Nucleon Form Factors at High Momentum Transfer

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Stony Brook University

INT Program INT17-68W
"The Flavor Structure of Nucleon Sea"
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Outline

Nucleon form factors at high momentum

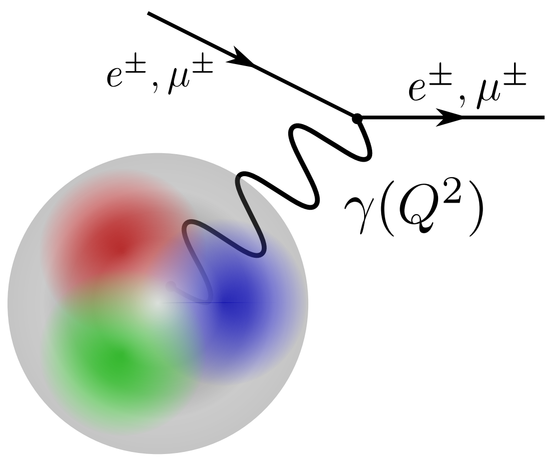
- Phenomenological motivation
  - *Form factors: JLab program, Perturbative Q^2-scaling limit*
  - *Lattice TMDs & qPDFs*
- Challenges for high-momentum nucleon structure
  - *Signal / noise for required kinematics*
  - *Boosted (momentum) smearing*
- Details of calculation
- Preliminary results and comparison to phenomenology
- Summary and Outlook

Nucleon Vector Form Factors

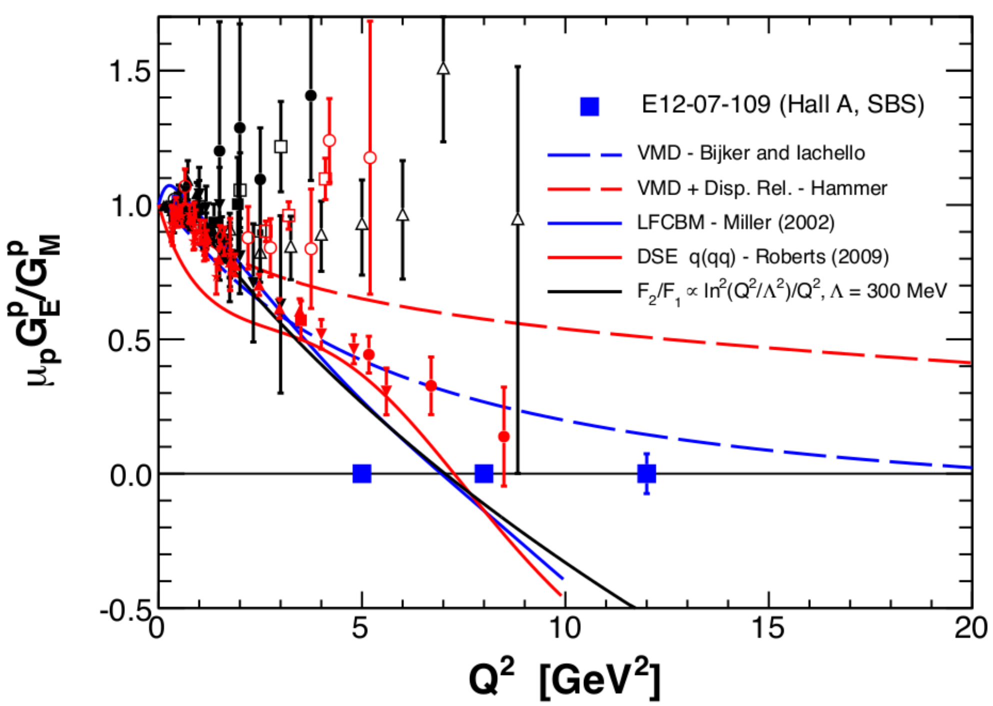
$$\langle P+q|\,\bar{q}\gamma^\mu q\,|P\rangle = \bar{U}_{P+q}\Big[F_1(Q^2)\gamma^\mu + F_2(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\Big]U_P$$

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2)$$

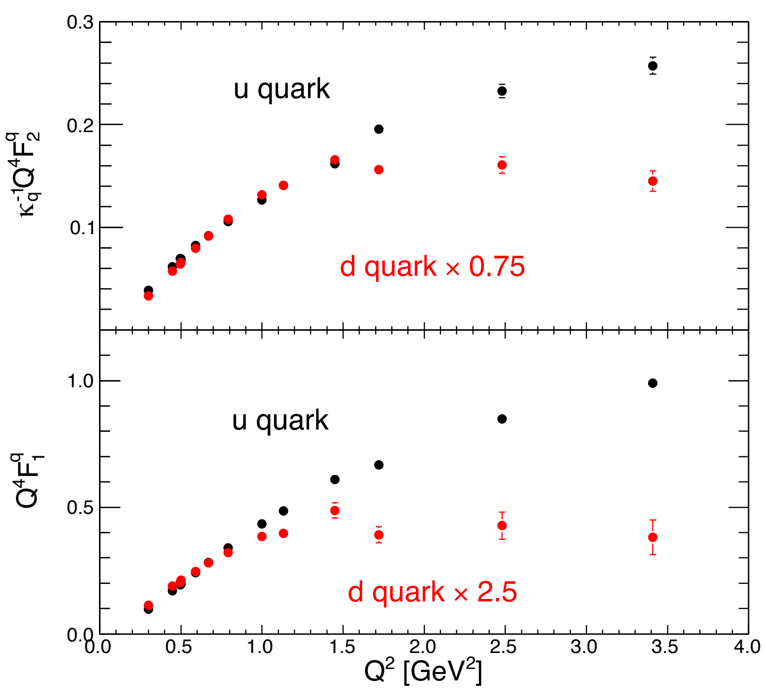
$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

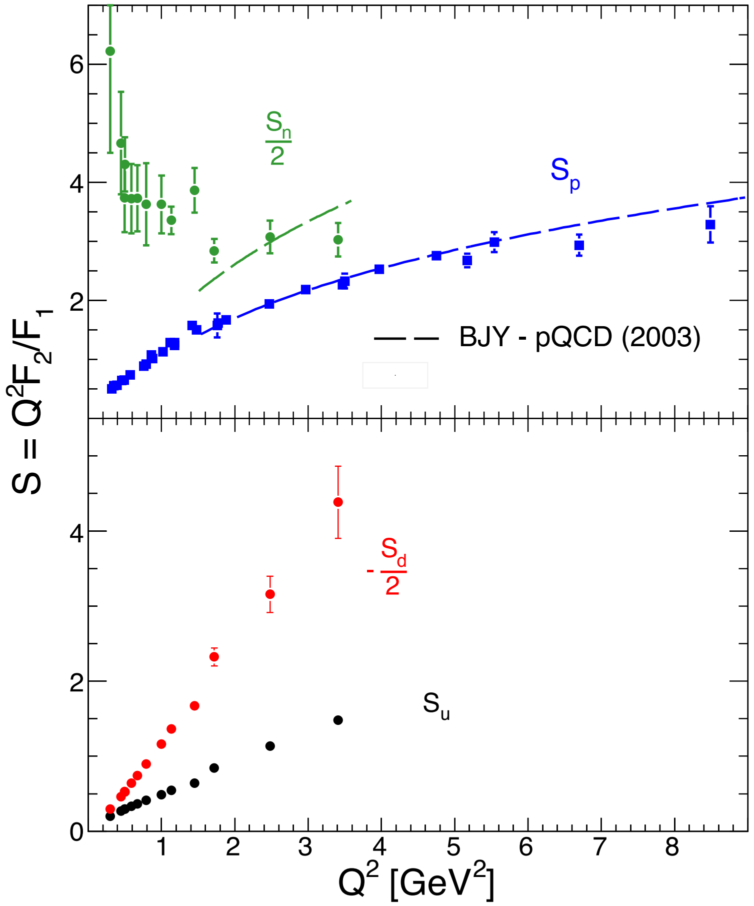


- JLab@12GeV + Super BigBite:
  explore form factors at Q^2 up to 18 GeV^2
  - (G_E/G_M) dependence
  - $(F_1\,/\,F_2)$ scaling at $Q^2 \to \infty$
  - *u-*, *d*-flavor contributions to form factors



[Research Mgmt. Plan for SBS(JLab Hall A)]

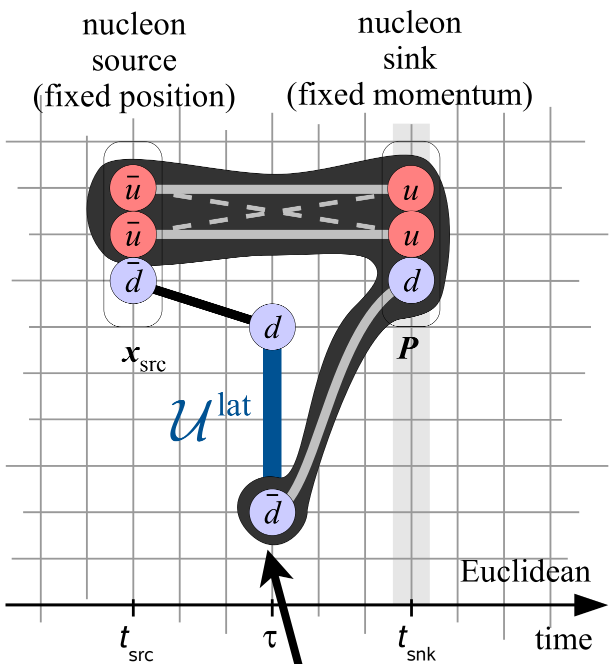




[G.D.Cates, C.W.de Jager, S.Riordan, B.Wojtsekhovski, PRL106:252003, arXiv:1103.1808]

Common Problem with TMD, qPDF

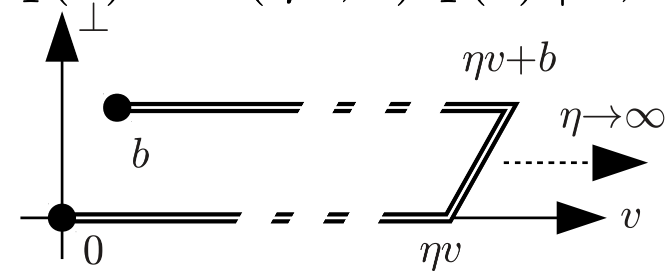
[B.Musch, M.Engelhardt, et al]
Non-local lattice operator



operator localized at Euclidean time τ

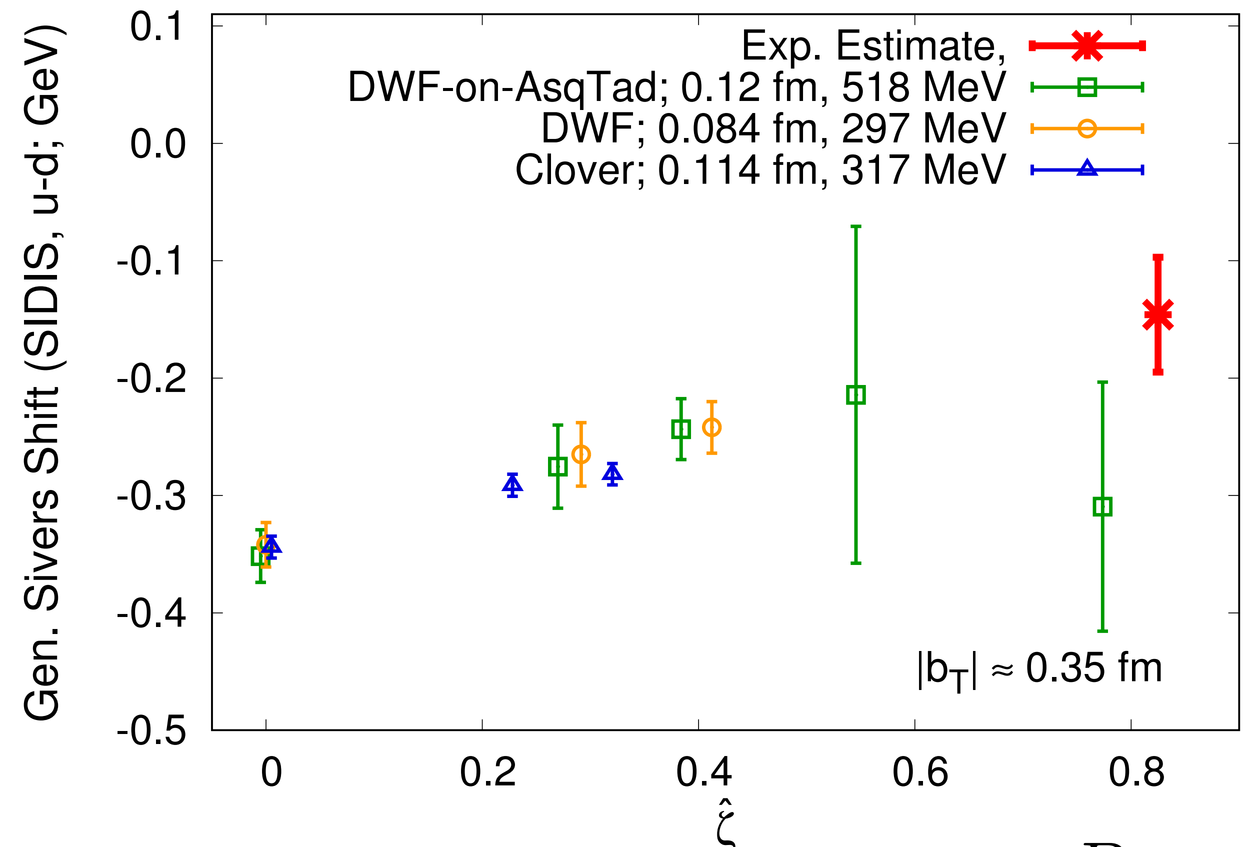
$$\Phi(b, P, S, \hat\zeta, \mu) = \frac{1}{2}\langle P, S|\,\bar q(0)\,\Gamma\,\mathcal{U}(\eta v, b)\,q(b)\,|P, S\rangle$$

with spacelike link path $\mathcal{U}$ =



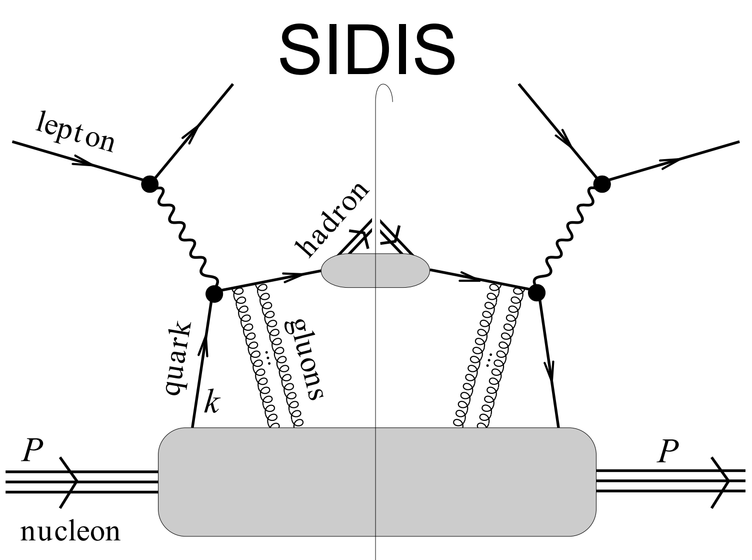
probes $k_\perp$-moments ("shifts") of TMDs

$$\sim \int dx \int d^2\vec{k}_\perp\, k_i\, f(x, \vec{k}_\perp)$$



LC limit of spacelike staple:
Collins-Soper parameter

$$\hat\zeta = \frac{P \cdot v}{m_N\,|v|} \to \infty$$

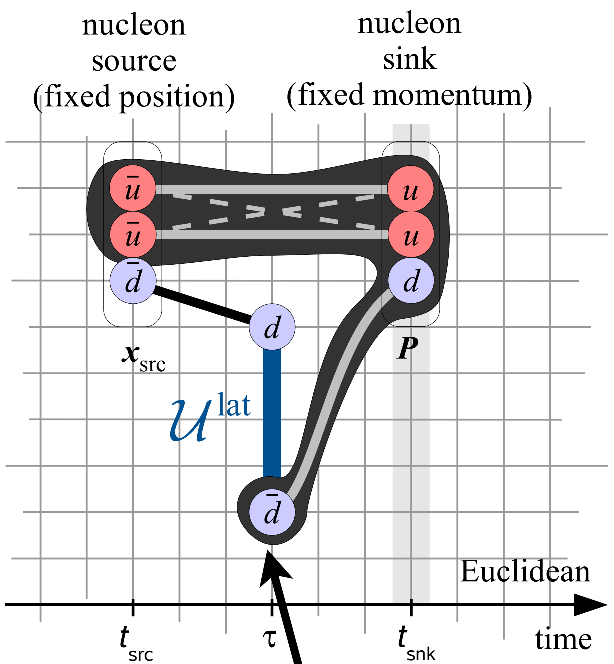


$$l + N(P) \longrightarrow l' + N(P_h) + X$$

Common Problem with TMD, qPDF

[B.Musch, M.Engelhardt, et al]

