Flavour Structure of the sea from lattice QCD

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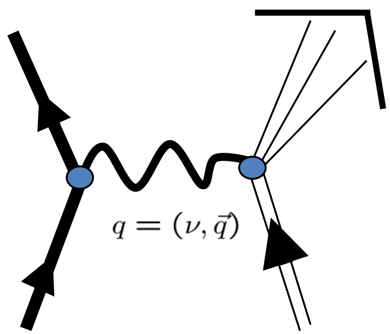
Outline

- Introduction
- Moments of the distributions
- Quark distributions and quark quasi-distributions
- Extracting quark distributions from the quasi-distributions
- Lattice results for unrenormalized distributions
- Nonperturbative Renormalization I: RI-MOM scheme
- Nonperturbative Renormalization II: The auxiliary field Approach
- Summary and outline

Introduction



Cross sections

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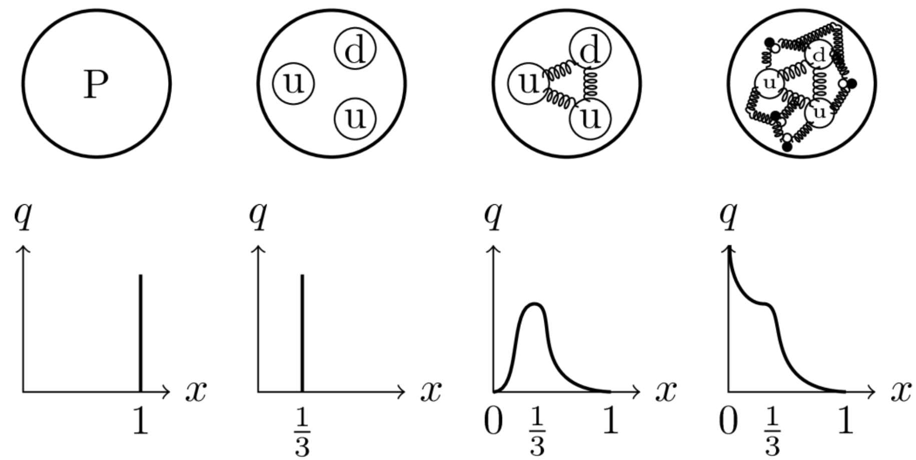
Structure Functions

↓

Quark and Gluon Distributions



Elastic

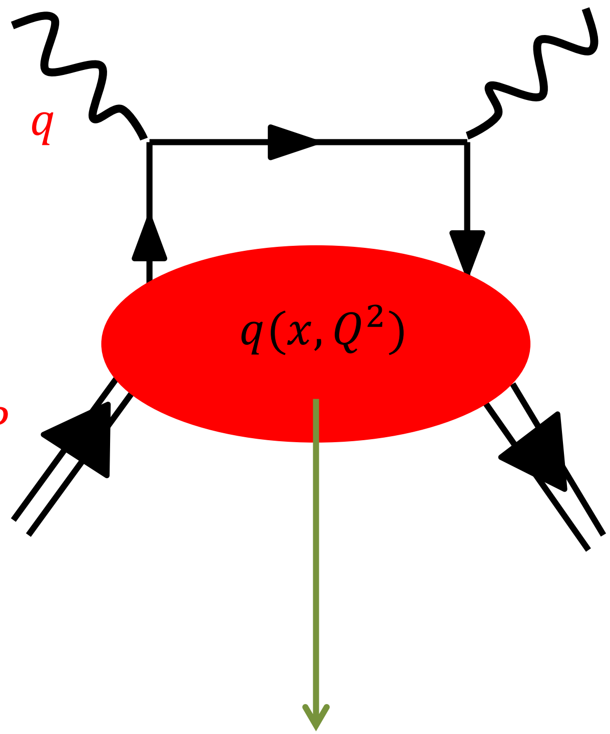
3 free quarks

3 bound quarks

Quarks + sea + self-interacting gluons

In the Bjorken limit $Q^2, \nu \to \infty,$ $x = \frac{Q^2}{2P \cdot q}$

$$W^2 = (P+q)^2 = M^2 + Q^2 \frac{(1-x)}{x}$$



Parton distributions

QCD + OPE

$$\int_0^1 dx x^{n-2} F_2(x, Q^2) = \sum_i a_n^{(i)} C_n^{(i)}(Q^2)$$

$$\langle P | \mathcal{O}_{\mu_1 \cdots \mu_n} | P \rangle = a_n P_{\mu_1} \cdots P_{\mu_2}$$

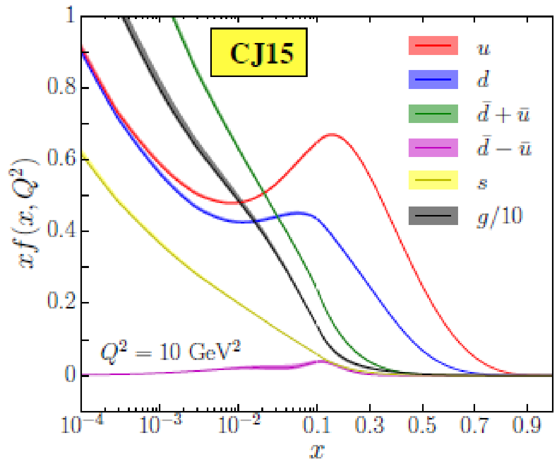
Moments of the parton distributions

$$a_n = \int dx\, x^{n-1} q(x)$$

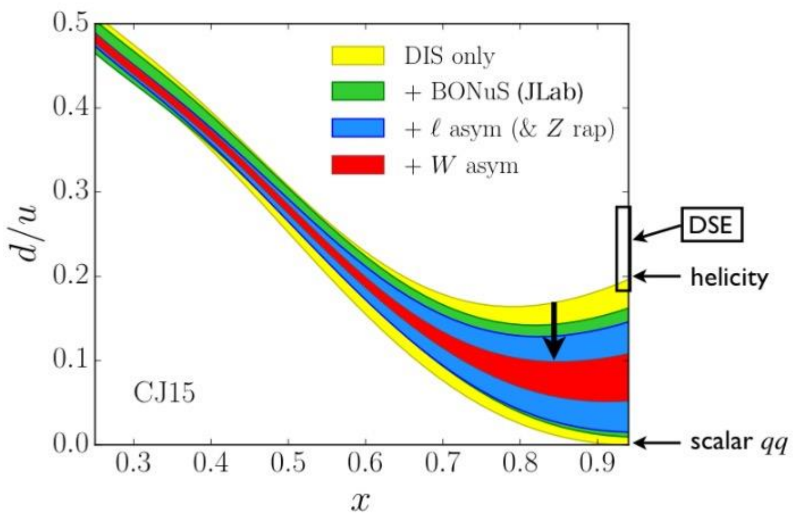
At Leading Order (LO) in pQCD,

$$F_2(x, Q^2) = x \sum_q e_q^2 q(x, Q^2)$$

The individual distributions



Giving a closer look



From W. Melnitchouk, presentation at QCD Down Under 2017

SU(6) symmetry: $d/u \to 1/2$

$S = 0$ qq dominance (colour-hyperfine interaction): $d/u \to 0$

$S_z = 0$ qq dominance (perturbative gluon exchange): $d/u \to 1/5$

DSE with qq correlations: $d/u \to$ 0.18-0.28

Extrapolated ratio at $x = 1$: 0.09 ± 0.03

No model can account for it

Can lattice say something about the large x region? Or the x dependence in general?

Antiquarks are not symmetric

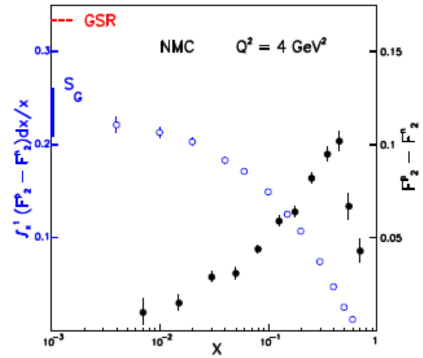
Expect $\bar{d} = \bar{u}$ if sea quarks are produced in $g \rightarrow q\bar{q}$

The Gottfried Sum Rule

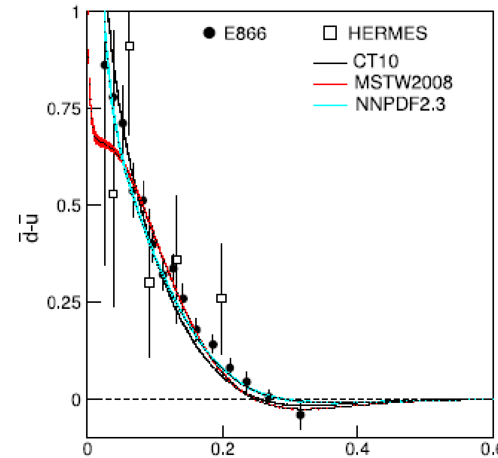
$$S_G = \int_0^1 [(F_2^p(x) - F_2^n(x))/x]dx$$

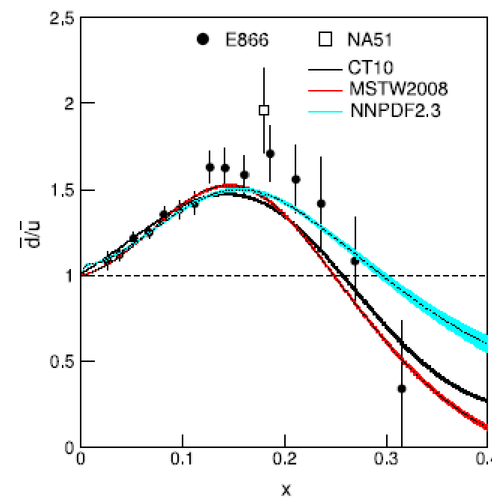
$$= \frac{1}{3} + \frac{2}{3}\int_0^1 (\bar{u}_p(x) - \bar{d}_p(x))\ dx$$

$$= \frac{1}{3} \quad (if\ \bar{u}_p = \bar{d}_p)$$



From JC Peng, EINN2015





New Muon Collaboration (NMC) obtains

$S_G = 0.235 \pm 0.026$

(Significantly lower than 1/3 !) $\Rightarrow \bar{d} \neq \bar{u}\,?$

5

Polarized sector: STAR data also consistent with an asymmetry in favor of u antiquarks

Can we explain these curves from first principles?

Lattice QCD

- Replace Euclidian space-time by 4-dimensional hypercubic lattice:

  quark fields on lattice sites,

  gluon fields on lattice links.

- Lattice as a regulator:

  UV cut-off: inverse of lat. spacing a^{-1},

  IR cut-off: inverse of lat. size L^{-1}.

