Towards a High Precision Calculation for the Polarized e+p Using N-Jettiness Subtraction

Hongxi Xing

In collaboration with R. Boughezal, F. Petriello and U. Schubert

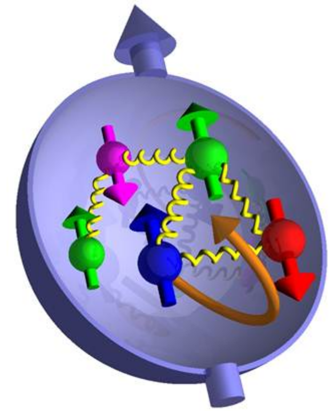
arXiv: 1704.05457, 1710.xxxxx

The Flavor Structure of Nucleon Sea, INT, Oct. 2-13, 2017

NORTHWESTERN UNIVERSITY

Argonne NATIONAL LABORATORY

Spin configuration of proton



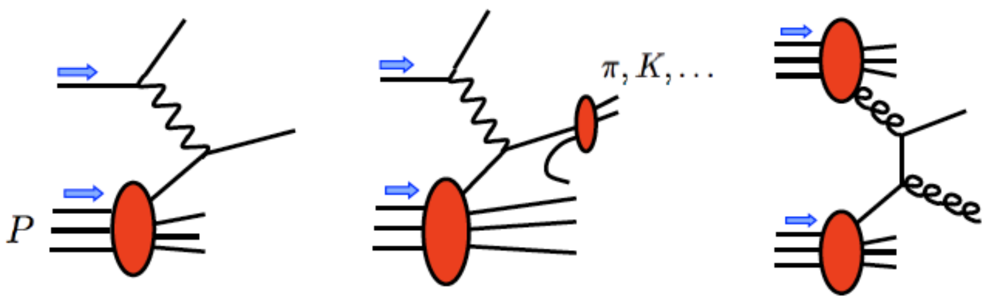
- Proton helicity sum rule

$$\frac{1}{2} = \frac{1}{2}\Delta\sum + \Delta G + L_q + L_g$$

quark spin $\Delta\sum = \int_0^1 dx \Delta f_q(x)$

gluon spin $\Delta G = \int_0^1 dx \Delta f_g(x)$

- Probes are used so far



DIS SIDIS pp (RHIC)

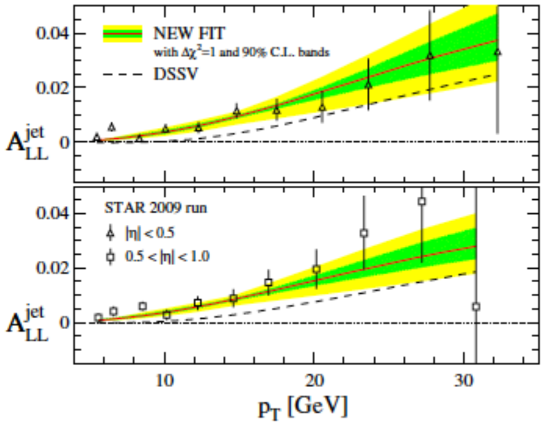
- QCD factorization for inclusive hadron production in pp

$$d\Delta\sigma = \sum_{a,b,c} \Delta f_a \otimes \Delta f_b \otimes d\Delta\hat{\sigma}_{ab\to c+X} \otimes D_c^h$$

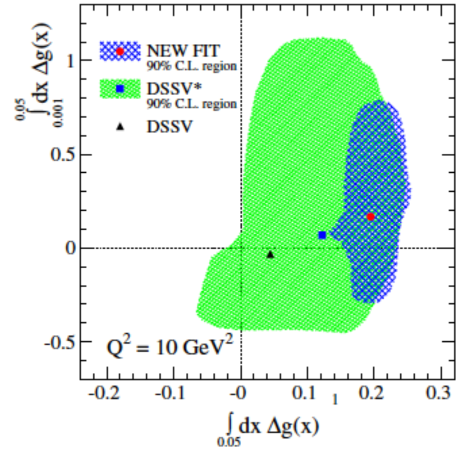
expand in α_s

Global extraction of helicity PDFs

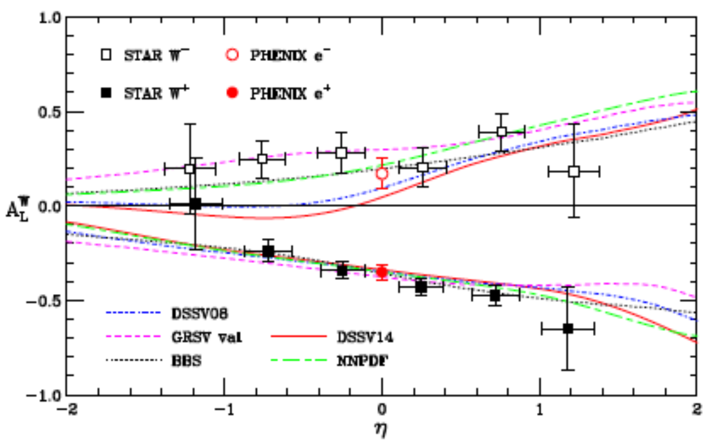
- Global fitting (GRV, DSSV, NNPDF …)



DSSV 2014

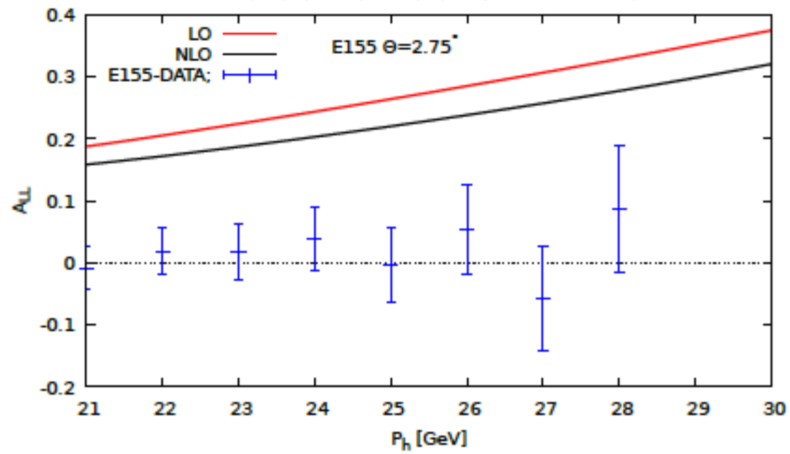


- NLO predictions



Ringer and Vogelsang, PRD 2015

Inclusive hadron in DIS



Hinderer, Schlegel, and Vogelsang, PRD 2017

Current state of the art for high orders in polarized collisions

❑ Leading twist (longitudinal polarized)

- NLO prompt photon, hadron in pp (Gordon, Vogelsang, 93, de Florian, 03; Jager, Stratmann, Vogelsang, 03)
- NLO jet in pp (de Florian, Frixione, Signer, Vogelsang, 98; Jager, Stratmann, Vogelsang, 04; Mukherjee, Vogelsang, 12)
- NLO W boson in pp (Ringer, Vogelsang 15)
- NLO inclusive hadron and jet in DIS (de Florian, Vogelsang 98, Hinderer, Schlegel, Vogelsang 17)
- NNLO DIS structure function (Zijlstra, van Neerven, 94)
- NNLO heavy flavor in DIS (Buza, Matiounine, Smith, van Neerven, 96)
- NNLO e+e- (Rijken, van Neerven, 97; Ravindran, van Neerven 98, 00)
- NNLO DY (Ravindran, Smith, van Neerven 04)
- …

❑ Twist-3 (transverse polarized)

- NLO weighted single transverse spin asymmetry in Drell-Yan (Vogelsang, Yuan 09, Chen, Ma, Zhang 16)
- NLO weighted single transverse spin asymmetry in SIDIS (Kang, Vitev, Xing 13, Dai, Kang, Prokudin, Vitev 14; Shinsuke 16)
- …

How to calculate high order corrections?

❑ Analytical calculation of high orders

- Integrate the final state phase space in d-dimension, extract and subtract divergences, derive analytical expressions for the hard part coefficients.
- Okay for “easy” processes, fast in numerical calculation, perfect for standard way of global fitting for PDFs/FFs.
- Need to take narrow cone approximation for full jet production, extremely hard for orders beyond NLO.

