n-$\bar{\mathrm{n}}$ oscillations beyond the quasi-free limit
or
n-$\bar{\mathrm{n}}$ oscillations in the presence of magnetic field

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Based on: Phys. Rev. D 95, 036004 (with A.R. Young)

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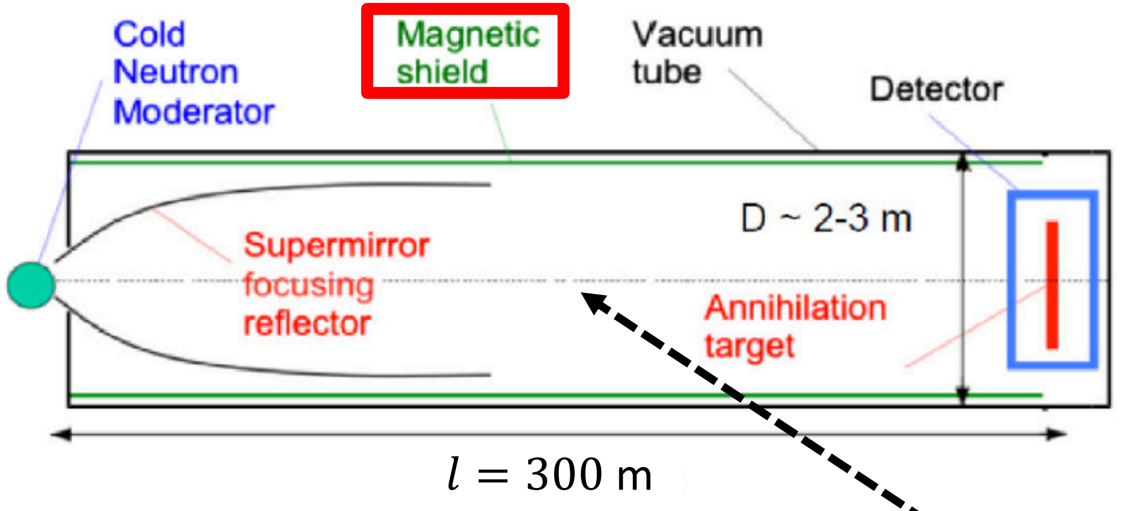
Contents

- Magnetic field (in the propagation region): much ado about nothing?
- Propagation region dynamics: mapping to spinor evolution problem
- Perturbative and non-perturbative analysis in noisy fields
- Exploring the use of NMR quantum control protocols

The magnetic field: much ado about nothing?

From D.G. Phillips II et al., Phys. Rep. 621, 1:

"For the magnetic shielding geometries of both the previous ILL experiment and the proposed experiment, … the dominant component of the residual magnetic field inside the shield is the component along the axis of the shield. The internal shield for the previous ILL experiment strongly suppressed transverse components of the magnetic field and rendered the longitudinal component sufficiently uniform that it could be largely compensated by a homogeneous external field generated by a coil wrapped on the outside of the shield. Once this major component to the residual field was removed, another set of coils were able to trim out the residual transverse fields. Current loops **for shield demagnetization**, an active compensation system for external magnetic field variations (including transverse fields), internal magnetometry, and removal of large external sources of magnetic field gradients were also required to ensure maintenance of the quasi-free condition. Since this experiment was performed a great deal has been learned about large volume magnetic shield technology in the course of R&D performed for experiments which search for the neutron electric dipole moment [125]."



Magnetically shielded propagation region

?