- Remove the regulator:

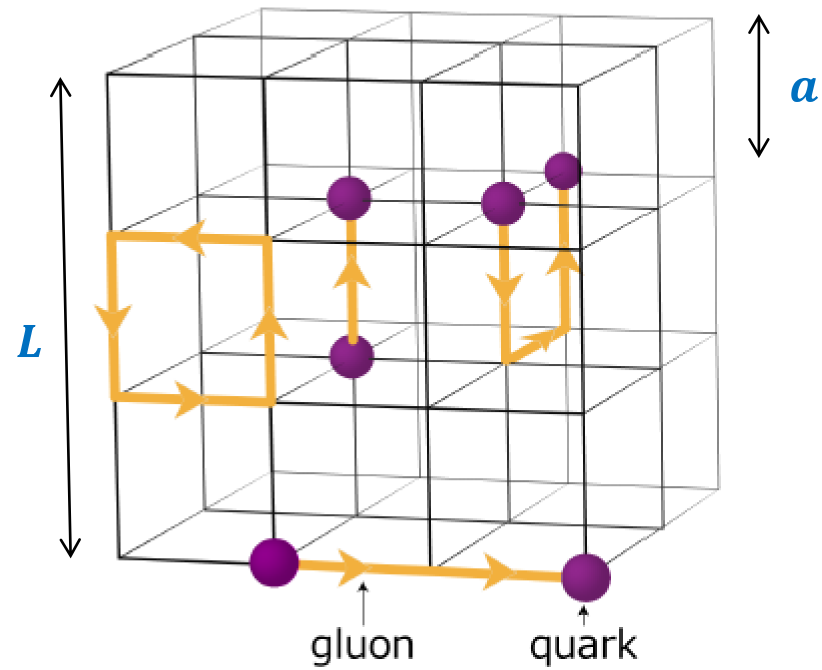
  continuum limit $a \to 0$,

  infinite volume limit $L \to \infty$.

- Gauge invariant objects:

  Wilson line: any path-ordered product of gauge link is gauge covariant,

  Wilson loops: the trace of a closed loop is gauge invariant



Source: JICFuS, Tsukuba

Moments of the distributions

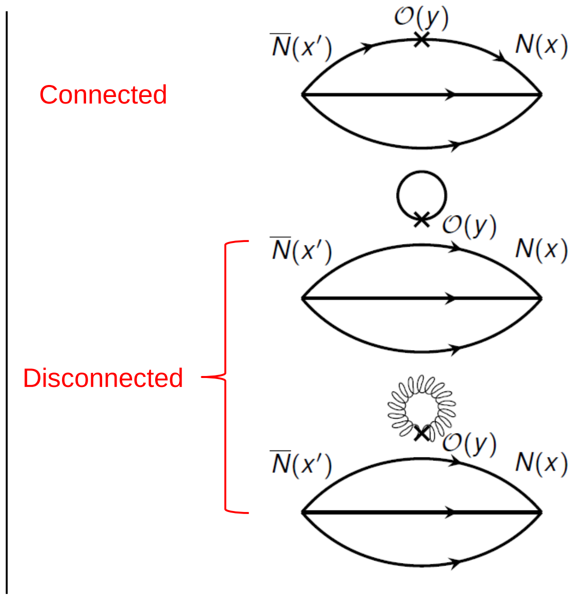
- If a sufficient number of moments are calculated, one can reconstruct the *x* dependence of the distributions;
- Hard to simulate high order derivatives on the lattice;
- Nevertheless, the first few moments can be calculated

Extracting the moments

$$C^{2pt}(\vec{P},t,t') = \frac{e^{-E_0(t-t')}}{2E_0}\langle\Omega|N(P)|0\rangle\langle 0|\overline{N}(P)|\Omega\rangle, \quad t \gg t'$$ (the two point function)

E_0 → Nucleon mass

$$\frac{C_\Gamma^{3pt}(t,\tau,t';\vec{P},\vec{P})}{C_\Gamma^{2pt}(t,t';\vec{P})} = \frac{Tr\left(\Gamma(\gamma_\mu P_0^\mu + m)\mathcal{O}_{00}(\gamma_\mu P_0^\mu + m)\right)}{2E\, Tr\left(\Gamma'(\gamma_\mu P_0^\mu + m)\right)}, \quad t \gg \tau \gg t'$$



Example: Proton spin decomposition

$$\langle N(p',s')|\mathcal{O}_A^{\mu,q}|N(p,s)\rangle = \bar{u}_N(p',s')g_A^q(Q^2)\gamma^\mu\gamma_5\, u_N(p,s)$$

$$\Delta\Sigma = g_A^{(0)} = \sum_q g_A^q(0) = \Delta u + \Delta d + \Delta s + \cdots$$

Total helicity carried by quarks

$$\langle N(p',s')|\mathcal{O}_V^{\mu\nu}|N(p,s)\rangle = \bar{u}_N(p',s')\Lambda_q^{\mu\nu}(Q^2)u_N(p,s)$$

$$\Lambda_q^{\mu\nu}(Q^2) = A_{20}^q(Q^2)\gamma^{\{\mu}P^{\nu\}} + B_{20}^q(Q^2)\frac{\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m} + C_{20}^q(Q^2)\frac{Q^{\{\mu}Q^{\nu\}}}{m}$$

$$\langle x\rangle^q = A_{20}^q(0)$$

Average fraction x of the nucleon momentum carried by quark q

The total quark angular momentum is given by

$$J^{quark} = \frac{1}{2}\sum_q\left(A_{20}^q(0) + B_{20}^q(0)\right) = \frac{1}{2}\Delta\Sigma + L^{quarks}$$

Orbital angular momentum carried by quarks

Similar expression can be obtained for the total angular momentum of gluons, J^{gluon}

In nonrelativistic quark model, spin of the proton is carried by quarks only

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s = 1$$

Experimentally, from hadron weak decays

$$g_A = \Delta u - \Delta d = 1.269(3) \quad \text{using } SU(2) \text{ symmetry}$$

$$a_8 = \Delta u + \Delta d - 2\Delta s = 0.586(31) \quad \text{using } SU(3) \text{ symmetry}$$

From measurements in polarized DIS, one obtains

$$\int_0^1 dx g_1(x, Q^2) = \frac{1}{18}(4\Delta u + \Delta d + \Delta s)$$

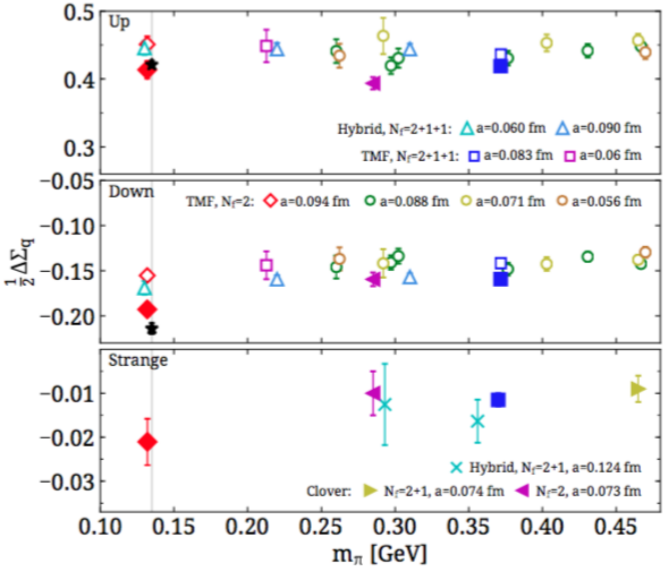
Early EMC (1988) data: $\Delta\Sigma \approx 0, \quad \Delta s \approx -(0.1 - 0.2)$

Spin sum rule

$$\frac{1}{2} = J^{quarks} + J^{gluons} = \frac{1}{2}\Delta\Sigma + L^{quarks} + \Delta G + L^{gluons}$$

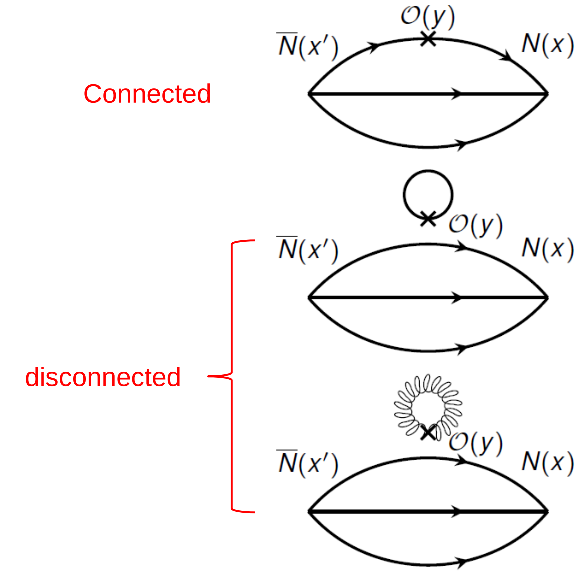
From where does the spin comes from?

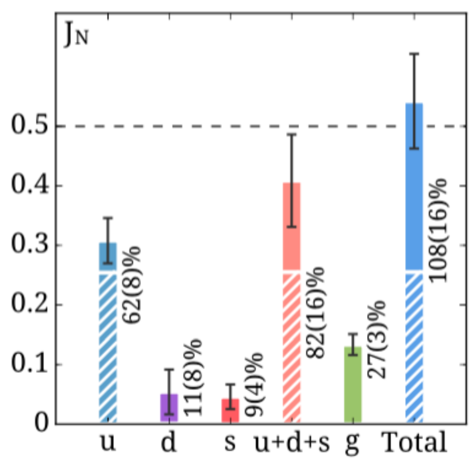
Results for $\mu = 2$ GeV

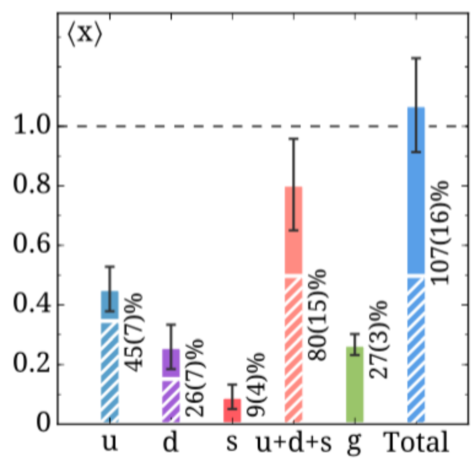


Open symbols: only connected contributions

Filled symbols: both connected and disconnected contributions







Total angular momentum

Average x: $\langle x \rangle$

	$\frac{1}{2}\Delta\Sigma$	J	L	$\langle x \rangle$
u	0.415(13)(2)	0.308(30)(24)	-0.107(32)(24)	0.453(57)(48)
d	-0.193(8)(3)	0.054(29)(24)	0.247(30)(24)	0.259(57)(47)
s	-0.021(5)(1)	0.046(21)(0)	0.067(21)(1)	0.092(41)(0)
g	-	0.133(11)(14)	-	0.267(22)(27)
tot.	0.201(17)(5)	0.541(62)(49)	0.207(64)(45)	1.07(12)(10)