Non-local lattice operator



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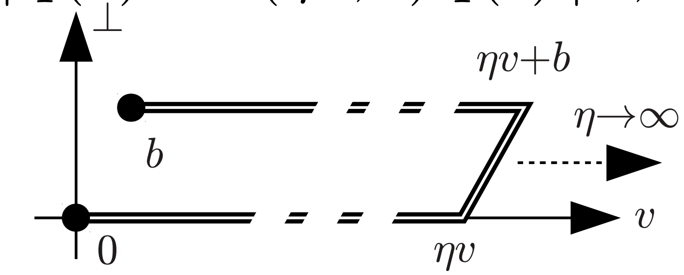
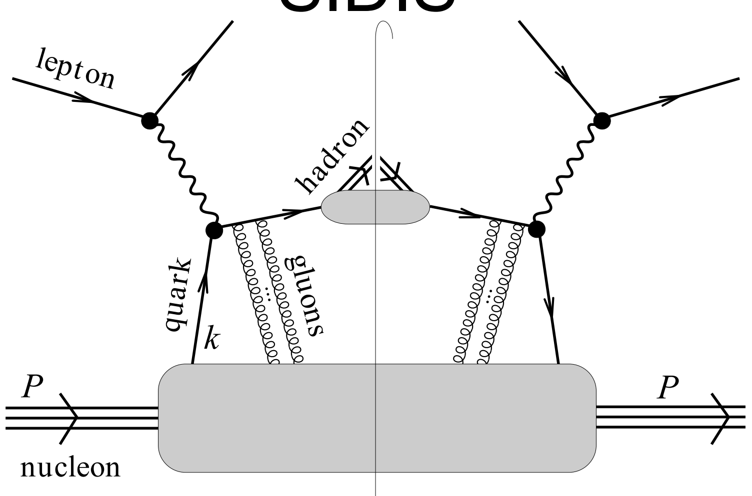
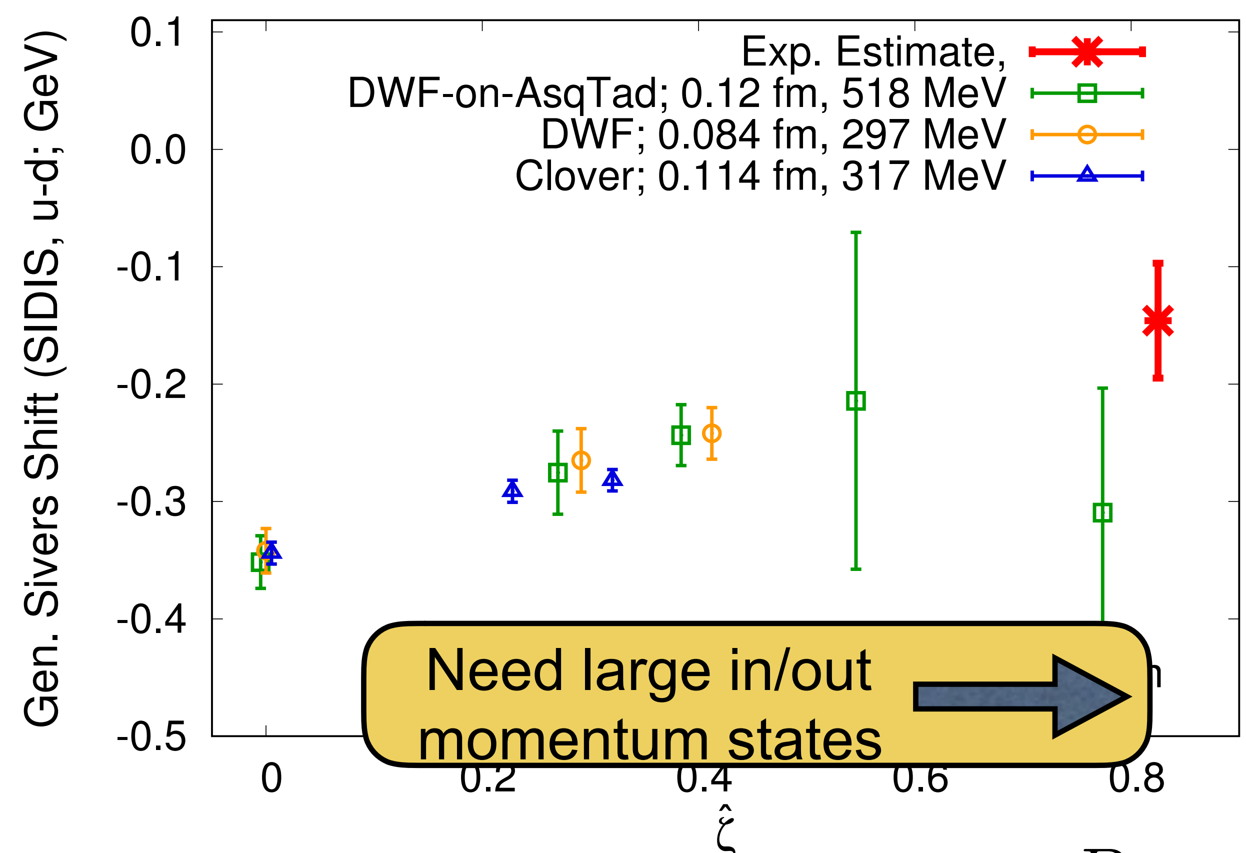


probes $k_\perp$-moments ("shifts") of TMDs $\sim \int dx \int d^2\vec{k}_\perp\, k_i\, f(x, \vec{k}_\perp)$

SIDIS



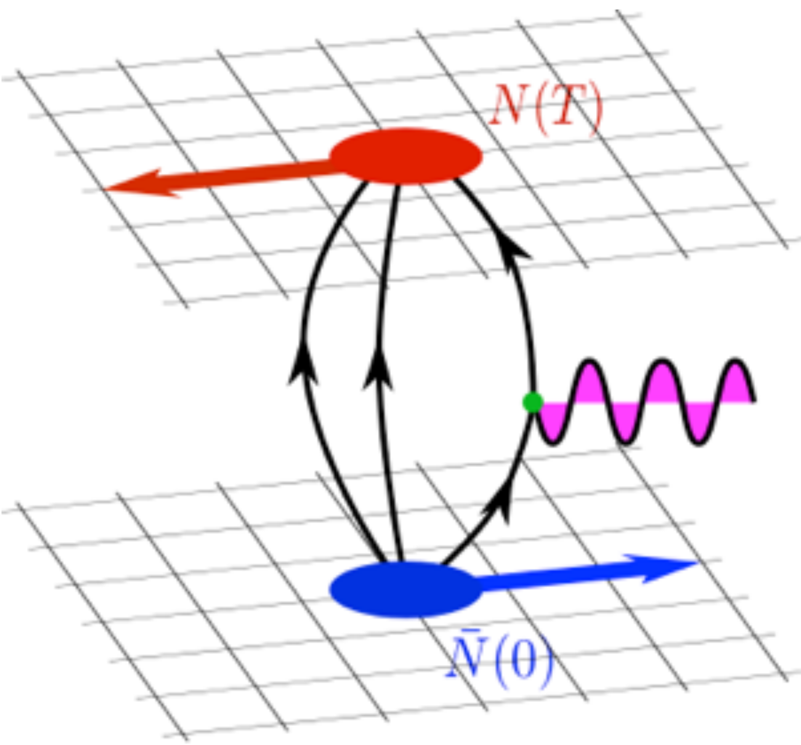
$$l + N(P) \longrightarrow l' + N(P_h) + X$$



LC limit of spacelike staple: Collins-Soper parameter

$$\hat{\zeta} = \frac{P \cdot v}{m_N\, |v|} \to \infty$$

Accessing Large Q^2 : Breit Frame



Minimize $E_{in,out}$ for required Q^2 :

$$Q^2 = (\vec{p}_{in} - \vec{p}_{out})^2 - (E_{in} - E_{out})^2$$

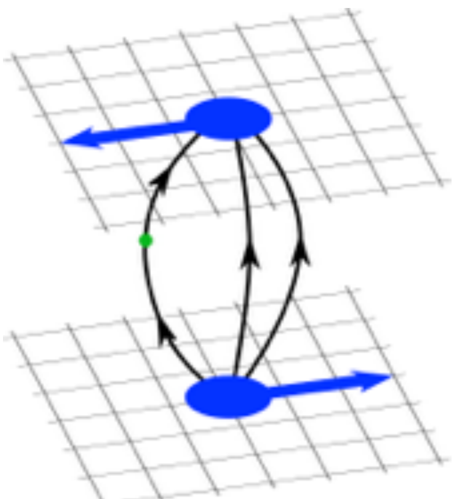
Back-to-back

$$Q^2 = 4\vec{p}^2$$

At right angle

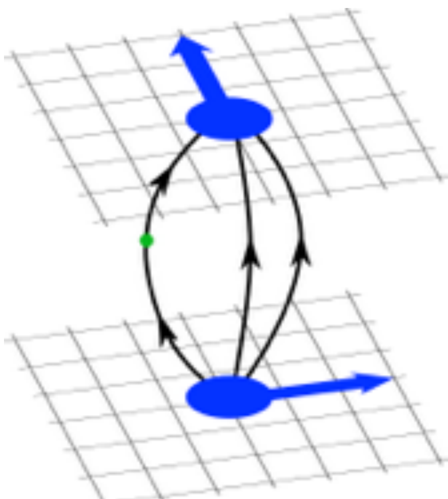
$$Q^2 = 2\vec{p}^2$$

For $Q^2 = 8\ GeV^2$



$$|\vec{p}| = \frac{1}{2}\sqrt{Q^2} \approx 1.4\ \mathrm{GeV}$$

$E_N \approx 1.8\ GeV$



$$|\vec{p}| = \sqrt{\frac{1}{2}Q^2} \approx 2.0\ \mathrm{GeV}$$

$E_N \approx 2.3\ GeV$

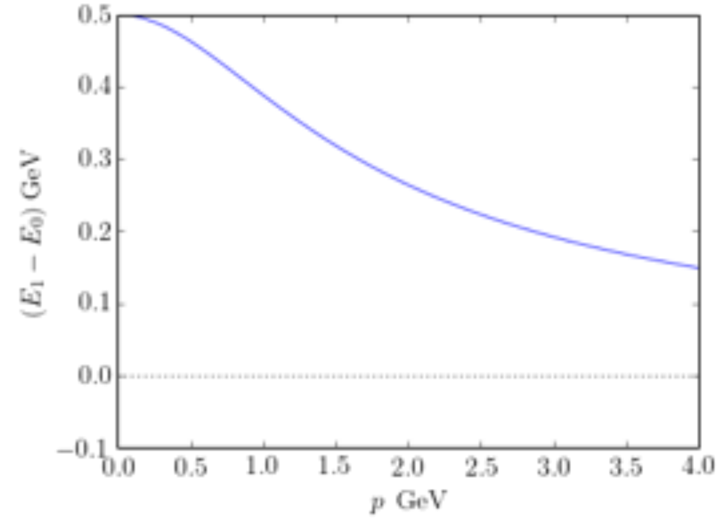
Challenges for Large Q^2 on a Lattice

- Stochastic noise : grows faster with T [Lepage'89]:

$$\begin{array}{lll} \text{Signal} & \langle N(T)\bar{N}(0)\rangle & \sim e^{-E_N T} \\ \text{Noise} & \langle |N(T)\bar{N}(0)|^2\rangle - |\langle N(T)\bar{N}(0)\rangle|^2 & \sim e^{-3m_\pi T} \\ \text{Signal/Noise} & & \sim e^{-(E_N - \frac{3}{2}m_\pi)T} \end{array}$$

- Excited states: boosting "shrinks" the energy gap

$$E_1 - E_0 = \sqrt{M_1^2 + \vec{p}^2} - \sqrt{M_2^2 + \vec{p}^2} < M_1 - M_0$$



  - In this work : use 2-exponential fits

Reduction of lattice correlator noise is crucial

Challenges for Large Q^2 on a Lattice (2)

- Discretization effects : *O(a¹)* for local operator

  O(a¹) improved vector-current operator

$$(V_\mu)_I = \bar{q}\gamma_\mu q + c_V\, a\, \partial_\nu(\bar{q} i \sigma_{\mu\nu} q)$$

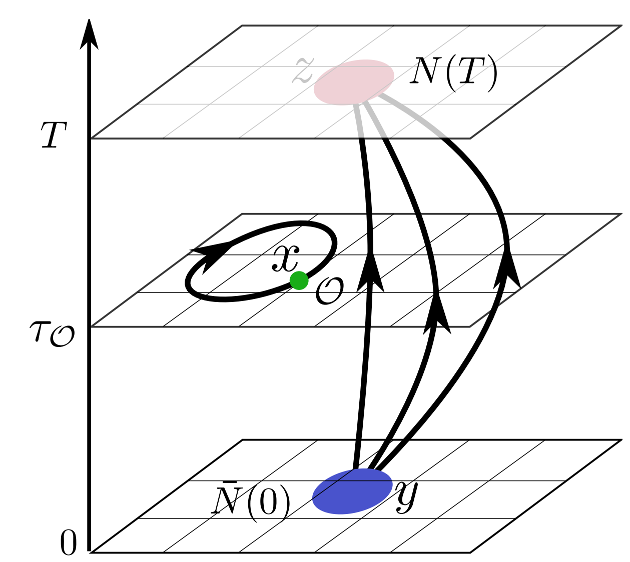
  improvement term is likely to grow with Q^2

- Disconnected contractions

  expensive: $\langle N' | J_\mu | N \rangle_{\text{disc}} \sim \langle \overline{N' N} \cdot \text{Tr}[\gamma_\mu \not{D}^{-1}] \rangle$

  negligible for small $Q^2 \lesssim 1$ GeV²
  [J. Green, S. Meinel, et al; PRD92:031501]

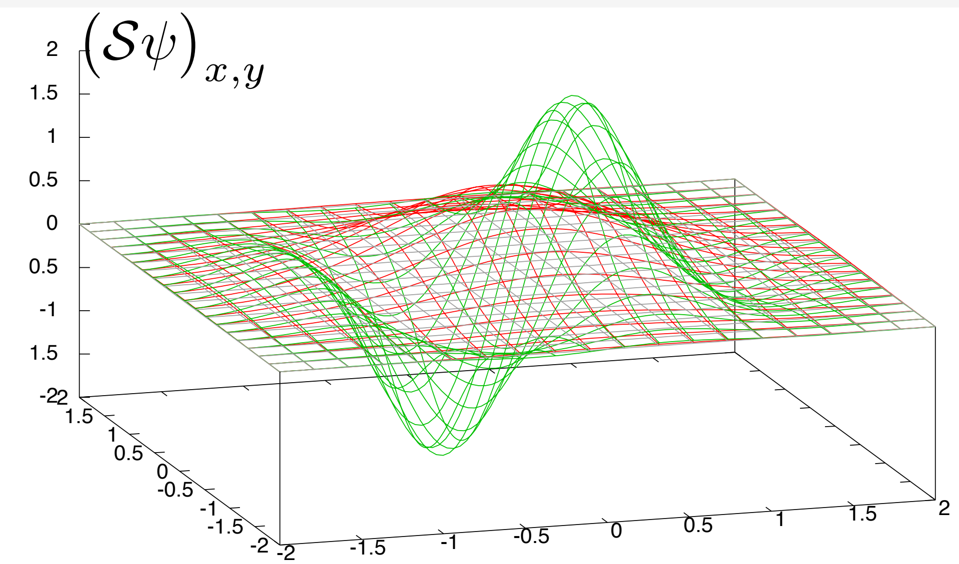
  - need to explore at $Q^2 \gtrsim 1$ GeV²
  - noise reduction for $\overline{N' N}$ is critical



High-momentum Hadron States on a Lattice

Nucleon operator is built from ≈Gaussian smeared quarks

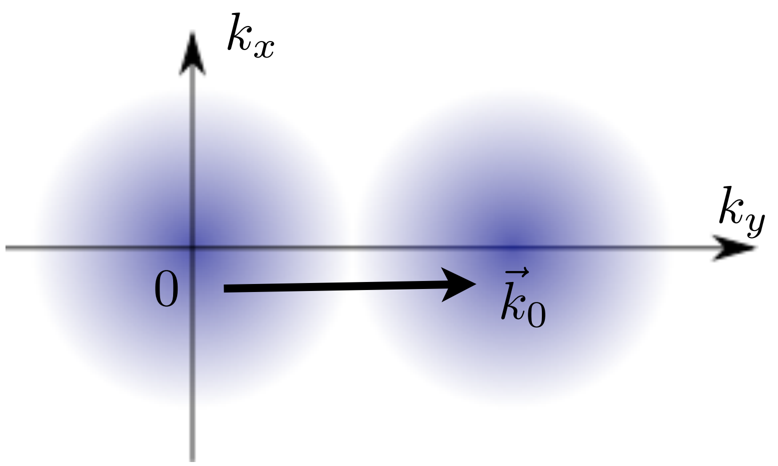
$$N_{\rm lat}(x) = (\mathcal{S}\,u)^a_x\,\big[(\mathcal{S}\,d)^b_x\,C\gamma_5\,(\mathcal{S}\,u)^c_x\big]\,\epsilon^{abc}$$



Gaussian shape in momentum space :
reduced overlap with quark WFs in a boosted nucleon

$$\mathcal{S}_{\text{at-rest}} = \exp[-\frac{w^2}{4}(i\vec\nabla)^2] \sim exp(-\frac{w^2\vec k^2_{\rm lat}}{4})$$