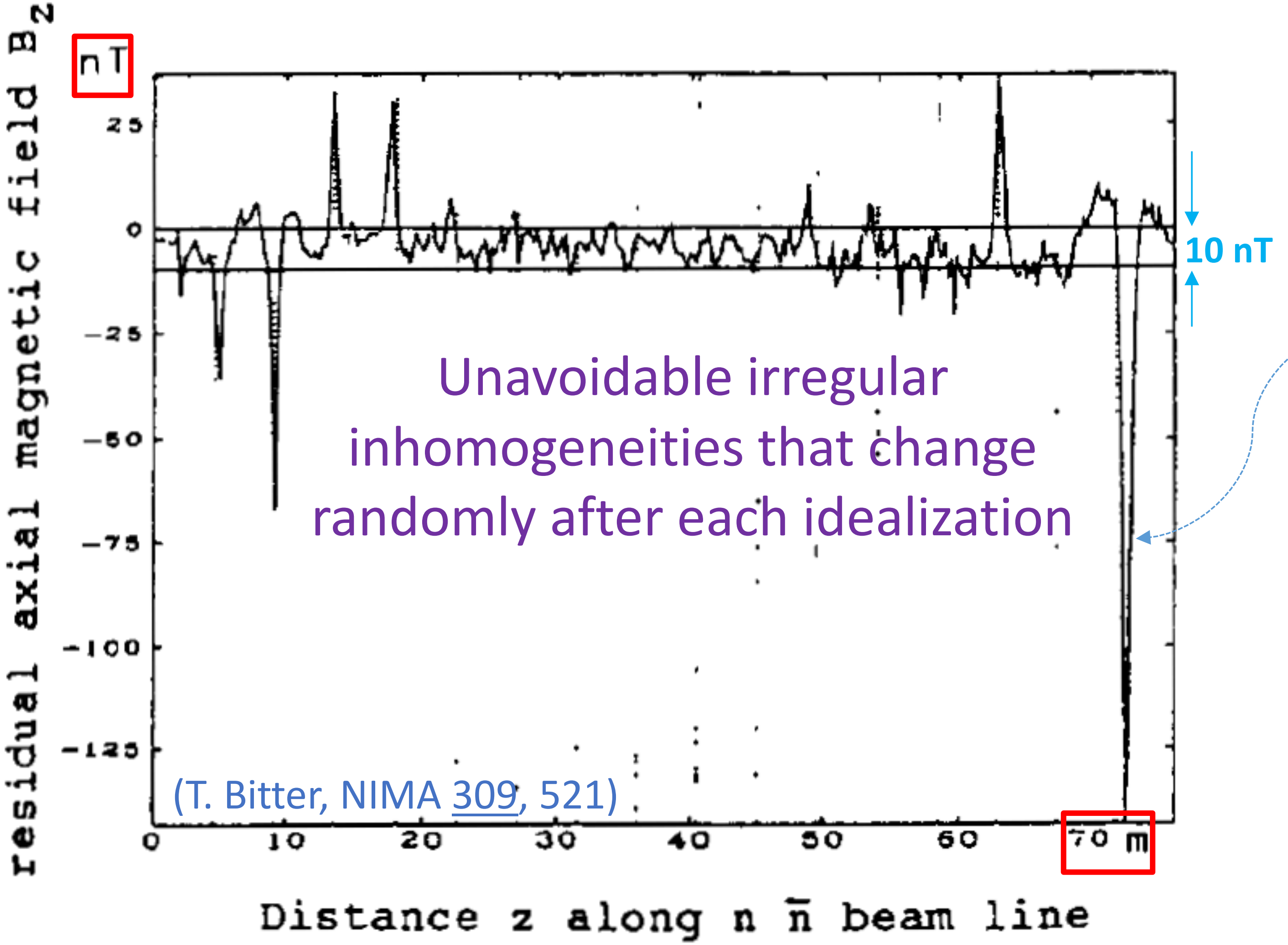
shield *idealization* = optimal adaptation of shield's magnetization to external fields

I. Altarev et al., Rev. Sci. Instrum. 85, 075106: "A magnetically shielded room with ultra low residual field and gradient"

The ideal case (no joints)

What do ILL field profiles look like? (After a daily shield idealization)

"… field variations are much worse than with no shield at all. At every joint and at every welding seem [sic] (three per tube segment) the magnetic flux leaks out and distorts the earth field considerably." (T. Bitter et al, NIMA 309, 521)



"The spikes visible at the beginning and at the end of the field profile do not noticeably disturb the neutron-antineutron oscillation process, so we did not bother to further suppress them."

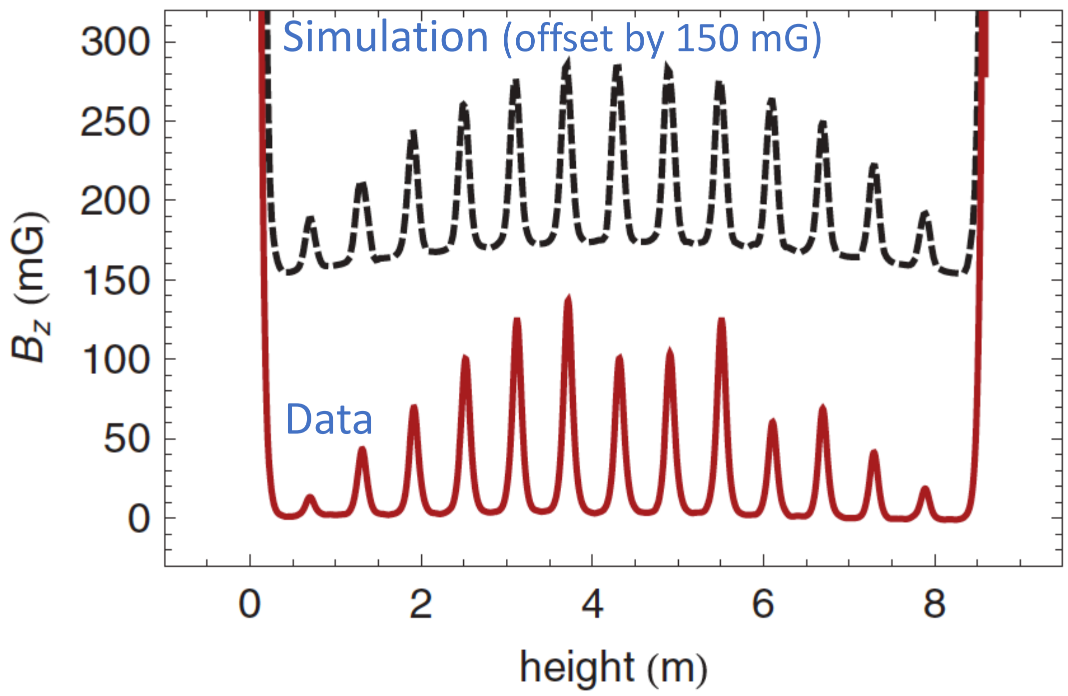
What constraints are placed on control of the residual magnetic field with a propagation region of increased length l?