❑ Monte Carlo computation of high orders

- Local subtraction
  - Dipole, Antenna, Sector decomposition …
  - Construct IR subtraction point by point in phase space, generate smooth integrand
- Global subtraction
  - qT subtraction, N-jettiness subtraction
  - Pick up a variable that captures all IR behaviors which can be computed in using simpler formalisms (CSS, SCET)
- Give all heavy duty works to computers

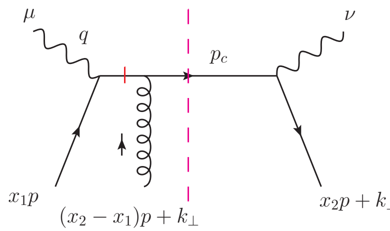
Transverse momentum weighted SSA in SIDIS

- Weighted cross section

Kang, Vitev, HX, PRD, 2013

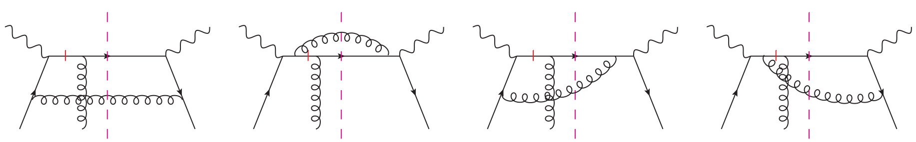
$$\frac{d\langle P_{h\perp}\Delta\sigma(S_\perp)\rangle}{dx_B dy dz_h} \equiv \int d^2P_{h\perp}\epsilon^{\alpha\beta}S_\perp^\alpha P_{h\perp}^\beta \frac{d\Delta\sigma(S_\perp)}{dx_B dy dz_h d^2P_{h\perp}}$$

- Leading order

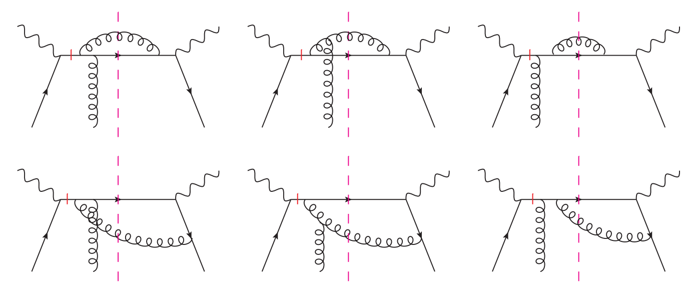


$$\frac{d\langle P_{h\perp}\Delta\sigma(S_\perp)\rangle}{dx_B dy dz_h} = -\frac{z_h\sigma_0}{2}\sum_q e_q^2 \int \frac{dx}{x}\frac{dz}{z} T_{q,F}(x,x) D_{q\to h}(z)\delta(1-\hat{x})\delta(1-\hat{z})$$

- Next-to-leading order



soft-pole



hard-pole

- QCD evolution of Qiu-Sterman function

$$\frac{\partial}{\partial \ln\mu^2}T_{q,F}(x_B,x_B,\mu^2) = \frac{\alpha_s}{2\pi}\int_{x_B}^1 \frac{dx}{x}\left\{T_{q,F}(x,x,\mu^2)C_F\left[\frac{1+\hat{x}^2}{(1-\hat{x})_+} + \frac{3}{2}\delta(1-\hat{x})\right] - N_c\delta(1-\hat{x})T_{q,F}(x,x,\mu^2)\right.$$
$$\left. + \frac{N_c}{2}\left[\frac{1+\hat{x}}{(1-\hat{x})_+}T_{q,F}(x,x\hat{x},\mu^2) - \frac{1+\hat{x}^2}{(1-\hat{x})_+}T_{q,F}(x,x,\mu^2)\right]\right\}.$$

- Complete next-to-leading order result

LO

$$
\frac{d\langle P_{h\perp}\Delta\sigma(S_\perp)\rangle}{dx_B dy dz_h} = -\frac{z_h\sigma_0}{2}\sum_q e_q^2 \int_{x_B}^1 \frac{dx}{x}\int_{z_h}^1 \frac{dz}{z} T_{q,F}(x,x,\mu^2) D_{q\to h}(z,\mu^2)\delta(1-\hat{a}tx)\delta(1-\hat{a}tz)
$$

$$
\begin{aligned}
&-\frac{z_h\sigma_0}{2}\frac{\alpha_s}{2\pi}\sum_q e_q^2 \int_{x_B}^1 \frac{dx}{x}\int_{z_h}^1 \frac{dz}{z} D_{q\to h}(z,\mu^2)\Bigg\{ \ln\left(\frac{Q^2}{\mu^2}\right)\left[\delta(1-\hat{x})T_{q,F}(x,x,\mu^2)P_{qq}(\hat{z})\right.\\
&\left.+\delta(1-\hat{z})P_{qg\to qg}\otimes T_{q,F}(x,x\hat{x},\mu^2)\right]\\
&+x\frac{d}{dx}T_{q,F}(x,x,\mu^2)\frac{1}{2N_c}\left[\frac{1-\hat{z}}{\hat{z}}+\frac{(1-\hat{x})^2+2\hat{x}\hat{z}}{\hat{z}(1-\hat{z})_+}-\delta(1-\hat{z})\left((1+\hat{x}^2)\ln\frac{\hat{x}}{1-\hat{x}}+2\hat{x}\right)\right]\\
&+T_{q,F}(x,x,\mu^2)\delta(1-\hat{z})\frac{1}{2N_c}\left[(2\hat{x}^2-\hat{x}-1)\ln\frac{\hat{x}}{1-\hat{x}}-2\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ +\frac{2\hat{x}(2-\hat{x})}{(1-\hat{x})_+}+2\frac{\ln\hat{x}}{1-}\right.\\
&+T_{q,F}(x,x,\mu^2)\delta(1-\hat{x})C_F\left[-(1+\hat{z})\ln\hat{z}(1-\hat{z})+2\left(\frac{\ln(1-\hat{z})}{1-\hat{z}}\right)_+ -\frac{2\hat{z}}{(1-\hat{z})_+}+2\frac{\ln\hat{z}}{1-\hat{z}}\right]\\
&+T_{q,F}(x,x,\mu^2)\frac{1}{2N_c\hat{z}}\left[\frac{2\hat{x}^3-3\hat{x}^2-1}{(1-\hat{x})_+(1-\hat{z})_+}+\frac{1+\hat{z}}{(1-\hat{x})_+}-2(1-\hat{x})\right]\\
&+T_{q,F}(x,x\hat{x},\mu^2)\delta(1-\hat{z})\frac{N_c}{2}\left[\ln\frac{\hat{x}}{1-\hat{x}}+2\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ -2\frac{\ln\hat{x}}{1-\hat{x}}-\frac{1+\hat{x}}{(1-\hat{x})_+}\right]\\
&+T_{q,F}(x,x\hat{x},\mu^2)\frac{1+\hat{x}\hat{z}^2}{(1-\hat{x})_+(1-\hat{z})_+}\left(C_F+\frac{1}{2N_c\hat{z}}\right)-T_{q,F}(x,x,\mu^2)6C_F\delta(1-\hat{x})\delta(1-\hat{z})\Bigg\}
\end{aligned}
$$

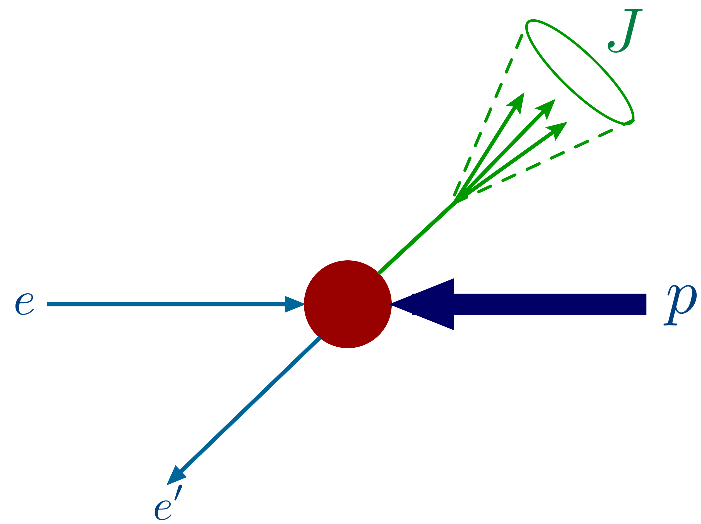
NLO

- Sivers effect at NLO in inclusive DIS

See talk by W. Vogelsang in Santa Fe jet workshop 2017

Inclusive jet production in DIS

- Semi-inclusive vs. inclusive
  - Semi-inclusive $\ell + p \to \ell' + jet + X$
    measure outgoing lepton
    hard scale: Q^2
  - Inclusive $\ell + p \to jet + X$
    Integrate over outgoing lepton
    Hard scale: jet p_T

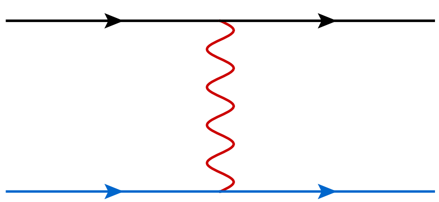


- QCD collinear factorization Kang, Metz, Qiu, Zhou, 11

$$\sigma^{\ell+p\to jet+X} = \sum_{a,b} f_{a/\ell} \otimes f_{b/p} \otimes \hat{\sigma}^{a+b\to jet+X}$$

- Leading order is trivial

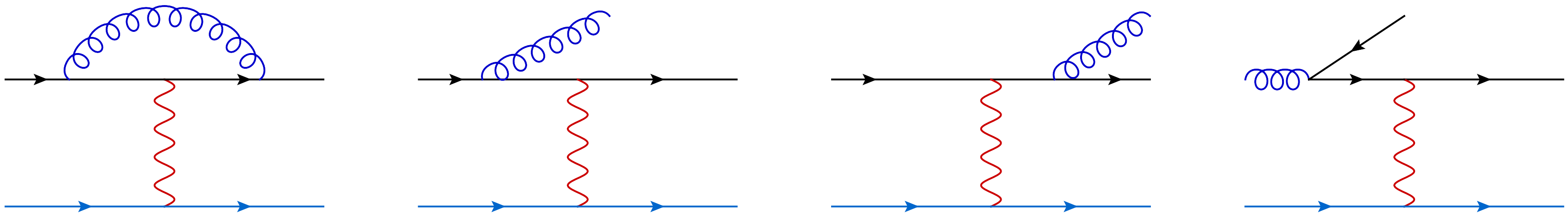
$$d\sigma^{LO} = \sum_q \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\left[f_{\ell/\ell}(x_1) f_{q/p}(x_2) d\hat{\sigma}^{(0)}_{q\ell} + f_{\ell/\ell}(x_1) f_{\bar{q}/p}(x_2) d\hat{\sigma}^{(0)}_{\bar{q}\ell}\right]$$