- First ever results at the physical point;
- Spin sum rule satisfied;
- Momentum sum rule satisfied;
- Slightly negative polarized strangeness

C. Alexandrou et al., arXiv: 1706.02973, PRL 119 (2017) 034503

Quark distributions

The most general form of the matrix element is:

$$\langle P|O^{\mu_1\mu_2\cdots\mu_n}|P\rangle = 2a_n^{(0)}\Pi^{\mu_1\mu_2\cdots\mu_n}$$

$$\Pi^{\mu_1\mu_2\cdots\mu_n} = \sum_{j=0}^{k}(-1)^j\frac{(2k-j)!}{2^j(2k)!}\{g\cdots gP\cdots P\}_{k,j}(P^2)^j$$

We use the following four-vectors

$$P = (P_0, 0, 0, P_3) \quad \lambda = (1,0,0,-1)/\sqrt{2} \quad \longrightarrow \quad \boxed{\lambda\cdot P = (P_0+P_3)/\sqrt{2} = P_+}$$

$$\lambda_{\mu_1}\lambda_{\mu_2}\left\langle P\middle|O^{\mu_1\,\mu_2}\middle|P\right\rangle = 2a_n^{(0)}\left(P^+P^+ - \lambda^2\frac{M^2}{4}\right) = 2a_n^{(0)}P^+P^+$$

In general, we have

$$\lambda_{\mu_1}\cdots\lambda_{\mu_n}\Pi^{\mu_1\cdots\mu_n} = (P^+)^n \quad \longrightarrow \quad \boxed{\langle P|O^{+\cdots+}|P\rangle = 2a_n^{(0)}(P^+)^n}$$

Taking the inverse Mellin transform

$$a_n^{(0)} = \int dx\, x^{n-1} q(x) \qquad\qquad q(x) = \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} dn\, x^{-n} a_n^{(0)}$$

Using $\quad a_n^{(0)} = \langle P|O^{+\cdots+}|P\rangle / 2(P^+)^n$

$$\Rightarrow \quad q(x) = \int_{-\infty}^{+\infty} \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \langle P\big|\bar{\psi}(\xi^-)\gamma^+ W(\xi^-,0)\psi(0)\big|P\rangle$$

$$W(\xi^-,0) = e^{-ig\int_0^{\xi^-} A^+(\eta^-)d\eta^-} \quad \text{(Wilson line)}$$

- Light cone correlations
- Equivalent to the distributions in the Infinite Momentum Frame
- Light cone dominated $\quad \xi^2 = t^2 - z^2 \sim 0$
- Not calculable on Euclidian lattice $\quad t^2 + z^2 \sim 0$

Quasi Distributions

X. Ji, "Parton Physics on a Euclidean Lattice," PRL 110 (2013) 262002.

Suppose we project outside of the light-cone:

$$\lambda = (0,0,0,-1) \qquad P = (P_0, 0, 0, P_3) \qquad \boxed{\lambda \cdot P = P_3}$$

We take n=2

= -1

$$\langle P|O^{33}|P\rangle = 2\tilde{a}_n^{(0)}(P^3P^3 - \lambda^2 P^2/4) = 2\tilde{a}_n^{(0)}((P^3)^2 + P^2/4)$$

Mass terms contribute

In general,

$$\langle P|O^{3\cdots 3}|P\rangle = 2\tilde{a}_{2k}^{(0)}(P_3)^{2k}\sum_{j=0}^{k}\mu^j\frac{(2k-j)!}{j!\,(2k-2j)!} \equiv 2\tilde{a}_{2k}(P_3)^{2k}$$

with $\mu = M^2/4(P_3)^2$

Defining

$$\tilde{a}_n^{(0)} = \int dx\, x^{n-1} \tilde{q}^{(0)}(x) \qquad \tilde{a}_n = \int dx\, x^{n-1} \tilde{q}(x)$$

Taking the inverse Mellin transform

$$\tilde{q}^{(0)}(x) = \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} dn\, x^{-n} \tilde{a}_n^{(0)} \qquad \tilde{q}(x) = \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} dn\, x^{-n} \tilde{a}_n$$

$$\tilde{q}(x) = \tilde{q}^{(0)}(\xi)/(1+\mu\xi^2) \; + antiquarks \qquad \xi = \frac{2x}{1+\sqrt{1+4\mu x^2}}$$

$$\tilde{q}(x, P_3) = \int_{-\infty}^{+\infty} \frac{dz}{4\pi} e^{izk_3} \langle P | \bar{\psi}(z) \gamma^3 W(z,0) \psi(0) | P \rangle$$

$$W(z,0) = e^{-ig\int_0^z A^3(z')dz'}$$

$$k_3 = xP_3$$

- Nucleon moving with finite momentum in the z direction
- Pure spatial correlation
- Can be simulated on a lattice

The light cone distributions:

$$x = \frac{k^+}{P^+}$$

$$0 \leq x \leq 1$$

Distributions can be defined in an Infinite Momentum Frame: P_3, P^+ goes to infinite

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Quasi distributions:

P_3 large but finite

Usual partonic interpretation is lost

$x < 0$ or $x > 1$ is possible

But they can be related to each other!

Extracting quark distributions from quark quasi-distributions

Infrared region untouched when going from
a finite to an infinite momentum

Infinite momentum: $P_3 \to \infty, \Lambda \ fixed$ (Λ is the UV regulator)

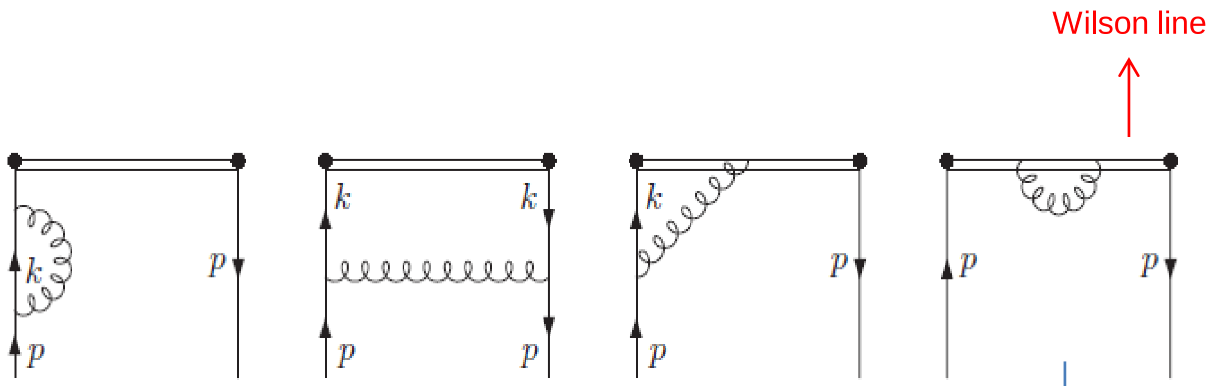
$$q(x,\mu) = q_{bare}(x)\left\{1 + \frac{\alpha_s}{2\pi} Z_F(\mu)\right\} + \frac{\alpha_s}{2\pi}\int_x^1 q^{(1)}\left(\frac{x}{y},\mu\right) q_{bare}(y)\frac{dy}{y} + \mathcal{O}(\alpha_s^2)$$

Finite momentum: $\Lambda \to \infty, P_3 \ fixed$

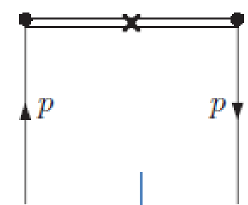
$$\tilde{q}(x,\Lambda,P_3) = q_{bare}(x)\left\{1 + \frac{\alpha_s}{2\pi}\widetilde{Z_F}(\Lambda,P_3)\right\} + \frac{\alpha_s}{2\pi}\int_{x/x_c}^1 \tilde{q}^{(1)}\left(\frac{x}{y},\Lambda,P_3\right) q_{bare}(y)\frac{dy}{y} + \mathcal{O}(\alpha_s^2)$$

$x_c \sim \Lambda/P_3$ Largest value at which the calculations are meaningful

Perturbative QCD in the continuum



Linear divergence comes
from this type of diagram



Mass counterterm introduced
to remove the linear div.

J. W. Chen, X. Ji and J. H. Zhang,
``Improved quasi parton distribution through Wilson line renormalization,''
arXiv:1609.08102.

T. Ishikawa, Y. Q. Ma, J. W. Qiu and S. Yoshida,
``Practical quasi parton distribution functions,''
arXiv:1609.02018.

W. Wang, S. Zhao and R. Zhu,
``A Complete Matching for Quasi Parton Distribution Functions at One-Loop Order,'' arXiv:1708.02458

I. W. Stewart and Y. Zhao,
``Matching the Quasi Parton Distribution in a Momentum Subtraction Scheme,'' arXiv:1709.04933

T. Ishikawa, Y. Q. Ma, J. W. Qiu and S. Yoshida,
``On the Renormalizability of Quasi Parton Distribution Functions,'' arXiv:1707.03107.

Solving for the quark distributions

$$q(x,\mu) = \tilde{q}(x,\Lambda,P_3) - \frac{\alpha_s}{2\pi}\tilde{q}(x,\Lambda,P_3)\delta Z_F^{(1)}\left(\frac{\mu}{P_3},\frac{\Lambda}{P_3}\right) - \frac{\alpha_s}{2\pi}\int_{-1}^{1} Z^{(1)}\left(\frac{x}{y},\frac{\mu}{P_3},\frac{\Lambda}{P_3}\right)\tilde{q}(x,\Lambda,P_3)\frac{dy}{|y|} + \mathcal{O}(\alpha_s^2)$$

Desired quantity

From pQCD

From lattice

Infrared physics is the same for $q(x)$ and $\tilde{q}(x)$

Matching affects the UV only

μ is the renormalization scale

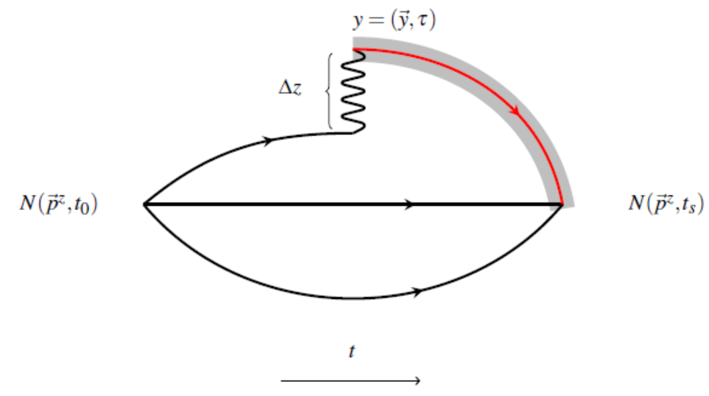
$\Lambda = \frac{1}{a}$ is the UV cut-off

X. Xiong, X. Ji, J.-H. Zhang and Y. Zhao,
"One loop matching for parton distributions:Nonsinglet case,"PRD90 (2014) 014051.