L dependence: reason for anxiety?

S. Dickerson et al., Rev. Sci. Instrum. 83, 065108

Axial field inside 14 segment shield

(Peaks at 13 joints)

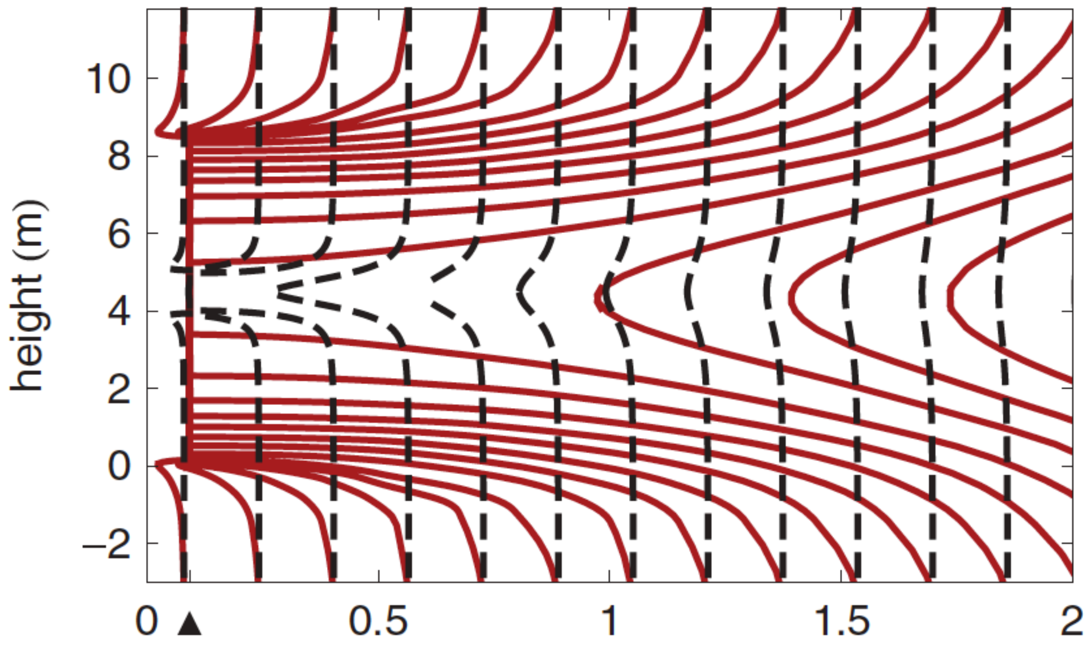


Increase in peak heights due to increase in magnetic field channeled within shield

Approximately quadratic dependence on L

Effect of shield length l on ambient field

(Simulation)



Radius of shields

Dashed line: 1.2 m shield

Solid line: 8.5 m shield (same length as 14 segment shield)

Propagation region dynamics: mapping to spinor evolution problem

- Starting point: Pauli-Schrödinger equation in rest frame of system

Larmor frequency vector $\vec{\omega}_L(t) = -\gamma\vec{B}(t)$

$n\bar{n}$ interaction

$$i\hbar\frac{d}{dt}\begin{pmatrix}\chi_n(t)\\ \chi_{\bar{n}}(t)\end{pmatrix} = \hbar\begin{pmatrix}\frac{1}{2}\vec{\sigma}\cdot\vec{\omega}_L & \delta\mathbb{I}_2\\ \delta\mathbb{I}_2 & -\frac{1}{2}\vec{\sigma}\cdot\vec{\omega}_L\end{pmatrix}\begin{pmatrix}\chi_n(t)\\ \chi_{\bar{n}}(t)\end{pmatrix}$$

- To lowest order in δ, $\bar{n}$ detection probability

$$P_{\bar{n}}(t) = \delta^2 t^2 \times \frac{1}{2}\int_0^t \frac{dt'}{t}\int_0^t \frac{dt''}{t}\mathrm{Tr}\left[\left(U_2^\dagger(t')\right)^2\left(U_2(t'')\right)^2\right]$$

Quasi-free estimate

Phase averaging suppression factor

where

$$U_2(t) = \exp_T\left[-\frac{i}{2}\int_0^t dt'\vec{\sigma}\cdot\vec{\omega}_L(t')\right]$$