Lepton in lepton: $f_{\ell/\ell}(x_1) = \delta(1-x_1)$

Partonic cross section: $d\hat{\sigma}^{(0)}_{q\ell} = d\Phi^{(0)}(p_3, p_4; p_1, p_2)|M_B|^2 J^{(1)}(p_4)$

Next-to-leading order



- Real and virtual corrections to q+lepton channel, gluon channel opens up

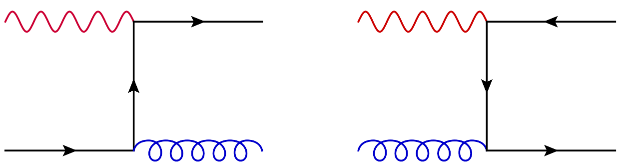
$$d\hat{\sigma}^{(1)}_{q\ell} = d\Phi^{(3)}(p_3,p_4,p_5;p_1,p_2)\left|M_R^{(q\ell)}\right|^2 J^{(2)}(p_4,p_5) + d\Phi^{(2)}(p_3,p_4;p_1,p_2)\left|M_V^{(q\ell)}\right|^2 J^{(1)}(p_4)$$

$$d\hat{\sigma}^{(1)}_{g\ell} = d\Phi^{(3)}(p_3,p_4,p_5;p_1,p_2)\left|M_R^{(g\ell)}\right|^2 J^{(2)}(p_4,p_5)$$

- We handle QED collinear divergence with standard dipole subtraction

Opens up new channel: Weizsacker-Williams photon

$$d\hat{\sigma}^{(1)}_{\gamma a} = d\Phi^{(2)}(p_4,p_5;p_\gamma,p_2)\left|M_B^{(\gamma a)}\right|^2 J^{(2)}(p_4,p_5)$$



- We handle QCD IR divergences with N-jettiness subtraction

N-jettiness

- N-jettiness is a global event shape variable designed to veto final state jets

Stewart, Tackmann, Waalewijn 0910. 0467

$$\tau_1 = \frac{2}{Q^2} \sum_i \min\{p_B \cdot q_i, p_J \cdot q_i\}$$

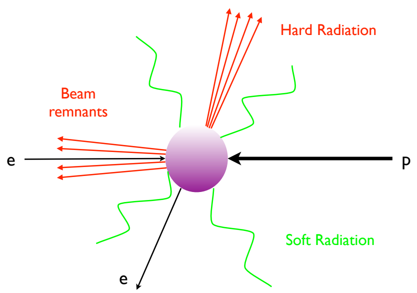
p_i momenta of initial state beams and final state jets

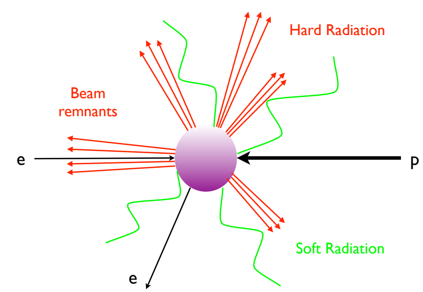
q_i momenta of all final state partons

Q^2 measure of the jet hardness

- Use N-jettiness to separate N jet event and more-than-N-jet event

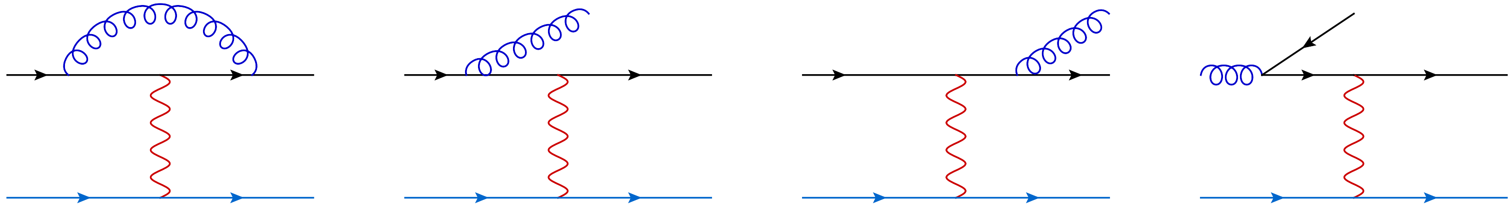
1 jet ← small τ_1 large → at least 2 jets





- $\tau_1 \ll 1$ forces an 1-jet final state, q_i must be soft or collinear to either the initial state beam or final state jet
- τ_1 controls all the IR behaviors for 1-jet

N-jettiness subtraction



❑ Introduce τ_1^{cut} to partition the phase space, identify IR behavior

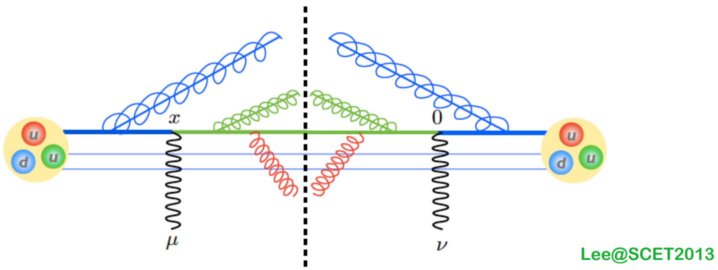
$$d\sigma^{NLO} = \int_0^{\tau_1^{cut}} d\tau_1 \frac{d\sigma^{NLO}}{d\tau_1} + \int_{\tau_1^{cut}}^{\infty} d\tau_1 \frac{d\sigma^{NLO}}{d\tau_1}$$
$$= \sigma^{NLO}\theta^{<} + \sigma^{NLO}\theta^{>}$$

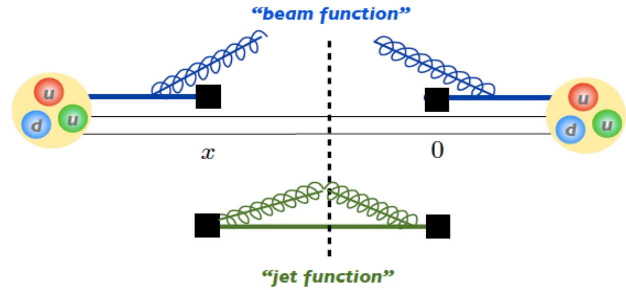
Boughezal, Focke, Liu, Petriello (2015); Gaunt, Stahlhofen, Tackmann, Walsh (2015)

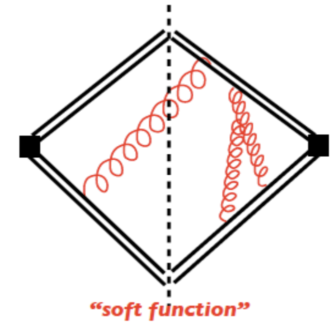
τ_1^{cut} has to be small, so we can safely neglect power corrections

- Below cut $\theta^{<} = \theta(\tau_1^{cut} - \tau_1)$
  - Virtual: τ_1 is zero
  - Real: the radiated gluon/quark is unresolved. Purely IR divergent region
  - Calculate this part from SCET
- Above cut $\theta^{>} = \theta(\tau_1 - \tau_1^{cut})$
  - Only real radiation contributes, the radiated gluon/quark is resolved, this region of phase space contains the tree diagram to the 2 jet process.
  - Tree level calculation

N-jettiness subtraction







❑ Factorization for 1-jettiness

$$\frac{d\Delta\sigma}{d\tau_1} = \Delta H \otimes \Delta B \otimes S \otimes J + \cdots$$

- Hard function: virtual correction, has been calculated up to two loops in DIS

$$\Delta H = H^+ - H^-$$

- Soft function: remains the same as unpolarized (R. Boughezal, X. Liu, F. Petriello, 15)
- Jet function: remains the same as unpolarized (Becher, Neubert 06, Becher, Bell 11)
- Beam function: available up to NNLO now!

$$\Delta B = B^+ - B^-$$

Polarized quark beam function

Operator definition

$$\Delta B_q(t,x,\mu) = \langle p_n(P^-),+|\theta(\omega)\bar{\chi}_n(0)\delta(t-\omega\hat{p}^+)\frac{\bar{n}\cdot\gamma\gamma_5}{2}[\delta(\omega-\overline{\mathcal{P}}_n)\chi_n(0)]|p_n(P^-),+\rangle$$

Composite quark operator

$$\chi_n(y) = W_n^\dagger(y)\xi_n(y)$$

Wilson line

$$W_n(y) = \left[\sum_{\text{perms}}\exp\left(-\frac{g}{\overline{\mathcal{P}}_n}\bar{n}\cdot A_n(y)\right)\right]$$

