in ESS 200 m flight path



L. Varriano/UT