Time-ordered exponential

Phase averaging suppression? Example of "longitudinal noise": $\vec{B} = b(t)\hat{k}$

Stationary Gaussian process

- Average of $P_{\bar{n}}$ over ensemble of neutrons studied

$$\langle P_{\bar{n}} \rangle = \delta^2 t^2 \times \int_{-1}^{+1} (1 - |x|)\ \cos(\langle \omega_L \rangle tx)\ \exp[-\chi(tx)]\, dx$$

Identical factor controls free inductive decay in NMR

with

$$\chi(\tau) = \int_0^\infty \left(\frac{\sin \frac{\omega\tau}{2}}{\frac{\omega}{2}} \right)^2 S_{\omega_L}(\omega) d\omega$$

where power spectral density

$$S_{\omega_L}(\omega) = \frac{1}{\pi} \int_0^\infty \left\langle \left(\omega_L(t + \frac{\tau}{2}) - \langle \omega_L \rangle \right) \left(\omega_L(t - \frac{\tau}{2}) - \langle \omega_L \rangle \right) \right\rangle \cos \omega\tau \, d\tau$$

Fourier transform of auto-correlation function of random process $\omega_L(t)$

Can quantum controls protocols within NMR combat phase averaging suppression?

Perturbative treatment of noisy magnetic field (in shielded region)

Ignore hiccups

- Treat $P_{\bar{n}}$ as functional of *random process* $\vec{\omega}_L$
- Minimalist stochastic model: assume $\vec{\omega}_L(t)$ is

  wide sense stationary

  (Due to active compensation)
- To 2nd order in small $\vec{\omega}_L$,

Wide sense stationary?

- Expectation value $\langle\vec{\omega}_L\rangle$ time independent
- Auto-correlation functions

  $$C_{\omega_L}^{(ij)}(t_1, t_2) = \langle(\omega_{L,i}(t_1) - \langle\omega_{L,i}\rangle)(\omega_{L,j}(t_2) - \langle\omega_{L,j}\rangle)\rangle$$

  depend on $\tau = t_2 - t_1$ only

$$\frac{\langle P_{\bar{n}}(t)\rangle}{\delta^2 t^2} = 1 - \tfrac{1}{12}|\langle\vec{\omega}_L\rangle|^2 t^2 - \int_0^\infty d\omega\, S_{\vec{\omega}_L}(\omega)\frac{F(\omega t)}{\omega^2}$$

$$S_{\vec{\omega}_L}(\omega) = \sum_{i=x,y,z} S_{\omega_{L,i}}(\omega)$$

Filter function $F(x) = 2\left[1 - \mathrm{sinc}^2\left(\frac{x}{2}\right)\right]$

Quasi-free propagation efficiency η (as in NIMA 309, 521)

Involves average of $\langle P_{\bar{n}}(t)\rangle$ over axial speed distribution $n(v)$ of neutrons

Average over $n(v)$

Substitute t in $\langle P_{\bar{n}}(t)\rangle$ by l/v

$$\eta \equiv \frac{\langle\langle P_{\bar{n}}(t)\rangle\rangle_v}{\delta^2\langle t^2\rangle_v} = \int_{v_{min}}^{\infty} \frac{P_{\bar{n},v}(v)}{\delta^2 l^2} n(v)dv \Big/ \int_{v_{min}}^{\infty} v^{-2} n(v)dv$$

Quasi-free estimate of $\langle\langle P_{\bar{n}}(t)\rangle\rangle_v$

Minimum speed to avoid collisions with drift vessel

Weak field

$$\eta = 1 - \tfrac{1}{12}|\langle\vec{\omega}_L\rangle|^2 \frac{\mu_{(-4)}}{\mu_{(-2)}} l^2 - \int_0^{\infty} d\omega \frac{S_{\vec{\omega}_L}(\omega)}{\omega^2} \int_{v_{min}}^{\infty} v^{-2} F\left(\frac{\omega l}{v}\right) n(v) \frac{dv}{\mu_{(-2)}}$$

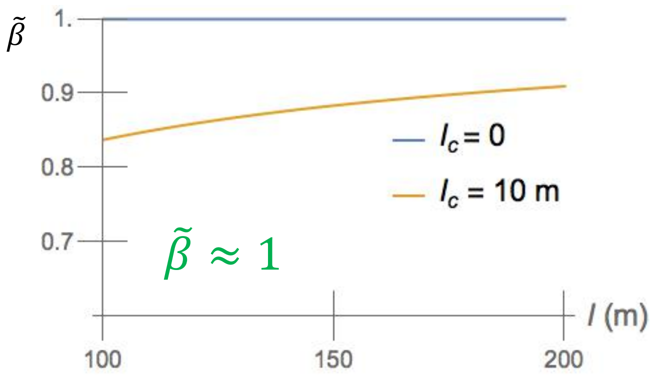
$$\mu_{(k)} = \int_{v_{min}}^{\infty} v^{-2} n(v)dv$$