C. Alexandrou, K. Cichy, V. Drach, E. Garcia-Ramos, k. Hadjiyiannakou, K. Jansen, FS and C. Wiese,
"A Lattice Calculation of Parton Distributions," PRD92 (2015) 014502.

Lattice QCD and the x dependence of the distributions

$$\frac{C^{3pt}(t,\tau,0;P_3)}{C^{2pt}(t,0;P_3)} = \frac{-iP_3}{E} h(P_3,z), \qquad 0 \ll \tau \ll t$$



$$C^{3pt}(t,\tau,0) = \langle N_\alpha(\vec{P},t)\mathcal{O}(\tau)\overline{N_\alpha}(\vec{P},0)\rangle$$

$$h(P_3,z) = \langle P|\bar{\psi}(z)\gamma_3 W_3(z,0)\psi(0)|P\rangle$$

$$\mathcal{O}(z,\tau,Q^2=0) = \sum_{\vec{y}} \bar{\psi}(y+z)\gamma_3 W_3(y+z,y)\psi(y)$$

Setup: $N_f = 2+1+1$ $\quad \beta = \frac{6}{g_0^2} = 1.95$ $\quad a \approx 0.082\, fm$

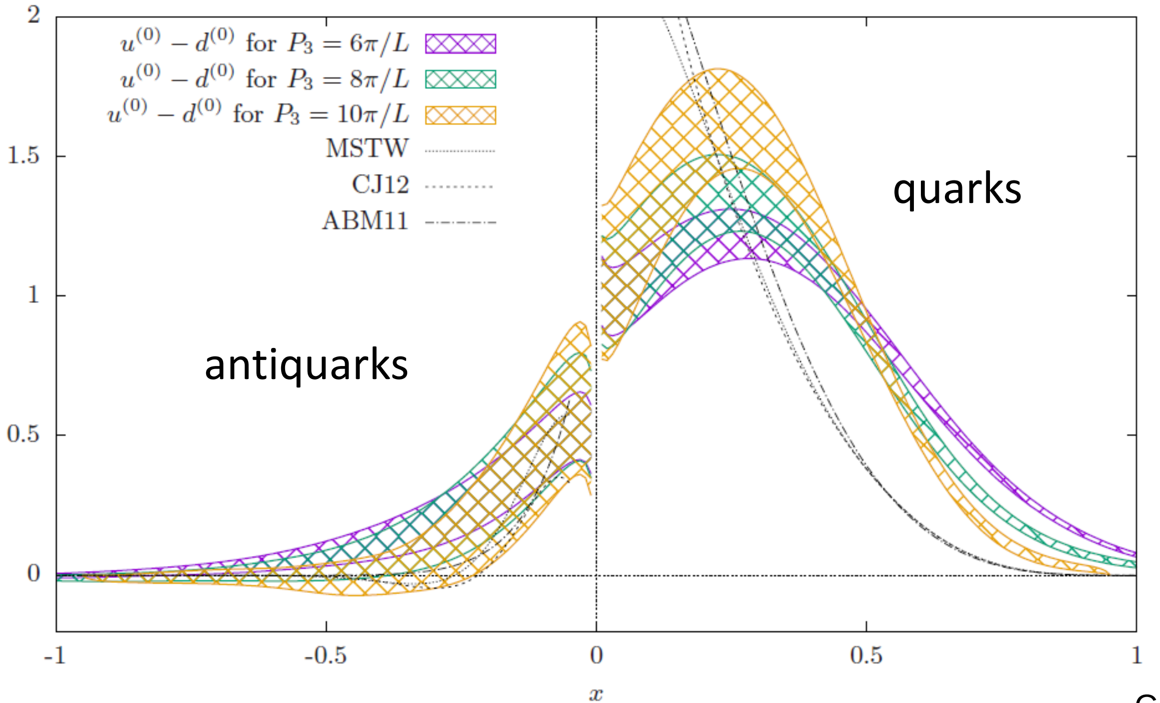
$32^3 \times 64$

Maximally twisted mass ensemble: $a\mu = 0.0055 \Longrightarrow m_{ps} \cong 370$ MeV

$$P_3 = \frac{2\pi}{L}, \frac{4\pi}{L}, \cdots$$

Not the physical point yet

Unpolarized distributions: $u(x) - d(x)$



$P_3 = 6\pi/L \approx 1.43$ GeV

$P_3 = 8\pi/L \approx 1.90$ GeV

$P_3 = 10\pi/L \approx 2.37$ GeV

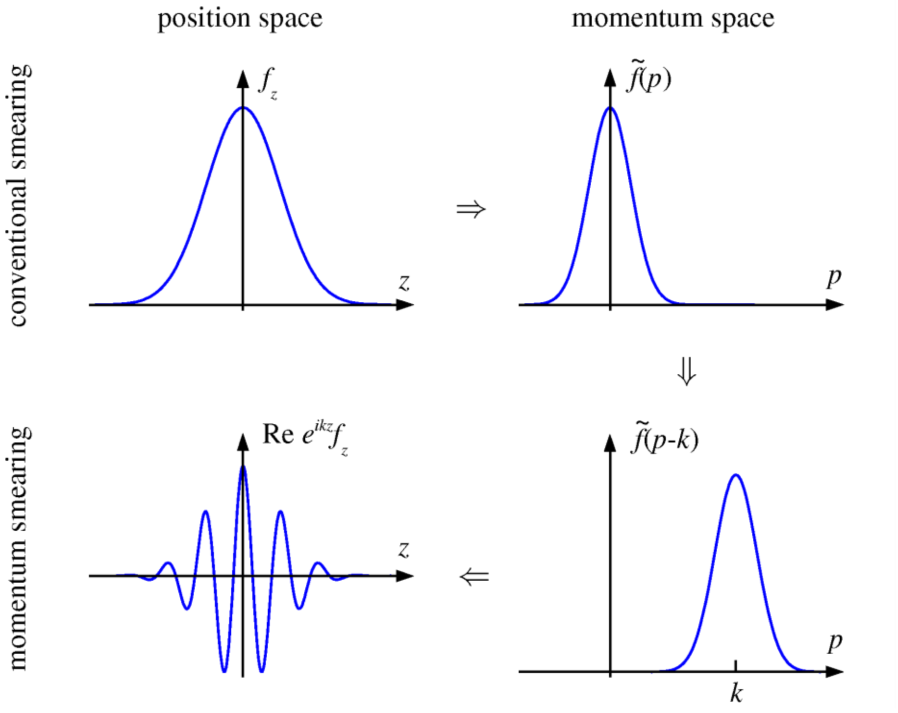
$$u(-x) - d(-x) = \bar{d}(x) - \bar{u}(x)$$

C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, FS and C.Wiese, ``Updated Lattice Results for Parton Distributions,'' arXiv:1610.03689, to appear in PRD

- 5 steps of HYP smearing in the gauge links;
- Momentum smearing in the quark fields allows to reach higher values of P_3;
- Matching and TMC applied;
- Bare matrix elements;
- Away from the physical point;

Momentum smearing

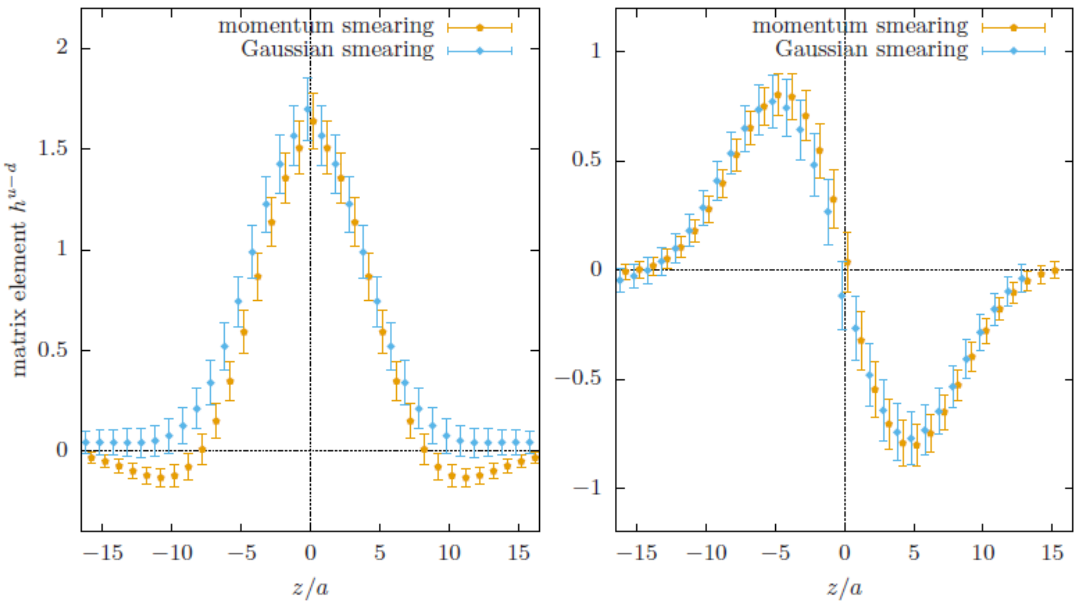
- We would like to study the PDFs at larger momenta
  Problem: poor signal
- Possible solution by Bali *et al.* in arXiv:1602.05525
- Alter Gaussian smearing so that in momentum space the desired momentum is modeled



$$S_M(k)\psi(x) = \frac{1}{1+8\kappa}\left[\psi(x) + \kappa \sum e^{ik\hat{j}} U_j(x)\psi(x+\hat{j})\right]$$

Gaussian and Momentum Smearing

- 30000 measurements for the case of Gaussian smearing;
- 150 measurements for the case of momentum smearing;
- We can now access larger values for the nucleon momentum;
- 150 measurements for the cases of $P_3 = \frac{6\pi}{L}, \frac{8\pi}{L}$;
- 300 measurements for the case of $P_3 = \frac{10\pi}{L}$.