Renormalization and RGE (double log resummation)

$$\Delta B_i^{bare}(t,z) = \int dt' Z_i(t-t',\mu)\Delta B_i(t',z,\mu)$$

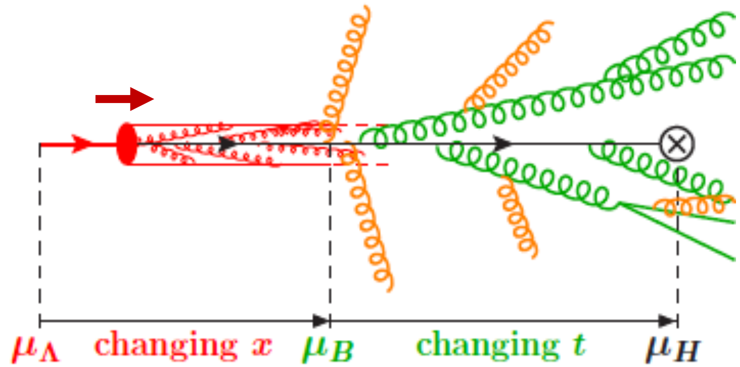
$$\mu\frac{d}{d\mu}\Delta B_i(t',z,\mu) = \int dt' \gamma_B^i(t-t',\mu)\Delta B_i(t',z,\mu)$$

- anomalous dimension

$$\gamma_B^i(t,\mu) = -\int dt'(Z_i)^{-1}(t-t',\mu)\mu\frac{d}{d\mu}Z_i(t',\mu)$$

$$\int dt'(Z_i)^{-1}(t-t',\mu)Z_i(t',\mu) = \delta(t)$$

Initial state radiation



- Single log resummation: DGLAP for polarized PDFs

$$\frac{d}{d\ln\mu^2}\Delta f_j = \Delta P_{jk} \otimes \Delta f_k$$

- Beam function matches to PDFs $\quad t >> \Lambda^2_{QCD}$

$$\Delta B_i(t,x,\mu) = \sum_j \int_x^1 \frac{d\xi}{\xi} \Delta I_{ij}\left(t, \frac{x}{\xi}\right) \Delta f_j(\xi,\mu)$$

t is the virtuality of the parton that enters the hard interaction

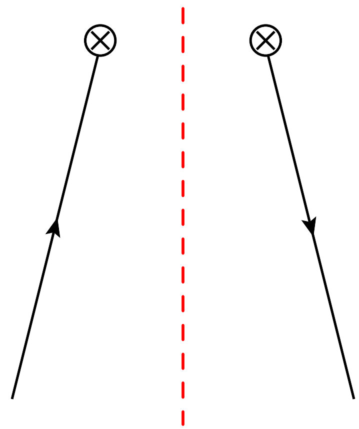
- Matching coefficient

ΔI_{ij} describes initial state radiation, can be computed perturbatively

❑ Calculate partonic beam function

$$\Delta B_{ij}(t,z,\mu)=\sum_k\int_z^1\frac{dz'}{z'}\Delta\mathcal{I}_{ik}(t,z',\mu)\Delta f_{kj}\left(\frac{z}{z'}\right)$$

❑ Leading order

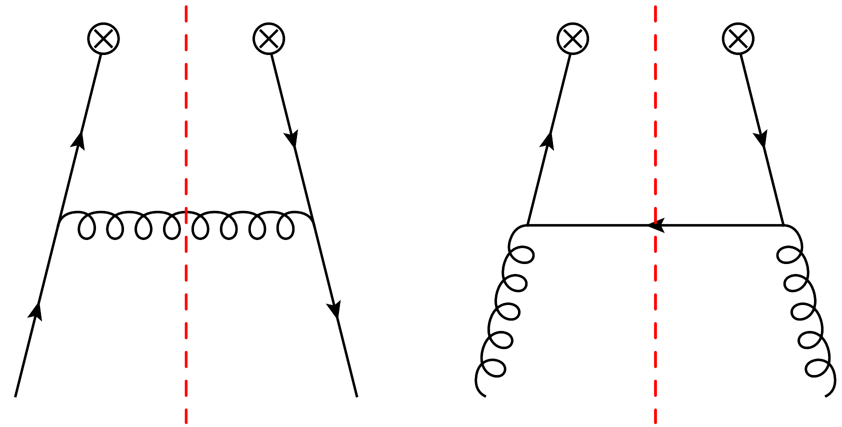


$$\Delta B^{(0)}_{qq}(t,z,\mu)=\langle q_n(p),+|\theta(\omega)\bar{\chi}_n(0)\delta(t-\omega\hat{p}^+)\frac{\bar{n}\cdot\gamma\gamma_5}{2}[\delta(\omega-\overline{\mathcal{P}}_n)\chi_n(0)]|q_n(p),+\rangle$$
$$=\delta(t)\delta(1-\omega/p^-)$$

$$\mathcal{I}^{(0)}_{qq}(t,z,\mu)=\mathcal{I}^{(0)}_{\bar{q}\bar{q}}(t,z,\mu)=\delta(t)\delta(1-z)$$

$$\mathcal{I}^{(0)}_{qg}(t,z,\mu)=\mathcal{I}^{(0)}_{gq}(t,z,\mu)=0$$

Next-to-leading order



$$\left(\frac{\alpha_s}{4\pi}\right)\Delta B_{qq}^{bare(1)}(t,z) = \frac{g^2}{N_c}\left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{\epsilon}\int d\mathrm{PS}^{(1)}\mathrm{Tr}\left[\frac{\bar{n}\cdot\gamma\gamma_5}{2}\ell\cdot\gamma\gamma^{\rho}\mathcal{P}_R p\cdot\gamma\gamma^{\sigma}\ell\cdot\gamma\right]d_{\rho\sigma}(k)\frac{1}{\ell^2}\frac{1}{\ell^2}\mathrm{Tr}[\mathrm{T^a T^a}]$$

- γ_5 in d-dimension – HVBM scheme

$$\gamma_5 \equiv \frac{i}{4!}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\sigma\gamma_\rho \quad \Longrightarrow \quad \{\gamma_5,\tilde{\gamma}_\mu\} = 0, \quad [\gamma_5,\hat{\gamma}_\mu] = 0$$

Maintain the four-dimension definition

anticommute in 4-dimension
commute in d-4 dimension

- Final state phase

$$\int d\mathrm{PS}^{(1)} = \int \frac{d^d k}{(2\pi)^{d-1}} d^d\ell\ \delta(k^2)\delta(\omega-\ell^-)\delta(t-\omega k^+)\delta^d(p-k-\ell)$$

$$= \frac{1}{(4\pi)^{2-\epsilon}}\frac{1}{\Gamma(-\epsilon)}\frac{1}{\omega}\int_0^{t\frac{1-z}{z}} d\hat{k}_\perp^2(\hat{k}_\perp^2)^{-1-\epsilon}$$

d-4 dimension momentum

- Bare quark beam function at NLO

UV divergence

$$\Delta B_{qq}^{bare(1)}(t,z) = \boxed{\frac{4}{\epsilon^2}C_F\delta(t)\delta(1-z) - \frac{4}{\epsilon}C_F\frac{1}{\mu^2}\mathcal{L}_0\left(\frac{t}{\mu^2}\right)\delta(1-z) + \frac{3}{\epsilon}C_F\delta(t)\delta(1-z)} - \frac{2}{\epsilon}C_F\delta(t)\Delta P_{qq}^{(0)}(z)$$
$$+ 4C_F\frac{1}{\mu^2}\mathcal{L}_1\left(\frac{t}{\mu^2}\right)\delta(1-z) + 2C_F\frac{1}{\mu^2}\mathcal{L}_0\left(\frac{t}{\mu^2}\right)\mathcal{L}_0(1-z)(1+z^2)$$
$$+ 2C_F\delta(t)\left[\mathcal{L}_1(1-z)(1+z^2) - \frac{1+z^2}{1-z}\ln z - 3(1-z) - \frac{\pi^2}{6}\delta(1-z)\right]$$

- renormalized quark beam function at NLO

IR divergence

$$\Delta B_{qq}^{(1)}(t,z,\mu^2) = \boxed{-\frac{2}{\epsilon}\delta(t)\Delta P_{qq}^{(0)}(z)} + 4C_F\frac{1}{\mu^2}\mathcal{L}_1\left(\frac{t}{\mu^2}\right)\delta(1-z) + 2C_F\frac{1}{\mu^2}\mathcal{L}_0\left(\frac{t}{\mu^2}\right)\mathcal{L}_0(1-z)(1+z^2)$$
$$+ 2C_F\delta(t)\left[\mathcal{L}_1(1-z)(1+z^2) - \frac{1+z^2}{1-z}\ln z - 3(1-z) - \frac{\pi^2}{6}\delta(1-z)\right]$$

- Matching coefficient at NLO

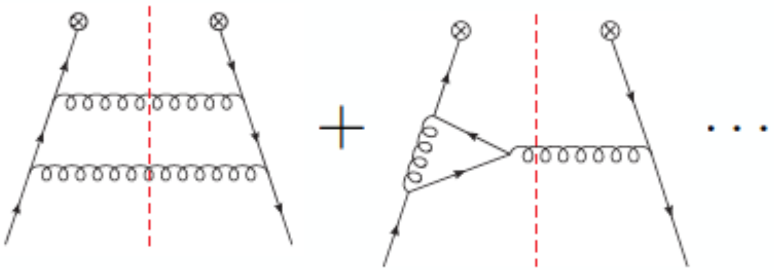
$$\Delta I_{qq}^{(1)}(z) = 2C_F\left[\mathcal{L}_1(1-z)(1+z^2) - \frac{1+z^2}{1-z}\ln z + (1-z) - \frac{\pi^2}{6}\delta(1-z)\right]$$

Finite!