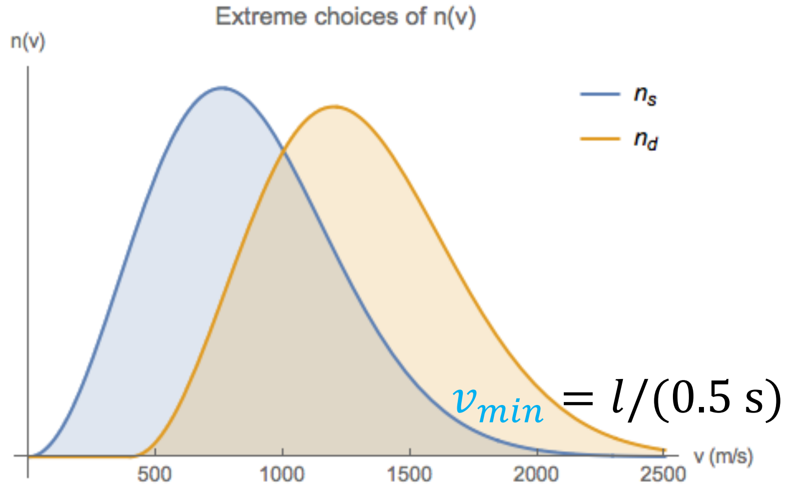
Flight path length l: apparent quadratic dependence

l-dependence of η (weak fields)

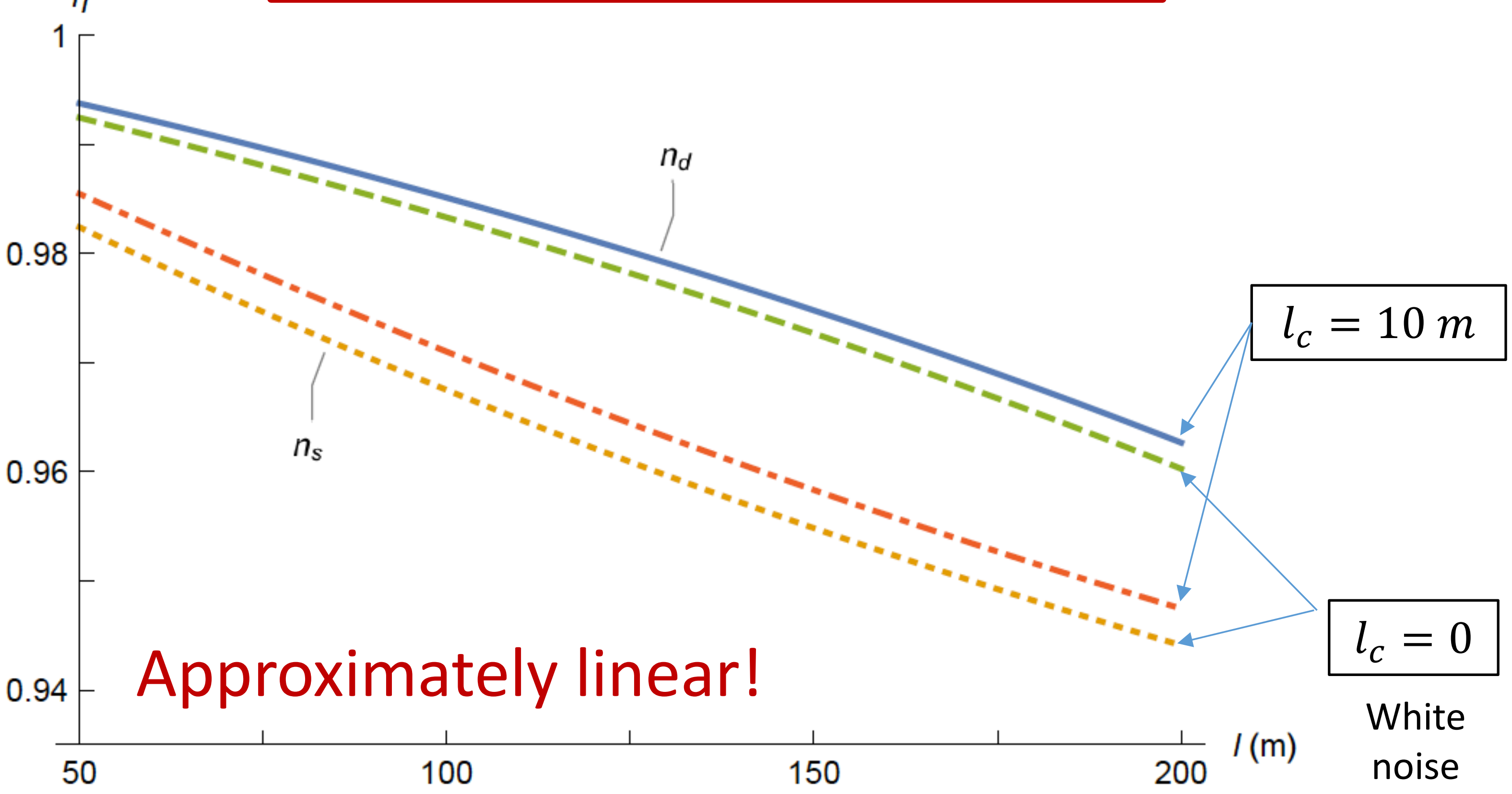
Markovian noise $\longrightarrow$ $\eta = 1 - \frac{1}{12}\left[\,|\langle\vec{\omega}_L\rangle|^2 + 2\tilde{\beta}\left(\frac{l_c}{l}\right)\sigma_L^2\,\right]\frac{\mu_{(-4)}}{\mu_{(-2)}}\; l^2$