SOLUTION: improve the overlap by shifting the spatial smearing operator in momentum space (*"momentum smearing"*)
[orig. B.Musch; first explored in G.Bali et al, 1602.05525]



$$\mathcal{S}_{\vec k_0} = \exp[-\frac{w^2}{4}(-i\vec\nabla - \vec k_0)^2] \sim \exp(-\frac{w^2(\vec k_{\rm lat} - \vec k_0)^2}{4})$$

Modified smearing operator

$$\big[\mathcal{S}_{\vec k_0}(\psi)\big]_x = e^{+\vec k_0 \vec x}\mathcal{S}(e^{-\vec k_0\vec y}\psi_y) \sim e^{+\vec k_0\vec x}\cdot\text{smooth fcn.}(x)$$

Modified covariant smearing operator in lattice*color space

$$\big[\mathcal{S}_{\vec k_0}\big]_{x,y} = e^{+i\vec k_0\vec x}\big[\mathcal{S}\big]_{x,y}e^{-i\vec k_0\vec y}$$

$\Longleftrightarrow$

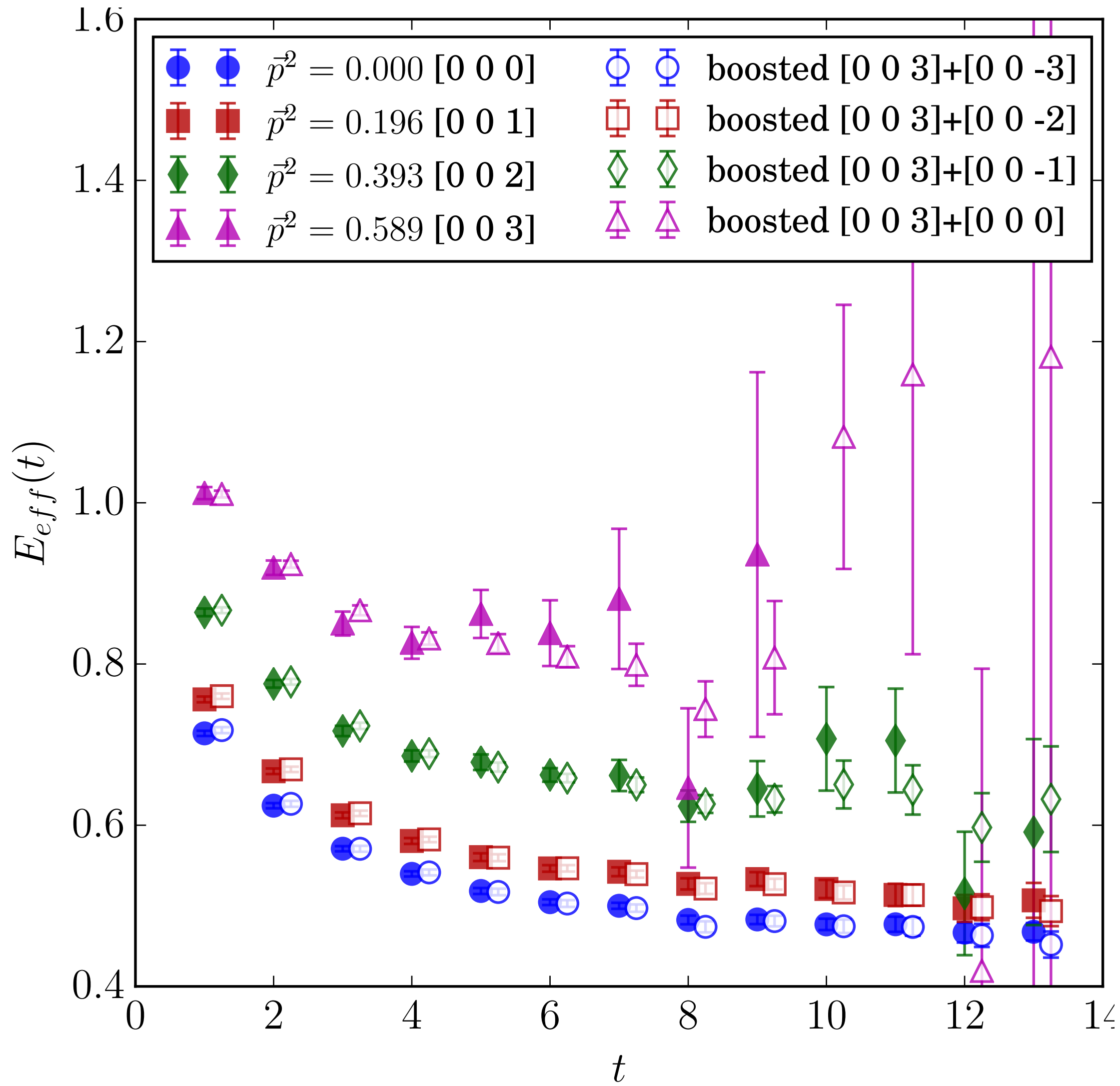
Smearing with twisted gauge links

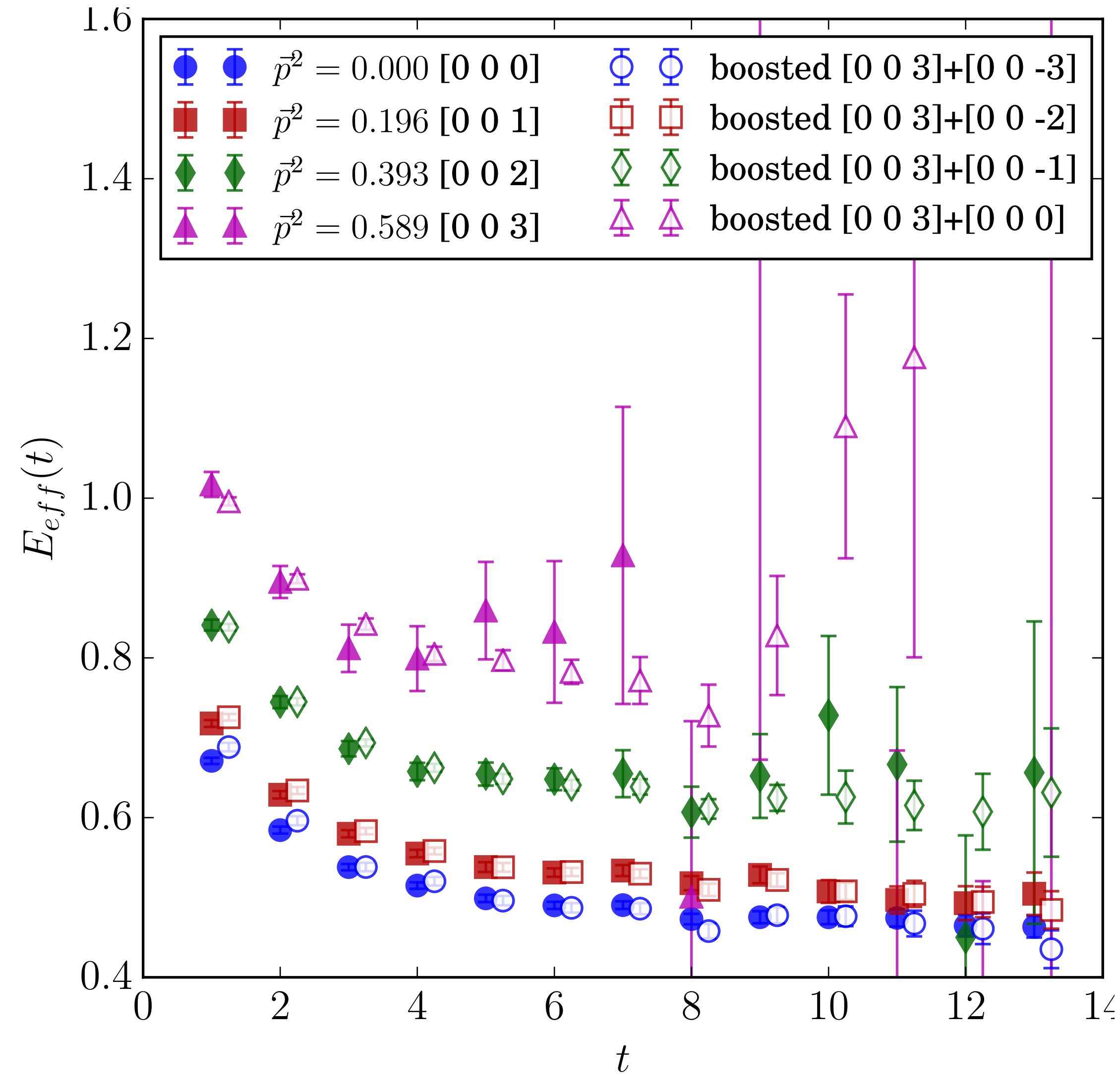
$$\Delta_{x,y} \longrightarrow e^{+i\vec k_0\vec x}\Delta_{x,y}e^{-i\vec k_0\vec y}$$

$$U_{x,\mu} \longrightarrow e^{-ik_\mu}U_{x,\mu}$$

Signal Gain : Traditional vs. Boosted Smearing

Nucleon Effective Energy: m_π = 300 MeV, a=0.094 fm, 32^3x64



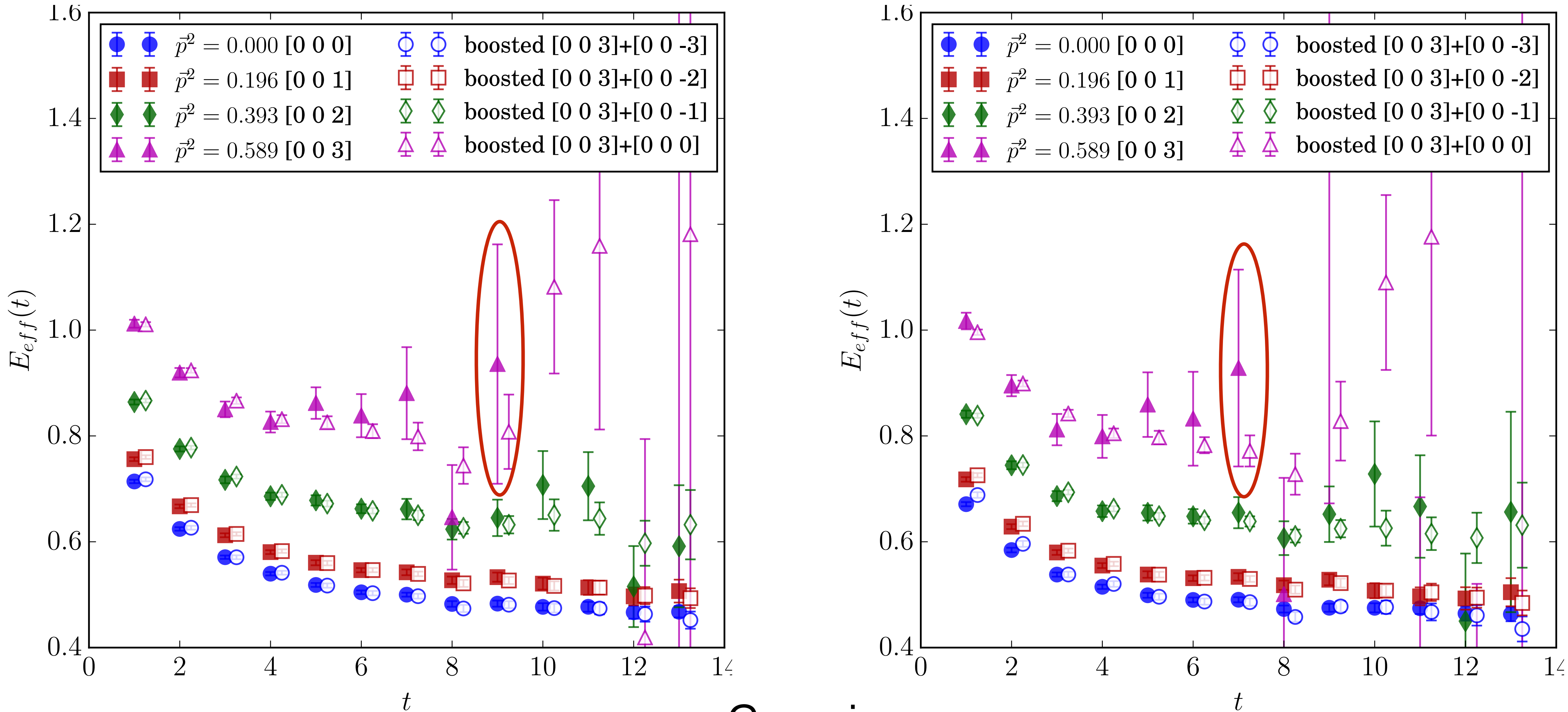


width ≈ 5a ⟵ Gaussian smearing ⟶ *w ≈ 6.6a*

- each quark is boosted with the same k=[0 0 1]
- *w ≈ 5.55a* chosen as ≈ optimal

Signal Gain : Traditional vs. Boosted Smearing

Nucleon Effective Energy: m_π = 300 MeV, a=0.094 fm, 32^3x64



width ≈ 5a ⟵ Gaussian quark trial wf ⟶ *w ≈ 6.6a*

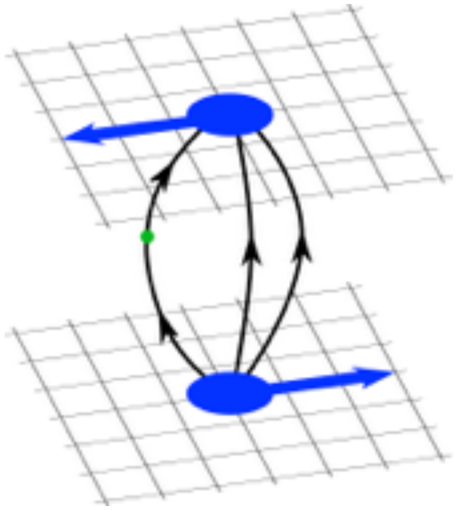
- each quark is boosted with the same k=[0 0 1]
- *w ≈ 5.55a* chosen as ≈ optimal

Preliminary Study: 2 Gauge Ensembles

Exploratory study with clover-improved Wilson action (WM/JLab)at $m_\pi \approx 300$ MeV

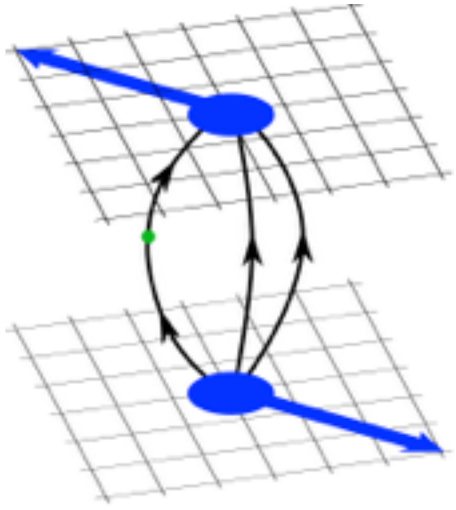
- 32^3x64
- a=0.094 fm
- *p_{min}=0.42 GeV*
- *tsep = (8 .. 12)a = 0.65 .. 0.97 fm*
- *boost-smear with [1,0,0]*
- *240*64=15,360 samples*

$$Q^2 \lesssim 6.1\,\mathrm{GeV}^2$$



- 32^3x96
- a=0.114 fm
- *p_{min}=0.34 GeV*
- *tsep = (6 .. 10)a = 0.68 .. 1.14 fm*
- *boost-smear with [1,1,0]*
- *210*96=20,160 samples*