Outline of NNLO calculation

R. Boughezal, F. Petriello, U. Schubert, **HX** PRD 2017.

- Generate all the diagrams and calculate the squared amplitude



- Integration-by-parts (IBP)

$$\Delta B_{ij}^{bare}(t,z) = \sum_{i=1}^{n} c_i(t,z) I_i(t,z)$$

- Differential Equation (DEQ)

$$\partial_x I_i = M_{ij}(x) I_j, \qquad x = t, z$$

- UV renormalization

$$\Delta B_{ij}^{bare}(t,z) = \int dt' Z_i(t-t',\mu) \Delta B_{ij}(t',z,\mu)$$

- IR regularization – matching to PDFs

$$\Delta B_{ij}(t,z,\mu) = \sum_k \Delta I_{ik}(t,z,\mu) \otimes \Delta f_{kj}(z)$$

1-jettiness subtraction for QCD IR divergences

- Phase space partition

$$d\sigma^{NLO} = \int_0^{\tau_1^{cut}} d\tau_1 \frac{d\sigma^{NLO}}{d\tau_1} + \int_{\tau_1^{cut}}^{\infty} d\tau_1 \frac{d\sigma^{NLO}}{d\tau_1}$$

- All QCD IR divergences are captured in the below-cut piece

$$\frac{d\sigma^{NLO}}{d\tau_1} = \int_0^1 dx \int d\Phi(p_3, p_4; p_1, p_2) \int dt_J dt_B dk_S \delta\left(\tau_1 - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_s}{Q}\right)$$
$$\times \sum_q B_q(t_B, x, \mu) H_q(\Phi_2, \mu) J_q(t_J, \mu) S(k_s, \mu) + \cdots$$
$$\approx \int_0^1 dx \int d\Phi(p_3, p_4; p_1, p_2) \sum_i \int_x^1 \frac{d\xi}{\xi} f_{i/p}(\xi, \mu) \Big\{ \mathcal{I}_{qi}^{(1)} \otimes H_q^{(0)} \otimes J_q^{(0)} \otimes S^{(0)} + \mathcal{I}_{qi}^{(0)} \otimes H_q^{(1)} \otimes J_q^{(0)} \otimes S^{(0)}$$
$$+ \mathcal{I}_{qi}^{(0)} \otimes H_q^{(0)} \otimes J_q^{(1)} \otimes S^{(0)} + \mathcal{I}_{qi}^{(0)} \otimes H_q^{(0)} \otimes J_q^{(0)} \otimes S^{(1)} \Big\}$$

- Above-cut piece is free of QCD IR divergence

$$\sigma^{\ell+p\to jet+X} = \sum_{a,b} f_{a/\ell} \otimes f_{b/p} \otimes \hat{\sigma}^{a+b\to jet+X}$$

Tree level two jet production, finite, no QCD regularization needed.
Collinear singularity of final state lepton remains.

QED collinear divergence

- Introduce a local counterterm and add Weizsacker-Williams contribution

$$\sigma^{NLO}\theta^{>} = \int d\Phi_3 \left[d\sigma^r - d\sigma^A\right] + \int d\Phi_2 \left[\int d\Phi_1 d\sigma^A + d\sigma^C\right]$$

finite, perform integral numerically in 4-dimension

Poles explicitly cancel

- Dipole subtraction

  - Matches singular behavior of $d\sigma^r$ exactly in d-dimension
  - Integrand is smooth, convenient for Monte Carlo integration
  - Exactly integrable over one-parton PS in d-dimension

a solution: dipole subtraction Catani & Seymour, hep-ph/9605323

$$d\sigma^A = \sum_{dipoles} d\sigma^B \otimes dV_{dipole}$$

- Dipole local subtraction

$$d\sigma^A = \mathcal{D}_q^{\ell\ell'}(p_3, p_4, p_5; p_1, p_2) F_J^{(2)}(\tilde{p}_4, p_5; \tilde{p}_\gamma, p_2)$$

$$D_q^{\ell\ell'}(p_3, p_4, p_5; p_1, p_2) = -\frac{1}{(p_1 - p_3)^2} \langle \tilde{p}_4, p_5; \tilde{p}_\gamma, p_2 | \mathbf{V}_q^{\ell\ell'} | \tilde{p}_4, p_5; \tilde{p}_\gamma, p_2 \rangle$$

$$\langle \mu | \mathbf{V}_q^{\ell\ell'} | \nu \rangle = -g^{\mu\nu} - \frac{4(\bar{z}_{\ell' q,\ell} p_3^\mu - z_{\ell' q,\ell} p_4^\mu)(\bar{z}_{\ell' q,\ell} p_3^\nu - z_{\ell' q,\ell} p_4^\nu)}{(p_1 - p_3)^2 x_{\ell' q,\ell}^2 \bar{z}_{\ell' q,\ell}} + \frac{\lambda_\ell}{2}(2 - x)\left[\epsilon_+^\mu(\tilde{p}_\gamma)^* \epsilon_+^\nu(\tilde{p}_\gamma) - \epsilon_-^\mu(\tilde{p}_\gamma)^* \epsilon_-^\nu(\tilde{p}_\gamma)\right]$$

- Matches the collinear behavior of the real part

$$D_q^{\ell\ell'}(p_3, p_4, p_5; p_1, p_2) \to \frac{\alpha}{2\pi} P_{\gamma\ell}(z) \langle \tilde{p}_4, p_5; \tilde{p}_\gamma, p_2 | \tilde{p}_4, p_5; \tilde{p}_\gamma, p_2 \rangle$$

- Cancels the pole in collinear counterterm (work with nonzero lepton mass)

$$\int d\Phi_1 d\sigma^A = \frac{\alpha}{2\pi} \sum_{\tau=\pm} \sum_{a=q,\bar{q},g} \int_0^1 dx \left\{ \ln\left[\frac{2p_\gamma \cdot \tilde{p}_4(1-x)}{x^3 m_\ell^2}\right] [P_{\gamma\ell}(x) + \tau \Delta P_{\gamma\ell}(x)] - \frac{1-x}{x} - \tau(1-x) \right\}$$
$$\times d\sigma_{\gamma a \to jX}(\tilde{p}_\gamma, p_2; \lambda_\gamma = \tau \lambda_\ell)$$

  - Collinear counter term

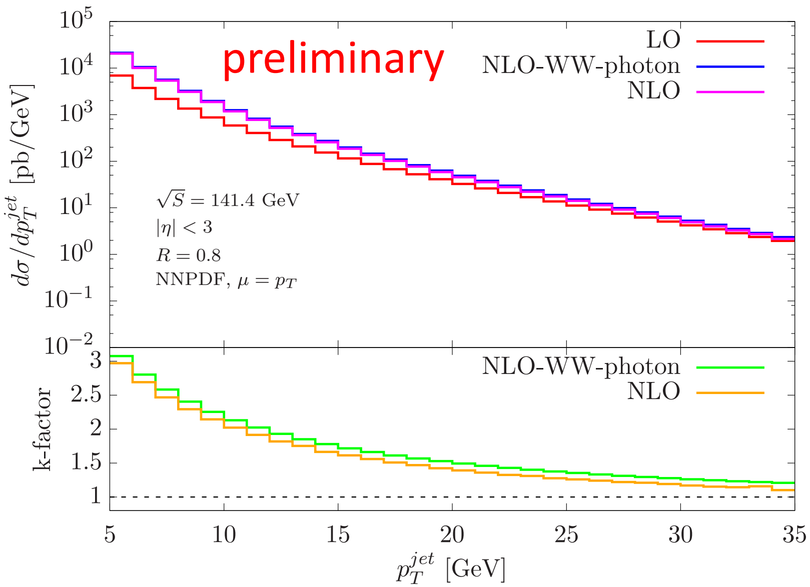
$$d\sigma^C = -\int dx f_{\gamma/\ell}(x, \mu^2) d\sigma_{\gamma a}^B$$

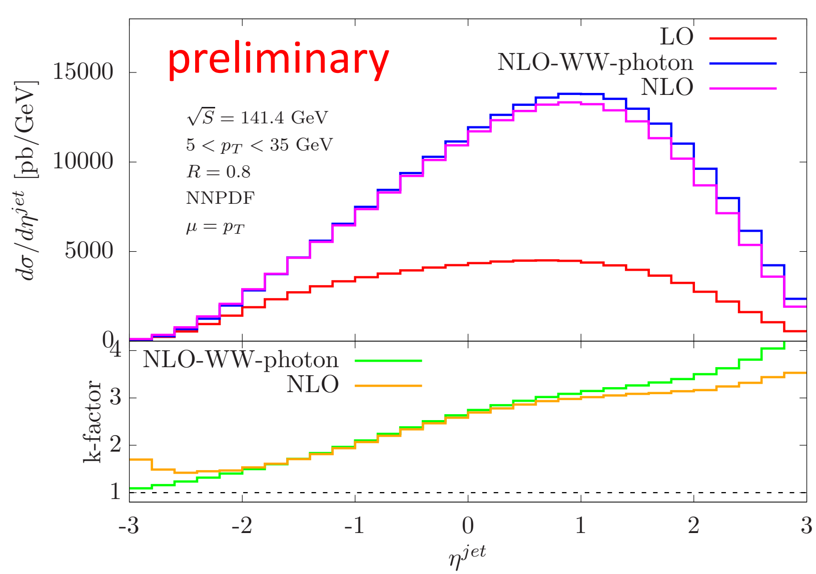
  - Weizsacker-Williams photon distribution in lepton

$$f_{\gamma/\ell}(x, \mu^2) = \frac{\alpha}{2\pi} P_{\gamma\ell}(x) \left[\ln\left(\frac{\mu^2}{x^2 m^2}\right) - 1\right]$$