Ingredients



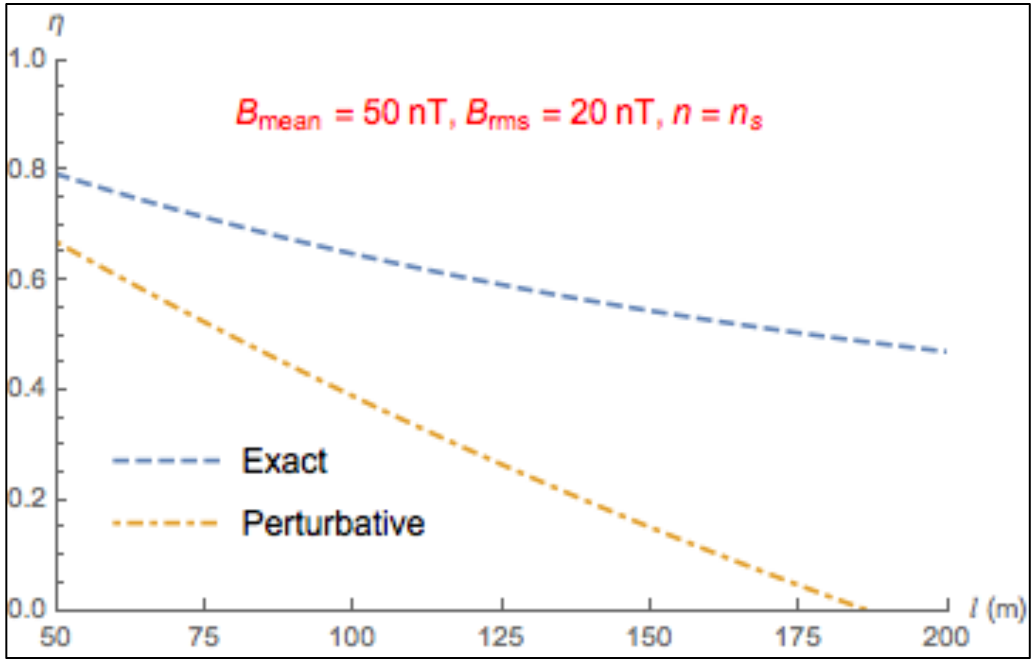


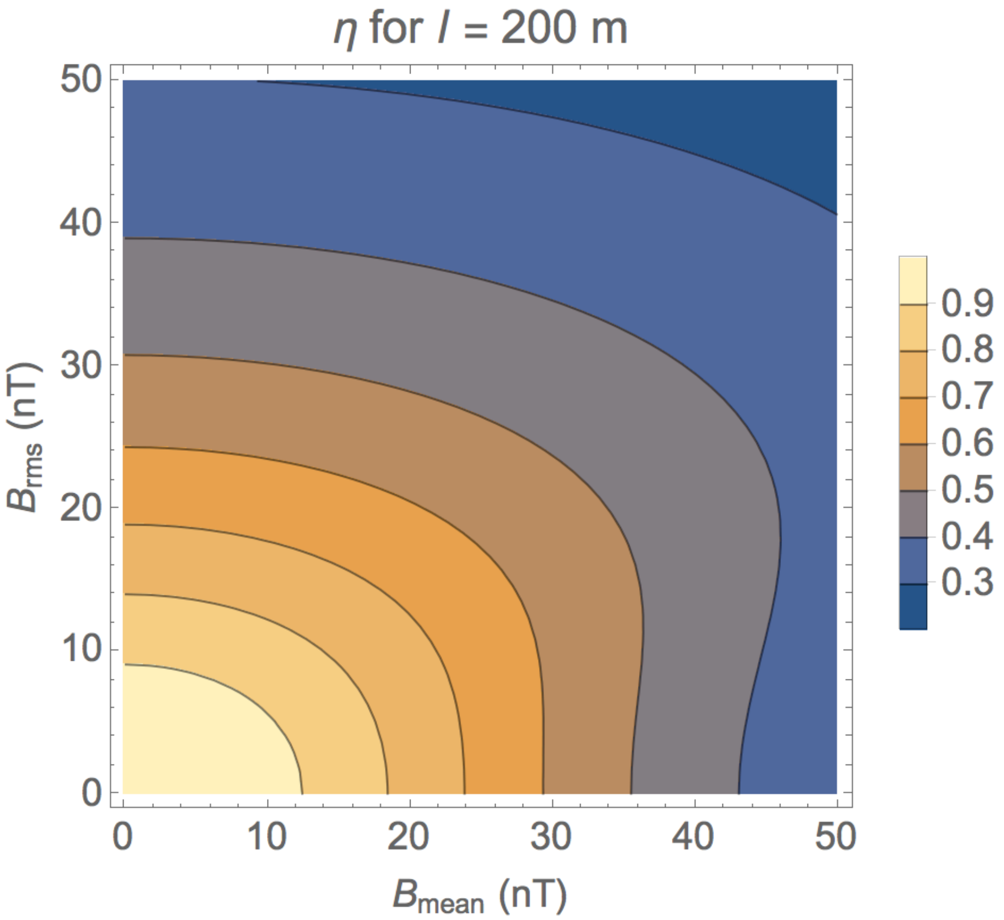
η versus l: $B_{mean} = \sqrt{2}(5\ \mathrm{nT}) = B_{rms}$