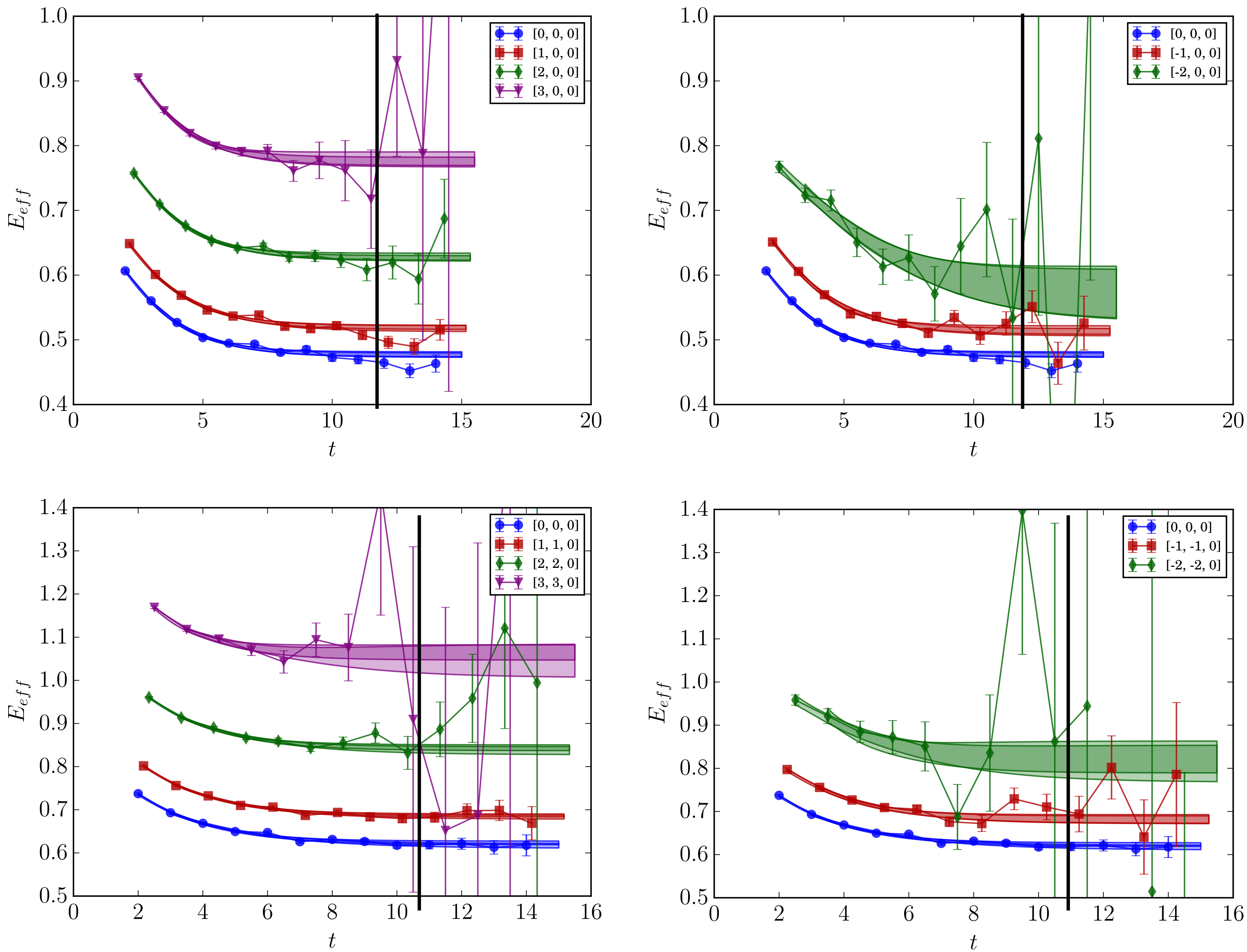
$$Q^2 \lesssim 8.3\,\mathrm{GeV}^2$$



- each quark is smeared with the same "boost" k=p/3

Effective Energy from Boost-Smeared C2pt

$p_N \upuparrows$ boost $\qquad$ $p_N \uparrow\downarrow$ boost



a=0.094 fm

kboost=[1,0,0]

Select source-sink sep (8 ..12)a =0.75 ..1.13 fm

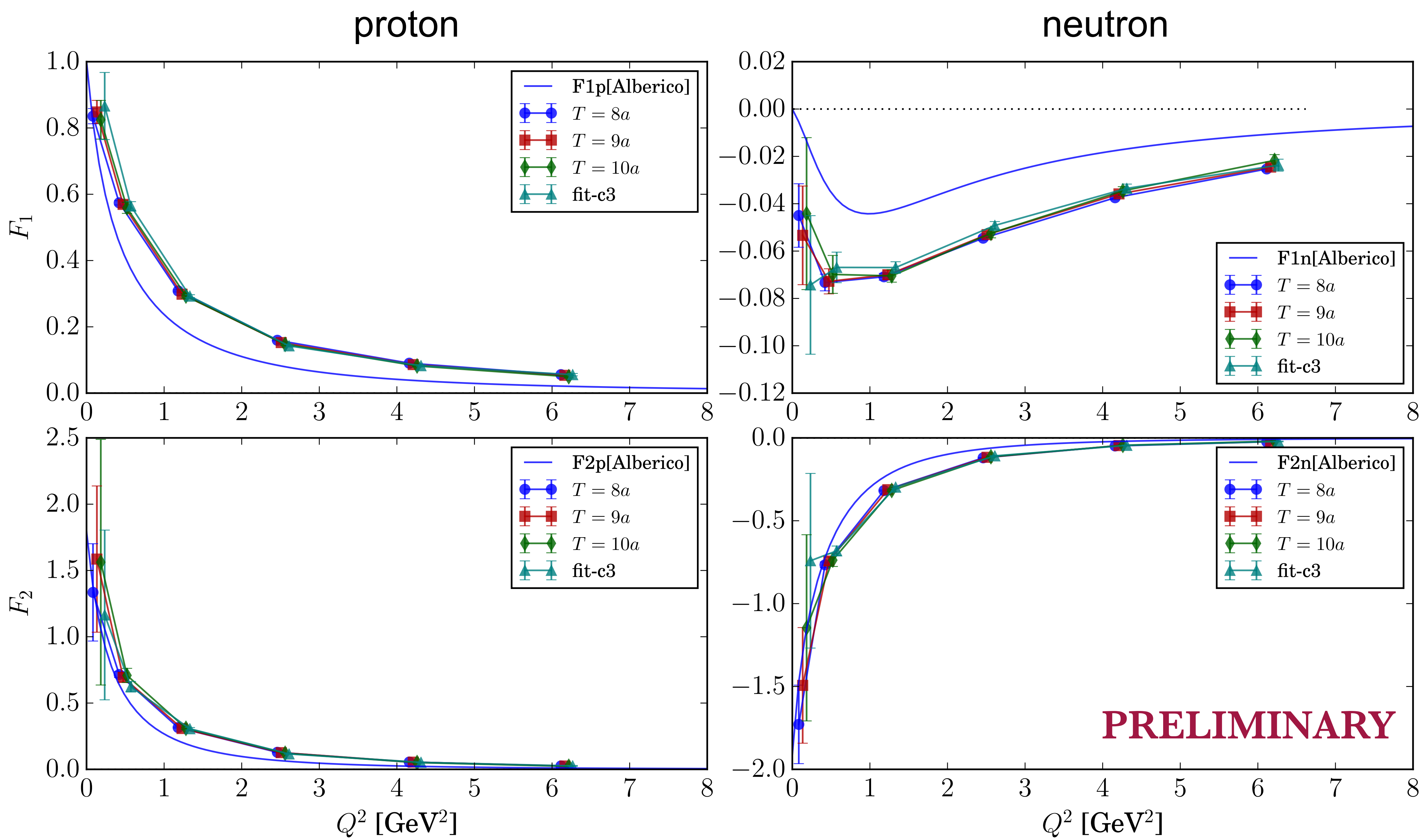
a=0.114 fm

kboost=[1,1,0]

Select source-sink sep (6 ..10)a =0.68 ..1.1 fm

Nucleon Form Factors at a=0.094 fm

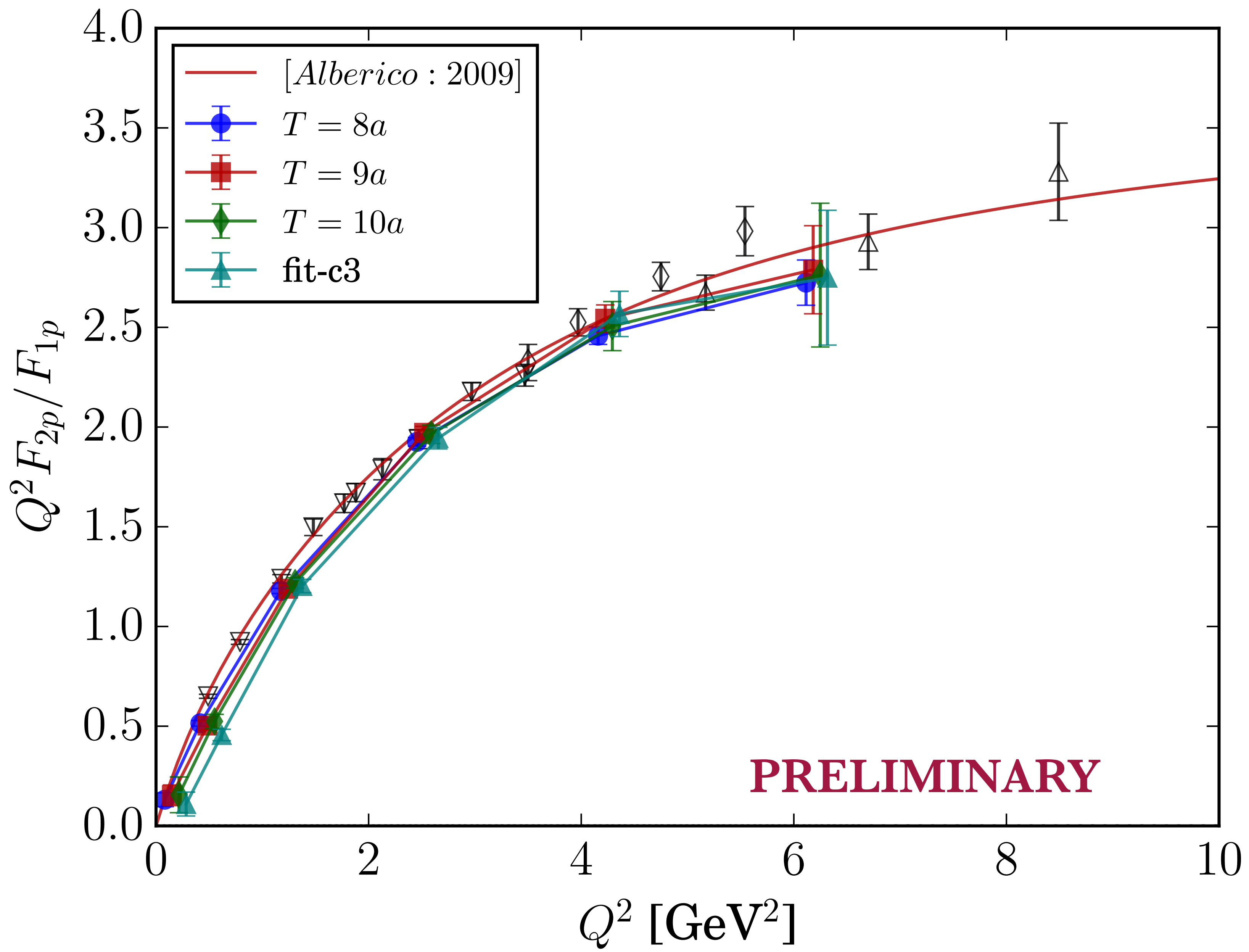
- No disconnected diagrams
- No discretization corrections

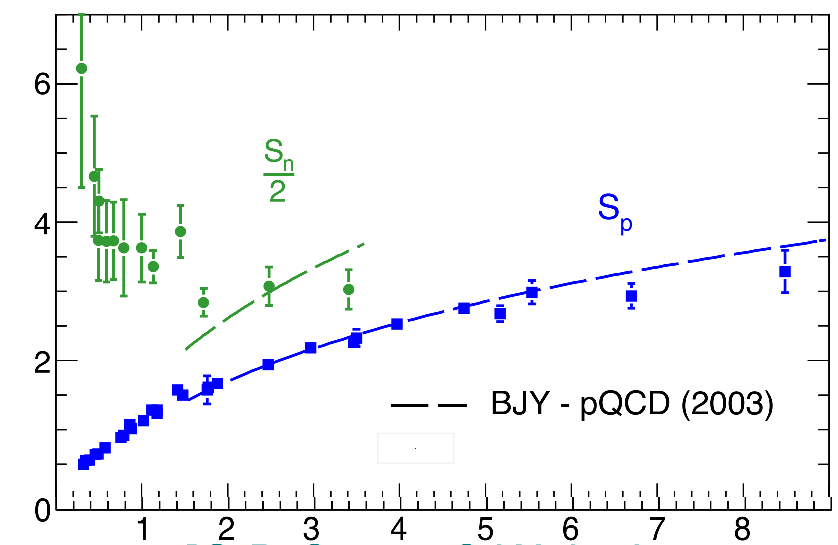


$$\langle P+q|\,\bar{q}\gamma^{\mu}q\,|P\rangle \;=\; \bar{U}_{P+q}\Big[\,F_1(Q^2)\,\gamma^{\mu} \;+\; F_2\,(Q^2)\frac{i\sigma^{\mu\nu}q_{\nu}}{2M_N}\Big]U_P$$

F_{2p}/F_{1p} Scaling

- *No disconnected diagrams*
- *No discretization corrections*



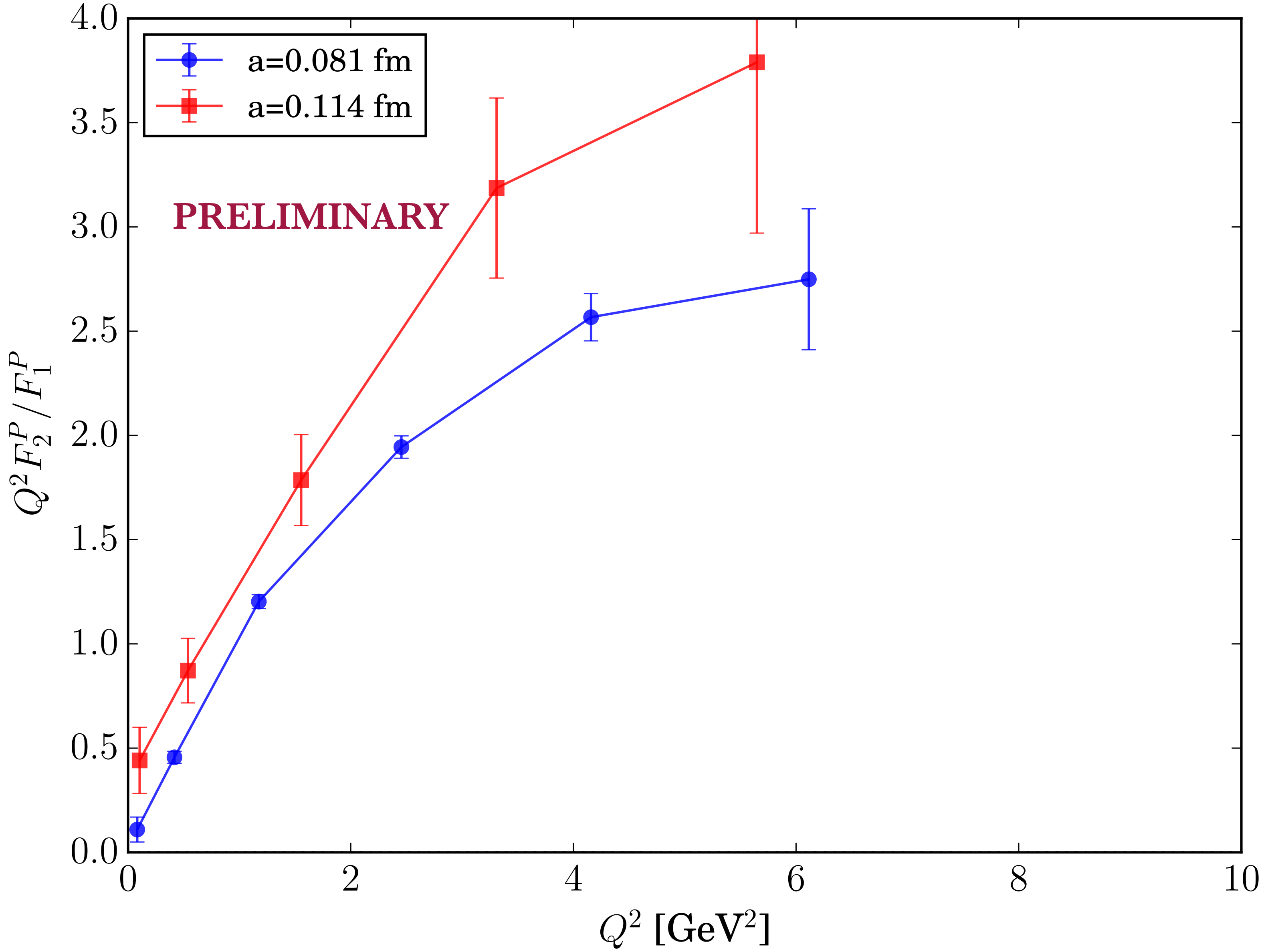


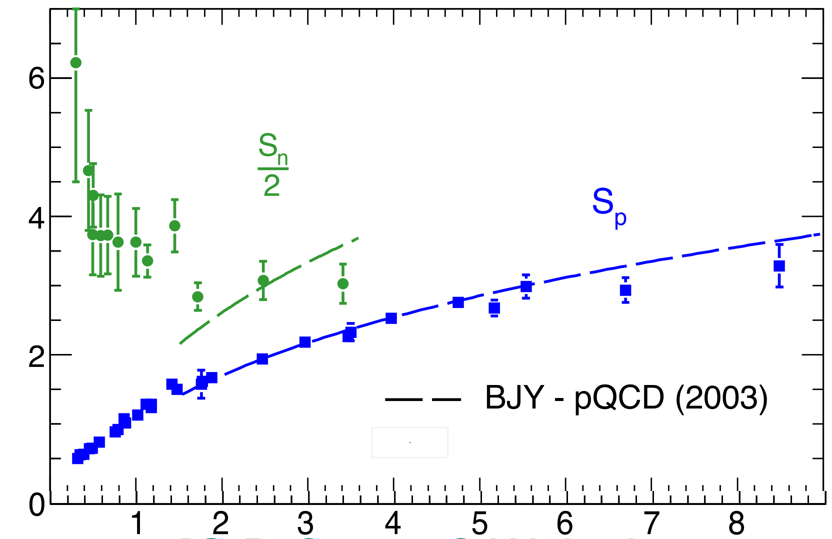
[G.D.Cates, C.W.de Jager, S.Riordan, B.Wojtsekhovski, PRL106:252003, arXiv:1103.1808]