EIC predictions

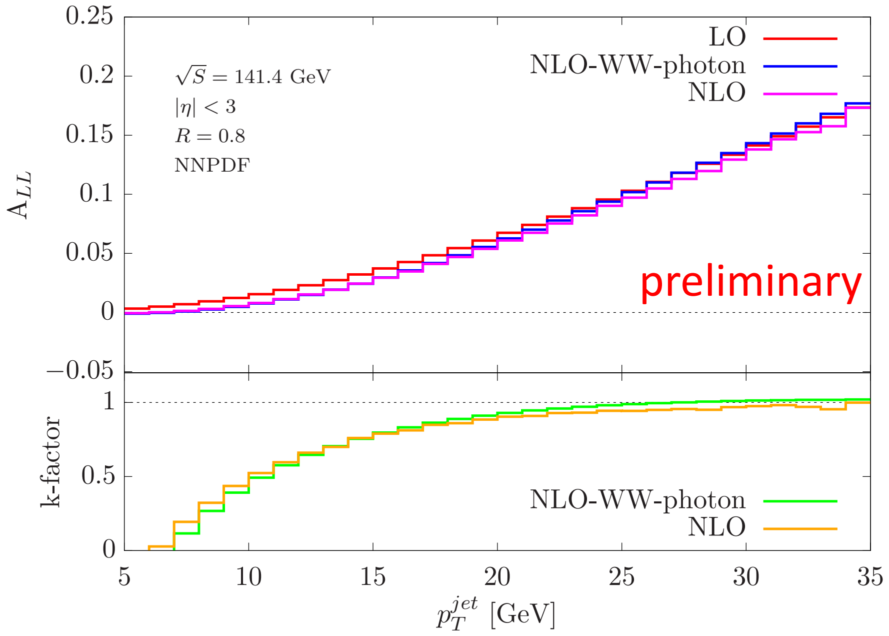
- NLO unpolarized cross section

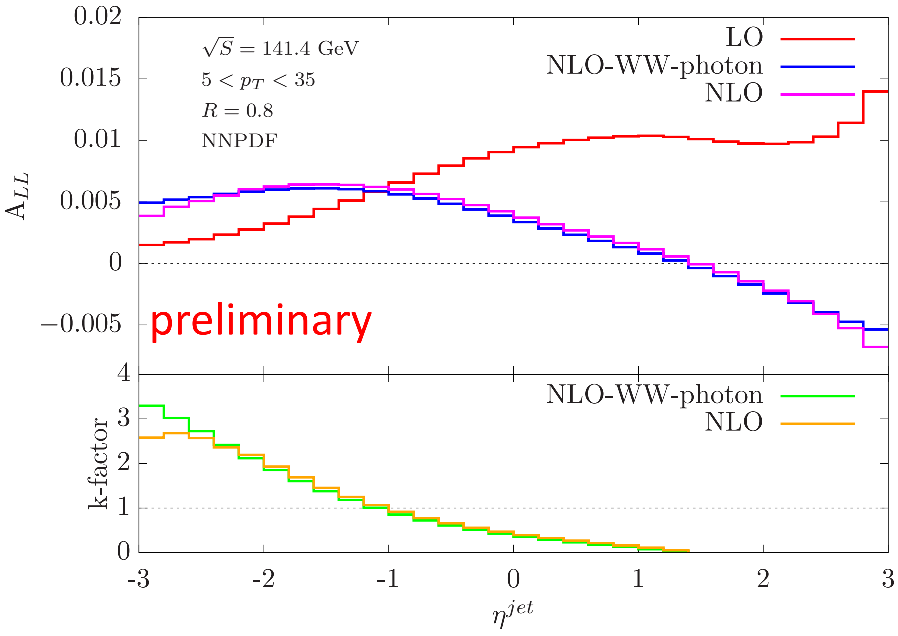




Significant NLO correction, dominant by WW-photon.

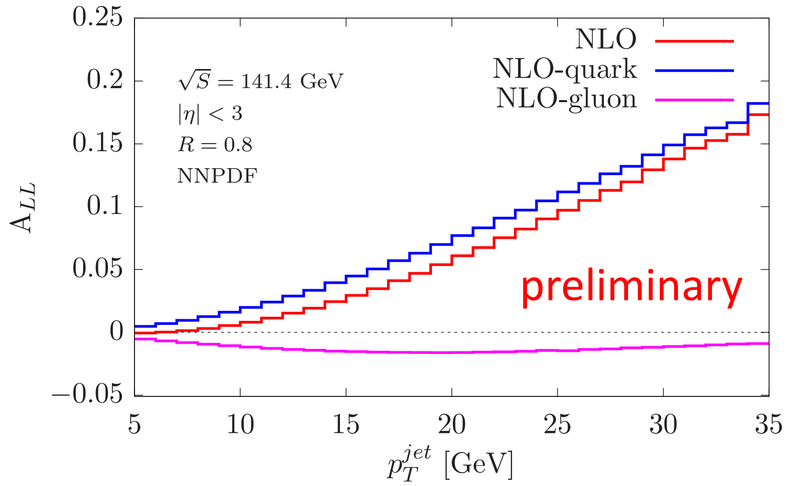
❑ Double longitudinal asymmetry



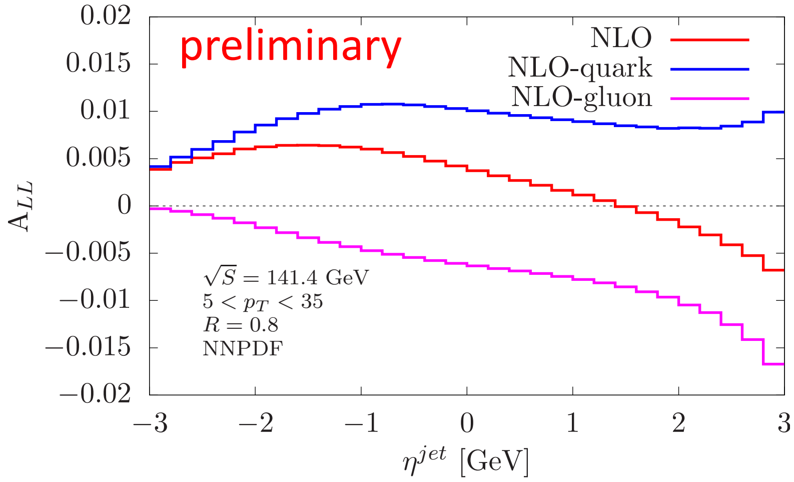


Large NLO corrections.

Fraction contributions

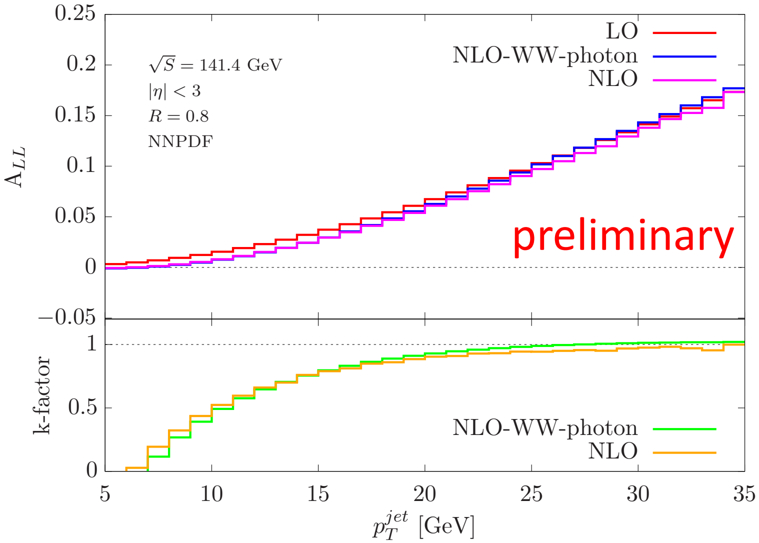


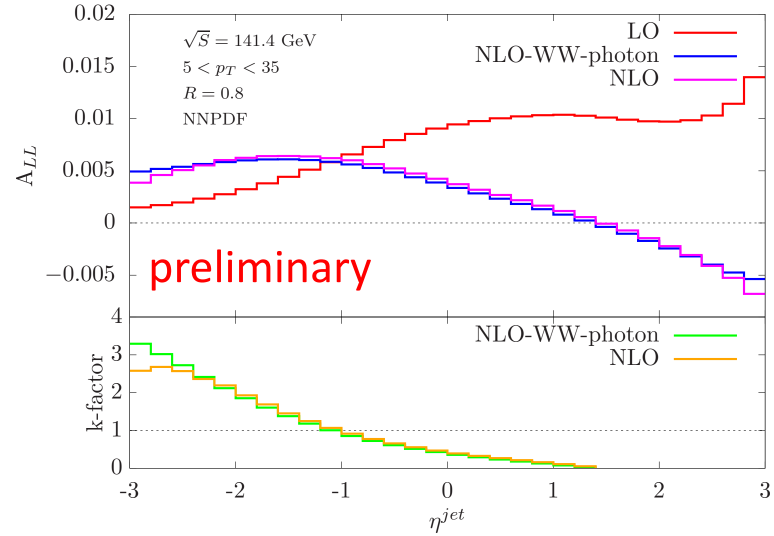
Sensitive to quark distribution.