Non-perturbative treatment of white Gaussian longitudinal noise

Can obtain exact $\langle P_{\bar{n}}(t)\rangle$ in closed analytic form (functional integrals Gaussian)





Inference: perturbative estimate too conservative

Influence of an elementary "quantum control protocol"

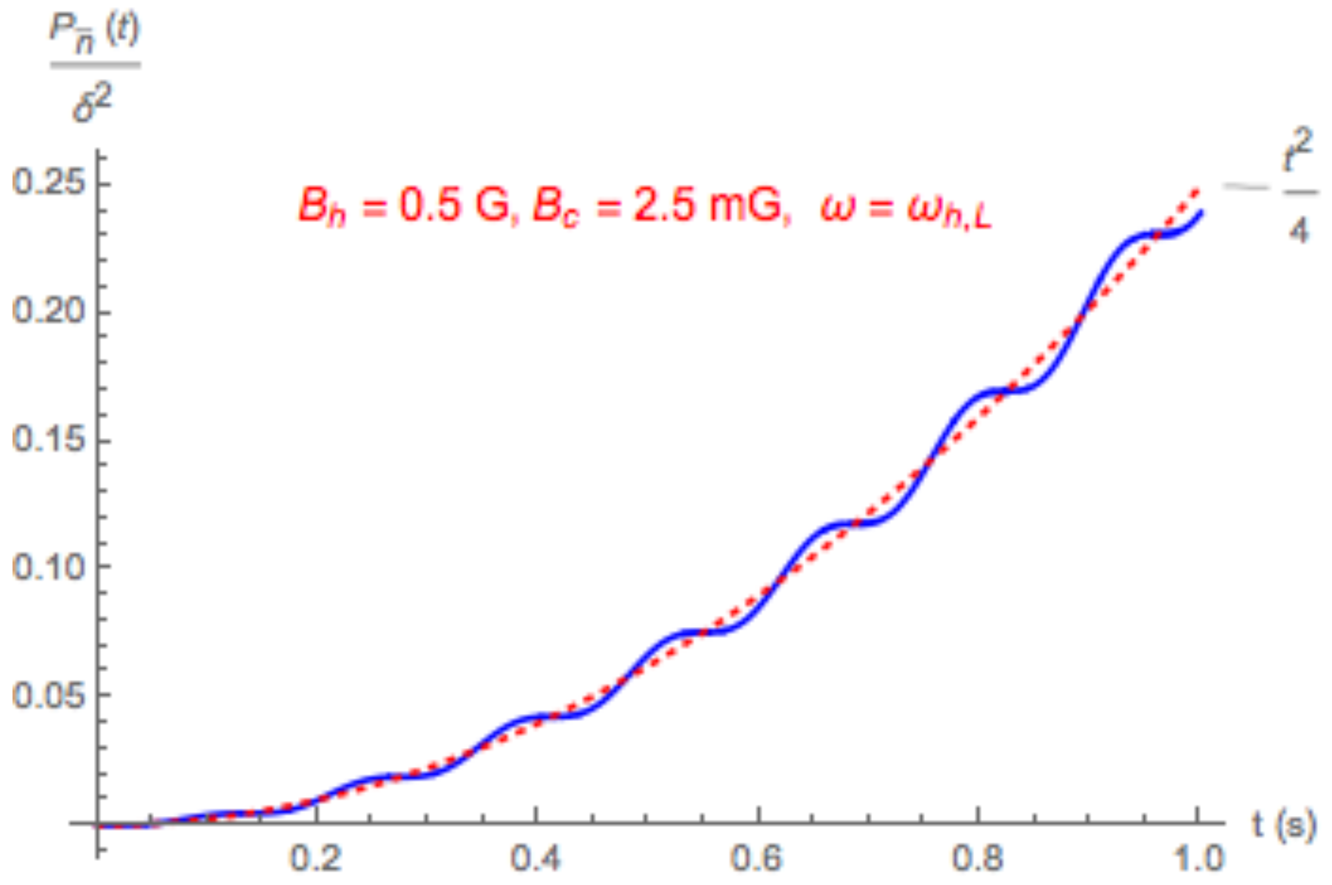
Rabi-like field configuration

$$\vec{B} = B_h \hat{k} + B_c(\cos \omega t\, \hat{\imath} + \sin \omega t\, \hat{\jmath})$$