- comparison to exp. data and pheno.parameterization [Alberico et al, PRC74:065204(2009)]
- expect $Q^2 F_1(Q^2)/F_2(Q^2) \sim \log[Q^2/\Lambda^2]$ scaling [Belitsky, Ji, Yuan (2003)]

F_{2p}/F_{1p} Scaling: a-Dependence

- *No disconnected diagrams*
- *No discretization corrections*





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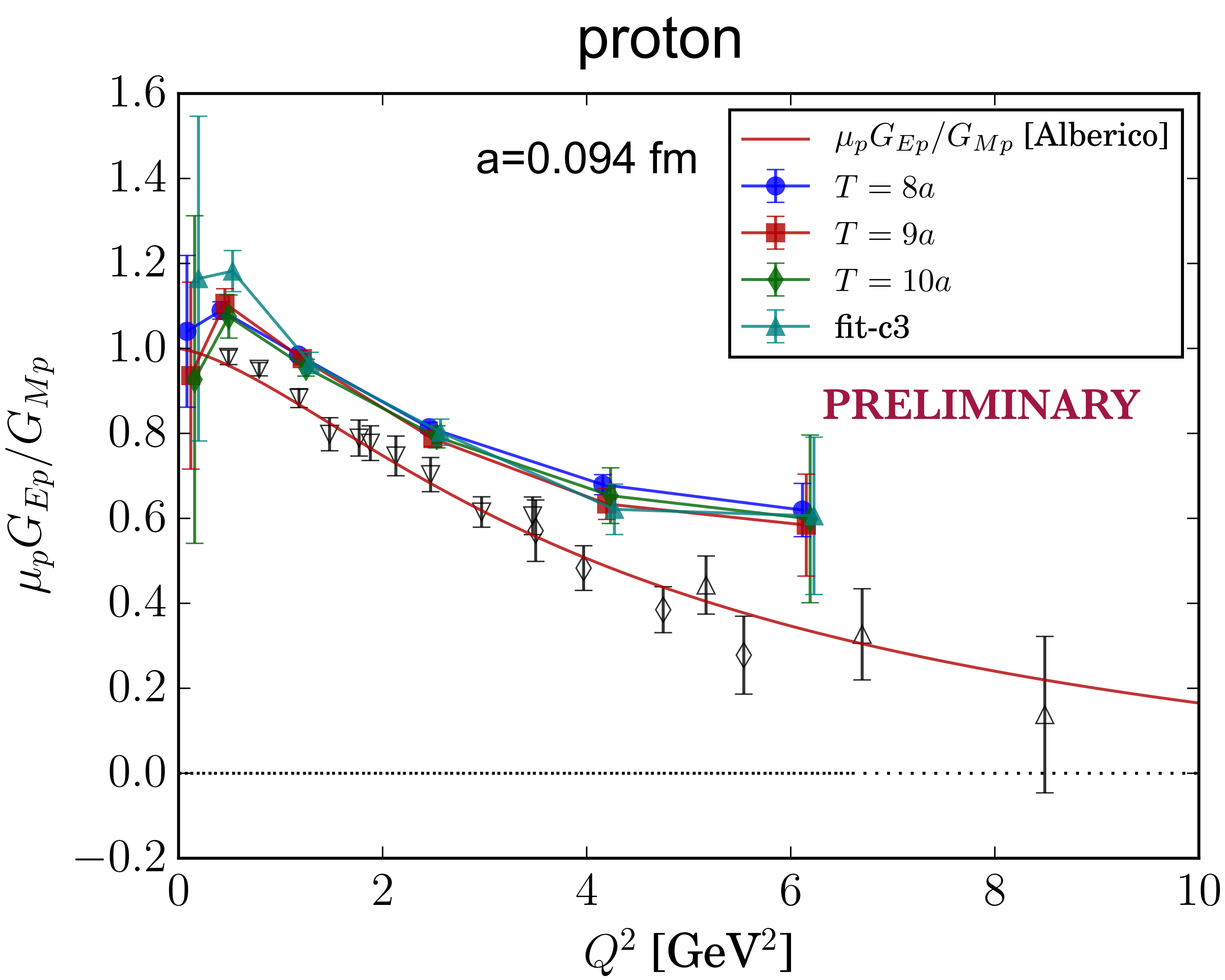
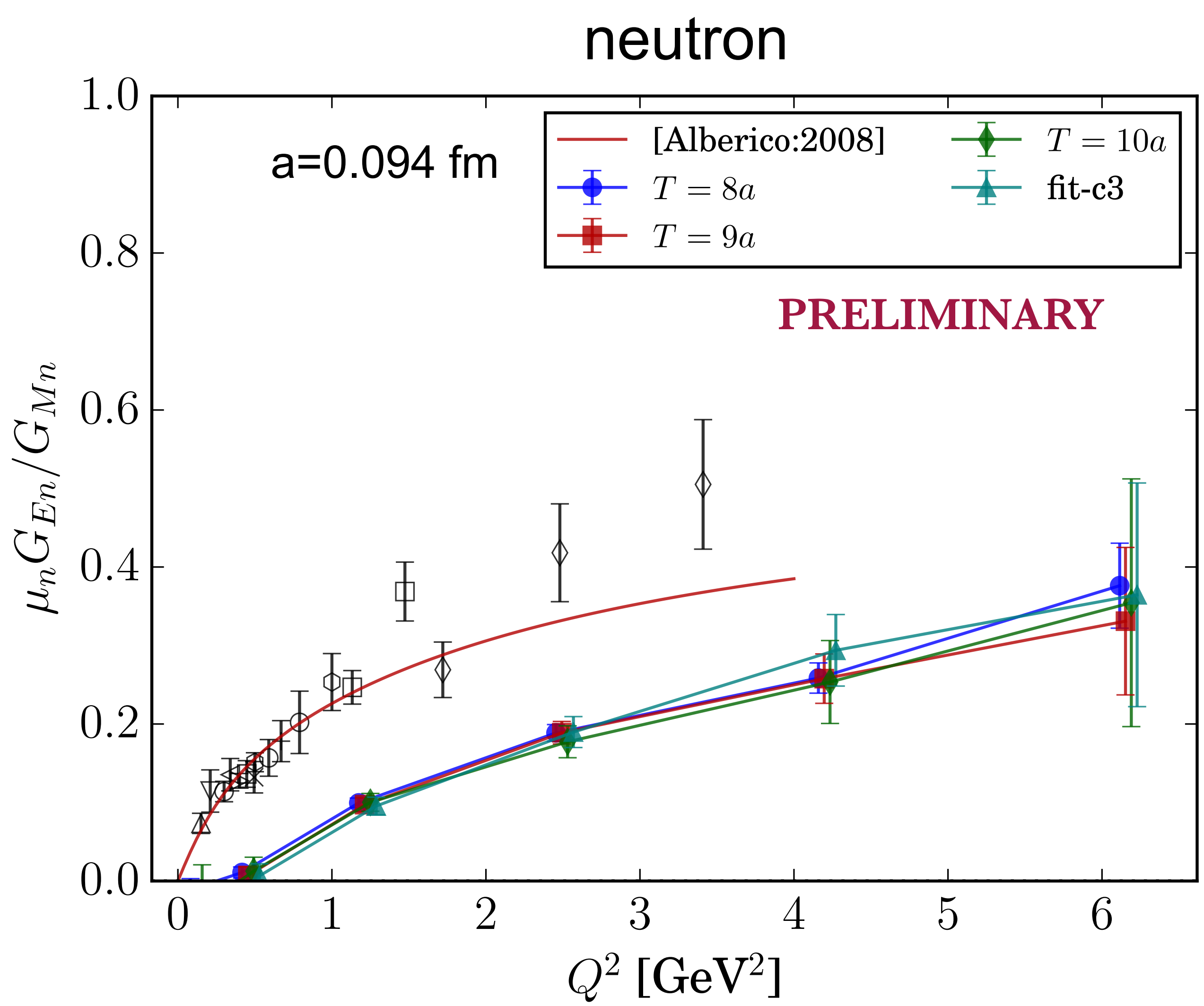
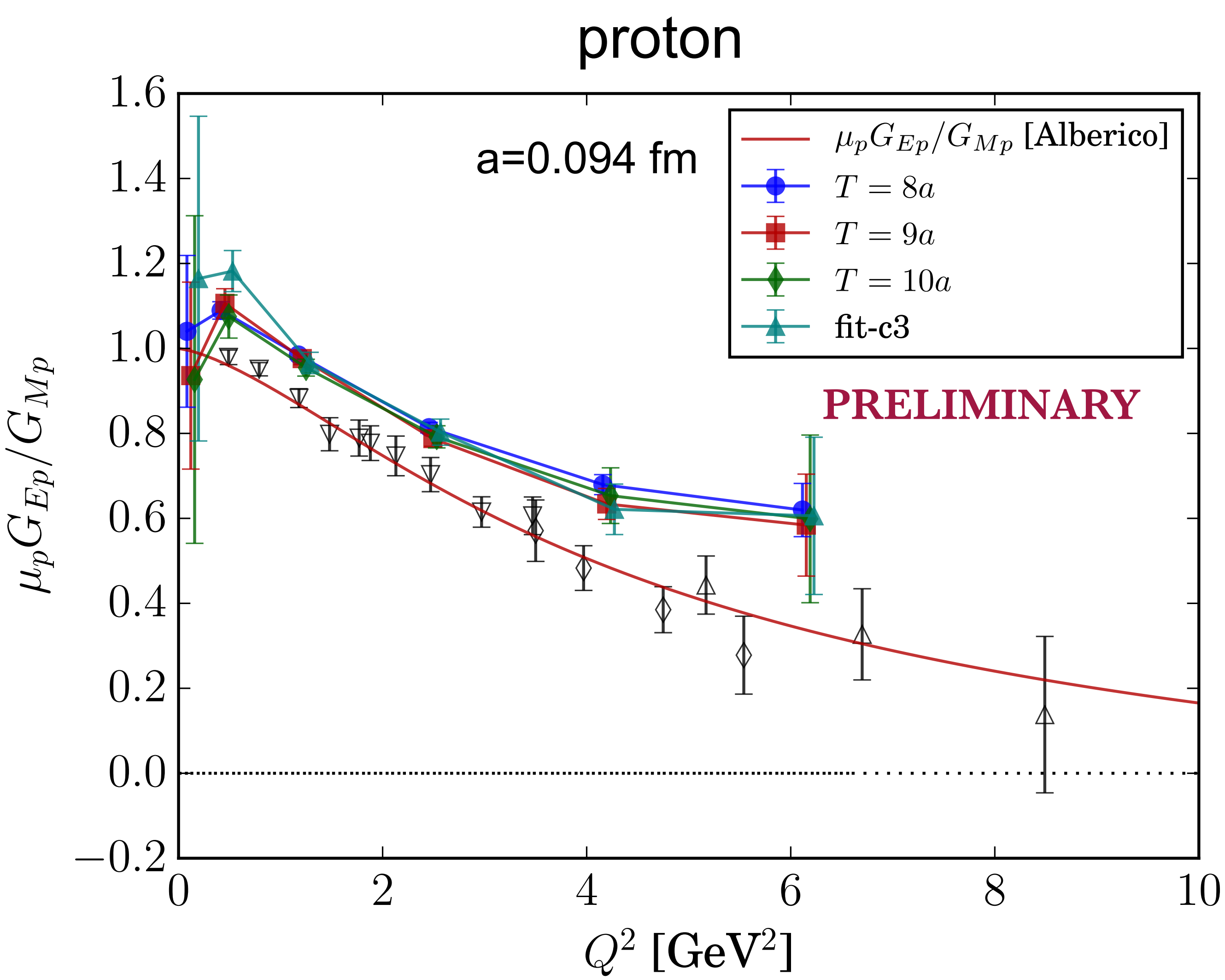