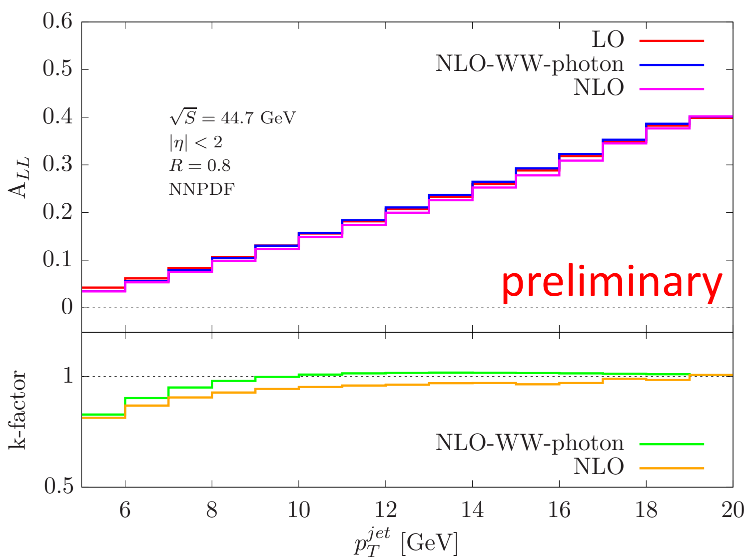


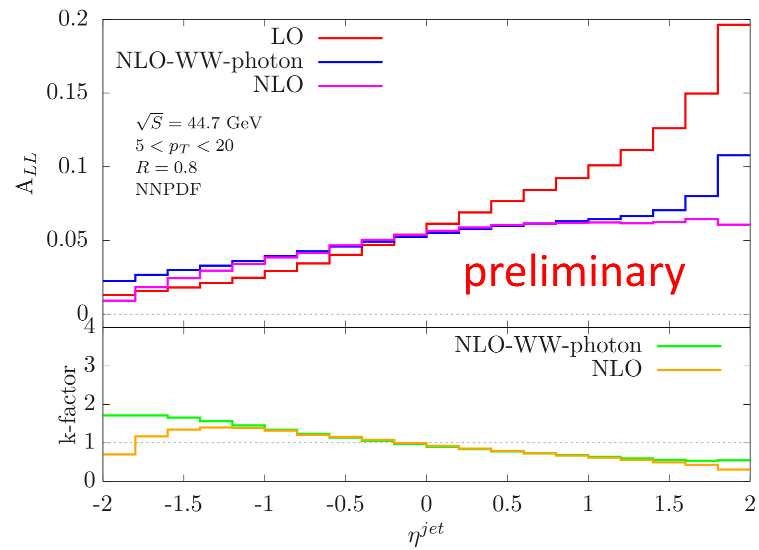
sensitive to gluon distribution.

❑ High energy vs. low energy

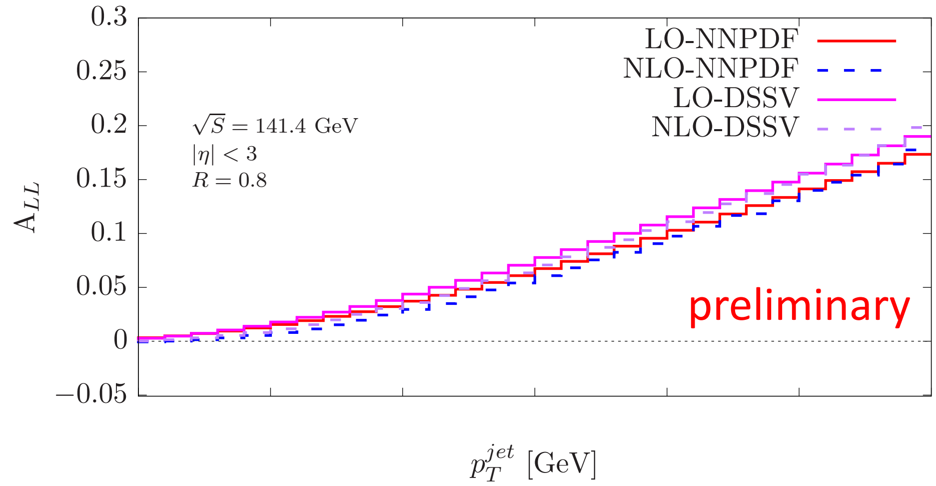


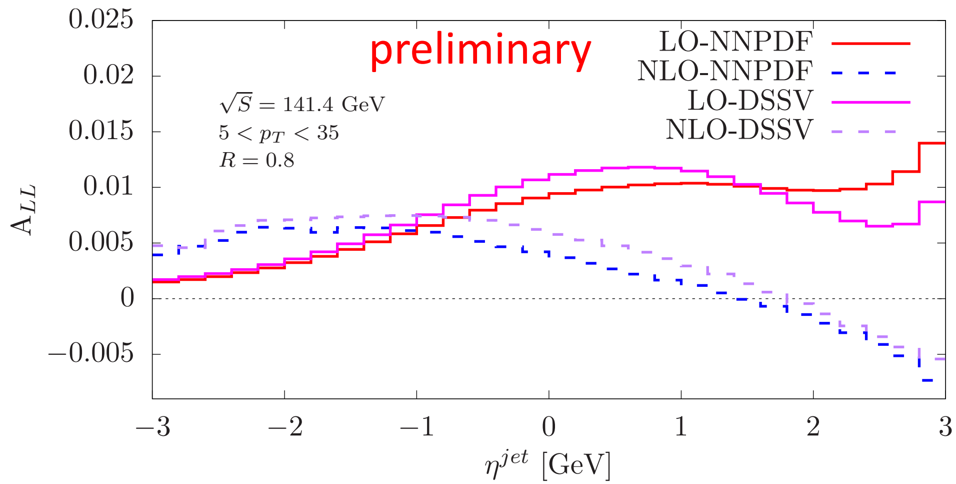






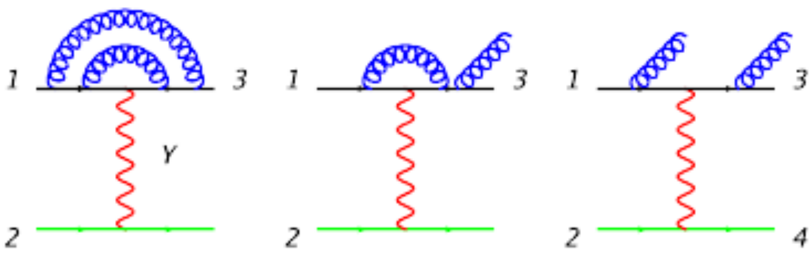
❑ NNPDF vs. DSSV





Outlook: DIS NNLO

❑ Higher fixed order results, progress on NNLO

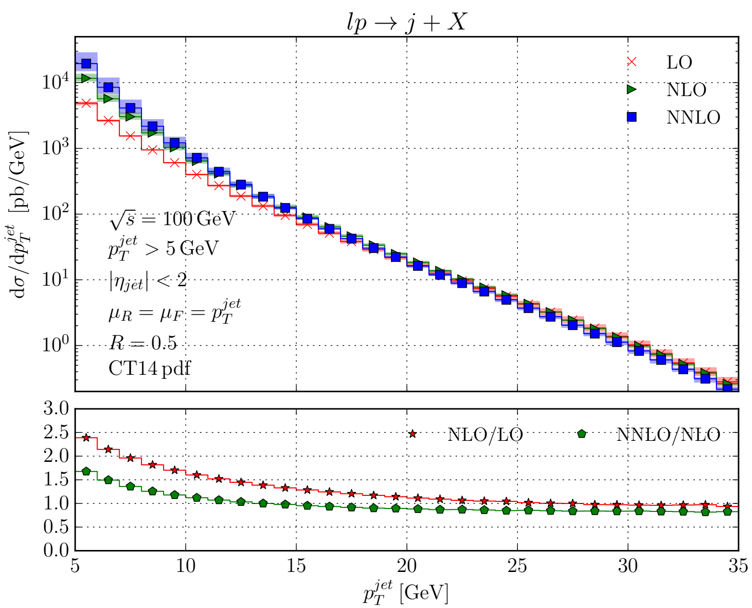


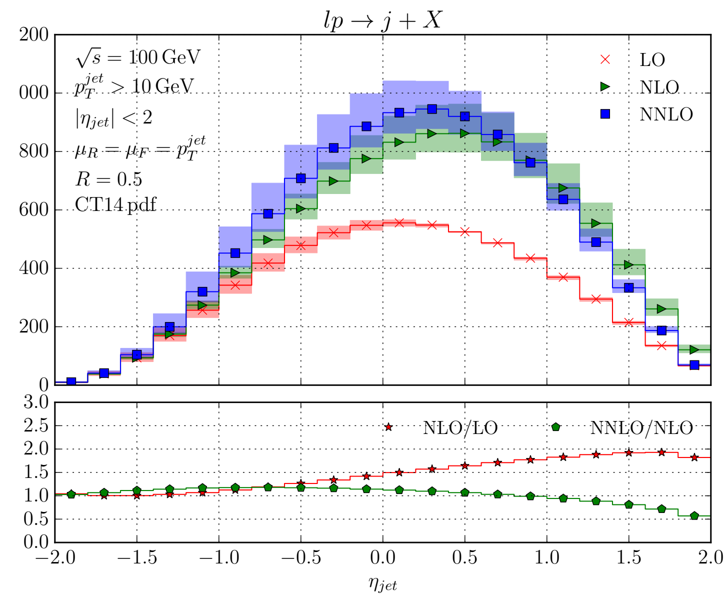
- Below cut $\theta_N^< = \theta(\tau_N^{cut} - \tau_N)$
  - VV: τ_N is zero
  - RV and RR: both additional radiations are unresolved. Purely IR divergent region
  - two loop, soft and collinear radiation
  - Calculate this part from SCET
- Above cut $\theta_N^> = \theta(\tau_N - \tau_N^{cut})$
  - In RR: at lease one of the two additional radiations that appear is resolved, this region of phase space contains the NLO correction to the 2 jet process.
  - In RV: the radiation has to be hard, this is NLO virtual correction to 2 jet production.
  - Calculate this part by recycling NLO 2 jet production.