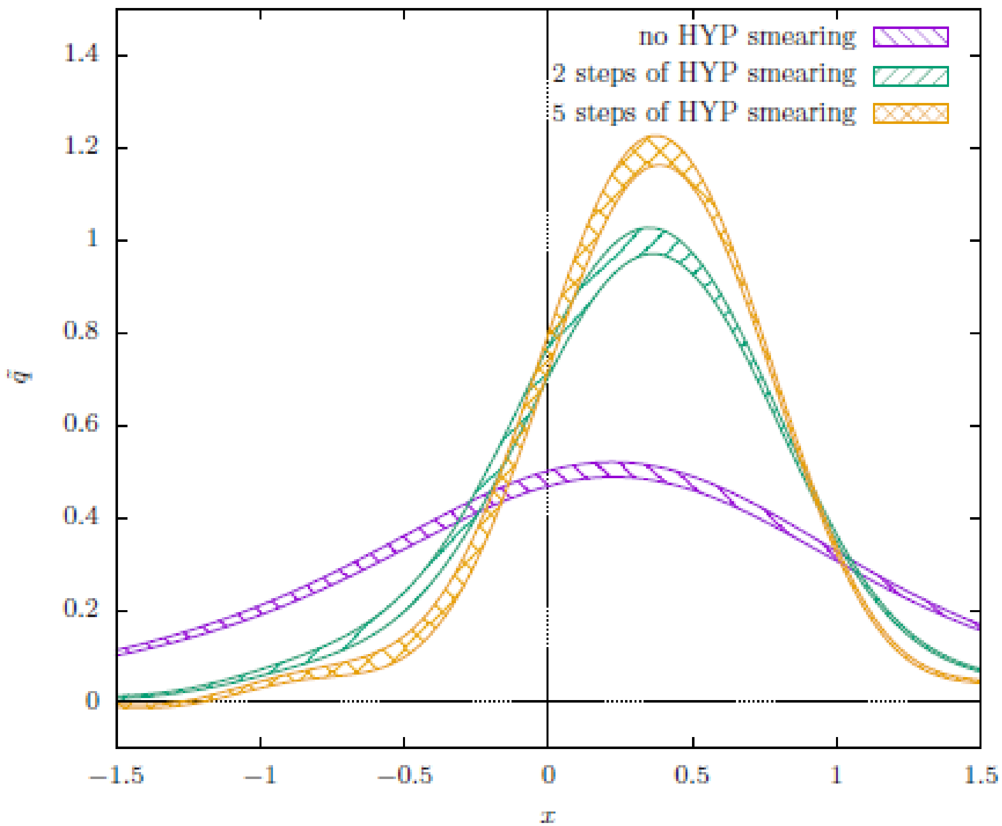


$$P_3 = \frac{6\pi}{L}$$

HYP Smearing

It replaces a given gauge link with some average over neighbouring links, *i.e*. ones from the hypercubes attached to it

Crude substitute for renormalization



Parameters

$$\alpha_s = \frac{6}{4\pi\beta} \approx 0.245$$

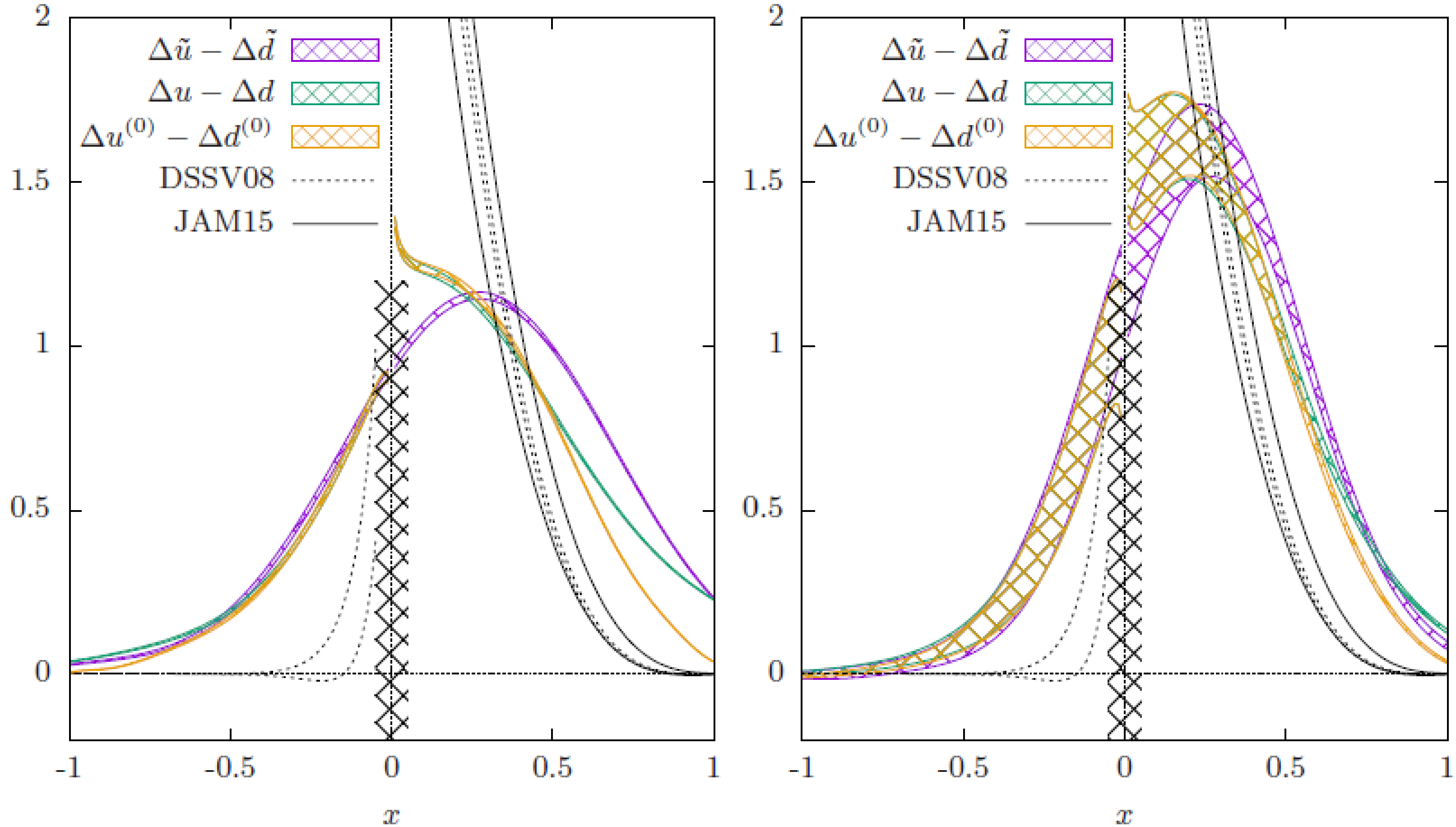
$$\Lambda = \frac{1}{a} \cong 2.5 \text{ GeV}$$

$$\tilde{q}(x, \Lambda, P_3) = 2P_3 \int_{-L/2}^{+L/2} \frac{dz}{4\pi} e^{-ixP_3 z} h(P_3, z)$$

$$P_3 = \frac{4\pi}{L}$$

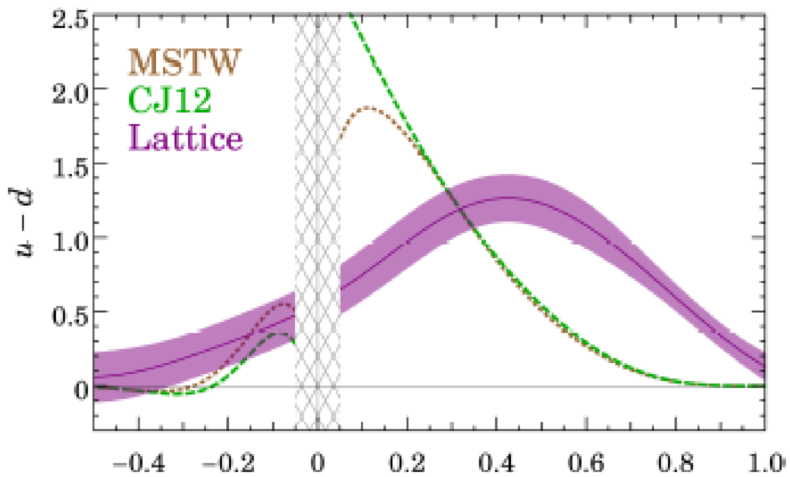
Helicity distribution $\Delta u(x) - \Delta d(x)$

Crossing relation: $\Delta\bar{q}(x) = \Delta q(-x)$ → $\Delta\overline{u}(x) > \Delta\overline{d}(x)$

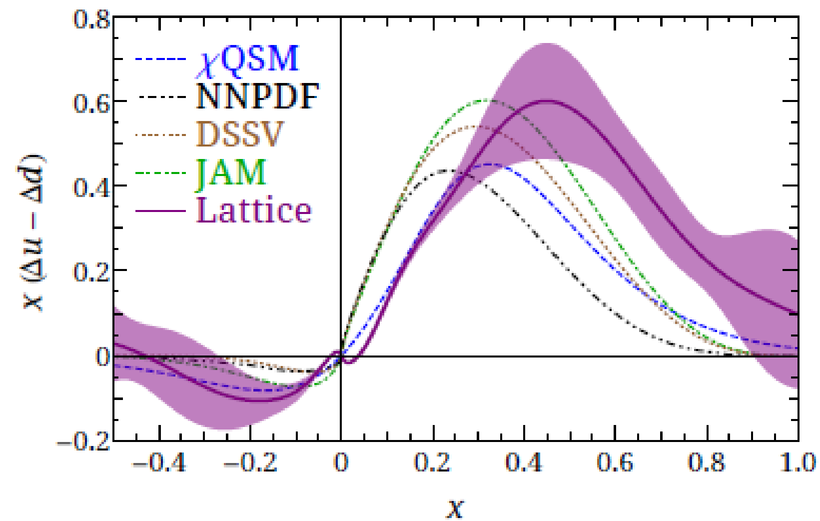


$P_3 = \frac{4\pi}{L}$ $\quad$ $P_3 = \frac{6\pi}{L}$

Only other results for the bare distributions



H. W. Lin et al.,
`Flavour Structure of the Nucleon Sea from Lattice QCD,''
Phys. Rev. D91 (2015) 054510
arXiv:1402.1462



J.-W. Chen et al.,
"Nucleon Helicity and Transversity Parton Distributions from Lattice QCD,''
Nucl. Phys. B911 (2016) 246
arXiv:1603.06664

$24^3 \times 48$

$a \approx 0.12 fm \qquad N_f = 2+1+1$

$m_{PS} \approx 310\ MeV$

Uses highly improved staggered quarks and HYP smearing

Integral of the distributions compared to the direct extraction of the moments

PDF type	P_3	Smear	Normalization antiquarks	quarks	total	$\langle x \rangle_q$	$\langle x^2 \rangle_q$
unpolarized	3	Gauss	0.187(55)	0.752(56)	0.94(11)	0.219(28)	0.134(12)
		mom	0.145(55)	0.750(53)	0.90(11)	0.240(32)	0.147(15)
	4	mom	0.130(77)	0.743(78)	0.87(15)	0.224(43)	0.116(20)
	5	mom	0.100(88)	0.798(98)	0.90(10)	0.234(46)	0.100(19)
helicity	3	Gauss	0.253(62)	0.920(58)	1.17(12)	0.249(29)	0.154(12)
		mom	0.184(47)	0.931(44)	1.11(9)	0.281(26)	0.154(11)
transversity	3	Gauss	0.175(99)	0.923(95)	1.10(19)	0.309(67)	0.163(35)
		mom	0.169(47)	0.878(44)	1.05(9)	0.276(26)	0.152(11)

By comparison, a direct calculation of the moments using the same ensemble gives:

C. Alexandrou et al.,
Nucleon form factors and moments of generalized parton distributions using $N_f=2+1+1$ twisted mass fermions,
Phys. Rev. D88 (2013), 014509

$$\langle x \rangle_q = 0.233(9)$$

$$g_A^{u-d} = 1.17(2) \qquad \langle x \rangle_{\Delta q} = 0.298(8)$$

$$g_T^{u-d} = 1.08(3) \qquad \langle x \rangle_{\delta q} = 0.316(12)$$

Physical point calculation should shift $\langle x \rangle_q$ to the left!