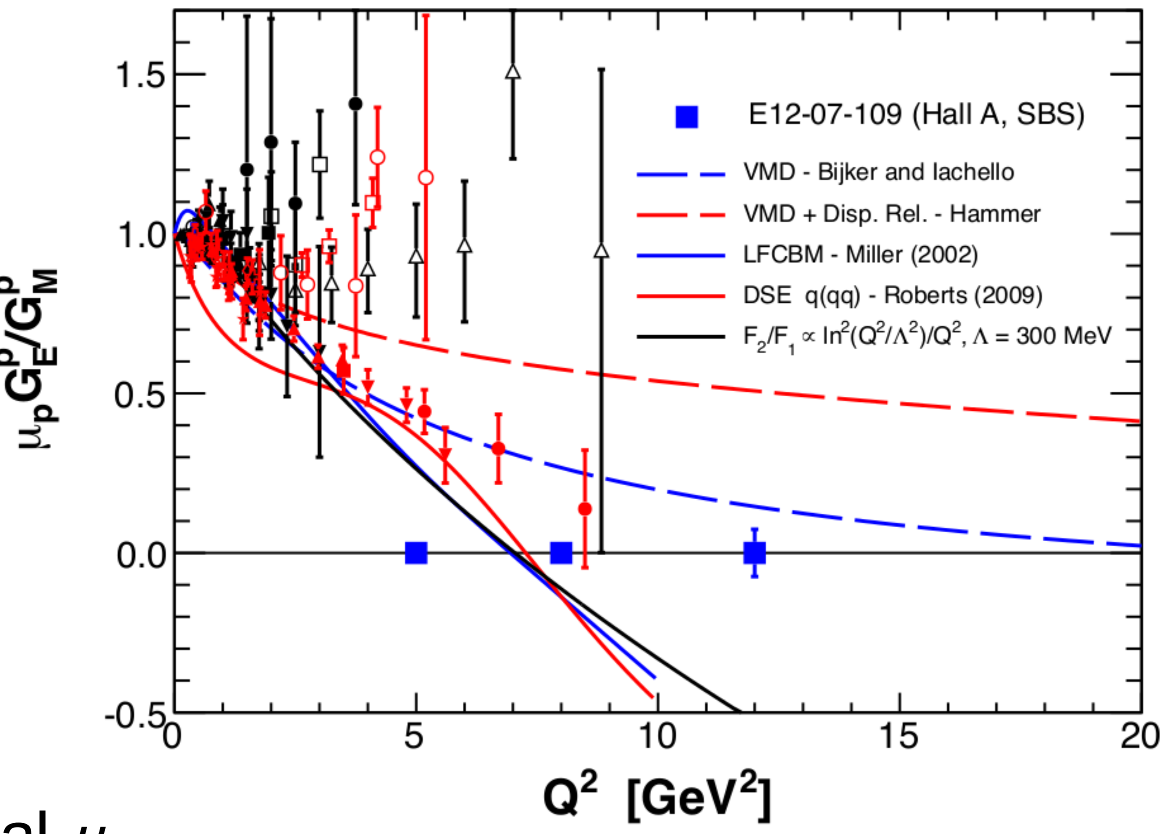


- lattice data are normalized by the physical $\mu_{p,n}$

G_{Ep}/G_{Mp} for Proton

- *No disconnected diagrams*
- *No discretization corrections*

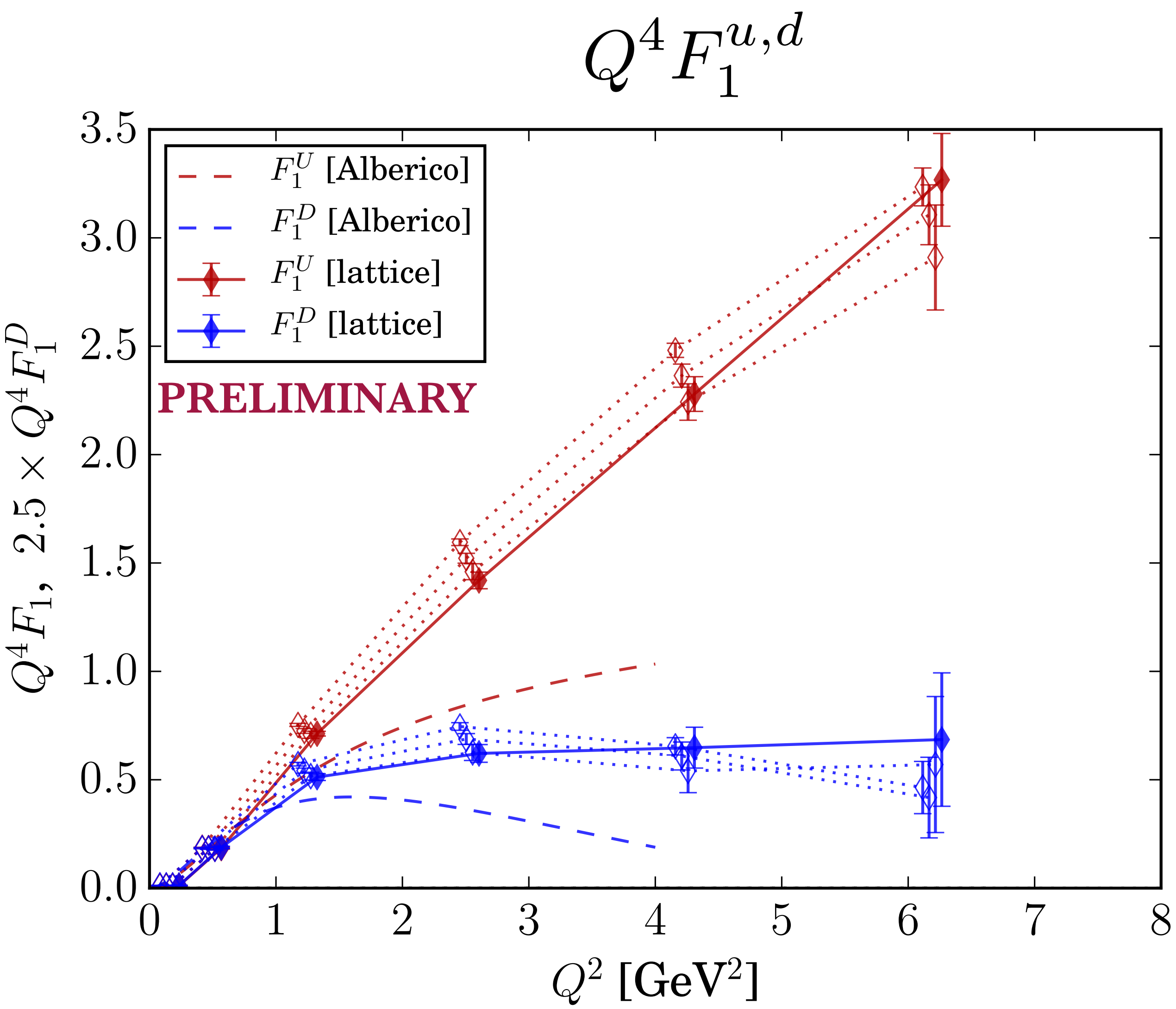


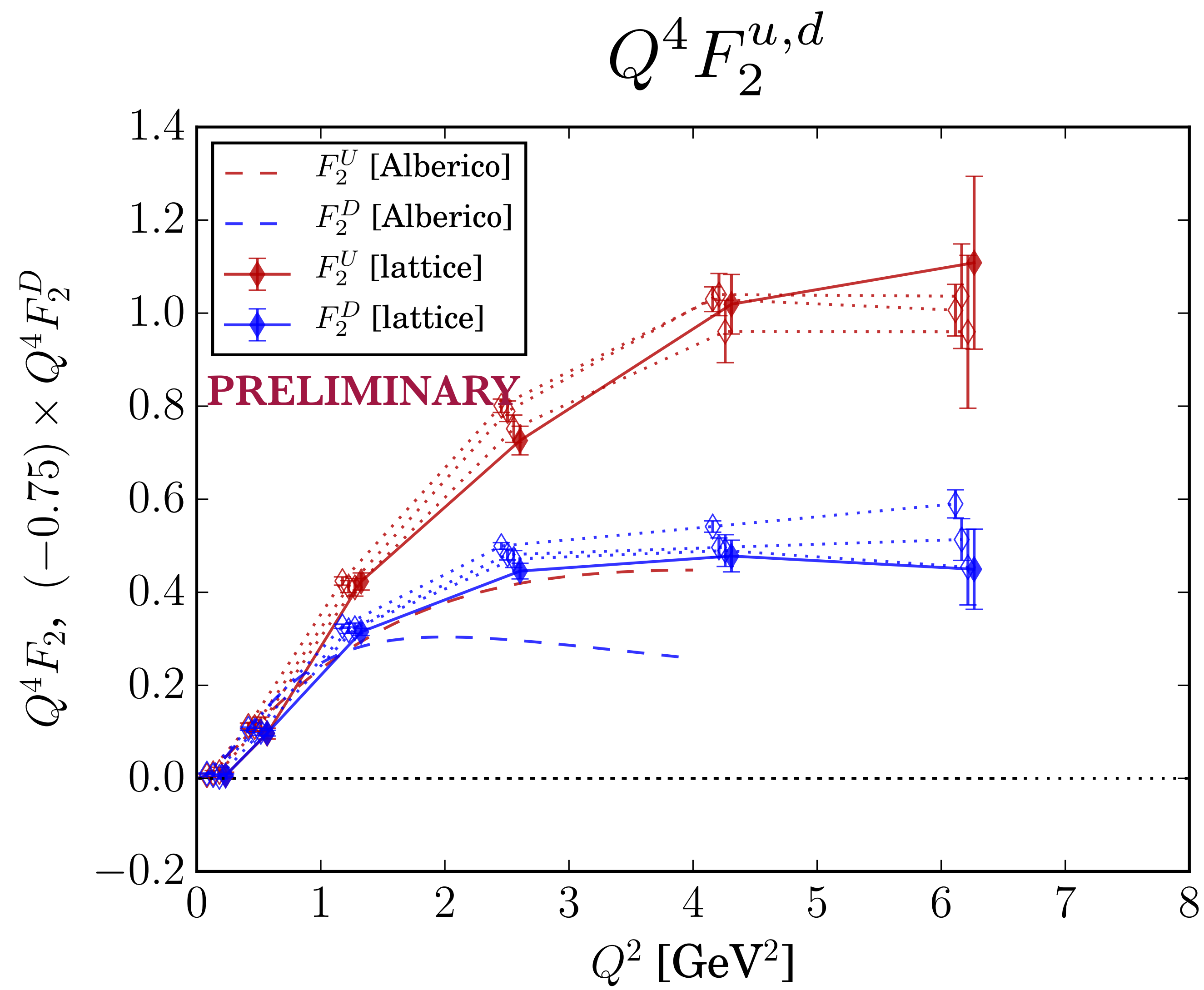


- lattice data are normalized by the physical $\mu_{p,n}$

Q^2 Dependence of F_1^u and F_1^d

- No disconnected diagrams
- No discretization corrections

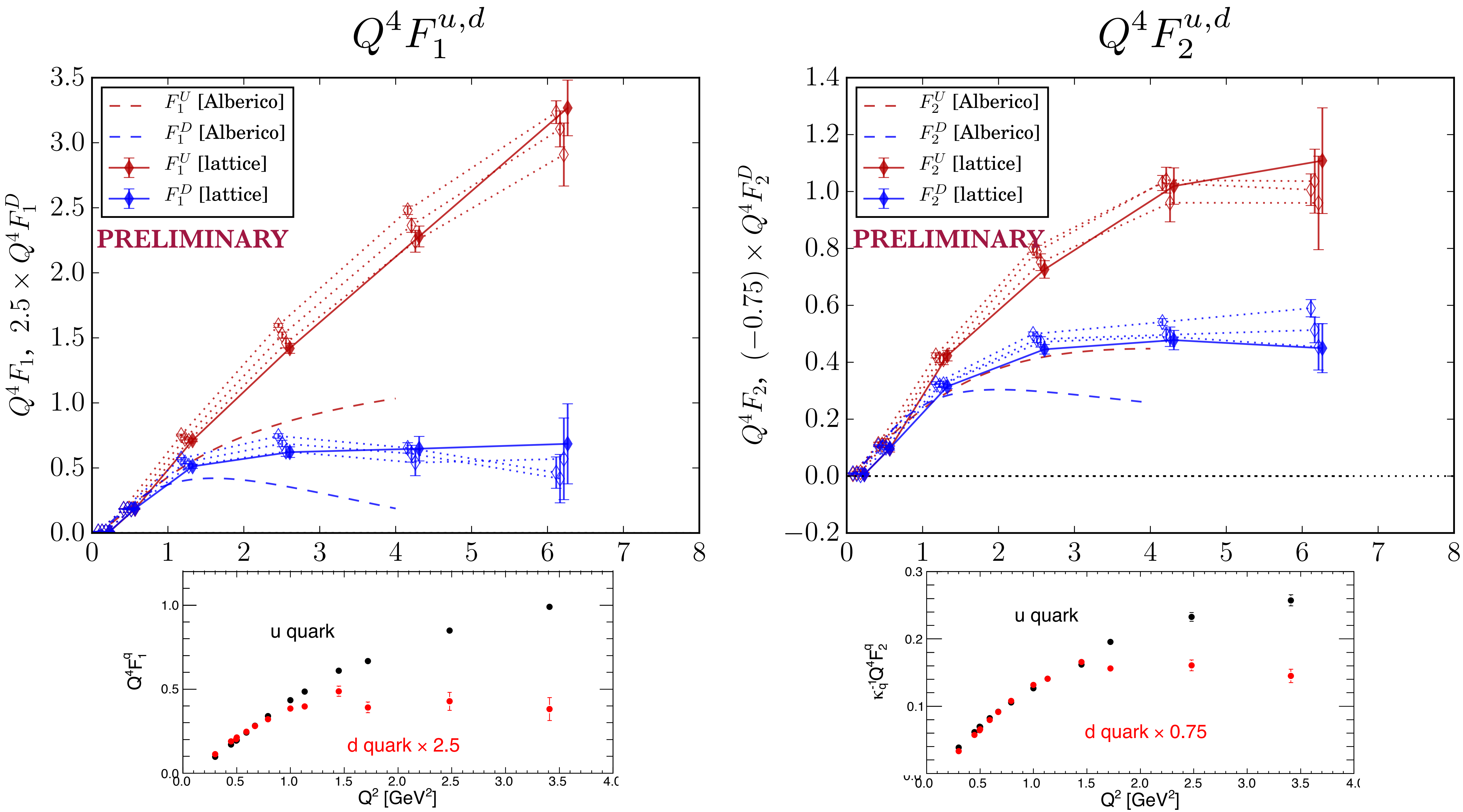




- expect $F_1(Q^2) \sim Q^4$, $F_2(Q^2) \sim Q^6$ scaling [Lepage, Brodsky (1979)]
- Both form factors overshoot experiment (x2-2.5)
- evidence for excited states

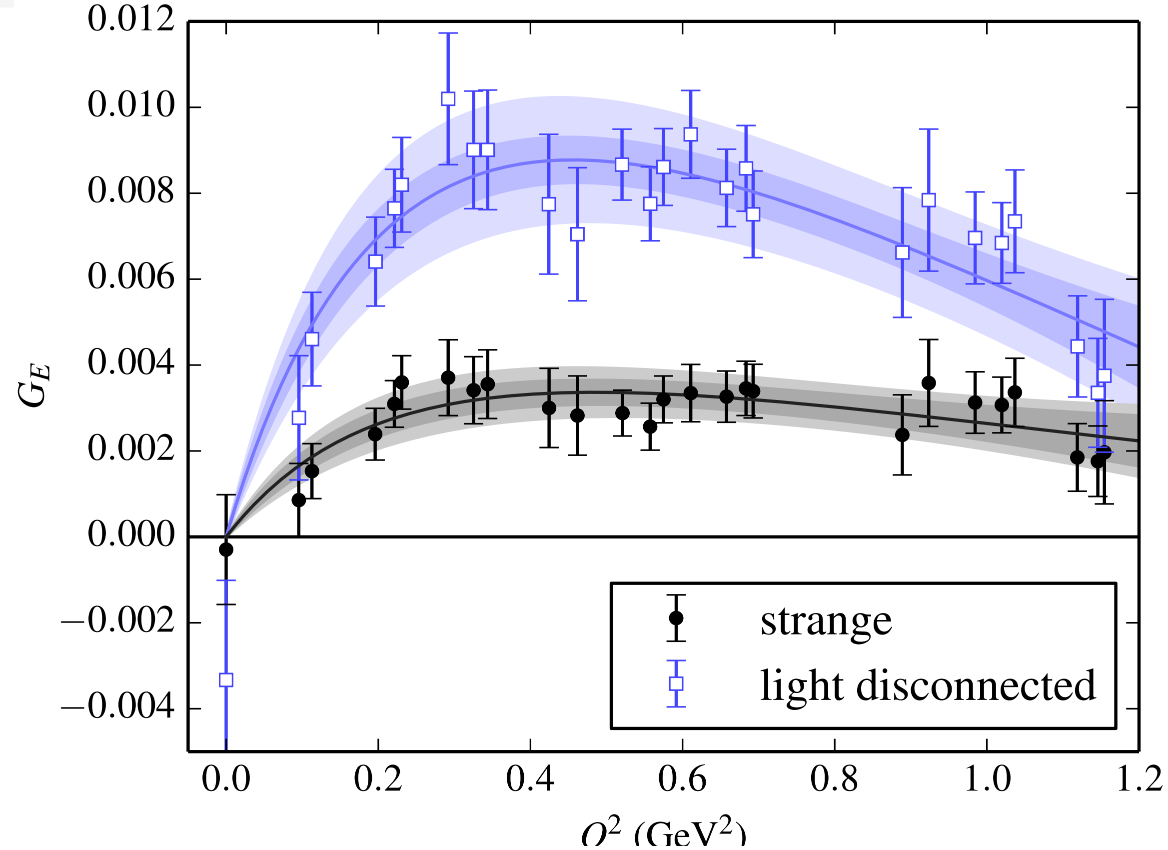
Light Flavor contributions to $F_{1,2}$

- No disconnected diagrams
- No discretization corrections



- Reproduce features of flavor dependence [G.D.Cates, et al, PRL106:252003(2011)]
- Larger form factors: nucleon (+ exc.states?) on a lattice is more "compact"
- Discretization effects?

Disconnected Nucleon FF's for up to ~1 GeV^2

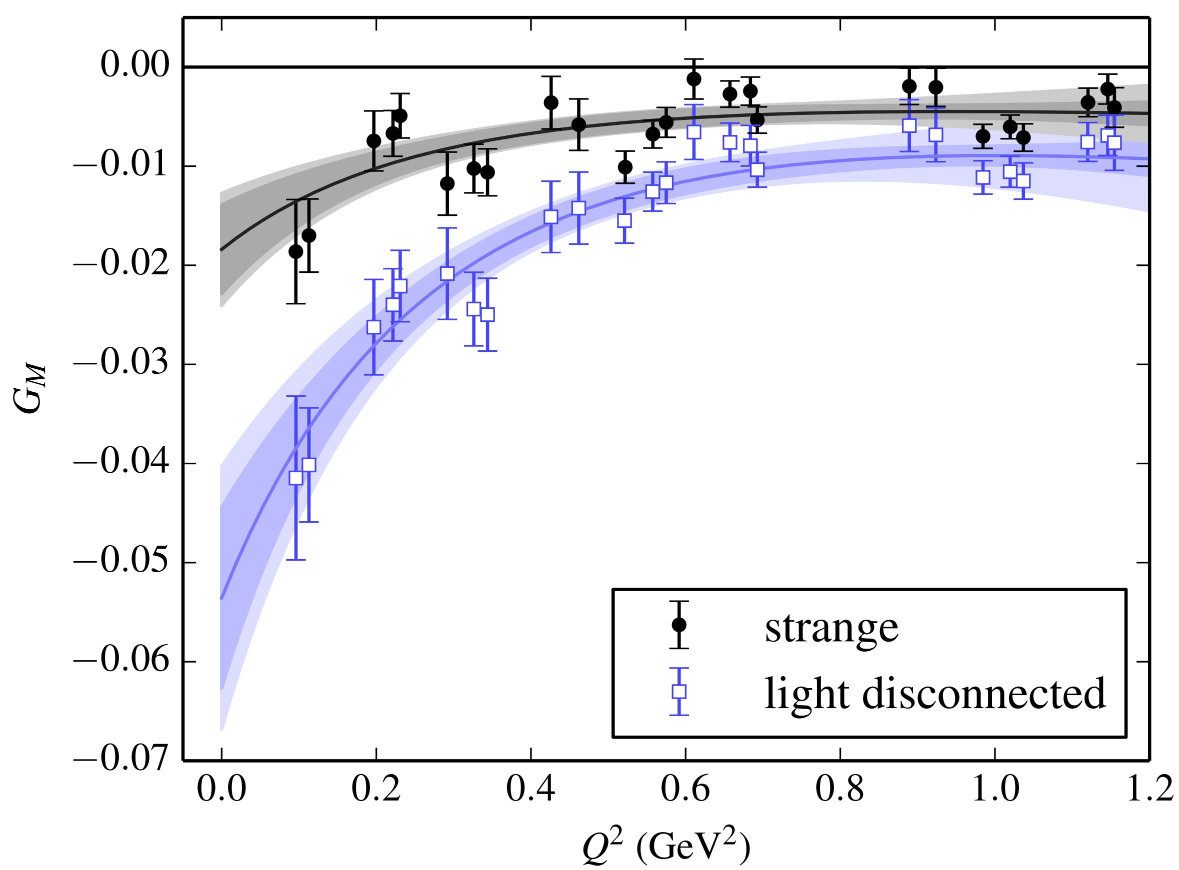


[J. Green, S. Meinel, et al; PRD92:031501]