(Phys. Rev. D 91, 096010)

$$\frac{P_{\bar{n}}(t)}{\delta^2} = \frac{\omega_{c,L}^2}{\omega_{c,L}^2 + \left(\omega - \omega_{h,L}\right)^2} \frac{t^2}{4}$$

$$+ \text{ oscillatory terms}$$



Demand *uniformity* of $B_h \longrightarrow$ other quantum control protocols (Rev. Mod. Phys. 76, 1037)

Carr-Pound-Meiboom-Gill-like spin-flip sequence

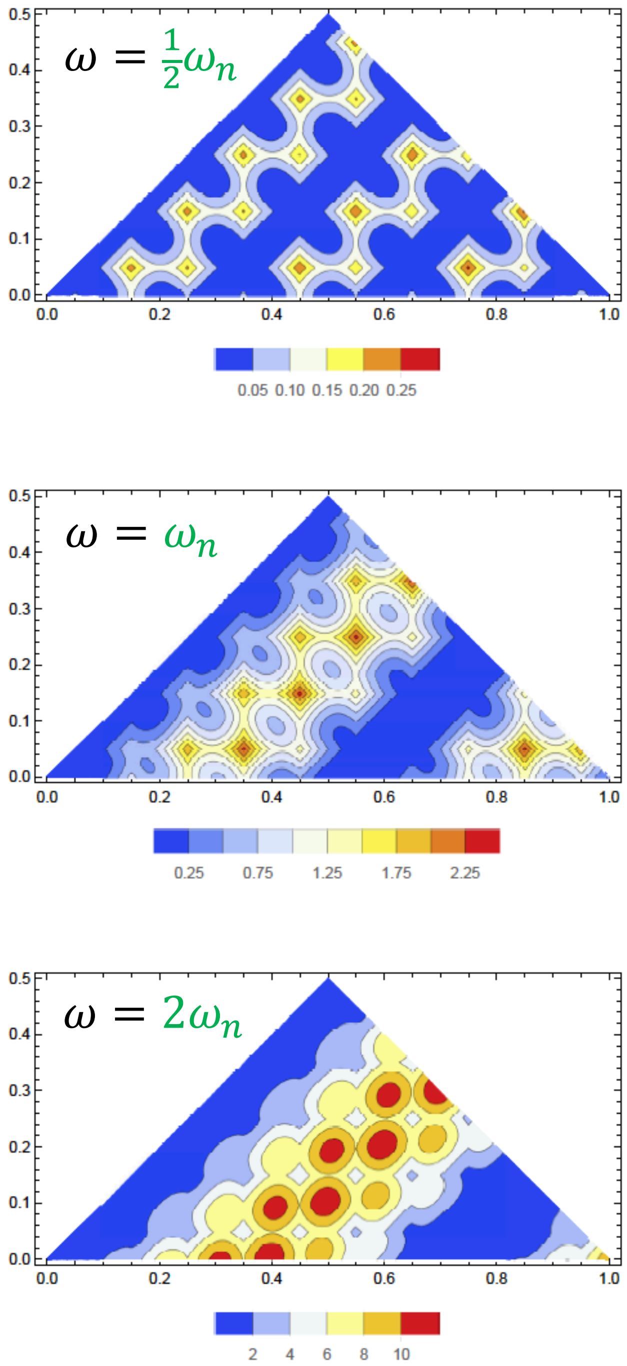
- For *uniform* "holding" field,

$$P_{\bar{n}}(t) = \delta^2 t^2 \times \mathrm{sinc}^2\left(\frac{\omega_L t}{2n}\right)$$

for n "bang-bang" spin flips

(Flips separated by interval $\Delta t = t/n$ with 1st flip at time $= \frac{1}{2}\Delta t$)

- Filter function is high-pass filter with cutoff frequency approximately equal to $\boldsymbol{\omega_n} \equiv \frac{2n}{t}$



Filter function

$n = 10$ flips

Conclusions

- Revisited the magnetic field studies of the ILL n-$\bar{n}$ experiment with a view to establishing scaling with size of the system. NOT a problem.
- Synthesis with quantum information methodologies developed since the ILL experiment could be productive

  Helpful in search for mirror neutrons at HFIR?

Thank you!