Origin of the large $x \leftrightarrow -x$ asymmetry

Matrix elements obeys the following relations:

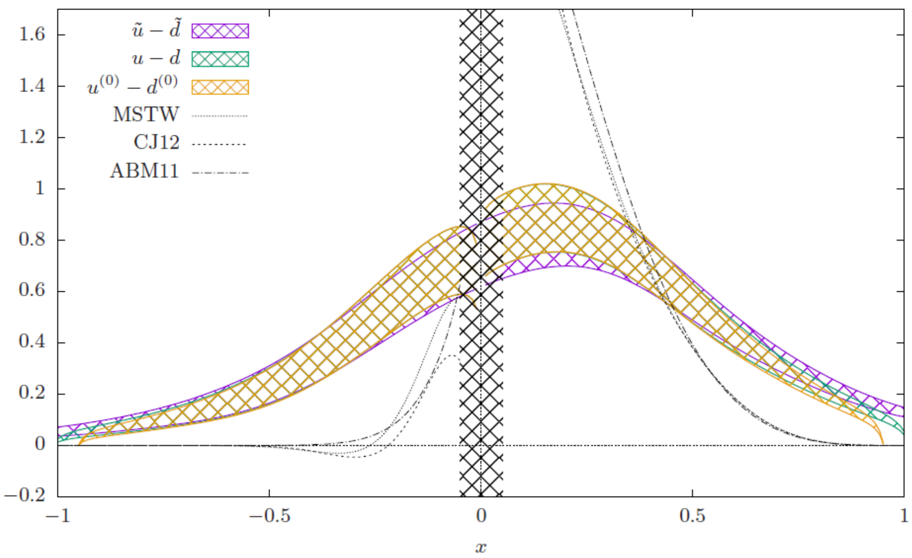
$$h(P_3, z) = h(P_3, -z)^\dagger$$

$$\Delta h(P_3, z) = \Delta h(P_3, -z)^\dagger$$

$$\delta h(P_3, z) = \delta h(P_3, -z)^\dagger$$

Imaginary part is odd under $z \rightarrow -z$

The asymmetry between x and $-x$ only appear because the imaginary part is an odd function



No HYP smearing in the gluon fields!!!

Renormalization seems to be fundamental for the asymmetry

Combined effect

Non-perturbative renormalization I

Proposed renormalization program described in:

C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, H. Panagoupolos, FS
“A complete non-perturbative renormalization prescription for quase-PDFs”, arXiv:1706.00265, NPB923 (2017) 394.

Important insights also from the lattice perturbative paper:

M. Constantinou and H. Panagopoulos,
“Perturbative renormalization of quasi-PDFs”, arXiv:1705.11193

Discovered mixing between the vector and scalar matrix elements (unpolarized PDF). This perturbative analysis is very important guidance to non-perturbative renormalization!

Similar non-perturbative renormalization procedure was also presented, almost simultaneously, in: Jiunn-Wei Chen, Tomomi Ishikawa, Luchang Jin, Huey-Wen Lin, Yi-Bo Yang, Jian-Hui Zhang, Yong Zhao, “ Parton distribution function with Non-perturbative renormalization from lattice QCD”, arXiv:1706.01295.

Features of the proposed renormalization programme:

- Removes the linear divergence that re-sums into a multiplicative exponential factor, $e^{-\delta m|z|+c|z|}$,
  δm is the strength of the divergence, operator independent
  c an arbitrary scale, fixed by the renormalization prescription.

- Takes away the logarithmic divergence with respect to the regulator, $\log(a\mu)$, where μ is the renormalization scale.

- Applies the necessary finite renormalization related to the lattice regularization.

- Unpolarized – eliminates the mixing between the vector operator and the twist-3 scalar operator; the two may be disentangled by the construction of a 2 x 2 mixing matrix.

Non-perturbative renormalization scheme: **RI'-MOM**

G. Martinelli, C. Pittori, C. T. Sachrajda, M. Testa and A. Vladikas, ``A General method for nonperturbative renormalization of lattice operators,'' Nucl. Phys. B 445 (1995) 81

Considered flavour non-singlet operators: $\mathcal{O}_\Gamma = \bar{u}(x)\Gamma\mathcal{P}e^{ig\int_0^z d\zeta\, A(\zeta)}\bar{d}(x)$, where $\Gamma = \gamma_\mu, \gamma_\mu\gamma_5, \gamma_\mu\gamma_\nu$

RI'-MOM renormalization conditions:

For the operator: $Z_q^{-1}\, Z_{\mathcal{O}} \frac{1}{12} Tr[\nu(p,z)(\nu^{Born}(p,z))^{-1}]|_{p^2=\bar{\mu}_0^2} = 1$

For the quark field: $Z_q = \frac{1}{12} Tr[\left(S(p)\right)^{-1} S^{Born}(p)]|_{p^2=\bar{\mu}_0^2}$

- Momentum p entering the vertex function is set to the RI' renormalization scale $\bar{\mu}_0$, chosen such that p_3 is the same as the nucleon boost P_3,
- $\nu(p,z)$ is the amputated vertex function of the operator,
- ν^{Born} is its tree-level value, $\nu^{Born}(p,z) = i\gamma_3\gamma_5 e^{ipz}$ for helicity,
- $S(p)$ is the fermion propagator ($S^{Born}(p)$ at tree-level)

- The vertex functions $\nu(p)$ contain the same linear divergence as the nucleon matrix elements.

- This is crucial, as it allows the extraction of the exponential together with the multiplicative Z-factor.

- $Z_{\mathcal{O}}$ can be factorized as $Z_{\mathcal{O}} = \bar{Z}_{\mathcal{O}}\, e^{+\delta m \frac{|z|}{a} - c|z|}$, where $\bar{Z}_{\mathcal{O}}$ is the multiplicative Z-factor of the operator. Already expected by Dotsenko & Vergeles, NPB 169 (1980) 527.

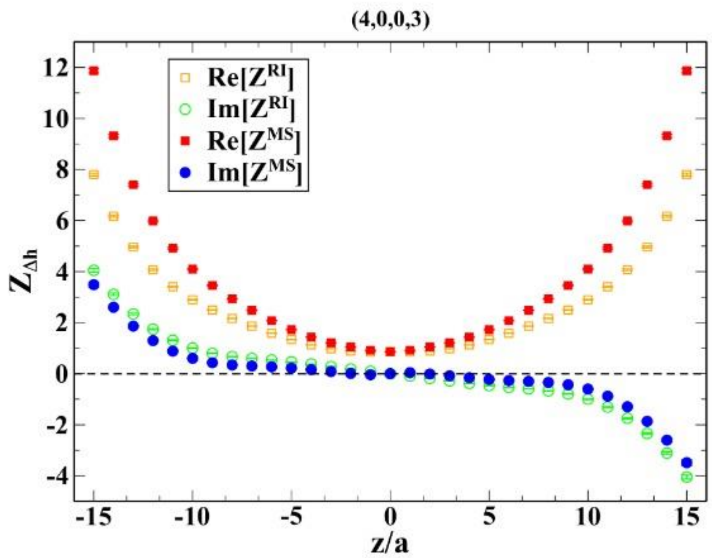
- Note that the exponential comes with a different sign compared to the nucleon matrix element ($Z_{\mathcal{O}}$ is related to the inverse of the vertex function).

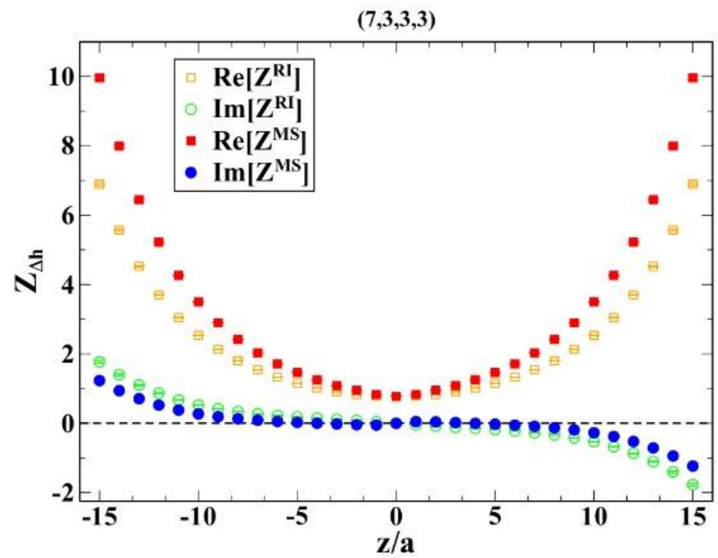
- Consequently, the above renormalization condition handles all the divergences which are present in the matrix element under consideration.

- In the absence of a Wilson line ($z = 0$), the renormalization functions reduce to the local currents, free of any power divergence, e.g. for helicity $Z_{\mathcal{O}}(z = 0) \equiv Z_A$.