- # Unpolarized scattering at NNLO

$$\sigma_{NNLO} = \color{blue}{\sigma_{NNLO}(\tau_N < \tau_N^{cut})} + \color{red}{\sigma_{NNLO}(\tau_N > \tau_N^{cut})}$$

SCET — Fixed order NLO for 2 jets





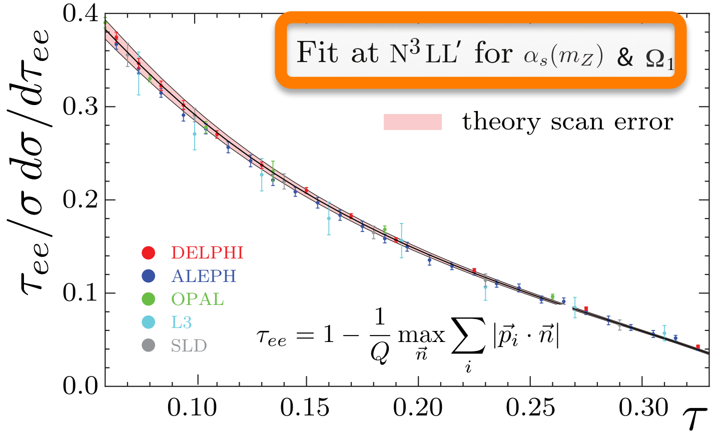
Abelof, Boughezal, Liu, Petriello, 16

- # Next: polarized scattering at NNLO

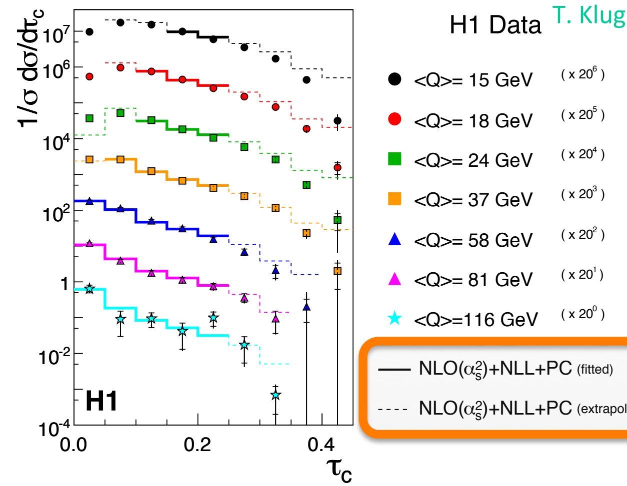
Global event shape

- Precision determination of strong coupling constant

$$\tau_{\rm DIS} = 1 - \frac{1}{E_J} \sum_{i \in \mathcal{H}_J} |\vec{p}_i \cdot \vec{n}|$$

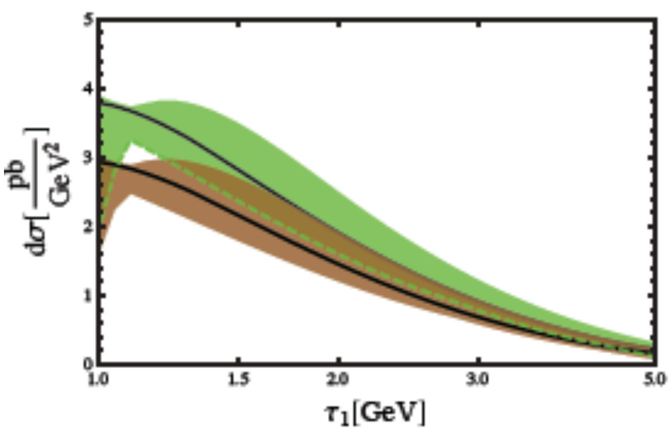


Bacher and Schwartz, Abbate et al.

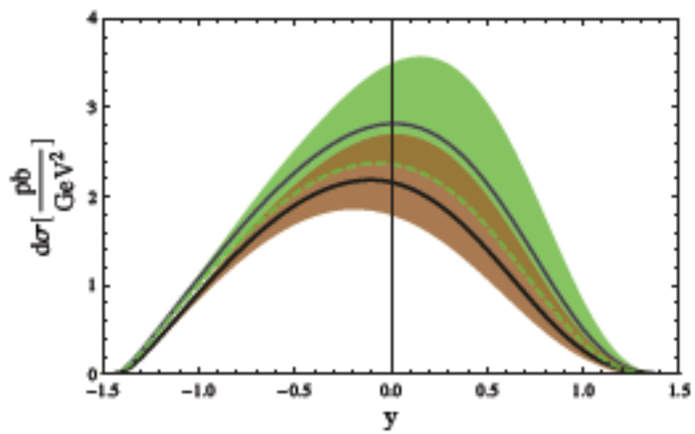


- 1-Jettiness distribution as a probe of nuclear effect

Kang, Liu, Mantry, Qiu



(f)Ur and Proton



(f) Proton and Ur

Event shape in polarized DIS

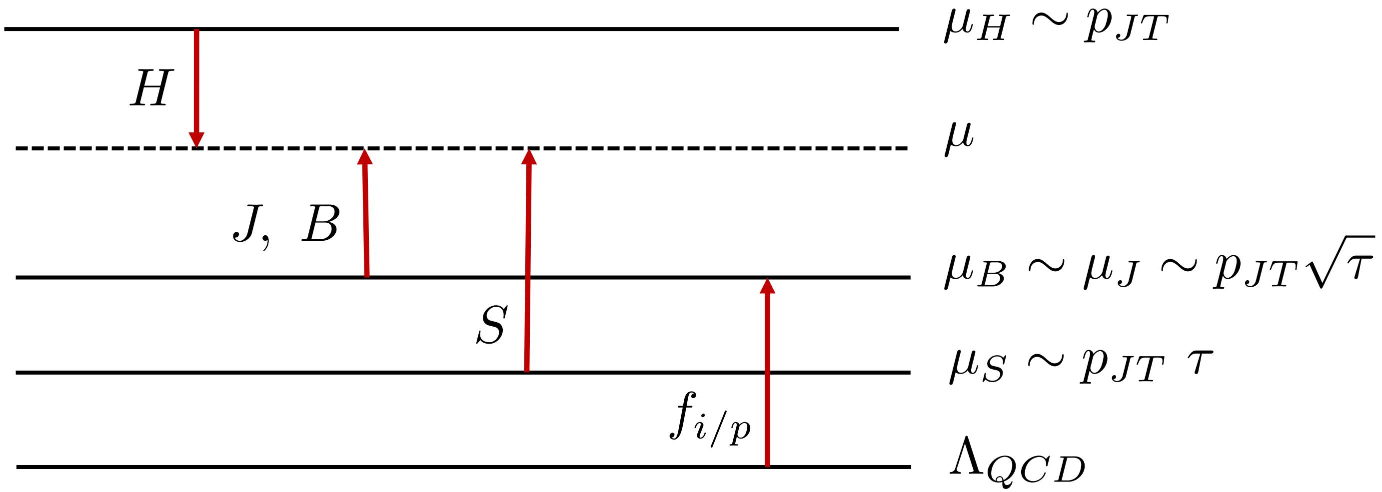
- Factorization and resummation

$$\frac{d\sigma}{d\tau} = \int d\Phi H(Q,x,\mu) U_H(Q;\mu,\mu_H) \int dt_B dt_J dk_s \delta\left(\tau - \frac{t_B}{Q^2} - \frac{t_J}{Q^2} - \frac{k_s}{Q}\right)$$
$$\times B(t_B,x,\mu_B) \otimes U_B(\mu,\mu_B) J_q(t_B,x,\mu_J) \otimes U_J(\mu,\mu_J)$$
$$\times U_S(\mu,\mu_S) S(k_S,\mu_S) + \frac{d\sigma^{nonsingular}}{d\tau}$$

- Factorization theorem sums singular (log enhanced) terms

  Large logs $\ln\tau \gg 1 \qquad \alpha_s \ln\tau \sim 1$

- Two loop beam functions enable N³LL resummation
- Precision test of QCD and accurate extraction of PDFs



Summary

- We calculated the matching coefficients between the polarized quark beam function and PDFs at two-loop order.
- We implemented 1-jettiness and dipole subtraction to handle all IR divergences in inclusive jet production at NLO.
- Further improvement will be done at NNLO for polarzied DIS.
- Global event shape will be also interesting in determining helicity parton distributions.

Summary

- We calculated the matching coefficients between the polarized quark beam function and PDFs at two-loop order.
- We implemented 1-jettiness and dipole subtraction to handle all IR divergences in inclusive jet production at NLO.
- Further improvement will be done at NNLO for polarzied DIS.
- Global event shape will be also interesting in determining helicity parton distributions.

Thanks!