N_f=2+1 dynamical fermions, $m_\pi \approx$ 320 MeV
(the "coarse" JLab Clover ensemble)

$$|(G_E^{u/d})_{\rm disc}| \lesssim 0.010 \text{ of } |(G_E^{u-d})_{\rm conn}|$$

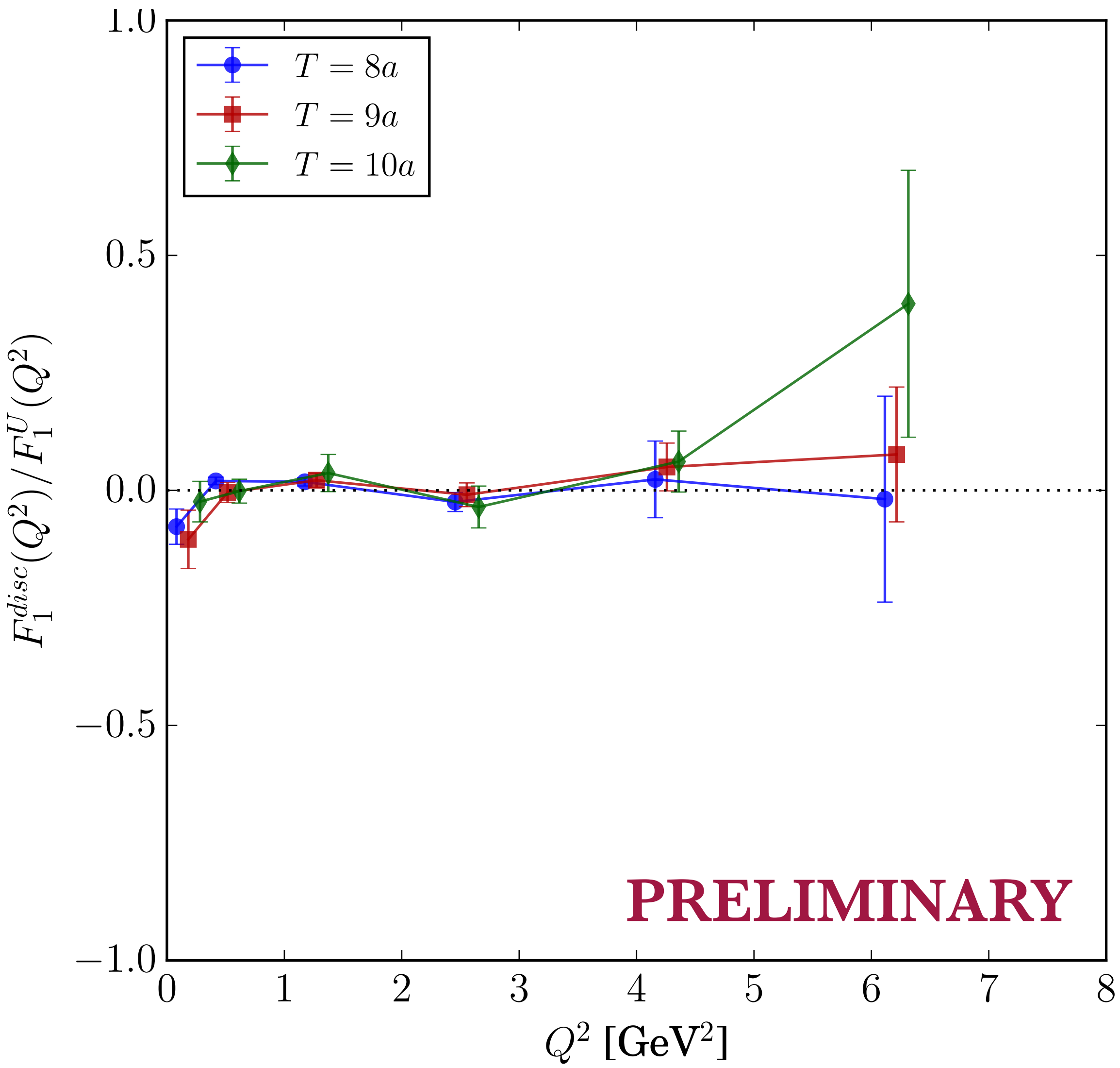
$$|(G_E^{s})_{\rm disc}| \lesssim 0.005 \text{ of } |(G_E^{u-d})_{\rm conn}|$$

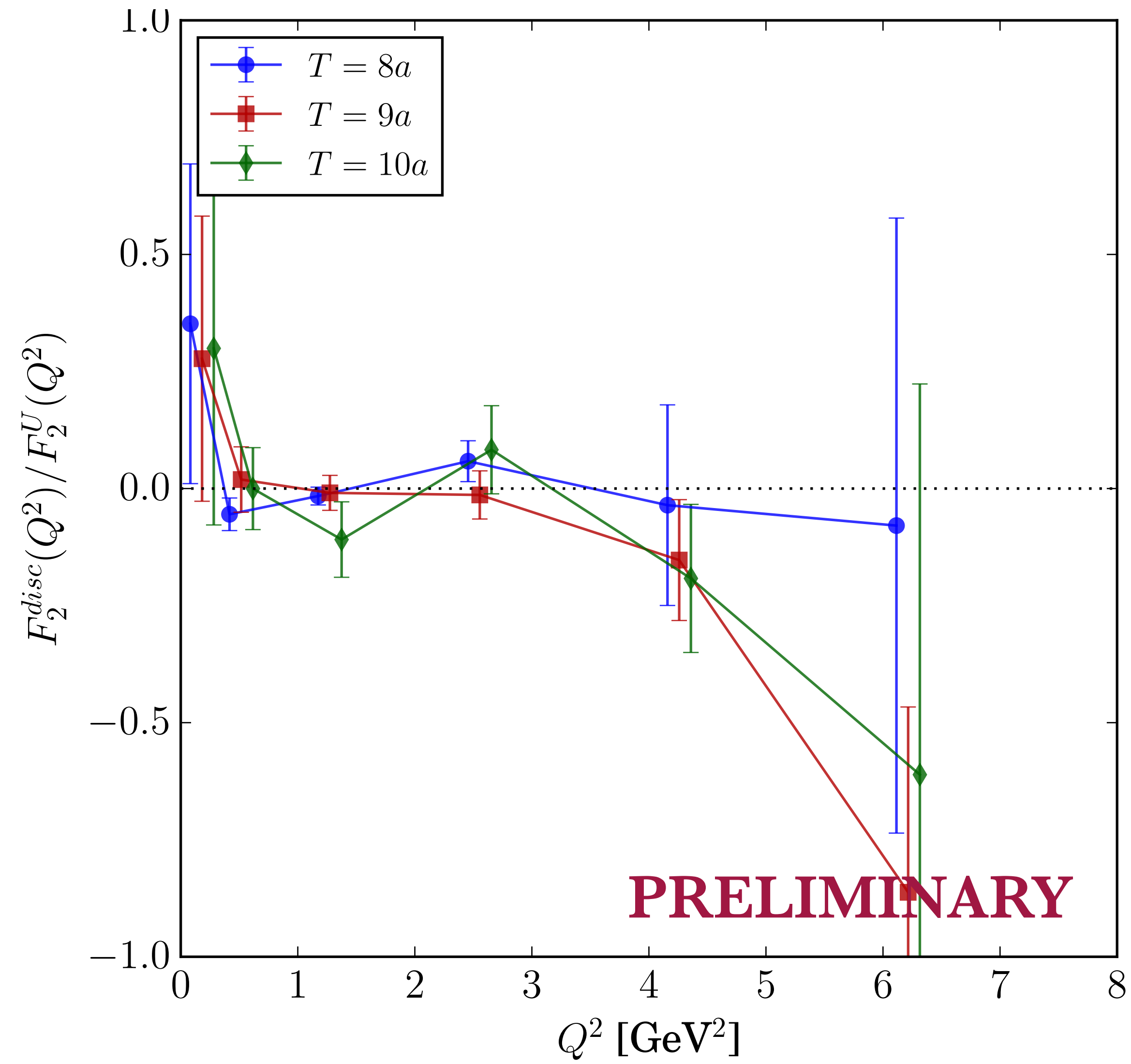


$$|(G_M^{u/d})_{\rm disc}| \lesssim 0.015 \text{ of } |(G_M^{u-d})_{\rm conn}|$$

$$|(G_M^{s})_{\rm disc}| \lesssim 0.005 \text{ of } |(G_M^{u-d})_{\rm conn}|$$

Disconnected Nucleon FF's: Relative Contribution





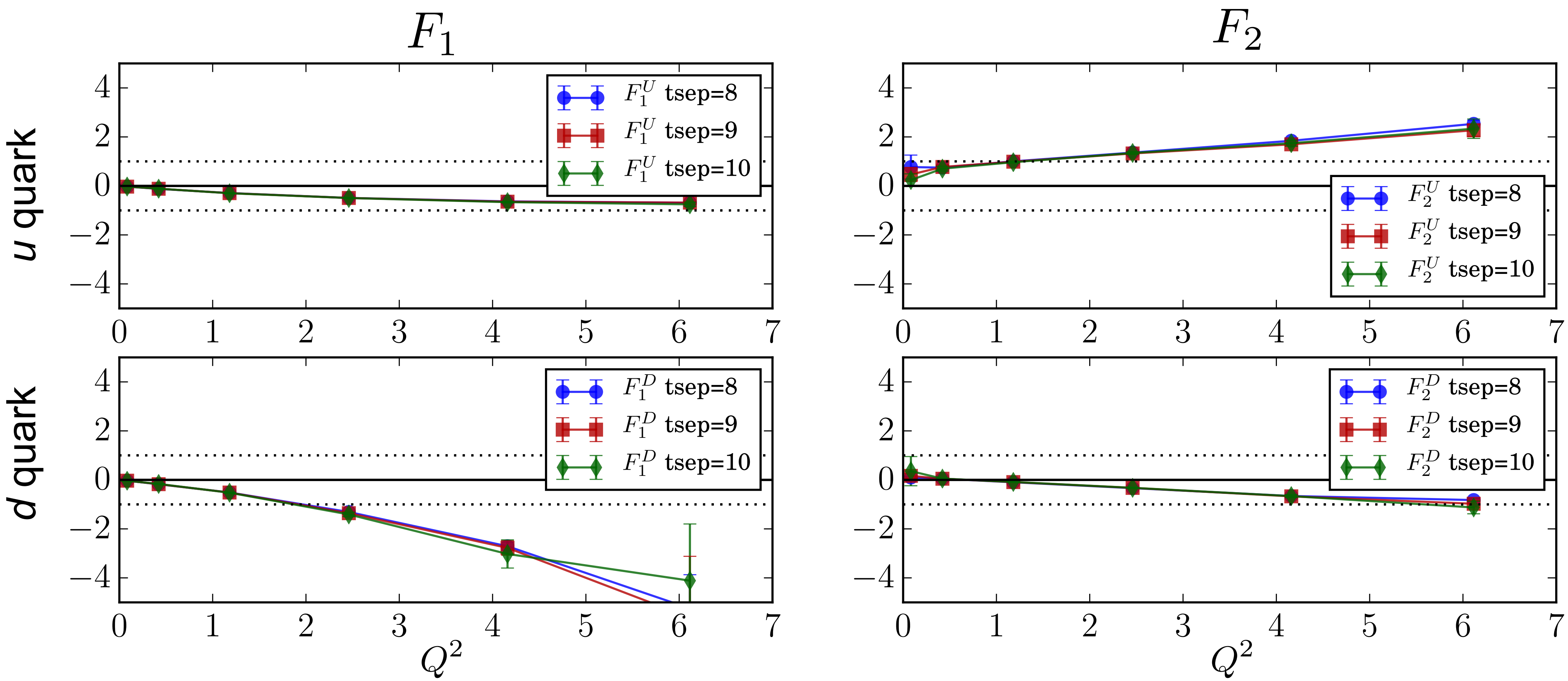
- a=0.094 fm ensemble
- Ratio of disconnected to connected(U) contributions
- Simplified preliminary analysis (plateau averages)

O(a) Vector Current Improvement

- *No disconnected diagrams*

Improved vector current $(V_\mu)_I = \bar{q}\gamma_\mu q + c_V\, a\partial_\nu \bar{q} i \sigma_{\mu\nu} q$

O(a¹) correction : form factors of $a\,\langle N|\partial_\nu(\bar{q} i \sigma^{\mu\nu} q)|N\rangle$



Relative magnitude of *O(a¹)* effects : {*O(a¹)* } / {*O(a⁰)* } form factors

- need improvement coefficient c_V : can be computed from current conservation

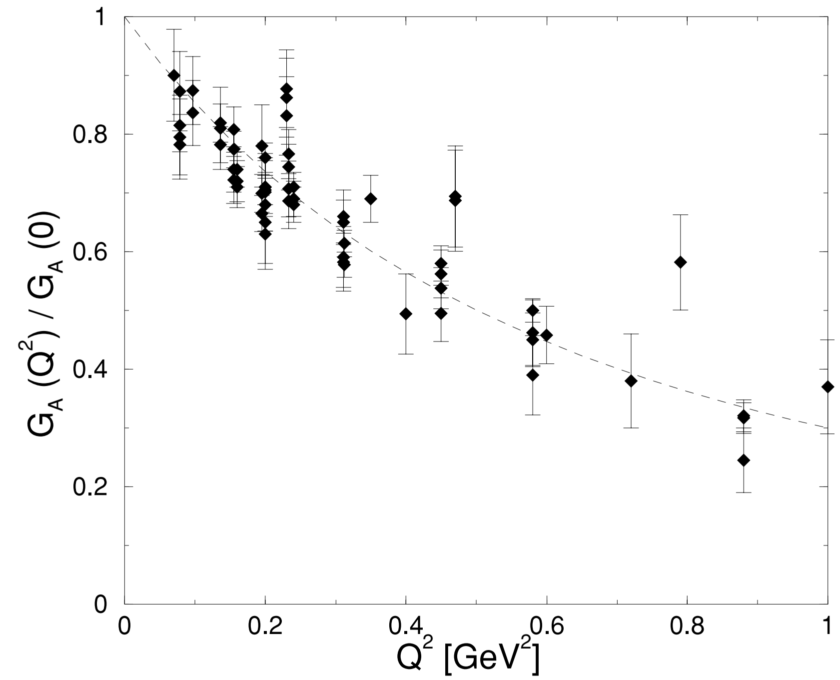
Axial Form Factors

- *No disconnected diagrams*
- *No discretization corrections*

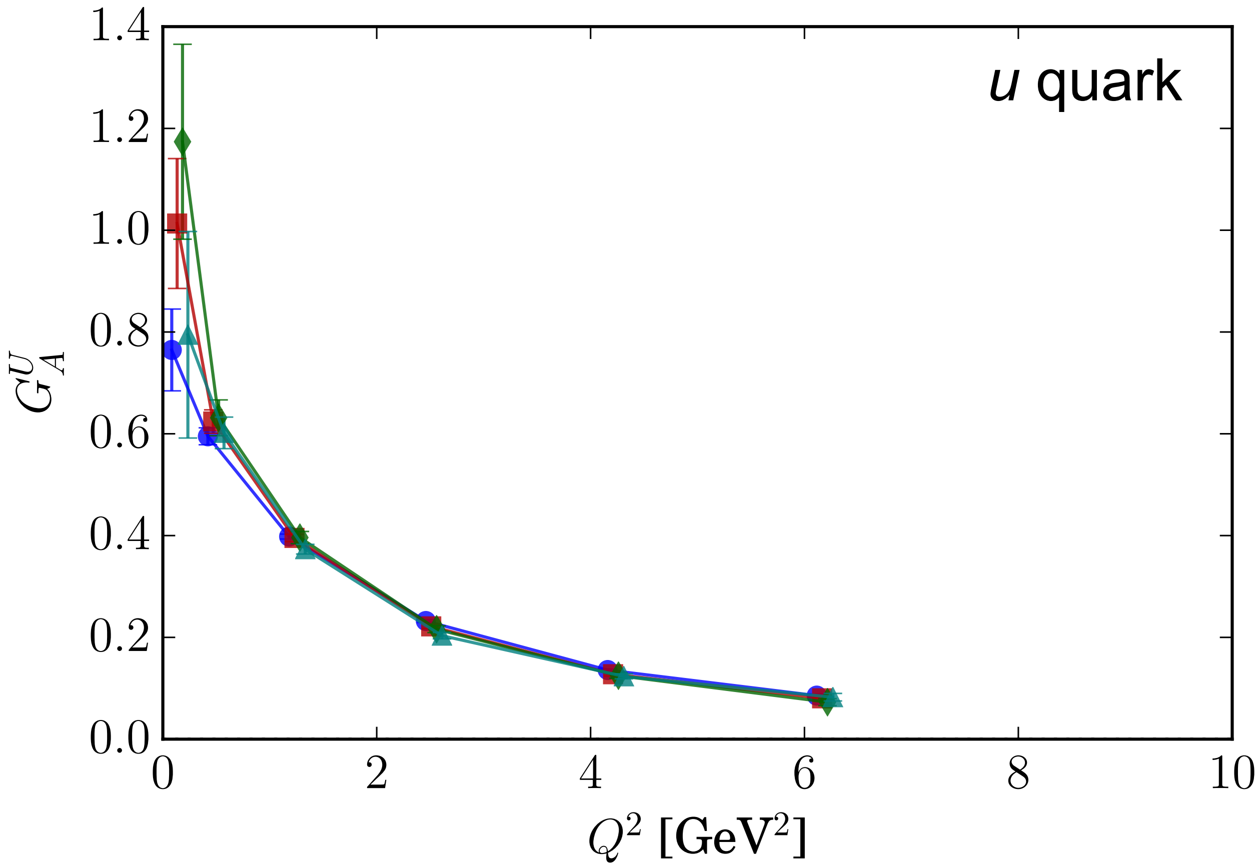
- *$G_A(Q^2)$* are measured in *ν*-scattering, ***π***-production;