Renormalization – helicity

$P_3 = 6\pi/32 \approx 1.43$ GeV





$\bar{\mu}_0 = \frac{2\pi}{32}\left(\frac{4}{2}+\frac{1}{4}, 0{,}0{,}3\right)$ $\bar{\mu}_0 = \frac{2\pi}{32}\left(\frac{7}{2}+\frac{1}{4}, 3{,}3{,}3\right)$

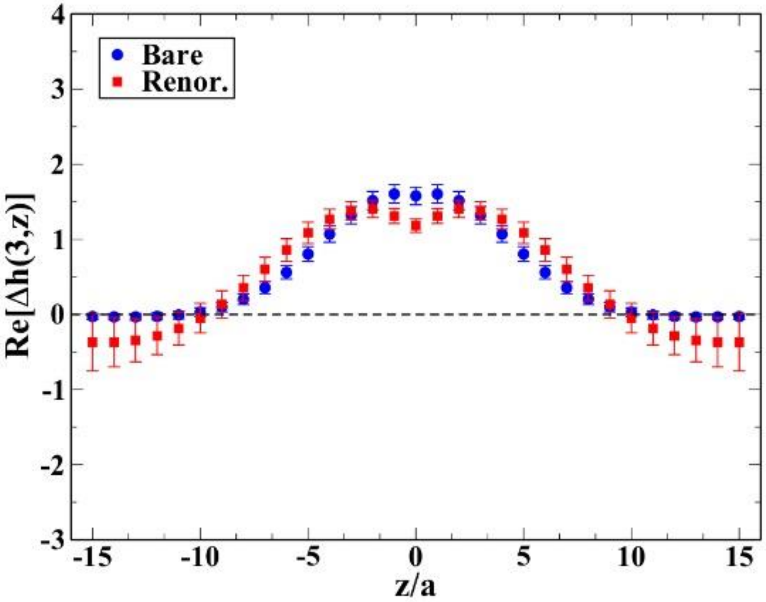
$z = 0 \Rightarrow Z_A \approx 0.86$ $z = 0 \Rightarrow Z_A \approx 0.77$

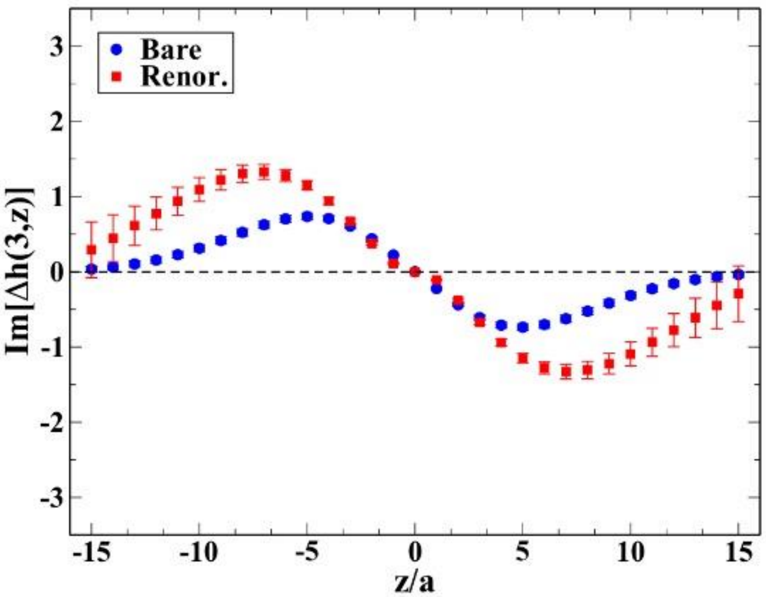
C. Alexandrou, M. Constantinou, H. Panagopoulos, PRD95 (2017) 034505: $Z_A = 0.75556(5)$

1-loop conversion factor from RI' to $\overline{MS}$ used, from M. Constantinou and H. Panagopoulos, arXiv: 1705.11193

- Perturbative Z-factor in DR and in the MS-scheme is real in all orders Thus, important two-loop contributions to the conversion factor, mainly in the imaginary part at large z
- Large lattice artefacts at high values of z/a. See C. Alexandrou et al. 1706.00265 for a detailed discussion on the uncertainties affecting the renormalization factors.

Comparison of bare and renormalized matrix elements

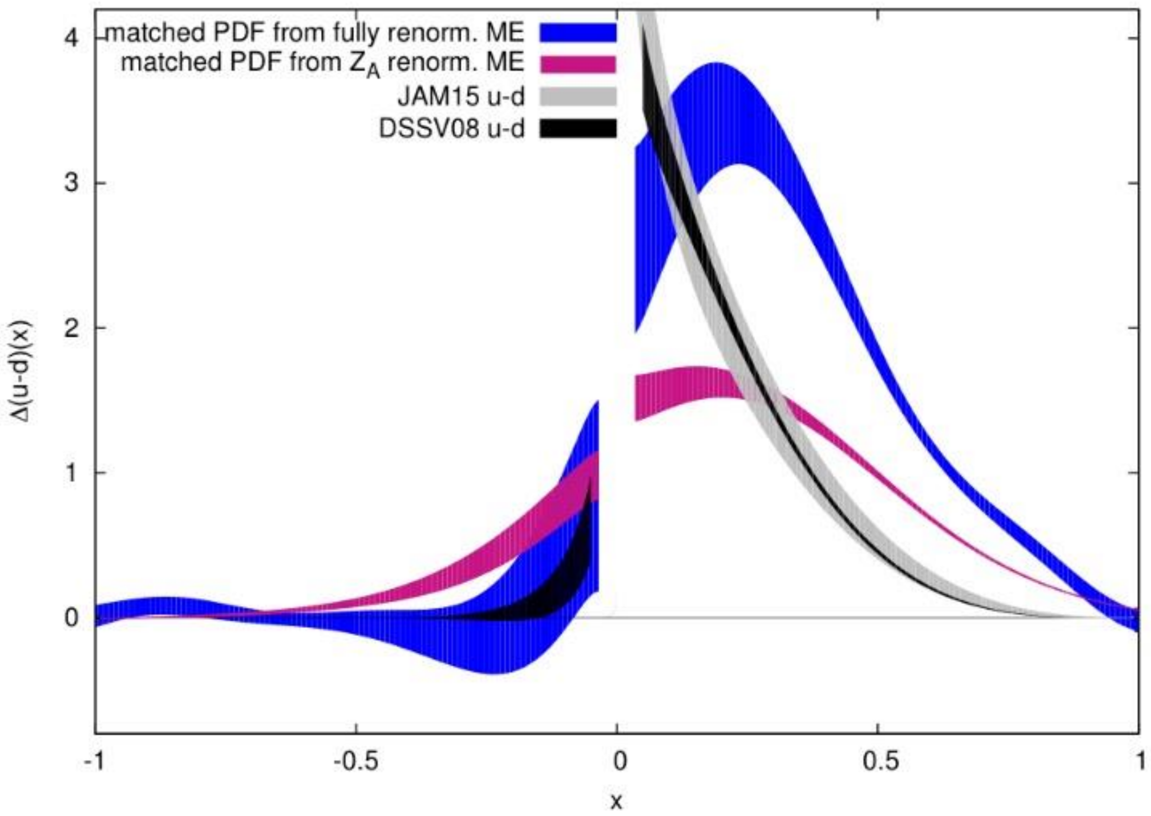




$$Re[\Delta h^{ren}] = Re\left[Z^{\overline{MS}}\right]Re[\Delta h^{bare}] - Im\left[Z^{\overline{MS}}\right]Im[\Delta h^{bare}]$$

$$Im[\Delta h^{ren}] = Re\left[Z^{\overline{MS}}\right]Im[\Delta h^{bare}] + Im\left[Z^{\overline{MS}}\right]Re[\Delta h^{bare}]$$

Isovector quark distribution in the $\overline{MS}$ scheme at 2 GeV



Helicity distributions

$$P_3 = \frac{6\pi}{L} \approx 1.43 \text{ GeV}$$

$$m_\pi \approx 370 \text{ MeV}$$

We still need to address:

- Cut-off and volume effects;
- Non-physical pion mass;
- Possible contamination of excited states;
- Extrapolation to infinite nucleon boost;
- Improvements in the renormalization functions.

Nonperturbative renormalization II: the auxiliary field approach

Based on J. Green talk given at the Lattice 2017, Granada, Spain
Jeremy Green, Karl Jansen, FS, arXiv: 1707.07152
See also the talk of Y. Zhao for a similar proposal 1706.08962

We want to renormalize $\mathcal{O}_\Gamma(x,\xi,n) \equiv \bar{\psi}(x+\xi n)\Gamma W(x+\xi n)\psi(x)$

Introduce an auxiliary scalar, colour triplet field $\zeta(\xi n)$ defined on the line $x+\xi n$ to simplify the renormalization of $\mathcal{O}_\Gamma(x,\xi,n)$

Modify the action to: $S = S_{QCD} + \int d\xi \bar{\zeta}(n \cdot D + m)\zeta$

In the continuum:
N. S. Craigie and H. Dorn, NPB185 (1981) 204
H. Dorn, Fortsch. Phys. 34 (1986) 11

So the propagator $\langle \zeta(\xi_2)\bar{\zeta}(\xi_1)\rangle = \theta(\xi_2-\xi_1)W(x_2,x_1)e^{-m(\xi_2-\xi_1)}$

In terms of a local bilinear field, $\phi \equiv \bar{\zeta}\psi$, one has for $m=0, \xi>0$ that

$$\mathcal{O}_\Gamma(x,\xi,n) = \left\langle \bar{\phi}(x+\xi n)\Gamma\phi(x)\right\rangle_\zeta$$

(expectation values over ζ fields)

In the end we have: $\mathcal{O}_\Gamma^R(x,\xi,n) = Z_\phi^2 e^{-m|\xi|}\mathcal{O}_{\Gamma'}(x,\xi,n)$

With $\Gamma' = \Gamma + r_{mix} sgn(\xi)\{\gamma\cdot n, \Gamma\} + r_{mix}^2 \gamma\cdot n\Gamma\gamma\cdot n,$ $\qquad \phi_R = Z_\phi(\phi + r_{mix}\gamma\cdot n\phi)$

Renormalization conditions

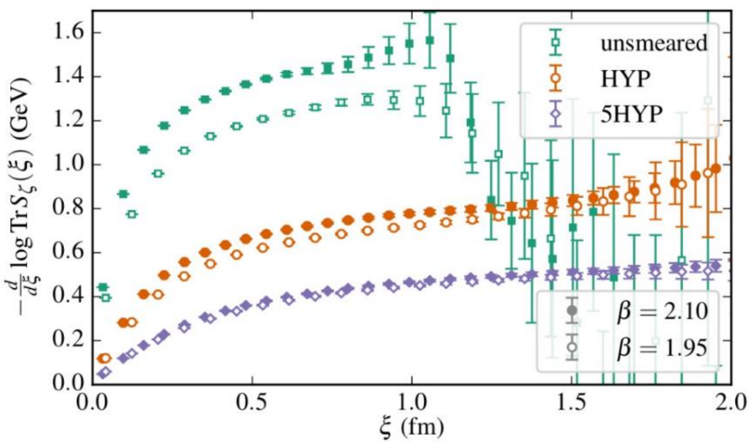
Compute $S_\zeta(\xi) = \langle \zeta(x+\xi n)\bar{\zeta}(x)\rangle_{QCD+\zeta} = \langle W(x+\xi n, x)\rangle_{QCD}$, the momentum space propagator $S_\psi(p)$, and the mixed Green function for ϕ: $G(\xi, p) = \int d^4x e^{ip\cdot x}\langle \zeta(\xi n)\phi(0)\bar{\psi}(x)\rangle_{QCD}$. Then apply the conditions:

$$-\frac{d}{d\xi}\log \mathrm{Tr}\, S_\zeta(\xi)\Big|_{\xi=\xi_0} + m = 0,$$

$$\left[\frac{Z_\zeta}{3}\,\mathrm{Tr}\, S_\zeta(\xi_0)\right]^2 = \frac{Z_\zeta}{3}\,\mathrm{Tr}\, S_\zeta(2\xi_0),$$

$$\frac{1}{6}\frac{Z_\phi^\pm}{\sqrt{Z_\zeta Z_\psi}}\Re\,\mathrm{Tr}\left[S_\zeta^{-1}(\xi_0)G^\pm(\xi_0,p_0)S_\psi^{-1}(p_0)\right] = 1$$

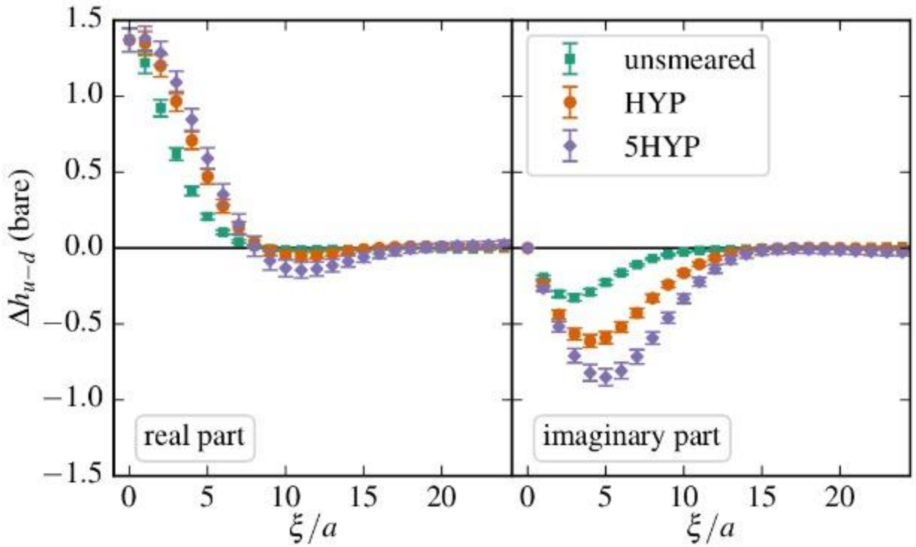
- First equation is sensitive to m, while the other two are construct to not depend on it;
- These conditions define a family of renormalization schemes at the scale p_0^2;
- Dependence on $|p_0|$ and $p_0 \cdot n/|p_0|$;
- RI´-MOM condition for S_ψ.



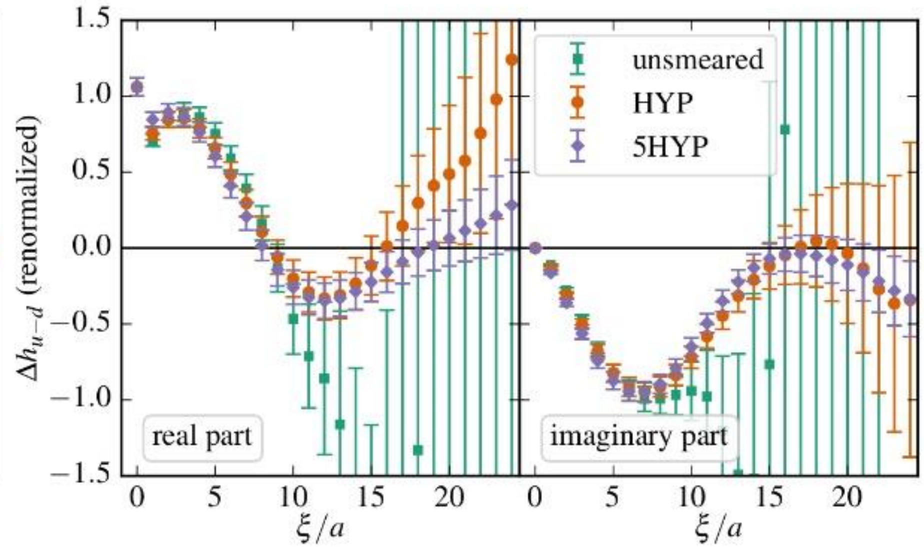
- m is determined from the 5 HYP;
- $\xi_0 = 0.6$ fm is chosen;
- For the helicity case, r_{mix} is negligible;
- Z_ϕ determinded in a similar way, for $p_0 \parallel n, p_0 \approx 1.85$ GeV

Setup and results

- Two lattice spacings used: $a \cong 0.082$ fm ($\beta = 1.95$), and $a \cong 0.064$ fm ($\beta = 2.10$)
- $m_\pi \approx 370$ MeV for both
- Helicity case used because r_{mix} is vanishingly small in this case



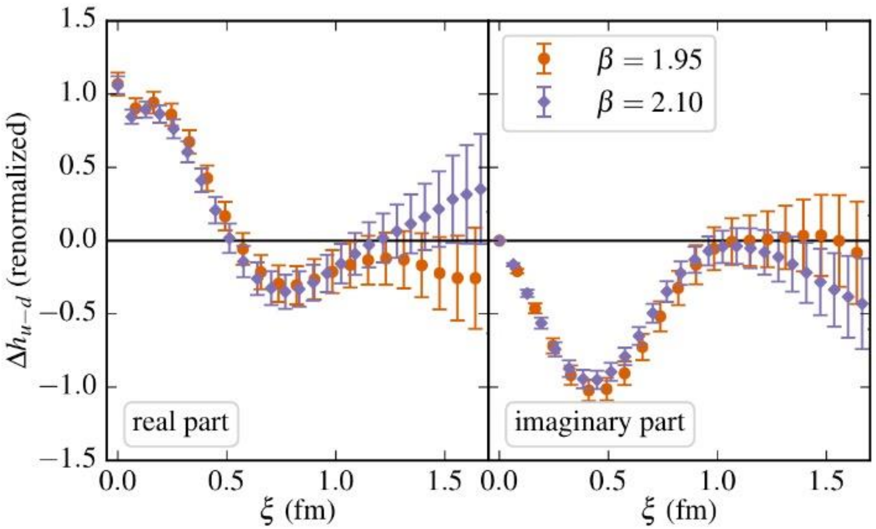
Before renormalization



After renormalization

After renormalization the tree link types shown above sit on top of each other

Results for the two different lattice spacings after renormalization



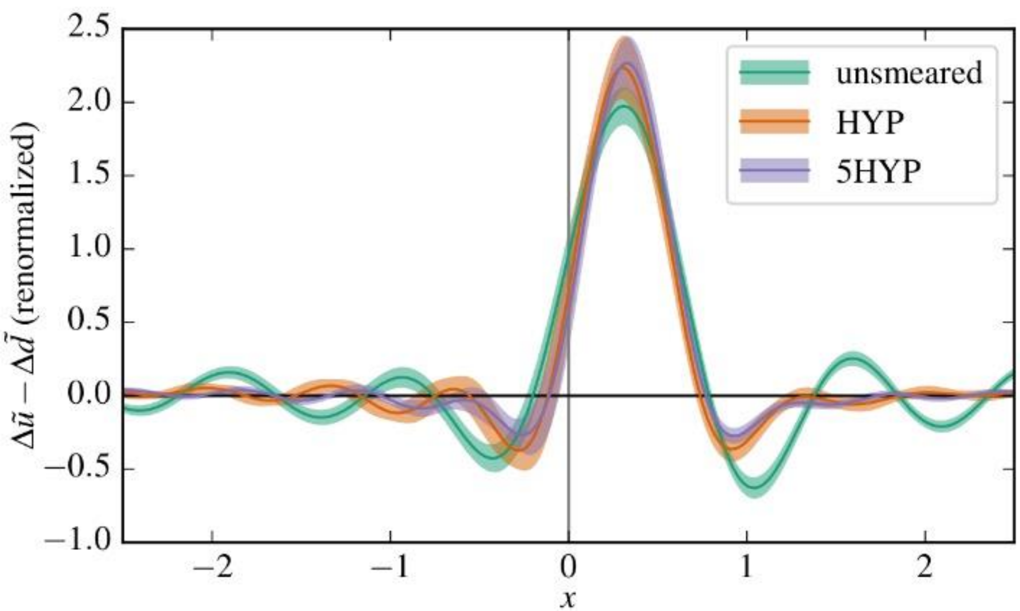
Linear divergence seems under control
Discretization effects are not too large

And the isovector helicity quasi-PDF:

Oscillations caused by the hard cut-off in the Fourier transform. It will be fixed in future studies.

For discussions about the long tail and oscillations, see Chen´s talk and Lin et al. 1708.05301

Conversion to $\overline{MS}$ still needs to be done



Summary

- Calculation of bare non-singlet quark distributions in lattice QCD at large values of the nucleon momentum;
- Asymmetry in the light antiquark distributions for all cases appears naturally;
- Calculated moments agree with previous calculation using a different method;
- A full renormalization prescription to handle all the divergences present in the matrix elements for the quasi-PDFs was presented;

  Standard logarithmic divergence handled with $\bar{Z}_{\mathcal{O}}$

  Power divergence renormalized with $e^{+\delta m \frac{|z|}{a} - c|z|}$

- For unpolarized, mixing between vector and scalar matrix elements – needs computation of a mixing matrix;
- For conversion to $\overline{MS}$, one needs to take care of truncation effects in the conversion factor. $Im\left[Z^{\overline{MS}}\right]$ should vanish for all z;

- Corrections modify the qPDFs in the right direction;
- We are running at the physical point, but the noise to ratio there is significantly worse;
- The long range has to be better understood after renormalization;
- Alternative nonperturbative renormalization was presented. The renormalization of the non-local operator is replaces by the renormalization of a local quark bilinear;
- Increasingly rapid progress in this field.

Thanks for the attention!

Minimum Bjorken x

$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

Largest momentum $|\vec{p}| = \frac{\pi}{a}$

Smallest momentum $|\vec{p}| = \frac{2\pi}{L}$

If the correlation lenght of the parton in th nucleon is $\sim 1/\Lambda_{QCD}$

$$\Delta z \Delta k_3 \sim 1 \longrightarrow \frac{1}{\Lambda_{QCD}} x_{min} P_3 \sim 1$$

So, in terms of the injected momentum, the minimal value of x is

$$x_{min} \sim \frac{\Lambda_{QCD}}{P_3}$$

Present approach is valid at intermediate and large x

cut imposed by the Lattice spacing