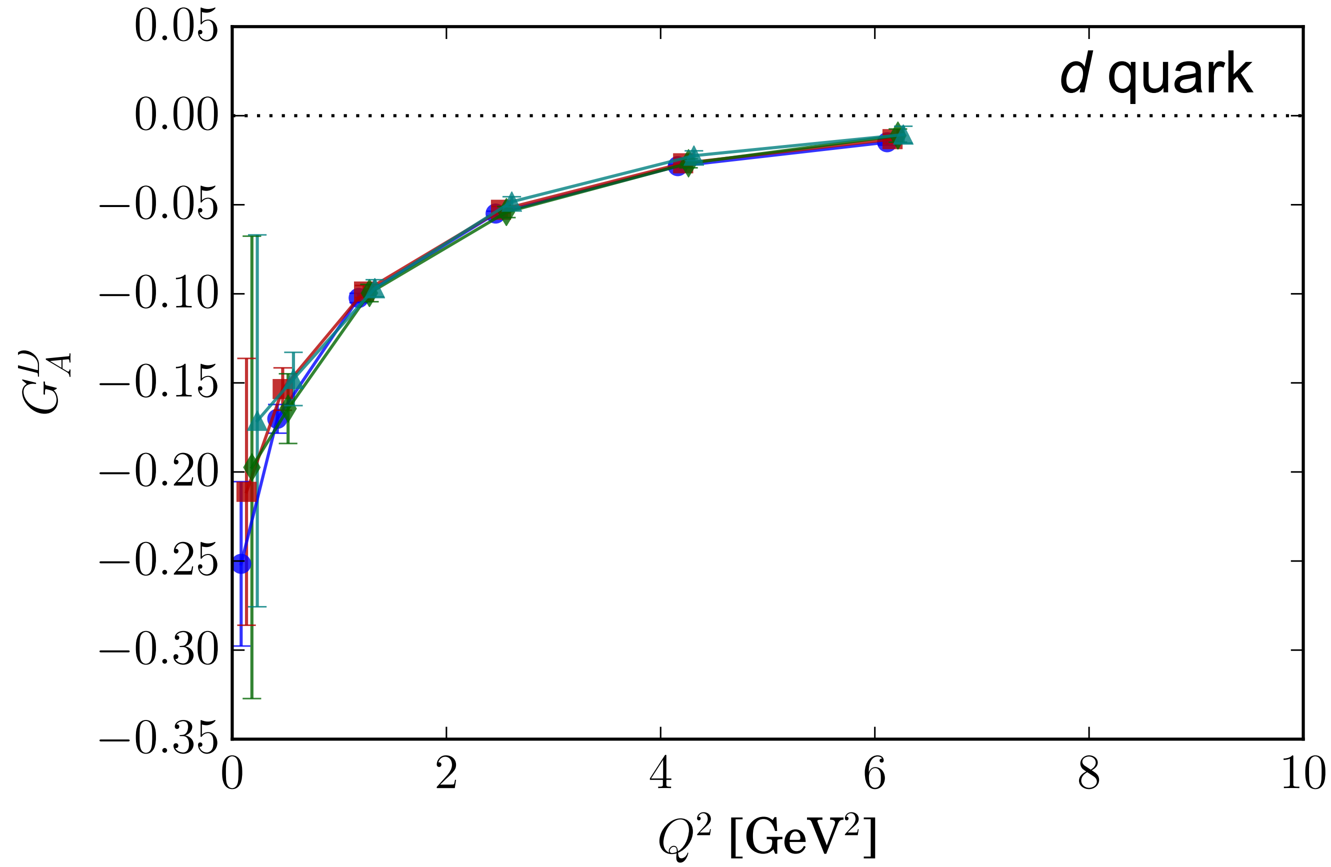
  implications for neutrino flux norm. (e.g. in IceCube)
- Axial radius *(r_A^2)*=12 / *m_A^2*: model dependence

  varying nuclear / G_A shape models: m_A=0.9 ... 1.4 GeV
- Reanalysis suggests large uncertainty in *$G_A(Q^2)$* [B.Bhattacharya,R.Hill,G.Paz, PRD84:073006(2011)]



[V.Bernard et at, J.Phys.G28:R1(2002)]





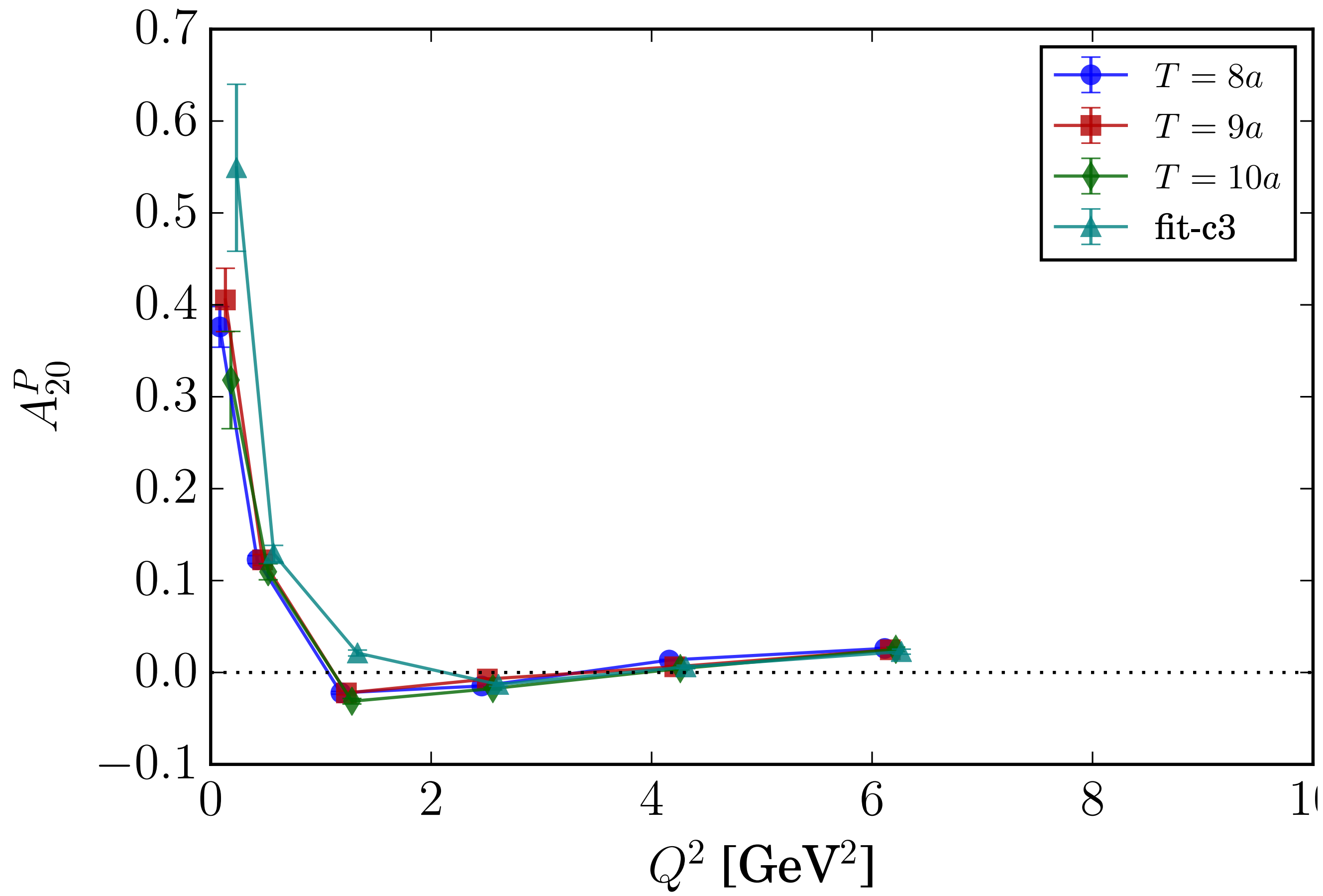
n=2 Generalized Form Factors

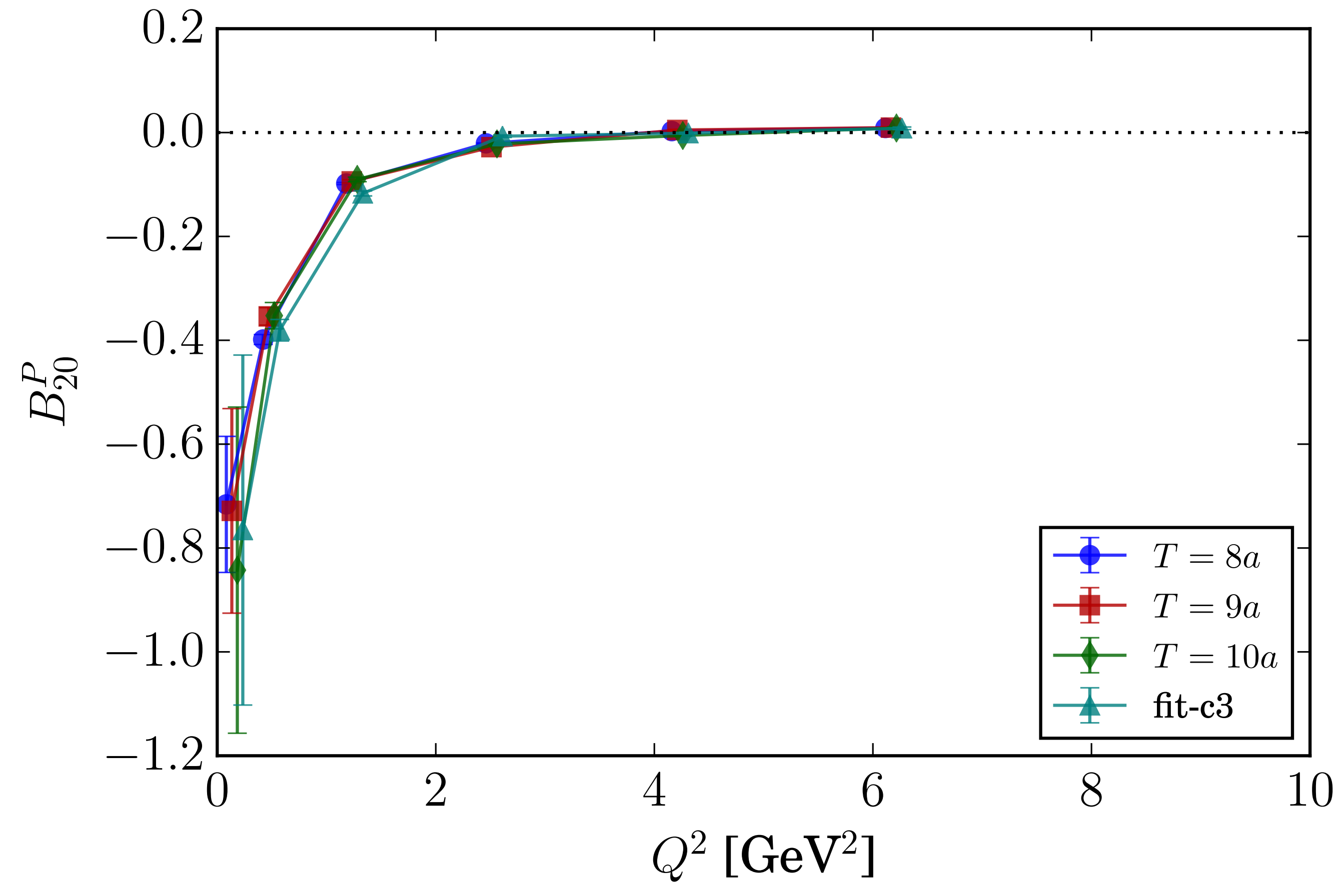
- *No disconnected diagrams*
- *No discretization corrections*

Generalized form factors: moments of GPDs

$$A_{20}(Q^2) = \int dx\, x\, H^q(x, \xi = 0, t = -Q^2)$$

$$B_{20}(Q^2) = \int dx\, x\, E^q(x, \xi = 0, t = -Q^2)$$





- Goal: constraints on GPD analysis from lattice

Summary and Outlook

- Initial results for high-momentum form factors with a new technique
  "momentum(boosted)" smearing : essential for studying relativistic hadrons on a lattice

- G_{Ep}/G_{Mp} agrees qualitatively with experiment;
  F_2/F_1 scaling agrees qualitatively with experiment, perturbative QCD
  agreement is (apparently?) independent of excited states

- Discretization effects grow quickly with Q^2
  Form factors on a=0.1 fm lattice: ~O(1) at Q^2=6 GeV^2
  Non-perturbative vector current improvement needed

- The new TMD and PDF programmes on a lattice (Lin, Engelhardt) depend on efficient and reliable evaluation relativistic nucleon matrix elements
  computing form factors is a "benchmark" for studying discretization and excited state effects for relativistic nucleons on a lattice

- Implications for neutrino physics (axial current) and constraining GPD

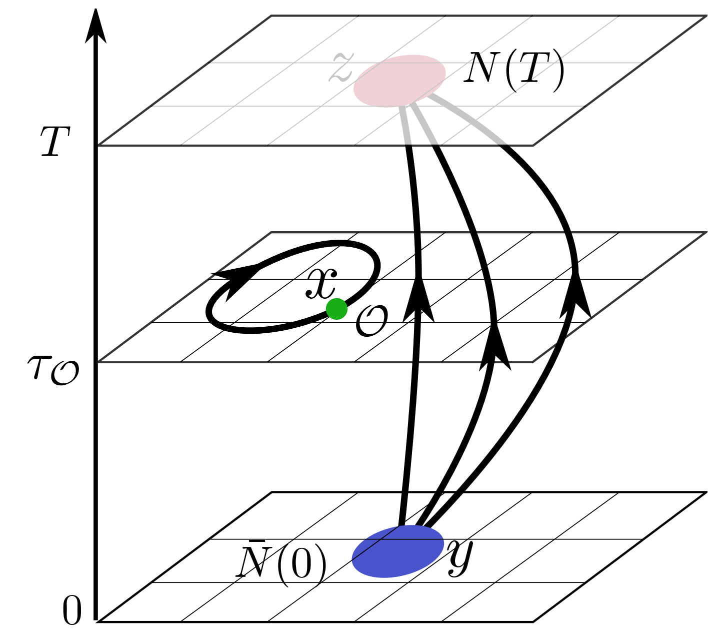
(BACKUP) Disconnected Quark Loops

- Stochastic evaluation: $\begin{cases} \xi(x) = \text{random } Z_2\text{-vector} \\ E\left[\xi^\dagger(x)\xi(y)\right] = \delta_{x,y} \end{cases}$

$$\sum_x e^{iqx} \not{D}^{-1}(x,x) \approx \frac{1}{N_{MC}} \sum_i^{N_{MC}} \xi_{(i)}^\dagger \left(e^{iqx} \not{D}^{-1} \xi_{(i)}\right)$$

$$\mathrm{Var}\left(\sum_x \not{D}^{-1}(x,x)\right) \sim \frac{1}{N_{MC}}$$

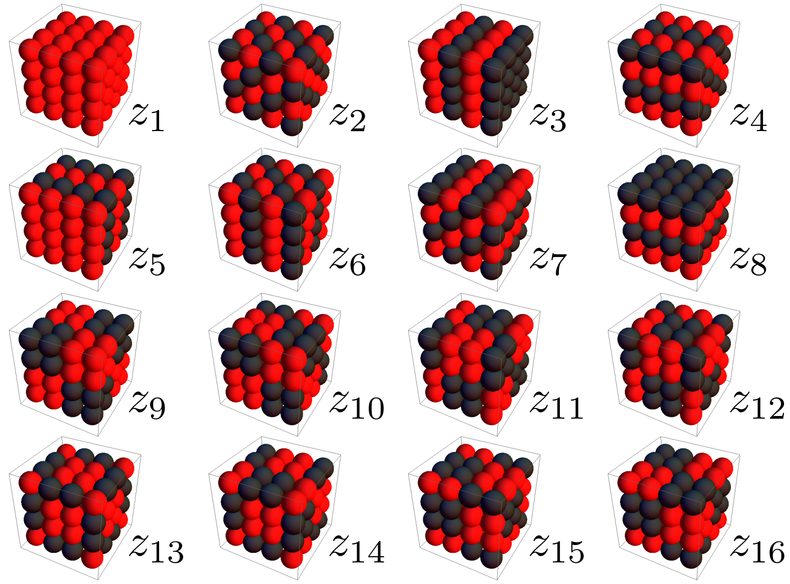
(contributions from $\not{D}^{-1}(x \neq y)$)



- Exploit $\not{D}^{-1}(x,y)$ FALLOFF to reduce $\sum_{x \neq y} |\not{D}^{-1}(x,y)|^2$:

Hierarchical probing method [K.Orginos, A.Stathopoulos, '13] :
In sum over $N=2^{nd+1}$ 3D(4D) Hadamard vectors,
near-(x,y) terms cancel:

$$\frac{1}{N} \sum_i z_i(x) z_i(y)^\dagger = \begin{cases} 0, & 1 \leq |x-y| \leq 2^k, \\ 1, & x = y \text{ or } 2^k < |x-y| \end{cases}$$



- Further decrease variance by deflating low-lying, long-range modes [A.Gambhir's PhD thesis]